

# Resurrecting the minimal renormalizable supersymmetric SU(5) model

*work in progress* with B. Bajc and S. Lavignac

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# Outline

- 1 Introduction
- 2 Minimal Renormalizable Supersymmetric SU(5) model
  - Assumptions
  - Structure and Content
- 3 Phenomenology
  - Light Higgs mass
  - Down-type quark masses
  - Colour Higgs triplet mass
- 4 Conclusions

# Beyond the Standard Model

*Standard Model* of *electroweak* and *strong interactions* despite its astonishing *phenomenological success* has some difficulties:

- \* *massive neutrinos*
- \* *dark matter*
- \* *hierarchy problem*
- \* *electric charge quantization*

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  - \* *dark matter*
  - \* *hierarchy problem*
  - \* *electric charge quantization*
- *new physics* beyond the Standard Model?

*Supersymmetric Grand Unification* is one of the most promising candidates, at the same time predicting also *proton decay*, existence of *magnetic monopole(s)* and some yet unobserved *new particles*.

The simplest GUT model is the *minimal renormalizable SU(5)*.

# Motivation

The *minimal renormalizable supersymmetric SU(5) model* excluded according to *Murayama-Pierce '01*, ...

- ▶ *gauge coupling unification* (no unification within MSSM  $\rightarrow$   
 $m_T \lesssim 1.4 \cdot 10^{15}$  GeV)
- ▶ *proton decay* ( $m_T \gtrsim 2.0 \cdot 10^{17}$  GeV)

Assumption:

*low-energy supersymmetry spectrum* - 3<sup>rd</sup> generation *sparticles* at the  $\mathcal{O}(1 \text{ TeV})$  scale, and *gauginos* around the EW scale ( $M_2 \approx 200 \text{ GeV}$ ,  $M_3/M_2 \simeq 3.5$ ).

Can all the *phenomenological constraints* be fulfilled by relaxing that request, allowing for some *more general superpartner mass spectrum*?

Can we bound the parameter space of allowed soft supersymmetry breaking terms in MSSM (*soft masses* and *trilinear a-terms*) and learn something about SUSY breaking?

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# Our starting points

- Why **minimal renormalizable** SUSY SU(5)?
    - \* *predictiveness* - probably the only way to ever test the high scale Yukawas (no SU(5) singlets used, small number of parameters  $\rightarrow$  the masses are calculable)
    - \* *smallness of terms*  $W \supset C \frac{Q_i Q_j Q_k L_l}{M_P}$ ;  $C \lesssim 10^{-7}$  experimental fact
  - perturbativity* (of couplings) at least up to the unification scale
  - soft terms* at the GUT scale **SU(5) invariant** (supergravity mediation)
  - studying the mass scales of the theory (*the effects of running*), not its flavor structure [the only constraint is *small FCNCs* ]
  - correcting the *down-sector quark masses* by generation dependent *supersymmetric thresholds* (a-terms)
- 
- neutrino masses (bilinear R-parity violation?)
  - dark matter (gravitino?)



# Scales in the theory

Decomposition

$$\text{SU}(5) \supset \overbrace{(\text{SM particles} \oplus \text{superpartners})}^{\text{MSSM}} \oplus \text{heavy thresholds}$$

light thresholds

Matching scales

$$\begin{aligned} \text{weak scale } \sim \mathcal{O}(m_Z) &\longrightarrow m_{\text{susy}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \approx \sqrt{m_{\tilde{u}_3} m_{\tilde{Q}_3}} && (\text{SM}) \\ m_{\text{susy}} \sim \mathcal{O}(10^3 - 10^4 \text{ TeV}) &\longrightarrow M_{\text{GUT}} && (\text{MSSM}) \\ M_{\text{GUT}} \sim \mathcal{O}(10^{16} \text{ GeV}) &\longrightarrow M_{\text{Planck}} && (\text{SU}(5)) \end{aligned}$$

Running of model parameters between matching scales (RGEs)

single scale effective theory 2-loop RGEs + 1-loop thresholds

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single scale effective theory **2-loop** RGEs + **1-loop** thresholds

