Dark Energy:
an Effective Field Theory approach

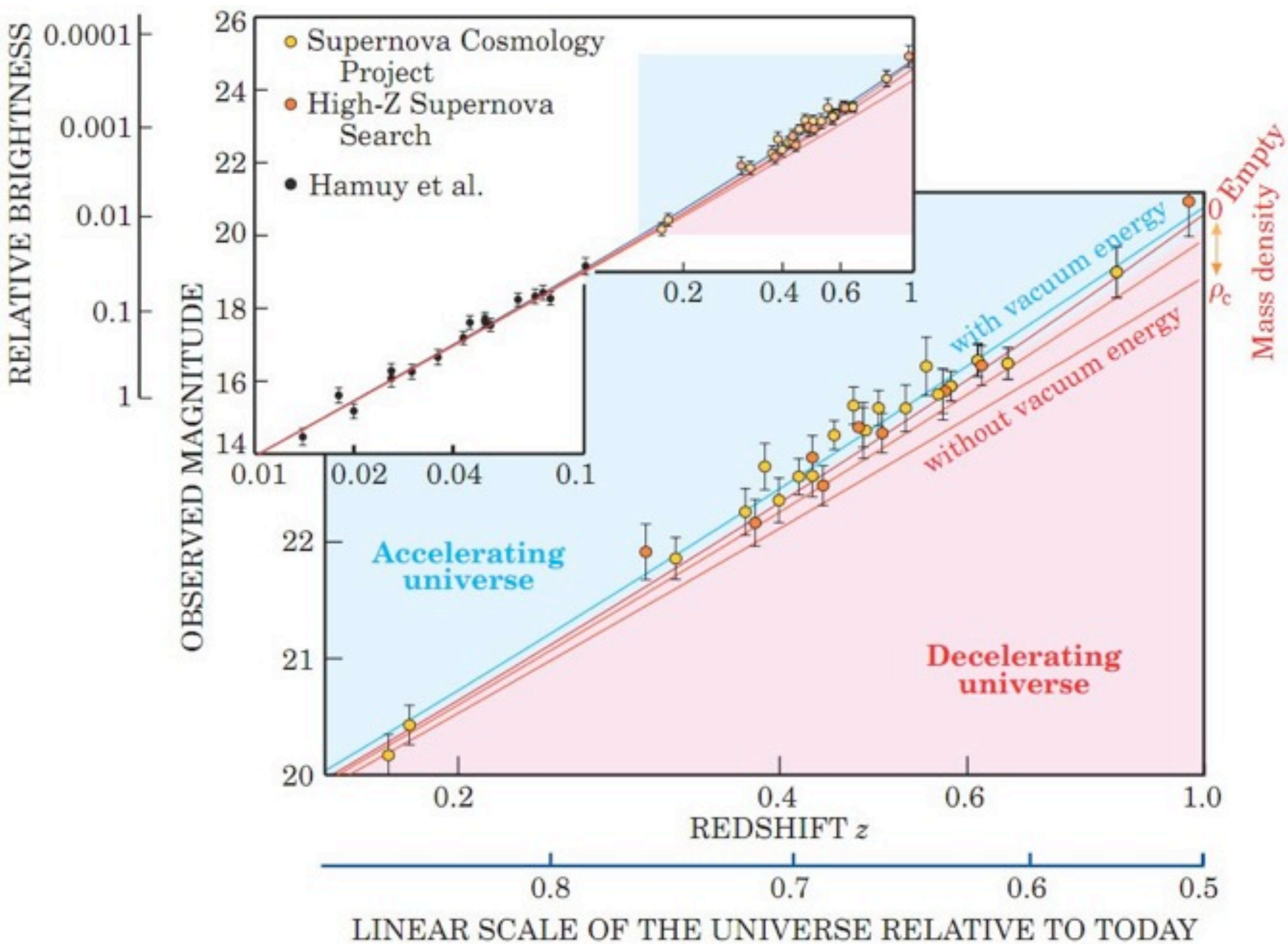
Federico Piazza

Paris Centre for Cosmological Physics

Gubitosi, F. P., Vernizzi, 1210.0201
Gleyzes, Langlois, F.P., Vernizzi, 1304.4840

Nobel Prize in Physics 2011

The Universe is accelerating!



The energy budget

Non-relativistic matter
(dark matter, baryons)

