Partial wave analysis of pion-induced reactions: 2π production

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PWA and Reaction Theory: Motivation

Aim: Extract properties of baryon resonances from experiment

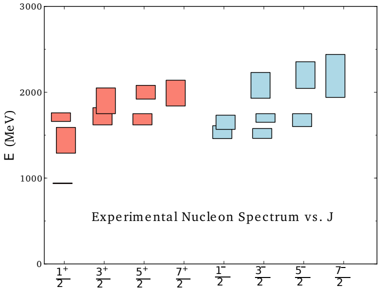
- accepted tool: PWA (decomposition in terms of J,P,I)
- PWA does not substitute dynamics: need to construct the reaction theory
- effective degrees of freedom: meson and baryons
- baryon resonance analysis: bring different pion- and photon-induced reactions together

Baryon resonance spectrum: where do we stand and where to move to ?

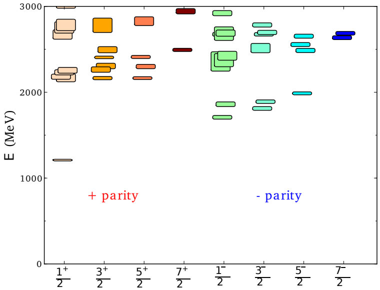
HADRON SPECTRUM COLLABORATION

Overall pattern of N* states

Expt. **** *** **



Lattice



Many more states in the lattice spectrum.

$N^* \to \pi N$ decays

PDG 2012 most information from
Carnegi-Mellon, KHU, SAID/GWU

N^*	$L_{2I\,2J}$	Overall	$Br(N^* \to \pi N)$
$N(939)$	P_{11}	★★★★	—
$N(1440)$	P_{11}	★★★★	55...67 %
$N(1520)$	D_{13}	★★★★	55...65 %
$N(1535)$	S_{11}	★★★★	35...55 %
$N(1650)$	S_{11}	★★★★	50...90 %
$N(1675)$	D_{15}	★★★★	35...45 %
$N(1680)$	F_{15}	★★★★	65...70 %
$N(1700)$	D_{13}	★★★	8 ... 17 %
$N(1710)$	P_{11}	★★★	5 ... 20 %
$N(1720)$	P_{13}	★★★★	9...14 %
$N(1870)$	D_{13}	★★★	10 ... 22 %
$N(1900)$	P_{13}	★★★	10 %
$N(2000)$	F_{15}	★★	9 %

- for $Br(N^* \to \pi N) < 20\%$ no general agreement between different analyses !
- indication for the existence is smaller for higher masses (more degrees of freedom at higher energies, many open channels)
- resonance/background separation is difficult

Baryon spectrum

Lattice

- more resonances than concluded from PWA
- there are many states above 2 GeV.

N^* with higher masses tend to have smaller πN coupling

- study photoproduction(gauge invariance, need πN amplitudes as input, enhanced background etc)
- come back to pion-induced experiments: increase experimental resolution.
- study pion-induced inelastic reactions
- investigate cascade reactions $N^* \to \pi N^* \to \pi\eta N^*$ etc. : multiparticle production

Baryon Spectrum: short summary

Short summary:

- photoproduction: very active, new data
- inelastic pion-induced inelastic reactions: good change to improve our knowledge on the resonance spectra.
- multiparticle production: study cascade reactions $N^* \to \pi N^* \to \pi\eta N^*$ etc. for higher resonance masses.

Reaction theory is wanted !

