The Slow Gravitino

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PLAN OF THE TALK

REMINDER: GLOBAL SUSY

In a global supersymmetric theory in flat space time, supersymmetry is broken spontaneously when the vacuum has non-zero energy.

$$[Q, Q] = H \qquad H|0> \neq 0 \Rightarrow Q|0> \neq 0$$

Preserving Lorentz invariance, for $N = 1$ SUSY in 4d

$$F \neq 0 \quad \text{or} \quad D \neq 0 \qquad \rightarrow H = F^2 + D^2$$

(For simplicity auxiliary fields F, D taken real).
Spontaneous breaking $\Rightarrow$ a goldstino is a spin $\frac{1}{2}$ field $(G_\alpha, \bar{G}_\alpha)$ in the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of the Lorentz group.
At quadratic order a kinetic term:

$$\mathcal{L}_G = -i\bar{G}\bar{\sigma}^\mu \partial_\mu G,$$

satisfies the Dirac equation

$$\bar{\sigma}^\mu \partial_\mu G = 0, \quad \sigma^\mu \partial_\mu \bar{G} = 0$$

REMINDER: LOCAL SUSY

$N = 1$ local supersymmetry for vierbein e^a_μ, gravitino $(\psi_{\mu\alpha}, \bar{\psi}_{\mu\alpha})$ of spin $\frac{3}{2}$ and the goldstino G fields:

$$\begin{aligned}
\delta e^a_\mu &= -\frac{1}{M_P}\left(i\bar{\epsilon}\bar{\sigma}^a\psi_\mu - i\epsilon\sigma^a\bar{\psi}_\mu\right) \\
\delta\psi_\mu &= -M_p 2\partial_\mu\epsilon \\
\delta\bar{\psi}_\mu &= -M_p 2\partial_\mu\bar{\epsilon} \\
\delta G &= \sqrt{2}F\epsilon \\
\delta\bar{G} &= \sqrt{2}F\bar{\epsilon}
\end{aligned}$$

Reminder: Local SUSY

$$\mathcal{L} = -\frac{1}{2M_P^2} eR - F^2 e - i\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - i\bar{G}\bar{\sigma}^\mu\partial_\mu G$$
$$-i\frac{F}{\sqrt{2}M_p}\left(\psi_\mu\sigma_\nu\bar{G} + \bar{\psi}_\mu\bar{\sigma}_\nu G\right) + \cdots$$

Problem solved by adding a combination :

$$\Delta\mathcal{L} = F^2 e - \frac{1}{2}\left(m_{\frac{3}{2}}\psi_\mu\sigma^{\mu\nu}\psi_\nu - m^*_{\frac{3}{2}}\bar{\psi}_\mu\sigma^{\mu\nu}\bar{\psi}_\nu - m_{\frac{3}{2}}GG - m^*_{\frac{3}{2}}\bar{G}\bar{G}\right)$$

Total Lagrangian invariant under supersymmetry variation

$$\delta\psi_\mu = -M_p\left(2\partial_\mu\epsilon - i m^*_{\frac{3}{2}}\sigma_\mu\bar{\epsilon}\right),$$

only if:

$$m_{\frac{3}{2}} = \frac{F}{\sqrt{3}M_p}.$$

REMINDER: LOCAL SUSY

Unitary gauge by performing the transformation

$$\psi_{\mu\alpha} \to \psi_{\mu\alpha} + \frac{\sqrt{2}M_P}{F}\partial_\mu G_\alpha + i\frac{1}{\sqrt{6}}\sigma_{\mu\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}} \,.$$

puts $G \to 0$ and leads to the Rarita-Schwinger Lagrangian for a massive gravitino:

$$\mathcal{L} = \frac{1}{2}(-i\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - m_{\frac{3}{2}}\psi_\mu\sigma^{\mu\nu}\psi_\nu - m^*_{\frac{3}{2}}\bar{\psi}_\mu\sigma^{\mu\nu}\bar{\psi}_\nu)$$

INTRODUCTION

We want to discuss propagation of a spin-$3/2$ in a fluid.
Fluids are omnipresent:

- radiation and matter in cosmology
- dark energy?
- dark sector breaking susy?
- heavy ion collisions (spin $3/2$ describes a hadron?)

