Mass Hierarchy discrimination: toy MC studies

A.Meregaglia (IPHC Strasbourg)

ORCA Workshop, Paris 17-18 April 2013

Introduction

- In this talk I will mostly describe the results of the paper:

  Mass hierarchy discrimination with atmospheric neutrinos in large volume ice/water Cherenkov detectors

  by

  D.Franco, C.Jollet, A.Kouchner, V.Kulikovskiy, A.Meregaglia, S.Perasso, T.Pradier, A.Tonazzo and V.Van Elewyck

  JHEP04(2013)008

- At the end additional results are shown using a specific possible ORCA effective mass profile.

Statistical method

Statistical method (1)

- To asses the mass hierarchy discrimination potential of the experiment, we used an extended unbinned log-likelihood ratio approach.
- To compute the likelihood we fixed the true hypothesis (either normal (NH) or inverted (IH) hierarchy) and we generated 1000 test experiments.
- Each test experiment is compared on event by event basis with the two model hypotheses (NH or IH) and the extended unbinned likelihood $\boldsymbol{L_j}$ (with j = NH or IH according to the model) is computed:

$$L_j = \frac{(e^{-\mu_j}\mu_j^n)}{n!} \times \prod_{i=1}^{n} \mathrm{pdf}_j(E_i, \theta_i)$$

Statistical method (1)

- To asses the mass hierarchy discrimination potential of the experiment, we used an extended unbinned log-likelihood ratio approach.
- To compute the likelihood we fixed the true hypothesis (either normal (NH) or inverted (IH) hierarchy) and we generated 1000 test experiments.
- Each test experiment is compared on event by event basis with the two model hypotheses (NH or IH) and the extended unbinned likelihood L_j (with j = NH or IH according to the model) is computed:

expected number of events in the model (j = NH or IH)

$$L_j = \frac{(e^{-\mu_j}\mu_j^n)}{n!} \times \prod_{i=1}^{n} \mathrm{pdf}_j(E_i, \theta_i)$$

number of observed events in the test experiment

probability of observing the i^{th} event with energy E_i and zenith angle θ_i

pdf depends on the model (j = NH or IH) and is obtained with high statistics MC simulations (1000 times the expected statistics)

Statistical method (2)

- The test statistics η is the logarithm of the extended unbinned likelihood ratio :

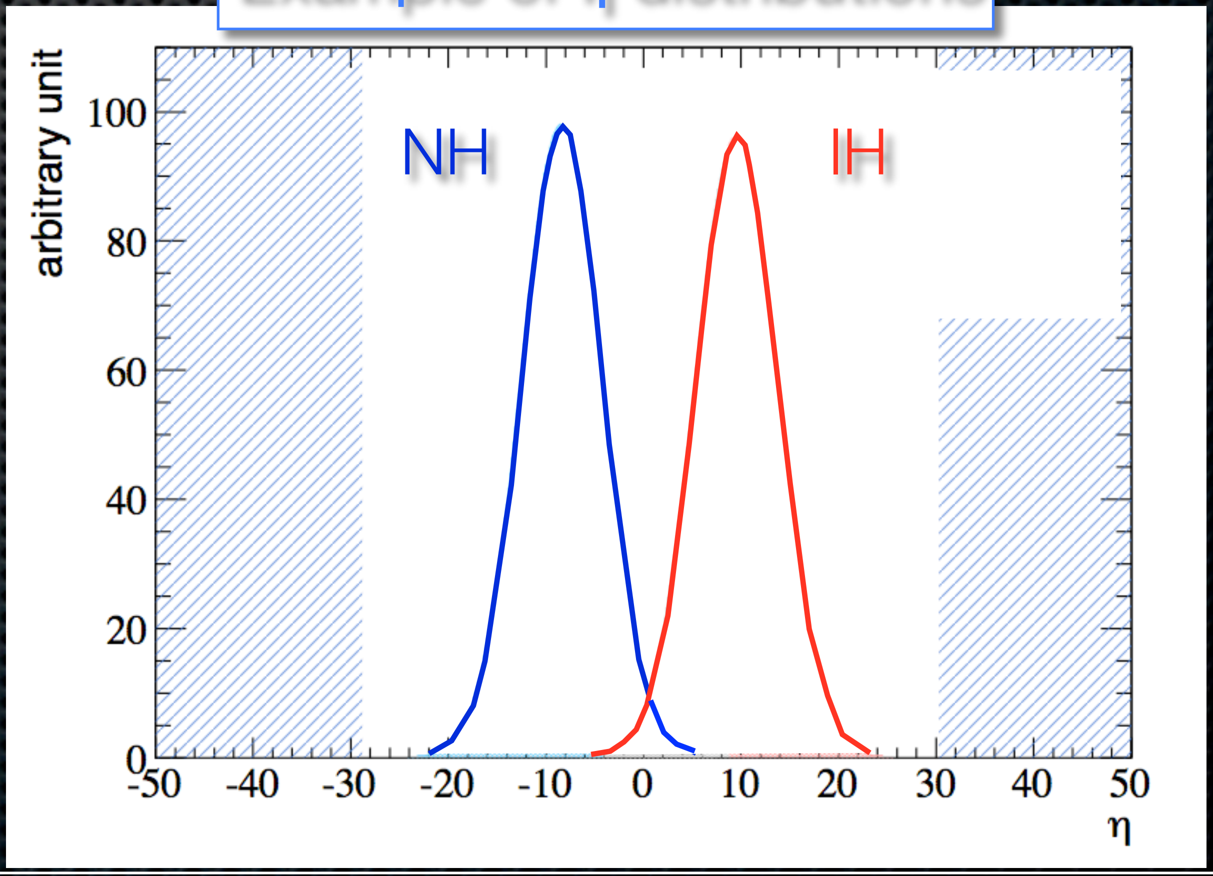
$$\begin{aligned}\eta &= \log(L_{\mathrm{IH}}/L_{\mathrm{NH}}) \\ &= -(\mu_{\mathrm{IH}} - \mu_{\mathrm{NH}}) + n\log(\mu_{\mathrm{IH}}/\mu_{\mathrm{NH}}) \\ &\quad + \sum_i \log(\mathrm{pdf}_{\mathrm{IH}}(E_i,\theta_i)/\mathrm{pdf}_{\mathrm{NH}}(E_i,\theta_i))\end{aligned}$$

Statistical method (2)

- The test statistics η is the logarithm of the extended unbinned likelihood ratio :

$$\begin{aligned}\eta &= \log(L_{\mathrm{IH}}/L_{\mathrm{NH}}) \\ &= -(\mu_{\mathrm{IH}} - \mu_{\mathrm{NH}}) + n\log(\mu_{\mathrm{IH}}/\mu_{\mathrm{NH}}) \\ &\quad + \sum_i \log(\mathrm{pdf}_{\mathrm{IH}}(E_i,\theta_i)/\mathrm{pdf}_{\mathrm{NH}}(E_i,\theta_i))\end{aligned}$$

Example of η distributions



Statistical method (2)

- The test statistics η is the logarithm of the extended unbinned likelihood ratio :

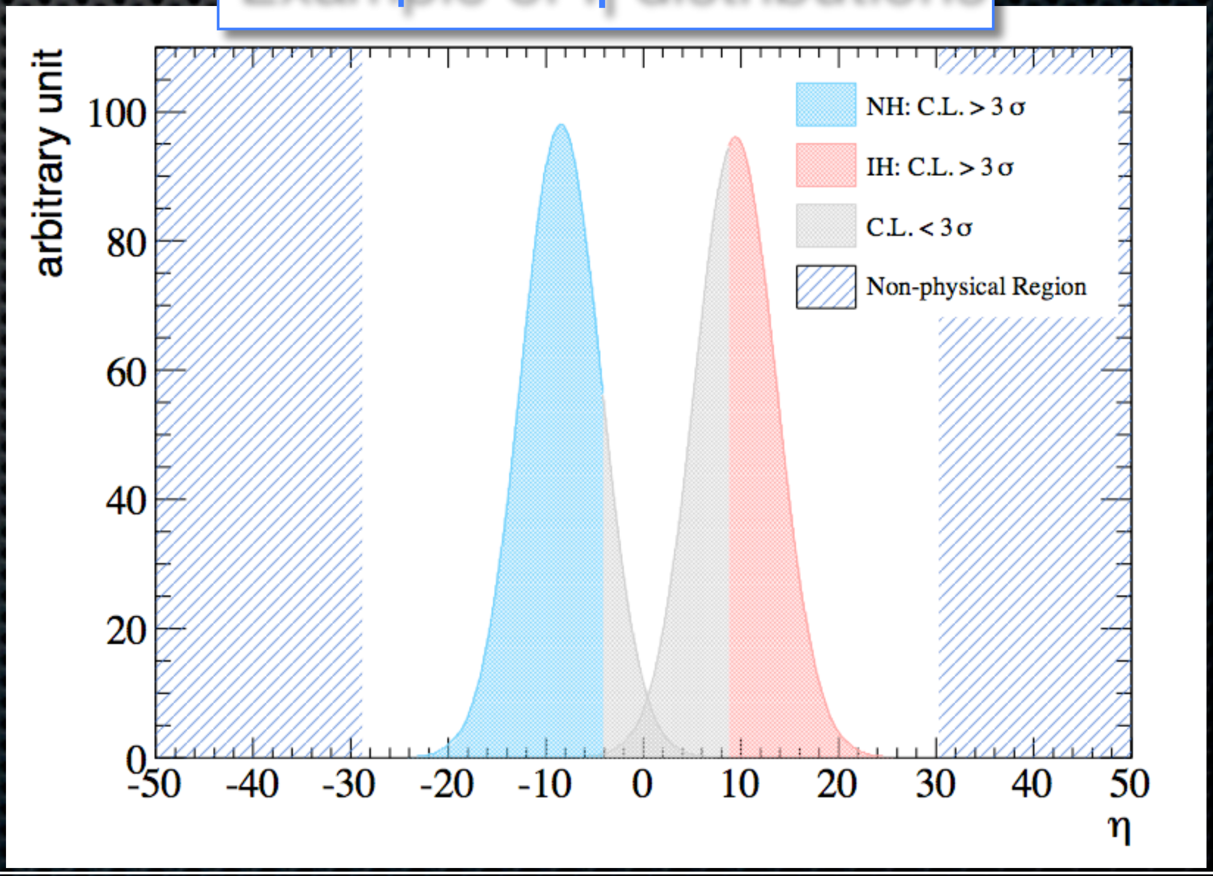
$$\eta = \log(L_{\rm IH}/L_{\rm NH}) = -(\mu_{\rm IH} - \mu_{\rm NH}) + n\log(\mu_{\rm IH}/\mu_{\rm NH}) + \sum_i \log(\mathrm{pdf}_{\rm IH}(E_i,\theta_i)/\mathrm{pdf}_{\rm NH}(E_i,\theta_i))$$

- The **p-value** i.e. the probability of reaching a definite C.L. is defined as the fraction of events satisfying:

$$\frac{N_t(\eta)}{N_{\rm NH}(\eta)+N_{\rm IH}(\eta)} > \alpha$$

where t is the true hypothesis and it corresponds to **either** NH **or** IH (e.g. for C.L of 3 σ, α = 0.9973002).

