

Primordial non-Gaussianities

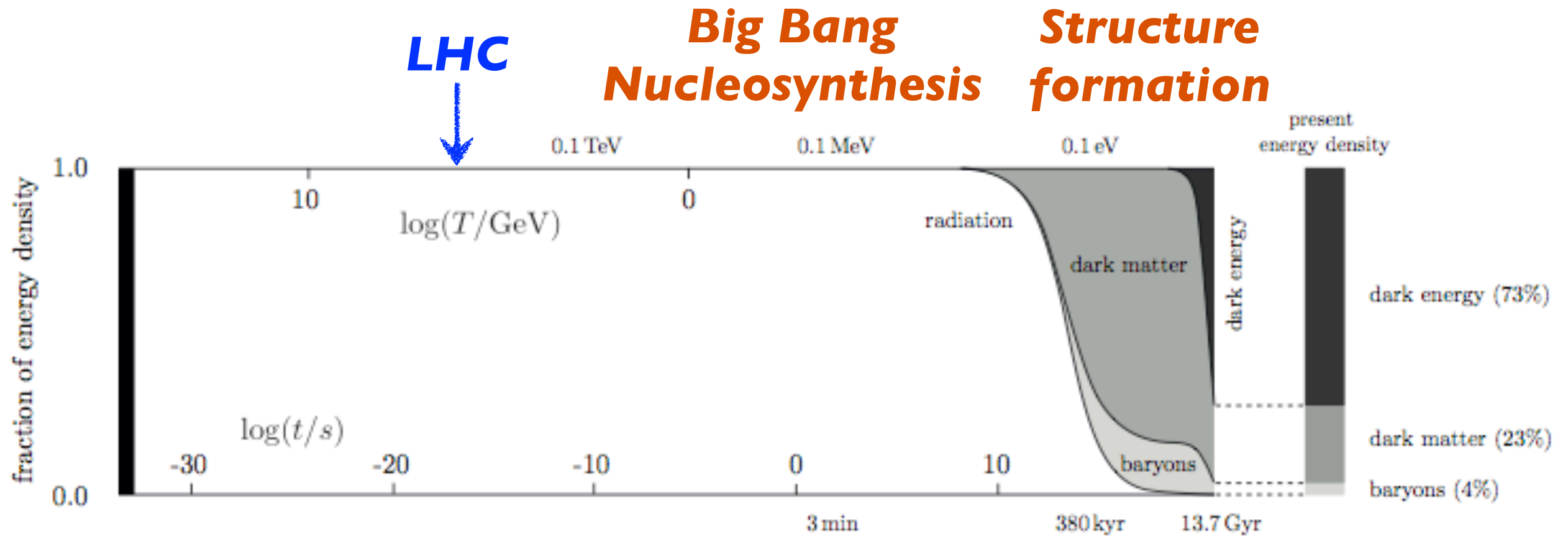
Sébastien Renaux-Petel

LPTHE - ILP

GDR Terascale, Montpellier.

May 14, 2013

Cosmic history



Inflation

Cosmic Microwave Background

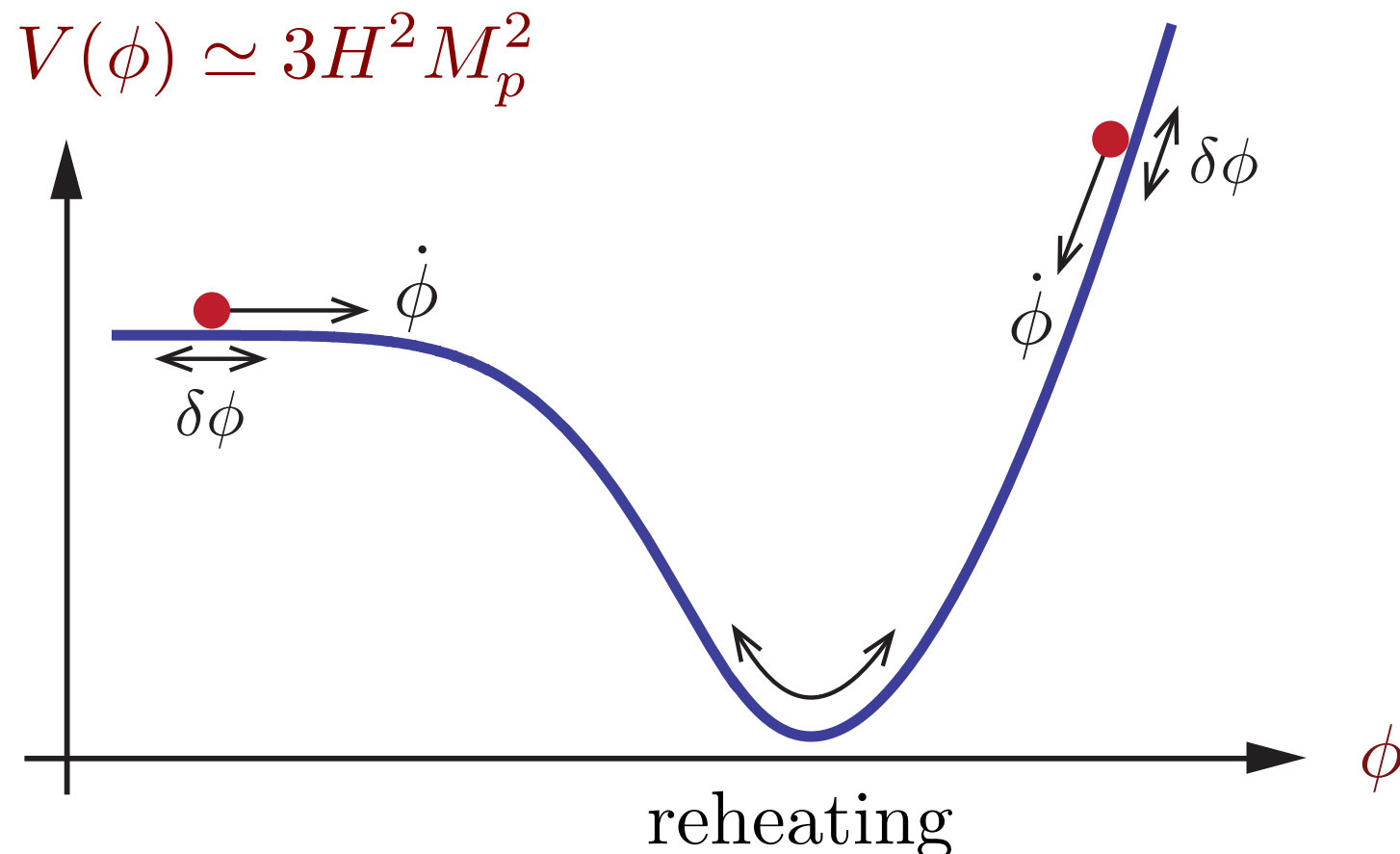
Inflation: a phase of accelerated expansion before the radiation era, that:

- solves the problems of the Hot Big Bang model
- generates the seeds of the large scale structures.

Slow-roll single field inflation

- Simplest set-up for prolonged phase of accelerated expansion: scalar field with flat potential in Planck units

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$



$$\epsilon \equiv \frac{M_{\text{pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$$

$$\eta \equiv M_{\text{pl}}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

Inflation generates



***scalar (density)
fluctuations***

$$P_{\zeta} \sim \frac{H^4}{M_{\text{pl}}^2 \dot{H}}$$



***tensor fluctuations
(Gravitational Waves)***

$$P_h \sim \frac{H^2}{M_{\text{pl}}^2}$$

power spectra (2-point correlations) measure:

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

Power spectra measurements constrain



***Scale-dependence of
scalar fluctuations***

$$n_s - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} \propto \frac{d}{dt} \left(\frac{H^4}{M_{\text{Pl}}^2 \dot{H}} \right)$$



