Statistical Transmutations in Doped Quantum Dimers

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Collaborators

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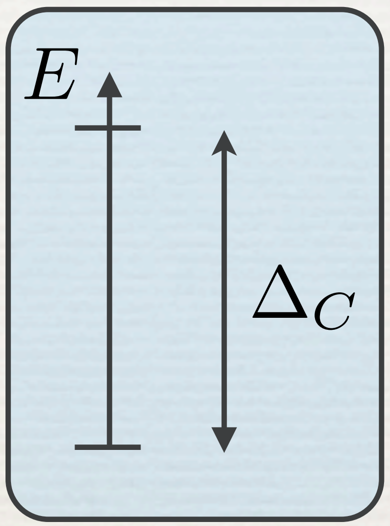
- Daniel Cabra (IFLP - La Plata / Argentina)
- Carlos Lamas (IFLP - La Plata / Argentina)

Scope

- Quantum Spin Liquids in Mott insulators
- Doped quantum dimer models
- Statistical transmutation
- Quantum phase transitions

Physical Motivations

What is the general situation for the QAF on lattices ?



Mott Insulators → Metals

Doping

$\delta > \Delta_C$

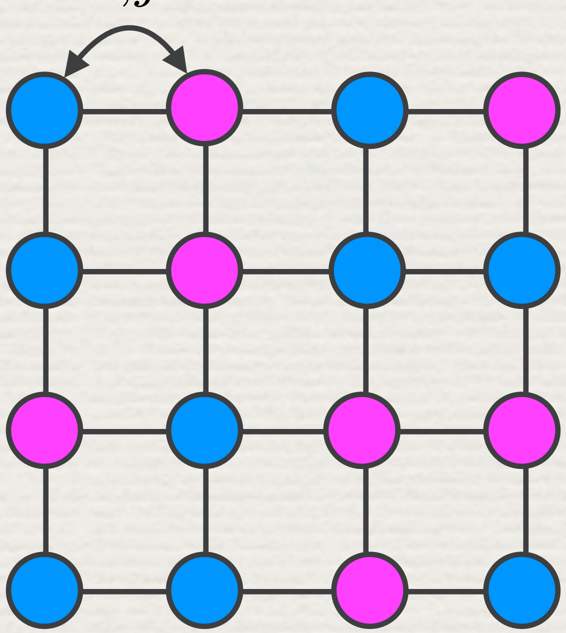
Cooling

?

Anderson (87)

High Tc Superconductors

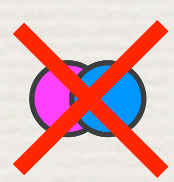
$$J \sum_{i,j} S_i \cdot S_j$$



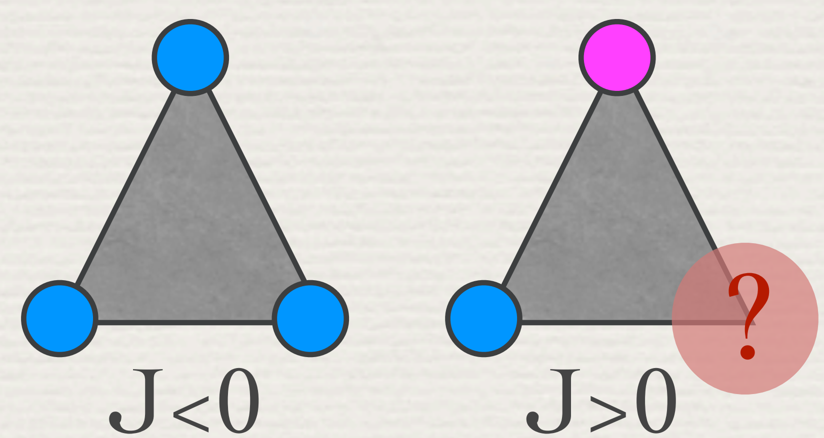
electron, spin up

electron, spin down

Colomb U >>1



Frustration:



What are the possible scenarios?

Heisenberg Model

$$\mathcal{H} = \underbrace{\underbrace{J \sum \mathbf{S}_i . \mathbf{S}_j}_{SU(2)} - h \sum S_i^z}_{U(1),\ \text{spin rotation along z-axis}}$$

+ spatial symmetries: translations, point group

Presence of a magnetic long range order: SU(2) broken

$$\langle \mathbf{S}_i . \mathbf{S}_j \rangle \simeq m_{\mathbf{q}}^2 \cos(\mathbf{q}.(\mathbf{r}_i - \mathbf{r}_j)) \quad as \quad |\mathbf{r}_i - \mathbf{r}_j| \to \infty$$

- 3D: up to Néel temperature
- 2D: only at T=0K

Low energy excitations: **Spin waves**

Other scenario: Spin liquids

No magnetic long range order

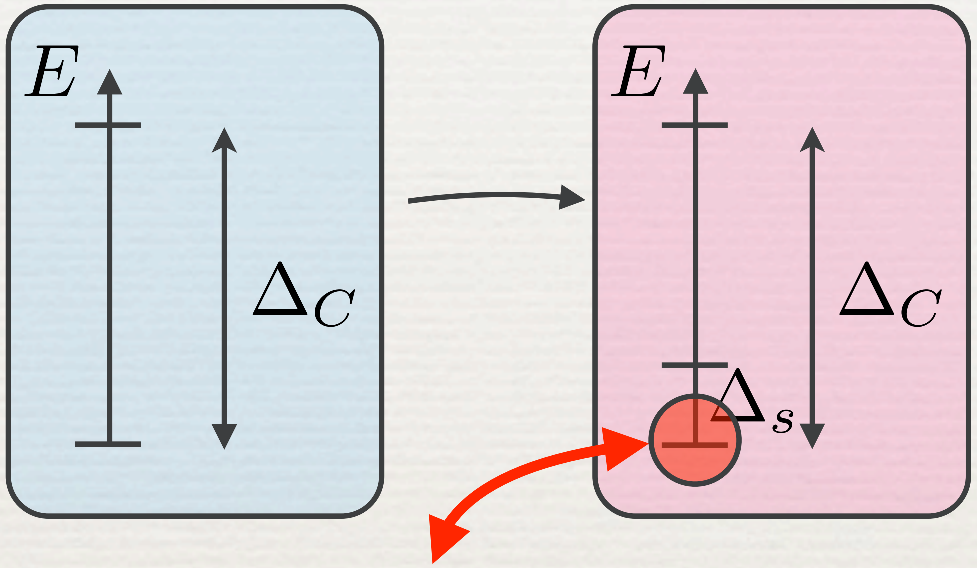
Shastry & Sutherland (81)

$$\langle \mathbf{S}_i . \mathbf{S}_j \rangle \simeq e^{-\frac{(\mathbf{r}_i - \mathbf{r}_j)}{\xi}}$$

Even at **T=0K**

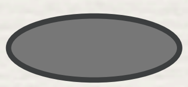
$$\Delta_s = \frac{(\mathbf{r}_i - \mathbf{r}_j)}{\xi / a}$$

SU(2) is preserved

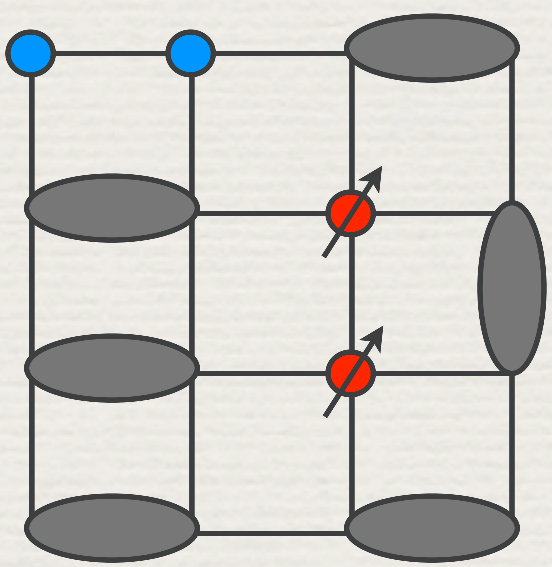


Ground state made of Spin Singlets !

S=0 states



$$= \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$$



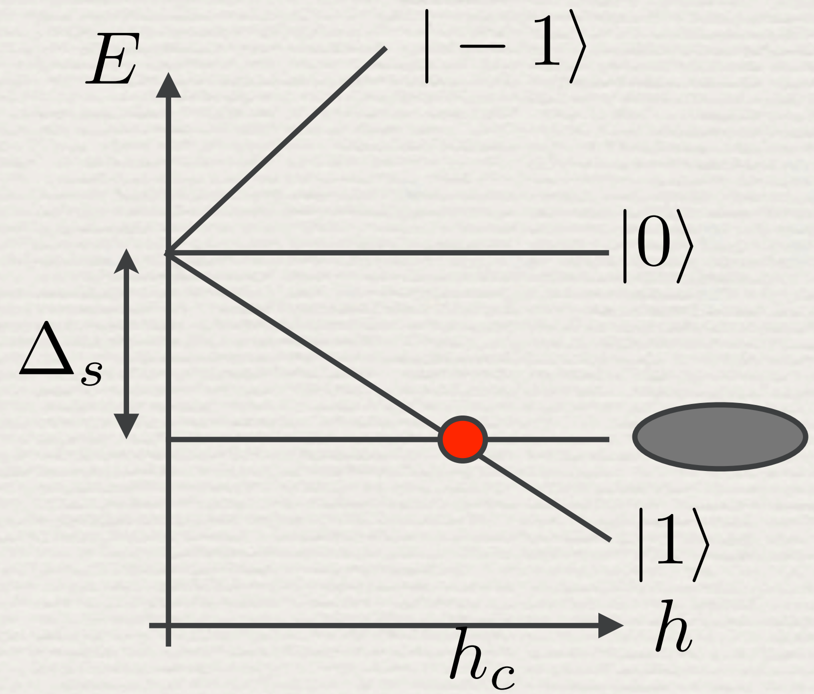
How to dope it ?