Work still on progress. I will present partial results.

FLUID AND LORENTZ SYMMETRY:

- Fluid: stress-energy tensor $< T^{\mu\nu} > \neq 0$
- Tensor with vev $\Rightarrow$ breaks Lorentz symmetry.
- $\Rightarrow$ Preferred frame (center of mass of the fluid).
- $\Rightarrow$ appearance of phonons, massless mode. For $T^{\mu\nu} = diag(\rho, p, p, p)$ the velocity is:

$$v_B^2 = \frac{p}{\rho}$$

FLUID AND GLOBAL SUSY

- Fluid: stress-energy tensor $< T^{\mu\nu} > \neq 0$
- Temperature treats differently bosons and fermions $\Rightarrow$ breaks SUSY.
- Ward-Takahashi identity $\Rightarrow$ spontaneous breaking
- $\Rightarrow$ appearance of goldstino, massless Majorana state has been named phonino.

For $T^{\mu\nu} = diag(\rho, p, p, p)$ the velocity is:

$$v = v_F = \frac{p}{\rho}$$

GOLDSTINO LAGRANGIAN

The goldstino equation of motion in a fluid:

$$T^{\mu\nu}\gamma_\mu\partial_\nu G = 0 \Rightarrow \gamma^0\partial_0 G - v\gamma^i\partial_i G = 0$$

This can be deriveded from the Lagrangian

$$\mathcal{L}_G = -\frac{i}{2\mathcal{T}^4}T^{\mu\nu}\bar{G}\gamma_\mu\partial_\nu G$$

Here, $\mathcal{T} = |\,\mathrm{Tr}\,\langle T^{\mu\nu}\rangle|^{\frac{1}{4}}$ has dimension of mass.

For $T^{\mu\nu} = -|F|^2\eta^{\mu\nu}$ the Lagrangian reduces to that of the usual goldstino of F-term case.

GRAVITINO-PHONINO MIXING:

Describing the system phonino-gravitino at the quadratic order and lowest order of an expansion in $\frac{\mathcal{T}}{M_p}$:

$$\begin{aligned}\mathcal{L} = & -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma^5\gamma_\nu\partial_\rho\psi_\sigma - \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\lambda}\bar{\psi}_\mu\gamma^5\gamma_\rho\gamma^\lambda\psi_\nu \\ & - \frac{i}{\sqrt{2}}\frac{\mathcal{T}^2}{M_P}\frac{T^{\mu\nu}}{\mathcal{T}^4}\bar{\psi}_\mu\gamma_\nu G \\ & + i\frac{T^{\mu\nu}}{2\mathcal{T}^4}\bar{G}\gamma_\mu\partial_\nu G + \frac{1}{4}\frac{T^{\mu\nu}n_{\mu\nu}}{\mathcal{T}^4}\bar{G}G\end{aligned}$$

GRAVITINO-PHONINO MIXING:

This Lagrangian is invariant under the supersymmetry transformations with Lorentz violating coefficients:

$$\delta G = \sqrt{2}\mathcal{T}^2 \varepsilon \,,$$

$$\delta\psi_\mu = -M_P(2\partial_\mu \varepsilon + i n_{\mu\nu}\gamma^\nu \bar{\varepsilon})$$

if $n_{\mu\nu}$ satisfies:

$$-\frac{1}{2}\epsilon^{\mu\nu\sigma\rho}\epsilon_\rho{}^{\lambda\gamma\kappa} n_{\nu\lambda} n_{\sigma\gamma} = \frac{T^{\mu\kappa}}{M_P^2}$$

