Nonequilibrium effects at the QCD phase transition

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with:
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Mini-Workshop QCD, Nantes, January 2013

Subatech

HIC for FAIR
Helmholtz International Center

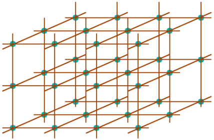
FIAS Frankfurt Institute for Advanced Studies

How to study the QCD phase diagram...

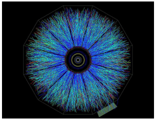
... be brave and solve

$$Z(T,\mu_B) = \int \mathcal{D}(A,q,q^\dagger)\mathrm{e}^{-S^E_{\mathrm{QCD}}}$$

ab initio and nonperturbatively,



... be strong and collide heavy ions at ultrarelativistic energies,



... be creative and study effective models of QCD.

$$\mathcal{L}_{\mathrm{eff}}$$

Being brave

The critical point in lattice QCD

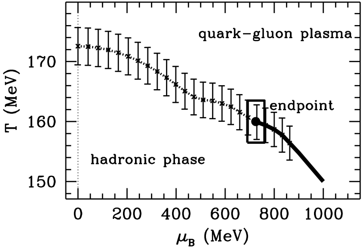
Crossover at $\mu_B = 0$ and $T = [145, 165]$ MeV

(Wuppertal-Budapest JHEP 1009 (2010),HotQCD PoS LATTICE2010 (2010))

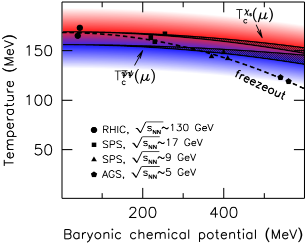
Fermionic sign problem at $\mu_B \neq 0 \rightarrow$ usual MC sampling fails!

Methods to explore the $T - \mu_B$–plane:

- Reweighting



(Z. Fodor, S.D. Katz, JHEP **0203** (2002))



(G. Endrodi, Z. Fodor, S. D. Katz, K. K. Szabo, JHEP **1104** (2011))

- Imaginary μ_B (de Forcrand, Philipsen): $\mu_B^c > 500$ MeV

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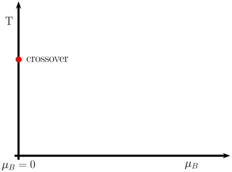

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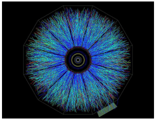

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Being strong

The critical point in heavy-ion collisions

Coupling of the order parameter to pions $g\sigma\pi\pi$ and protons $G\sigma\bar{p}p \Rightarrow$ fluctuations in multiplicity distributions

$$\langle(\delta N)^2\rangle \propto \langle(\Delta\sigma)^2\rangle \propto \xi^2$$

ξ: correlation length of fluctuations of the order parameter, diverges at the CP

(M. Stephanov, K. Rajagopal, E. Shuryak, PRL **81** (1998), PRD **60** (1999))

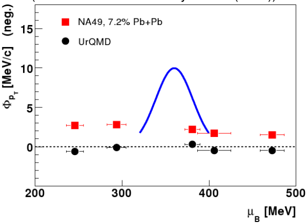
Higher cumulants are more sensitive to the CP

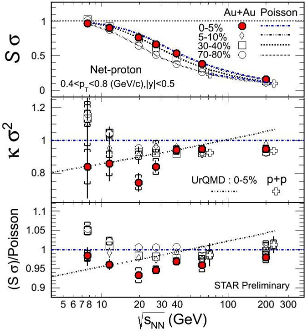
$$\langle(\delta N)^3\rangle \propto \xi^{4.5}$$

$$\langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \propto \xi^7$$

(M. Stephanov, PLB **102** (2009), PRL **107** (2011))

(NA49 collaboration J. Phys. G **35** (2008))





(STAR collaboration, QM2012)

Being strong

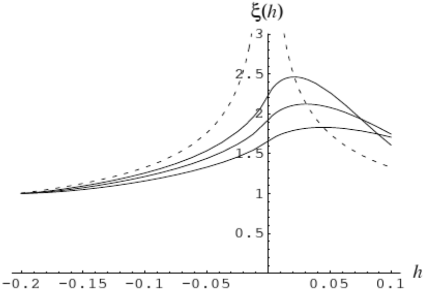
The critical point in dynamic systems

long relaxation times near a critical point $\Rightarrow$ critical slowing down
$\Rightarrow$ the system is driven out of equilibrium
phenomenological equation:

$$\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)})$$

with $\Gamma(m_\sigma) = \frac{A}{\xi_0}(m_\sigma \xi_0)^z$
$z = 3$
(dynamic) critical exponent

$\Rightarrow \xi \sim 1.5 - 2.5$ fm



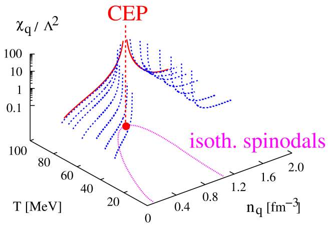
(B. Berdnikov and K. Rajagopal, PRD **61** (2000)); D.T.Son, M.Stephanov, PRD **70** (2004); M.Asakawa, C.Nonaka, Nucl. Phys. A774 (2006))

Being strong

Effects at the first order phase transition

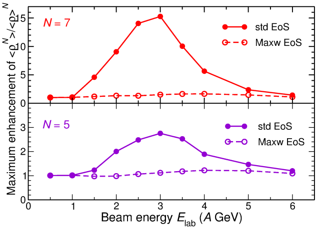
- Instability of slow modes at the spinodal lines (**spinodal decomposition**)