- Removing electrons (holons)
- Magnetic field h (spinon)

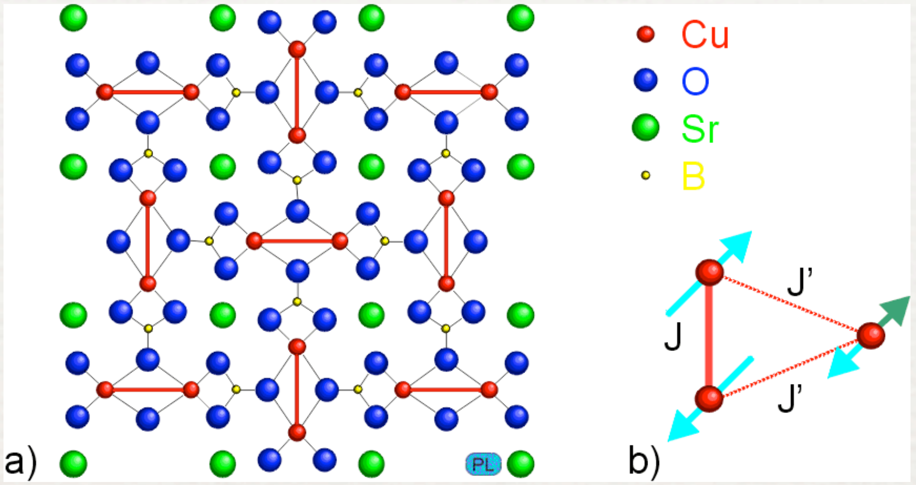
$|1\rangle = \uparrow\uparrow$

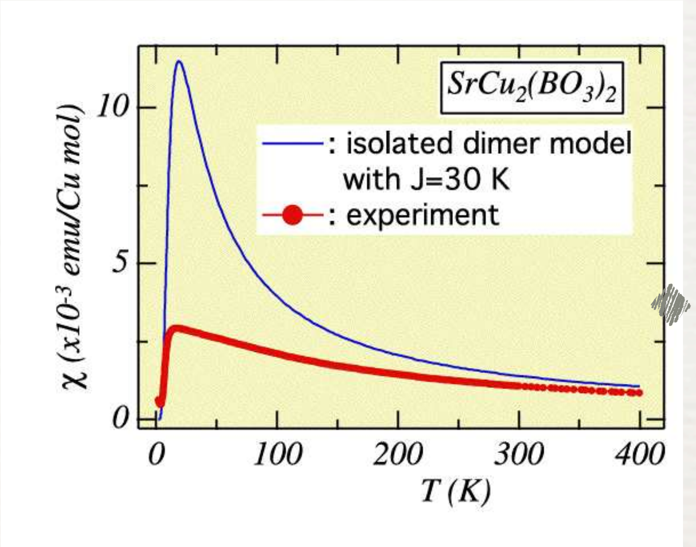
$|-1\rangle = \downarrow\downarrow$

$$|0\rangle = \frac{\uparrow\downarrow + \downarrow\uparrow}{\sqrt{2}}$$



Dimers in the nature: $SrCu_2(BO_3)_2$

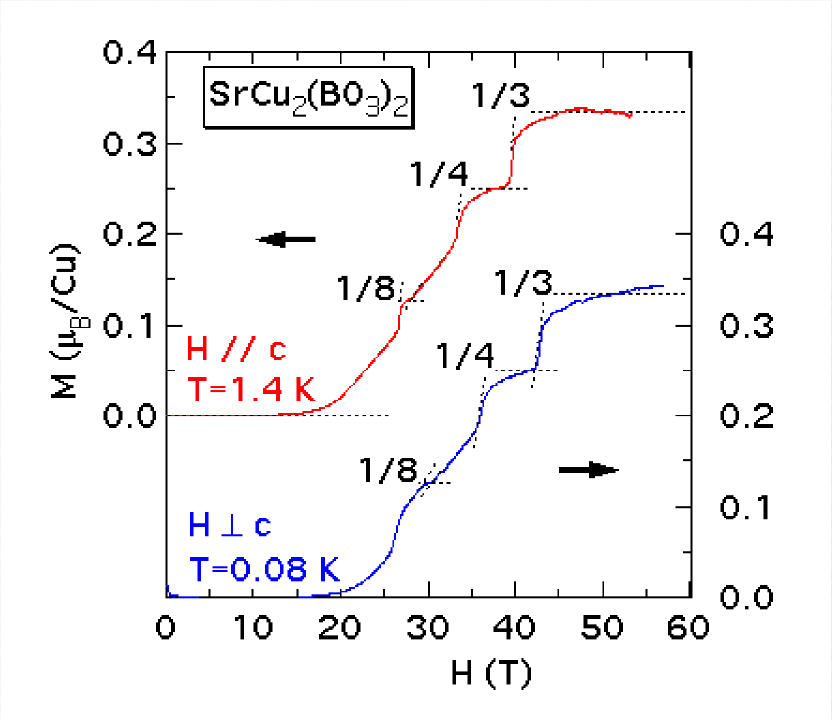




No magnetic long range order

$$\chi(T) \simeq T^{-1/2} e^{-\Delta_s/T}$$

Shastry & Sutherland (81), Kageyama *et al.* (2005)

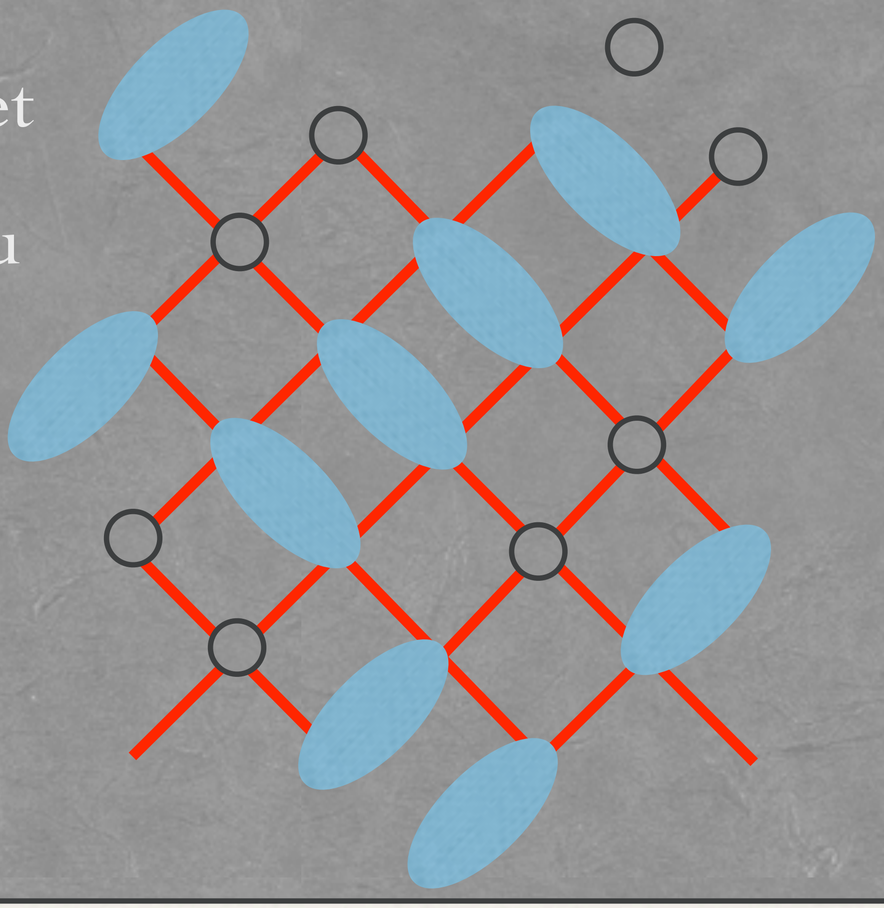


- Static dimer background
- Bose-condensation of triplets
- Exotic phases: SF, SS