# Content

## 1. Higgs sector:

*adjoint representation:*  $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\begin{aligned}
 24_H &= \underbrace{(8, 1, 0)}_{m_8(m_\Sigma)} \oplus \underbrace{(1, 3, 0)}_{m_3(m_\Sigma)} \oplus \underbrace{(1, 1, 0)}_{m_1} \oplus \underbrace{(3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, \frac{5}{6})}_{m_V} \\
 &= \begin{bmatrix} O + \frac{2}{\sqrt{30}}\sigma & X \\ X & T - \frac{3}{\sqrt{30}}\sigma \end{bmatrix}
 \end{aligned}$$

*fundamental & antifundamental representation:*  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

$$5_H = \underbrace{(3, 1, -\frac{1}{3})}_{m_T} \oplus \underbrace{(1, 2, \frac{1}{2})}_{m_H}, \quad \bar{5}_H = \underbrace{(\bar{3}, 1, \frac{1}{3})}_{m_T} \oplus \underbrace{(1, 2, -\frac{1}{2})}_{m_H}$$

# Content

## 2. Gauge sector:

$$24_g = (8, 1, 0) \oplus (1, 3, 0) \oplus (1, 1, 0) \oplus \underbrace{(3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, \frac{5}{6})}_{m_V}$$

## 3. Matter (Yukawa) sector:

$$10_i = \underbrace{(3, 2, \frac{1}{6})}_{m_{\hat{Q}_i}} \oplus \underbrace{(\bar{3}, 1, -\frac{2}{3})}_{m_{\hat{u}_i^c}} \oplus \underbrace{(1, 1, 1)}_{m_{\hat{e}_i^c}} \quad , \quad \bar{5}_i = \underbrace{(\bar{3}, 1, \frac{1}{3})}_{m_{\hat{d}_i^c}} \oplus \underbrace{(1, 2, -\frac{1}{2})}_{m_{\hat{L}_i}}$$

# Structure of the theory (high energy)

## Higgs sector superpotential

$$W_H = \frac{\mu}{2} \text{Tr} \mathbf{24}_H^2 + \sqrt{30} \frac{\lambda}{3} \text{Tr} \mathbf{24}_H^3 + \eta \bar{\mathbf{5}}_H \left( \mathbf{24}_H + 3 \frac{\langle \sigma \rangle}{\sqrt{30}} \right) \mathbf{5}_H$$

## Yukawa sector superpotential

$$W_Y = \bar{\mathbf{5}}_i Y_5^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H + \frac{1}{8} \mathbf{10}_i Y_{10}^{ij} \mathbf{10}_j \mathbf{5}_H \quad , \quad i=1,2,3$$

$$\left. \begin{aligned} m_T &= \frac{5}{\sqrt{30}} \eta \langle \sigma \rangle \\ m_\Sigma &= m_8 = m_3 = 5\mu = 5\lambda \langle \sigma \rangle \\ m_1 &= \mu = \lambda \langle \sigma \rangle \\ m_V &= \frac{5}{\sqrt{30}} g_{\text{GUT}} \langle \sigma \rangle \end{aligned} \right\} \Rightarrow \text{perturbativity } (m_T, m_\Sigma \lesssim m_V)$$

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# Structure of the theory (low energy)

## MSSM superpotential

$$W_{\text{MSSM}} = \mathbf{u}^c Y_u \mathbf{Q} \mathbf{H}_u + \mathbf{d}^c Y_d \mathbf{Q} \mathbf{H}_d + \mathbf{e}^c Y_e \mathbf{L} \mathbf{H}_d + \mu \mathbf{H}_u \mathbf{H}_d$$

## soft Lagrangian (SUSY)

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = & m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d \\ & + \tilde{Q}^\dagger m_{\tilde{Q}}^2 \tilde{Q} + \tilde{u}^c m_{\tilde{u}^c}^2 \tilde{u}^{c\dagger} + \tilde{e}^c m_{\tilde{e}^c}^2 \tilde{e}^{c\dagger} + \tilde{L}^\dagger m_{\tilde{L}}^2 \tilde{L} + \tilde{d}^c m_{\tilde{d}^c}^2 \tilde{d}^{c\dagger} \\ & + \frac{1}{2} \left( M_1 \tilde{b} \tilde{b} + M_2 \tilde{w} \tilde{w} + M_3 \tilde{g} \tilde{g} \right) + \text{h.c.} \\ & + \tilde{u}^c A_u \tilde{Q} H_u + \tilde{d}^c A_d \tilde{Q} H_d + \tilde{e}^c A_e \tilde{L} H_d + b H_u H_d + \text{h.c.} \end{aligned}$$



