The Virgo detector

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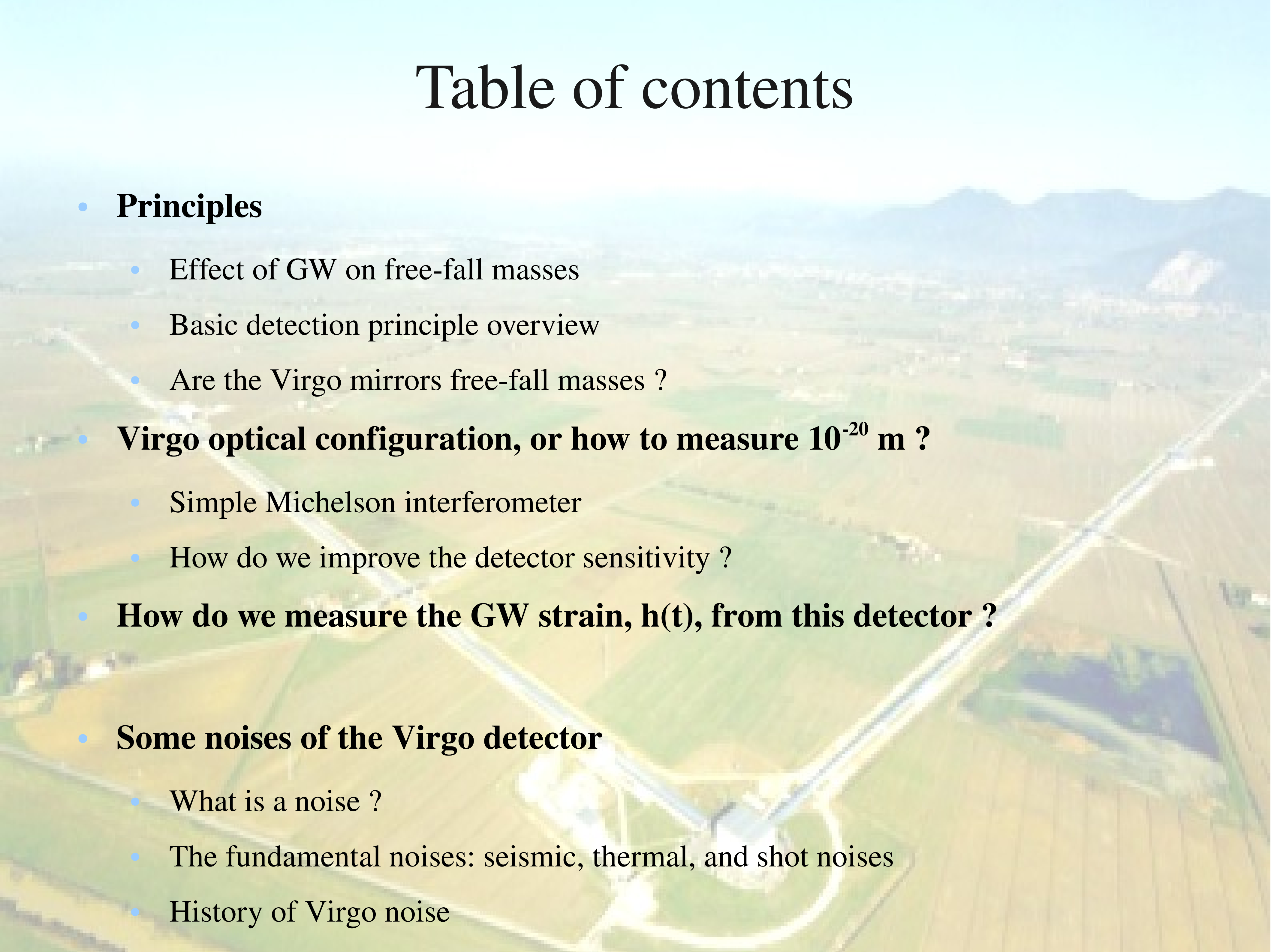
GraSPA summer school 2013



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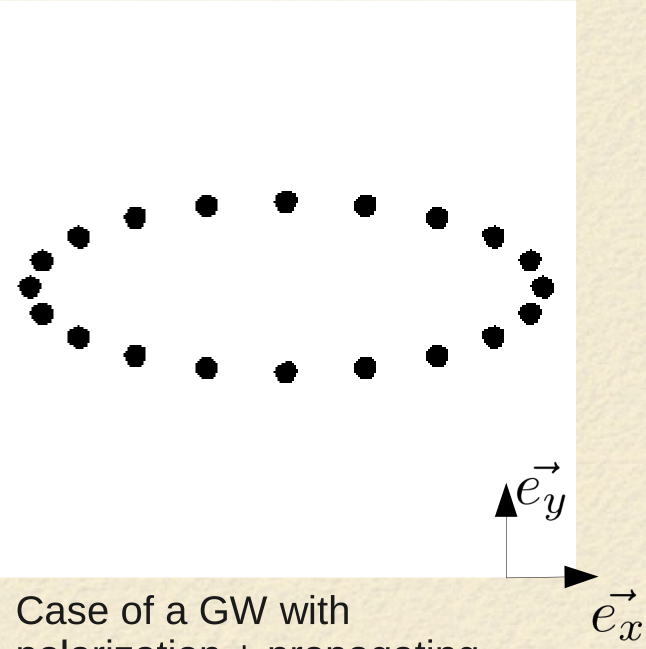
Reminder: effect of a GW on free masses

A gravitational wave (GW) modifies the distance between free-fall masses



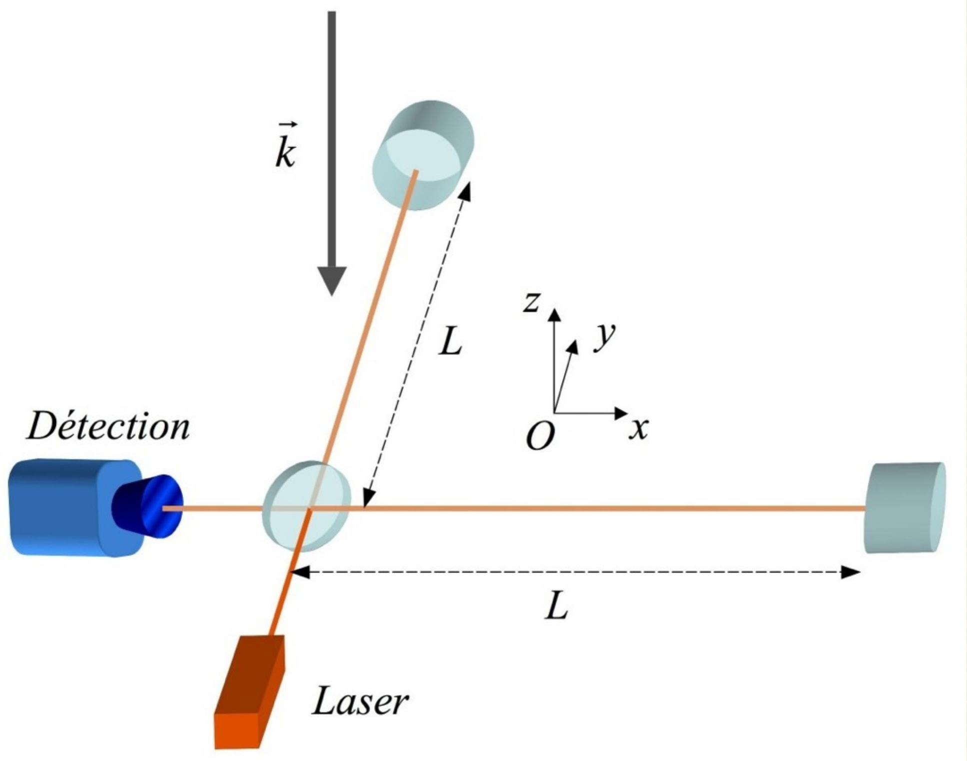
Case of a GW with polarization + propagating along z

$$\delta x(t) \;=\; -\delta y(t) \;=\; \tfrac{1}{2}\, h(t)\, L_0$$

Typical amplitude of a GW crossing the Earth:

$h \sim 10^{-23}$

A general overview of the Virgo detector



$$\Delta L(t) = l_{x(}t) - l_y(t)$$

$$\begin{aligned}\delta\Delta L(t) &= \delta l_x(t) - \delta l_y(t)\\ &= \frac{1}{2}\,h(t)\,L_0 \;-\; -\frac{1}{2}\,h(t)\,L_0\\ &= h(t)\,L_0\end{aligned}$$

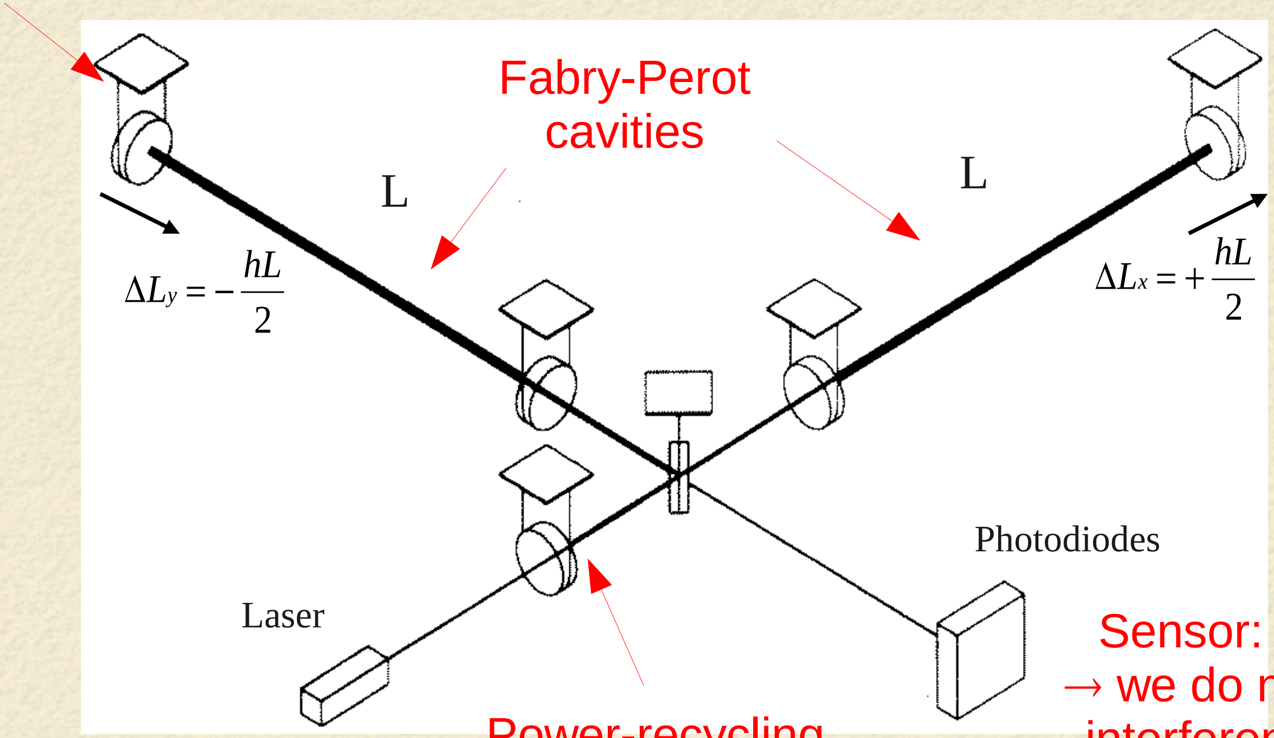
3 km arms !

Typical amplitude of a differential arm length variations when a GW crosses the Earth:

$$\delta\Delta L \sim 10^{-23} \times 3000$$

$$\delta\Delta L \sim 3 \times 10^{-20}\ \text{m} \qquad \sim \frac{\text{size of a proton}}{100000}$$

Virgo: a more complicated interferometer



Suspended mirrors

Fabry-Perot cavities

Power-recycling cavity

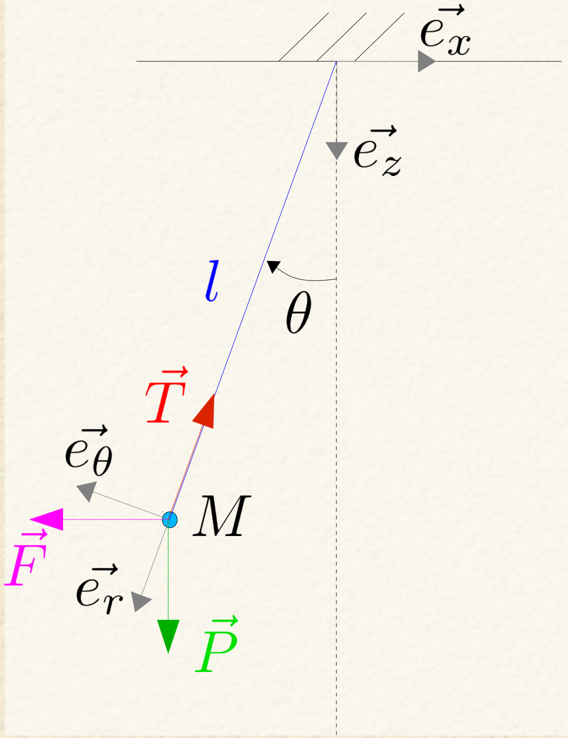
Infra-red laser
P ~ 100 W

Sensor: photodiode
→ we do not image the interference pattern !

Why are the Virgo mirrors free masses ?

We want the mirrors (mass M) to be free falling masses: $a = 0$

- In the case of sinusoidal regime: $\underline{x} = x_0 e^{-\jmath\omega t} \rightarrow a = -\omega^2 x_0 = 0$



Assuming that $\theta \ll 1$, we have $x = l\theta$

Newton's law, $M\vec{a} = \sum \vec{F}$, projected onto $\vec{e_\theta}$:

$$Ml\ddot{\theta} = -Mg\sin(\theta) + Fcos(\theta)$$

$$\ddot{x} + \omega_0^2 x = \frac{F}{M} \quad \text{with } \omega_0 = \sqrt{\frac{g}{l}}$$

In the case of sinusoidal regime: $(\omega_0^2 - \omega^2)x_0 = \frac{F_0}{M}$

If $\omega \gg \omega_0$, then $-\omega^2 x_0 = \frac{F_0}{M} = \text{a}$

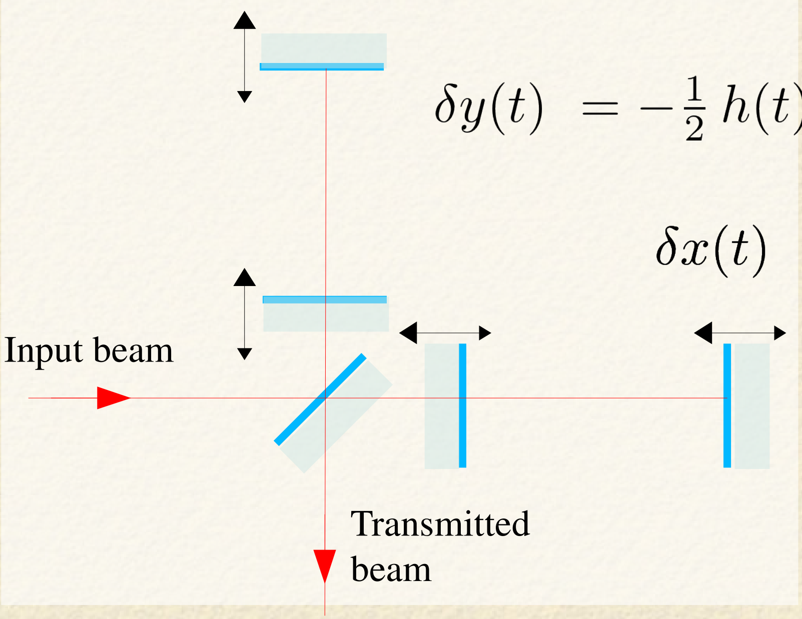
→ Mass M can be considered as free along x if $\omega \gg \omega_0$