In the unitary gauge, G is set to zero through the supersymmetry transformation:

$$\psi_\mu \to \psi_\mu + \frac{\sqrt{2}M_P}{\mathcal{T}^2}\partial_\mu G + i\frac{M_P}{\sqrt{2}\mathcal{T}^2} n_{\mu\nu}\gamma^\nu \bar{G} \,.$$

THE NEW LAGRANGIAN

For a perfect fluid:

$$T_{\mu\nu} = \rho \left[v\eta_{\mu\nu} + (1+v)u_\mu u_\nu \right]$$

a perfect fluid with four-velocity u^μ such that $u_\mu u^\mu = -1$.
In the fluid center of mass reference frame, corresponding to $u^\mu = (1,0,0,0)$.
For the measure of Lorentz symmetry violation, we use:

$$\epsilon_{LV} \equiv 1 + \frac{p}{\rho} = 1 + v\,.$$

The Lorentz invariant solution corresponds to $v = -1$.

The new Lagrangian

The modified Rarita-Schwinger Lagrangian can be written as:

$$\mathcal{L} = \frac{1}{2}\bar{\psi}_\mu S^{\mu\nu}\psi_\nu$$

where $S^{\mu\nu}$ can be split into a kinetic and mass term:

$$S^{\mu\nu} = -i(\gamma^\mu\gamma^\nu + \eta^{\mu\nu})\not{\partial} + i\gamma^\nu\partial^\mu - i\gamma^\mu\partial^\nu + S_m^{\mu\nu}$$

where

$$S_m^{\mu\nu} = m\left[\gamma^\mu\gamma^\nu + \eta^{\mu\nu} + \frac{3\epsilon_{LV}}{4-3\epsilon_{LV}}(r^\mu t^\nu + t^\mu r^\nu)\right]$$

with the notation

$$t^\mu = -u^\mu u^\nu \gamma_\nu$$
$$r^\mu = \gamma^\mu + u^\mu u^\nu \gamma_\nu$$

for the projection of the gamma matrices along and orthogonal to u^μ.

PARAMETERS

The gravitino Lagrangian depends on the three parameters

$$u^\mu \,, \qquad \rho \,, \qquad \text{and} \qquad \epsilon_{LV}$$

or

$$u^\mu \,, \qquad m \,, \qquad \text{and} \qquad n$$

where

$$m = n\,(1 - \frac{3}{4}\epsilon_{LV})$$

and n is given for the fluid by:

$$n^2 = \frac{\rho}{3M_P^2}$$

(Notice that n is equal to the Hubble parameter of an FRW metric that would be generated by having $T^{\mu\nu}$ on the r.h.s of Einstein equations).

MAKE OF $\psi_{\mu\alpha}$ A SPIN-$\frac{3}{2}$ STATE

Fierz and Pauli,

$$\psi_{\mu\alpha} \to (\frac{1}{2},\frac{1}{2}) \otimes (\frac{1}{2},0) = (1,\frac{1}{2}) \oplus (0,\frac{1}{2})$$

Irreducible spin $\frac{3}{2}$ representation IFF the additional spin $\frac{1}{2}$ components are projected out
The $(0,\frac{1}{2})$ is removed by imposing

$$\bar{\sigma}^{\mu}\psi_{\mu} = 0$$

The representation $(1,\frac{1}{2})$ has 6 d.o.f. each. To reduce the number of d.o.f. to 4 we impose

$$\partial^{\mu}\psi_{\mu\alpha} = 0 \Rightarrow p^{\mu}\psi_{\mu\alpha} = 0\ .$$

THE DEGREES OF FREEDOM IN A FLUID

ψ_μ has 8 degrees of freedom:

- 2 helicity-3/2 dof:

$$\mathcal{P}^{\mu\nu}_{3/2}\psi_\nu = \left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \frac{k^\mu k^\nu}{k^2} + \frac{1}{2}(r^\mu - \frac{\not k k^\mu}{k^2})(r^\nu - \frac{\not k k^\nu}{k^2})\right]\psi_\nu$$