  (I. Mishustin, PRL **82** (1999); C. Sasaki, B. Friman, K. Redlich, PRD **77** (2008))



- Significant amplification of density irregularities

  (J. Steinheimer, J. Randrup, PRL **109** (2012))



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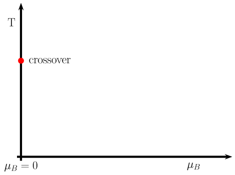

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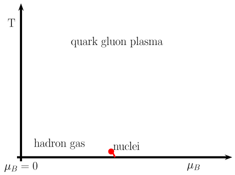

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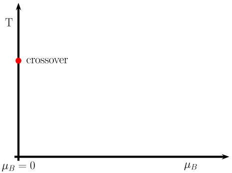

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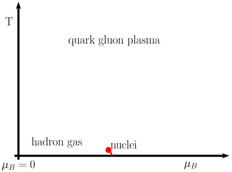

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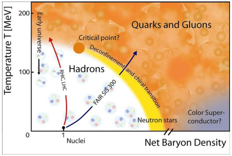

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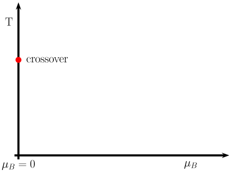

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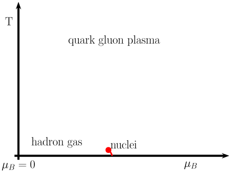

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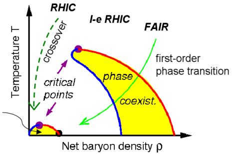

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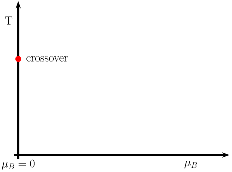

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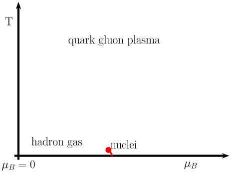

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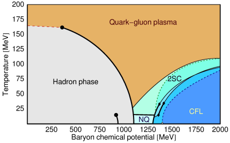

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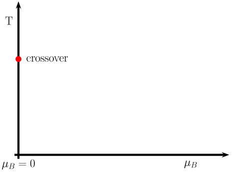

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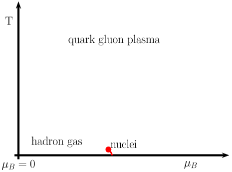

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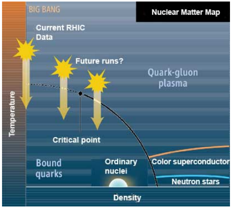

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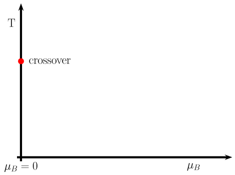

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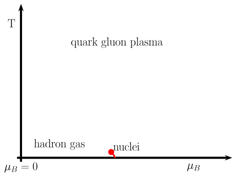

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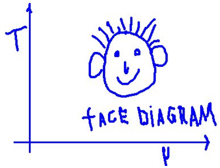
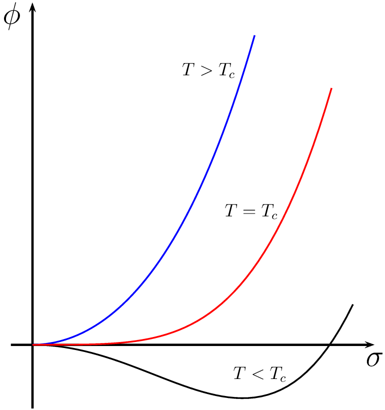


Critical point

- $m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \to 0$
- Correlation length diverges
  $\xi = \frac{1}{m_\sigma} \to \infty$
- Universality classes
  for QCD: $\mathcal{O}(4)$ Ising model in
  3d $\Rightarrow \langle \sigma^2 \rangle \propto \xi^2$
- Renormalization group
- Critical opalescence

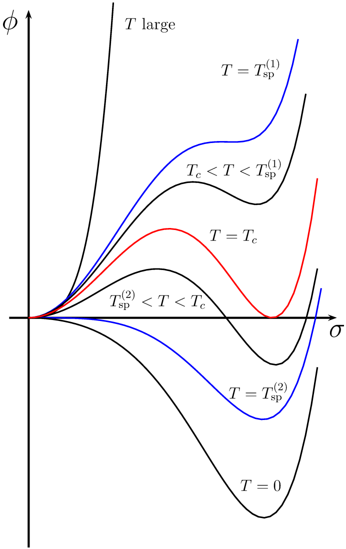


$\Rightarrow$ Large event-by-event fluctuations in thermal systems!

First order phase transitions

- Two degenerate minima separated by a barrier
- Latent heat
- Phase coexistence
- Supercooling effects in nonequilibrium situations
- Nucleation
- Spinodal decomposition

(I.N.Mishustin, PRL **82** (1999); Ph.Chomaz, M.Colonna, J.Randrup, Physics Reports **389** (2004))



$\Rightarrow$ (Large) fluctuations in single events in nonequilibrium situations!

Fluctuations are different, but all are interesting!

- Crossover: remnants of the $\mathcal{O}(4)$ criticality.