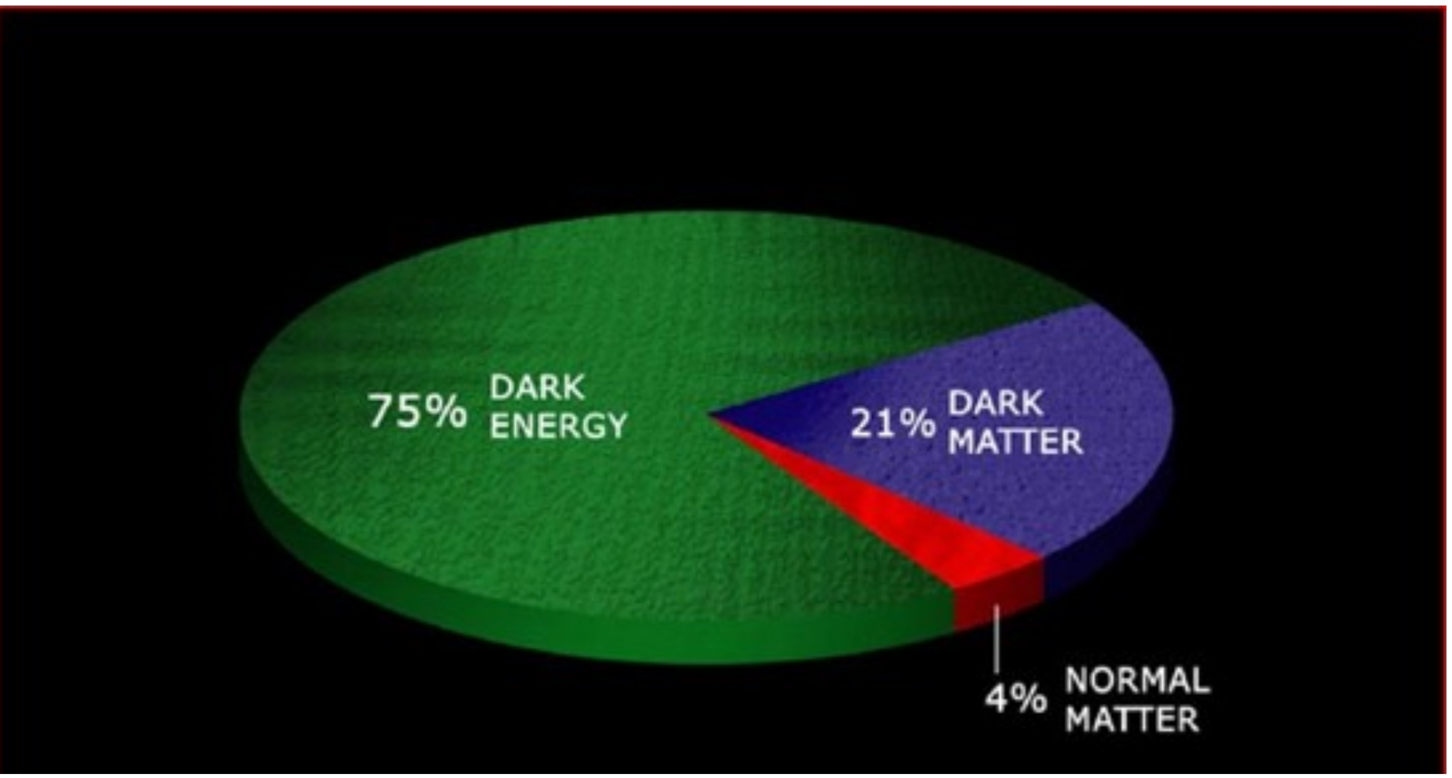
$$p_{\rm M} \sim 0$$

$$\Omega_{\rm M} \sim 0.3$$

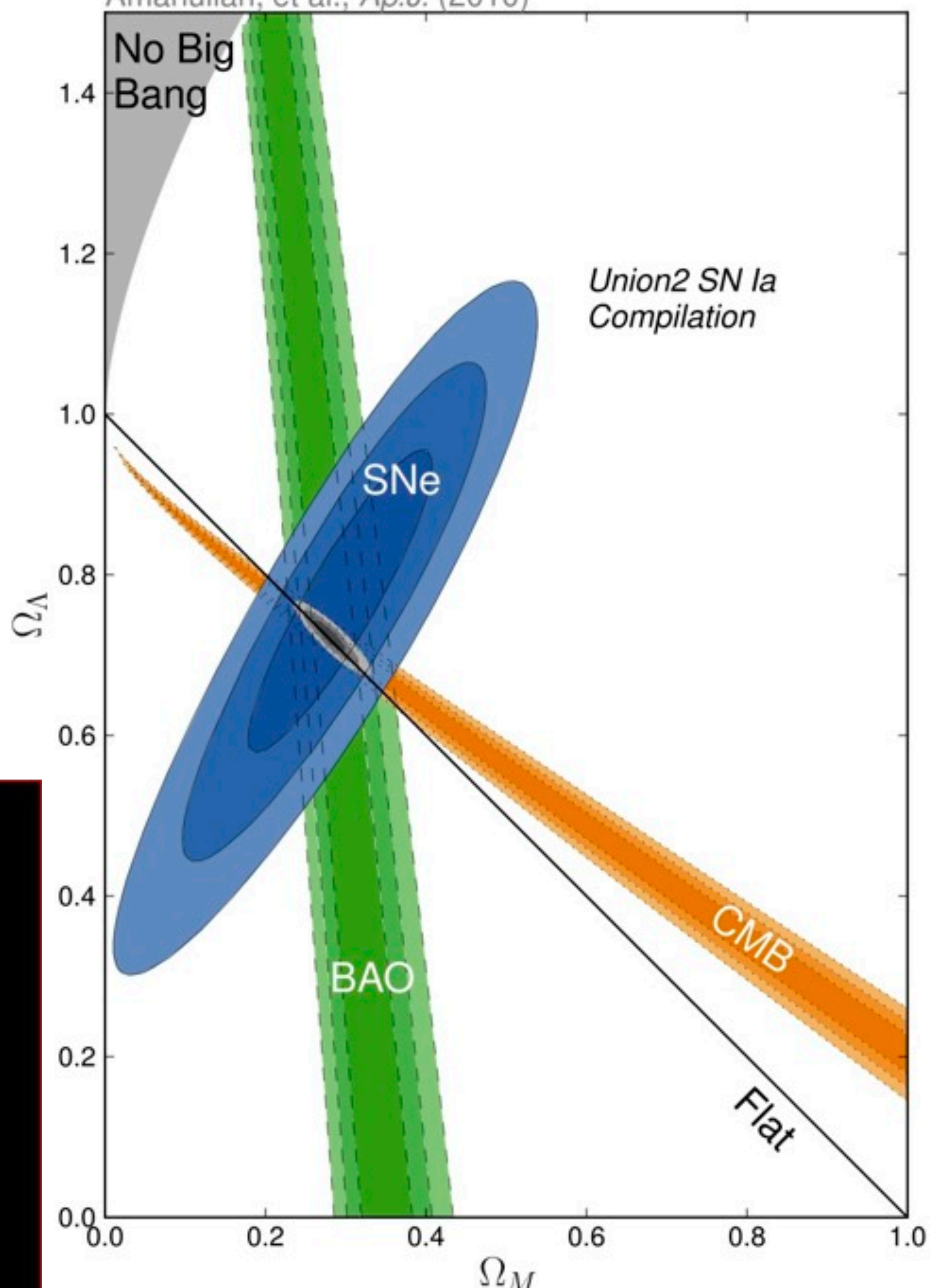
Dark Energy

$$p_{\rm DE} < -\frac{1}{3}\rho_{\rm DE}$$

$$\Omega_{\rm DE} \sim 0.7$$



Supernova Cosmology Project
Amanullah, et al., *Ap.J.* (2010)



Beyond the Cosmological Constant...

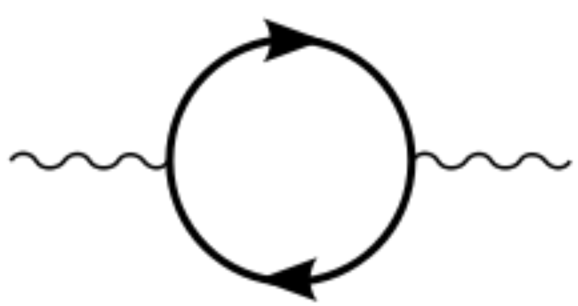
The first obvious candidate for Dark Energy is a cosmological constant