The case of the Virgo mirrors

$$\left.\begin{array}{r} g = 9.81\ \mathrm{m.s}^{-2} \\ l = 0.7\ \mathrm{m} \end{array}\right\} \quad f_0 \sim 0.6\ \mathrm{Hz}$$

$$(M \sim 20\ \mathrm{kg})$$

→ Mirrors can be considered as free for frequencies larger than ~10 Hz



$$\delta y(t) \ = -\tfrac{1}{2}\, h(t)\, L_y$$

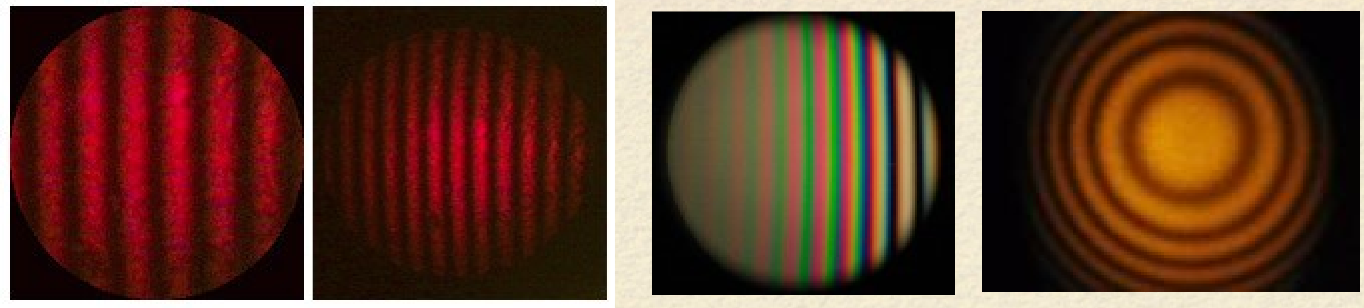
$$\delta x(t) \ = \tfrac{1}{2}\, h(t)\, L_x$$

$$\begin{aligned} \delta\Delta L &= \delta x(t) - \delta y(t) \\ &= h(t)\, L_0 \end{aligned}$$

$$\mathrm{h} \sim 10^{-23} \qquad L_0 = 3\,\mathrm{km}$$

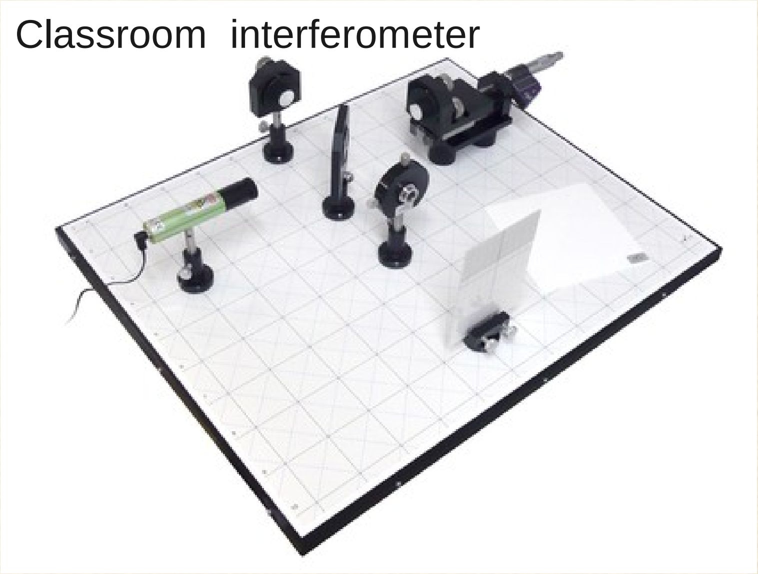
$$\longrightarrow \ \delta\Delta L \sim 3 \times 10^{-20}\ \mathrm{m}$$

How and for what did you use interferometers ?



Wavelength of monochromatic source

Sodium doublet wavelength separation

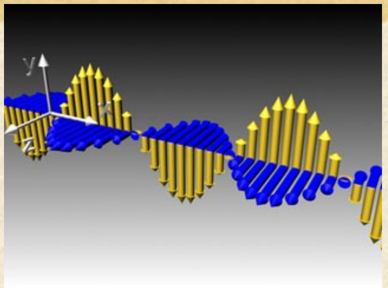


Classroom interferometer

Virgo interferometer
Pisa, Italy

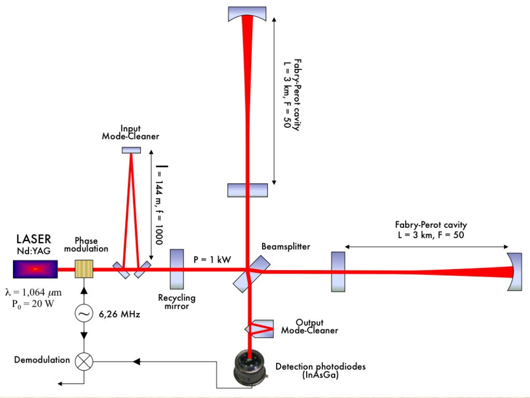
Virgo optical configuration

- Reminder about planes waves



- How do we “observe” ΔL with a Michelson interferometer ?
  - Measurement of a power variations
  - From power variations to ΔL (or to gravitational wave amplitude h)

- Improving the interferometer:
  - How do we increase the power on the beam-splitter mirror ?
  - How do we amplify the phase offset between the arms ?



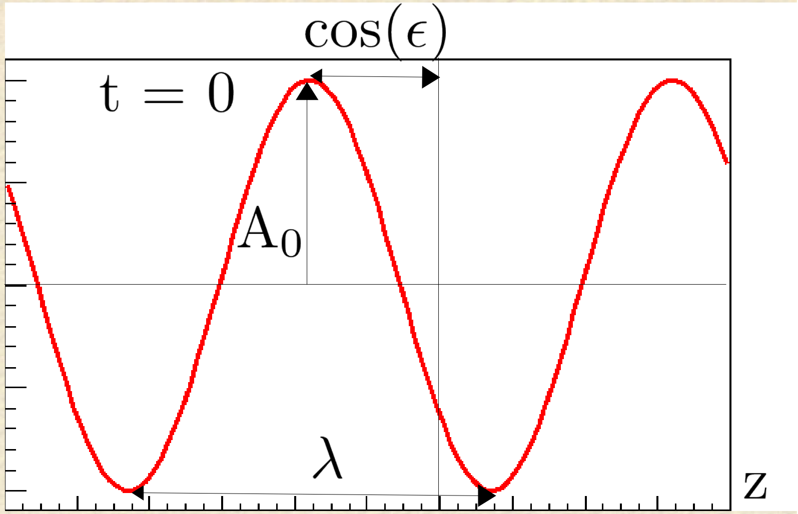
Description of plane waves

- Plane wave propagating along z, with speed c

$$A(z,t) = A_0 \cos(kz - \omega t + \epsilon) \qquad (\text{since } \vec{k}\vec{r} = k\,z)$$

$$\begin{cases} A_0 & \text{amplitude} \\ \lambda & \text{wavelength (m)} \\ k = \frac{2\pi}{\lambda} & \text{wave number (rad/m)} \\ \omega = kc & \text{angular frequency (rad/s)} \end{cases}$$

  - Average power: $\mathrm{P} \propto A_0^2$



- Complex form

$$U(z,t) = A_0 e^{\jmath(kz-\omega t+\epsilon)}$$
$$= \underline{A}_0 e^{\jmath(kz+\epsilon)} \qquad \text{with} \quad \underline{A}_0 = A_0 e^{-\jmath\omega t}$$

--> simpler algebraic calculations, for example $\mathrm{P} \propto |U|^2 = UU^*$

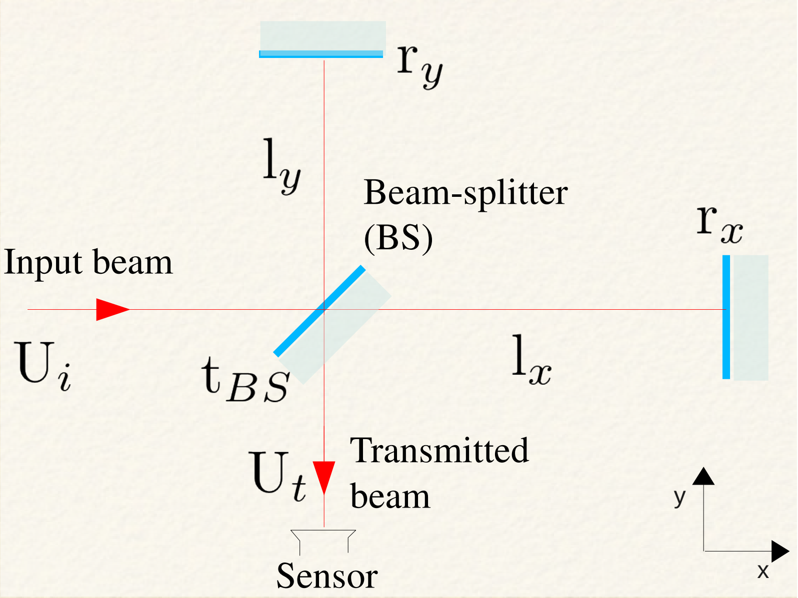
--> real plane wave is the real part: $\Re\big(U(z,t)\big) = A(z,t)$

- Plane waves do not exist but they are a good approximation of many waves in localized region of space

How do we “observe” ΔL with a Michelson interferometer ?

- Input wave $U_i(x,t) = \underline{A_i} e^{jkx}$
  $= \underline{A_i}$ on BS
- BS located at (0,0)
- Sensor located at $(0,-y_s)$
- Amplitude reflection and transmission coefficients: r and t

→ We are interested in the beam transmitted by the interferometer: it is the sum of the two beams (fields) that have propagated along each arm.



Around the beam-splitter mirrors:

- Radius of curvature of the beams ~ 1400 m
- Size of the beams ~ few cm

→ The beams can be approximated by plane waves

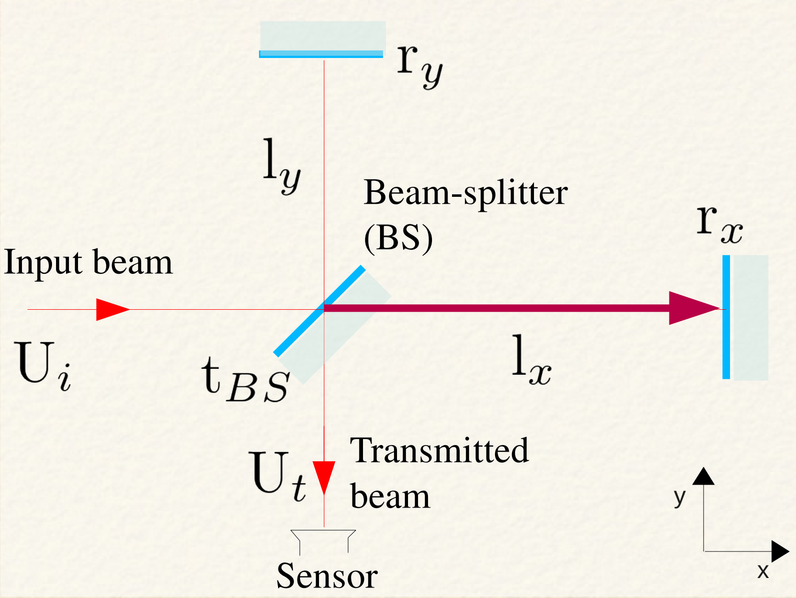
How do we “observe” ΔL with a Michelson interferometer ?

- Input wave $U_i(x,t) = \underline{\mathcal{A}}_i e^{\jmath kx}$

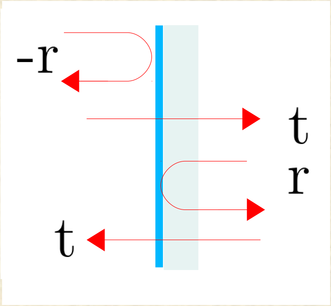
  $= \underline{\mathcal{A}}_i$ on BS

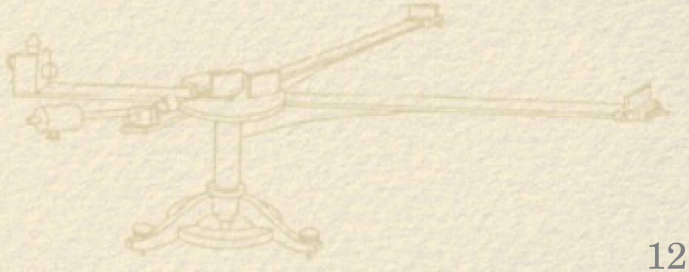
- Beam propagating along x-arm:

$$U_{tx} = \underline{\mathcal{A}}_i \, t_{BS} \, e^{\jmath k l_x} \;\; ......$$



Sign convention for amplitude reflection and transmission coefficients





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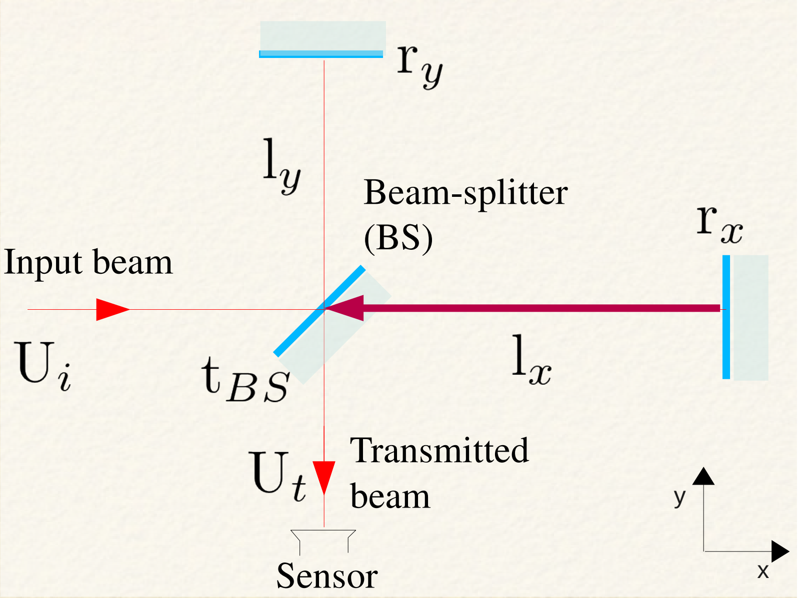
- Input wave $U_i(x,t) = \underline{A}_i e^{\jmath kx}$

$$= \underline{A}_i \quad \text{on BS}$$

- Beam propagating along x-arm:

$$U_{tx} = \underline{A}_i \, t_{BS} \, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \ldots\ldots$$



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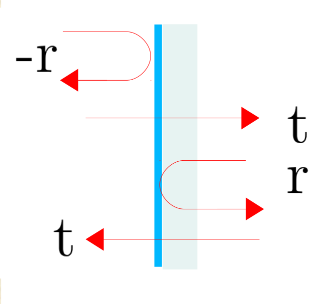

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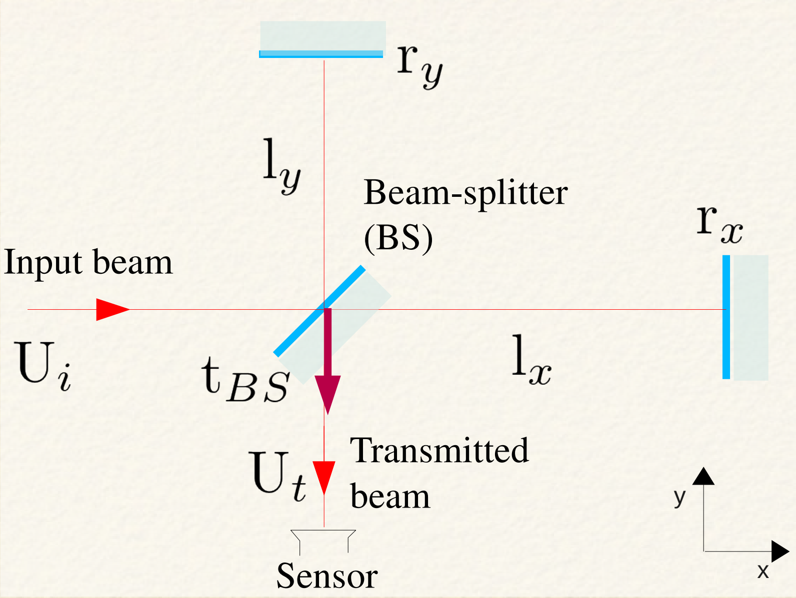
$$U_{tx} = \underline{A}_i \, t_{BS} \, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \quad r_{BS} \, e^{-\jmath k y_s}$$



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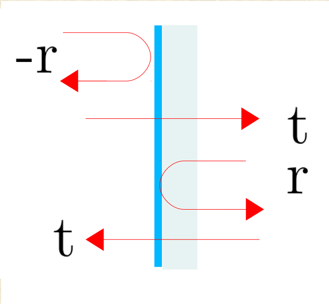
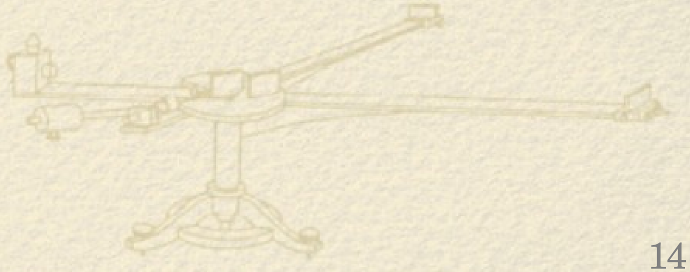




How do we “observe” ΔL with a Michelson interferometer ?