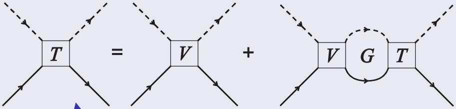
- SAID/GWU
- Giessen
- MAID
- Valencia
- EBAC
- BoGa
- Zagreb

Giessen model for baryon analysis

Giessen model. PRC71, 055206 (2005)

Bethe-Salpeter in K-matrix: dynamical model: based on eff. L_{mBB}

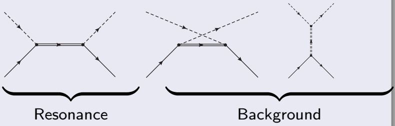
T-matrix



multidimensional T-matrix

$$T = \begin{pmatrix} T_{\gamma N,\gamma N} & T_{\gamma N,\pi N} & T_{\gamma N,K\Lambda} & \cdots \\ T_{\pi N,\gamma N} & T_{\pi N,\pi N} & T_{\pi N,K\Lambda} & \cdots \\ T_{K\Lambda,\gamma N} & T_{K\Lambda,\pi N} & T_{K\Lambda,K\Lambda} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

Interaction terms V_{ij}



How many channels?

$\gamma N \to \gamma N$	$\pi N \to \pi N$
$\gamma N \to \pi N$	$\pi N \to 2\pi N$
$\gamma N \to \eta N$	$\pi N \to \eta N$
$\gamma N \to \omega N$	$\pi N \to \omega N$
$\gamma N \to K\Lambda$	$\pi N \to K\Lambda$
$\gamma N \to K\Sigma$	$\pi N \to K\Sigma$

K-matrix approximation:

To solve Bethe-Salpeter equation take the imaginary part of the propagator:

$$\int dq \frac{1}{q^2 - m^2 \pm i\varepsilon} = P \int dq \frac{1}{q^2 - m^2} \mp i\pi \int dq \delta(q^2 - m^2)$$

where all intermediate particles are on-shell.

main features

- neglect dispersive part of self energy
- Minkowsky space
- resonance parameters: coupling constants at interaction Lagrangians
- respect fundamental symmetries (e.g. chiral symmetry)

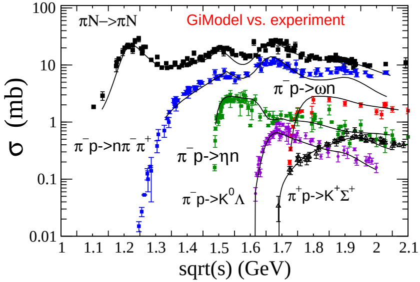
Partial wave version of optical theorem

constraints on partial wave cross sections

$$Im T^{JP}_{\pi N\to\pi N} = \frac{k^2}{4\pi}\left(\sigma^{JP}_{\pi N\to\pi N} + \sigma^{JP}_{\pi N\to 2\pi N} + \sigma^{JP}_{\pi N\to\eta N} + \sigma^{JP}_{\pi N\to\omega N} + \sigma^{JP}_{\pi N\to K\Lambda} + \sigma^{JP}_{\pi N\to K\Sigma} + \dots\right)$$

all reaction data are linked

$\rightarrow$ need for coupled-channel unitary analysis

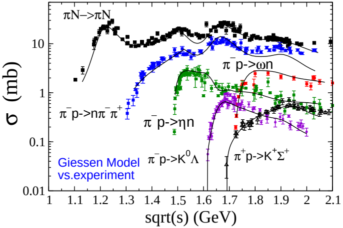


$$T = \begin{pmatrix} T_{\gamma N,\gamma N} & T_{\gamma N,\pi N} & T_{\gamma N,K\Lambda} & \cdots \\ T_{\pi N,\gamma N} & T_{\pi N,\pi N} & T_{\pi N,K\Lambda} & \cdots \\ T_{K\Lambda,\gamma N} & T_{K\Lambda,\pi N} & T_{K\Lambda,K\Lambda} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$\leftarrow$ Giessen Model vs experimental data

$\pi N \to 2\pi N$ reactions

$\pi N \to 2\pi N$: strong contribution to the πN inelasticity.



$\pi N \to 2\pi N$ strong resonance contributions

All what we know about N^* couplings to ρN, $\pi\Delta$, σN channel is due to Manley, Arndt, Goradia, Teplitz PRD**30**,(1984) 904.