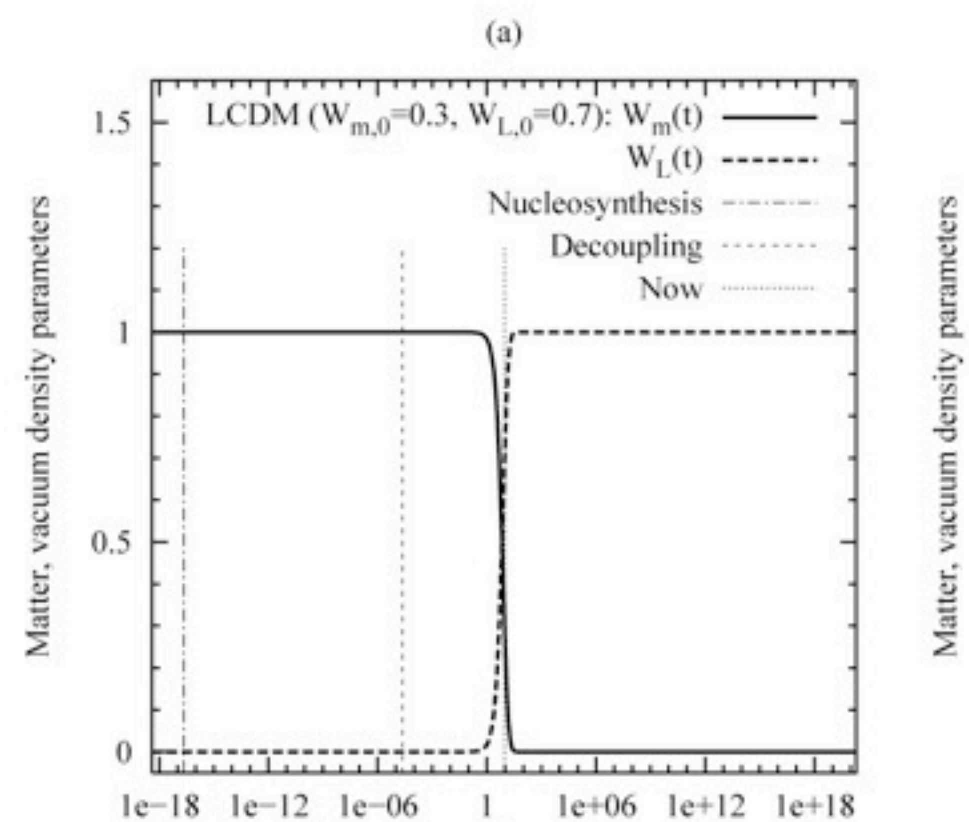
$$S = \int \sqrt{-g}\,\left(M_{\mathrm{Pl}}^2 R - \Lambda\right)$$

However:

Naturalness problem (perhaps $\Lambda = 0$ is better than $\Lambda \sim (10^{-3}\mathrm{eV})^4$)

Coincidence problem (why $\rho_{\mathrm{DE}} \sim \rho_{\mathrm{M}}$ now?!).





Beyond the Cosmological Constant...

The first obvious candidate for Dark Energy is a cosmological constant

$$S = \int \sqrt{-g}\ \left(M_{\rm Pl}^2 R - \Lambda\right)$$

$$DE \neq \Lambda$$

$$\Downarrow$$

there is a new propagating degree of freedom in the theory ϕ

- There is `no shortage’ of dark energy and modified gravity (DE) models (>5000 papers on Spires)

- Each one with its motivations, number and type of parameters etc...

- EUCLID and BigBoss will be sensitive to dynamical properties of DE

- Need for a Unifying and Effective description of DE

Ideally...

- A limited number of effective operators, each one responsible for an observable dynamical feature (e.g. flavor-changing neutral currents in physics beyond Standard Model)

- There is `no shortage' of dark energy and modified gravity (DE) models (>5000 papers on Spires)

- Each one with its motivations, number and type of parameters etc...

- EUCLID and BigBoss will be sensitive to dynamical properties of DE

$$S[\phi, g_{\mu\nu}, \Psi_m]$$

Background

$$\phi = \phi_0(t)$$
$$ds^2 = -dt^2 + a^2(t)dx^2$$
$$\rho_m = \rho_m(t)$$

Expand in perturbations

$$\delta\rho_m(t, \vec{x}) \Longleftrightarrow \delta\phi(t, \vec{x})$$

physics beyond Standard Model)

- There is `no shortage’ of dark energy and modified gravity (DE) models (>5000 papers on Spires)

- Each one with its motivations, number and type of parameters etc...

- EUCLID and BigBoss will be sensitive to dynamical properties of DE

- Need for a Unifying and Effective description of DE

Ideally...

- A limited number of effective operators, each one responsible for an observable dynamical feature (e.g. flavor-changing neutral currents in physics beyond Standard Model)

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

One possible strategy: (Weinberg \`08, Park, Zurek and Watson \`10, Bloomfield and Flanagan \`11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

However:

1) Expansion in number of fields is not necessarily meaningful

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

C ... gan `11)

A ... ion

$$\begin{aligned} V &= V_1\phi + V_2\phi^2 + V_3\phi^3 + V_4\phi^4 \\ &= V_2\delta\phi^2 + V_3\phi_0(t)\delta\phi^2 + 6V_4\phi_0^2(t)\delta\phi^2 \end{aligned}$$

All terms potentially important in cosmological perturbation theory!

H

1) Expansion in number of fields is not necessarily meaningful

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively ``perturbations'' but not always so...

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EI … ative expansion

$$\partial\phi^4,\ \Box\phi\partial\phi^2,\ \text{etc.}$$

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively ``perturbations'' but not always so...

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S=\int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{F}[\phi,g^{\mu\nu}]\right]+S_m[g_{\mu\nu},\Psi_m]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EF … ative expansion

$$\partial\phi^4,\ \Box\phi\partial\phi^2,\ \text{etc.}$$

Ghost Condensate

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively ``perturbations'' but not always so...

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant E ... ative expansion

$$\partial\phi^4, \quad \Box\phi\partial\phi^2, \quad \text{etc.}$$

Ghost Condensate Galileons

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively ``perturbations'' but not always so...

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively ``perturbations'' but not always so...
3) Only halfway through the work to be done (background first + expand..)

EFT: a theory for the relevant low-energy d.o.f.

Examples:

1) QCD: quarks and gluons $\Rightarrow$ nucleons and pions at low energies

EFT: a theory for the relevant low-energy d.o.f.

Examples:

1) QCD: quarks and gluons → nucleons and pions at low energies

2) EW theory: 4 massless vector bosons, 2 complex scalars etc.

UNITARY GAUGE ↓

3 massive vector bosons, 1 massive ``Higgs'' field etc.

EFT: a theory for the relevant low-energy d.o.f.

Examples:

1) QCD: quarks and gluons ⟹ nucleons and pions at low energies

2) EW theory: 4 massless vector bosons, 2 complex scalars etc.

UNITARY GAUGE ⟹

3 massive vector bosons, 1 massive ``Higgs'' field etc.

3) Cosmology: ...Cosmological Perturbations!

Unitary Gauge in Cosmology

The Effective Field Theory of Inflation (Creminelli et al. `06, Cheung et al. `07)

Main idea: scalar degrees of freedom are `eaten' by the metric. Ex:

$$\phi(t,\vec{x}) \to \phi_0(t) \quad (\delta\phi = 0) \qquad -\frac{1}{2}\partial\phi^2 \to -\frac{1}{2}\dot{\phi}_0^2(t)\; g^{00}$$

Bennett et al, 2012 (Final WMAP paper) constrain inflation EFT operators

Unitary Gauge in Cosmology

The Effective Field Theory of Inflation (Creminelli et al. `06, Cheung et al. `07)

Main idea: scalar degrees of freedom are `eaten' by the metric. Ex:

$$\phi(t,\vec{x}) \to \phi_0(t) \quad (\delta\phi = 0) \qquad -\frac{1}{2}\partial\phi^2 \to -\frac{1}{2}\dot{\phi}_0^2(t)\ g^{00}$$

Our Recipe for Dark Energy: (Gubitosi, F.P., Vernizzi 2012)

1) Assume WEP (universally coupled metric $S_m[g_{\mu\nu},\Psi_i]$)

2) Write the most generic action for $g_{\mu\nu}$ compatible with the residual un-broken symmetries (3-diff).

The Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

Genuine 4-dim covariant terms are still allowed, but will in general be multiplied by functions of time cause time translations are broken

The Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

General functions of time are allowed

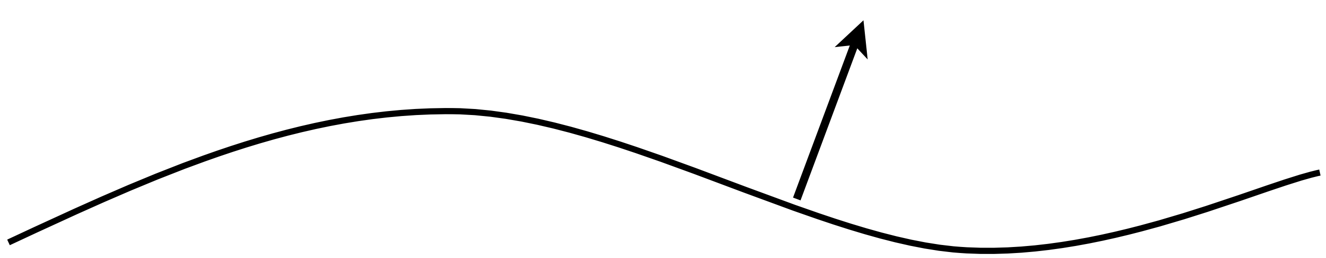
The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

...as well as tensors with free `0' indices

Essentially: contractions with $n_\mu = -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi^2)}}$



The Action: main message

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

The Action: main message

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S^{(2)}_{DE}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

Example:

$$\partial\phi^2 \, R \;=\; \dot{\phi}_0^2(-1+\delta g^{00})(R^{(0)}+\delta R) \;=\; \dot{\phi}_0^2 \left[-R + R^{(0)}(t) + R^{(0)}(t) g^{00} + \delta g^{00} \delta R \right]$$

The Action: main message

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

Example:

$$\partial\phi^2\, R \;=\; \dot\phi_0^2(-1+\delta g^{00})(R^{(0)}+\delta R) \;=\; \dot\phi_0^2\left[-R + R^{(0)}(t) + R^{(0)}(t)g^{00} + \delta g^{00}\delta R\right]$$

The Action: main message

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S^{(2)}_{DE}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

Example:

$$\partial\phi^2 \, R \;=\; \dot{\phi}_0^2(-1+\delta g^{00})(R^{(0)}+\delta R) \;=\; \dot{\phi}_0^2 \left[-R + R^{(0)}(t) + R^{(0)}(t) g^{00} + \delta g^{00} \delta R \right]$$

The Action: main message

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

Example:

$$\partial\phi^2\, R \;=\; \dot\phi_0^2(-1+\delta g^{00})(R^{(0)}+\delta R) \;=\; \dot\phi_0^2\left[-R+R^{(0)}(t)+R^{(0)}(t)g^{00}+\delta g^{00}\delta R\right]$$

The Action: main message

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

Example:

$$\partial\phi^2 \, R \;=\; \dot{\phi}_0^2(-1+\delta g^{00})(R^{(0)}+\delta R) \;=\; \dot{\phi}_0^2 \left[-R + R^{(0)}(t) + R^{(0)}(t) g^{00} + \delta g^{00} \delta R \right]$$

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$\begin{aligned} c &= \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D) \\ \Lambda &= \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D) \end{aligned}$$

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$\begin{aligned} c &= \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D) \\ \Lambda &= \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D) \end{aligned}$$

Generally Related to post-newtonian parameters

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$\begin{aligned} c &= \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D) \\ \Lambda &= \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D) \end{aligned}$$

Generally Related to post-newtonian parameters

"Bare" Planck Mass

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$\begin{aligned} c &= \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D) \\ \Lambda &= \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D) \end{aligned}$$

Generally Related to post-newtonian parameters

$$H^2 = \frac{1}{3fM^2}(\rho_m + \rho_D)$$

$$\dot{H} = -\frac{1}{2fM^2}(\rho_m + \rho_D + p_m + p_D)$$

"Bare" Planck Mass

Defined by the modified Friedman equations

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$c \;=\; \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D)$$

$$\Lambda \;=\; \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D)$$

Generally Related to post-newtonian parameters

"Bare" Planck Mass

Defined by the modified Friedman equations

Matter + Dark matter (in practice $\rho_m \propto a^{-3}$)

$$H^2 = \frac{1}{3fM^2}(\rho_m + \rho_D)$$

$$\dot{H} = -\frac{1}{2fM^2}(\rho_m + \rho_D + p_m + p_D)$$

The Action: perturbations

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

Explicitly quadratic in the perturbations:

$$S_{DE}^{(2)} = \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K - \frac{\bar{M}_2^2}{2} \delta K^2 - \frac{\bar{M}_3^2}{2} \delta K_\mu{}^\nu \delta K^\mu{}_\nu + \dots$$

The Action: perturbations

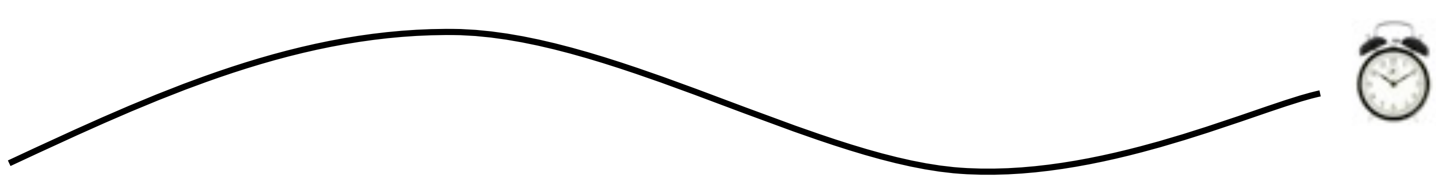
$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Explicitly quadratic in the perturbations:

$$S_{DE}^{(2)} = \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\,\delta g^{00}\delta K - \frac{\bar{M}_2^2}{2}\,\delta K^2 - \frac{\bar{M}_3^2}{2}\,\delta K_\mu{}^\nu \delta K^\mu{}_\nu + \ldots$$

Extrinsic curvature: $n_\mu = -\dfrac{\partial_\mu\phi}{\sqrt{-(\partial\phi^2)}}$ $\quad h_{\mu\nu} \equiv g_{\mu\nu} + n_\mu n_\nu$

$$K_{\mu\nu} = h_\mu{}^\sigma \nabla_\sigma n_\nu \qquad \delta K_{\mu\nu} = K_{\mu\nu} - H h_{\mu\nu}$$



The Action: perturbations

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

Explicitly quadratic in the perturbations:

$$S_{DE}^{(2)} = \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K - \frac{\bar{M}_2^2}{2} \delta K^2 - \frac{\bar{M}_3^2}{2} \delta K_\mu{}^\nu \delta K^\mu{}_\nu + \ldots$$

Action in standard form (no ambiguities, field redefinitions)