Doping the singlet GS

Increasing H $\Rightarrow$ Triplet

Magnetization Plateau



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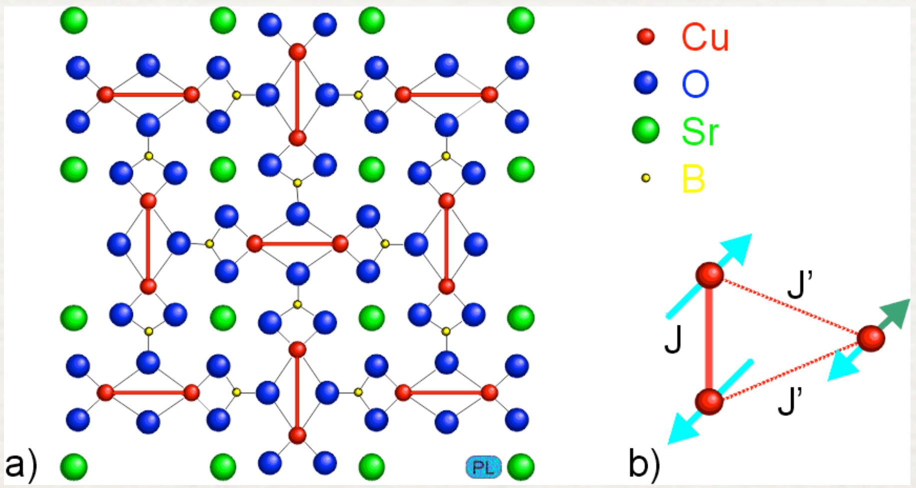

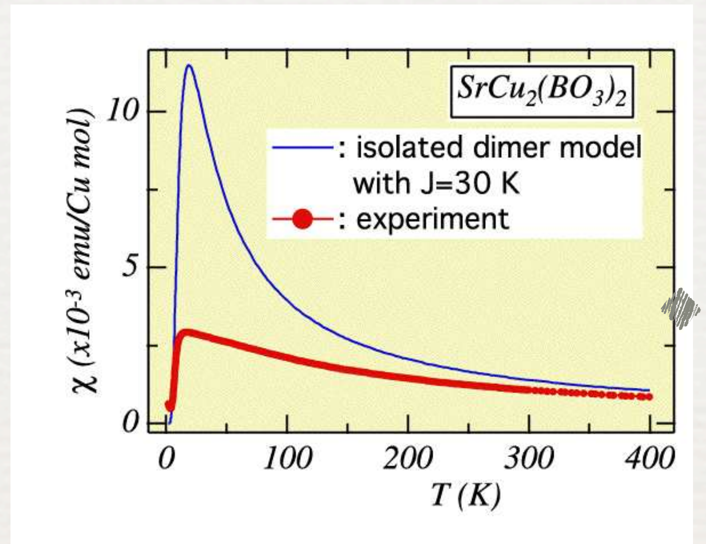

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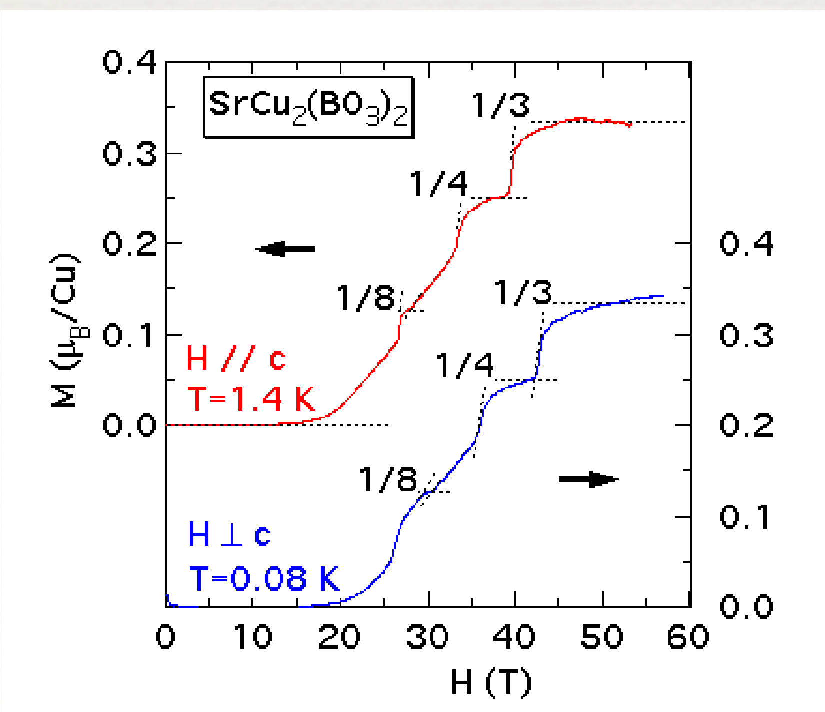

- Static dimer background
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Doping the singlet GS

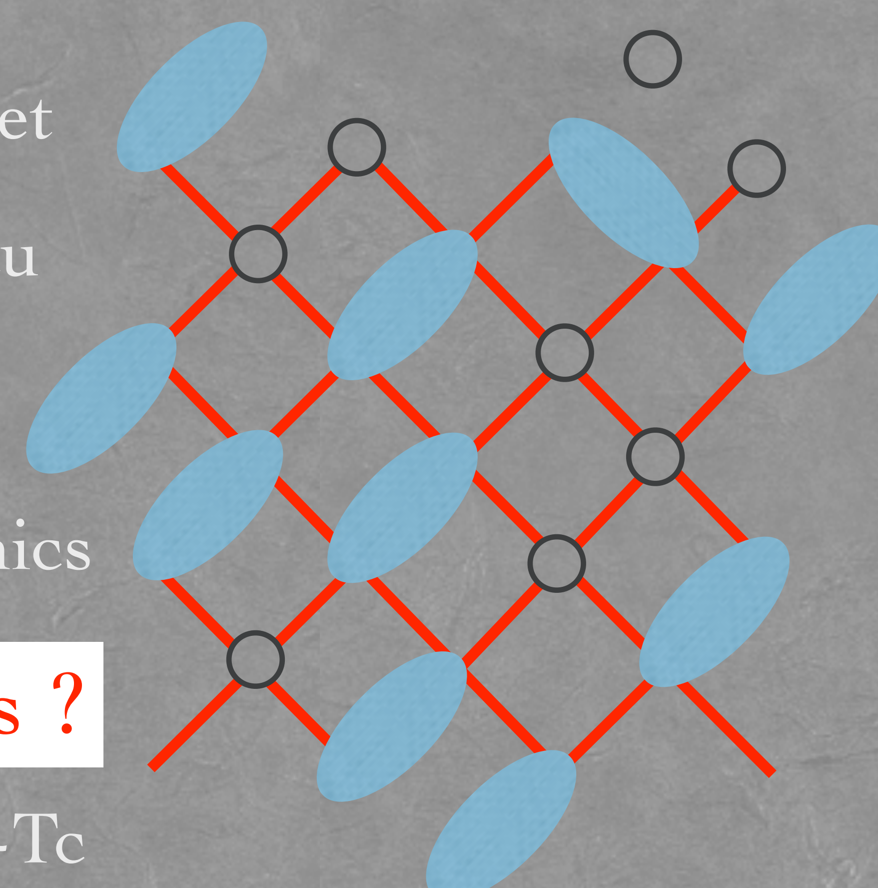
Increasing H ⇒ Triplet

Magnetization Plateau

Holon quantum dynamics

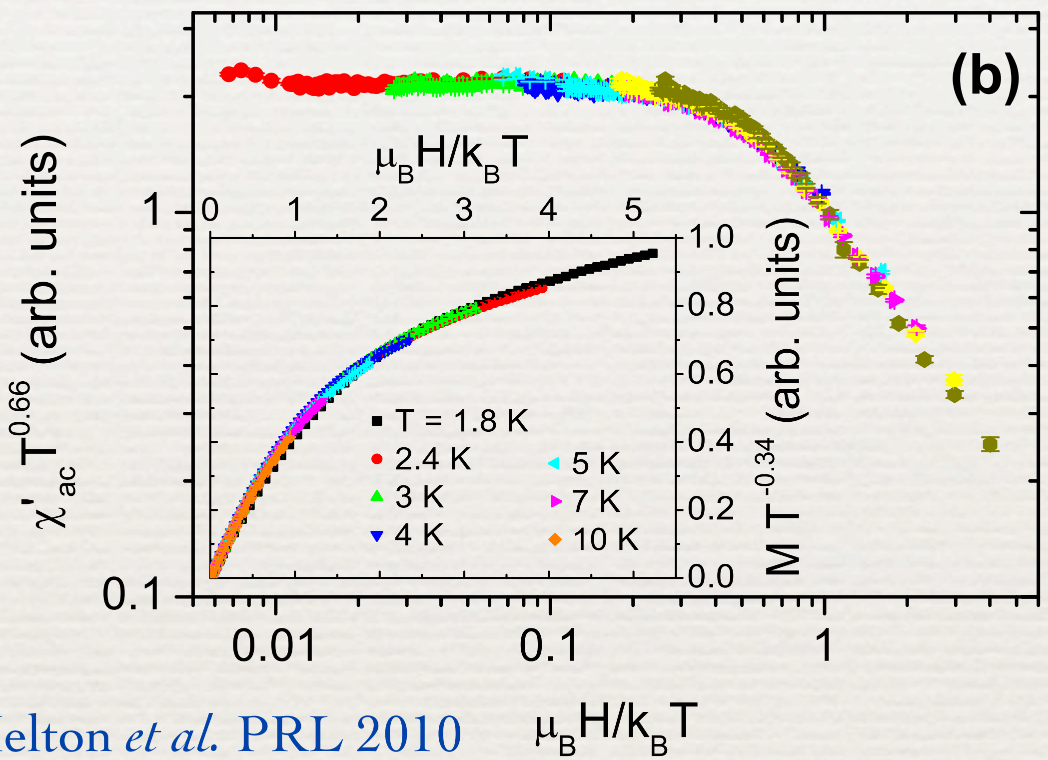
Statistics of Holons ?