- based on 240000 events from old bubble-chamber experiments W = 1.2...2 GeV: $\approx$ 9000 events per energy/angular (θ, ϕ) bin
- non-unitary
- amplitudes $\pi N \to \rho N$, $\pi\Delta$ at fixed isobar masses

$\pi N \to 2\pi N$ reaction

New data came late (most of them are total X-sections) (I.Strakovsky, GWU) but not suited for $N^* \to \rho N$

- W=1221 to 1356 MeV
  $\pi^+ p \to \pi^+\pi^+ n$ PNPI (1978)
  $\pi^+ p \to \pi^+\pi^+ n$ TRIUMF (1991)
  $\pi^\pm p \to \pi^\pm\pi^\pm n$ TRIUMF (1998)
  $\pi^+ p \to \pi^+\pi^0 p$ LAMPF (1994)
  $\pi^+ p \to \pi^-\pi^+ p$ CERN (1990)
- W=1213 to 1527 MeV
  $\pi^- p \to \pi^0\pi^0 n$ BNL(2004)
- W=1257 to 1302 MeV
  $\pi^\pm p \to \pi^\pm\pi^\pm n$ TRIUMF (1998)(events)
- W=1300 to 1302 MeV
  $\pi^- p \to \pi^+\pi^- n$ PSI (1993)

PDG 2010

$N(1520)$ DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	0.55 to 0.65
Γ_2	$N\eta$	$(2.3 \pm 0.4) \times 10^{-3}$
Γ_3	$N\pi\pi$	40–50 %
Γ_4	$\Delta\pi$	15–25 %
Γ_5	$\Delta(1232)\pi$, S-wave	5–12 %
Γ_6	$\Delta(1232)\pi$, D-wave	10–14 %
Γ_7	$N\rho$	15–25 %

- Manley, Arndt PRD30(1984): strong ρN-coupling
- $m_{1520} << 1.7$ GeV$(m_\rho + m_N)$
- important for in-medium ρ-meson properties

$P_{11}(1710)$: problems

$N(1710)\ P_{11}$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: $* * *$

Most of the results published before 1975 were last included in our 1982 edition, Physics Letters **111B** 1 (1982). Some further obsolete results published before 1984 were last included in our 2006 edition, Journal of Physics, G **33** 1 (2006).

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

$N(1710)$ BREIT-WIGNER MASS

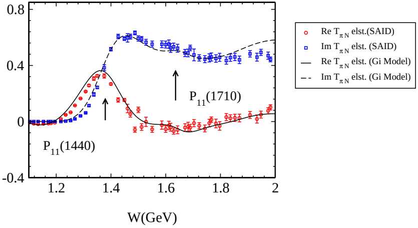
VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
1680 to 1740 (≈ 1710) OUR ESTIMATE				
1717±28	MANLEY	92	IPWA	$\pi N \to \pi N$ & $N\pi\pi$
1700±50	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
1723± 9	HOEHLER	79	IPWA	$\pi N \to \pi N$

PDG 2010:

Br(πN)≈ 10 to 20 %

Br($2\pi N$)≈ 40 to 90 %

Br($K\Lambda$)≈ 5 to 25 %



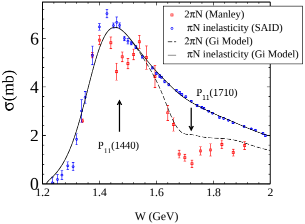
Modern GWU (SAID) PWA:

no signal around 1710 MeV !

Giessen Model: $P_{11}(1710)$:

Br(πN) ≈ 3%

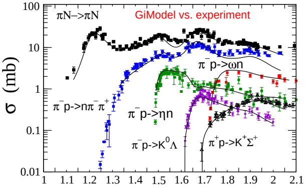
$P_{11}(1710)$: problems



Optical theorem :

$$\left[\frac{4\pi}{k_{\rm cm}^2} Im T^{JI}_{\pi N} - \sigma^{JI}_{\pi N\to\pi N}\right] = \sigma^{JI}_{\pi N\to 2\pi N} + \sigma^{JI}_{\pi N\to\eta N} + \sigma^{JI}_{\pi N\to\omega N} + \sigma^{JI}_{\pi n\to K\Lambda} + \sigma^{JI}_{\pi N\to K\Sigma} + \dots$$