Examples

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K \ + \frac{M_3^4}{6} (\delta g^{00})^3 + \dots \right)$$

Examples

$$S=\int\sqrt{-g}\left(\frac{M^2}{2}fR-\Lambda-cg^{00}+\frac{M_2^4}{2}(\delta g^{00})^2-\frac{\bar m_1^3}{2}\delta g^{00}\delta K\ +\frac{M_3^4}{6}(\delta g^{00})^3+\dots\right)$$

Non-minimally coupled scalar field

$$S=\int d^4x\sqrt{-g}\left[\frac{M^2}{2}F(\phi)R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right]$$

$$f(t)=F(\phi_0(t))\ ,\quad \Lambda(t)=V(\phi_0(t))\ ,\quad c(t)=\dot\phi_0^2(t)$$

Examples

$$S=\int\sqrt{-g}\left(\frac{M^2}{2}fR-\Lambda-cg^{00}+\frac{M_2^4}{2}(\delta g^{00})^2-\frac{\bar{m}_1^3}{2}\delta g^{00}\delta K+\frac{M_3^4}{6}(\delta g^{00})^3+\dots\right)$$

K-essence

(Amendariz-Picon et al., 2000)

$$S=\int d^4x\sqrt{-g}P(\phi,X)\qquad X\equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Expansion: $X=\dot{\phi}_0^2(t)(-1+\delta g^{00})$

$$\Lambda(t)=c(t)-P(\phi_0(t),\dot{\phi}_0^2(t))\ ,\quad c(t)=\left.\frac{\partial P}{\partial X}\right|_{\phi=\phi_0,\,X=\dot{\phi}_0^2}\ ,$$

$$M_n^4(t)=\left.\frac{\partial^n P}{\partial X^n}\right|_{\phi=\phi_0,\,X=\dot{\phi}_0^2}\qquad (n\geq 2)$$

Examples

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{M_3^4}{6}(\delta g^{00})^3 + \ldots\right)$$

"Galilean Cosmology" (Chow and Khoury, 2009)

$$S = \int d^4x \sqrt{-g}\left[\frac{M^2}{2} e^{-2\phi/M} R - \frac{r_c^2}{M}(\partial\phi)^2 \Box\phi\right]$$

$$f(t) = e^{-2\frac{\phi_0}{M}} \quad , \quad \Lambda(t) = -\frac{r_c^2}{M}\dot{\phi}_0^2(\ddot{\phi}_0 + 3H\dot{\phi}_0)\ , \quad c(t) = \frac{r_c^2}{M}\dot{\phi}_0^2(\ddot{\phi}_0 - 3H\dot{\phi}_0)\ ,$$

$$M_2^4(t) = -\frac{r_c^2}{2M}\dot{\phi}_0^2(\ddot{\phi}_0 + 3H\dot{\phi}_0)\ , \quad M_3^4(t) = -\frac{3r_c^2}{4M}\dot{\phi}_0^2(\ddot{\phi}_0 + H\dot{\phi}_0)\ , \quad \bar{m}_1^3(t) = -\frac{r_c^2}{M} 2\dot{\phi}_0^3\ ,$$

More examples, more operators...

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K \ + \frac{M_3^4}{6} (\delta g^{00})^3 + \dots \right)$$

"Generalized Galileons" (≡ Horndeski)

(Deffayet et al., 2011)

$$\mathcal{L}_2 = A(\phi, X) \ ,$$

$$\mathcal{L}_3 = B(\phi, X) \Box\phi \ ,$$

$$\mathcal{L}_4 = C(\phi, X) R - 2 C_{,X}(\phi, X) \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] \ ,$$

$$\mathcal{L}_5 = D(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} D_{,X}(\phi, X) \left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right] \ ,$$

Gleyzes, Langlois, F.P., Vernizzi, 1304.4840

More examples, more operators...

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K \ + \frac{M_3^4}{6} (\delta g^{00})^3 + \dots \right)$$

Only 1 more quadratic operator!

“Generalized Galileons” (≡ Horndeski)

(Deffayet et al., 2011)

$$\mathcal{L}_2 = A(\phi, X) \ ,$$

$$\mathcal{L}_3 = B(\phi, X) \Box \phi \ ,$$

$$\mathcal{L}_4 = C(\phi, X) R - 2 C_{,X}(\phi, X) \left[(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] \ ,$$

$$\mathcal{L}_5 = D(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} D_{,X}(\phi, X) \left[(\Box \phi)^3 - 3 (\Box \phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right] \ ,$$

The Power of EFT of DE

- Completely democratic (quintessence and modified gravity on same foot)
- Mixing with gravity studied systematically

Mixing with gravity:

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply "Stueckelberg trick"
and go to Newtonian Gauge

$$t \to t + \pi(x)$$

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

Expand at quadratic order and retain only kinetic operators (2 derivatives):

$$\dot{\Psi}^2\,, \quad (\vec{\nabla}\Psi)^2\,, \quad \text{etc.}$$

Modified Gravity $\approx$ Kinetic mixing $\quad \dot{\Psi}\dot{\pi}\,, \quad \vec{\nabla}\Psi\vec{\nabla}\pi\,, \quad \text{etc.}$

Mixing with gravity:

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

Apply "Stueckelberg trick" and go to Newtonian Gauge

$$t \to t + \pi(x)$$

$$ds^2 = -(1+2\Phi) dt^2 + a^2 (1 - 2\Psi) \delta_{ij} dx^i dx^j$$

Expand at quadratic order and retain only kinetic operators (2 derivatives):

$$\dot{\Psi}^2 \,, \quad (\vec{\nabla}\Psi)^2 \,, \quad \text{etc.}$$

Modified Gravity $\approx$ Kinetic mixing $\quad \dot{\Psi}\dot{\pi} \,, \quad \vec{\nabla}\Psi \vec{\nabla}\pi \,, \quad \text{etc.}$

Ex: $\quad \delta g^{00} \delta K \simeq \dot{\Psi}\dot{\pi} + \nabla\Phi\nabla\pi$ $\Rightarrow$ Anisotropic stress, Renormalized Newton Constant etc.

The Power of EFT of DE

- Completely democratic (quintessence and modified gravity on same foot)
- Mixing with gravity studied systematically
- Stability, speed of sound etc.