Connection with High-Tc

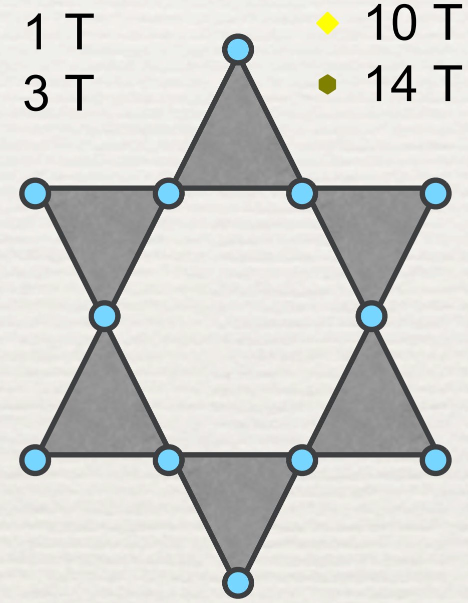


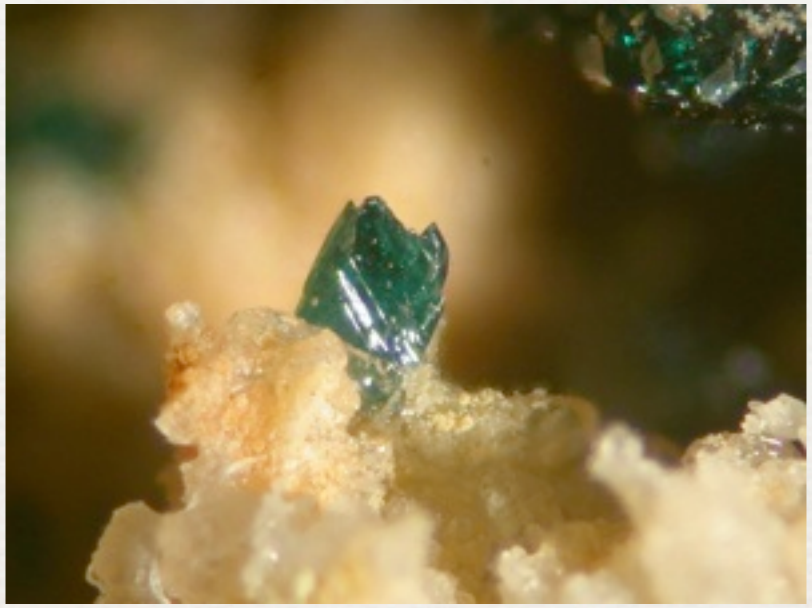
Dimers in the nature: $ZnCu_3(OH)_6Cl_2$

Most frustrated spin-1/2 system



Helton *et al.* PRL 2010



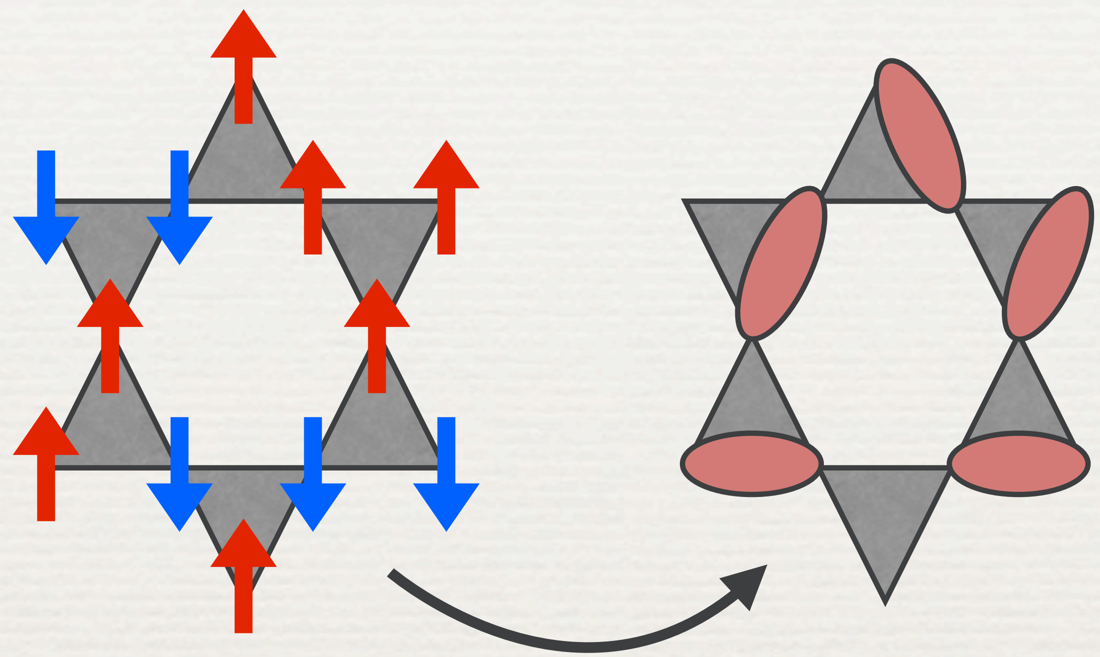


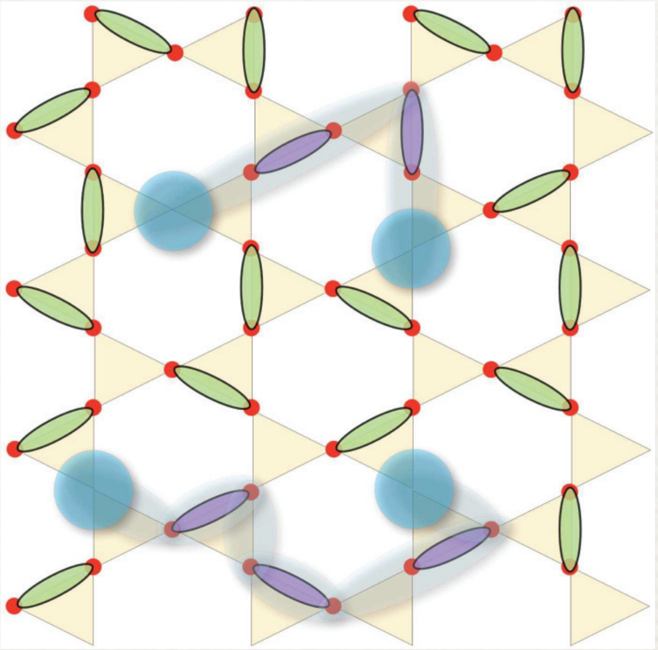
$$\chi(T) \simeq H/T^{1+0.66}$$

- "Ideal" spin-1/2 QAF
- Non magnetic order up to 50mK
  Mendels *et al.* PRL 2007
- Finite susceptibility (Z_2 liquid?)

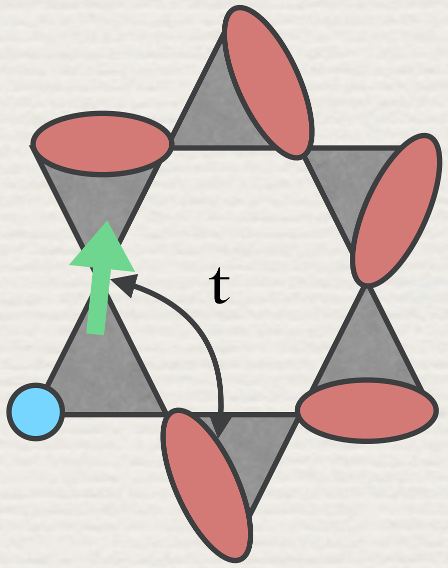
GS at the proximity of a QCP and/or influenced by disorder

Impurities in the Kagomé Spin-1/2 QAF





Real system: Zinc impurities ranging in 6% to 10%



- Hopping term added in the effective H
  Hao & Tcherbyshyov (2010)
- Possible comparison to NMR experiments

What is the parent insulating GS?