  (V. Skokov, B. Stokic, B. Friman and K. Redlich, PRC **82** (2010), V. Skokov, B. Friman and K. Redlich, PRC **83** (2011), V. Skokov, B. Friman and K. Redlich, PLB **708** (2012))

- Critical point: divergent event-by-event fluctuations in thermodynamic equilibrium.
- First order phase transition: large nonstatistical fluctuations in η/p_T spectra in individual events.

Motivation: Heavy-ion collisions are dynamic, inhomogeneous and finite in space and time.

? Can nonequilibrium effects become strong enough to develop signals of the first order phase transition?

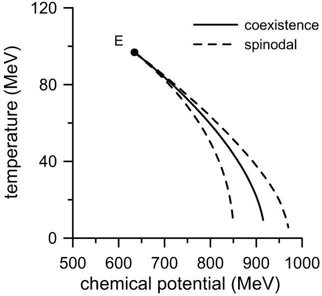
? Do enhanced equilibrium fluctuations at the critical point survive the dynamics?

The linear sigma model with constituent quarks

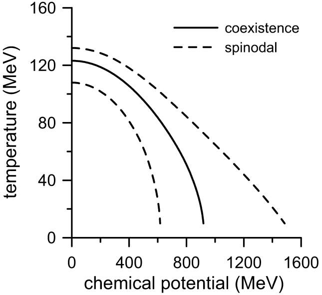
$$\mathcal{L} = \overline{q}\left[i\gamma^{\mu}\partial_{\mu} - g\left(\sigma + i\gamma_5\tau\vec{\pi}\right)\right]q + 1/2\left(\partial_{\mu}\sigma\right)^2 + 1/2\left(\partial_{\mu}\vec{\pi}\right)^2 - U\left(\sigma,\vec{\pi}\right)$$

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - h_q\sigma - U_0$$

$g = 3.3$: crossover at $\mu = 0$



$g = 5.5$: first order pt at $\mu = 0$



$g = 3.63$: critical point at $\mu = 0$

(O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, PRC **64** (2001); C.E. Aguiar, E.S. Fraga, T. Kodama, J.Phys.G **32** (2006))

Nonequilibrium chiral fluid dynamics - NχFD

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Fluid dynamic expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T^{\mu\nu}_{\mathrm{q}} = S^\nu = -\partial_\mu T^{\mu\nu}_\sigma$$

- Nonequilibrium equation of state

$$p = p(e, \sigma)$$

$\Longrightarrow$ Selfconsistent approach within the two-particle irreducible effective action!

(MN, S. Leupold, C. Herold, M. Bleicher, PRC **84** (2011))

The two-particle irreducible (2PI) effective action

Resummation of subdiagrams $\rightarrow$ full propagators
For the σ mean field and the full quark propagators S^{ab}

$$\Gamma[\sigma, S] = S_{\rm cl}[\sigma] - i\mathrm{Tr}\ln S^{-1} - i\mathrm{Tr}S_0^{-1}S + \Gamma_2[\sigma, S]\,,$$

equation of motion for σ and S^{ab}

$$\frac{\delta\Gamma[\sigma, S]}{\delta\sigma^a} = 0 \quad \text{and} \quad \frac{\delta\Gamma[\sigma, S]}{\delta S^{ab}} = 0$$

give conserving transport equations if the self-energy is given by

$$-i\Sigma^{ab}(x, y) = -\frac{\delta\Gamma_2[\sigma, S]}{\delta S^{ab}(x, y)}\,.$$

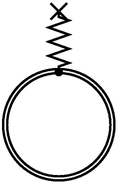
Dyson-Schwinger equation for S^{ab}

$$(i\partial\!\!\!/ - m_f)S^{ab}(x, y) - i\int_{\mathcal{C}} \mathrm{d}^4 z\Sigma^{ac}(x, z)S^{cb}(z, y) = i\delta_{\mathcal{C}}^{ab}(x - y)$$

(J. M. Luttinger, J. C. Ward, Phys. Rev. **118** (1960); G. Baym, L. P. Kadanoff, Phys. Rev. **124** (1961); G. Baym, Phys. Rev. **127** (1962))

The 2PI effective action

$$\Gamma_2[\sigma, S] = g \int_{\mathcal{C}} \mathrm{d}^4 x \mathrm{tr}(S^{++}(x,x)\sigma^+(x) + S^{--}(x,x)\sigma^-(x))$$

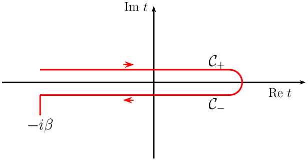


equation of motion for the σ mean field

$$-\frac{\delta S_{\mathrm{cl}}[\sigma]}{\delta \sigma^a} = \frac{\delta \Gamma_2[\sigma, S]}{\delta \sigma^a} = g \mathrm{tr} S^{aa}(x,x)$$

the effective action along the contour

$$\begin{aligned}\Gamma[\sigma, S] = & g \mathrm{tr} S^{++}_{\mathrm{th}}(x,x)\Delta\sigma(x) - \frac{T}{V} \ln Z_{\mathrm{th}} \\ & + \int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x) \\ & + \frac{i}{2} \int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x)\mathcal{I}[\bar{\sigma}](x,y)\Delta\sigma(y)\end{aligned}$$



with $\Delta\sigma = \sigma^+ - \sigma^-$ and $\bar{\sigma} = 1/2(\sigma^+ + \sigma^-)$ on the contour.