Tensor-to-Scalar ratio

$$r \equiv \frac{P_h}{P_\zeta}$$

Planck constraints

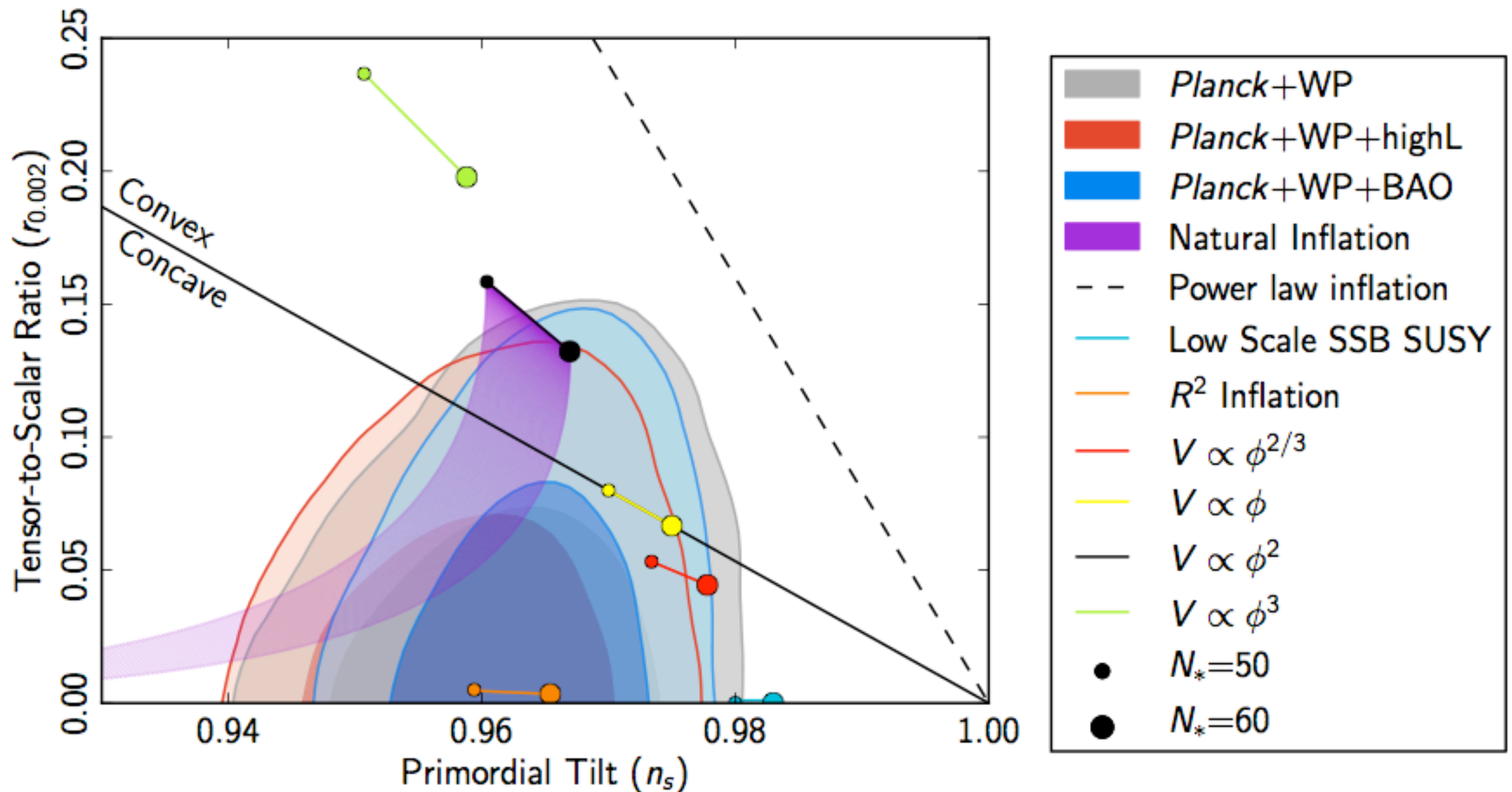


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

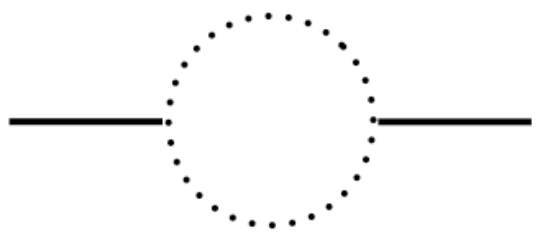
Microphysical origin of inflation?

- Physics at the energy scale of inflation is unknown!

Observational probe of very high-energy physics

- Candidate physical theories motivate much more complicated dynamics than the simplest scenarios:

- ***eta-problem*** (like Higgs hierarchy problem)
- ***multiple fields***
- ***non-standard kinetic terms***
- ***modified gravity***
- alternatives to inflation, e.g. curvaton



A Feynman diagram representing a tadpole. It consists of a central circle with a dotted line, connected to two horizontal solid lines extending outwards.

$$m_{\phi}^2 \sim \Lambda_{\text{uv}}^2 \gg H^2$$

- Plethora of inflationary models. How can we learn more?

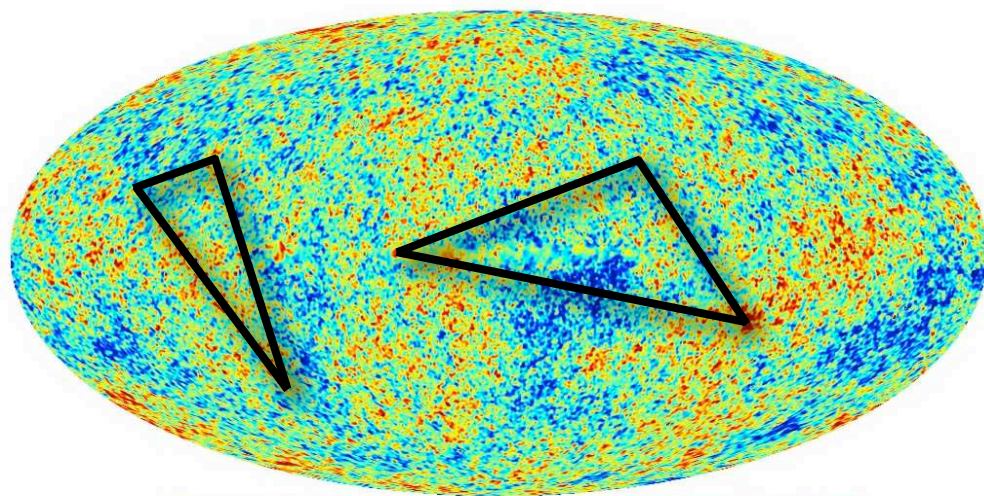
Non-Gaussianity



probes physics beyond

$$*H(t)*$$

3-point correlations



direct measurement of
inflaton interactions

Primordial non-Gaussianities

- Gaussian approximation: freely propagating particles
- Non-Gaussianities measure the ***interactions*** of the field(s) driving inflation. ***Discrimination amongst models*** which are degenerate at the linear level

Particle physics



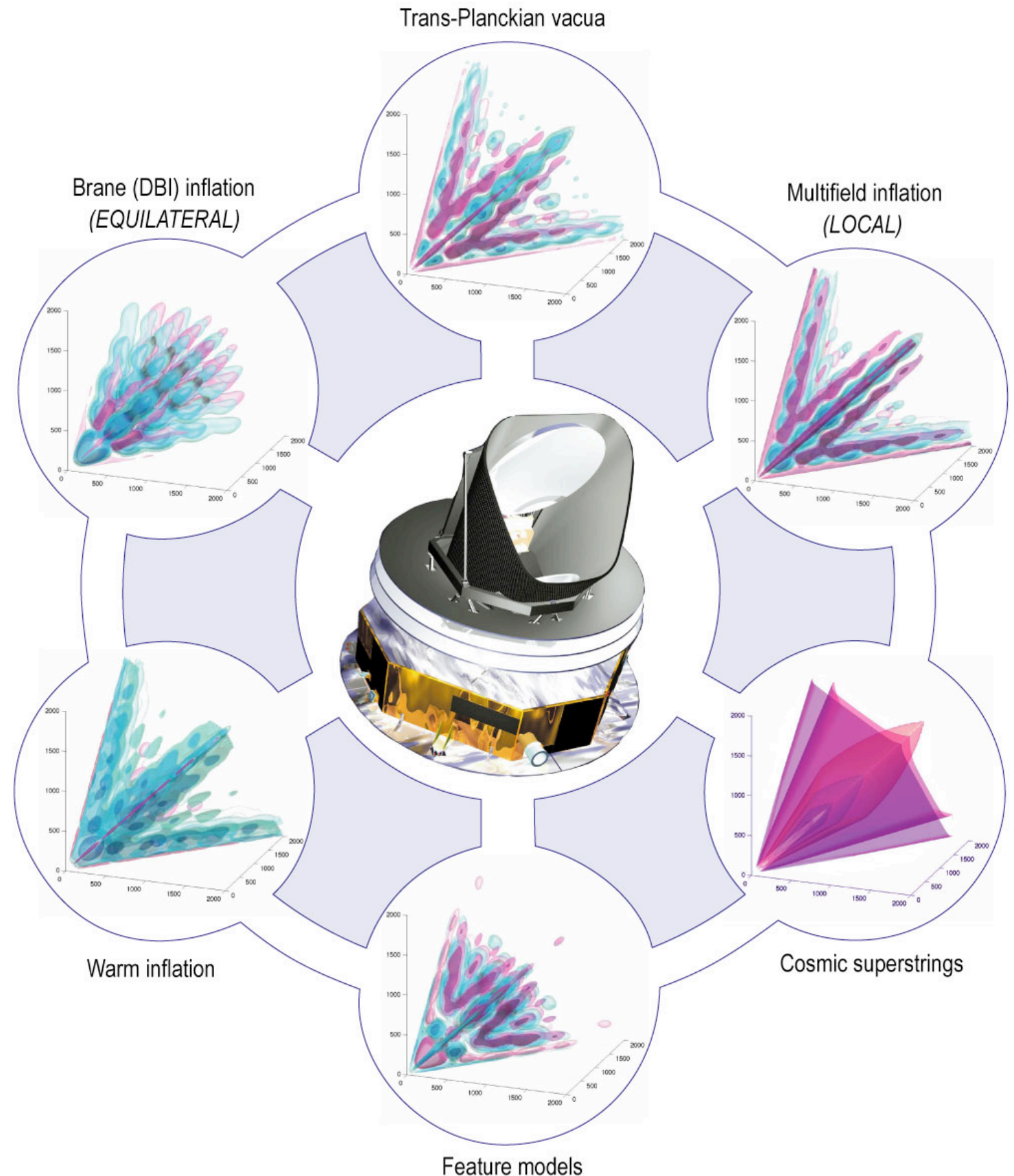
Cosmology



Primordial non-Gaussianities

‘Happy families are all alike;
every unhappy family is
unhappy in its own way.’