- PDG values are inconsistent with modern PWA SAID
- large difference between πN inelasticity and $2\pi N$ channel - cannot be fully absorbed into $K\Lambda$ or $K\Sigma$
- Giessen model: decay $P_{11}(1710) \to \eta N$
- $P_{11}(1710)$ modern status: only indirect indications: more data is needed



$\pi N \to 2\pi$ channel in the first resonance energy region

Roper resonance N(1400) properties:

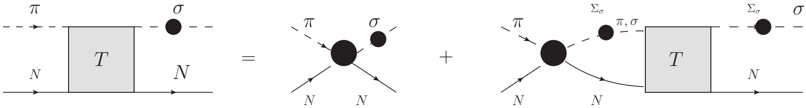
- Manley & Saleski PRD30 904, $Br(\Delta\pi) = 22\%$ $Br(\sigma N) = 9\%$
- Vrana et al PRPL328, $Br(\Delta\pi) = 16\%$ $Br(\sigma N) = 12\%$
- Sarantsev et al PLB659,94, $Br(\Delta\pi) = 17\%$ $Br(\sigma N) = 21\%$

- Julich Model: PRC62: pion exchange is responsible for a large amount of attraction: P(1440) is dynamically generated
- Crytal Ball PRL91(2003): PWA of the $2\pi^0$-subsystem: σ-meson production via pion exchange is small
- Crytal Ball PRL69(2004): measurement of the $\pi N \to 2\pi^0 N$-reaction: no direct evidence for a strong σN subchannel

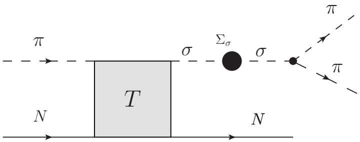
$\pi N \to 2\pi$ channel in the first resonance energy region

BSE in the isobar approximation:

system of coupled-channel integral equations

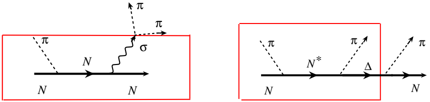


$\pi N \to 2\pi N$ amplitude from BSE



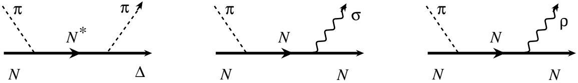
Next step: improve description of the $2\pi N$ channel

$\pi N \to 2\pi N$ reaction via ρN, $\pi\Delta$ channels

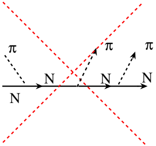


Assumptions

- decays $N^* \to \rho N$, σN, $\pi\Delta$ drive the $\pi N \to 2\pi N$ channel



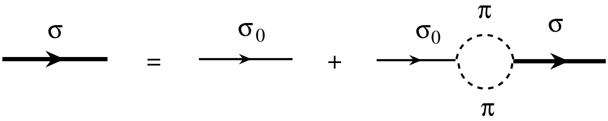
- two-step diagrams are neglected



σ-meson dynamics

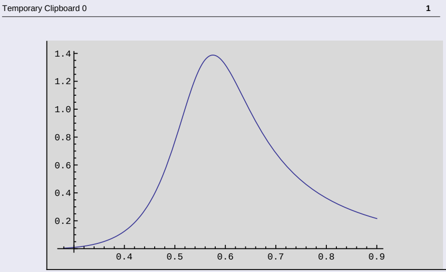
propagator of the σ-meson

$$D(s) = \frac{1}{s - m_\sigma^2 + i\Sigma_\sigma(s)}$$

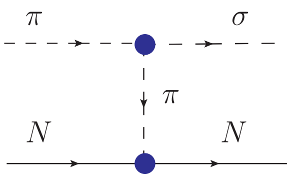


$$A_\sigma(s) = \frac{1}{\pi} \frac{\Sigma_\sigma(s)}{(s - m_\sigma^2)^2 + \Sigma_\sigma(s)^2}$$