The 2PI effective action - term by term

$$\Gamma[\sigma, S] = -\frac{T}{V}\ln Z_{\mathrm{th}} + g\,\mathrm{tr} S^{++}_{\mathrm{th}}(x,x)\Delta\sigma(x) + \int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x) + \frac{i}{2}\int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x)\mathcal{I}[\bar{\sigma}](x,y)\Delta\sigma(y)$$

equilibrium properties, equation of state: $-\frac{T}{V}\ln Z_{\mathrm{th}}$

lowest order in the eq. of motion for the σ field: $g\,\mathrm{tr} S^{++}_{\mathrm{th}}(x,x)\Delta\sigma(x)$

dissipative processes: $\int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x)$

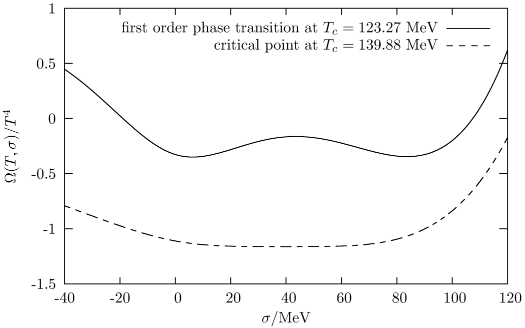
origin of fluctuations: $\frac{i}{2}\int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x)\mathcal{I}[\bar{\sigma}](x,y)\Delta\sigma(y)$

The 2PI effective action

Equilibrium properties

$$\Omega_{\mathrm{eff}} = -\frac{T}{V}\ln Z_{\mathrm{th}} = -d_q T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left(1 + \exp\left(-\frac{E}{T}\right)\right) + U(\sigma, \vec{\pi})$$

with dynamically generated quark masses $E = \sqrt{p^2 + g^2\sigma^2}$



The 2PI effective action

Lowest order

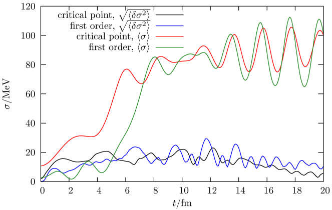
$$\mathrm{tr} S_{\mathrm{th}}^{++}(x,x) = 2 d_q m_q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{n_{\mathrm{F}}(p)}{E} = \rho_s$$

equation of motion for the sigma field:

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s = 0$$

equivalent to

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + \frac{\delta \Omega_{\mathrm{eff}}}{\delta \sigma} = 0$$



(I. N. Mishustin and O. Scavenius, PRL **83** (1999); K. Paech, H. Stöcker and A. Dumitru, PRC **68** (2003))

The 2PI effective action

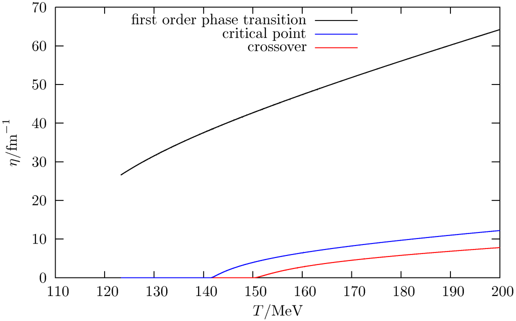
The damping coefficient

Solve $\int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x)$ explicitly for the zero-mode:

$$\eta = g^2 \frac{d_q}{\pi}\left(1 - 2n_{\mathrm{F}}\left(\frac{m_\sigma}{2}\right)\right)\frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$

below T_c:
$\sigma \to \pi\pi$
$\Rightarrow$
$\eta = 2.2/\mathrm{fm}$

(T. S. Biro and C. Greiner, PRL **79** (1997))



The 2PI effective action

The origin of fluctuations

imaginary part of Γ is interpreted as stochastic fluctuations

$$\begin{aligned}\exp[-\frac{1}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \Delta\sigma(x)\mathcal{I}(x,y)\Delta\sigma(y)] \\ = \int \mathcal{D}\xi P[\xi] \exp[i\int \mathrm{d}^4x \xi(x)\Delta\sigma(x)]\end{aligned}$$

$P[\xi]$ Gaussian measure with

$$\begin{aligned}\langle\xi\rangle &= 0 \\ \langle\xi(t)\xi(t')\rangle &= \mathcal{I}^{-1}(t,\mathbf{x};t',\mathbf{y})\end{aligned}$$

in white noise approximation:

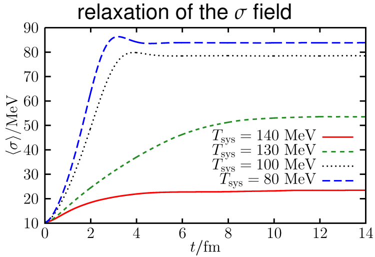
$$\langle\xi(t)\xi(t')\rangle = \frac{1}{V}\delta(t-t')m_\sigma\eta \coth\left(\frac{m_\sigma}{2T}\right)$$

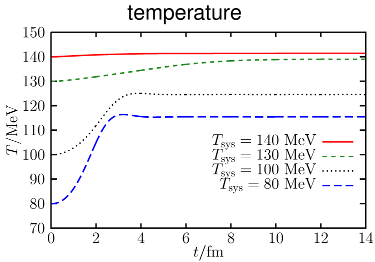
Evolution in a box

- Nonexpanding, finite heat bath
- Initialize the sigma field in equilibrium at $T > T_c$
- Initialize the energy density at a $T_{\mathrm{sys}} < T_c$
- Update sigma field on the grid according to the Langevin equation