Example of η distributions



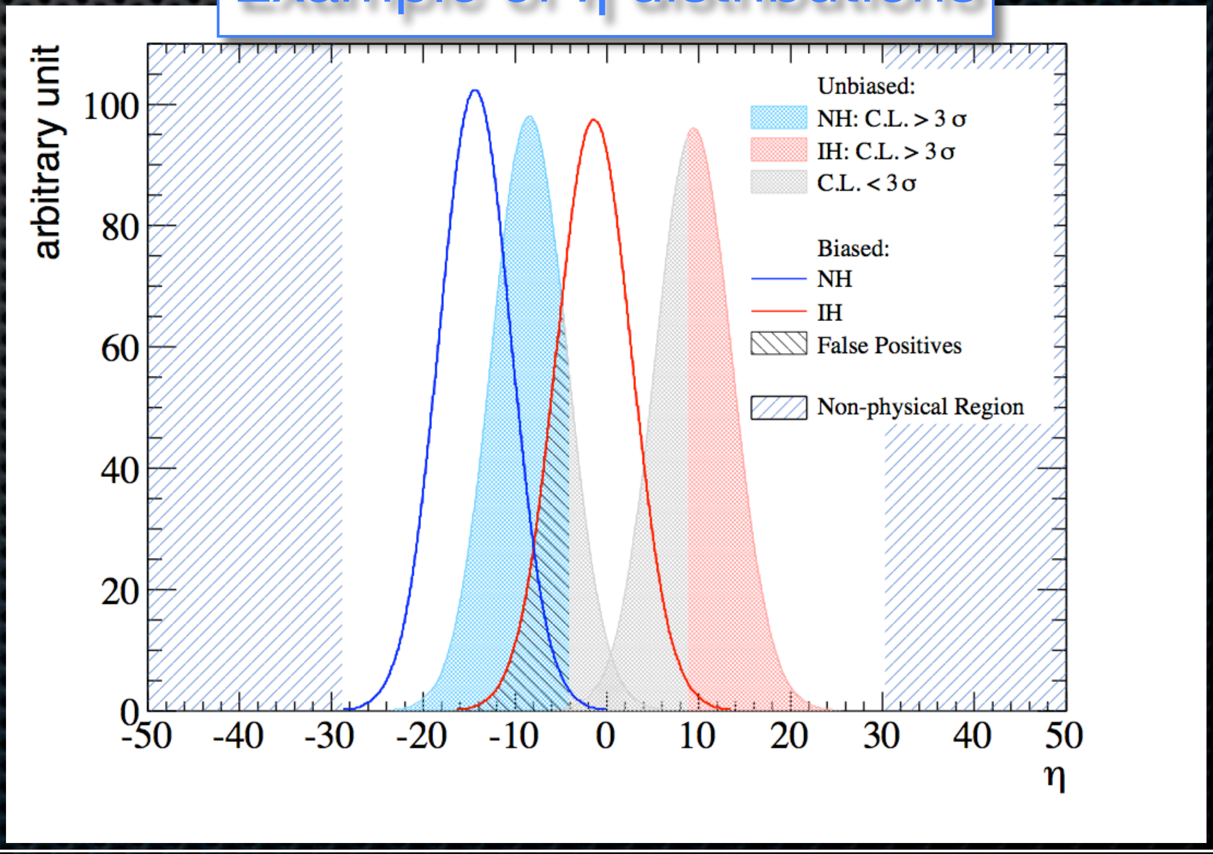
Statistical method (3)

- A way to introduce the systematics is assuming that the true and the model hypotheses have different parameters (exactly what happens in real life when we compute the likelihood ratio starting from the measured data and using the “best fit” parameters for the model).

- This introduces shifts in our distributions which lead to false positive results (the wrong hierarchy discovery is claimed) or unphysical ones (the results are at more than 5 σ from the expected Gaussians).

- The p-value is computed in this case subtracting the unphysical results and the false positive ones.

Example of η distributions



Toy MC

Toy MC: basic ingredients

- To setup a full MC chain the following ingredients are needed:

Neutrino Fluxes

Oscillation Probabilities

Earth density profile

Neutrino cross sections

Detector specific information on the event reconstruction

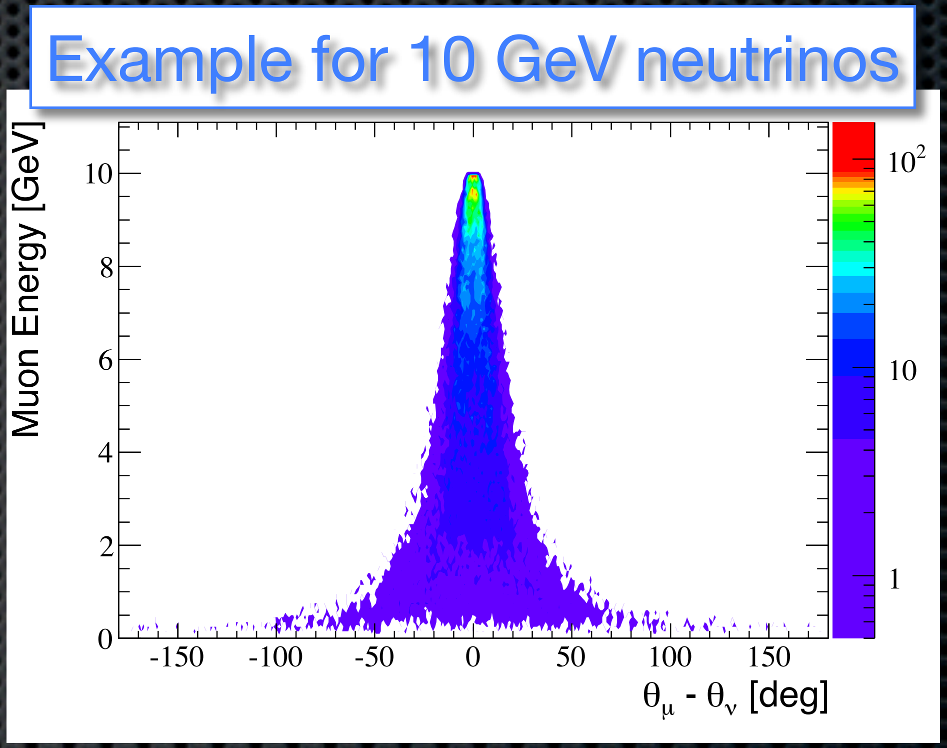
Toy MC: basic ingredients

- To setup a full MC chain the following ingredients are needed:

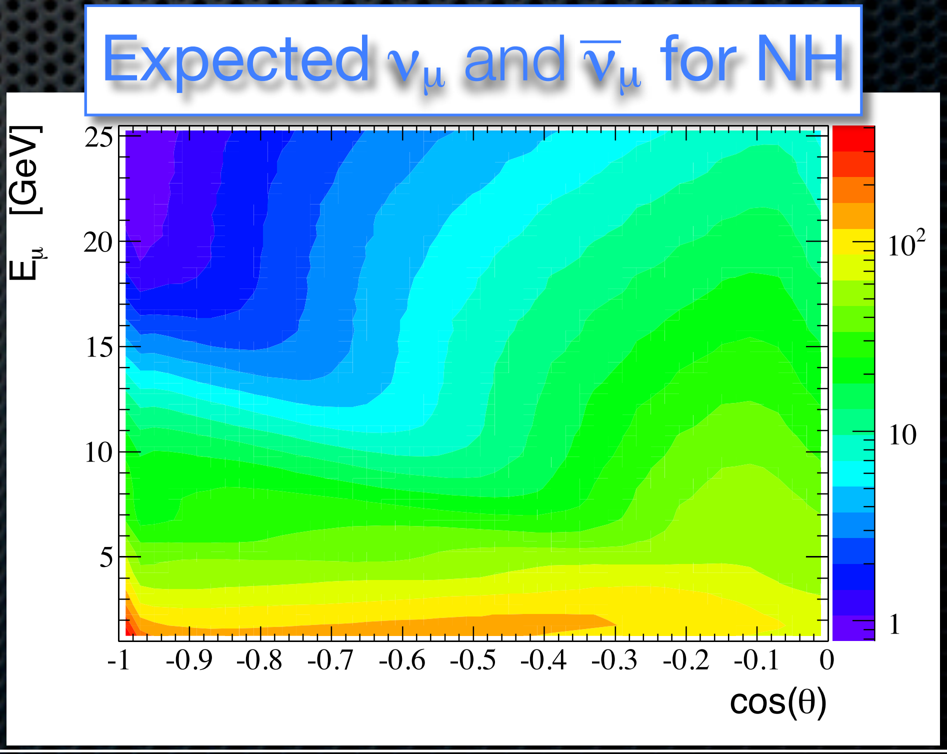
Neutrino Fluxes	→	Honda as base option - comparison with FLUKA and Bartol.
Oscillation Probabilities	→	GLoBES
Earth density profile	→	PREM (in GLoBES) - 1000 steps per baseline - 50 baselines (steps of 0.02 in the zenith angle θ)
Neutrino cross sections	→	GLoBES
Detector specific information on the event reconstruction	→	Muon energy reconstruction only Energy threshold at 5 GeV

Toy MC: kinematics

- Since we use muon reconstructed energy and angle, the correct kinematics has to be take into account.
- Neutrino interactions are simulated with GENIE.