spectral function A_σ



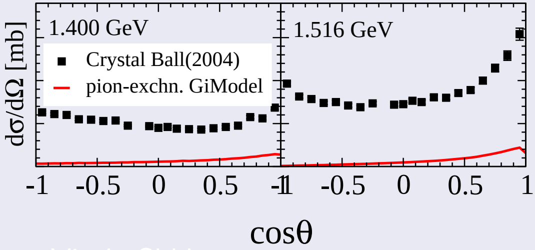
t-channel pion exchange: σN how large?



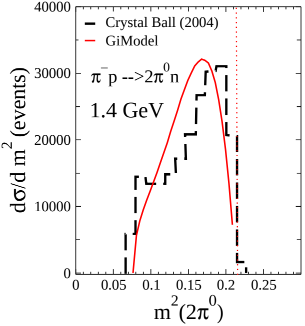
- coupling constants are well fixed
- $g_{\pi NN} = 13$, $g_{\sigma\pi\pi} = 2$ correspond to $m_\sigma^0 = 600\text{MeV}$, $\Gamma_{\sigma\pi\pi} = 600\text{MeV}$
- contribution from the t-channel diagram is well fixed

- shed light on the σ-meson dynamics
- background mechanism in $\pi N \to 2\pi N$ reaction

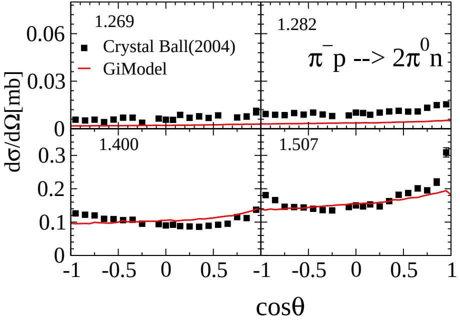
t-channel pion exchange: very small !

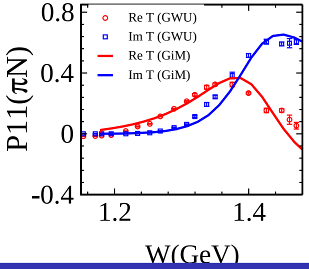


Giessen Model vs. Crystal Ball data



Roper resonance





- good description of the $\pi^- p \to 2\pi^0 n$ data
- three-body unitarity is maintained

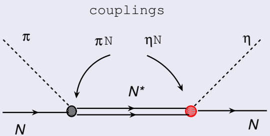
$$\mathrm{Im}\, T^{11}_{\pi N} = \frac{k^2}{4\pi}\left(\sigma^{11}_{\pi N} + \sigma^{11}_{2\pi^0 N}\right)$$

Another possibility: inelastic reactions

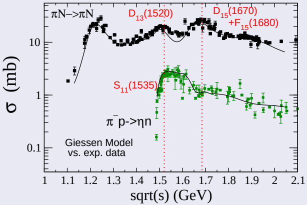
inelastic $\pi N \to \eta N$, ωN, ρN... etc scattering

My argument:

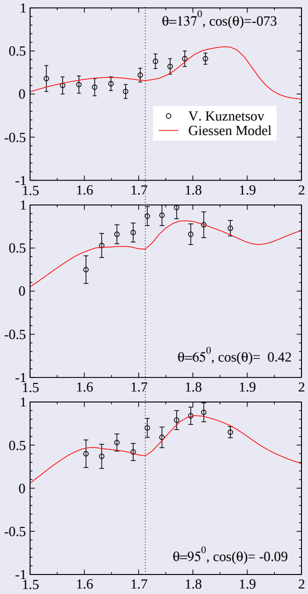
- resonance contribution to e.g. η-production: $\frac{d\sigma}{d\Omega} \sim g^2_{\pi NN^*} g^2_{\eta NN^*}$
- signals from N* with small πN coupling can be visible provided $g^2_{\eta NN^*}$ is large