Model building v.s. General treatment (with Gubitosi, Piazza, 2012)

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar m_1^3}{2}\delta g^{00}\delta K \ + \frac{M_3^4}{6}(\delta g^{00})^3 + \dots\right)$$

Find, once and for all, the action for the scalar degree of freedom:

$$S_\pi \overset{\text{kinetic}}{=} \int a^3 \Bigg\{ \left[c + 2M_2^4 + \frac{3}{4}\frac{\dot f^2}{f} M^2 - \frac{3}{2}\bar m_1^3 \frac{\dot f}{f} + \frac{3}{4}\frac{\bar m_1^6}{M^2}\right]\dot\pi^2$$
$$- \left[c + \frac{3}{4}\frac{\dot f^2}{f} M^2 - \frac{1}{2}\bar m_1^3 \frac{\dot f}{f} - \frac{1}{4}\frac{\bar m_1^6}{M^2} + \frac{1}{2}\left(\dot{\bar m}_1^3 + H \bar m_1^3\right)\right]\frac{(\vec\nabla \pi)^2}{a^2}\Bigg\}$$

And address, once and for all, all questions of stability, speed of sound and deviations from GR:

$$1 - \gamma = \frac{1}{2}\frac{(M^2 \dot f^2 + \bar m_1^3 \dot f)/f}{c + M^2 \dot f^2/f + \frac{1}{2}(\dot{\bar m}_1^3 + H \bar m_1^3)}$$

$$G_{\rm eff} = \frac{1}{8\pi M^2 f}\frac{c + M^2 \dot f^2/f + \frac{1}{2}(\dot{\bar m}_1^3 + H\bar m_1^3)}{c + \frac{3}{4}M^2 \dot f^2/f - \frac{1}{2}\bar m_1^3 \dot f/f - \frac{1}{4}\bar m_1^6/M^2 + \frac{1}{2}(\dot{\bar m}_1^3 + H \bar m_1^3)}$$

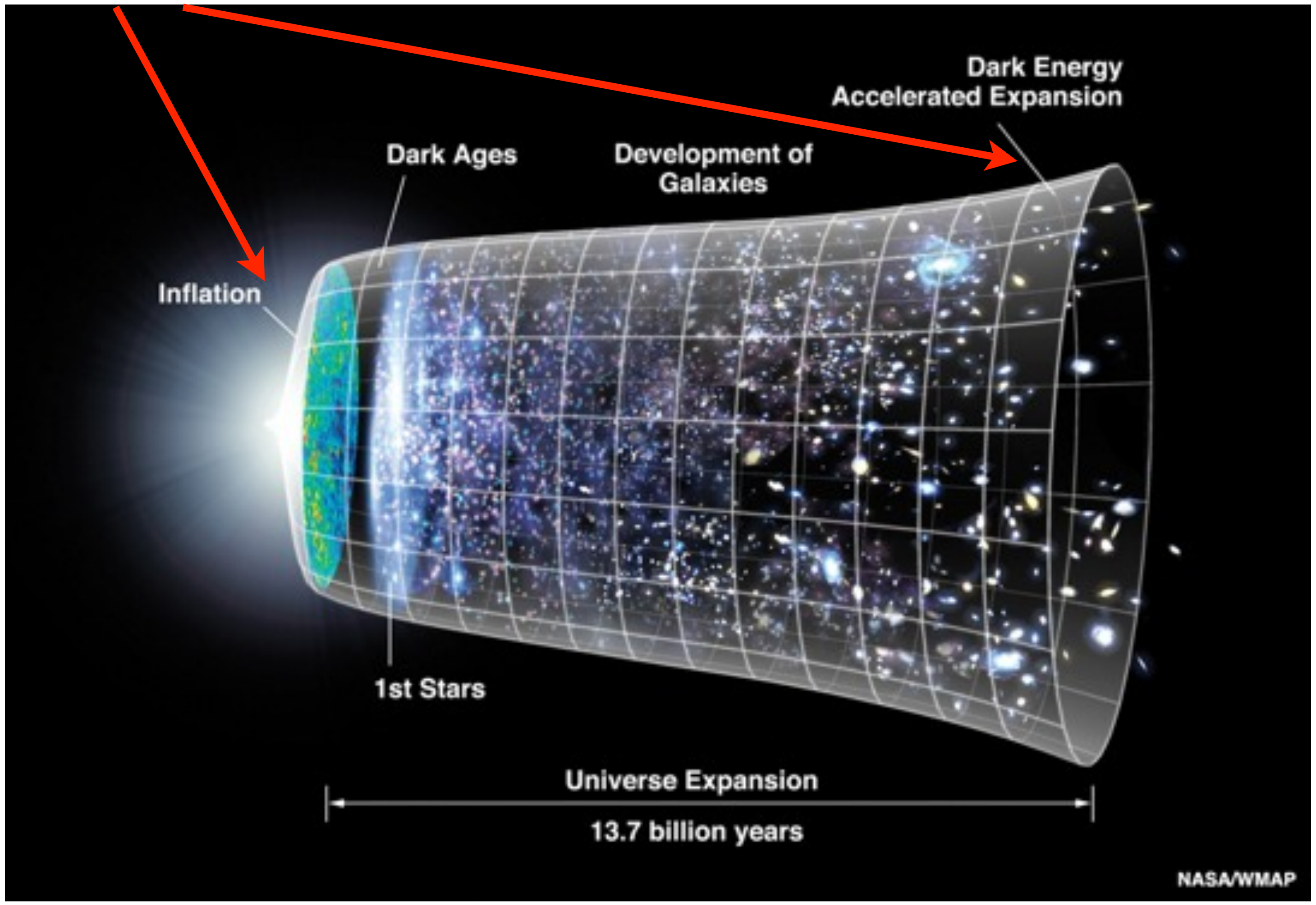
Conclusions

- Unifying framework for dark energy/modified gravity
- Effective language: cosmological perturbations as the relevant d.o.f.
- Dark energy and modified gravity on the same foot
- Unambiguous way to address mixing, stability, speed of sound etc.
- See also Bloomfield et al. 1211.7054. Much work in progress