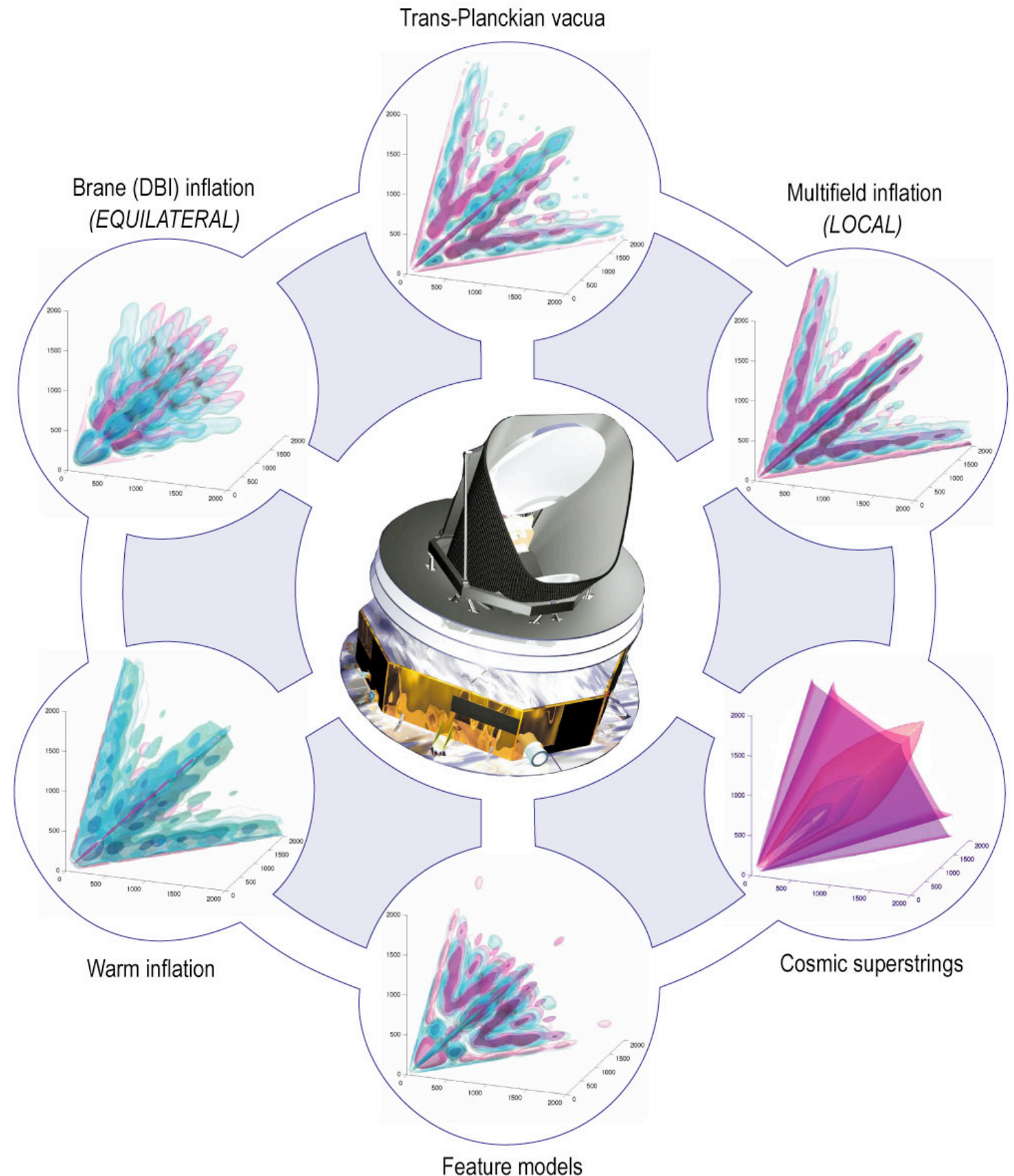
Anna Karénine, Tolstoi



Primordial non-Gaussianities

Gaussian distributions are all alike; every non-Gaussian distribution is non-Gaussian in its own way.

Cosmologist.



Maldacena's 2003 result

Very small non-Gaussianities (much more quantitative statement actually!)

UNDER HYPOTHESES

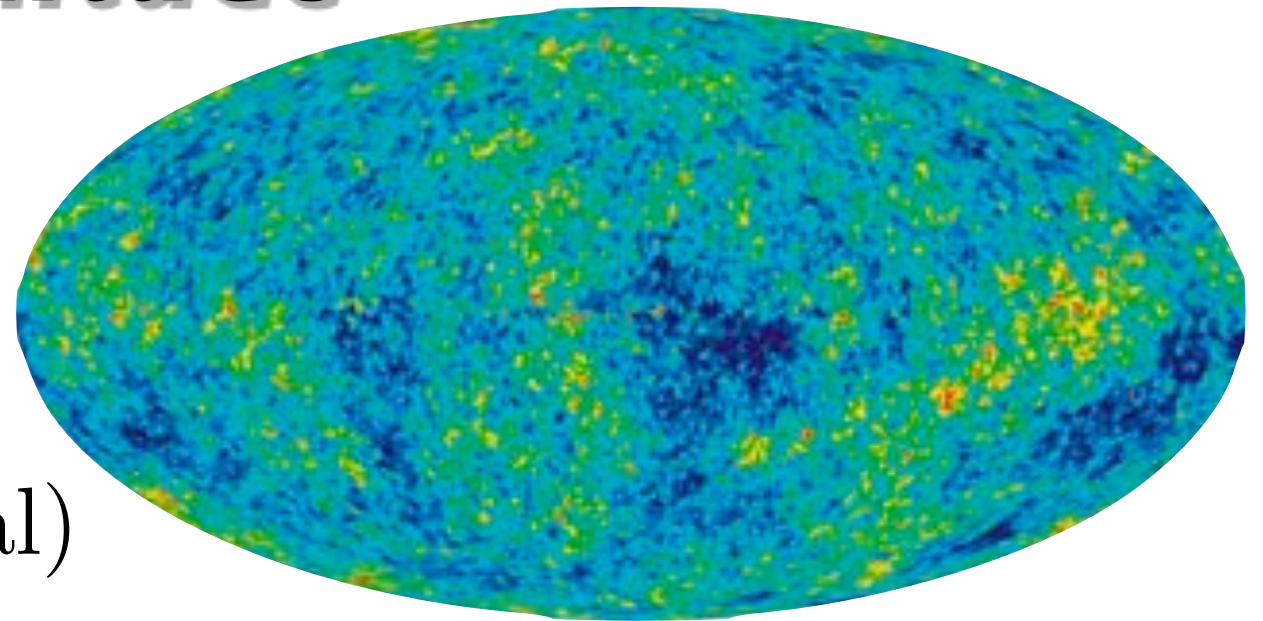
- Single field
- Standard kinetic term
- Slow-roll
- Initial vacuum state
- Einstein gravity

It is now clear that violating any of these assumptions might lead to observably large NGs.