- Including this in the toy MC, we have the full chain to compute the number of expected ν_μ and $\overline{\nu}_\mu$ for a given mass hierarchy in each energy and azimuthal angle bin, for a defined mass (1 Mt in our case).



Asymmetry

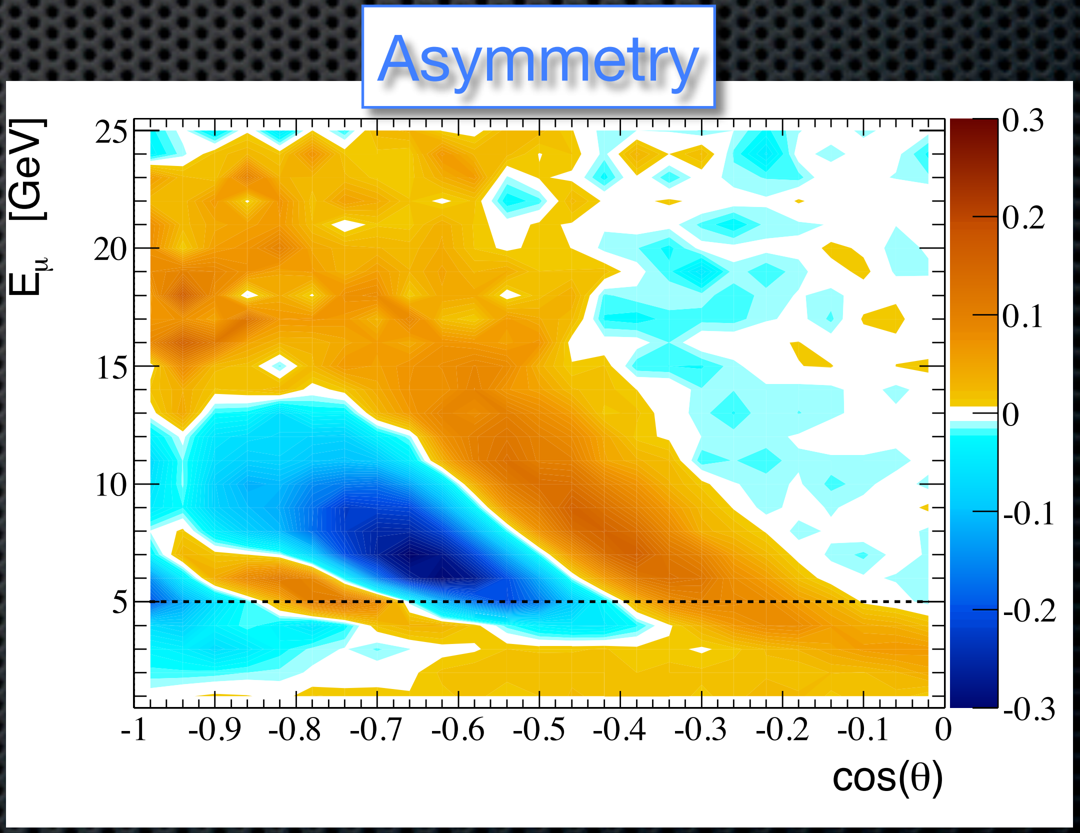
- Simply looking at the differences in the matrixes generated in case of NH and IH it is possible to identify the region where the effect is more observable.

- We defined the asymmetry as:

$$2 \times \frac{M_{\mathrm{NH}} - M_{\mathrm{IH}}}{M_{\mathrm{NH}} + M_{\mathrm{IH}}}$$

where M_{NH} and M_{IH} are the number of expected events in each energy/angle bin for each mass hierarchy.

- The region between 5 and 10 GeV is the one where the effect is more evident.



Oscillation parameters (1)

- We used the following values for the reference oscillation parameters:

Parameter	Value
Δm^2_{21}	$(7.58^{+0.22}_{-0.26})\times 10^{-5}$ eV^2
Δm^2_{31}(NH)	$(2.45 \pm 0.09)\times 10^{-3}$ eV^2
Δm^2_{31}(IH)	0.13×10^{-3} eV^2 - Δm^2_{31}(NH)
$\sin^2(2\theta_{12})$	$0.849\ ^{+0.071}_{-0.059}$
$\sin^2(2\theta_{13})$	0.096 ± 0.013
$\sin^2(2\theta_{23})$	$0.974^{+0.026}_{-0.032}$

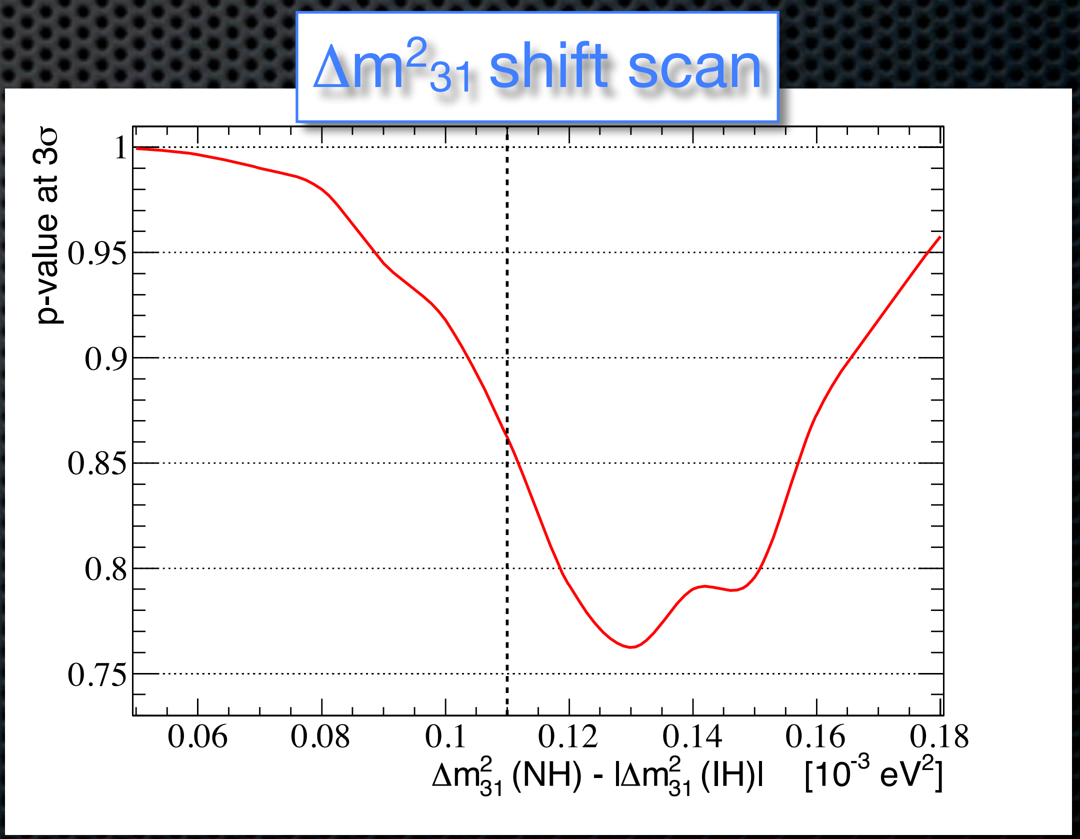
- Although most of the values used are just the best fit taken from the PGD or other papers, a specific discussion is needed for some of them:
  - δ_{cp} has ben set to 0.
  - θ_{23} has ben chosen in the preferred first octant. Test in the second octant showed the robustness of our results under this respect.
  - The choice of the value of Δm^2_{31} in IH is made in order to be sensible and conservative at the same time (see next slide).

Oscillation parameters (2)

- The difference between the values of Δm^2_{31} for NH and IH can be computed analytically as (JHEP08(2012)058):

$$\begin{aligned}\delta m^2_{31} &= \Delta m^2_{31}(\mathrm{NH}) - |\Delta m^2_{31}(\mathrm{IH})| \\ &= 2\Delta m^2_{21}(\cos^2\theta_{12} - \cos\delta_{CP}\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23})\end{aligned}$$

- The dependence on the unknown δ_{cp} makes however the value non univocally assigned.
- Standard fit studies leave the value of Δm^2_{31} as a free parameter overcoming this problem.
- This approach is not feasible in our toy MC therefore a scan is performed.
- The result corresponding to the minimum p-value, not far form the global best fit (NewJ.Phys.13(2011)109401), is conservatively used.
- The effect is almost negligible once the exposure is large enough and the p-value ~ 1.