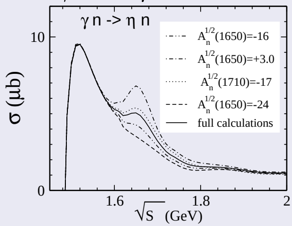
- signals from N* with small πN coupling are less shadowed by contributions from N* with large πN coupling: no clean signal from $D_{13}(1520)$, $D_{15}(1680)$, $F_{15}(1680)$ in $\pi N \to \eta N$

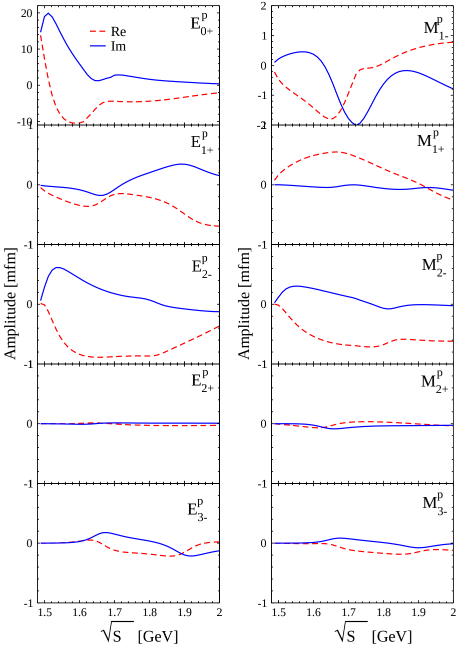


asymmetry $\gamma n \to \eta n$



total $\gamma n \to \eta n$





$\gamma p \to \eta p$
photoproduction
multipoles

$\gamma n^* \to \eta n$

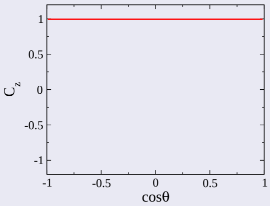
Which resonance is contributing at 1.68 GeV

measurement of the $C_z = \frac{d\sigma(\lambda_\gamma=\uparrow,\lambda_n^z=\uparrow)-d\sigma(\lambda_\gamma=\downarrow,\lambda_n^z=\uparrow)}{d\sigma(\lambda_\gamma=\uparrow,\lambda_n^z=\uparrow)+d\sigma(\lambda_\gamma=\downarrow,\lambda_n^z=\uparrow)}$ asymmetry is very important

- assuming the dominant S_{11} and P_{11} resonance contributions
- then the reaction proceeds from initial $\lambda_{in} = \frac{1}{2}$ helicity

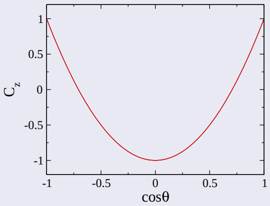
S_{11}

electric interaction (E_0-multipole): no spin flip $C_z(\theta)=1$



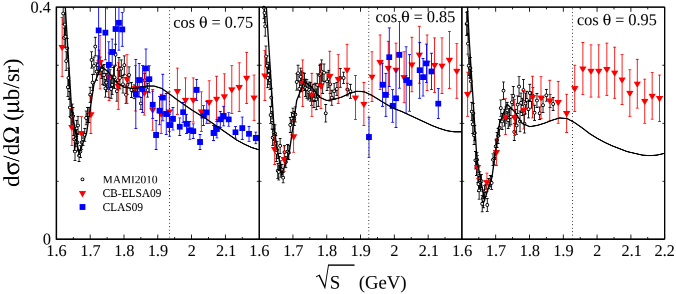
P_{11}

magnetic interaction (M_{1-}-multipole): spin flip at 90^0: $C_z(\theta) = 1 - 2\sin^2(\theta)$



measurement of the C_z might be extremely useful to pin down

Shklyar et al PRC87 (2013)



$\gamma p \rightarrow \eta p$ above 1.89 GeV