A venturesome path

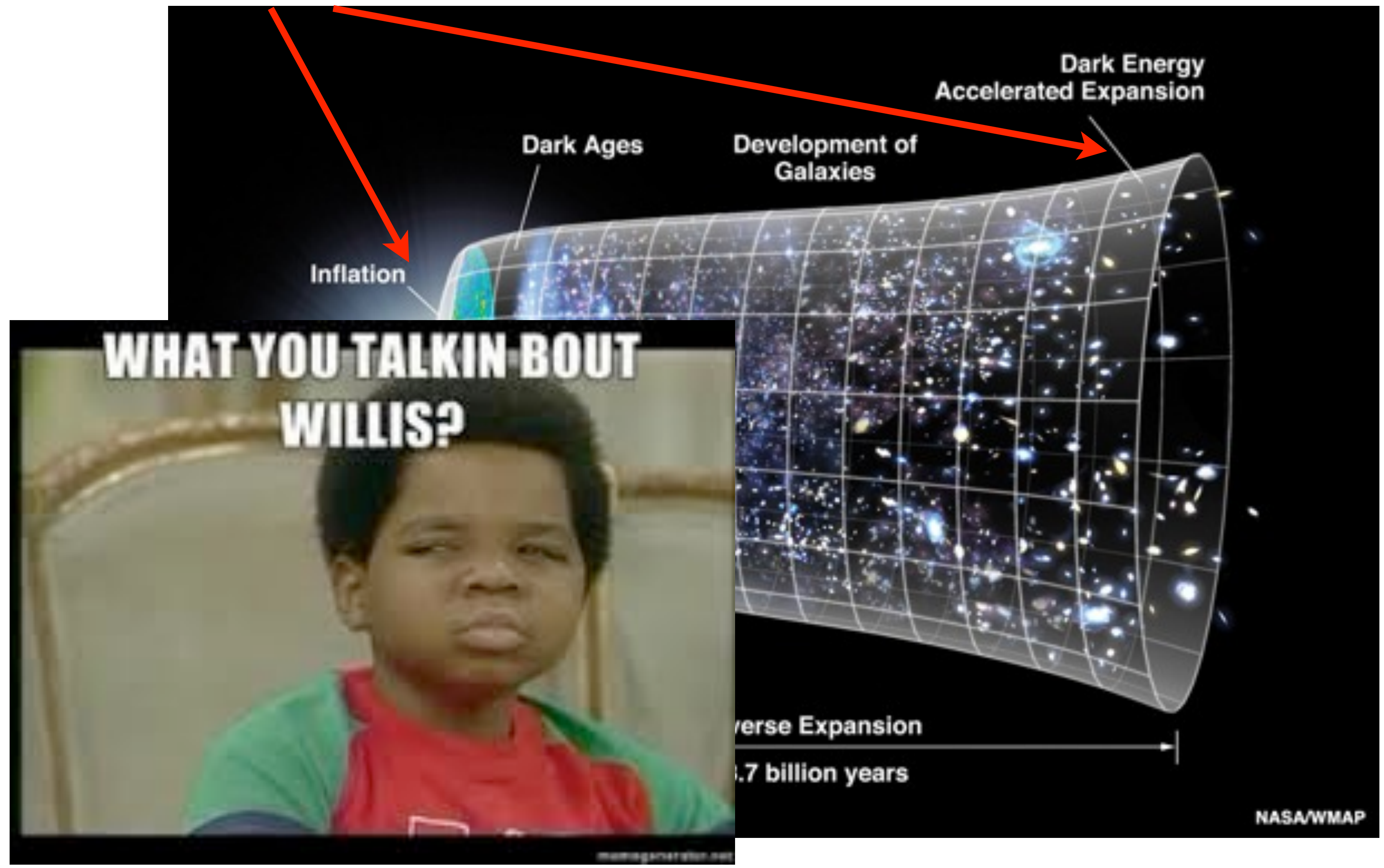
A venturesome path

The Universe has accelerated... twice!



A venturesome path

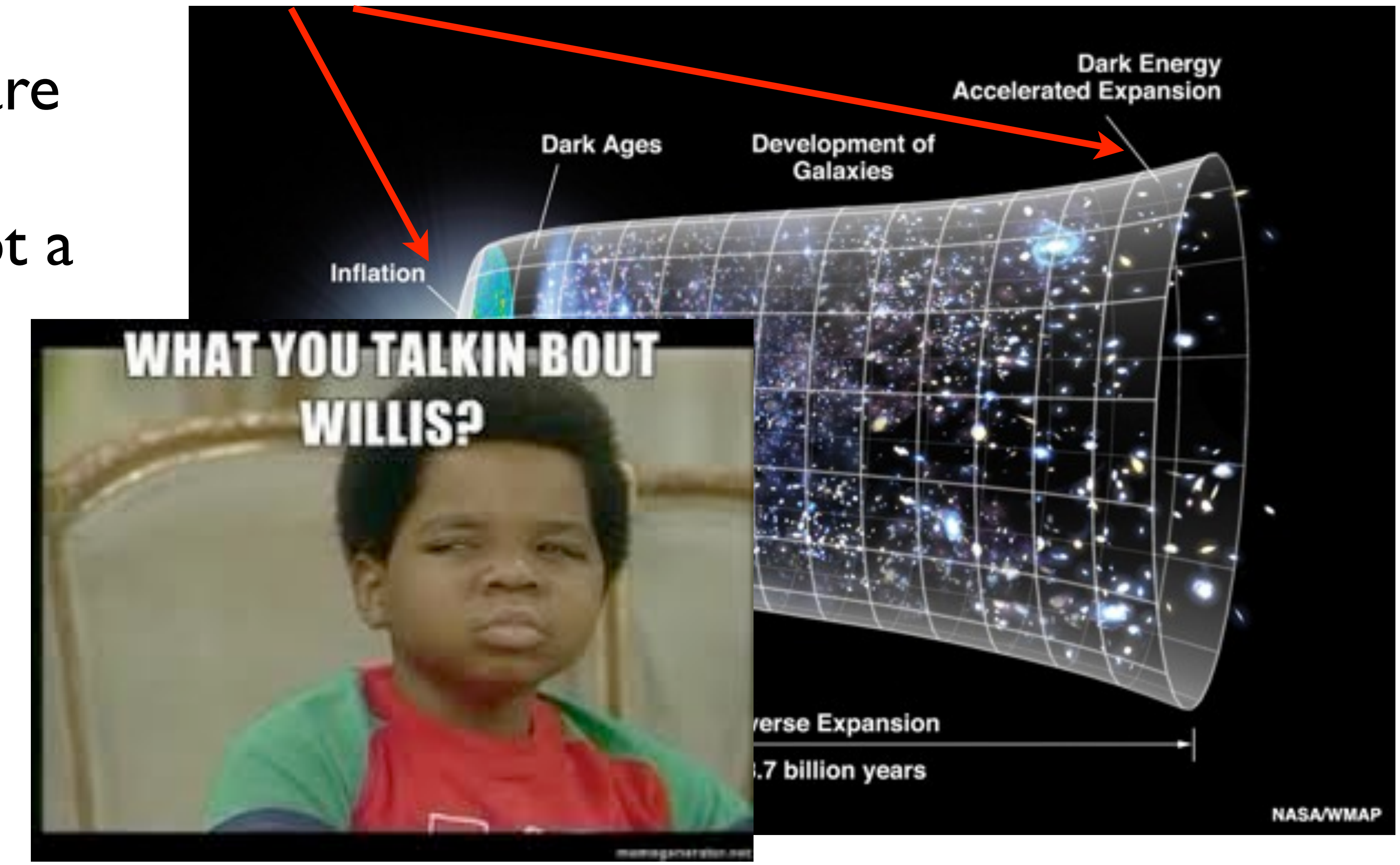
The Universe has accelerated… twice!



A venturesome path

The Universe has accelerated... twice!