Equilibration for a heat bath with reheating

Critical point





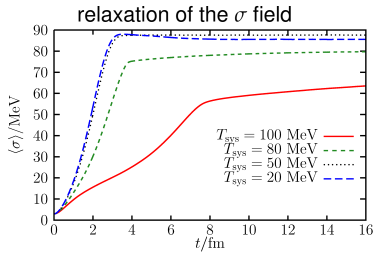
$T_c = 139.8$ MeV

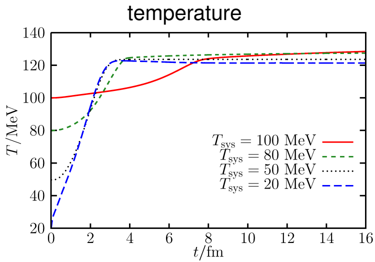
- During relaxation of the σ-field the temperature of the heat bath increases.
- Coupled dynamics equilibrate at a given $T_{\rm eq}$ and $\sigma_{\rm eq}$.
- Green curves correspond to scenarios with $T_{\rm eq}$ near T_c.
  $\Rightarrow$ Critical slowing down!

(MN, S. Leupold, M. Bleicher, PLB **711** (2012))

Equilibration for a heat bath with reheating

First order phase transition





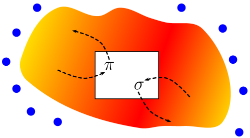
$T_c = 123.3$ MeV

- Strong reheating during relaxation of the σ-field.
- Long (initial) relaxation times for $T_{\rm sys}$ close to the phase transition.
- Except for the scenario with $T_{\rm sys} = 20$ MeV the heat bath reheats to $T > T_c$.
- System gets trapped in metastable states.

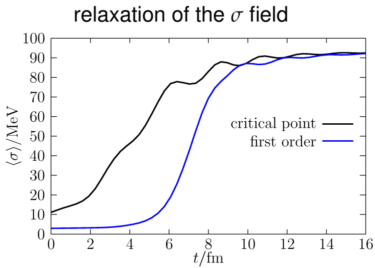
(MN, S. Leupold, M. Bleicher, PLB **711** (2012))

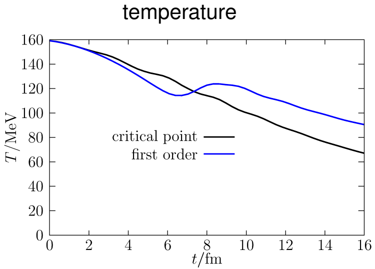
Fluid dynamic expansion of the heat bath

- Very simple initial conditions: almond-shaped initial temperature distribution, sigma field and energy density in equilibrium at $T(x)$
- 3+1d fluid dynamic expansion
- Update sigma field on the grid according to the Langevin equation
- Very good energy conservation



Reheating and supercooling





- Oscillations at the critical point
- Supercooling of the system at the first order phase transition
- Reheating effect visible at the first order phase transition

MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

Intensity of sigma fluctuations

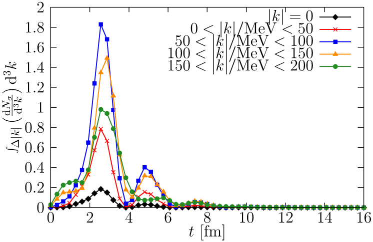
in single events

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{(\omega_k^2|\sigma_k|^2 + |\partial_t\sigma_k|^2)}{(2\pi)^3 2\omega_k}$$

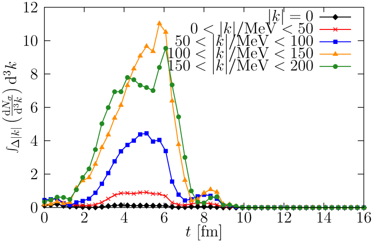
$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

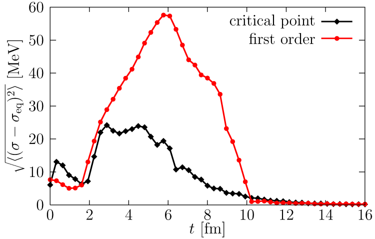
$$m_\sigma = \sqrt{\partial^2 V_{\mathrm{eff}}/\partial\sigma^2|_{\sigma=\sigma_{\mathrm{eq}}}}$$

critical point



first order phase transition





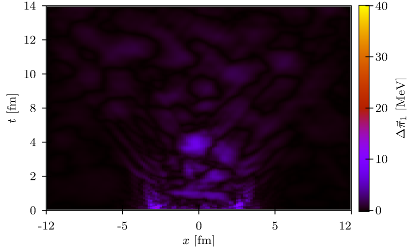
deviation from equilibrium

MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

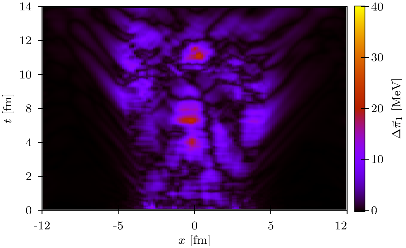
Pion fluctuations

So far: pion fluctuations were not considered and $\vec{\pi} = \langle \vec{\pi} \rangle = 0$.
Propagate pion fluctuations, too:

critical point



first order phase transition



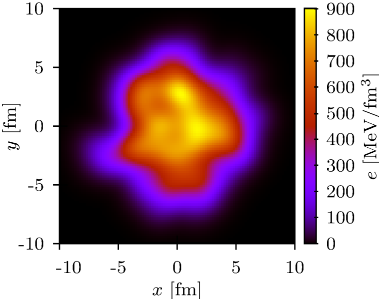
Larger pion fluctuations in a scenario with a first order phase transition!