# SU(5)-invariant boundary conditions at the GUT scale

$$\alpha_1(M_{\text{GUT}}) = \alpha_2(M_{\text{GUT}}) = \alpha_3(M_{\text{GUT}}) \equiv \alpha_{\text{GUT}}$$

$$Y_u(M_{\text{GUT}}) = Y_u^T(M_{\text{GUT}})$$

$$Y_d(M_{\text{GUT}}) = Y_e^T(M_{\text{GUT}})$$

$$A_u(M_{\text{GUT}}) = A_u^T(M_{\text{GUT}})$$

$$A_d(M_{\text{GUT}}) = A_e^T(M_{\text{GUT}})$$

$$M_1(M_{\text{GUT}}) = M_2(M_{\text{GUT}}) = M_3(M_{\text{GUT}}) \equiv M_{1/2}$$

$$\begin{aligned}
 m_{\tilde{Q}_i}(M_{\text{GUT}}) = m_{\tilde{u}_i^c}(M_{\text{GUT}}) = m_{\tilde{e}_i^c}(M_{\text{GUT}}) &\equiv \tilde{m}_{10_i} \\
 m_{\tilde{L}_i}(M_{\text{GUT}}) = m_{\tilde{d}_i^c}(M_{\text{GUT}}) &\equiv \tilde{m}_{\bar{5}_i}
 \end{aligned}
 \quad (i = 1, 2, 3)$$

All the *splittings* within SU(5) representations are only due to *running*!

# Theoretical and experimental constraints

- \* *Higgs mass* ( $m_h \simeq 125.7 \text{ GeV}$ )
- \* correct *down-sector fermion mass* relations ( $\delta m_d, \delta m_s, \delta m_b$ )
- \* *vacuum* (meta)*stability* (UFB 1,2,3 and CCB 1,2,3)
- \* gauge coupling *unification*
- \* *perturbativity* ( $m_T, m_\Sigma \lesssim m_V \ll M_{\text{Planck}}$ )
- \* *proton lifetime* bounds  $\tau_p^{\text{exp}}(p \rightarrow K^+ \bar{\nu}) > 2.3 \times 10^{33} \text{ yrs} \rightarrow m_T \gtrsim \dots$  ,  
 $\tau_p^{\text{exp}}(p \rightarrow \pi^0 e^+) > 13 \times 10^{33} \text{ yrs} \rightarrow m_V \gtrsim \dots$
- \* *LEP* and *LHC* bounds on sfermion and gaugino masses  
 $(m_{\tilde{Q}_{1,2}}, m_{\tilde{g}} \gtrsim 1 \text{ TeV}; m_{\tilde{Q}_3}, m_{\tilde{\chi}} \gtrsim 300 \text{ GeV})$

# Mass of the light Higgs

*matching scale* between SM and MSSM RGEs

$$m_{\text{susy}} \equiv \sqrt{m_{\tilde{t}_1}(m_{\text{susy}})m_{\tilde{t}_2}(m_{\text{susy}})} \approx \sqrt{m_{\tilde{u}_3}(m_{\text{susy}})m_{\tilde{Q}_3}(m_{\text{susy}})}$$

is determined by (2-loop) SM running of the *Higgs quartic coupling*

$$V_{\text{Higgs}} = -m_h^2 H_u^\dagger H_u + \frac{\lambda}{2} (H_u^\dagger H_u)^2$$

with the *matching condition*

$$\lambda(m_{\text{susy}}) = \overbrace{\left( \frac{3}{5} g_1^2(m_{\text{susy}}) + g_2^2(m_{\text{susy}}) \right) \frac{\cos^2(2\beta)}{4}}^{\text{tree-level}} +$$

$$+ \underbrace{\frac{6(\lambda_t \sin \beta)^4}{(4\pi)^2} \left( \frac{X_t}{m_{\text{susy}}} \right)^2 \left[ 1 - \frac{1}{12} \left( \frac{X_t}{m_{\text{susy}}} \right)^2 \right]}_{\text{1-loop stop-mixing contribution to Higgs mass}} + \dots$$

$$\text{1-loop stop-mixing contribution to Higgs mass} \leq \frac{6(\lambda_t \sin \beta)^4}{(4\pi)^2} \times 3$$