A simple example and orders of magnitude

$$\frac{\delta T}{T} \sim \zeta \sim 10^{-5}$$

$$\zeta = \zeta_G + \frac{3}{5} f_{NL}^{loc} \zeta_G^2 \quad (\text{local})$$



- **Constraints:** $f_{NL}^{loc} = 37.2 \pm 19.9$ (68% CL) WMAP 9

Gaussianity already tested to better than 0.1%

- **Planck:** $f_{NL}^{loc} = 2.7 \pm 5.8$ (68% CL)
- Slow-roll single field prediction: $f_{NL}^{loc} = \mathcal{O}(\epsilon, \eta) \approx 10^{-2}$

Non-Gaussianities

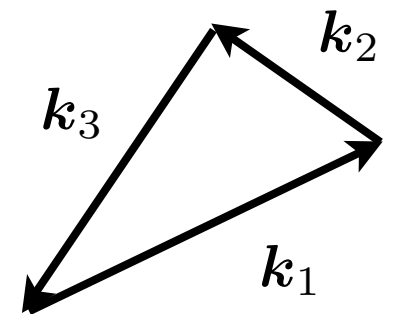
- Beyond the power spectrum:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = P_\zeta(k_1) (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2)$$

- Higher-order connected, n-point functions:

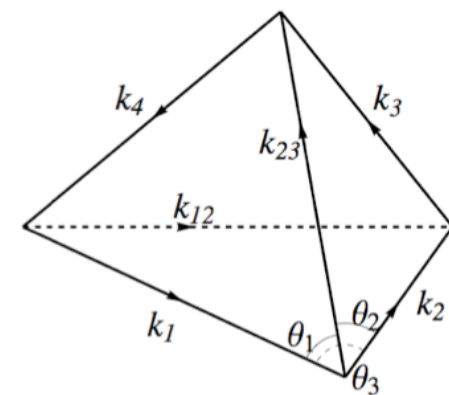
3 point: bispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = B_\zeta(k_1, k_2, k_3) (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$



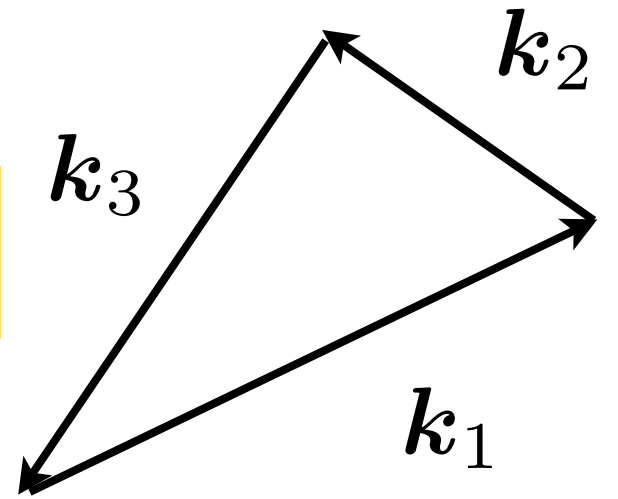
4 point: trispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c = T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) (2\pi)^3 \delta^3\left(\sum_i \mathbf{k}_i\right)$$



The bispectrum

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = (2\pi)^7 \delta\left(\sum_{i=1}^3 \mathbf{k}_i\right) \mathcal{P}_\zeta^2 \frac{S(k_1, k_2, k_3)}{(k_1 k_2 k_3)^2}$$



→ $f_{NL} \sim S$ dimensionless measure of the **amplitude** of the bispectrum

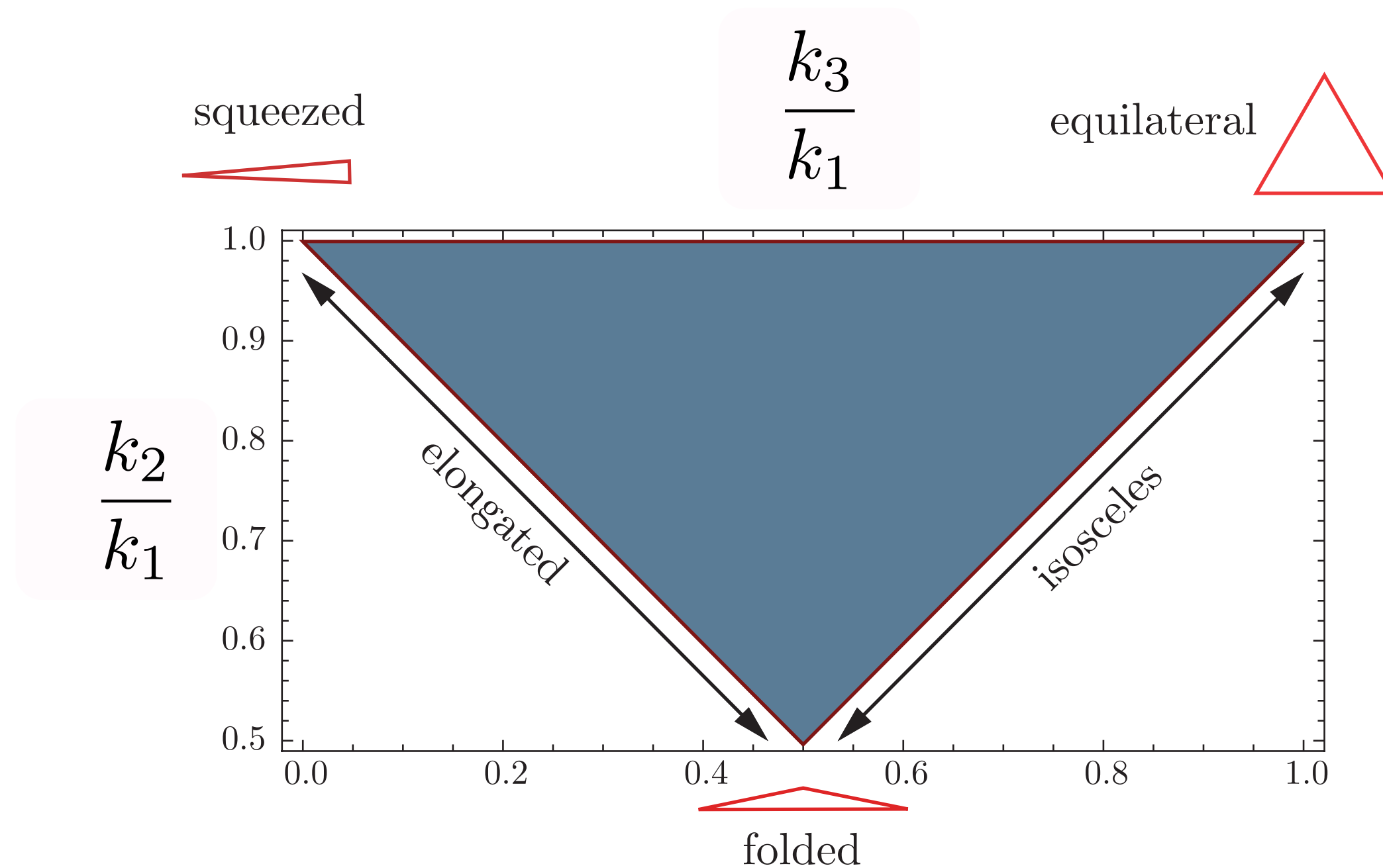
→ **Scale-dependence** (growing or shrinking on small scales?)

→ **Sign** (more or less cold spots?)

*Each of these features
can rule out large
classes of models*

→ **Shape** (dependence on the configuration of triangles)