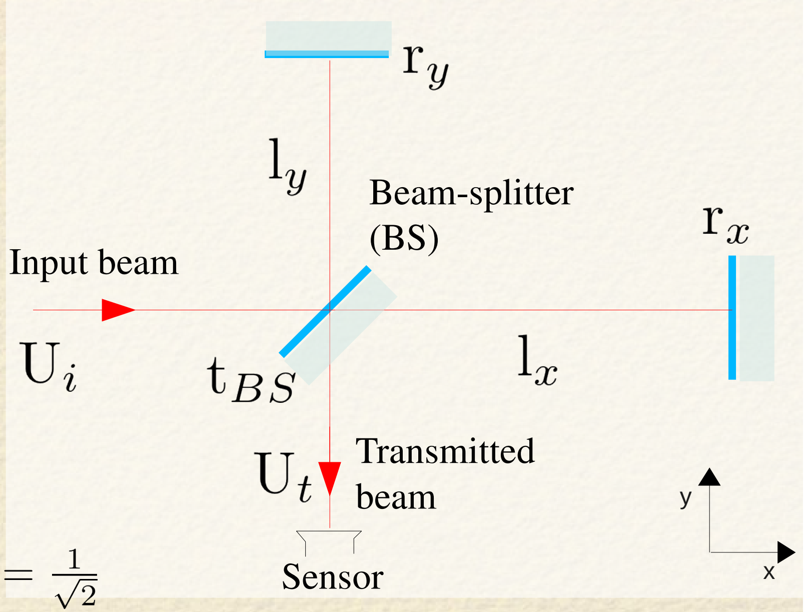
- Input wave $$U_i(x,t) = \underline{\mathcal{A}}_i e^{\jmath k x}$$ $$= \underline{\mathcal{A}}_i \quad \text{on BS}$$

- Beam propagating along x-arm:

$$U_{tx} = \underline{\mathcal{A}}_i \, t_{BS} \, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \quad r_{BS} \, e^{-\jmath k y_s}$$

$$= \underline{\mathcal{A}}_i \, t_{BS} \, r_{BS} \, (-r_x) \, e^{2\jmath k l_x} \, e^{-\jmath k y_s}$$

$$= \frac{\underline{\mathcal{A}}_i}{2} \times \underbrace{\left(- r_x \, e^{2\jmath k l_x} \right)}_{\text{Complex reflection of the x-arm}} e^{-\jmath k y_s} \quad \text{with } t_{BS} = r_{BS} = \frac{1}{\sqrt{2}}$$



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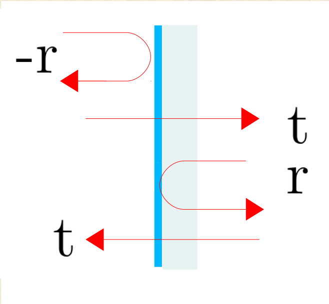
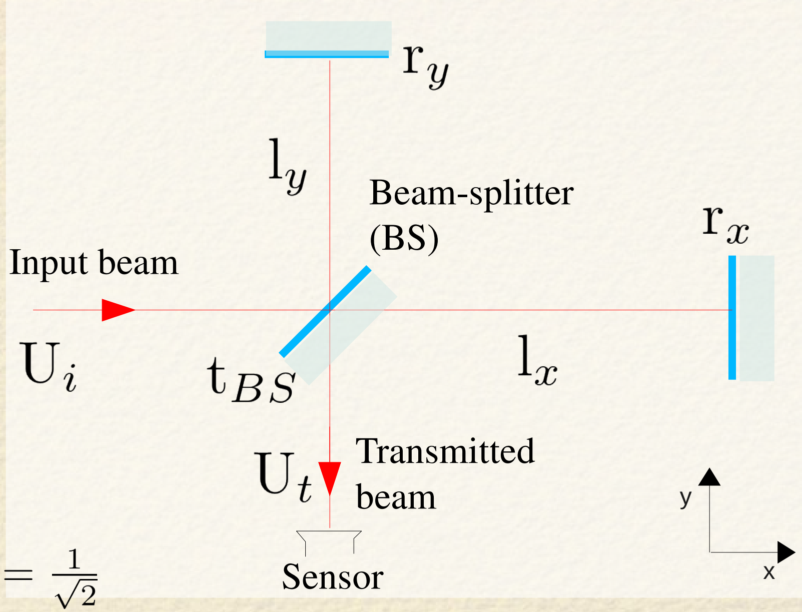


How do we “observe” ΔL with a Michelson interferometer ?



- Input wave $U_i(x,t) = \underline{\mathcal{A}}_i e^{\jmath kx}$

$$= \underline{\mathcal{A}}_i \quad \text{on BS}$$

- Beam propagating along x-arm:

$$U_{tx} = \underline{\mathcal{A}}_i\, t_{BS}\, e^{\jmath k l_x} \quad (-r_x) e^{\jmath k l_x} \quad r_{BS}\, e^{-\jmath k y_s}$$

$$= \underline{\mathcal{A}}_i\, t_{BS}\, r_{BS}\, (-r_x)\, e^{2\jmath k l_x}\, e^{-\jmath k y_s}$$

$$= \frac{\underline{\mathcal{A}}_i}{2} \times \underbrace{\left(- r_x\, e^{2\jmath k l_x} \right)}_{\text{Complex reflection of the x-arm}} e^{-\jmath k y_s} \quad \text{with } t_{BS} = r_{BS} = \frac{1}{\sqrt{2}}$$

$+$

- Beam propagating along y-arm:

$$U_{ty} = -\frac{\underline{\mathcal{A}}_i}{2} \times \underbrace{\left(- r_y\, e^{2\jmath k l_y} \right)}_{\text{Complex reflection of the y-arm}} e^{-\jmath k y_s}$$

- Transmitted field:

$$U_t = U_{tx} + U_{ty}$$

$$= \frac{\underline{\mathcal{A}}_i}{2}\, e^{-\jmath k y_s} \left(r_y\, e^{2\jmath k l_y} \; - \; r_x\, e^{2\jmath k l_x} \right)$$

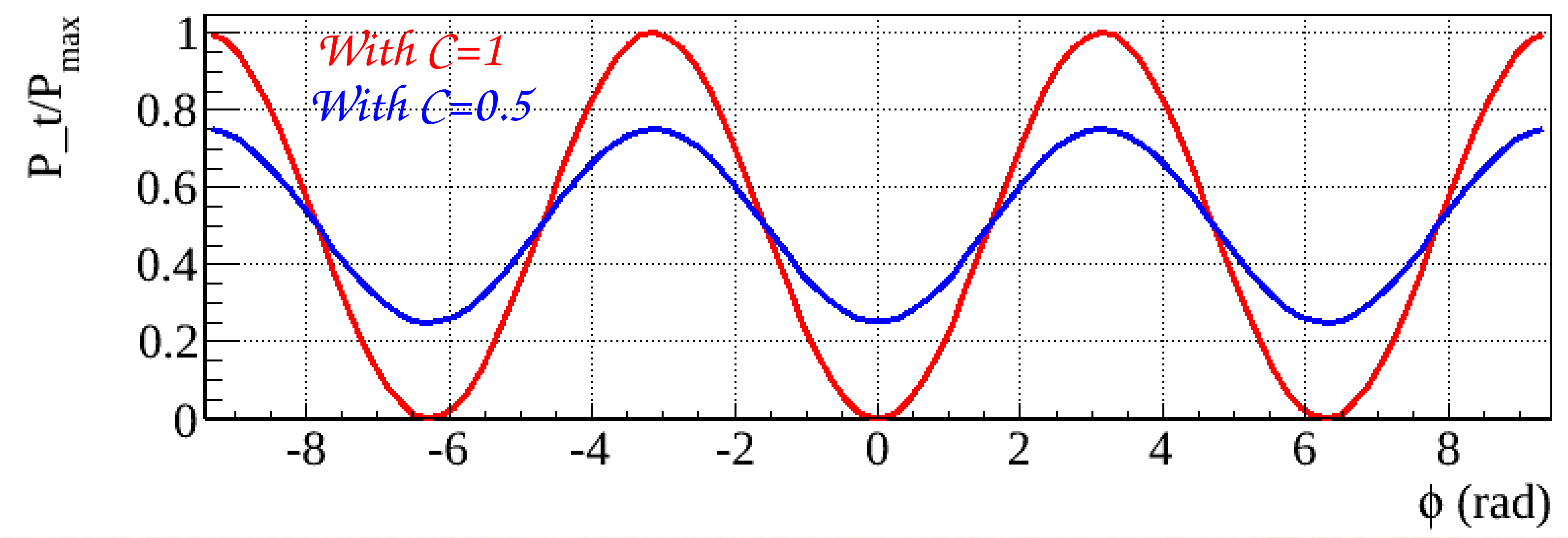
Power transmitted by a simple Michelson

- Transmitted field: $U_t = \frac{\mathcal{A}_i}{2} e^{-\jmath k y_s} \left(r_y \, e^{2\jmath k l_y} - r_x \, e^{2\jmath k l_x} \right)$
- Calculation of the transmitted power:

$$P_t \propto |U_t|^2 = \frac{P_{max}}{2} \left(1 - C \cos(\phi)\right) \qquad \text{where } \phi = 2k(l_y - l_x)$$

$$C = 2 \frac{r_x r_y}{r_x^2 + r_y^2}$$

$$P_{max} = \frac{P_i}{2}(r_x^2 + r_y^2)$$

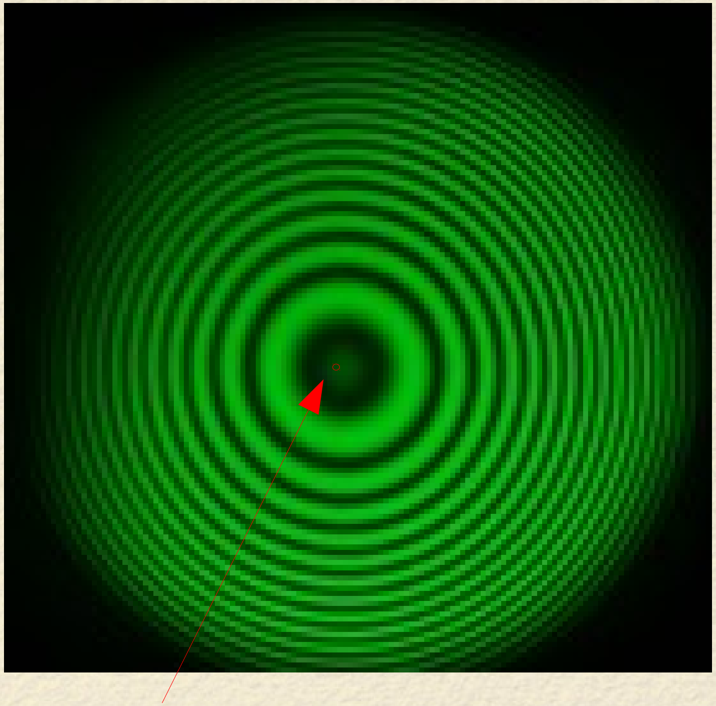


What power does Virgo measure ?

- In general, the beam is not a plane wave but a spherical wave

  → interference pattern

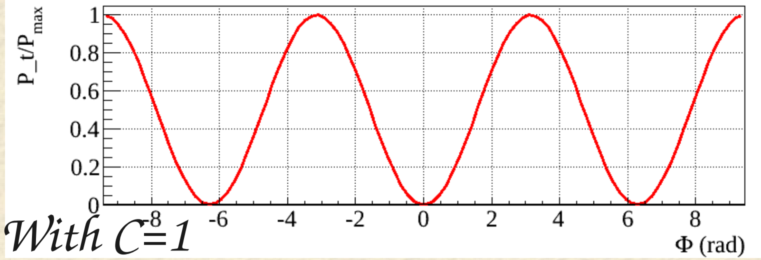
  (and the complementary pattern in reflection)



Equivalent size of Virgo beam

- Virgo interference pattern much larger than the beam size: ~1 m between 2 two consecutive fringes

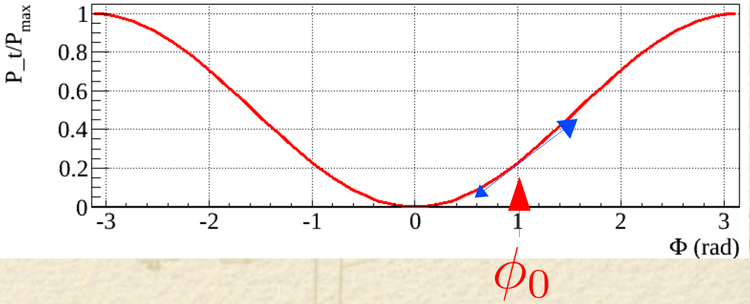
  → we do not study the fringes in nice images !