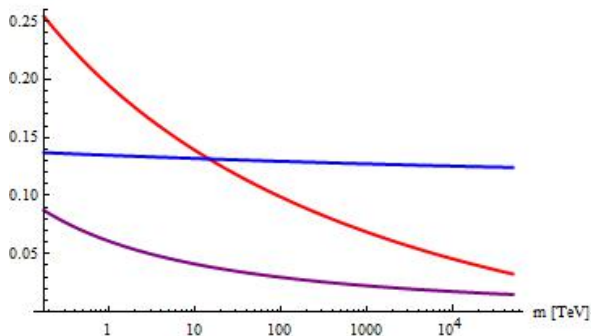
# Mass of the light Higgs

2-loop SM running of couplings

$$\text{RED} = \lambda$$

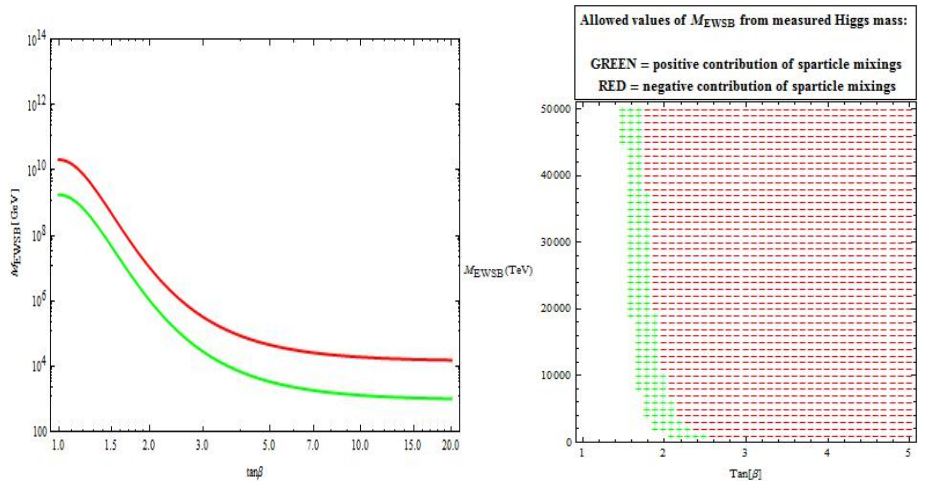
$$\text{BLUE} = (g_2^2 + 3/5 g_1^2) / 4$$

$$\text{PURPLE} = 6 h_t^4 / (4\pi)^2 \times 3$$



# Mass of the light Higgs

For each  $\tan\beta$  exist a *minimal*  $m_{\text{susy}}$  which fits the measured Higgs mass



# Mismatch between measured and SU(5)-respecting masses

SM running between  $m_Z$  and  $m_{\text{susy}}$  and MSSM running between  $m_{\text{susy}}$  and  $M_{\text{GUT}}$  for *leptonic* and *down sector Yukawas*

$$\mathbf{y}_e, \mathbf{y}_\mu, \mathbf{y}_\tau : \quad m_Z \xrightarrow{\text{SM}} m_{\text{susy}} \xrightarrow{\text{MSSM}} M_{\text{GUT}} \quad \left( \begin{array}{l} \text{no susy threshold corr.} \\ \alpha_2 \text{ instead of } \alpha_3 \text{ depen.} \end{array} \right)$$

SU(5) symmetry + minimal renormalizable model  $\rightarrow$  charged lepton and down-type quark masses (Yukawas) equal at the GUT scale

$$\text{minimal renormalizable SU(5) model} \rightarrow \begin{cases} m_e(M_{\text{GUT}}) = m_d(M_{\text{GUT}}) \\ m_\mu(M_{\text{GUT}}) = m_s(M_{\text{GUT}}) \\ m_\tau(M_{\text{GUT}}) = m_b(M_{\text{GUT}}) \end{cases}$$

$$\mathbf{y}_d, \mathbf{y}_s, \mathbf{y}_b : \quad M_{\text{GUT}} \xrightarrow{\text{MSSM}} m_{\text{susy}} \xrightarrow{\text{SM}} m_Z$$

$\rightarrow$  *wrong mass relations* when run down to the electroweak scale

$$\left. \begin{array}{l} m_e(m_Z)/m_d(m_Z) \\ m_\mu(m_Z)/m_s(m_Z) \\ m_\tau(m_Z)/m_b(m_Z) \end{array} \right\} = \text{wrong} \rightarrow \text{threshold corrections needed}$$