The shape of the bispectrum



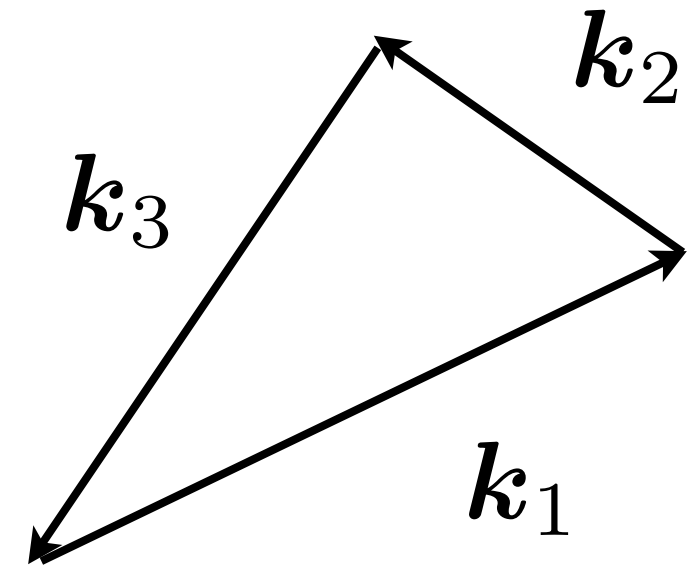
$$k_3 \leq k_2 \leq k_1$$

The shape of the bispectrum

In most (almost scale-invariant) models

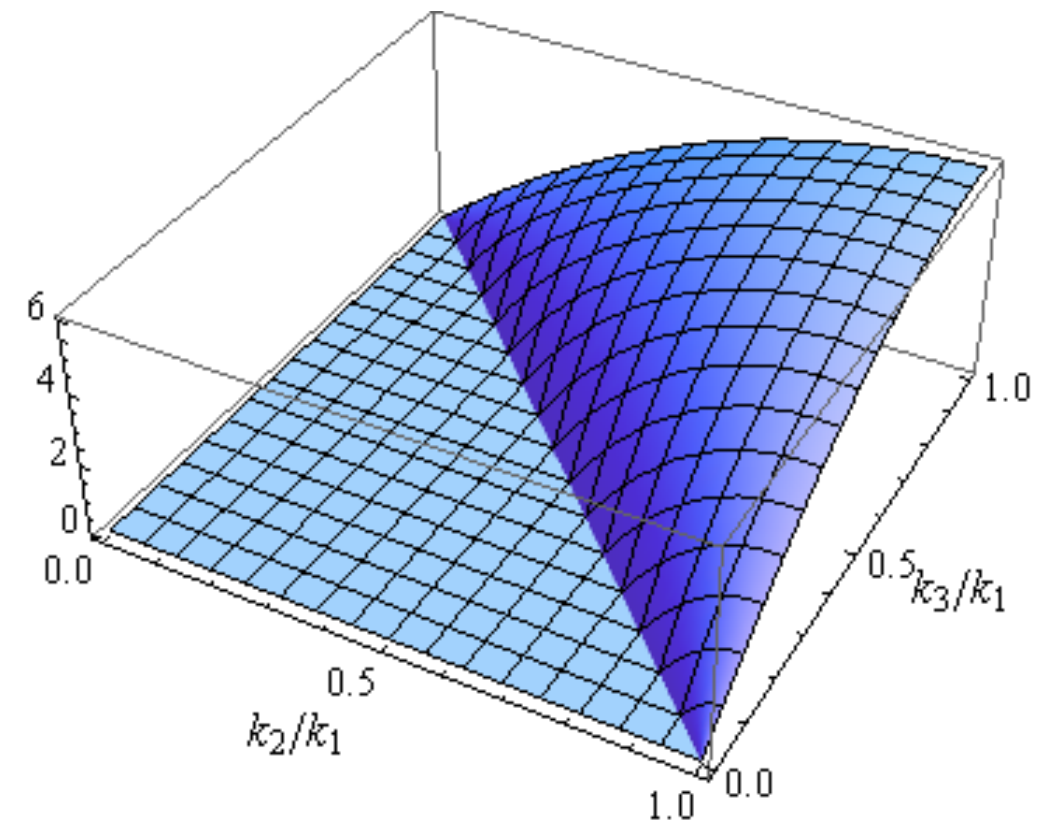
$$S(k_1, k_2, k_3)$$

- only depends on the **ratios between the norms of the wavevectors**
- has the same properties than the observed angular bispectrum

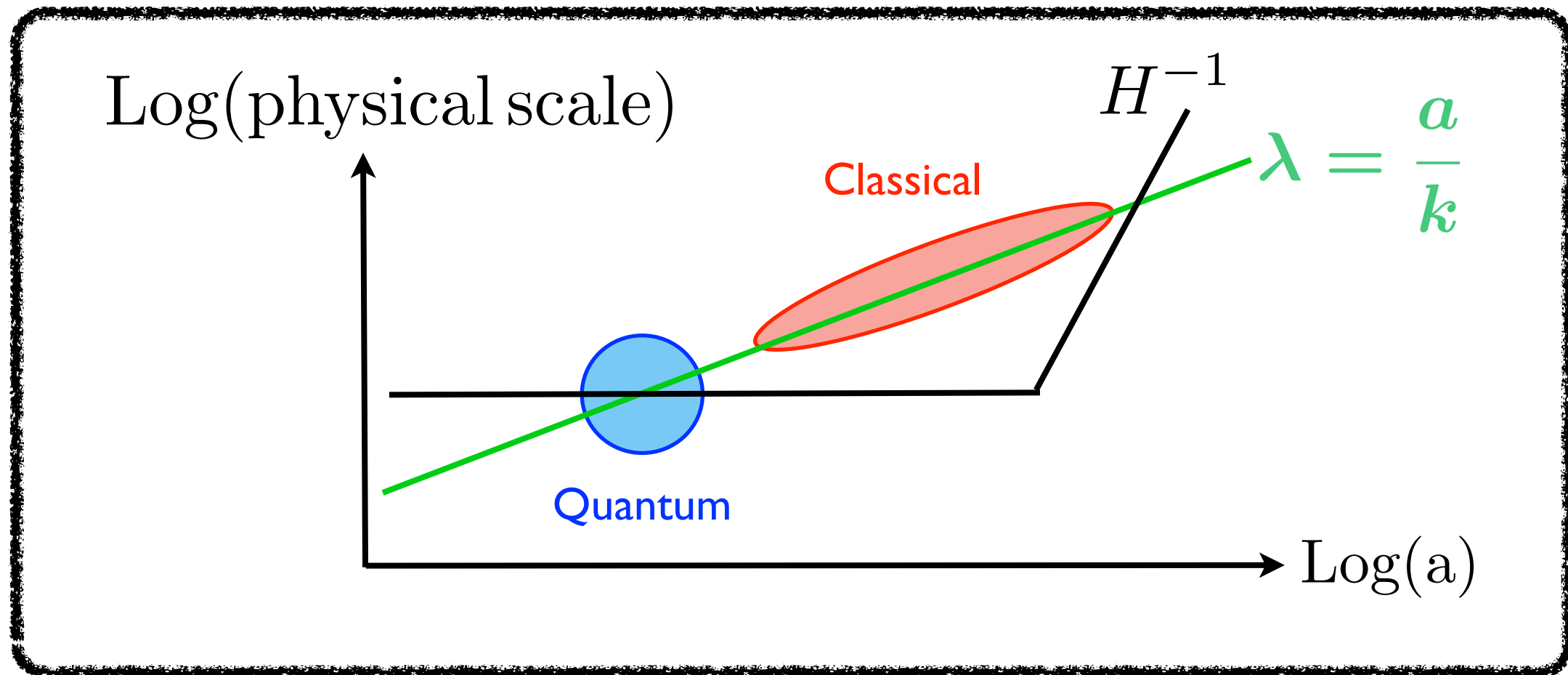


Babich et al (04)

Fergusson & Shellard (08)



Inflationary physics and shapes of non-Gaussianities

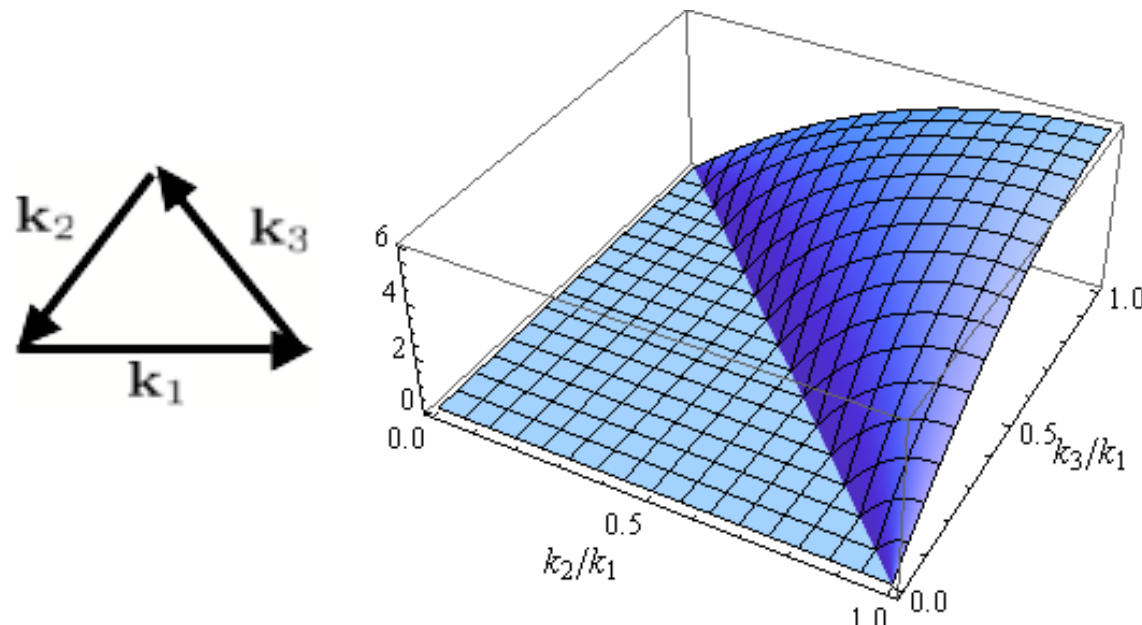


$$S(k_1, k_2, k_3) = f_{NL}^{loc} \left(\frac{k_1^2}{k_2 k_3} + 2 \text{ perm.} \right) + 3 f_{NL}^{eq} \left(- \left(\frac{k_1^2}{k_2 k_3} + 2 \text{ perm.} \right) + \left(\frac{k_1}{k_2} + 5 \text{ perm.} \right) - 2 \right)$$

The diagram illustrates the momentum configurations for the two terms in the equation. The first term, f_{NL}^{loc} , is associated with a triangle of momenta k_1, k_2, k_3 where k_1 is the largest side. The second term, $3 f_{NL}^{eq}$, is associated with a triangle of momenta k_1, k_2, k_3 where k_1 is the smallest side.