Realistic initial conditions

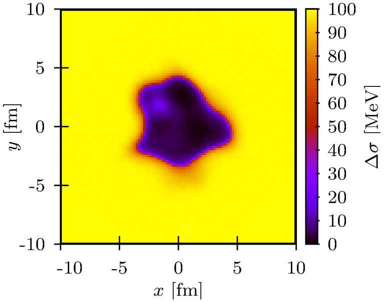
initial conditions from the hybrid UrQMD+hydro approach
(profiles from Pb+Pb at $E_{\mathrm{lab}} = 40A$ GeV)

(H.Petersen et al. PRC **78** (2008))

energy density e



sigma field σ



Dynamic domain formation

First order phase transition

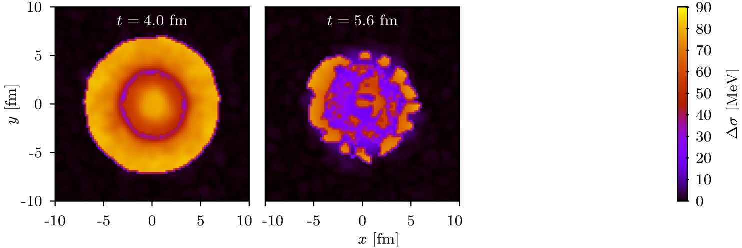
Sigma field fluctuations: $\Delta\sigma = \sqrt{(\sigma - \sigma_{\mathrm{eq}})^2}$

- Highly supercooled state at $t = 4.0$ fm/c.

Dynamic domain formation

First order phase transition

Sigma field fluctuations: $\Delta\sigma = \sqrt{(\sigma - \sigma_{\mathrm{eq}})^2}$

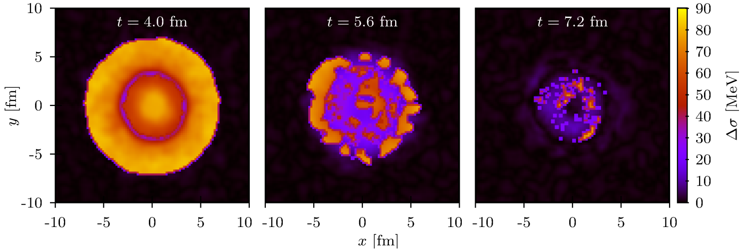


- Highly supercooled state at $t = 4.0$ fm/c.
- Dynamic formation of domains at $t = 5.6$ fm/c.

Dynamic domain formation

First order phase transition

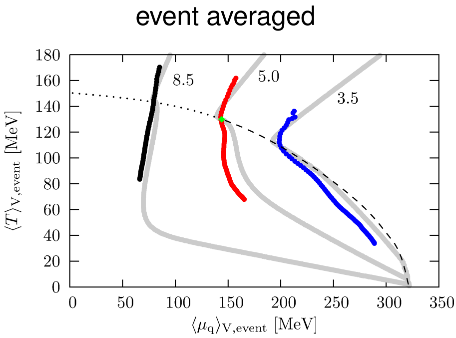
Sigma field fluctuations: $\Delta\sigma = \sqrt{(\sigma - \sigma_{\mathrm{eq}})^2}$



- Highly supercooled state at $t = 4.0$ fm/c.
- Dynamic formation of domains at $t = 5.6$ fm/c.
- Dynamic decay of domains at $t = 7.2$ fm/c.

This could lead to non-statistical fluctuations in hadron multiplicities.

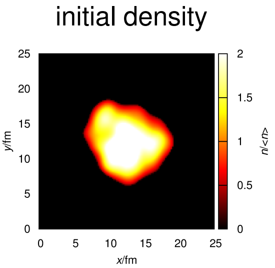
Trajectories and isentropes at finite μ_B

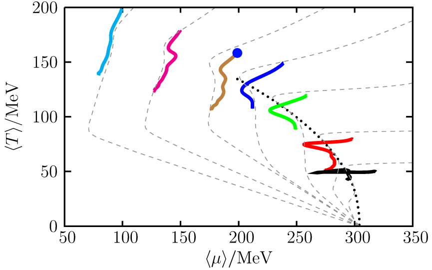


- Grey: equilibrium isentropes ($s/n = \mathrm{const.}$), color: $\langle T \rangle$-$\langle \mu_{\mathrm{B}} \rangle$ trajectories from simulations.
- Fluid trajectories differ from the (equilibrium) isentropes due to interaction with the fields.