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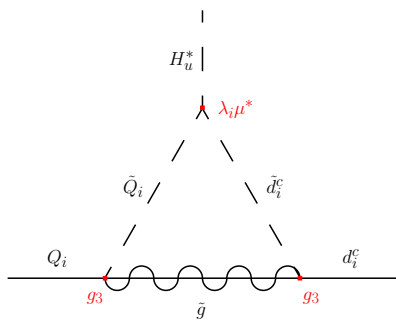
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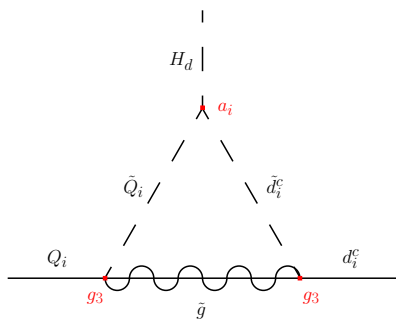
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# Correcting light fermion masses with a-terms

Diagrams for the finite corrections to the quark Yukawa couplings.



$$W_{\text{MSSM}} \supset \mu \mathbf{H}_u \mathbf{H}_d + \lambda_i \mathbf{Q}_i \mathbf{d}_i^c \mathbf{H}_d$$



$$\mathcal{L}_{\text{soft}} \supset a_i \tilde{\mathbf{Q}}_i \tilde{\mathbf{d}}_i^c \mathbf{H}_d$$

# Correcting light fermion masses with a-terms

$$\begin{aligned}
 \delta m_i &\equiv m_i^{\text{exp}} - m_i^{\text{SU(5)}} = -\frac{2\alpha_3}{3\pi} v \cos \beta m_{\tilde{g}} I_3 \left[ \overbrace{m_{\tilde{d}_{R_i}}^2}^{m_{d^c}^2}, \overbrace{m_{\tilde{d}_{L_i}}^2}^{m_{\tilde{Q}_i}^2}, m_{\tilde{g}}^2 \right] \lambda_i X_i \\
 &\approx -\frac{2\alpha_3}{3\pi} v \cos \beta \underbrace{\frac{m_{\tilde{g}}}{\tilde{m}_i} I_1 \left[ \left( \frac{m_{\tilde{g}}}{\tilde{m}_i} \right)^2 \right]}_{\substack{|\wedge \\ 0.57 \text{ at } \frac{m_{\tilde{g}}}{\tilde{m}_i} = 2.12}} \frac{\lambda_i X_i}{\tilde{m}_i} \propto \frac{\alpha_3 (m_{\text{susy}}) a_i}{\tilde{m}_i}
 \end{aligned}$$

$$X_t \equiv \frac{a_t}{\lambda_t} - \frac{\mu}{\tan \beta} \approx \frac{a_t}{\lambda_t}$$

$$X_b \equiv \frac{a_b}{\lambda_b} - \mu \tan \beta \approx \frac{a_b}{\lambda_b}$$

$$X_\tau \equiv \frac{a_\tau}{\lambda_\tau} - \mu \tan \beta \approx \frac{a_\tau}{\lambda_\tau}$$

Problem is accommodating the measured **b quark mass** - apart from using large **a-terms** other sources can not account for more than 10 % of required corrections.

# Vacuum (meta)stability

1. *absolute vacuum stability*  $\rightarrow$  our vacuum is a *global minimum*

$$\text{UFB 1,2,3} \rightarrow m_{H_u}^2 > 0$$

$$\text{CCB 1,2,3} \rightarrow \frac{|a_i|}{\lambda_i} \sim |X_i| \not\lesssim \sqrt{3(m_{H_d}^2 + m_{\tilde{Q}_i}^2 + m_{\tilde{d}_i^c}^2)}$$

2. *vacuum metastability*  $\rightarrow$  our vacuum only a *local minimum*, but its *lifetime* longer than the age of the Universe

$$\text{CCB 1,2*,3*} \rightarrow |\lambda_i X_i| \sim |a_i| \lesssim \sqrt{m_{H_d}^2 + m_{\tilde{Q}_i}^2 + m_{\tilde{d}_i^c}^2}$$