Strange critical behavior connected to holons

Doped Quantum Dimer models

Quantum Dimer Models

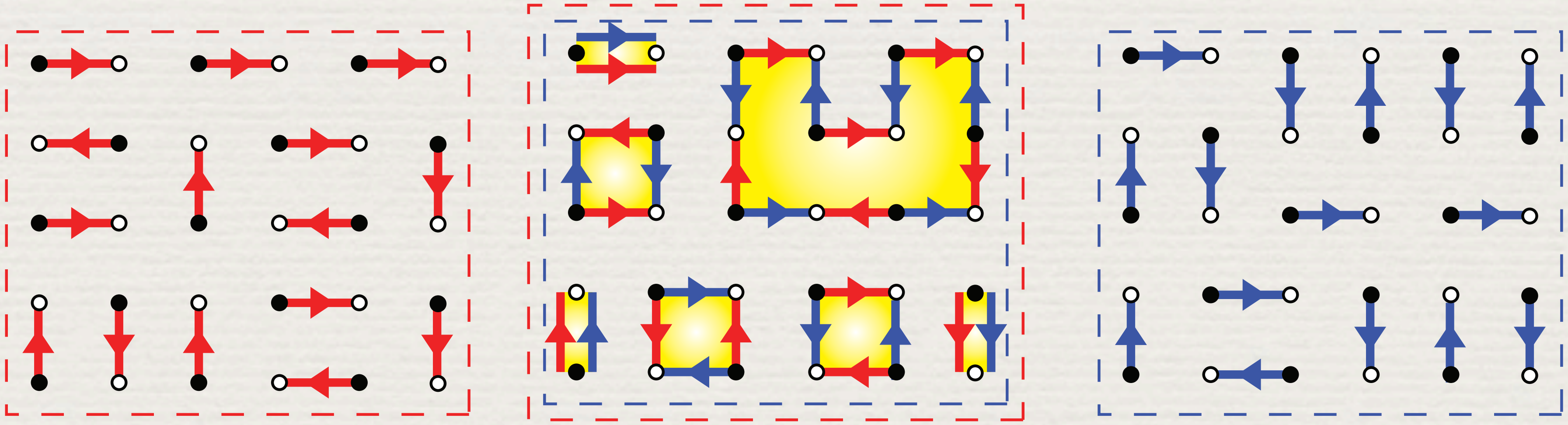
Effective models derived from microscopic systems

- Heisenberg Rokhsar & Kivelson (88), Moessner & Sondhi (01), **AR** *et al.* (11)
- Spin-orbital Vernay, **AR**, Becca, Mila (06)

Quantum Dimer Model: projection onto the singlet subspace

 $= \alpha(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$\alpha = \frac{1}{\sqrt{2}}$$



φ $\langle\varphi|\psi\rangle$ ψ

$$O_{\phi,\psi} = Id + 2\alpha^4 A + 2\alpha^6 B + \cdots$$

Sutherland (88)

$$\mathcal{H}^{\text{eff}}_{\phi,\psi} = \langle\phi|\mathcal{H}|\psi\rangle \simeq O_{\phi,\psi}$$

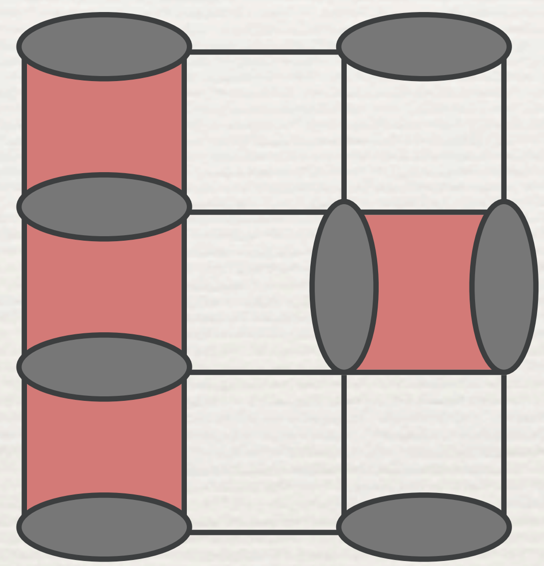
Quantum Dimer Models

Dimer background quantum dynamics

Rokhsar & Kivelson (88)

$$\mathcal{H} = v(|\ \rangle\langle\ | + |\ \rangle\langle\ |) - J(|\ \rangle\langle\ | + |\ \rangle\langle\ |)$$

- v: Potential term



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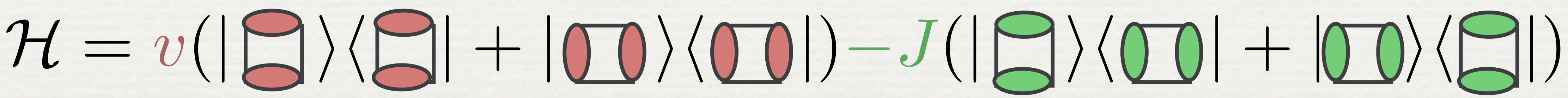
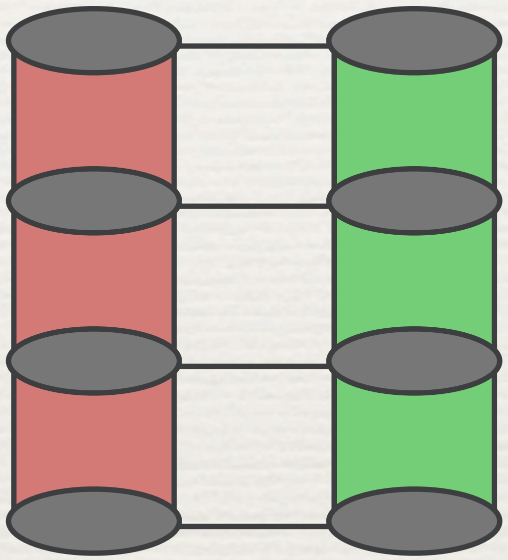


- v: Potential term
- J: Kinetic term



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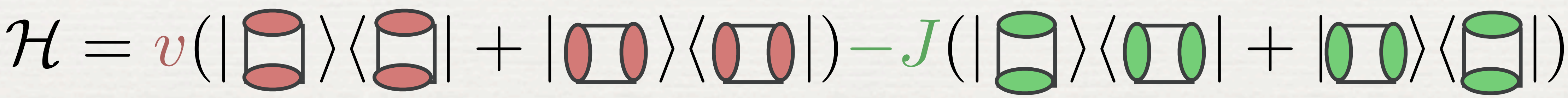
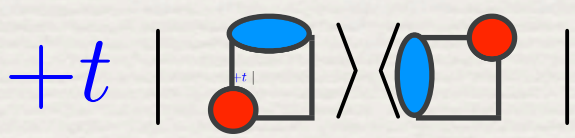
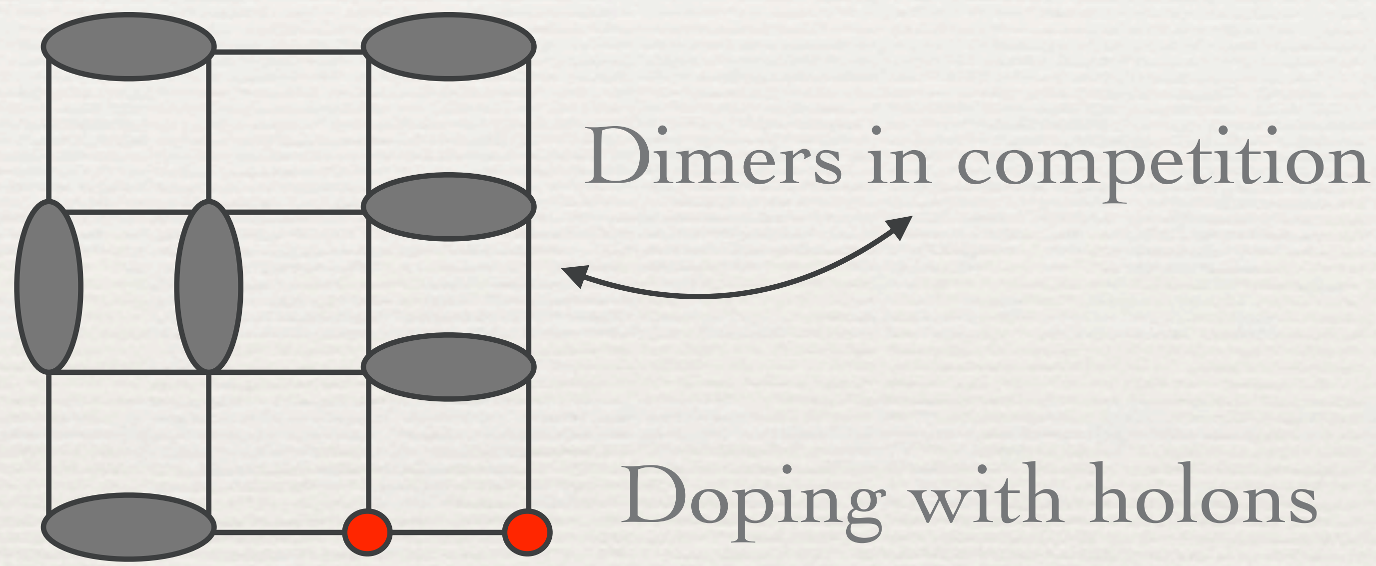


- v: Potential term
- J: Kinetic term
- t: Holon hopping term





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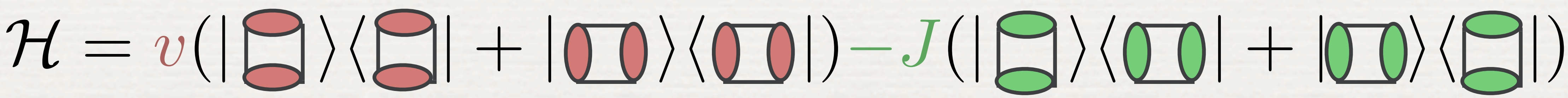