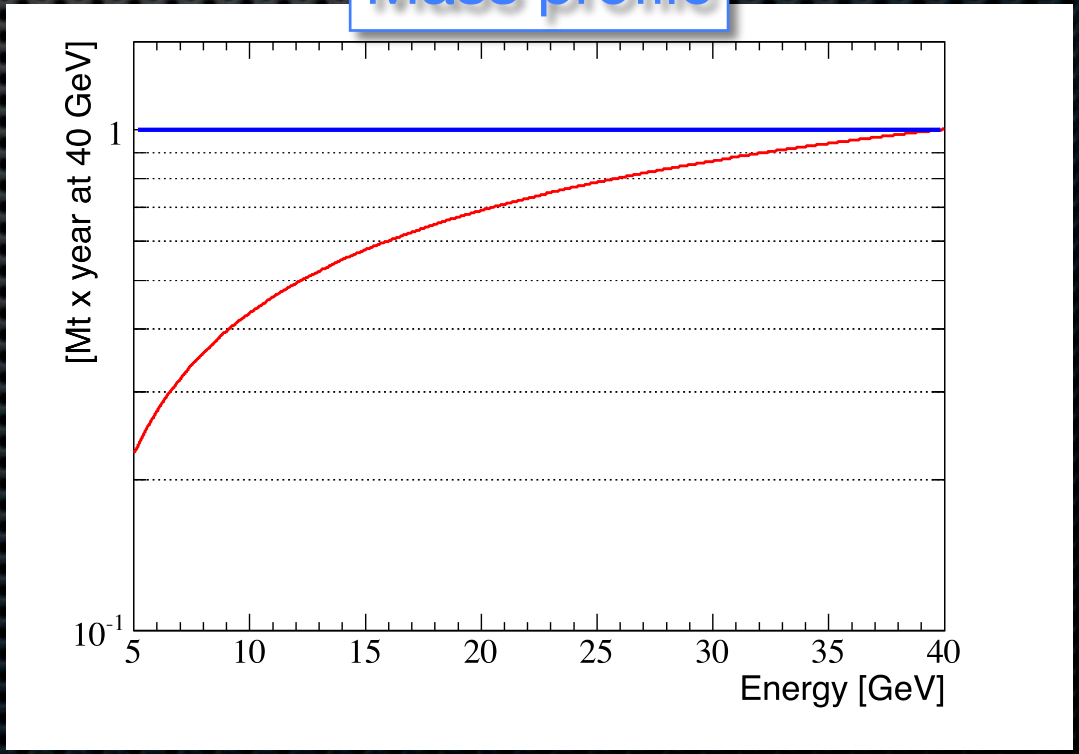


Results

Ideal case (1)

- We first studied the p-value as a function of the exposure for ideal conditions (perfect angular and energy resolution).
- No parameters systematics (identical oscillation parameters for test and model hypotheses).
- The neutrino interaction kinematics is however included as explained before.
- We considered two different mass profiles: flat and energy dependent (PINGU-like (arXiv:1205.7071)).
- The effective exposure was computed fixing for each profile 1 Mt at 40 GeV.

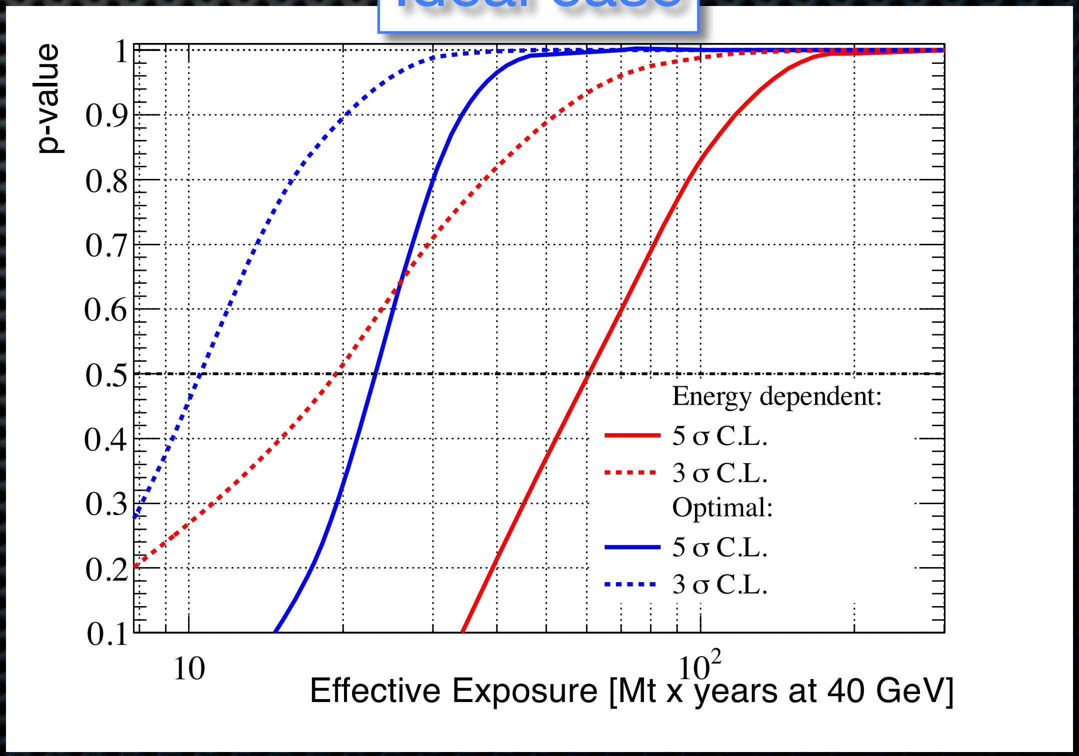
Mass profile



Ideal case (2)

- The results obtained can be interpreted as the comparison between two different mass profiles with the same normalization.
- To transform them in exposure [years] they have to be divided by the real effective mass at 40 GeV.
- This means that a “less favourable” profile could still yield better results if its overall normalization is larger enough.
- If the “acceptable threshold” is set at a p-value of 0.5, 60 Mt × year [at 40 GeV] are needed at 5 σ C.L. for the energy dependent profile.
- The flat profile reduces the needed exposure by about a factor of 3.
- In the following **an exposure of 170 Mt x years** [at 40 GeV] (p-value ~ 1 at 5 σ C.L.) **is assumed**.

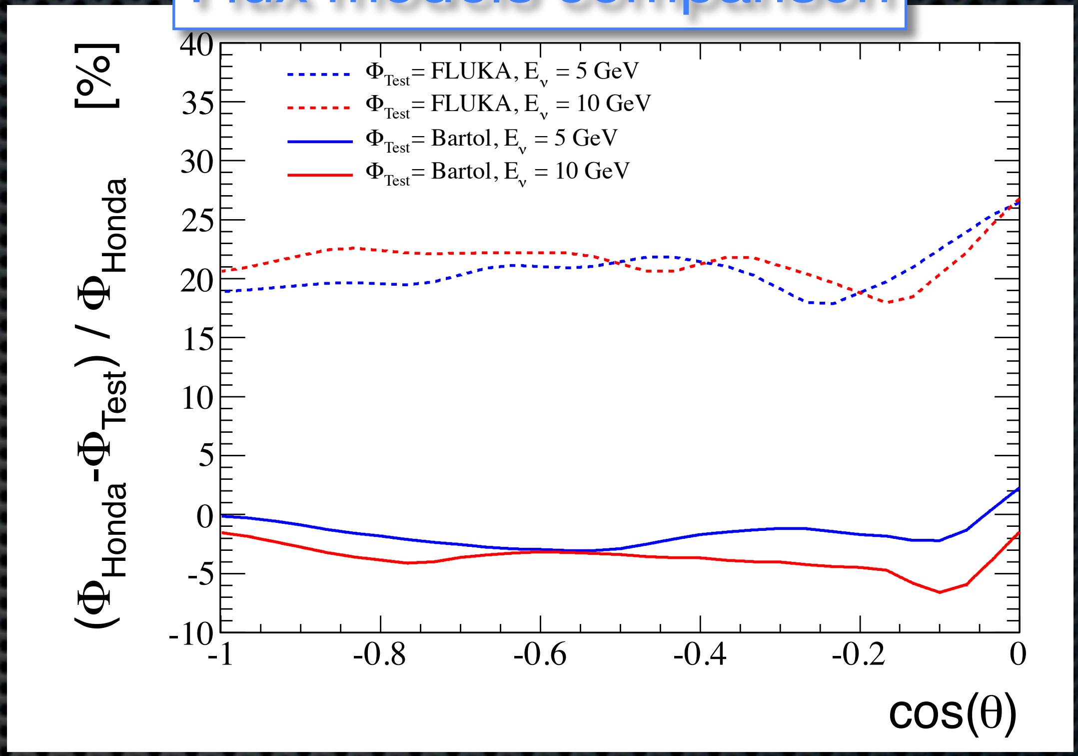
Ideal case



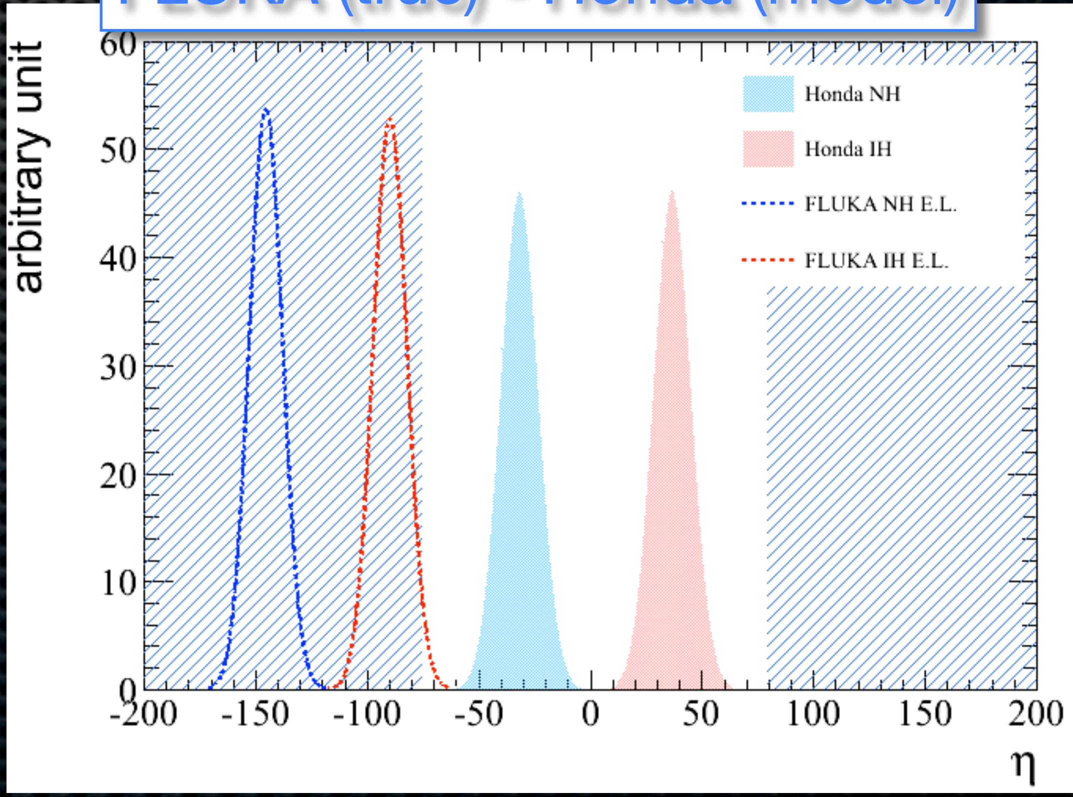
Flux models

- We studied the impact of different flux models namely FLUKA, Bartol and Honda.
- Although the agreement in shape above 5 GeV is at the level of 5%, the normalization of the FLUKA model differs by more than 20%.
- Assuming the Honda flux as model and the FLUKA one as true hypothesis as explained in the statistical method description, the p-value is reduced to almost 0.
- This is mainly due to the fact that we use an extended likelihood (constraint on the normalization).