\**more complicated situation, numerical analysis required*

# Vacuum (meta)stability

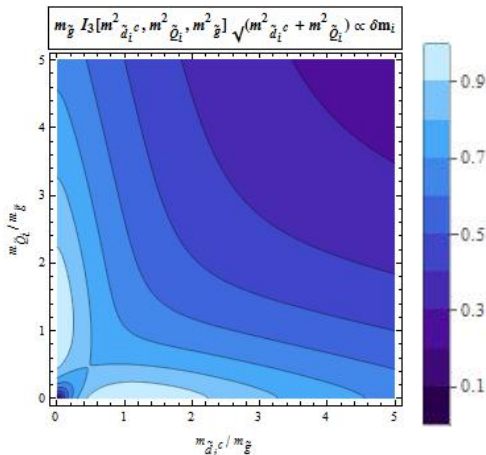
$$\delta m_i \approx -\frac{2\alpha_3}{3\pi} v \cos\beta m_{\tilde{g}} I_3[m_{\tilde{d}_i^c}^2, m_{\tilde{Q}_i}^2, m_{\tilde{g}}^2] \sqrt{m_{\tilde{d}_i^c}^2 + m_{\tilde{Q}_i}^2 + m_{\tilde{L}_i}^2} \frac{\lambda_i X_i}{\sqrt{m_{\tilde{d}_i^c}^2 + m_{\tilde{Q}_i}^2 + m_{\tilde{L}_i}^2}}$$

LOOP FUNCTION:

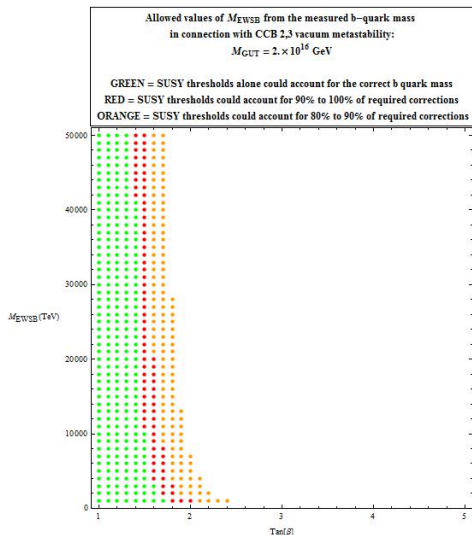
$$m_{\tilde{g}} I_3[m_{\tilde{d}_i^c}^2, m_{\tilde{Q}_i}^2, m_{\tilde{g}}^2] \sqrt{m_{\tilde{d}_i^c}^2 + m_{\tilde{Q}_i}^2 + m_{\tilde{L}_i}^2}$$

$$\text{III} \\ \delta m_i \left[ \frac{m_{\tilde{d}_i^c}^c}{m_{\tilde{g}}}, \frac{m_{\tilde{Q}_i}}{m_{\tilde{g}}} \right] \leq 1 \text{ for } m_{\tilde{d}_i^c}^c \ll m_{\tilde{Q}_i} = m_{\tilde{g}}$$

$$\frac{\lambda_i X_i}{\underbrace{\sqrt{m_{\tilde{d}_i^c}^2 + m_{\tilde{Q}_i}^2 + m_{\tilde{L}_i}^2}}_{\gtrsim \mathcal{O}(1)}}$$



# Vacuum (meta)stability



Combining corrections to  $b$ -quark mass and vacuum metastability constraints

# Gauge coupling Unification

No unification in MSSM  $\rightarrow$  high-energy thresholds  $m_T, m_8, m_3, m_V$  required

single scale ( $m_{\text{susy}}$ ) MSSM **2-loop** RGEs + **1-loop** thresholds

# Gauge coupling Unification

No unification in MSSM  $\rightarrow$  high-energy thresholds  $m_T$ ,  $m_8$ ,  $m_3$ ,  $m_V$  required

single scale ( $m_{\text{susy}}$ ) MSSM **2-loop** RGEs + **1-loop** thresholds

$$\begin{aligned}
 \left[ \overbrace{m_V^2}^{m_T^2} (m_3 m_8)^{1/2} \right]^{1/3} &= M_{\text{GUT}} \times \exp \left[ \frac{\pi}{18} (5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})_{2\text{-loop}} (M_{\text{GUT}}) \right] \\
 &\times \left( \frac{m_{\text{susy}}^2}{m_{\tilde{w}} m_{\tilde{g}}} \right)^{1/9} \underbrace{\prod_{i=1}^3 \left( \frac{m_{\tilde{u}_i^c} m_{\tilde{e}_i^c}}{m_{\tilde{Q}_i}^2} \right)^{1/36}}_{\substack{\lambda \\ \mathcal{O}(1)}}
 \end{aligned}$$