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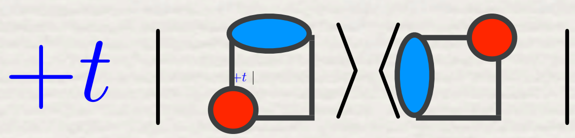
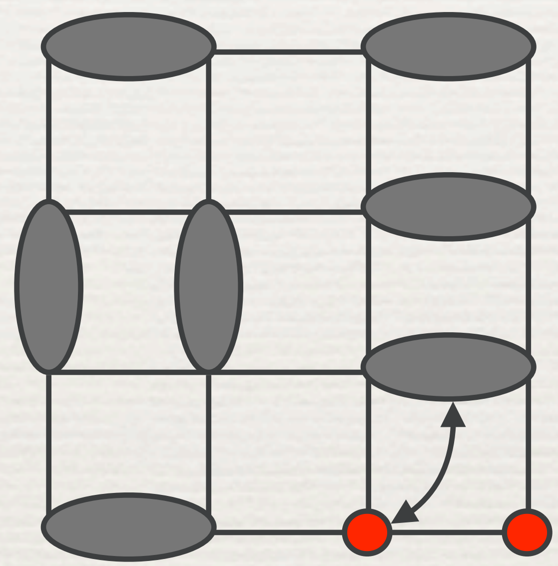
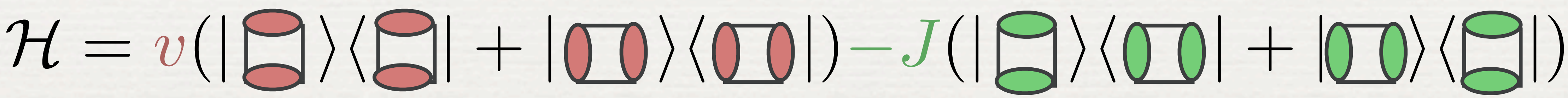




Quantum Dimer Models

Dimer background quantum dynamics

Rokhsar & Kivelson (88)



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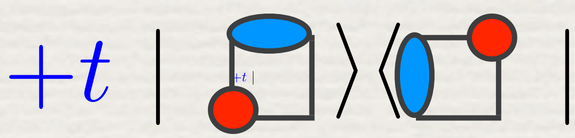

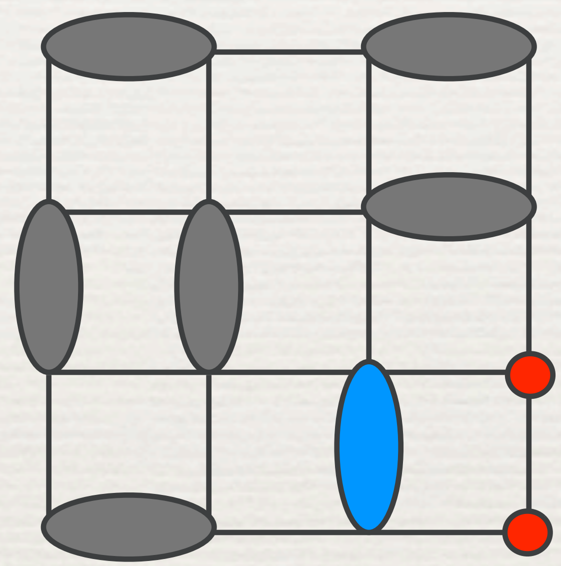

Quantum Dimer Models

Dimer background quantum dynamics

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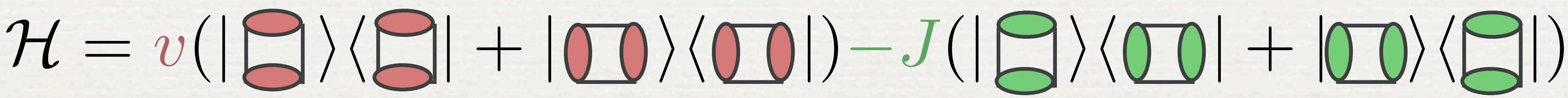

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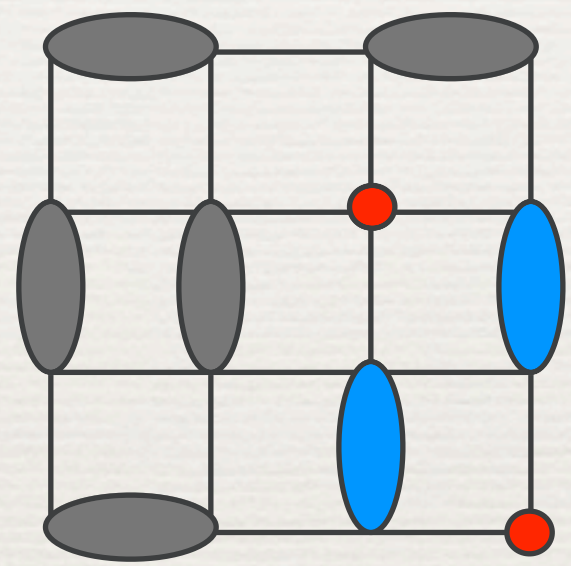
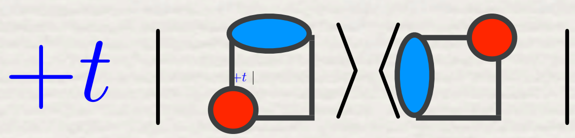




But what are the holon properties?

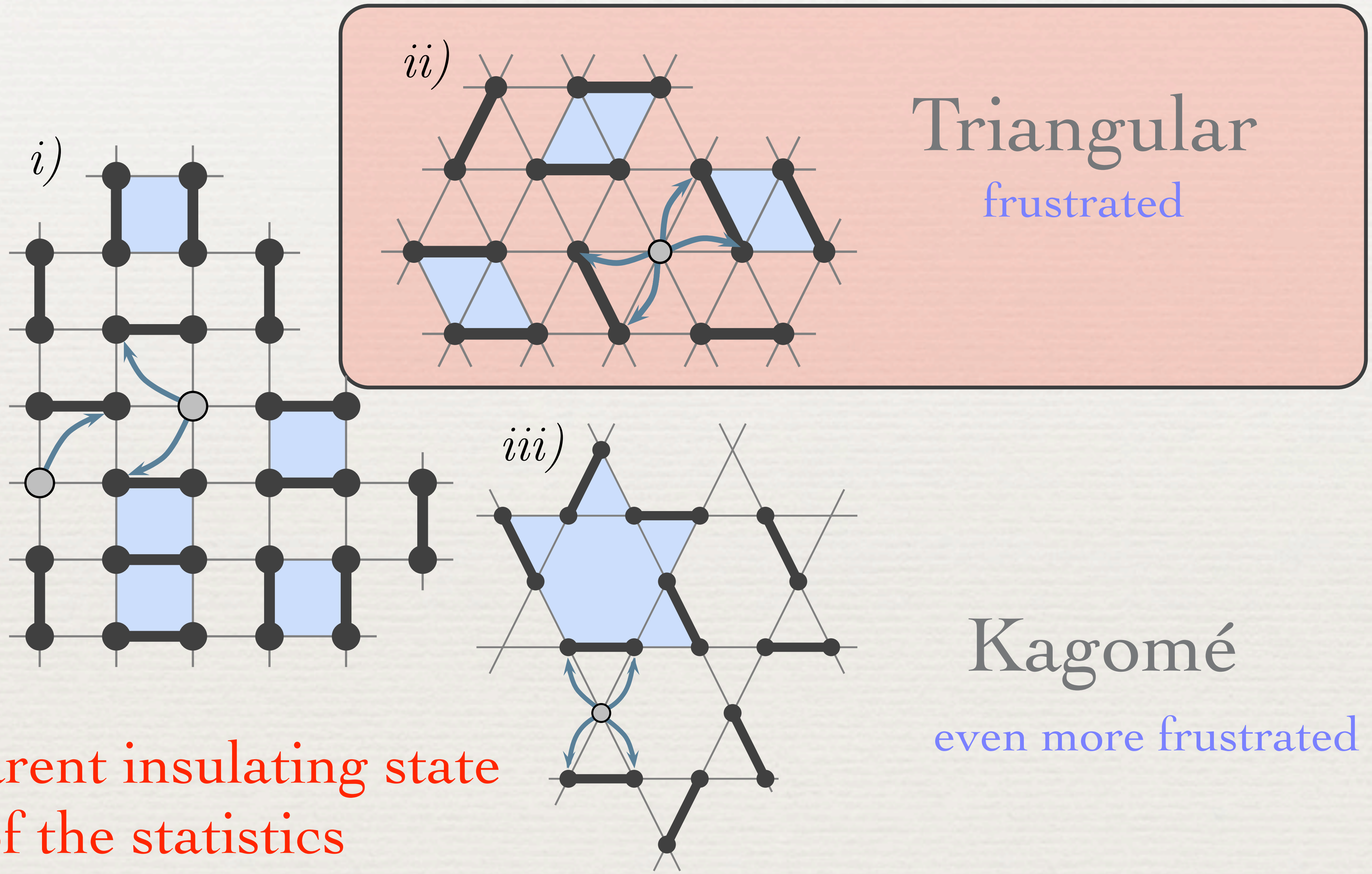
$$\mathcal{H} = \mathcal{H}_v + \mathcal{H}_J + \mathcal{H}_t$$

- Analitycal methods (Jordan-Wigner transformation)
- Exact diagonalizations, Quantum Monte-Carlo

AR *et al.* PRL(2012), PRB(2013)

Square

not frustrated



- role of the parent insulating state
- importance of the statistics
- effect of the frustration

How to properly define the holon statistics?