With C=1

Freely swinging mirrors

Setting a working point →



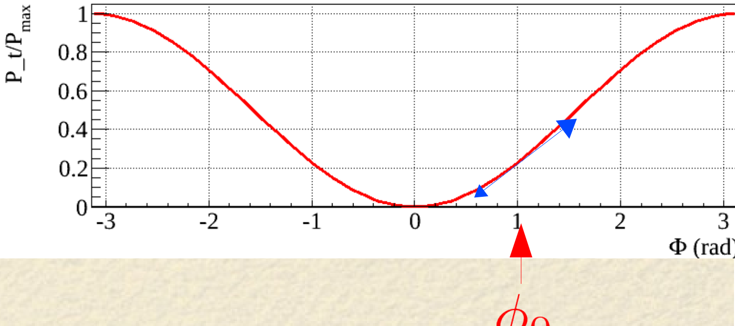
Controlled mirror positions

From the power to the gravitational wave

$$P_t = \frac{P_i}{2}\left(1 - C\,\cos(\phi)\right) \qquad \text{where } \phi = 2\frac{2\pi}{\lambda}(l_y - l_x)$$

- Around the working point:

$$\left.\frac{\mathrm{d}P_t}{d\phi}\right|_{\phi_0} = \frac{P_i}{2}C\,\sin(\phi_0) \qquad \text{where } \phi_0 = \frac{4\pi}{\lambda}\Delta L_0$$



- Power variations as function of small differential length variations:

$$\delta P_t = \frac{P_i}{2}C\,\sin(\phi_0)\delta\phi$$

$$\delta P_t = P_i\,C\,\frac{2\pi}{\lambda}\sin\left(\frac{4\pi}{\lambda}\Delta L_0\right)\delta\Delta L$$

$$\boxed{\delta P_t \propto \delta\Delta L = hL_0 \quad \text{around the working point !}}$$

From the power to the gravitational wave

- Around the working point:

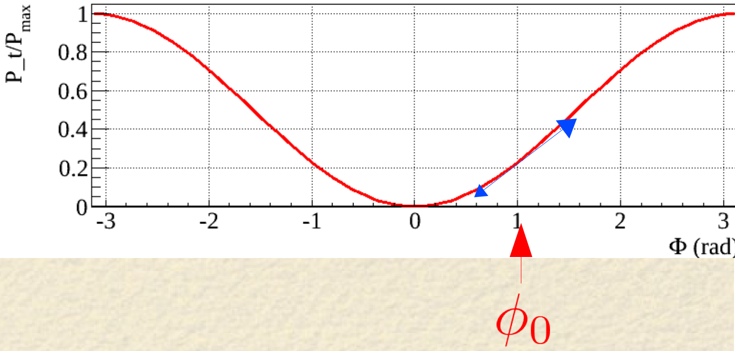
$$\delta P_t = \underbrace{P_i\, C\, \frac{2\pi}{\lambda}\, \sin\left(\frac{4\pi}{\lambda}\Delta L_0\right)}\, \delta\Delta L$$

$$\delta P_t = \text{(Interferometer response)} \times \delta\Delta L$$

(W/m)

Measurable physical quantity

Physical effect to be detected



Improving the interferometer sensitivity

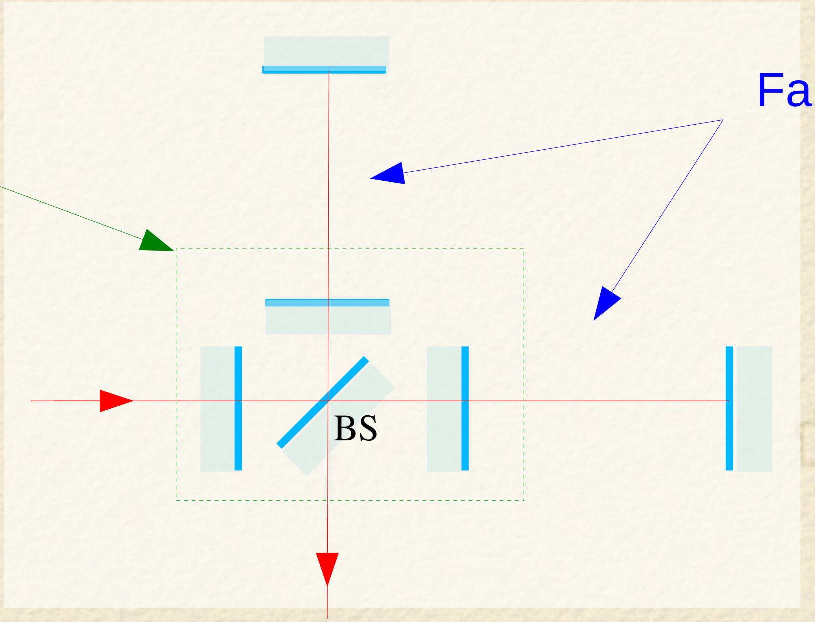
$$\delta P_t = P_i\, C\, \sin\left(\frac{2\pi}{\lambda}\Delta L_0\right) \underbrace{\left(2\,k\,\delta\Delta L\right)}_{\delta\phi}$$

Increase the input power

Increase the phase difference between the arms for a given differential arm length variation

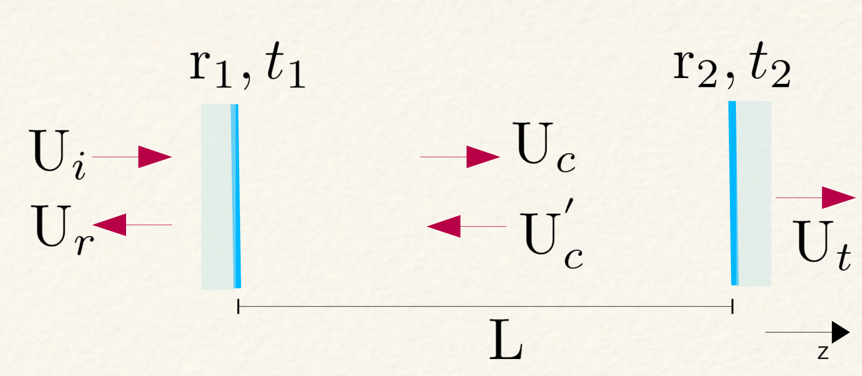
Recycling **cavity**

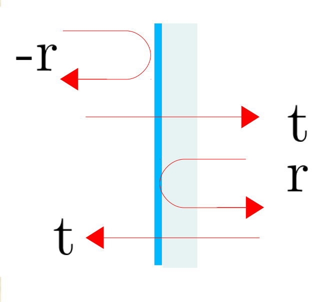
Fabry-Perot **cavities** in the arms



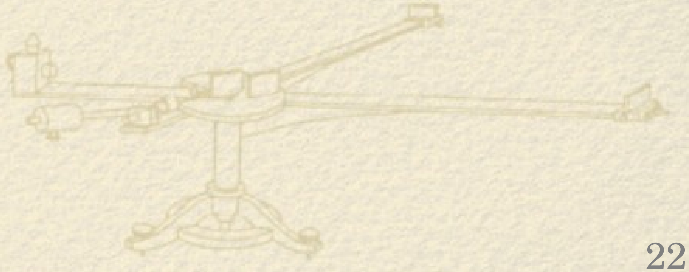
Optical cavity with two mirrors

- Cavity made of two plane infinite mirrors, in front of each other.





Sign convention for amplitude reflection and transmission coefficients



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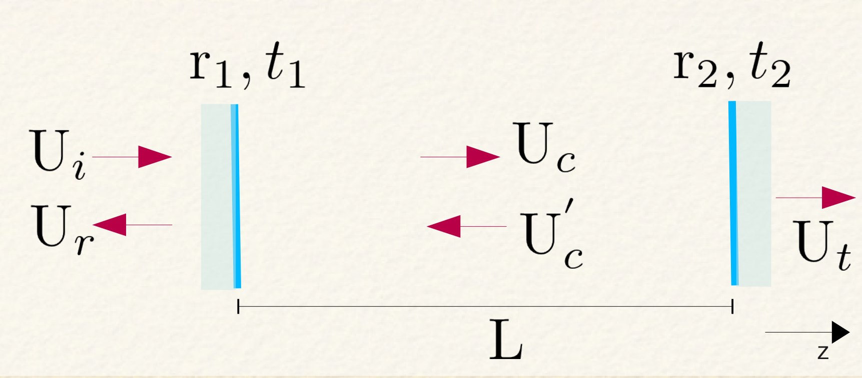
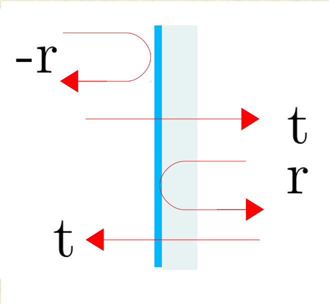


$$
\begin{aligned}
U_i &= A_i\, e^{\jmath(kz-\omega t)} & U_c^{'} &= A_c^{'}\, e^{\jmath(-kz-\omega t)} \\
U_c &= A_c\, e^{\jmath(kz-\omega t)} & U_r &= A_r\, e^{\jmath(-kz-\omega t)} \\
U_t &= A_t\, e^{\jmath(kz-\omega t)} & &
\end{aligned}
$$

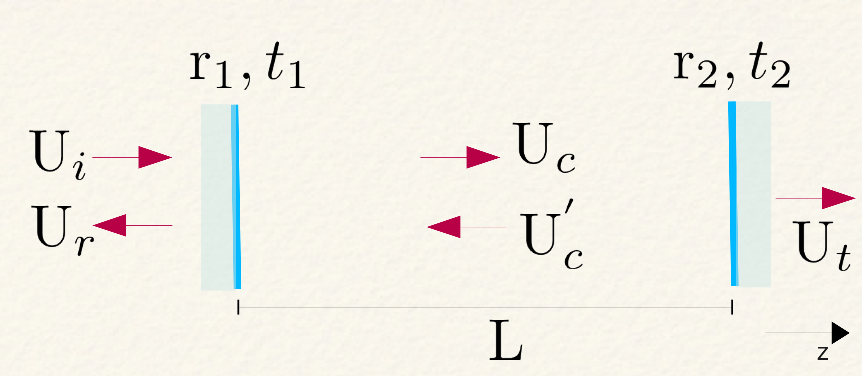


Sign convention for amplitude reflection and transmission coefficients

- Relations between the fields at input and output of the cavity:

$$
\left\{
\begin{aligned}
U_c(z=0) &= t_1\, U_i(0) \;-\; r_1\, U_c^{'}(0) \\
U_r(z=0) &= t_1\, U_c^{'}(0) \;+\; r_1\, U_i(0) \\
U_t(z=L) &= t_2\, U_c(L) \\
U_c^{'}(z=L) &= -r_2\, U_c(L)
\end{aligned}
\right.
\quad\longrightarrow\quad
\left\{
\begin{aligned}
A_c &= t_1\, A_i \;-\; r_1\, A_c^{'} \\
A_r &= t_1\, A_c^{'} + r_1\, A_i \\
A_t\, e^{\jmath kL} &= t_2\, A_c\, e^{\jmath kL} \\
A_c^{'}\, e^{-\jmath kL} &= -r_2\, A_c\, e^{+\jmath kL}
\end{aligned}
\right.
$$

In Virgo, the beam is resonant inside the cavities



- Cavity field as function of input field:

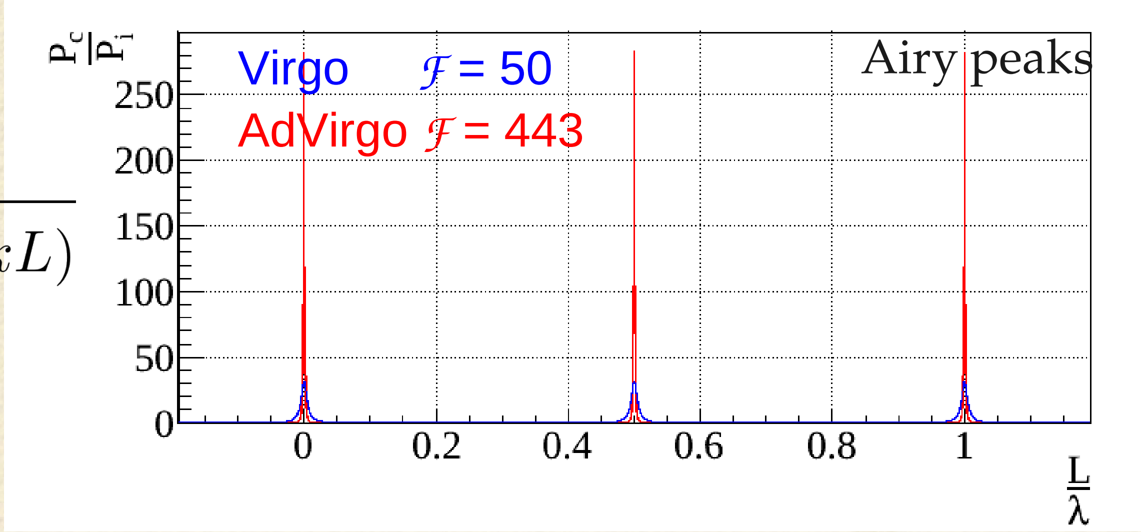
$$A_c = \frac{t_1}{1 - r_1 r_2 e^{2\jmath kl}} A_i$$

- Power in the cavity:

$$P_c \propto |A_c|^2$$

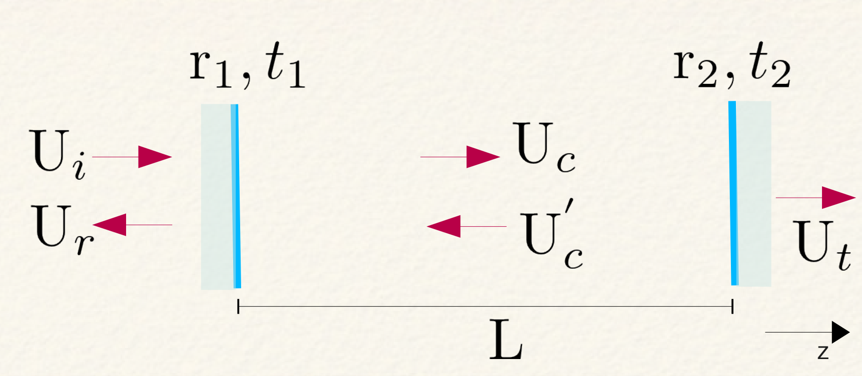
$$= P_i \frac{t_1^2}{(1 - r_1 r_2)^2} \frac{1}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2(kL)}$$

Finesse $\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$



Virgo cavity at resonance: $L = n\frac{\lambda}{2} \quad (n \in \mathbb{N})$

Field reflected by a Virgo arm cavity



- Reflected field as function of input field:

$$A_r = \frac{-r_2\, e^{2\jmath kL} + r_1}{1 - r_1 r_2 e^{2\jmath kl}} A_i$$

- Power reflected by the cavity, with $\mathrm{r}_2 \sim 1$

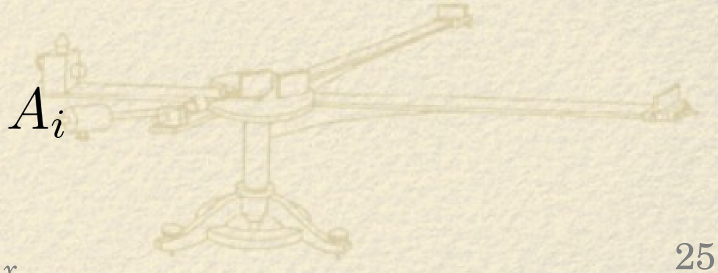
$$P_r \propto |A_r|^2$$
$$= P_i$$

- Phase of the field reflected by one arm cavity around resonance:

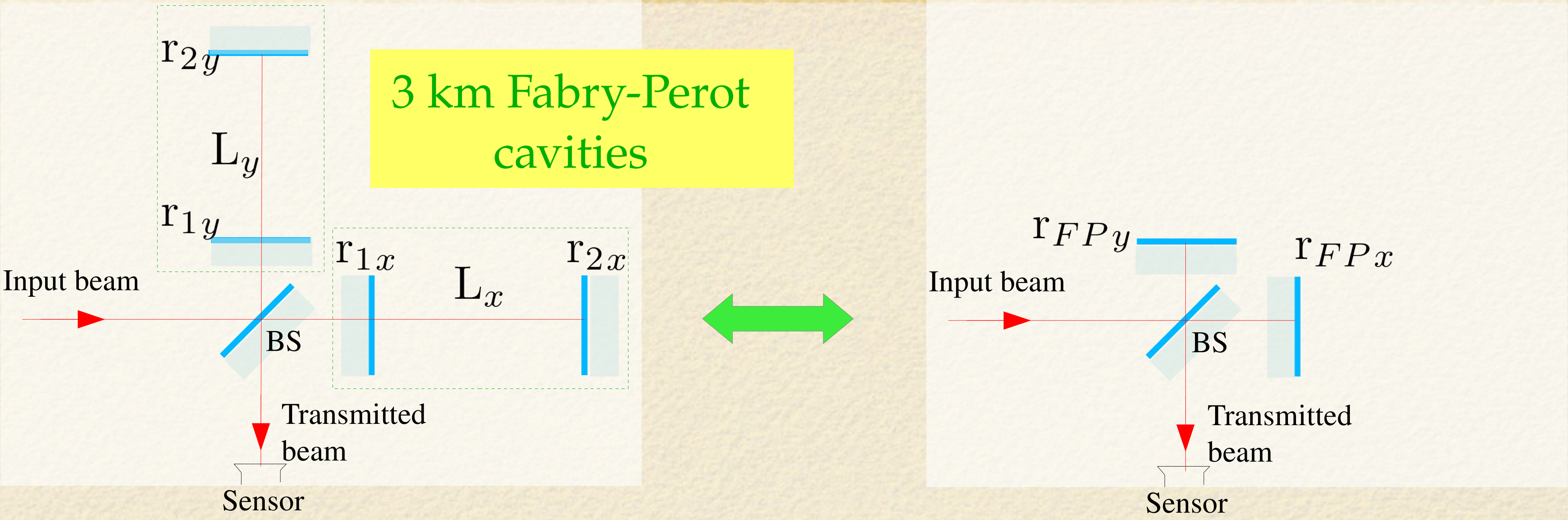
Cavity around resonance $L = n\,\frac{\lambda}{2} + \delta L \quad (n \in \mathbb{N})$

$$\phi = \arg\left(\frac{A_r}{A_i}\right) = \pi + \frac{1+r_1}{1-r_1}\, 2\,k\,\delta L$$

Field reflected by the x-arm: $A_{rx} = -1 \times e^{\jmath \frac{1+r_1}{1-r_1} 2k\delta L_x} A_i$



How do we amplify the phase offset ?