Inflationary physics and shapes of non-Gaussianities

Equilateral type (quantum)



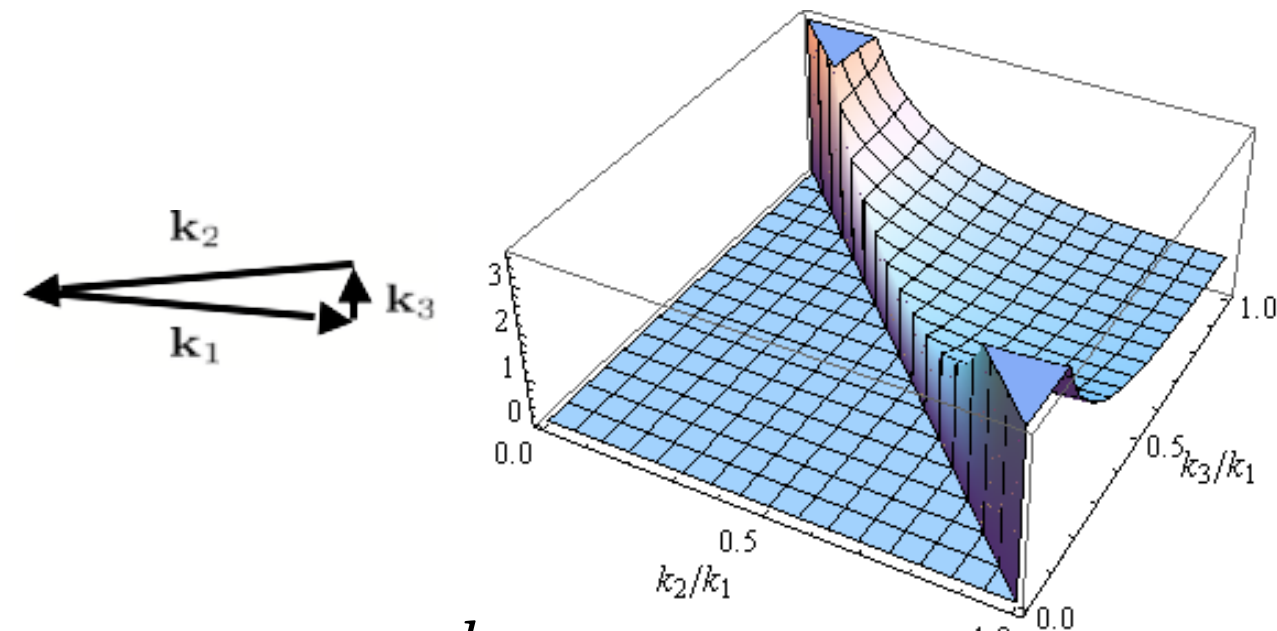
WMAP $f_{NL}^{eq} = 51 \pm 136$

Planck $f_{NL}^{eq} = -42 \pm 75$

Non-standard kinetic terms:
DBI, low sound speed models.

**Good understanding
with EFT of inflation**

Local type (classical)



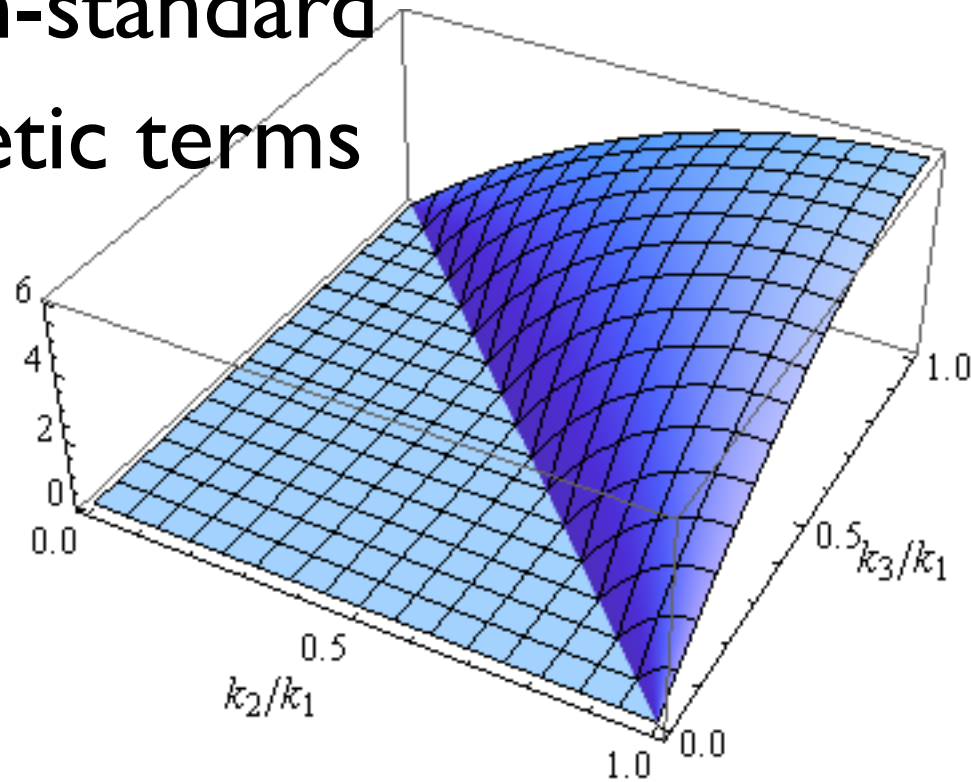
WMAP $f_{NL}^{loc} = 37.2 \pm 19.9$

Planck $f_{NL}^{loc} = 2.7 \pm 5.8$

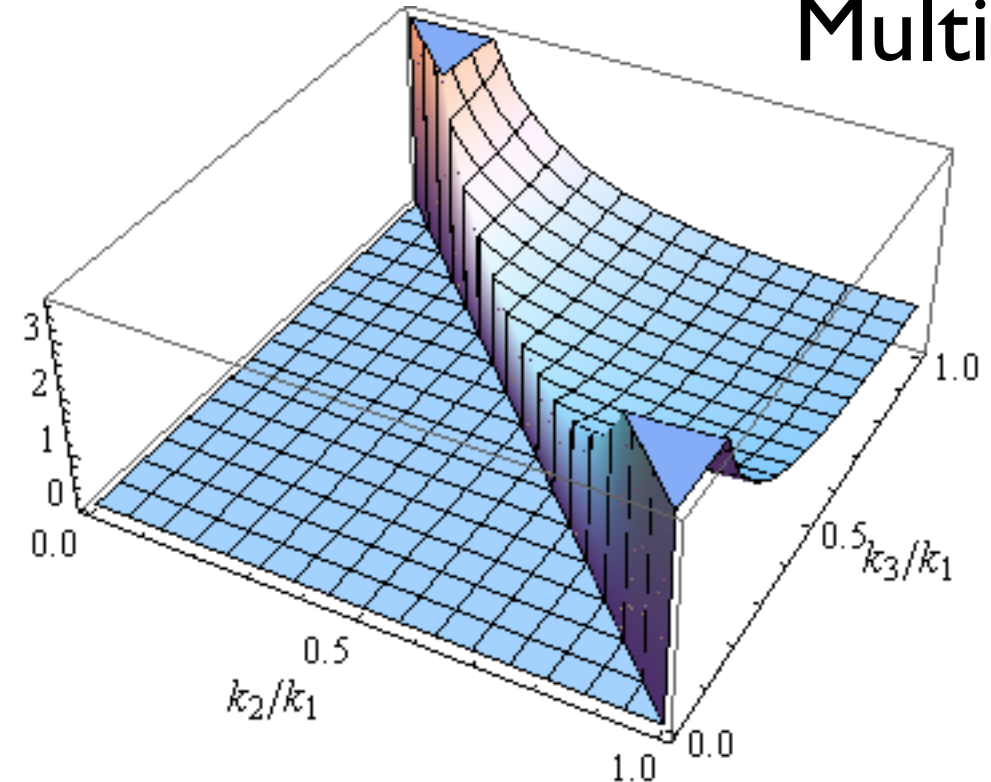
Multiple degrees of freedom:
Multifield inflation, curvaton,
ekpyrotic...