# Gauge coupling Unification

No unification in MSSM  $\rightarrow$  high-energy thresholds  $m_T$ ,  $m_8$ ,  $m_3$ ,  $m_V$  required

single scale ( $m_{\text{susy}}$ ) MSSM **2-loop** RGEs + **1-loop** thresholds

$$\begin{aligned}
 m_T^0 &\simeq 10^{15} \text{ GeV} \times \left( \frac{m_{\text{susy}}}{1 \text{ TeV}} \right)^{5/6} \\
 m_T &= M_{\text{GUT}} \times \exp \left[ \frac{5\pi}{6} (-\alpha_1^{-1} + 3\alpha_2^{-1} - 2\alpha_3^{-1})_{2\text{-loop}} (M_{\text{GUT}}) \right] \\
 &\times \underbrace{\left( \frac{m_3}{m_8} \right)^{5/2}}_{\substack{\parallel \\ 1}} \underbrace{\left( \frac{m_{\tilde{W}}}{m_{\tilde{g}}} \right)^{5/3}}_{\substack{\uparrow \\ m_{\tilde{g}} \approx m_{\text{susy}}}} \underbrace{\prod_{i=1}^3 \left( \frac{m_{\tilde{Q}_i}^4}{m_{\tilde{u}_i^c}^3 m_{\tilde{e}_i^c}} \frac{m_{L_i}^2}{m_{d_i^c}^2} \right)^{1/12}}_{\substack{\} \\ \mathcal{O}(1)}} \left( \frac{m_h^4 m_A}{m_{\text{susy}}^5} \right)^{1/6}
 \end{aligned}$$

Light  $m_T$  mediates too fast proton decay.

Large  $m_{\text{susy}}$  poses the opposite problem:  $m_T$  can be too heavy (perturbativity).

# Gauge coupling Unification

$$|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{m_Z^2}{2} \quad (\text{tree-level EWSB condition at } m_{\text{susy}})$$

$$m_{\tilde{h}} = |\mu|$$

$$m_A = \sqrt{(\mu^2 + m_{H_d}^2)(1 + 1/\tan^2 \beta)} \approx \sqrt{(m_{H_d}^2 - m_{H_u}^2) \frac{\tan^2 \beta + 1}{\tan^2 \beta - 1}}$$

$$\frac{m_{\tilde{w}}}{m_{\tilde{g}}} \approx \frac{\alpha_2(m_{\tilde{g}})}{\alpha_3(m_{\tilde{g}})} \frac{\alpha_3(M_{\text{GUT}})}{\alpha_2(M_{\text{GUT}})}$$

$$m_T^0 \simeq 10^{15} \text{ GeV} \times \left(\frac{m_{\text{susy}}}{1 \text{ TeV}}\right)^{5/6}$$

$$\equiv$$

$$m_T = M_{\text{GUT}} \times \exp \left[ \frac{5\pi}{6} (-\alpha_1^{-1} + 3\alpha_2^{-1} - 2\alpha_3^{-1})_{2\text{-loop}} (M_{\text{GUT}}) \right]$$

$$\times \underbrace{\left(\frac{m_3}{m_8}\right)^{5/2}}_{\equiv 1} \underbrace{\left(\frac{m_{\tilde{w}}}{m_{\tilde{g}}}\right)^{5/3}}_{\substack{\uparrow \\ m_{\tilde{g}} \approx m_{\text{susy}}}} \underbrace{\prod_{i=1}^3 \left(\frac{m_{\tilde{Q}_i}^4}{m_{\tilde{U}_i^c}^3 m_{\tilde{E}_i^c}} \frac{m_{\tilde{L}_i}^2}{m_{\tilde{D}_i^c}^2}\right)^{1/12}}_{\substack{\approx 1 \\ \mathcal{O}(1)}} \left(\frac{m_{\tilde{h}}^4 m_A}{m_{\text{susy}}^5}\right)^{1/6}$$