3 km Fabry-Perot cavities

(With $r_2 \sim 1$)

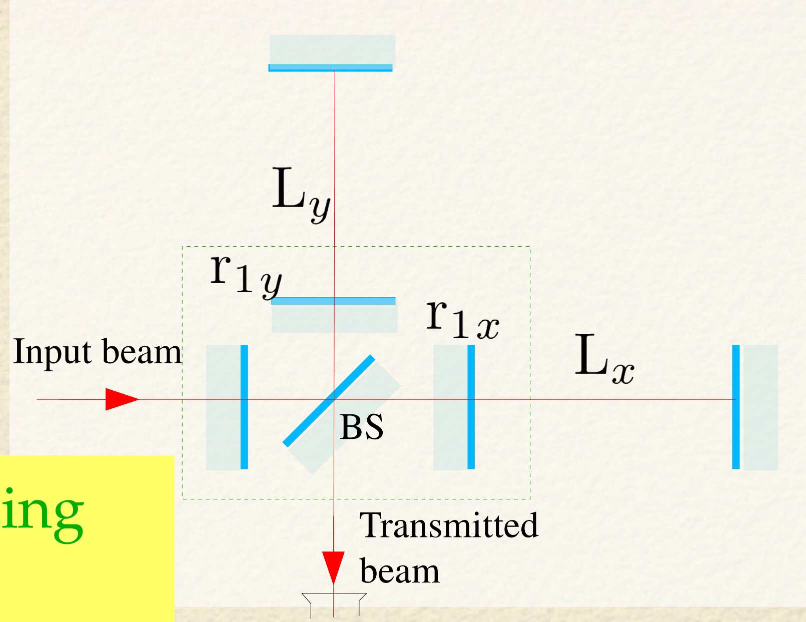
$$r_{FPx} = -1 \times e^{J \frac{1+r_{1x}}{1-r_{1x}} 2\, k\, \delta L_x}$$

~number of round-trips in the arm
~300 for AdVirgo

(instead of $r_{armx} = -1 \times e^{J 2k(L_x+\delta L_x)}$ in the arm of a simple Michelson)

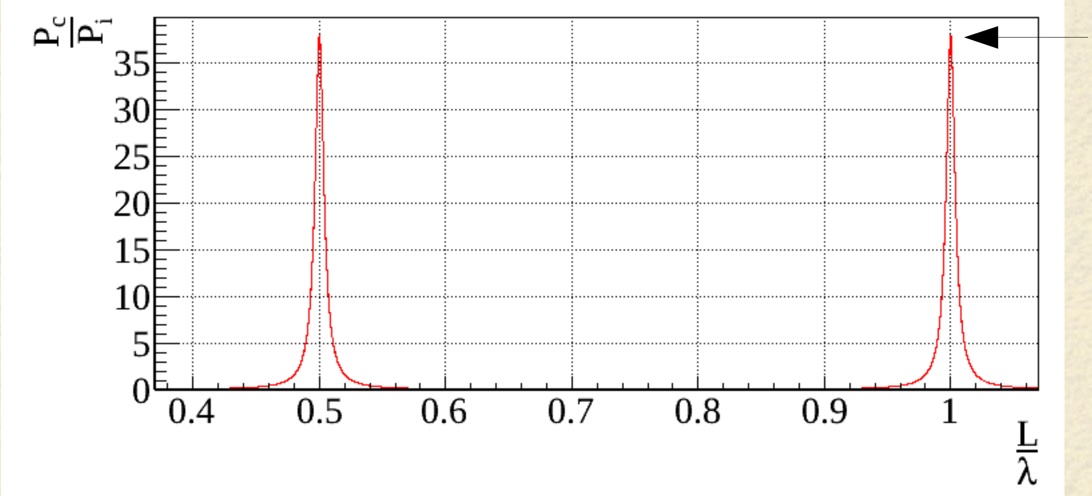
How do we increase the power on BS ?

Detector working point close to a dark fringe
→ most of power go back towards the laser



Power recycling cavity

Resonant power recycling cavity



$G_{PR} = 38 \quad (r_{PR}^2 = 0.95)$

→ input power on BS increased by a factor 38 !

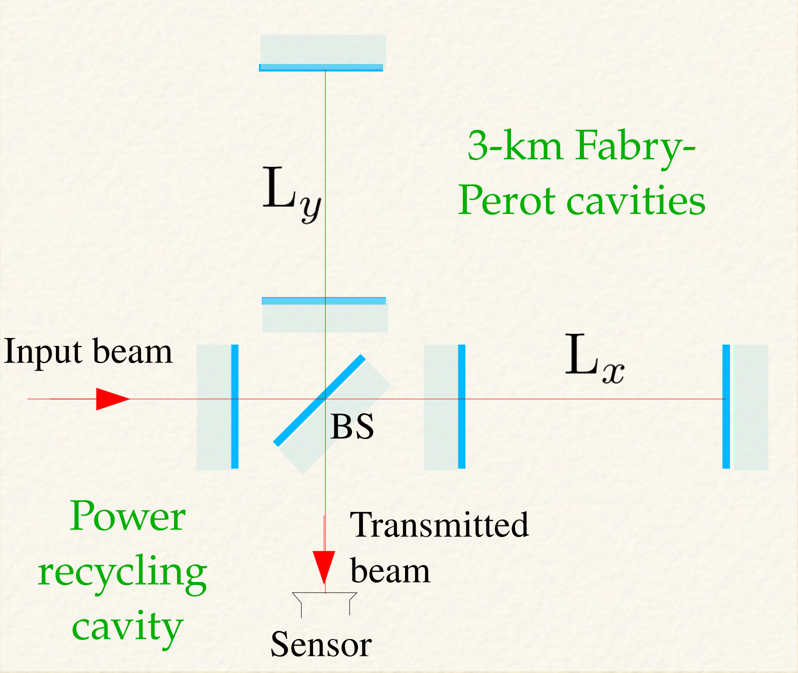
The improved interferometer response

- **Response of simple Michelson:**

$$\delta P_t = P_i \, C \, \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) \delta\Delta L$$

$$\delta P_t = (\text{Michelson response}) \times \delta\Delta L$$

(W/m)

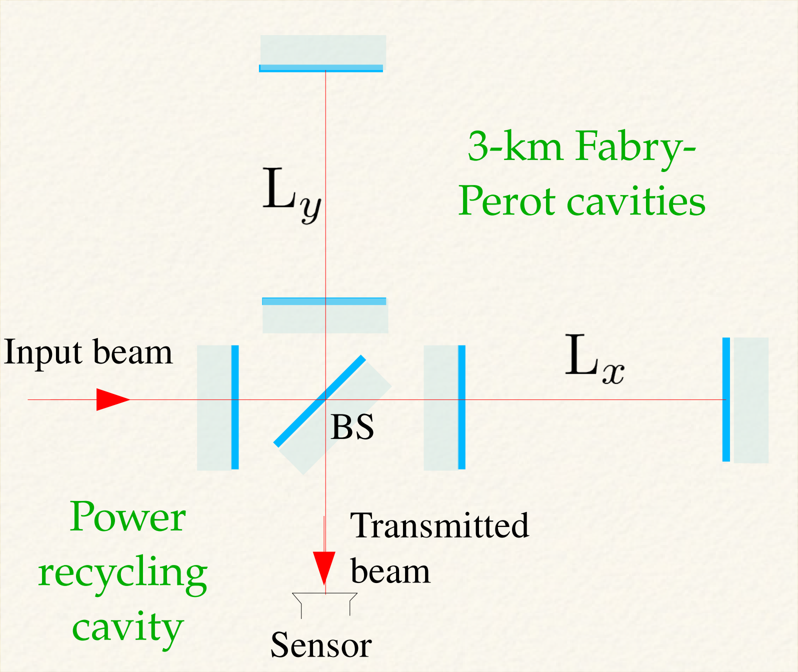


- **Response of recycled Michelson with Fabry-Perot cavities:**

$$\delta P_t = \underbrace{G_{PR}}_{\sim 38} P_i \, C \, \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) \underbrace{\frac{1 + r_1}{1 - r_1}}_{\sim 300} \delta\Delta L$$

For the same $\delta\Delta L$, δP_t has been increased by a factor ~ 12000.

A hint of AdvancedVirgo sensitivity



- Response of recycled Michelson with Fabry-Perot cavities:

$$\delta P_t = G_{PR}\, P_i\, C\, \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda}\Delta L_0\right) \frac{1+r_1}{1-r_1} \delta\Delta L$$

Laser wavelength: $\lambda = 1.064\,\mu\text{m}$
Input power: P$_i$ ~ 100 W
Interferometer contrast: $C \sim 1$
Input mirror reflection: $r_1 = \sqrt{0.986}$
Working point: $\Delta L_0 \sim 10^{-11}$ m
Power recycling gain: $G_{PR} \sim 38$

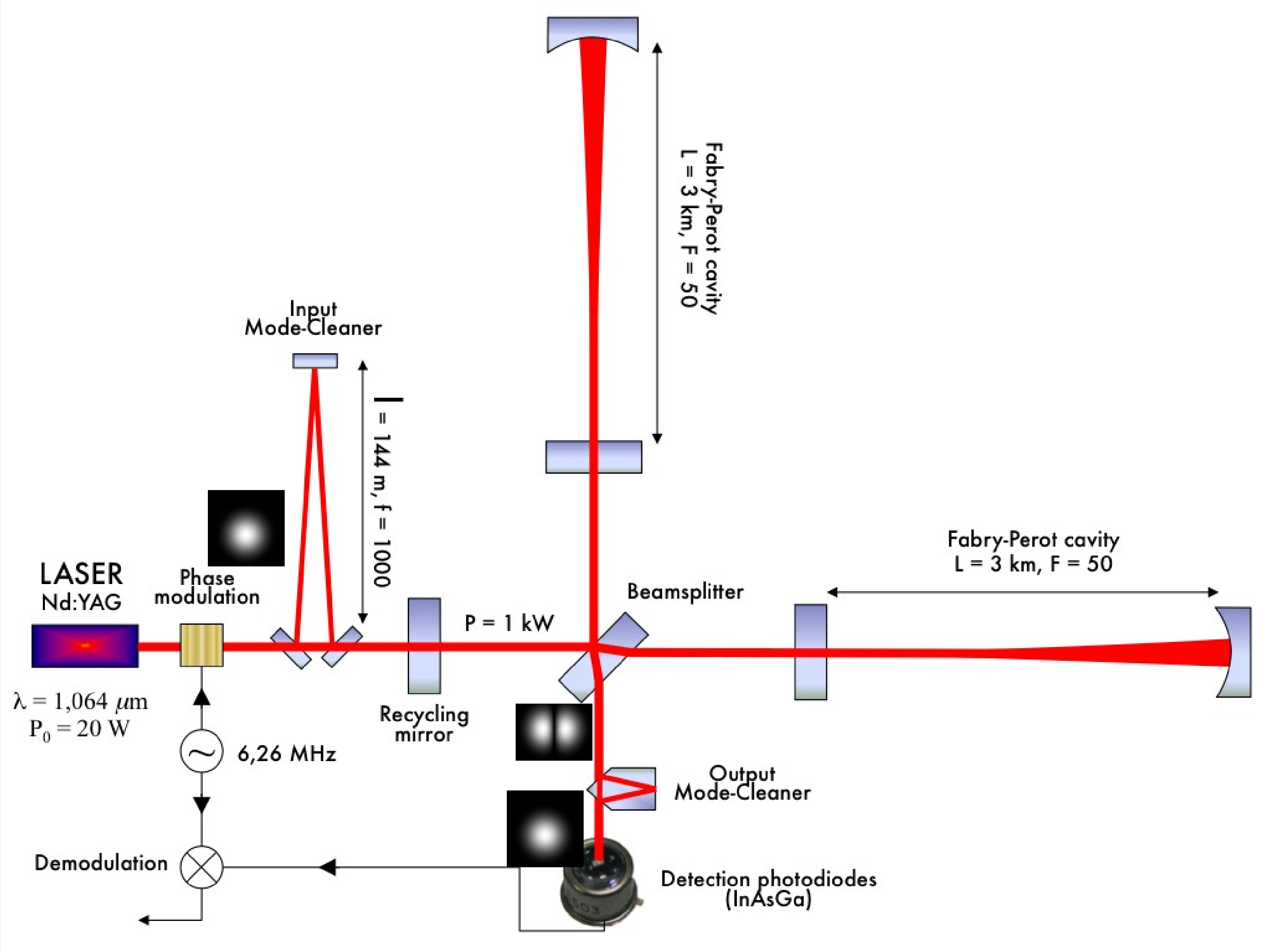
Power noise: $\delta P_{t,min} \sim 0.1\,\text{nW}$ $\longrightarrow$

$$\delta\Delta L_{min} \sim 5 \times 10^{-20}\ \text{m}$$

$$\rightarrow h_{min} = \frac{\delta\Delta L_{min}}{L} \sim 10^{-23}$$

In reality, the detector response depends on frequency...

Optical layout of Virgo

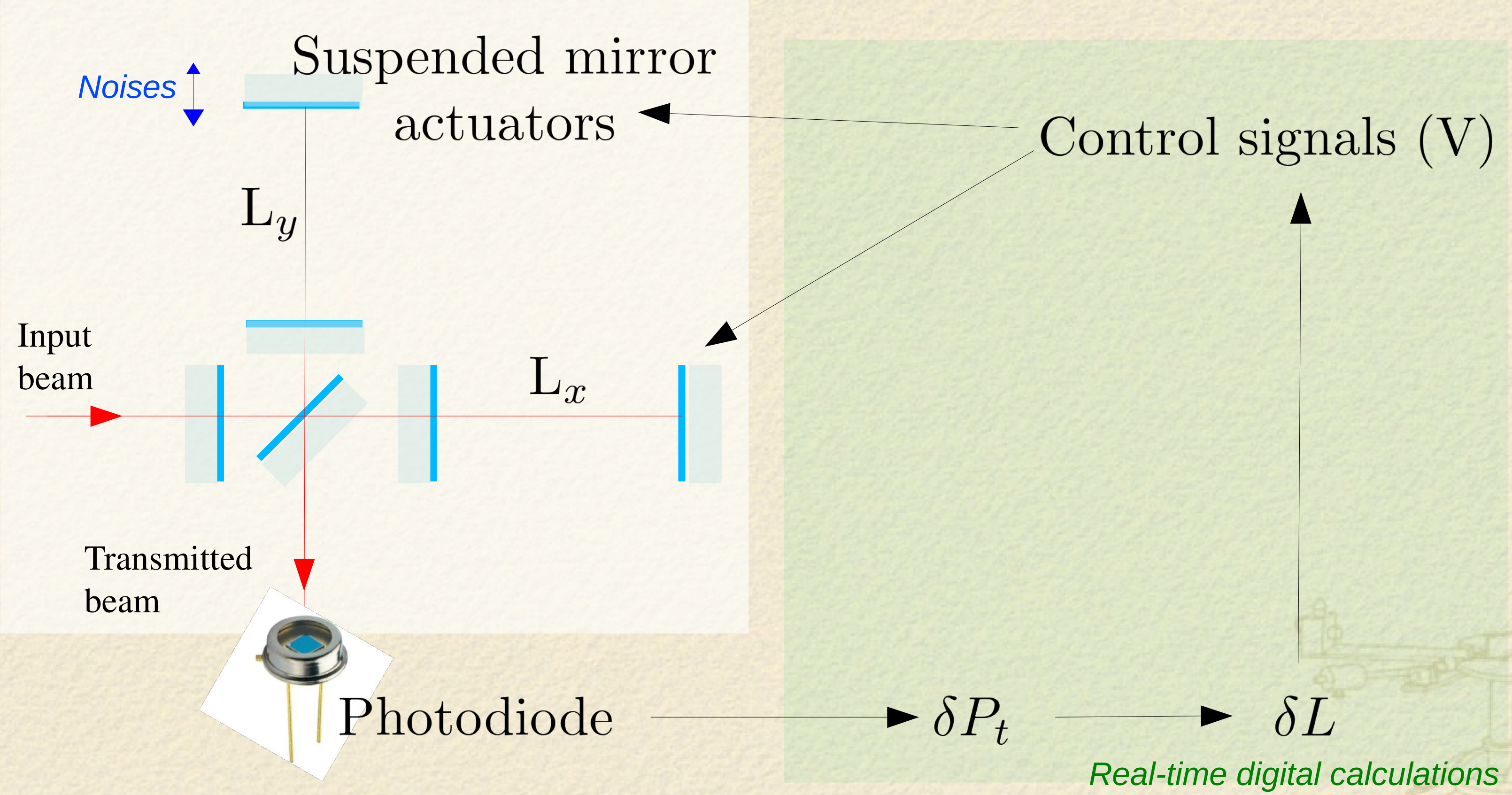


How do we control the working point ?