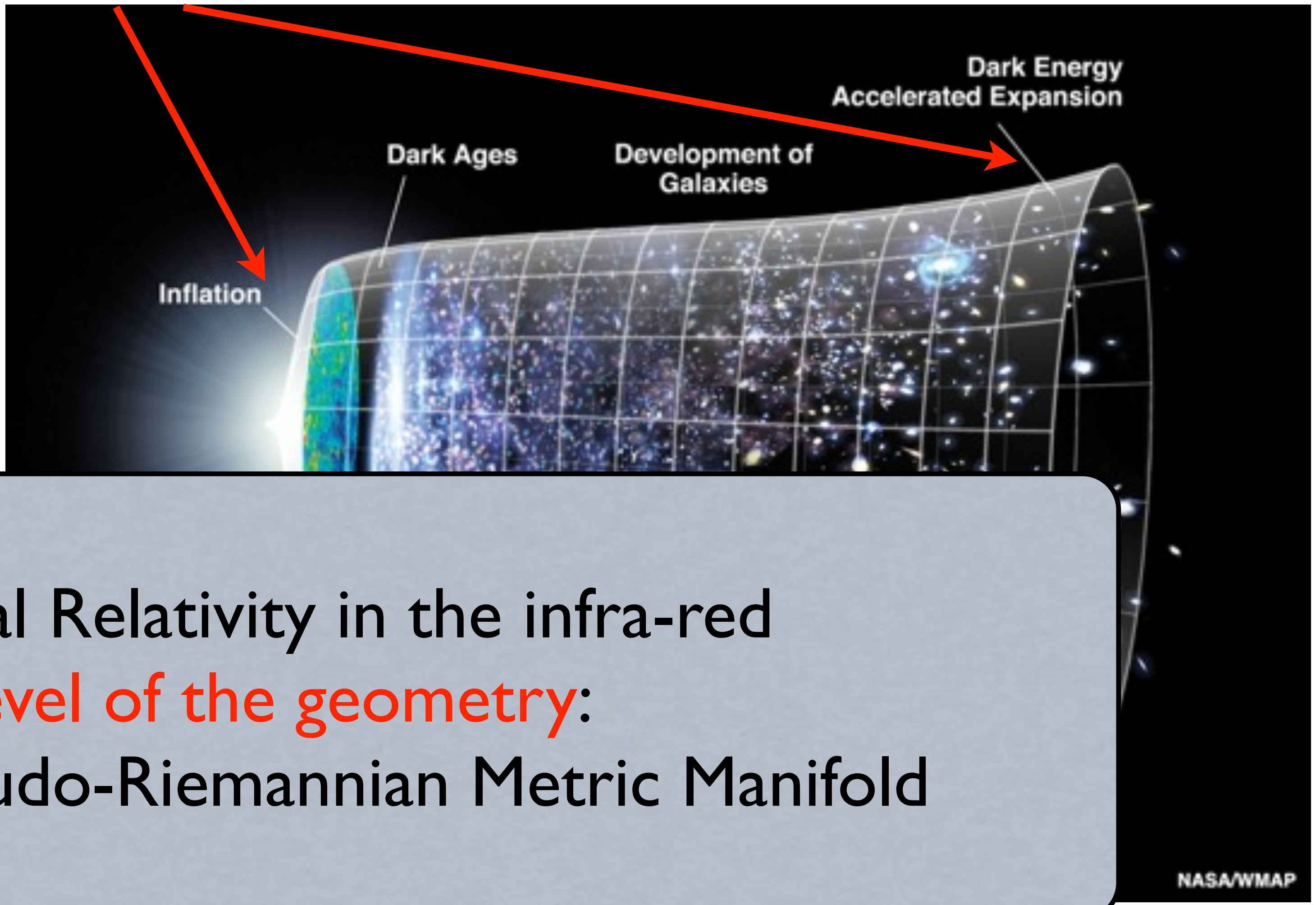
Both DE and Inflation are (at face value) Infra-red issues. Can't we attempt a Unifying description?



A venturesome path

The Universe has accelerated... twice!

Both DE and Inflation are (at face value) Infra-red issues. Can't we attempt a Unifying description?



Modify General Relativity in the infra-red
at the level of the geometry:
Go beyond the Pseudo-Riemannian Metric Manifold

0904.4299 (JMPA)

0907.0765 (NJP, "best of 2009")

0910.3949 (PLB, with S. Nesseris and S. Tsujikawa)

0910.4677, 1204.4099, in progress...

Rest of the talk: not concerned about this. Unifying approach for DE and more conventional modifications of gravity.

Mixing with gravity 1: Brans-Dicke

$$S=\int\sqrt{-g}\left(\frac{M^2}{2}fR-\Lambda-cg^{00}+\frac{M_2^4}{2}(\delta g^{00})^2-\frac{\bar{m}_1^3}{2}\delta g^{00}\delta K+\frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2=-(1+2\Phi)dt^2+a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S\stackrel{\text{kinetic}}{=}\int M^2 f\left[-3\dot{\Psi}^2-2\vec{\nabla}\Phi\vec{\nabla}\Psi+(\vec{\nabla}\Psi)^2+c\,\dot{\pi}^2-c(\vec{\nabla}\pi)^2+3(\dot{f}/f)\dot{\Psi}\dot{\pi}+(\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi-2\vec{\nabla}\Psi)\right]$$

Mixing

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2 f \left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 + c\,\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 + 3(\dot{f}/f)\dot{\Psi}\dot{\pi} + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi)\right]$$

Mixing

De-mixing = conformal transformation

$$\Phi_E = \Phi + \frac{1}{2}(\dot{f}/f)\pi$$

$$\Psi_E = \Psi - \frac{1}{2}(\dot{f}/f)\pi$$

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S^{\text{kinetic}} = \int M^2 f\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 + c\,\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 + 3(\dot{f}/f)\dot{\Psi}\dot{\pi} + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi)\right]$$

Newtonian limit $\longrightarrow$

$$1 - \gamma \equiv \frac{\Phi - \Psi}{\Phi} = \frac{M^2\dot{f}^2/f}{2(c + M^2\dot{f}^2/f)}$$

anisotropic stress

$$G_{\text{eff}} = \frac{1}{8\pi M^2 f}\,\frac{c + M^2\dot{f}^2/f}{c + \frac{3}{4}M^2\dot{f}^2/f}$$

dressed Newton constant

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} fR - \Lambda - cg^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} \right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2 \left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 \right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

Mixing

De-mixing ≠ conformal transformation

$$\Phi_E = \Phi + \frac{\bar{m}_1^3}{2M^2}\pi$$

$$\Psi_E = \Psi + \frac{\bar{m}_1^3}{2M^2}\pi$$

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

Mixing

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2} T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

Mixing

Speed of Sound of DE

$$c_s^2 = \frac{c + \frac{1}{2}(H\bar{m}_1^3 + \dot{\bar{m}}_1^3) - \frac{1}{4}\bar{m}_1^6/M^2}{c + \frac{3}{4}\bar{m}_1^6/M^2}$$

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} fR - \Lambda - cg^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

$$1 - \gamma = \frac{\Phi - \Psi}{\Phi} = 0$$

NO anisotropic stress

Newtonian limit $\longrightarrow$

$$G_{\rm eff} = \frac{1}{8\pi M^2 f}\left(1 - \frac{\bar{m}_1^3}{4cM^2}\right)^{-1}$$

dressed Newton constant

Detailed model building v.s. general treatments

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K \ + \frac{M_3^4}{6} (\delta g^{00})^3 + \ldots \right)$$

More interesting operators to come...

(Gleyzes, Langlois, F.P., Vernizzi, in progress)

Conclusions

- Unifying framework for dark energy/modified gravity
- Effective language: cosmological perturbations as the relevant d.o.f.
- Dark energy and modified gravity on the same foot
- Unambiguous way to address mixing, stability, speed of sound etc.
- See also Bloomfield et al. 1211.7054. Much work in progress