Flux models comparison



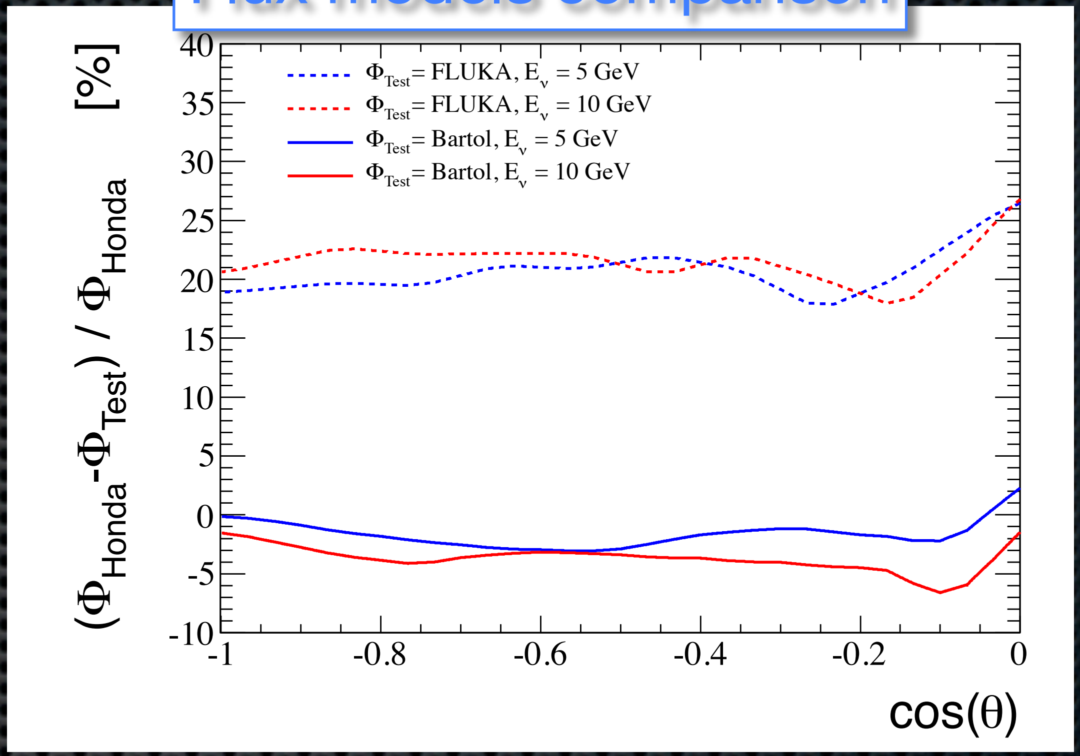
FLUKA (true) - Honda (model)



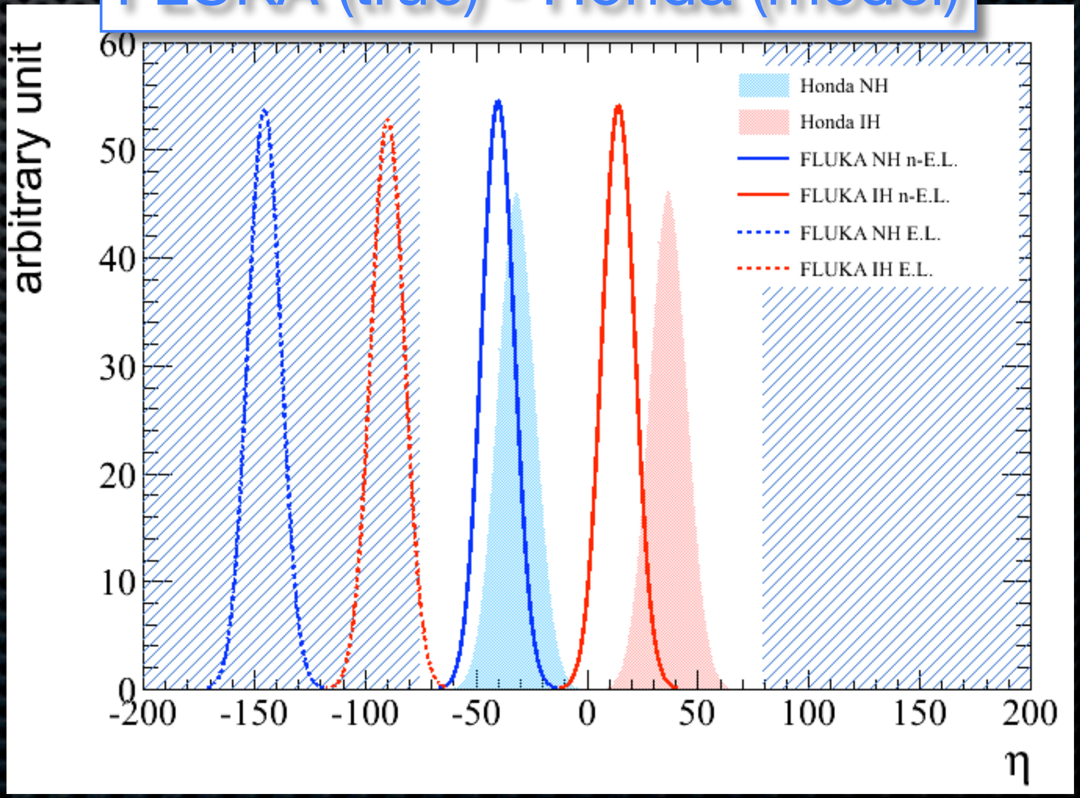
Flux models

- We studied the impact of different flux models namely FLUKA, Bartol and Honda.
- Although the agreement in shape above 5 GeV is at the level of 5%, the normalization of the FLUKA model differs by more than 20%.
- Assuming the Honda flux as model and the FLUKA one as true hypothesis as explained in the statistical method description, the p-value is reduced to almost 0.
- This is mainly due to the fact that we use an extended likelihood (constraint on the normalization).
- In case of a non-extended likelihood the p-value is recovered to about 0.65.

Flux models comparison



FLUKA (true) - Honda (model)



Extended or non-extended likelihood?

- Removing the extended component makes the test statistics independent on the expected number of events (reducing in general the discrimination power).

extended unbinned likelihood

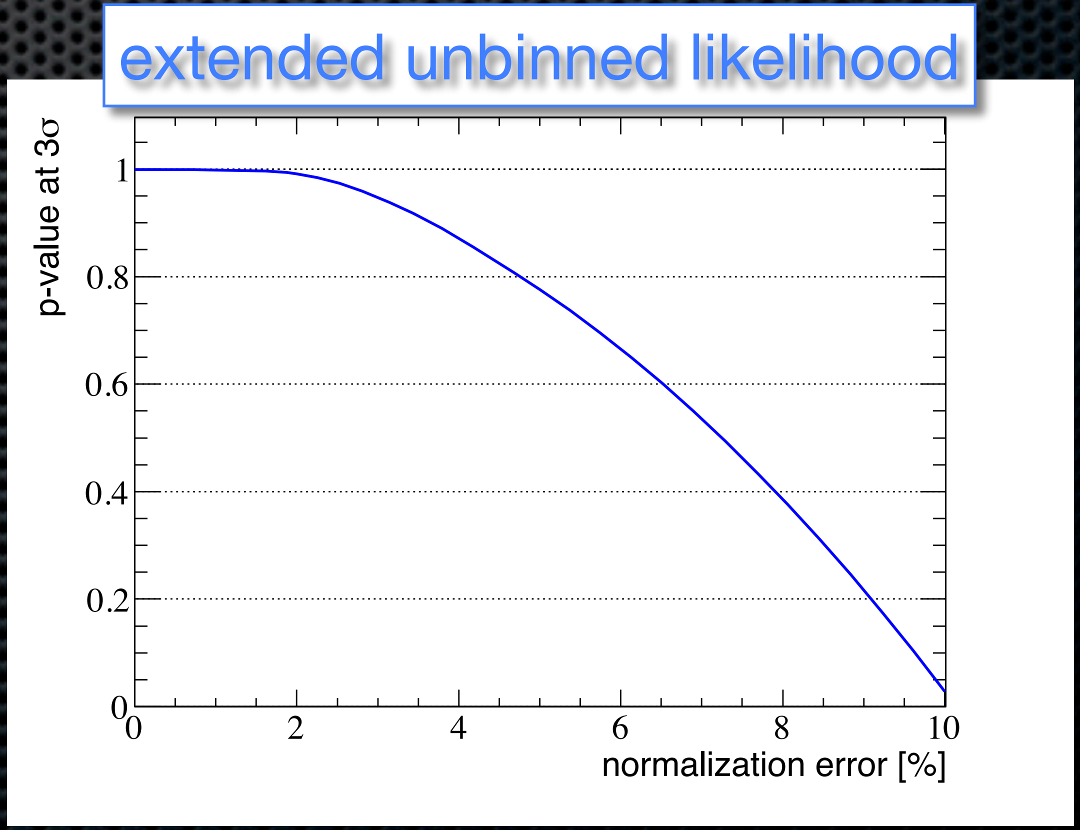
$$L_j = \frac{(e^{-\mu_j}\mu_j^n)}{n!} \times \prod_{i=1}^{n} \text{pdf}_j(E_i, \theta_i)$$