We want $\Delta L_0 = 10^{-11}\,\mathrm{m}$ to be (almost) fixed !

Control loop done for noises with f between ~10 Hz and ~100 Hz

Precision of the control ~ 10^{-16} m



Noises

Suspended mirror actuators

Control signals (V)

L_y

Input beam

L_x

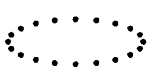
Transmitted beam

Photodiode → δP_t → δL

Real-time digital calculations

From the data to the GW strain h(t)...

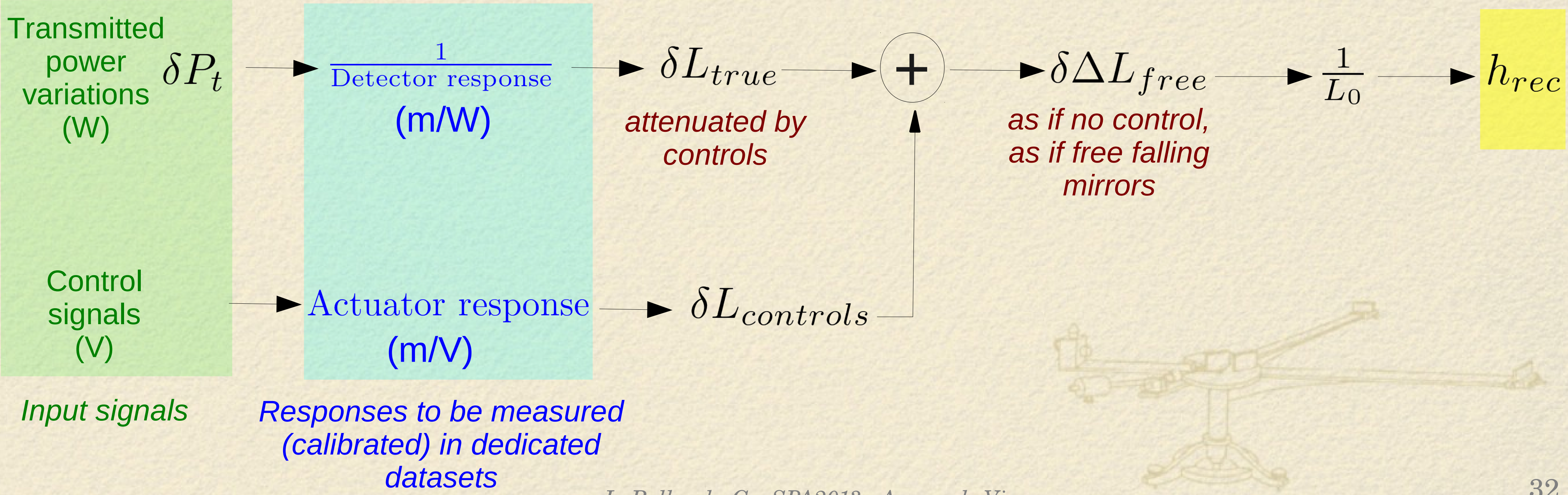
For free falling masses, $h(t) = \frac{\delta \Delta L(t)}{L_0}$



→ this condition is valid for the suspended mirrors above ∼ 100 Hz.

At lower frequencies, the controls attenuate the noise...
but also the gravitational wave signal !

→ the control signals contain information on h(t)



Transmitted power variations (W) δP_t → $\frac{1}{\text{Detector response}}$ (m/W) → δL_{true} → $+$ → $\delta \Delta L_{free}$ → $\frac{1}{L_0}$ → h_{rec}

attenuated by controls

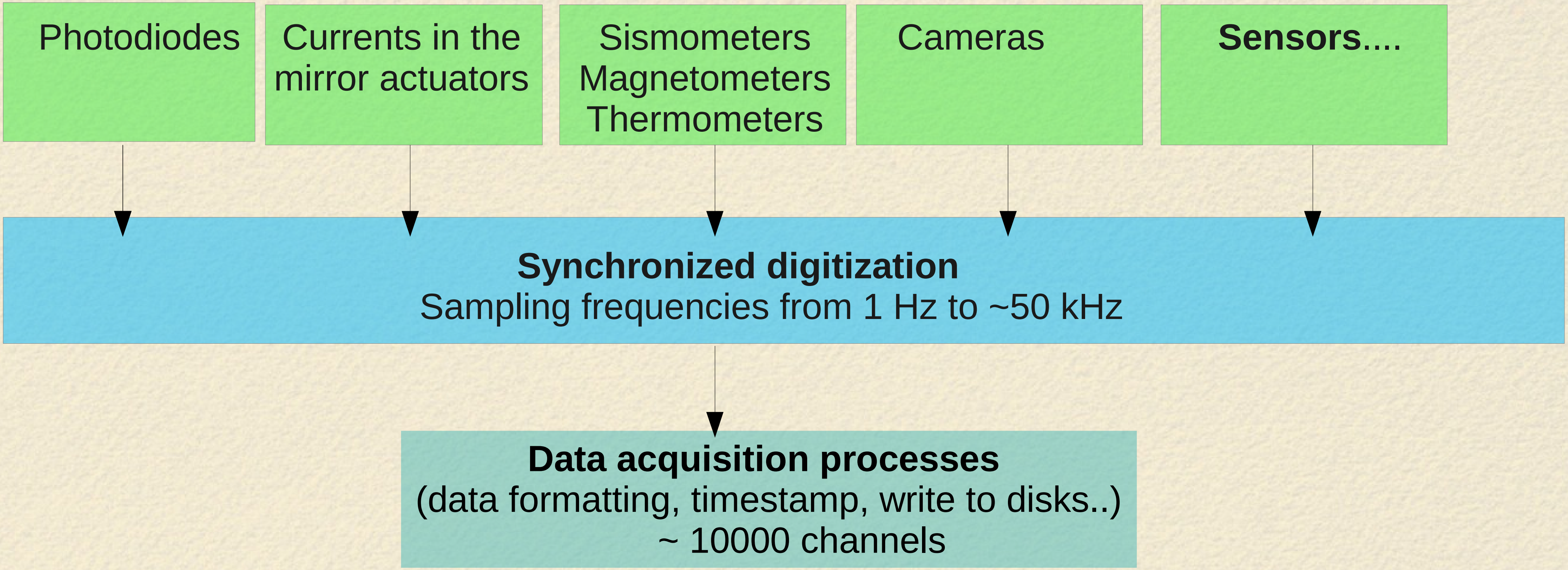
as if no control, as if free falling mirrors

Control signals (V) → Actuator response (m/V) → $\delta L_{controls}$ → $+$

Input signals

Responses to be measured (calibrated) in dedicated datasets

AdVirgo data acquisition summary



Photodiodes

Currents in the mirror actuators

Sismometers
Magnetometers
Thermometers

Cameras

Sensors....

Synchronized digitization
Sampling frequencies from 1 Hz to ~50 kHz

Data acquisition processes
(data formatting, timestamp, write to disks..)
~ 10000 channels

Continuous flow of ~2 TBytes/day (20 to 40 MBytes/s)
Disk space on Virgo site: ~400 TB for 6 months of data

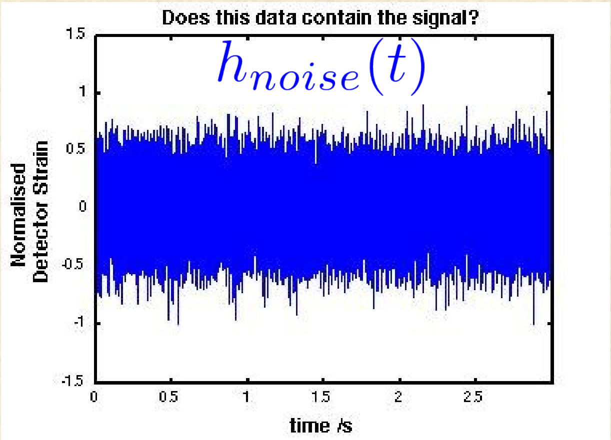
Longer storage: data sent via Ethernet to computing centers (Lyon, Bologna)

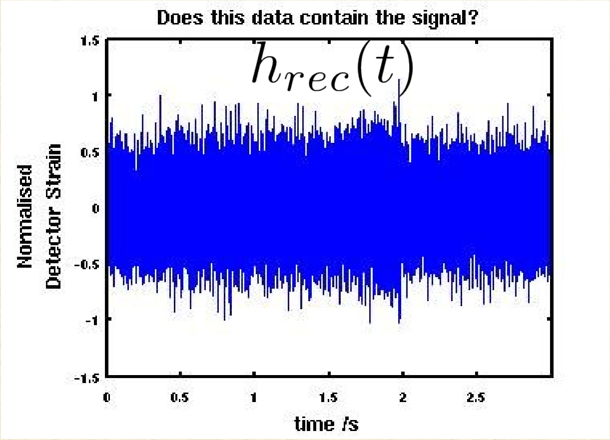
Virgo noises

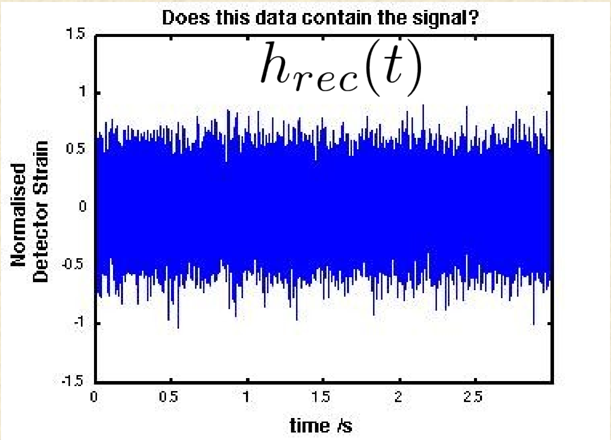
What is a noise in Virgo ?

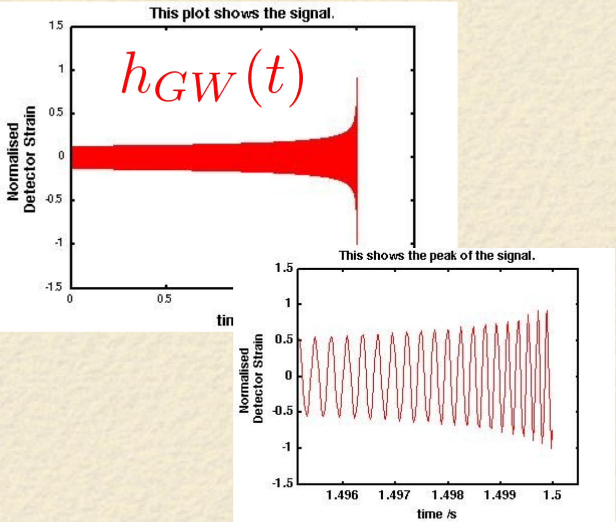
- Stochastic (random) signal that contributes to the signal h_mes(t) but does not contain information on the gravitational wave strain h_GW(t)

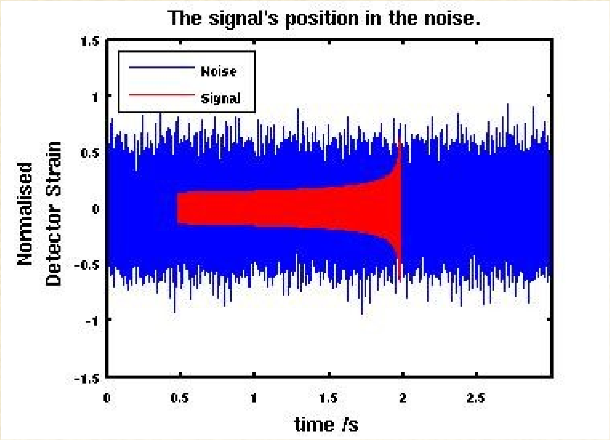
$$h_{rec}(t) = h_{noise}(t) + h_{GW}(t)$$

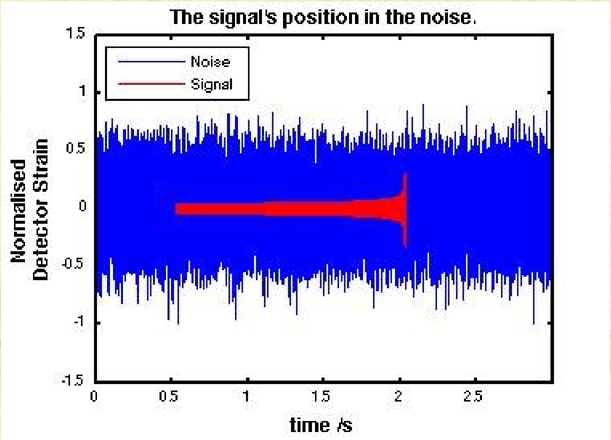










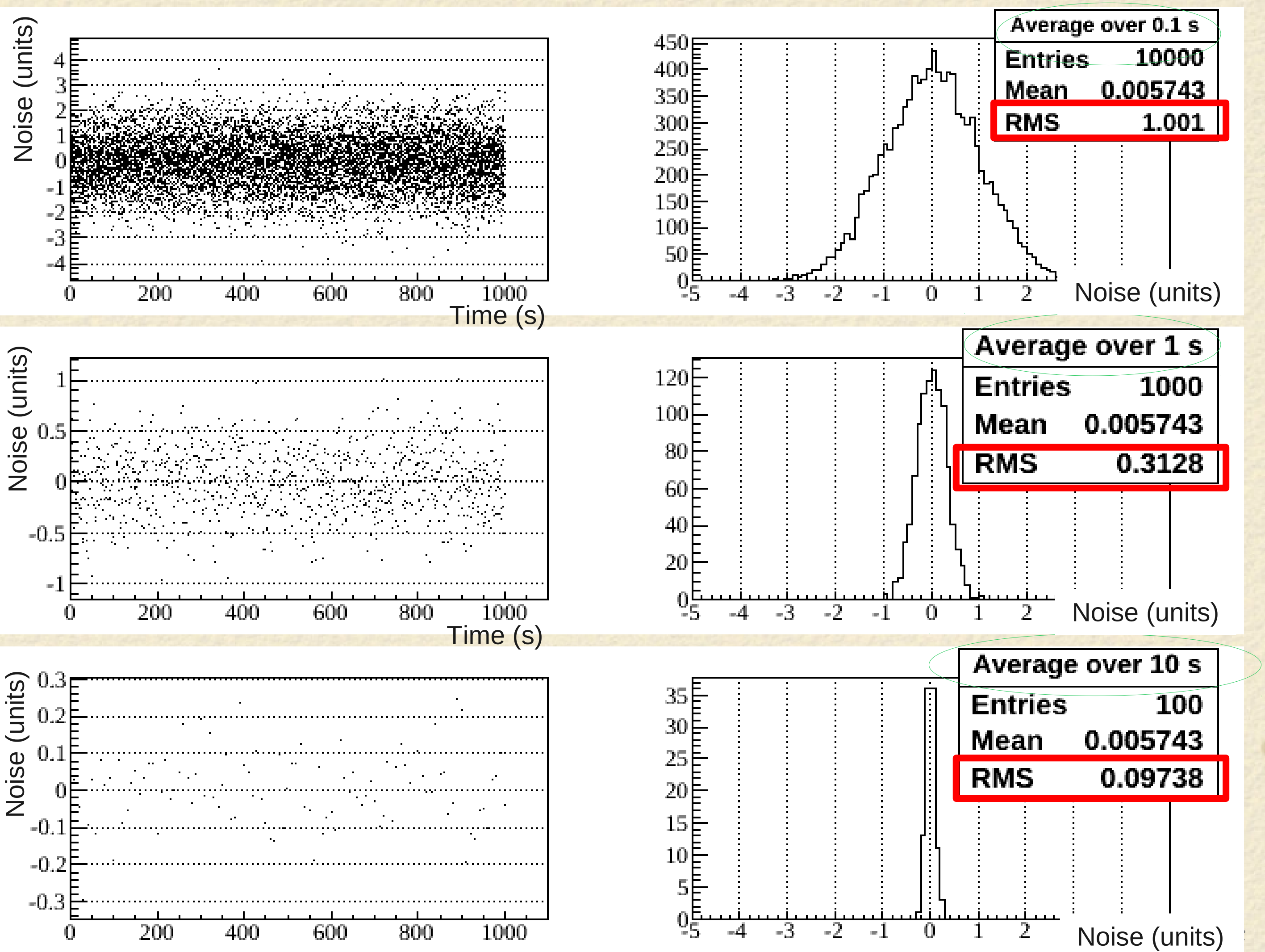


Extracted from Black Hole Hunter: http://www.blackholehunter.org/

How do we characterize a noise ?