Evolution of quark number density

Crossover - critical point



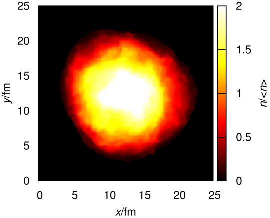


(work in progress C. Herold, MN, I. Mishustin, M. Bleicher)

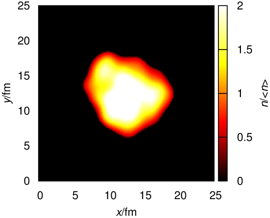
Evolution of quark number density

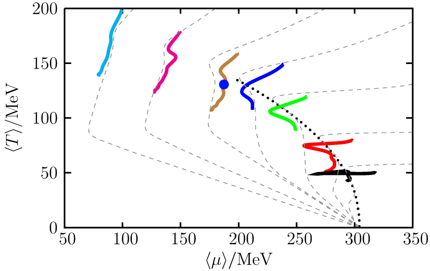
Crossover - critical point

during the evolution



initial density



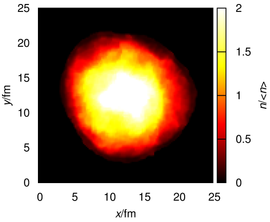


(work in progress C. Herold, MN, I. Mishustin, M. Bleicher)

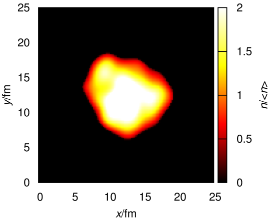
Evolution of quark number density

Crossover - critical point

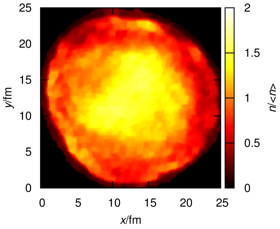
during the evolution

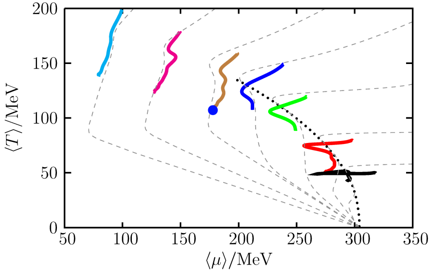


initial density



after $t = 12$ fm



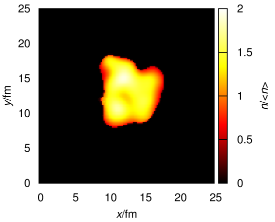


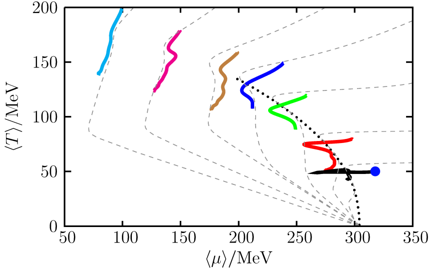
(work in progress C. Herold, MN, I. Mishustin, M. Bleicher)

Formation of high quark number density domains

First order phase transition

initial density



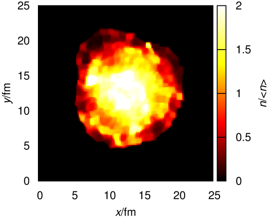


(work in progress C. Herold, MN, I. Mishustin, M. Bleicher)

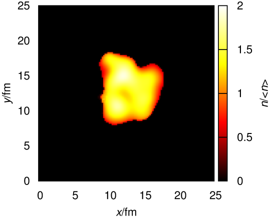
Formation of high quark number density domains

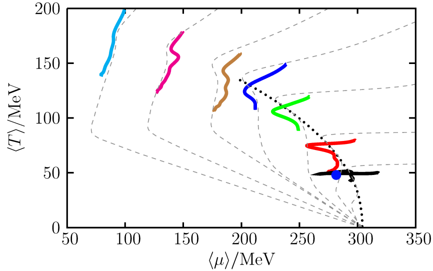
First order phase transition

during the evolution



initial density



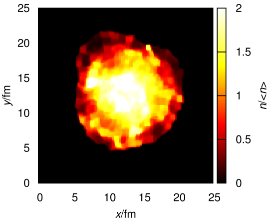


(work in progress C. Herold, MN, I. Mishustin, M. Bleicher)

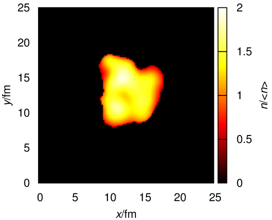
Formation of high quark number density domains

First order phase transition

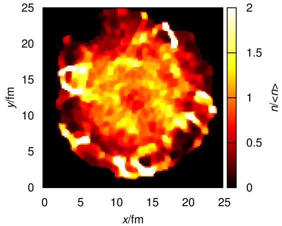
during the evolution

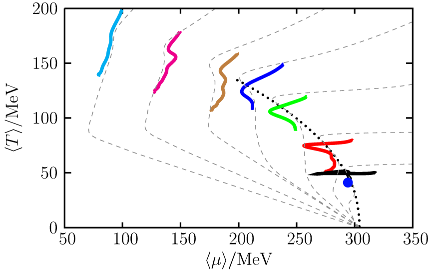


initial density



after $t = 12$ fm

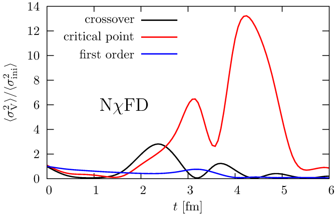




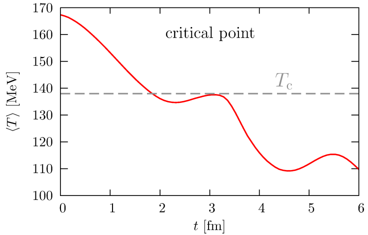
(work in progress C. Herold, MN, I. Mishustin, M. Bleicher)

Dynamic enhancement of event-by-event fluctuations

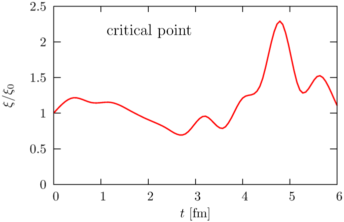
Event-by-event fluctuations



temperature



correlation length from
$G(r) \propto \exp(-r/\xi)$



- Dynamic correlation length grows up to $\simeq$ 2.5 fm.
- Enhanced event-by-event fluctuations of the order parameter, $\langle\sigma_V^2\rangle \propto \xi^2$.
- Initial fluctuations are washed out during the first 1 fm.
- Delay between the averaged T_c and the peak in ξ and $\langle\sigma_V^2\rangle$.