→

non-extended unbinned likelihood

$$L_j = \prod_{i=1}^{n} \text{pdf}_j(E_i, \theta_i)$$

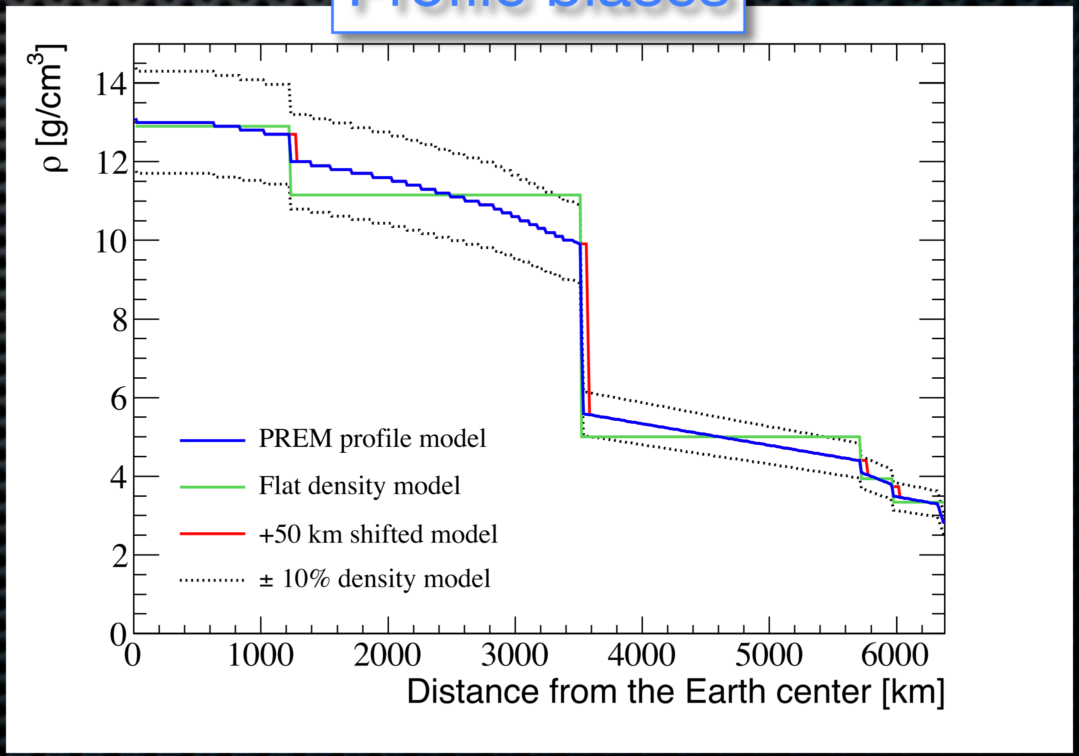
- To understand in which validity range the extended likelihood is better we used the Honda flux as model and the Honda flux with an introduced normalization error as true hypothesis.
- Assuming a threshold of 0.5 on the p-value, the extended likelihood is convenient if the flux is known with an error smaller than 7.5%.
- This could be probably done in the experiment itself, anchoring the flux at high energy (> 20 GeV).



Earth density profile

- To understand the impact of the earth density profile uncertainties, we modified it by hand in the GLoBES code.
- Biases introduced are much larger than the known uncertainties and even unphysical but the idea was to use them as true hypothesis and see how much the p-value would be reduced compared to the model (PREM).
- Even in these extreme cases the p-values is almost unaffected.
- PREM parameters have therefore **a negligible impact** on the mass hierarchy determination.

Profile biases



Neutrino oscillation parameters

REMARK: In an experiment the neutrino oscillation parameters are evaluated with a data fit. **Our goal** is just **identify the critical parameters** (large impact on the mass hierarchy determination) which have to be left free in the final fit.

- We applied the same method on the neutrino oscillation parameters changing the true hypothesis by $\pm 1\ \sigma$.

Neutrino oscillation parameters

REMARK: In an experiment the neutrino oscillation parameters are evaluated with a data fit. **Our goal** is just **identify the critical parameters** (large impact on the mass hierarchy determination) which have to be left free in the final fit.

- We applied the same method on the neutrino oscillation parameters changing the true hypothesis by $\pm 1\ \sigma$.

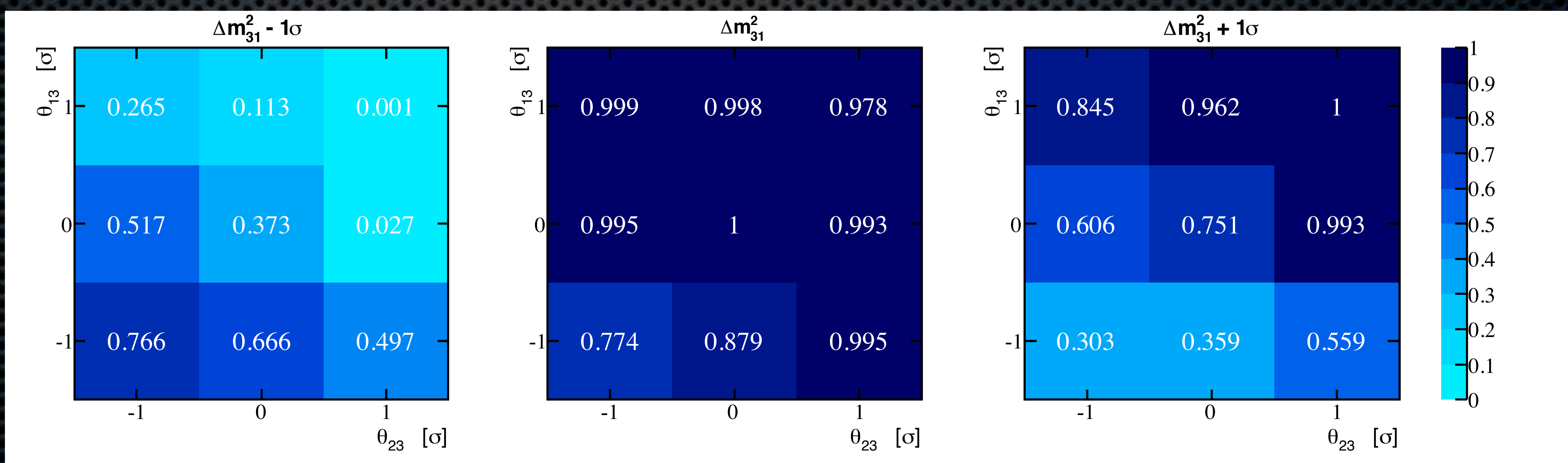
Solar sector → **Negligible impact** on the p-value seen studying the correlation between θ_{12}, Δm^2_{21}, and Δm^2_{31}.

Neutrino oscillation parameters

REMARK: In an experiment the neutrino oscillation parameters are evaluated with a data fit. **Our goal** is just **identify the critical parameters** (large impact on the mass hierarchy determination) which have to be left free in the final fit.

- We applied the same method on the neutrino oscillation parameters changing the true hypothesis by ±1 σ.

Atmospheric sector → **Important dependence** of the p-value on the correlation between θ_{23}, θ_{13}, and Δm^2_{31}.

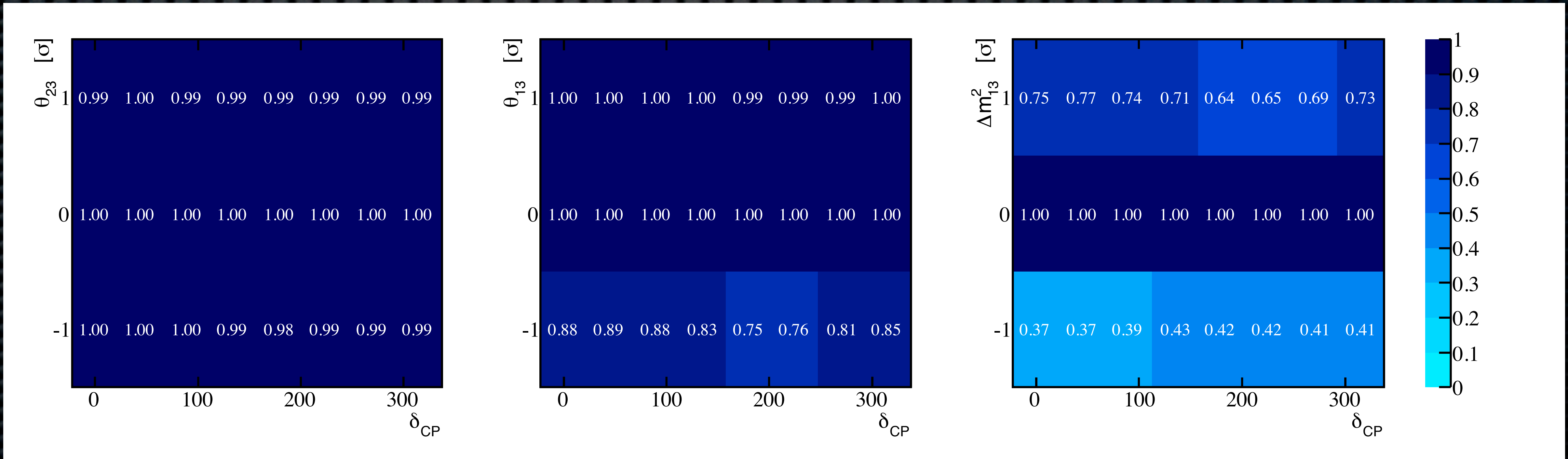


Neutrino oscillation parameters

REMARK: In an experiment the neutrino oscillation parameters are evaluated with a data fit. **Our goal** is just **identify the critical parameters** (large impact on the mass hierarchy determination) which have to be left free in the final fit.