- 2 helicity-1/2 dof : $\psi_{\frac{1}{2}} = r^\mu \psi_\mu$.
- 4 irrelevant helicity-1/2 to project-out :

$$\tilde{\pi}^\mu_2 \psi_\mu = t^\mu \psi_\mu$$

$$\tilde{\pi}^\mu_3 \psi_\mu = (r^\mu - \frac{3\not k k^\mu}{k^2})\psi_\mu$$

This decomposition can be expressed explicitly as:

$$\psi^\mu = \psi^\mu_{3/2} - \frac{1}{3} r^\mu \psi_{\frac{1}{2}} - t^\mu t^\nu \psi_\nu - \frac{1}{6}(r^\mu - 3\frac{\not k k^\mu}{k^2})(r^\nu - 3\frac{\not k k^\nu}{k^2})\psi_\nu \,.$$

THE NEW CONSTRAINTS

The usual Rarita-Schwinger constraints:

$$\gamma^\mu \psi_\mu = 0 \qquad p^\mu \psi_\mu = 0$$

The new constraints projecting out four irrelevant degrees of freedom:

$$T^{\mu\nu}\gamma_\mu\psi_\nu = -2iM_P^2(\partial_\mu S_m^{\mu\nu})\,\psi_\nu$$

and

$$[k^\nu + (\not{k} - \frac{m}{(1-\frac{3}{4}\epsilon_{LV})})r^\nu]\psi_\nu = 0$$

For a fluid at rest $\partial_\mu S_m^{\mu\nu} = 0$

GENERAL EQUATIONS OF MOTION

For the longitudinal mode:

$$(r^\mu t^\nu + t^\mu r^\nu)\partial_\mu \psi_\nu + im\psi_{\frac{1}{2}} \quad = -\tfrac{3i}{2} n(v\psi_{\frac{1}{2}} - t^\lambda \psi_\lambda) \, .$$

while the transverse mode satisfies:

$$(\gamma^\nu \partial_\nu + im)\psi^\mu_{3/2} = \frac{3in^2}{4k^2}(r^\mu r^\lambda + 3\mathbf{r}^{\mu\lambda})[(1+v)\partial_\lambda t^\gamma \, \psi_\gamma + v\partial_\lambda \psi_{\frac{1}{2}} - \partial_\rho (t^\lambda \psi_\lambda)] \, .$$

In general the equations are coupled.

PERFECT FLUID

Diagonalisation:

$$(\gamma^0\partial_0 - v\,\gamma^i\partial_i)\psi_{\frac{1}{2}} - im\psi_{\frac{1}{2}} = 0$$

Andt the transverse part satisfies the decoupled equation

$$(\gamma^0\partial_0 - \gamma^i\partial_i)\psi^\mu_{3/2} - im\,\psi^\mu_{3/2} = 0$$

Dirac equations satisfied by the longitudinal helicity-1/2 mode and transverse helicity-3/2 modes with the same mass

$$m = \frac{\sqrt{3}}{4M_P}\left|\frac{p - \frac{\rho}{3}}{\sqrt{\rho}}\right|$$

but different velocities. For $\rho = -p = F^2$ we recover the usual F-term case.

THE PROPAGATOR

$$G^{\mu\nu} = \frac{\Pi^{\mu\nu}_{3/2}}{p^2 + m^2} + \frac{\Pi^{\mu\nu}_{1/2}}{m^2 + v^2 k^2 + q^2} .$$

where the two polarizations can be written :

$$\Pi^{\mu\nu}_{3/2} = (m - \not{p})\mathcal{P}^{\mu\nu}_{3/2} = \Pi^{\mu\nu}_{RS} + \frac{2}{3} C^{\mu}(\not{p} + m)C^{\nu}$$

and

$$C^{\mu} = \gamma^{\mu} + \frac{p^{\mu}}{m} - \frac{3}{2}(r^{\mu} - \frac{\not{k}k^{\mu}}{k^2}) .$$

and $\Pi^{\mu\nu}_{RS}$ is the Rarita-Schwinger polarisation tensor:

$$\Pi^{\mu\nu}_{RS} = (m - \not{p})[\eta^{\mu\nu} + \frac{1}{3}\gamma^{\mu}\gamma^{\nu} + 2\frac{p^{\mu}p^{\nu}}{3m^2} + \frac{\gamma^{\mu}p^{\nu} - \gamma^{\nu}p^{\mu}}{3m}] .$$