AR *et al.* PRL(2012), PRB(2013)

Complex structure of the Hamiltonian

- We start with the unfrustrated case J>0, t>0, bosons $\mathcal{H}_a$

$$\begin{tikzcd}[column sep=huge, row sep=large]
\mathcal{H}_a \arrow[rr, leftrightarrow, bend left, "t \leftrightarrow -t"] \arrow[d, leftrightarrow, "\mathcal{F} \leftrightarrow \mathcal{B}"'] & & \mathcal{H}_b \arrow[d, leftrightarrow, "\mathcal{F} \leftrightarrow \mathcal{B}"] \\
\mathcal{H}_c \arrow[rr, leftrightarrow, bend right, "t \leftrightarrow -t"'] & & \mathcal{H}_d
\end{tikzcd}$$

$$\mathcal{H} = \mathcal{H}_v + \mathcal{H}_J + \mathcal{H}_t$$

Four Hamiltonian classes

$$\begin{tikzcd}[column sep=huge]
\mathcal{H}_a \arrow[r, leftrightarrow, bend left, "J \leftrightarrow -J", "\mathcal{F} \leftrightarrow \mathcal{B}"'] & \mathcal{H}_{a'}
\end{tikzcd}$$

same spectrum !

Depending the sign of J, bosons transmutes in fermions !!!

2D Jordan-Wigner transformation

Fradkin (88), Wang (92)

- From fermionic to fractional statistics:

Fermion

$c_i c_j^+ + c_i^+ c_j = \delta_{i,j}$

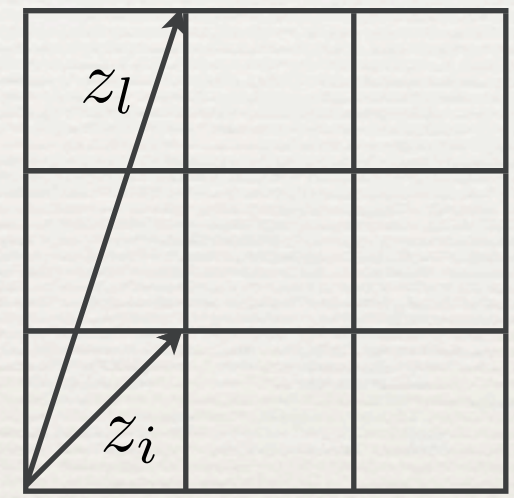
Anyons

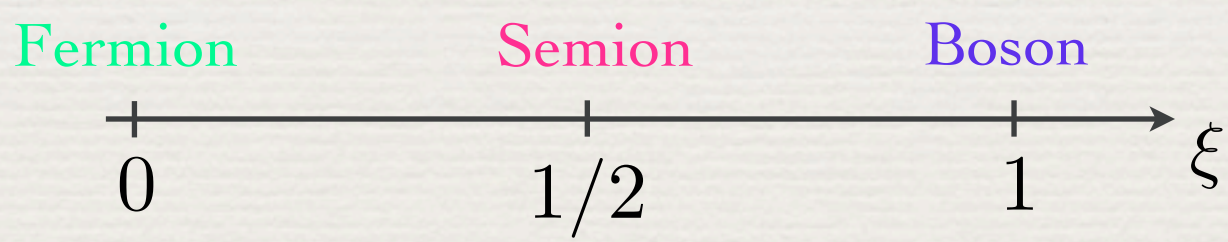
$$a_i^\dagger = e^{i\phi_i} c_i^\dagger$$

$a_i a_j^+ + e^{i\xi\pi} a_i^+ a_j = \delta_{i,j}$

$$-i2\xi \sum_{l \neq i} n_l \arg(z_i - z_l)$$

$\arg(z_i - z_j) = \arg(z_j - z_i) \pm \pi$



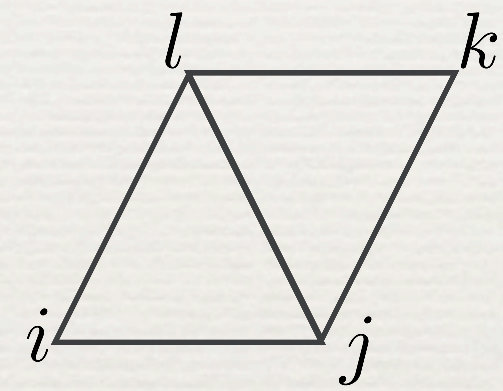


- Anyons are hard-core thanks to the Pauli principle of initial fermions
- Mean-field: a flux tube $\xi\phi_0$ attached on *each* electron

Aharonov-Bohm phase when 2 anyons are exchanged: $e^{\frac{e}{\hbar}\oint \vec{A}\cdot\vec{dl}} = e^{i\xi\pi}$

Application to $\mathcal{H} = \mathcal{H}_v + \mathcal{H}_J + \mathcal{H}_t$

- Rewrite dimers $b^{+}_{i,j} = \frac{1}{\sqrt{2}}\left(c^{+}_{i,\uparrow}c^{+}_{j,\downarrow} - c^{+}_{i,\downarrow}c^{+}_{j,\uparrow}\right)$
- Impose constraint on site i $\quad a^{+}_i a_i + \sum_{j\in n.n.} b^{+}_{i,j} b_{i,j} = 1$
- Write down the Hamiltonian



Dimer Kinetic $\quad h^{(J)}_{i,j,k,l} = -J(b^{+}_{i,j}b^{+}_{k,l}b_{j,k}b_{l,i} + h.c.)$

Dimer Potential $\quad h^{(v)}_{i,j,k,l} = v(b^{+}_{i,j}b_{i,j}b^{+}_{k,l}b_{k,l} + b^{+}_{j,k}b_{j,k}b^{+}_{l,i}b_{l,i})$

- Perform the Jordan-Wigner $\quad a_i = e^{-i\phi_i}c_i \qquad b_{i,j} = e^{-i(\phi_i+\phi_j)}\tilde{b}_{i,j}$

$$h^{(J)}_{i,j,k,l} \to h^{(\tilde{J})}_{i,j,k,l} = -\tilde{J}(\tilde{b}^{+}_{i,j}\tilde{b}^{+}_{k,l}\tilde{b}_{j,k}\tilde{b}_{l,i} + h.c.) \qquad \tilde{J} = -J \quad \text{(with fixed gauge)}$$

$$h^{(v)}_{i,j,k,l} \to h^{(\tilde{v})}_{i,j,k,l} = \tilde{v}(\tilde{b}^{+}_{i,j}\tilde{b}_{i,j}\tilde{b}^{+}_{k,l}\tilde{b}_{k,l} + \tilde{b}^{+}_{j,k}\tilde{b}_{j,k}\tilde{b}^{+}_{l,i}\tilde{b}_{l,i}) \qquad \tilde{v} = -v$$

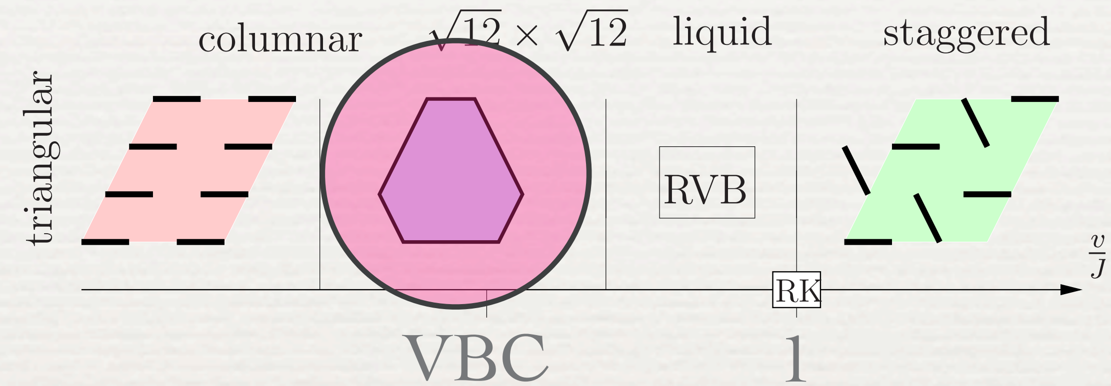
Equivalence proved

$$\boxed{\begin{array}{c}\tilde{J} \leftrightarrow -J \\ \mathcal{F} \leftrightarrow \mathcal{B}\end{array}}$$