Inflationary physics and shapes of non-Gaussianities

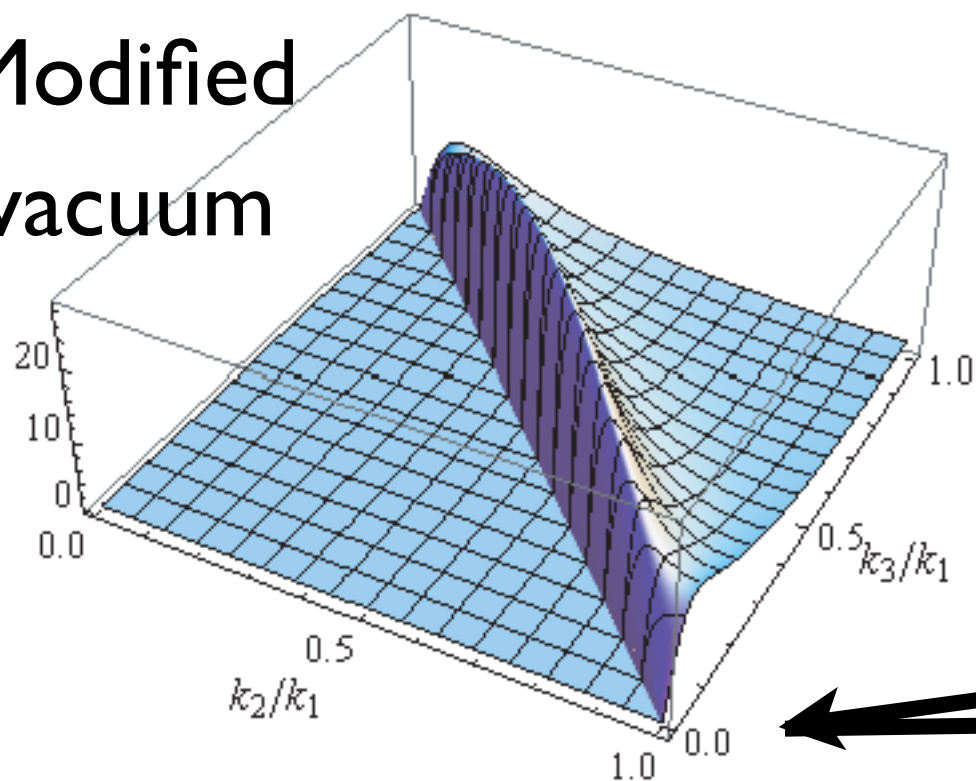
Non-standard
kinetic terms



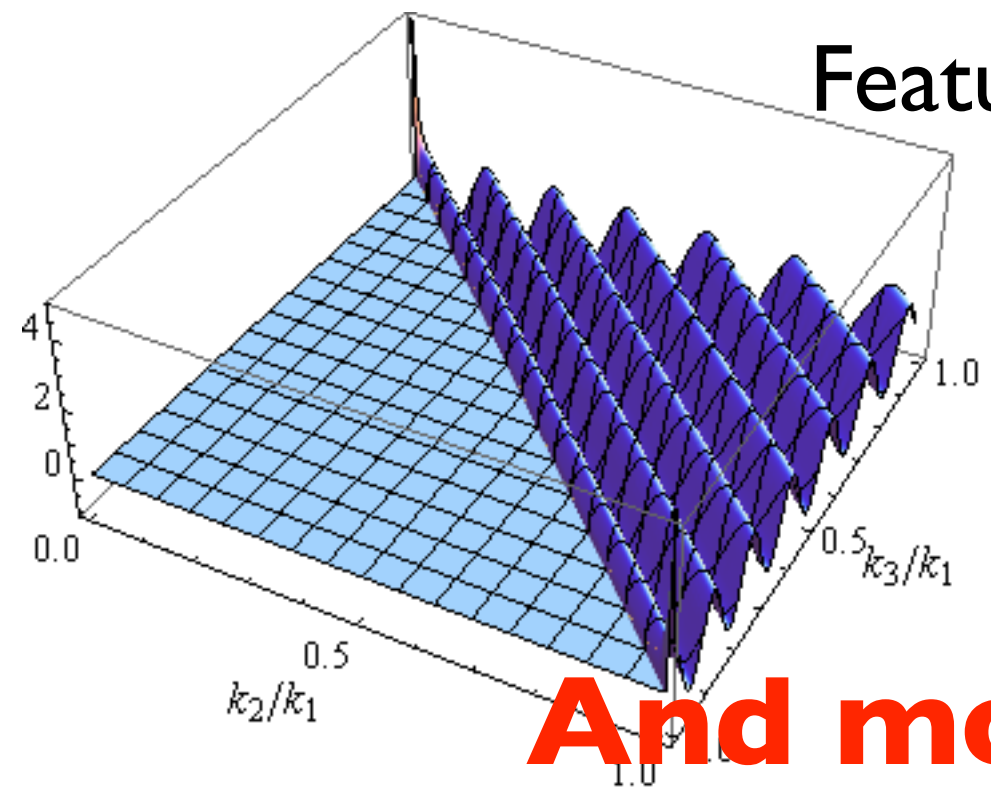
Multifield



Modified
vacuum



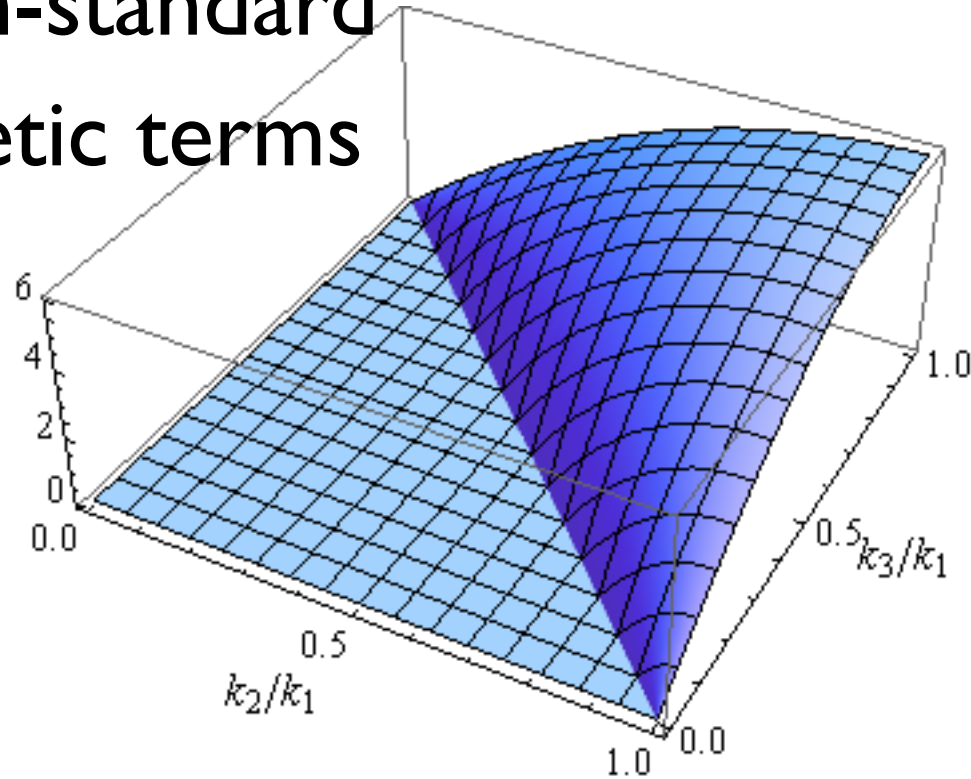
Features



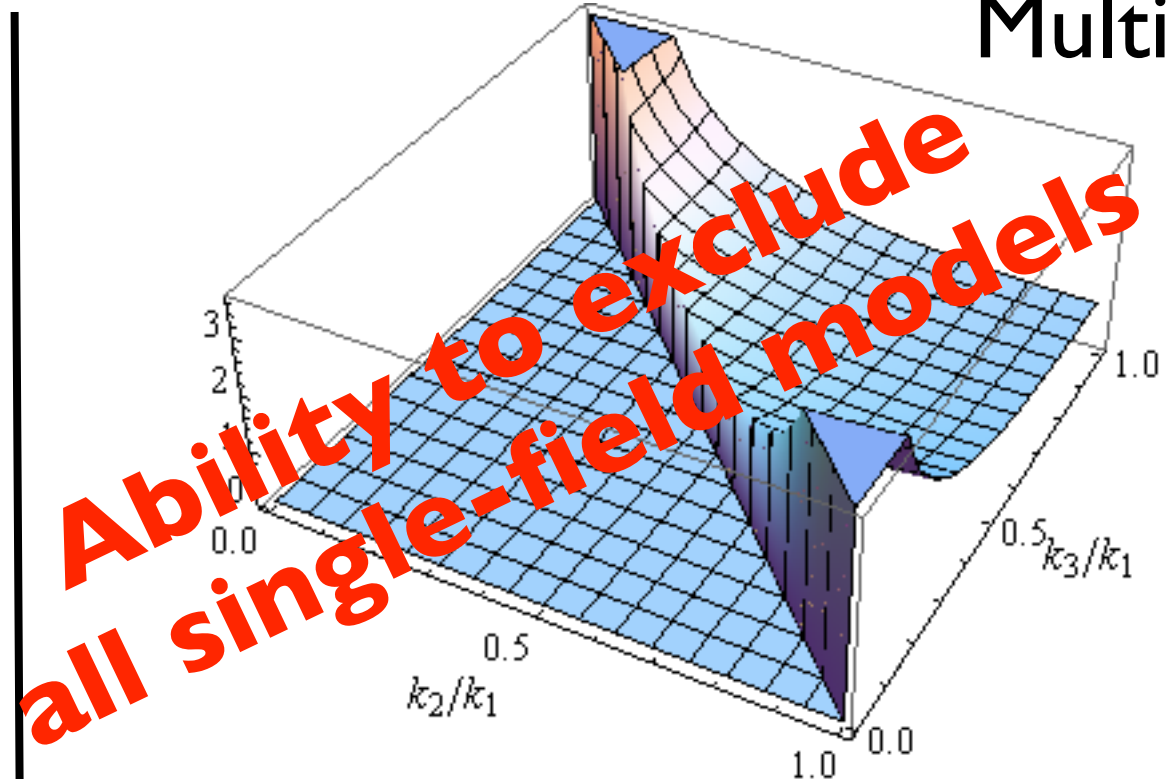
And more!

Inflationary physics and shapes of non-Gaussianities

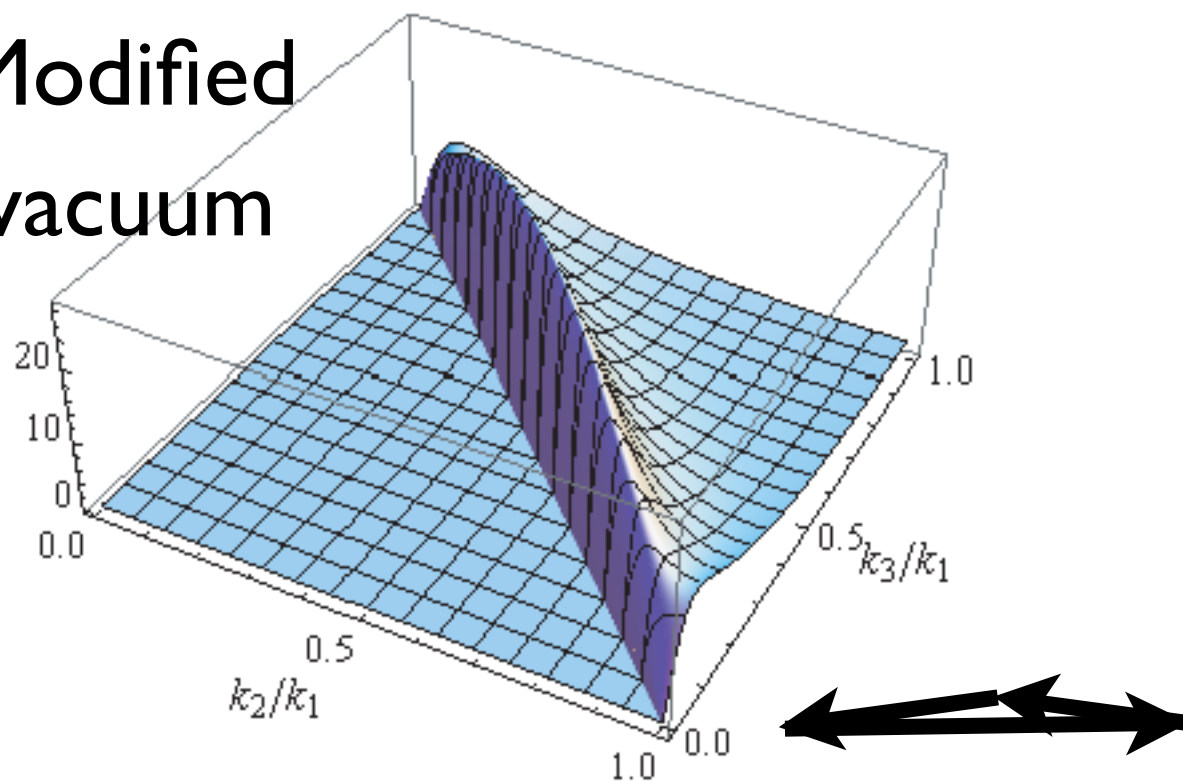
Non-standard kinetic terms



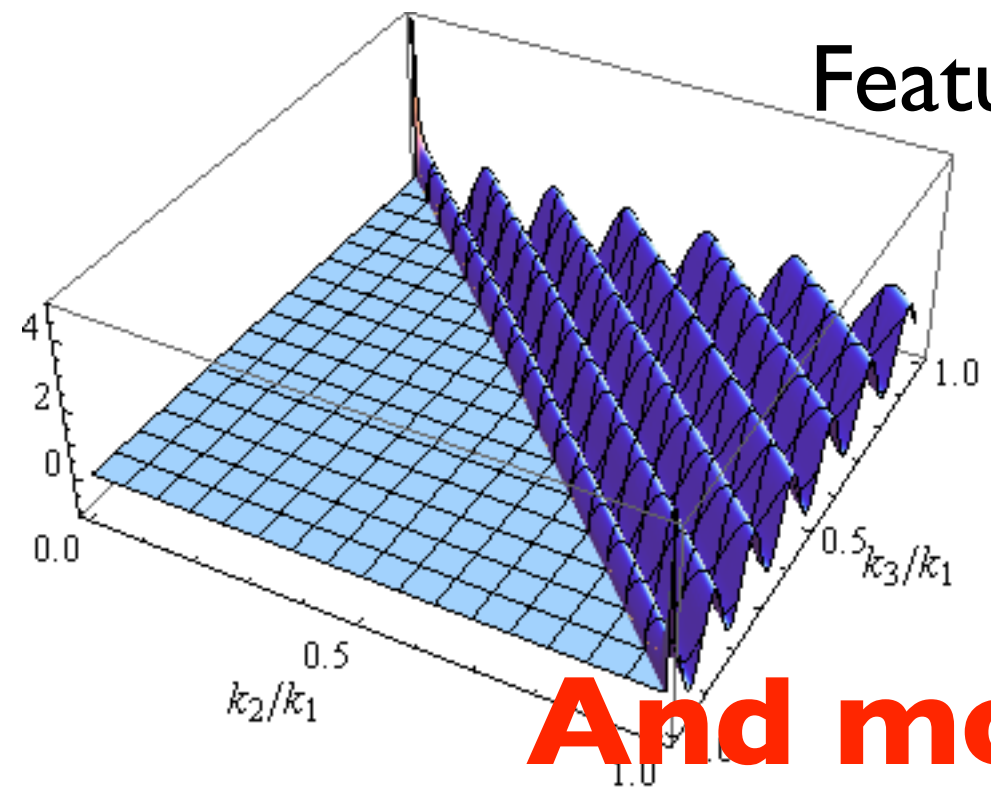
Multifield



Modified vacuum



Features



And more!

Single field consistency relation

Any single-clock inflation (irrespective of kinetic terms, potential etc)

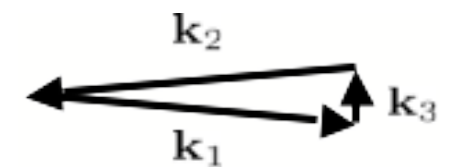
Maldacena (03), Creminelli & Zaldarriaga (04)



$$f_{NL}^{sq}(k_1) = \frac{5}{12}(1 - n_s(k_1))$$

with

$$f_{NL}^{sq}(k_1) \equiv \lim_{k_3 \rightarrow 0} f_{NL}(k_1, k_2, k_3)$$



Remember $n_s = 0.9603 \pm 0.0073$ (68% CL) Planck

If $f_{NL}^{sq} \gtrsim 1$ of primordial origin is robustly detected,
all single field models would be ruled out!

Planck implications

$f_{NL}^{loc} = 2.7 \pm 5.8$  Constrain multi-field effects

$f_{NL}^{eq} = -42 \pm 75$  Lower bound on the
inflaton speed of sound

$f_{NL}^{orth} = -25 \pm 39$ $c_s \geq 0.02$ (95% CL)

Strong constraints on light hidden sector fields coupled to the inflaton via operators suppressed by a high mass scale.

Assassi et al, 2013.

$$\Lambda > 10^5 H$$

$$\Lambda > 10^2 H$$

depending on assumptions on the hidden sector

The simplest inflationary models are in full agreement with data and have passed very stringent tests.