- We applied the same method on the neutrino oscillation parameters changing the true hypothesis by ±1 σ.

δ_{cp} dependence → **Weak dependence** of the p-value on the correlation between δ_{cp} - θ_{23}, δ_{cp} - θ_{13}, and δ_{cp} - Δm^2_{31}.



Energy threshold

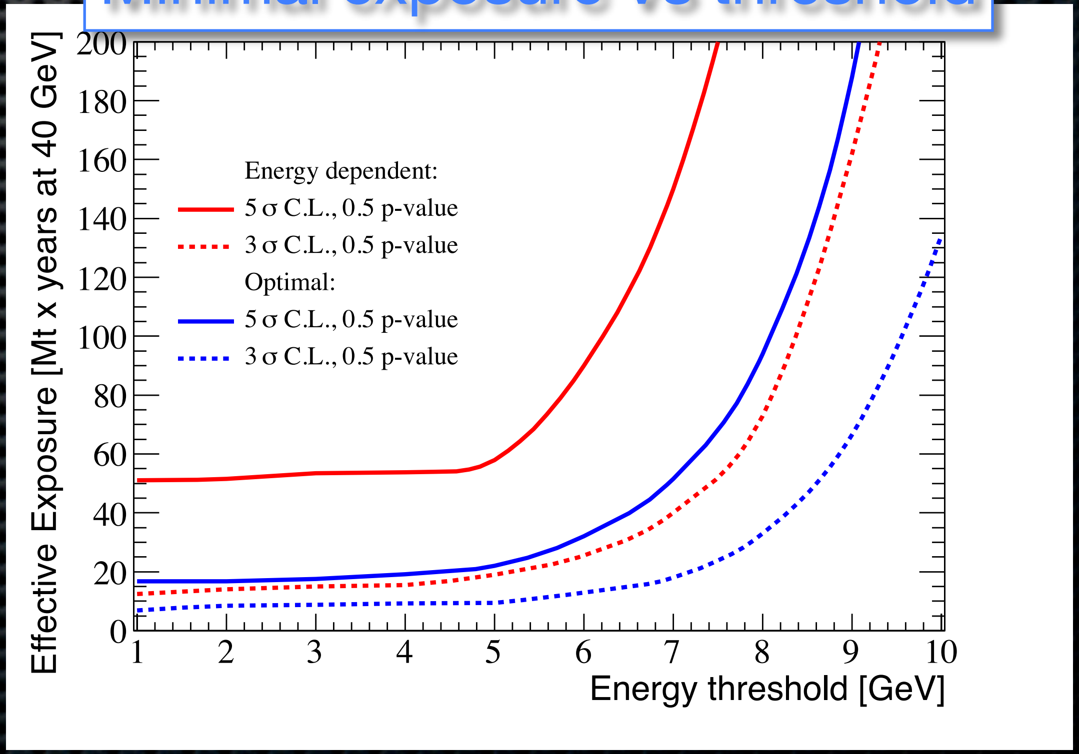
- Another important point we investigated is the minimal exposure required to have a p-value of 0.5 as a function of the muon energy threshold.

- **No** large improvement (order of 20%) is expected lowering the threshold below 5 GeV.

- In addition below 5 GeV the energy resolution degradation is expected and cross section uncertainties have to be considered.

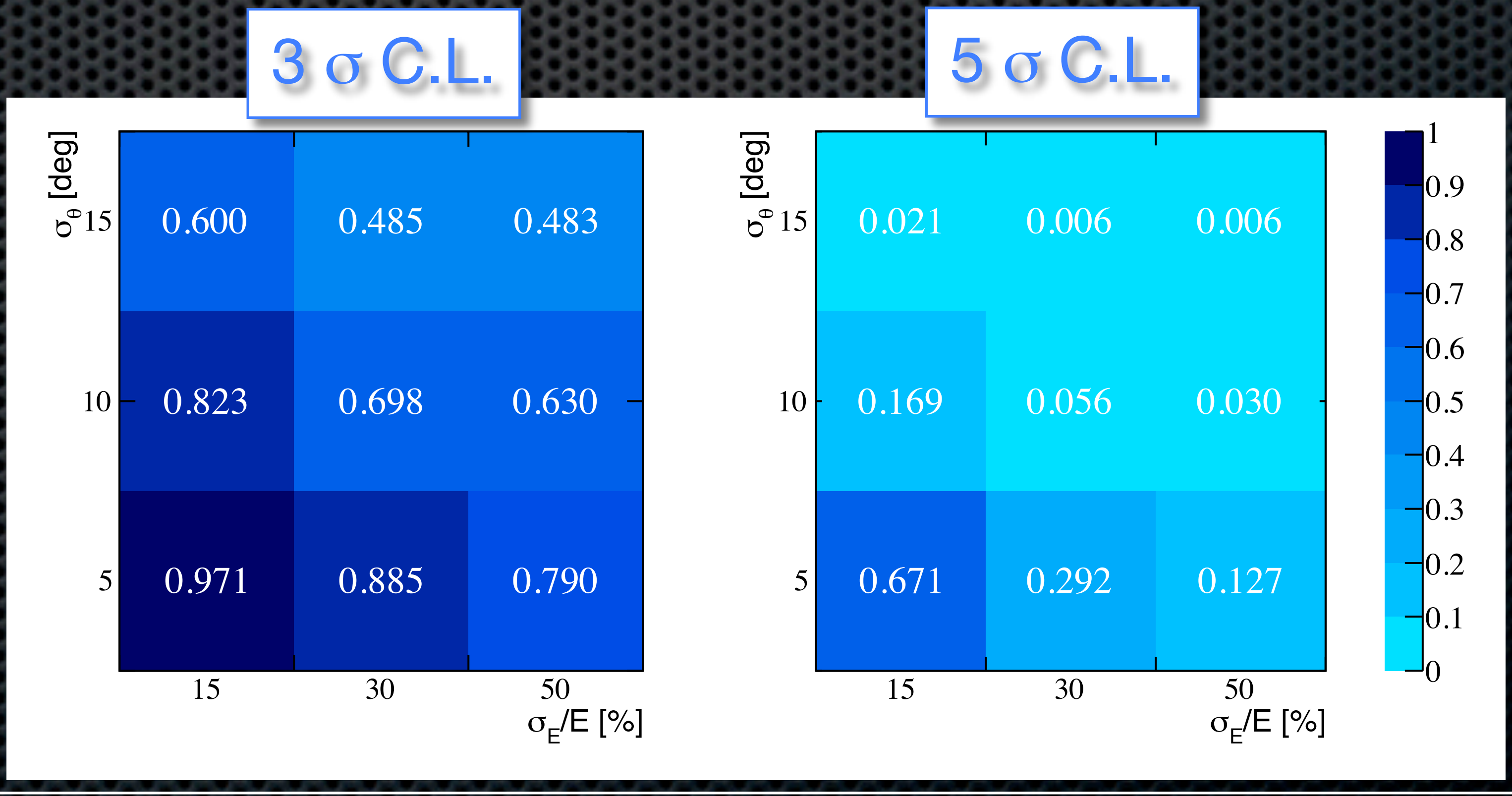
- **Rapid** degradation of the sensitivity for thresholds above 5 GeV.

Minimal exposure Vs threshold



Detector resolution

- Detector **energy** and **angular** resolutions were introduced.
- Since we did not have real detector input from ORCA or PINGU, arbitrary combinations of resolutions were tested.
- The reference p-values obtained in ideal detector conditions are 1 and 0.992 at 3 σ and 5 σ C.L. respectively.
- Introducing the detector resolution could **degrade** a lot the discrimination power.