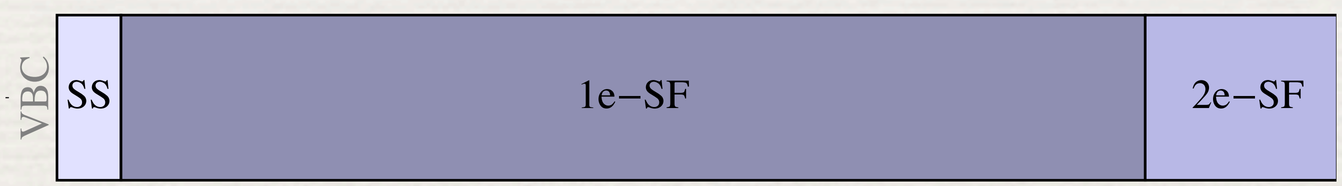
Quantum phase transitions

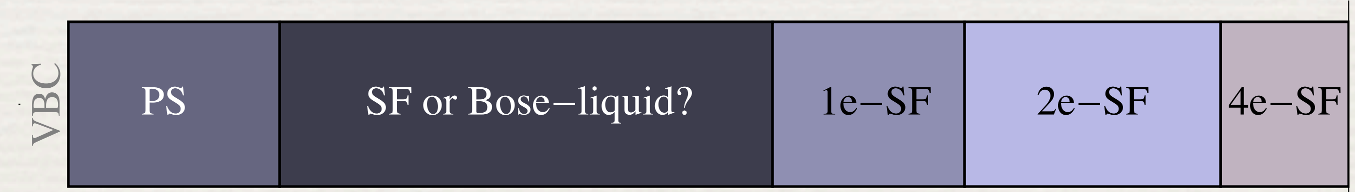
$$\mathcal{H} = \mathcal{H}_v + \mathcal{H}_J + \mathcal{H}_t$$

- Zero doping at J>0, rich phase diagram

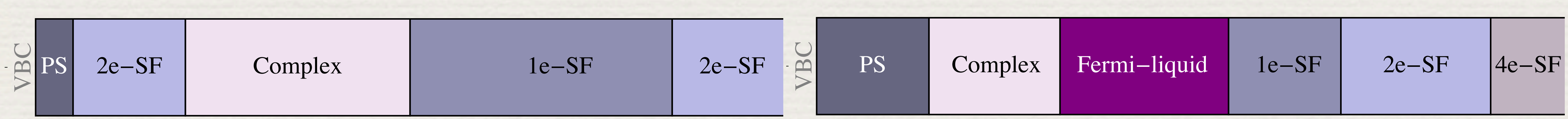


- Under doping : generic 1-e superconducting phase



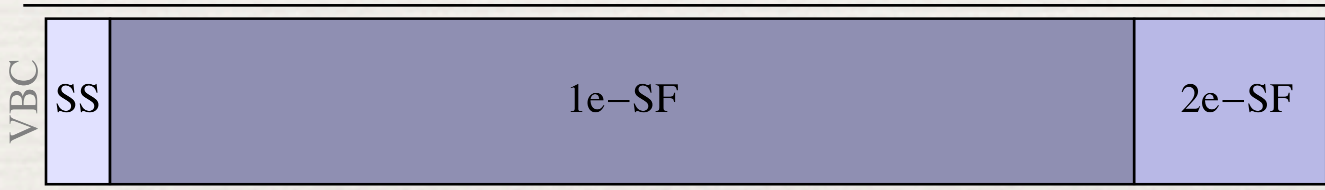


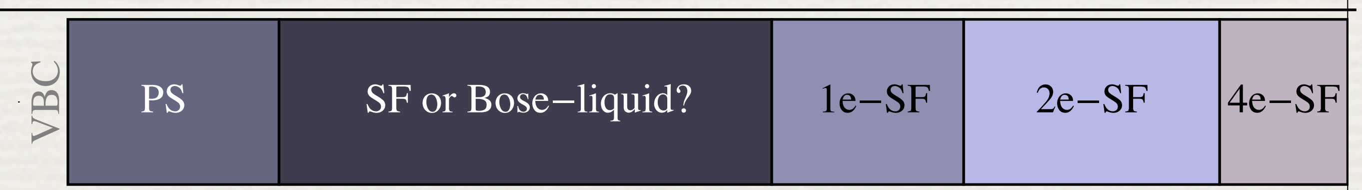
$$\begin{tikzcd} \mathcal{H}_a \arrow[r, bend left, "t \to -t"] \arrow[d, "\mathcal{B} \to \mathcal{F}"] & \mathcal{H}_b \arrow[d, "\mathcal{B} \to \mathcal{F}"] \\ \mathcal{H}_c \arrow[r, bend right, "t \to -t"] & \mathcal{H}_d \end{tikzcd}$$



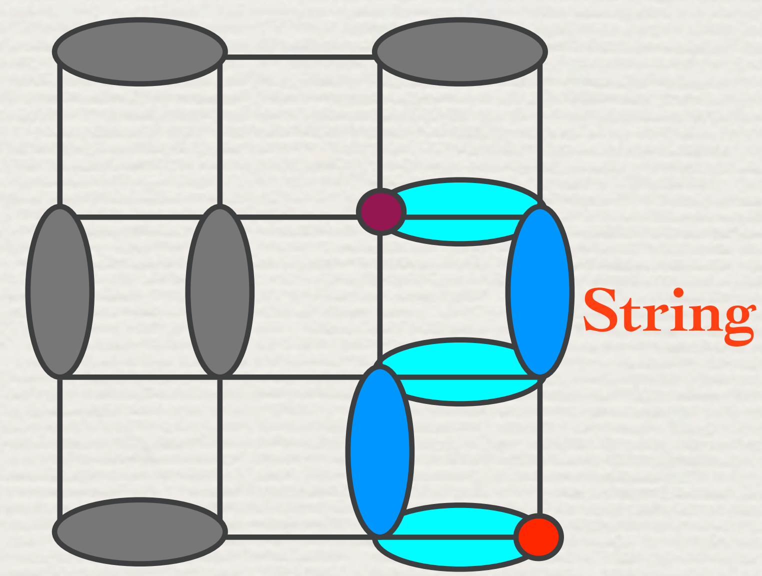
Quantum phase transitions

Observables→ Phases↓	κ	$\langle b^\dagger_{i,j} b_{i,j} b^\dagger_{k,l} b_{k,l}\rangle$	$\langle a^\dagger_k a^\dagger_l a_i a_j\rangle$	$\langle a^\dagger_i S_{i,j} a_j\rangle$	sgn_B	sgn_F	Flux periodicity
PS	<0						
VBC	>0	LR	SR	SR			$2e$
SS	>0	LR	LR	SR	1	0	$2e$
$2e$-SF	>0	SR	LR	SR	$0 < \mathrm{sgn}_B < 1$	$0 < \mathrm{sgn}_F < 1$	$2e$
e-SF	>0	SR	LR (weak)	LR	1	0	$2e$
Bose liquid	>0	SR	SR	SR	1	0	$2e$
Fermi liquid	>0	SR	SR	SR	0	1	$2e$
"Complex" phase	>0	SR	SR	SR	$0 < \mathrm{sgn}_B < 1$	$0 < \mathrm{sgn}_F < 1$	$2e$

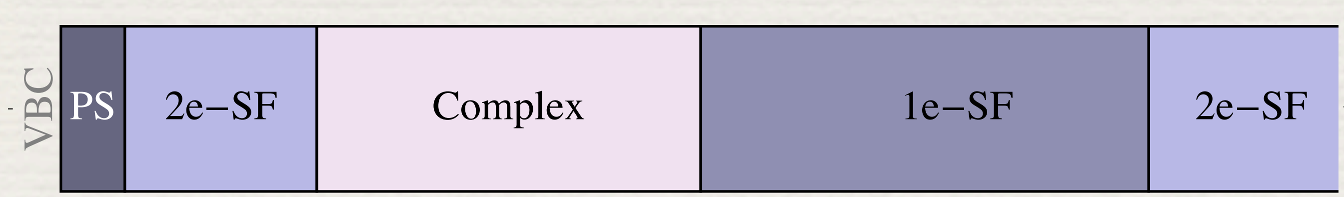


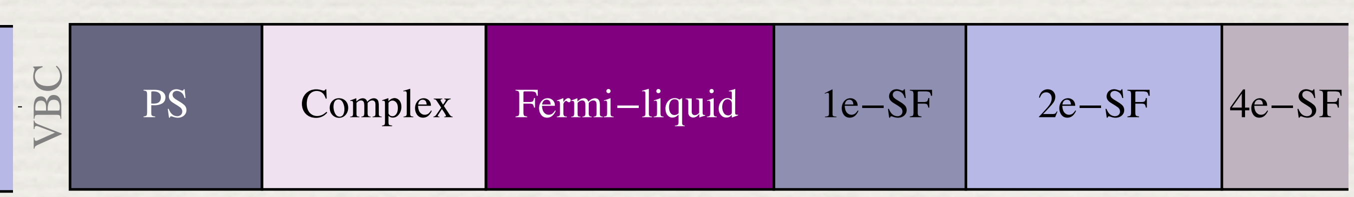


$$\begin{tikzcd} \mathcal{H}_a \arrow[r, bend left, "t \to -t"] \arrow[d, bend right, "\mathcal{B} \to \mathcal{F}"'] & \mathcal{H}_b \arrow[d, bend left, "\mathcal{B} \to \mathcal{F}"] \\ \mathcal{H}_c \arrow[r, bend right, "t \to -t"'] & \mathcal{H}_d \end{tikzcd}$$



e-SF phase





Thank you !