The propagator

$$\Pi^{\mu\nu}_{1/2} = -\frac{2}{3}\left[C^\mu - \frac{3}{4}\epsilon_{LV}(t^\mu + \frac{p^\mu}{m})\right](\not{p} + m - \epsilon_{LV}\not{k})\left[C^\nu - \frac{3}{4}\epsilon_{LV}(t^\nu + \frac{p^\nu}{m})\right]$$
$$+ \frac{3}{4}\epsilon_{LV}(m^2 + v^2k^2 + q^2)\frac{\not{k}}{mk^2}(t^\mu p^\nu - p^\mu t^\nu)$$

Note that the part corresponding to the spin-1/2 components of the spinor-vector has a pole for $m^2 + v^2k^2 + q^2 = 0$ due to a different dispersion relation.

OUTLOOK

- We generalised the case of F-term $T^{\mu\nu} = -F^2\eta^{\mu\nu}$ to the case of a general $T_{\mu\nu}$
- We get simple formulae for the perfect fluid case:

$$m = \frac{\sqrt{3}}{4M_P}\left|\frac{p - \frac{\rho}{3}}{\sqrt{\rho}}\right|$$

- computation of the propagator opens the road for phenomenological studies
- Lorentz-violation for spin 0, 1/2, 1 studied. Here, open the spin 3/2 case: limits on ϵ_{LV}.

BACK-UP 1

A contribution, $m_{3/2}^{curv}$, from the curvature of space-time and it is induced by the total stress-energy tensor $T^{\mu\nu}_{total}$. For a fluid at rest with stress-energy tensor $T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$.
The null energy condition implies that ϵ_{LV}, defined in (**??**), is positive thus forbids superluminal sound velocities.
If $T^{\mu\nu} = \mathrm{diag}(\rho, p, p, p) = T^{\mu\nu}_{total}$ then

$$m_{3/2}^{curv} = \frac{\sqrt{\rho}}{3M_P} \tag{1}$$

therefore

$$m_{3/2} = \frac{\sqrt{\rho}}{3M_P}\left(1 - \frac{3}{4}\epsilon_{LV}\right) \leq m_{3/2}^{curv} \tag{2}$$

in contradiction with our assumption. The equality is reached for the de Sitter solution corresponding to $T^{\mu\nu} = -|F|^2\eta^{\mu\nu}$.

BACK-UP 2

As in the usual F-term $T^{\mu\nu}_{total}$ receives a cancelling contribution :

- An approximate cancellation through the addition of a cosmological constant. $T^{\mu\nu}_{(total)} = T_{\mu\nu} - \Lambda\eta_{\mu\nu} = \mathrm{diag}(\rho + \Lambda, p - \Lambda, p - \Lambda, p - \Lambda)$ gives for $\Lambda = p$:

$$\frac{m^{curv}_{3/2}}{m_{3/2}} = \frac{\sqrt{\epsilon_{LV}}}{(1 - \frac{3}{4}\epsilon_{LV})} \tag{3}$$

  which implies $\epsilon_{LV} \ll 1$. Such a small number is anyway also expected in phenomenological applications given the strong experimental limits.
- An exact cancellation can be engineered for the perfect fluid at rest for arbitrary ϵ_{LV}. A non-dynamical object with negative tension $\mathrm{diag}(-\rho - \Lambda, 0, 0, 0)$ will be introduced in addition to the cosmological constant. Such objects appear in string theory for instance as orientifold, here point-like.