$$\equiv$$

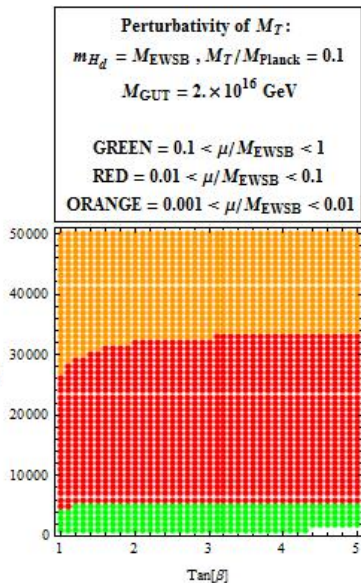
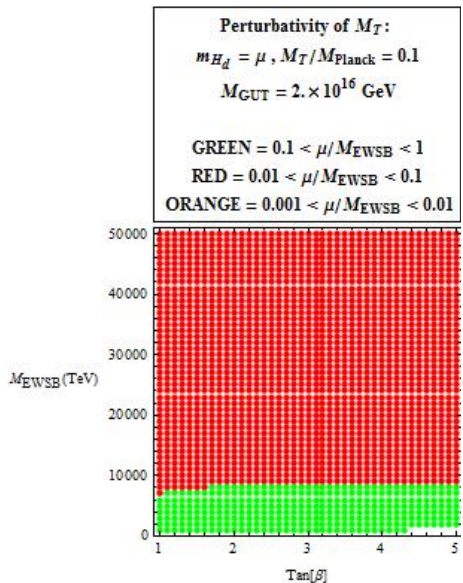
$$1$$

$$m_{\tilde{g}} \approx m_{\text{susy}}$$

$$\approx 1$$

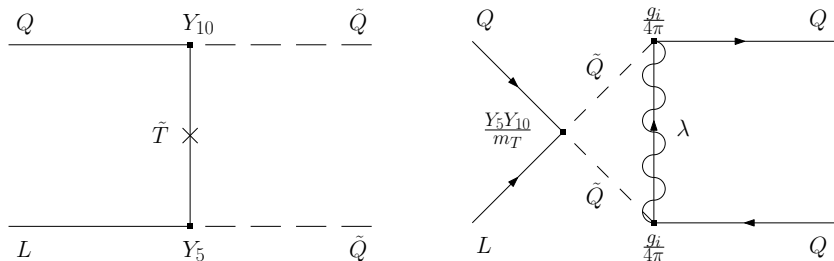
$$\mathcal{O}(1)$$

# Perturbativity



# Proton decay bounds

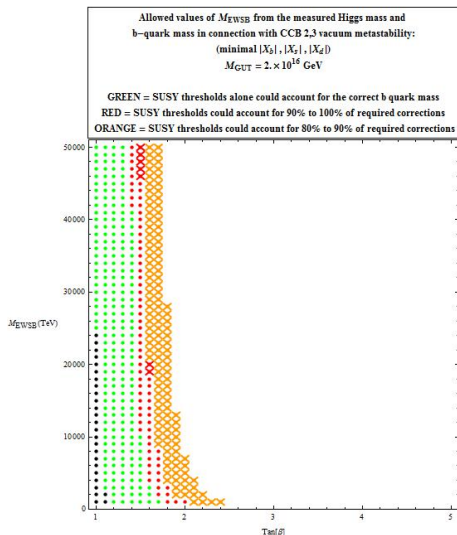
Dimension 5 *colour Higgs triplet exchange* operators get *dressed* by *winos* which leads to Weinberg type 4-fermion effective operators



Dominant proton decay channel is  $p \rightarrow K^+ \bar{\nu}$  that scales approximately as

$$\tau(p \rightarrow K^+ \bar{\nu}) \propto \left( \frac{\tilde{m}_i}{m_{\tilde{w}} / \tilde{m}_i l_1(m_{\tilde{w}}^2 / \tilde{m}_i^2)} \right)^2 \left( \frac{2 \tan \beta}{1 + \tan^2 \beta} \right)^2 m_T^2$$

# Results & Conclusions



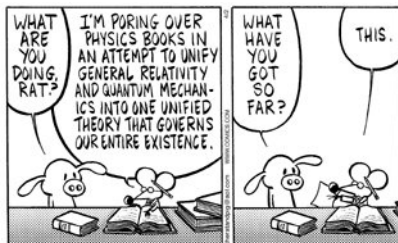
## POINTS TO TAKE HOME:

- \*  $m_h + \delta m_b + \text{vacuum metastability}$
- \* strong correlation between  $\tan \beta$  and allowed  $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$
- \* importance of *2-loop* running for  $\lambda$

*work in progress ...*

## TO DO list:

- \* numerical analysis of the CCB 2,3 vacuum metastability
- \* *RGEs* for *a-terms* and *soft masses* → check of *SU(5)* boundary conditions



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# Thank you for your attention!