Hypothesis: - we are looking for a constant signal S_0 in the data
- data are noisy (Gaussian noise)

Data points of noise only **Projection of noise data**



Gaussian distribution:

$$N e^{-\frac{1}{2}\frac{(x-<x>)^2}{\sigma_x^2}}$$

The mean value of the noise stays around 0
The mean value of the signal stays around S_0.

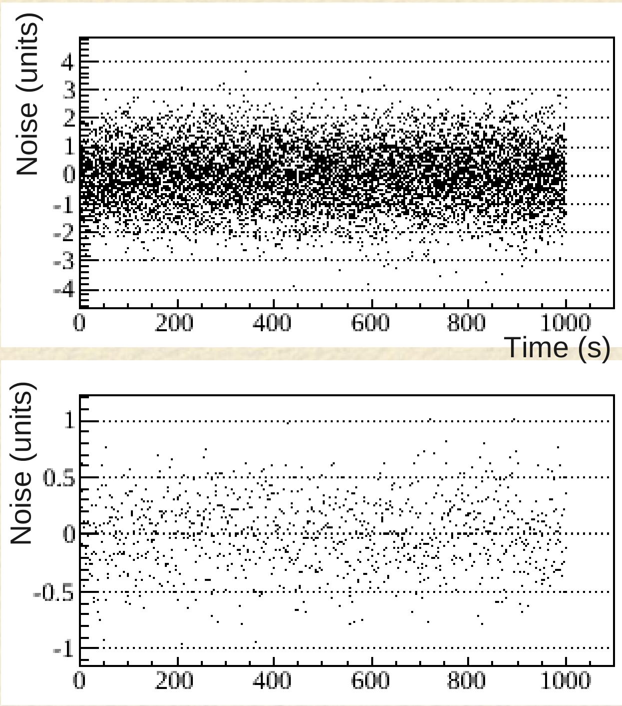
The variations of the noise decrease when the data are averaged over longer time

$$\sigma_{noise} \propto \frac{1}{\sqrt{\text{average duration}}}$$

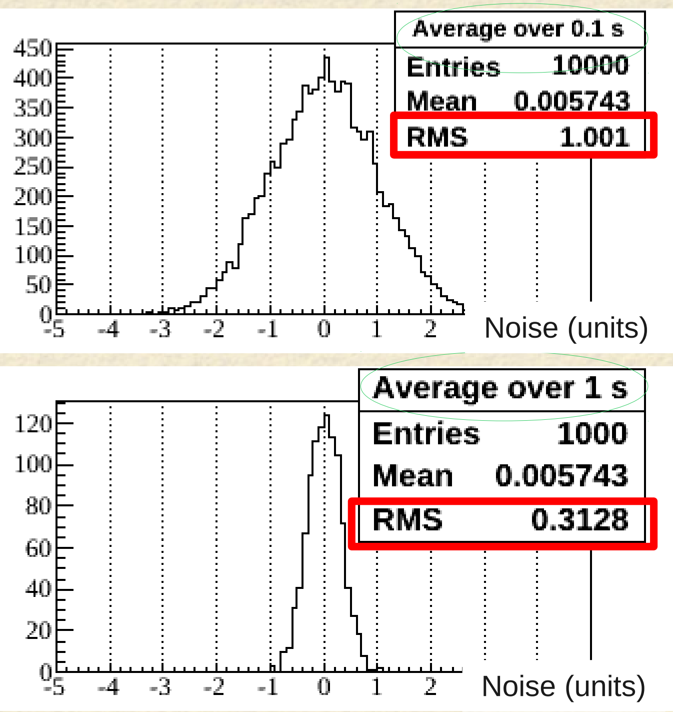
→ What is important to characterize a noise is it dispersion σ_{noise} !

How do we characterize a noise ?

Data points of noise only



Projection of noise data



The variations of the noise decrease when the data are averaged over longer time

$$\sigma_{noise} \propto \frac{1}{\sqrt{\text{average duration}}}$$

↓

The noise can be characterized by the coefficient of proportionality D

$$\underbrace{\sigma_{noise} = \frac{D}{\sqrt{\text{average duration}}}}$$

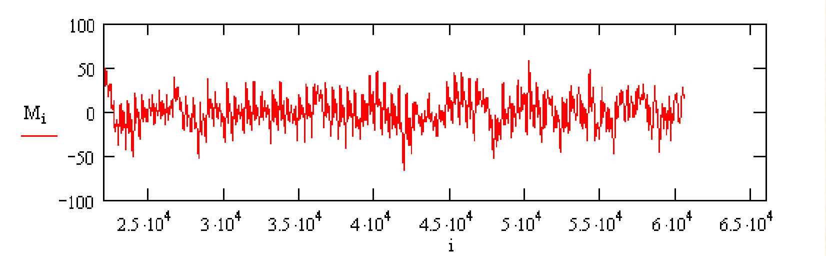
$$D \text{ is in } \frac{\text{Data units}}{\sqrt{\text{Hz}}}$$

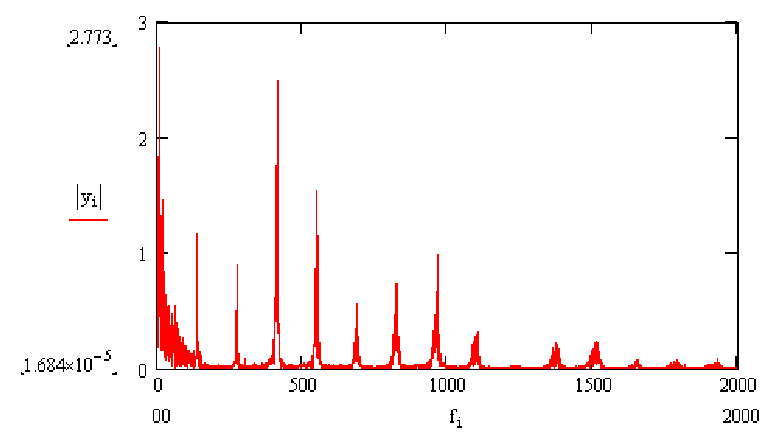
its absolute value is equal to the dispersion of the noise when it is averaged over 1 s.

How do we characterize a noise ...in frequency-domain?

A signal can be decomposed in different frequency components.

$$S(t) \xrightarrow{\text{Fourier transform}} A(f) \text{ and } \Phi(f)$$





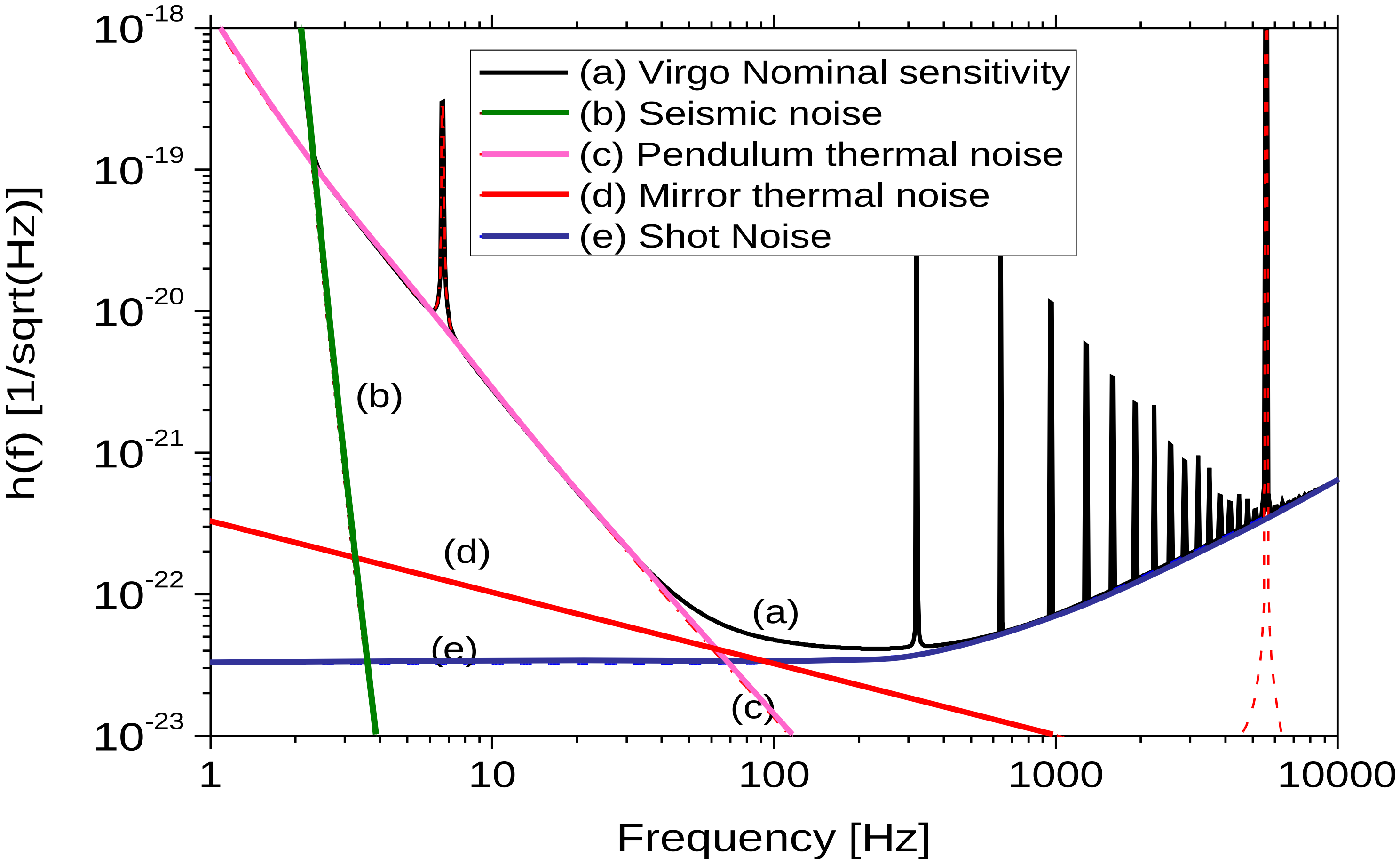
Doing the same for the noise, we can characterize the variation of $A_{noise}(f)$ at a given f.

→ $D(f)$ (amplitude spectral density).

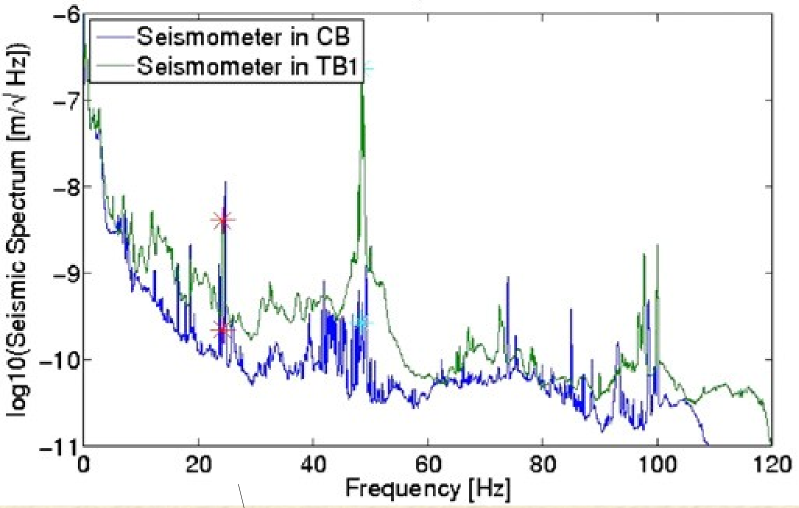
$D(f)$ is also in $\frac{\text{Data units}}{\sqrt{\text{Hz}}}$

What is the noise level of Virgo ?

Noise level of $h_{rec}(t)$, shown as function of frequency

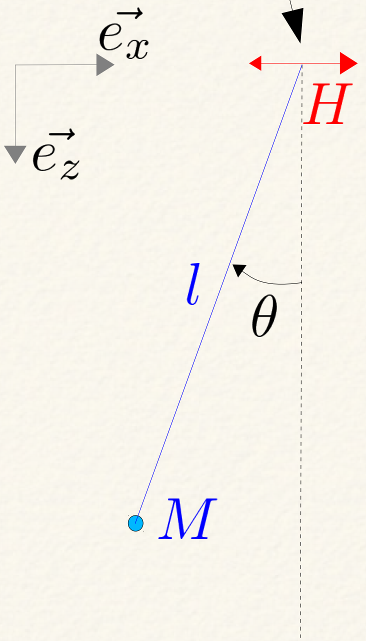


Seismic noise and suspended mirrors



Ground vibrations up to ~1 µm at low frequency decreasing down to ~ 10 pm at 100 Hz

$\gg 10^{-19}$ m needed to detect GW !!



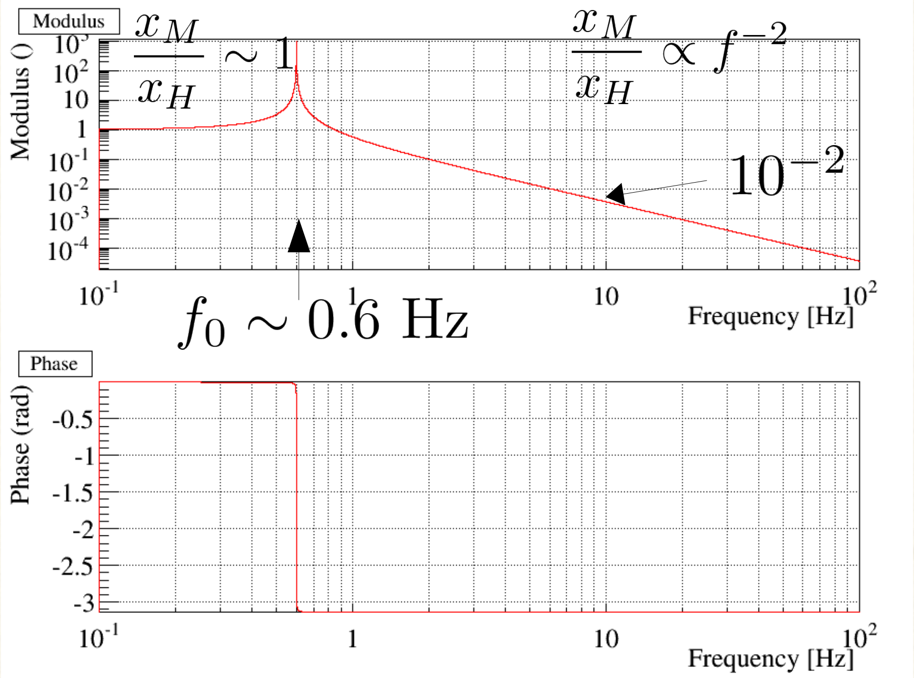
Assuming

$\delta x_H \ll 1$ and sinusoidal

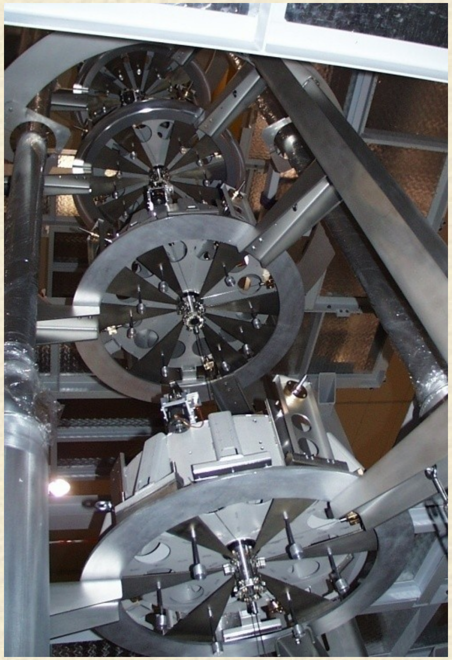
and $\theta \ll 1$:

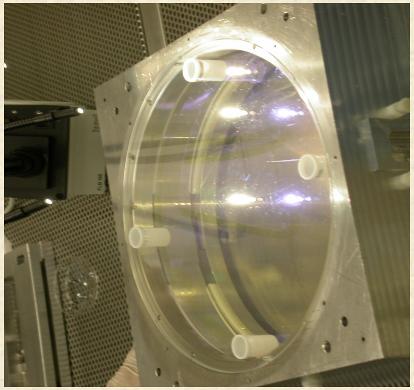
$$\underline{x_M} = \underline{\mathcal{H}} \times \underline{x_H}$$

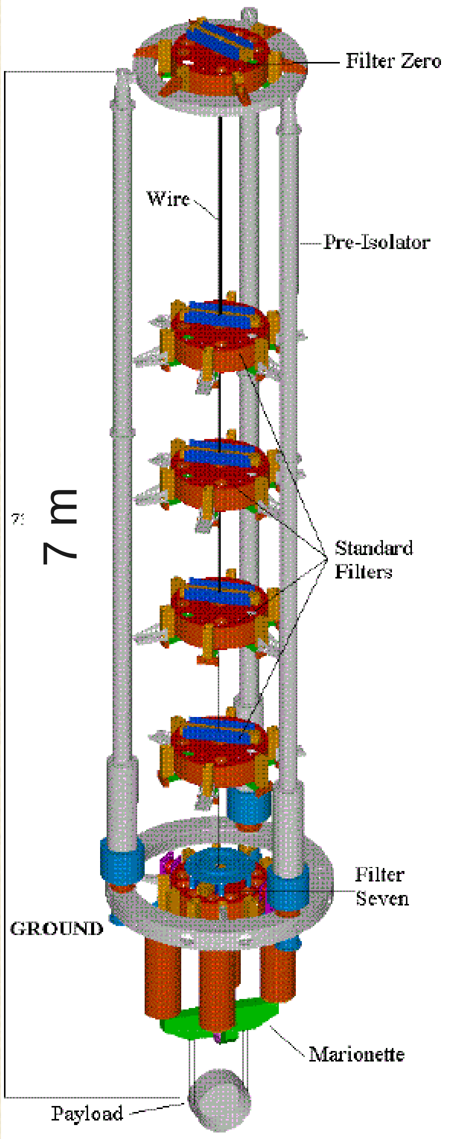
Transfer function



Seismic noise and the Virgo suspension







- **Passive attenuation:** 7 pendulum in cascade

$$\text{At 10 Hz: } \frac{x_{mirror}}{x_{ground}} \sim \left(10^{-2}\right)^7 = 10^{-14}$$

$$\rightarrow x_{mirror} \sim 10^{-23}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$$

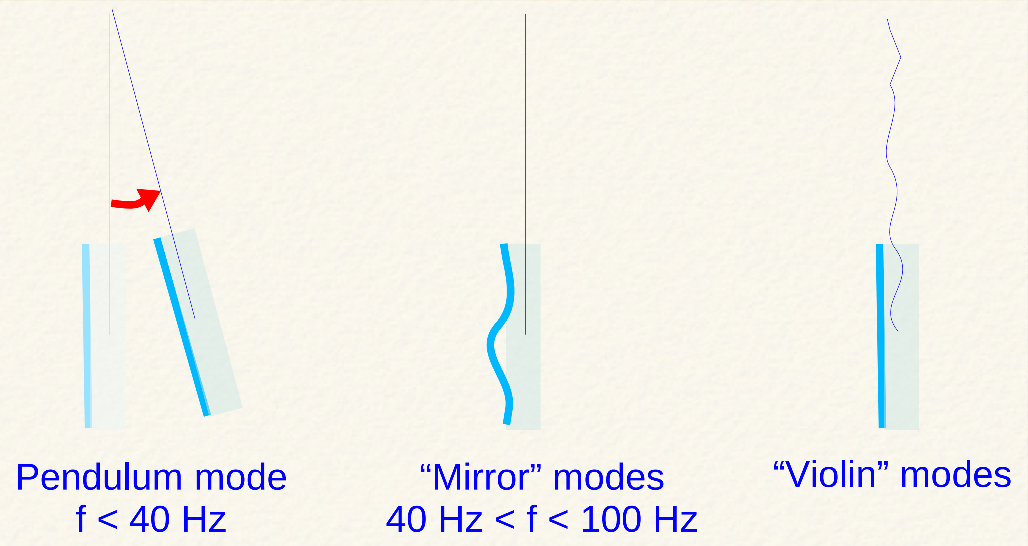
It would directly modifies the positions of the mirror surfaces, and thus $\delta\Delta L$ and $h_{rec}(t)$!

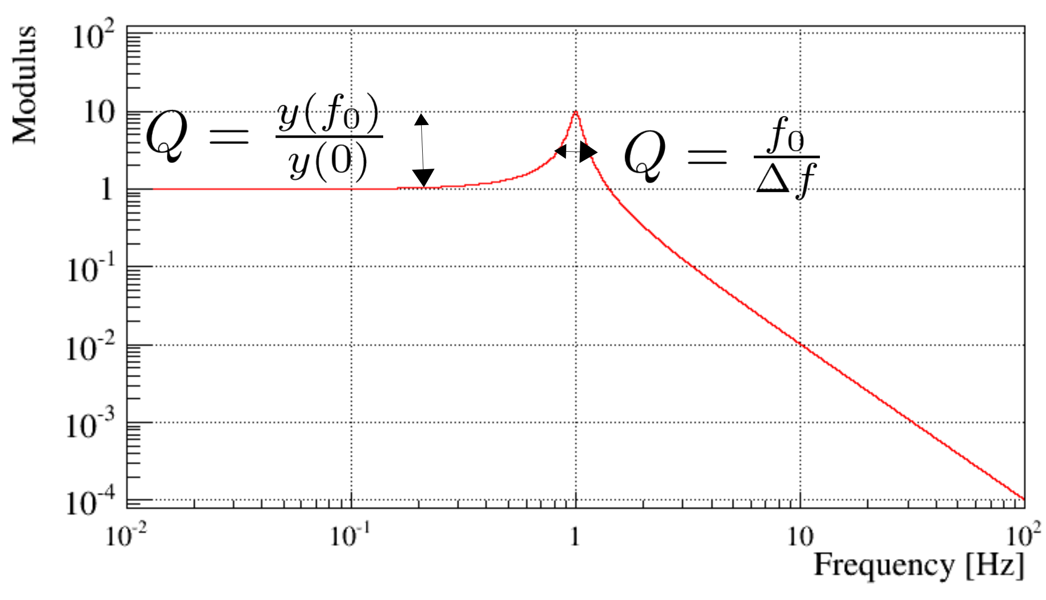
- **Active controls** at low frequency
  - Accelerometers or interferometer data
  - Electromagnetic actuators
  - Control loops

Some noises: thermal noise

- Microscopic thermal fluctuations

--> dissipation of energy through excitation of the macroscopic modes of the mirror





It directly modifies the positions of the mirror surfaces, and thus $\delta\Delta L$ and $h_{rec}(t)$!

- We want high quality factors Q to concentrate all the noise in a small frequency band

What is the shot noise ?

- Fluctuations of arrival times of photons (quantum noise)

Power received by the photodiode: P_t

$$\rightarrow \quad N = \frac{P_t}{\mathrm{h}\nu} \text{ photons/s on average.}$$

Standard deviation on this number: $\sigma_N = \sqrt{N}$

$$\rightarrow \quad \sigma_{P_t} = \sigma_N \times \mathrm{h}\nu = \sqrt{\frac{P}{\mathrm{h}\nu}}\mathrm{h}\nu = \sqrt{P_t \mathrm{h}\nu}$$

Virgo laser: $\lambda = 1.064\,\mu\mathrm{m} \rightarrow \nu = \frac{\mathrm{c}}{\lambda} \sim 2.8 \times 10^{14}\,\mathrm{Hz}$

Working point: $P_t \sim 80\,\mathrm{mW} \quad \rightarrow \quad \sigma_{P_t} = 0.1\,\mathrm{nW}/\sqrt{\mathrm{Hz}}$

→ a variation of power is interpreted as a variation of distance

$$\delta P_t = (\text{Virgo response}) \times L_0 \times h \qquad h_{equivalent} = \frac{1}{L_0}\frac{\sigma_{P_t}}{(\text{Virgo response})}$$

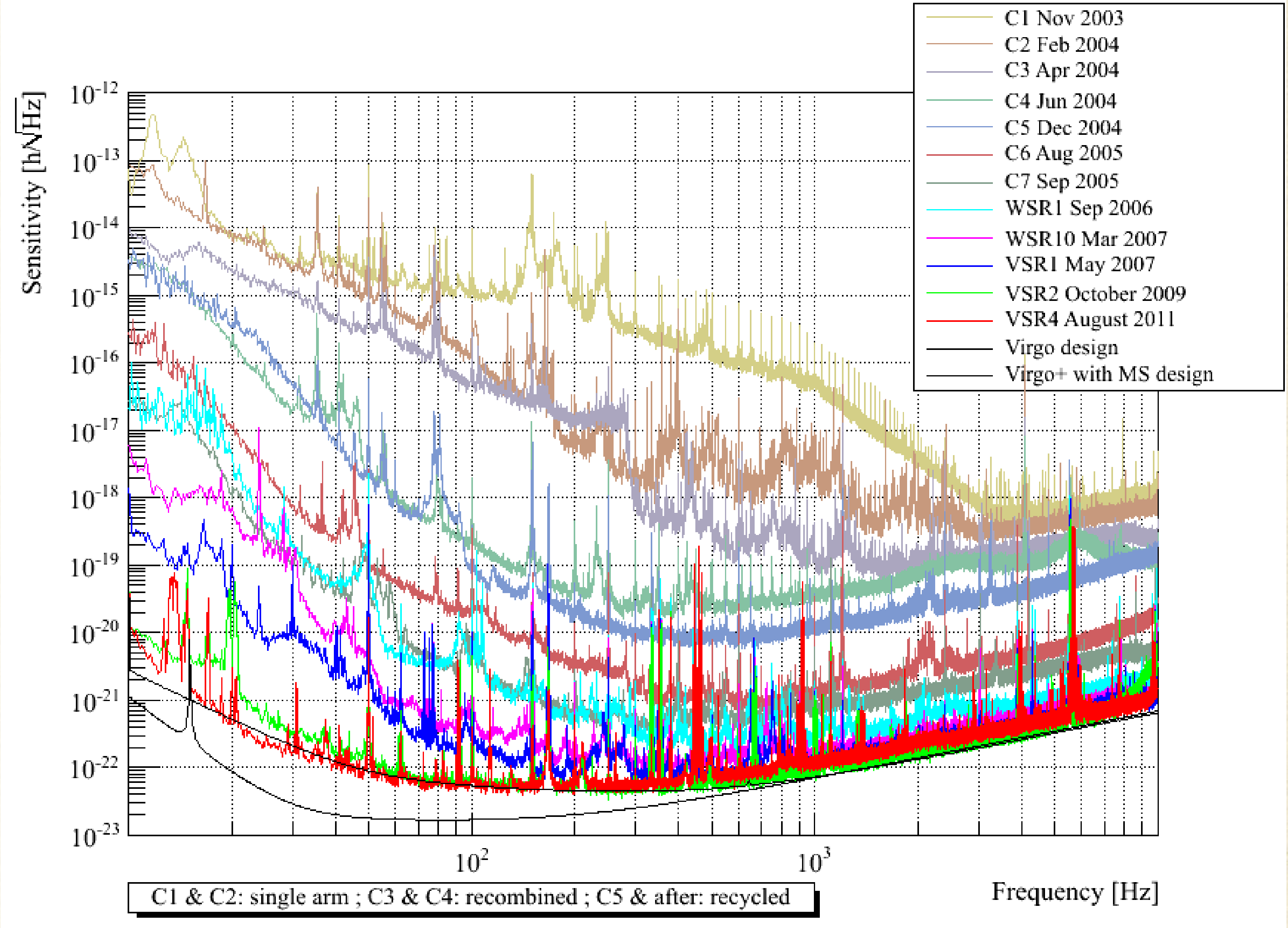
(in W/m)

Some other noises

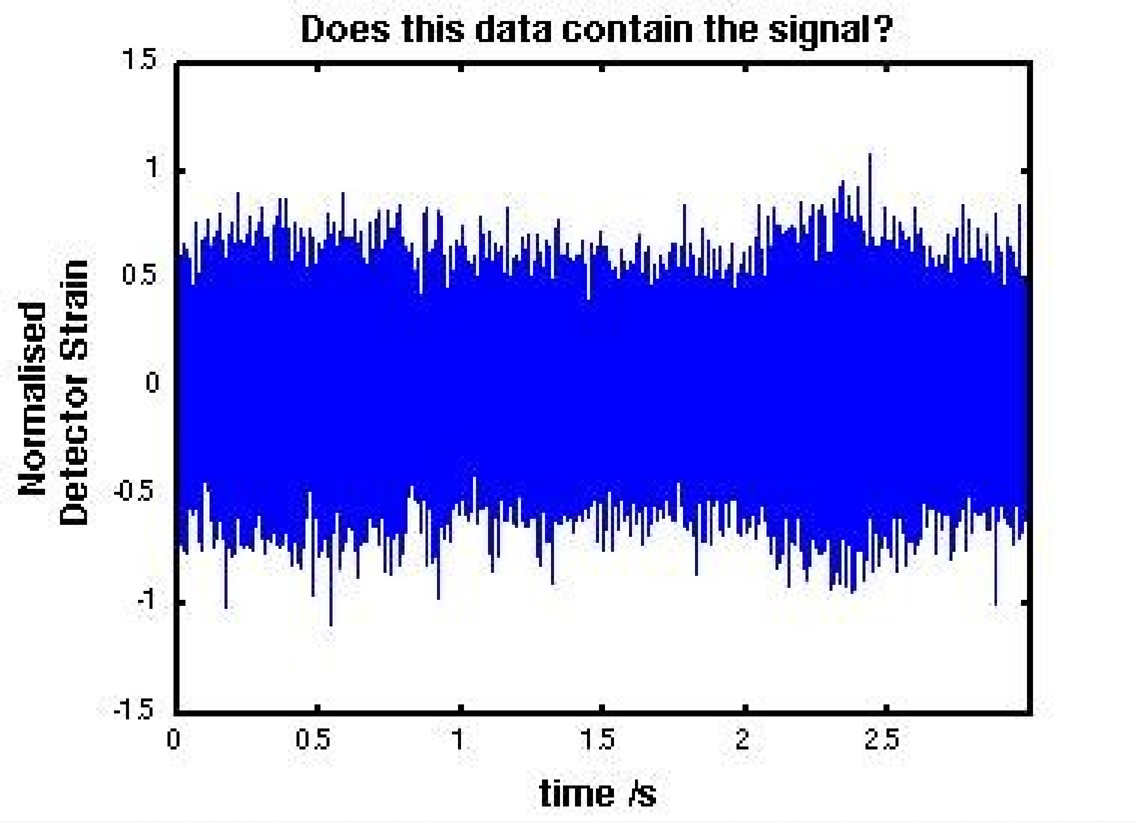
- Acoustic vibrations and refraction index fluctuations
    - Main elements installed in vacuum
- Laser: amplitude, frequency, jitter noise
    - Lots of control loops to reduce these noises
- Electronics noise

    - Challenge for the electronicians to measure down to 0.1 W/sqrt(Hz)
- Non-linear noise from diffuse light
    - Need dedicated optical elements with specific mechanical modes

History of Virgo noise curve



Noises are not always stationary...



“Glitches” are impulses of noise. They might look like a transient GW signal...

→ Now it is time to play with the data analysis !