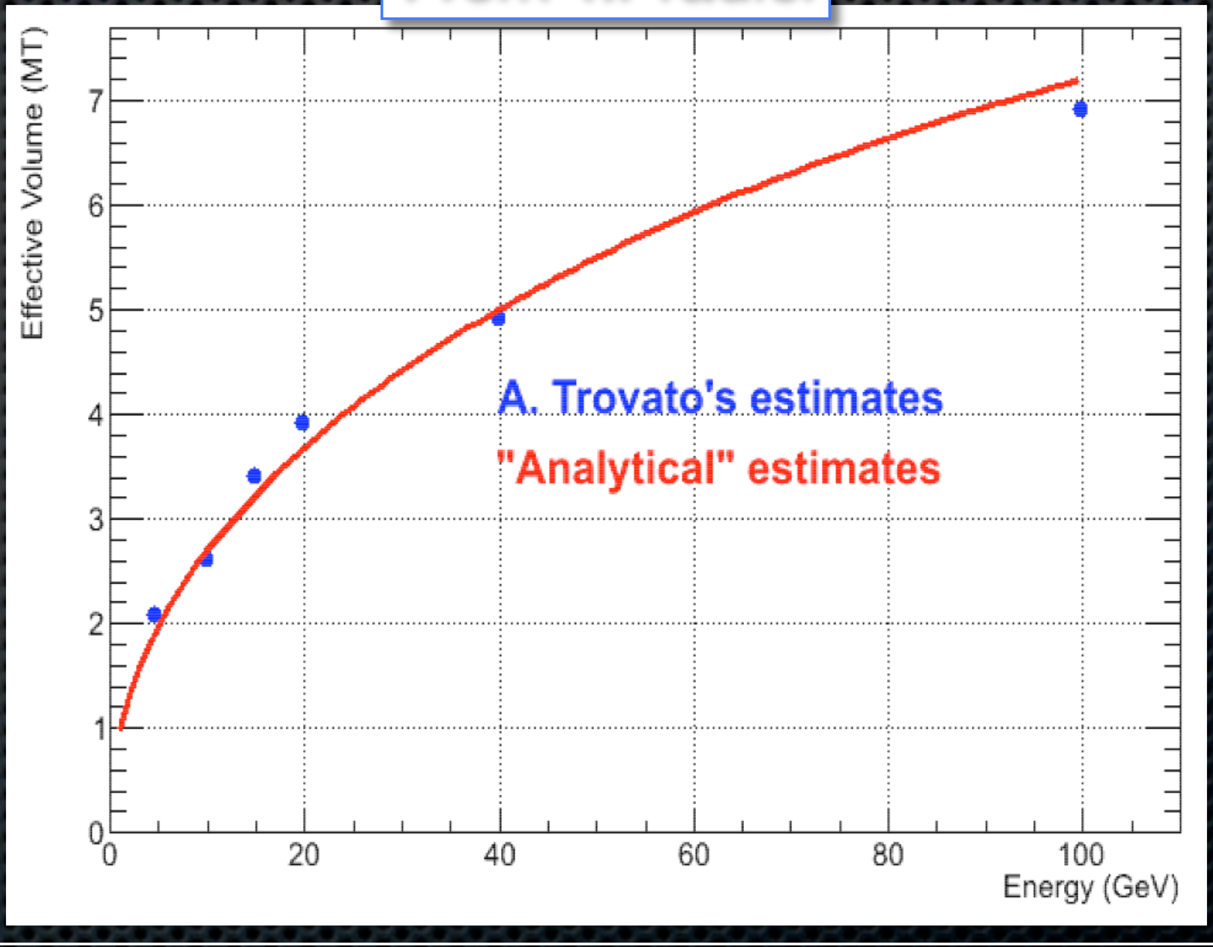


Some Results for ORCA

Mass profile

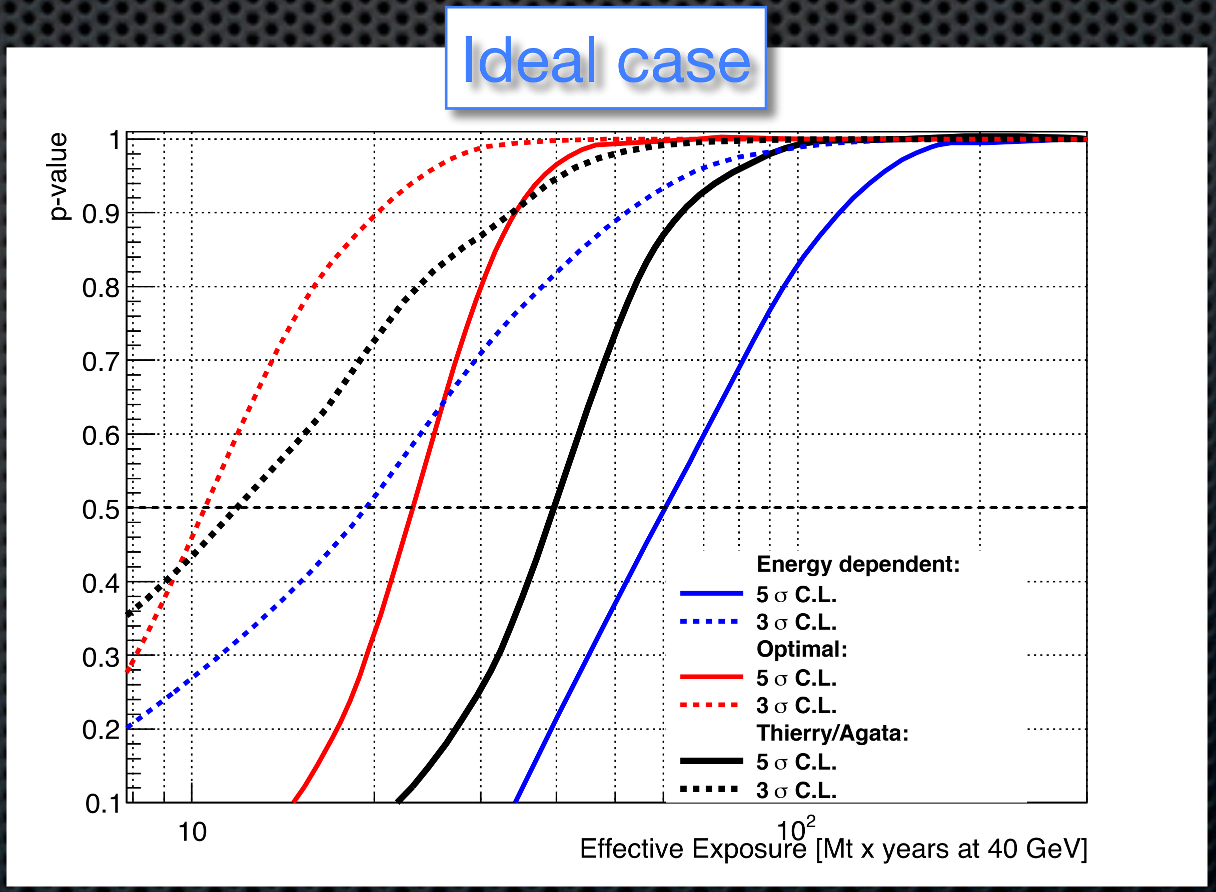
- We have shown that the sensitivity of the experiment depends strongly on the mass profile.
- We used the one found by A.Trovato with a full MC simulation (and confirmed independently by T.Pradier using analytical estimates) to compute the sensitivity curve in **ideal conditions**.

From T.Pradier



Experimental sensitivity (1)

- The choice adopted in our paper of normalizing the effective exposure at 1 Mt at 40 GeV was made in order to compare different mass profiles **regardless** of their absolute normalization.
- Using the ORCA profile we found a curve in-between the optimal profile (energy-independent) and the PINGU-like one.



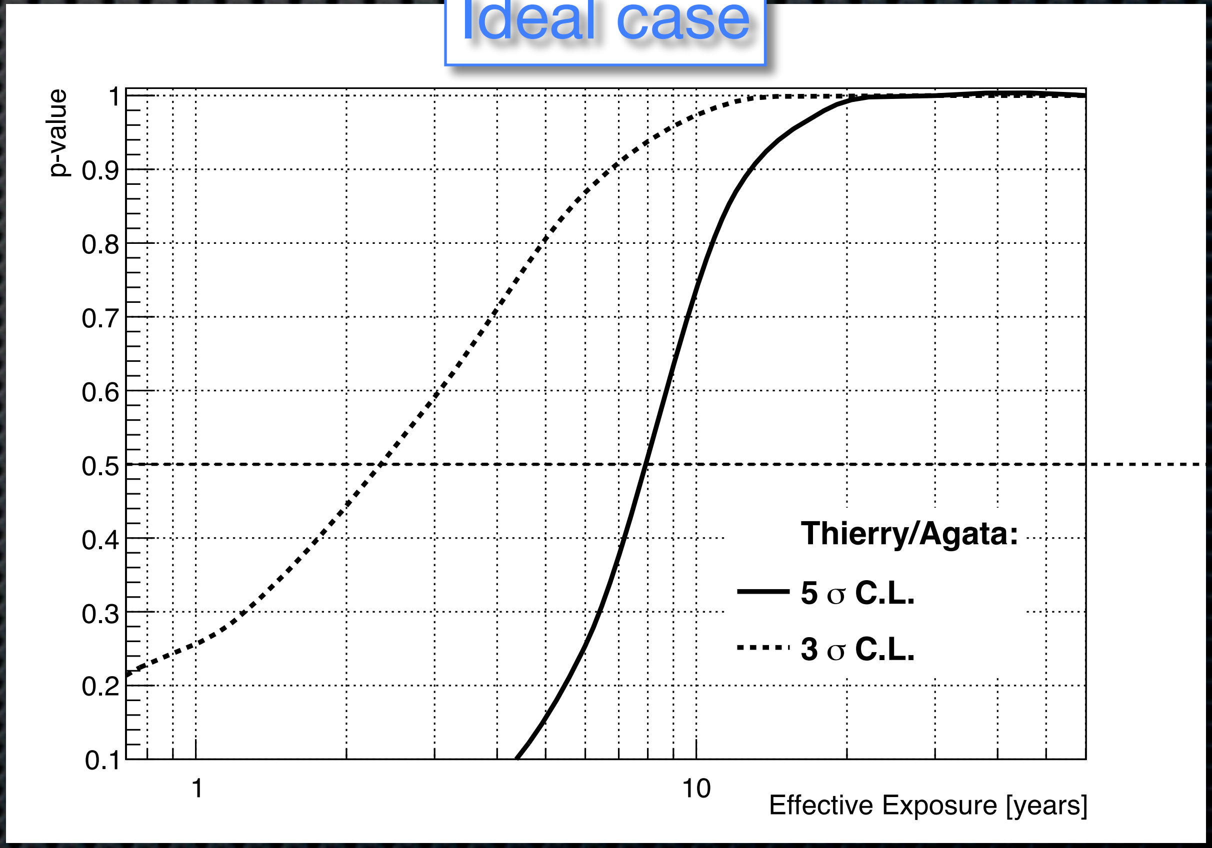
Experimental sensitivity (2)

- Taking the mass at 40 GeV (i.e. 5 Mton) we can convert the exposure into years, showing that a p-value of 0.5 can be obtained in 2.4 and 8 years at 3 σ and 5 σ C.L. respectively.

- This result can be **improved**:
  - reconstructing the hadronic shower
  - lowering the energy threshold

- On the other hand the sensitivity is **degraded** including:
  - the detector resolutions
  - the parameters systematics
  - the background

Ideal case



Need of reliable detector MC full simulation

Conclusions

- A toy MC based on an unbinned likelihood ratio test statistics has been developed to assess the neutrino mass hierarchy discrimination potential.
- The dependence on different mass profiles has been studied.
- The impact of model uncertainties such as flux models, oscillation parameters or Earth density profile was evaluated.
- Some arbitrary detector resolutions were introduced to study the sensitivity degradation.
- In order to correctly evaluate the experimental sensitivity a full detector MC is needed to introduce the **real event reconstruction**.
- In addition the **background** has to be considered.