Not included (yet)

not presented:

- Polyakov-loop extended NχFD

  (C. Herold, MN, I. Mishustin, M. Bleicher, arXiv:1301.1214)

- Effects of the inhomogeneity of the system

  (MN, C. Herold, M. Bleicher, arXiv:1301.2577)

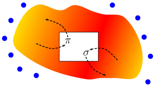
not included:

- Quantum dynamics (e. g. J. Berges et al., PRL **107** (2011) 061301)
- Fluid dynamic fluctuations and viscosities

  (J. Kapusta, B. Mueller, M. Stephanov, PRC **85** (2012); Acta Phys. Polon. B **43** (2012))

- Final state interaction

Summary

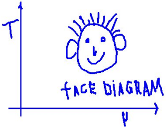


- Dynamic domain formation (σ and n) at the first order phase transition.

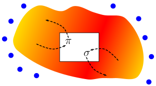
? Can nonequilibrium effects become strong enough to develop signals of the first order phase transition?

- Dynamic correlation length ξ grows up to $\simeq$ 2.5 fm.
- Dynamic enhancement of event-by-event-fluctuations of the order parameter (σ) at the critical point.

? Do enhanced equilibrium fluctuations at the critical point survive the dynamics?



Summary



- Dynamic domain formation (σ and n) at the first order phase transition.

✓ Yes, nonequilibrium dynamics can lead to signals at the first order phase transition!

- Dynamic correlation length ξ grows up to $\simeq$ 2.5 fm.
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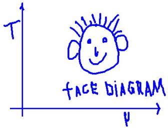
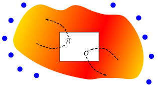
? Do enhanced equilibrium fluctuations at the critical point survive the dynamics?



Summary

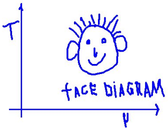


- Dynamic domain formation (σ and n) at the first order phase transition.

✓ Yes, nonequilibrium dynamics can lead to signals at the first order phase transition!

- Dynamic correlation length ξ grows up to $\simeq$ 2.5 fm.
- Dynamic enhancement of event-by-event-fluctuations of the order parameter (σ) at the critical point.

✓ Yes, the critical fluctuations develop even in the situation of a nonequilibrium, dynamic simulation of heavy-ion collisions!



Energy-momentum conservation

for the full propagator:

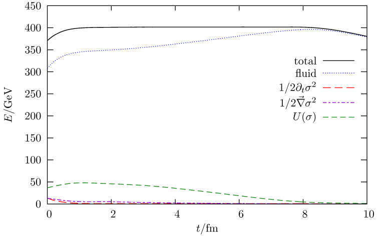
$$\partial_\mu (T_{\mathrm{q}}^{\mu\nu} + T_\sigma^{\mu\nu}) = 0$$

HERE, approximation of an ideal fluid and the source term

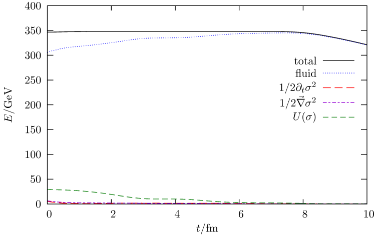
$$\begin{aligned}\partial_\mu T_{\mathrm{q}}^{\mu\nu} =& g\,\mathrm{tr} S_{\mathrm{th}}^{++}(x,x) \\ &= -\partial_\mu T_\sigma^{\mu\nu} = S^\nu\end{aligned}$$

(MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962)

first order phase transition



critical point

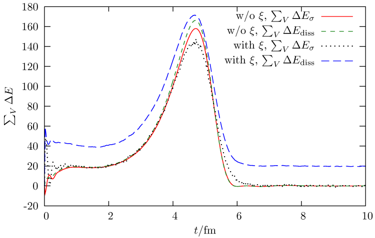


Energy transfer between the field and the heat bath

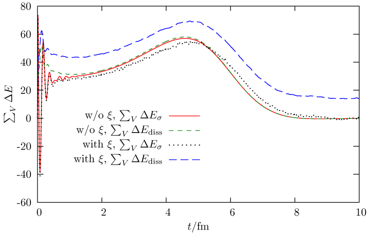
$$\Delta E_{\text{diss}} \simeq -\partial_\mu T_\sigma^{\mu 0} \Delta t = (g\rho_s + \eta \partial_t \sigma)\partial_t \sigma \Delta t$$

The total energy of the σ field

$$E_\sigma = 1/2\partial_t \sigma^2 + 1/2\vec{\nabla}\sigma^2 + U(\sigma)$$



first order phase transition
quench from $T = 160$ MeV to
$T = 100$ MeV



critical point
quench from $T = 160$ MeV to
$T = 130$ MeV