Reaction-diffusion approach in soft diffraction

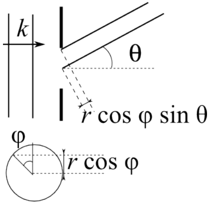
Rodion Kolevatov

in collaboration with *Konstantin Boreskov* (ITEP) and *Larissa Bravina* (Oslo)

Based on EPJC71-1757(1105.3673), 1212.0691 and A.B.Kaidalov Phys.Rep. 50N3 (1979) 157.

Outline

Diffraction of light



Amplitude of the diffracted wave:

$$A(\theta) \sim \int f(r,\theta) e^{ikr\cos\phi\sin\theta} r dr d\phi$$

$$A(\theta) \sim \int f(\vec{r}) e^{i\vec{k}_\perp r} d^2\vec{r}, \quad k_\perp \equiv k \sin\theta$$

The intencity: $I(\theta) \sim A(\theta)^2$

Hankel transform:

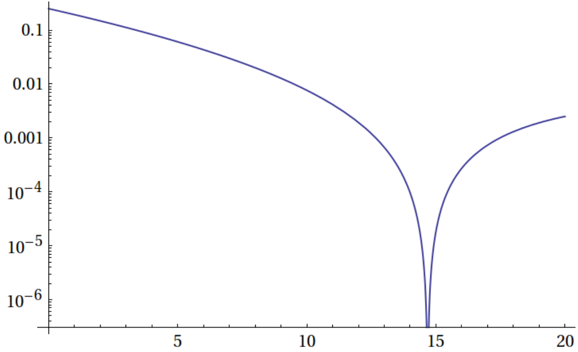
$$g(q) = 2\pi \int_0^\infty f(r) J_0(2\pi q r) r dr; \quad f(r) = 2\pi \int_0^\infty q(q) J_0(2\pi q r) q dq$$

$$f(r) = \theta(a-r) \quad \Rightarrow \quad g(q) = \frac{a J_1(2\pi a q)}{q}$$

$$f(r) = e^{-\pi r^2} \quad \Rightarrow \quad g(q) = e^{-\pi q^2}$$

Diffraction of light

$f(\vec{r}) \sim \theta(R-r)$ (a round hole) $\Rightarrow \quad I \sim (RJ_1(k_\perp R)/k_\perp)^2$



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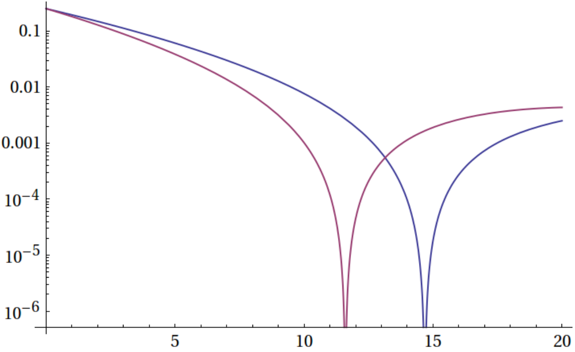

With growth of hole radius R the fall is steeper and the “dip” moves to lower $k_\perp$

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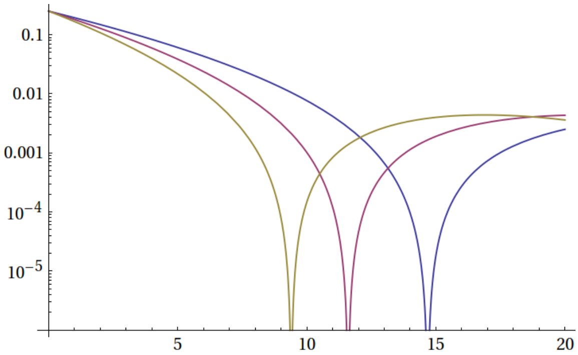

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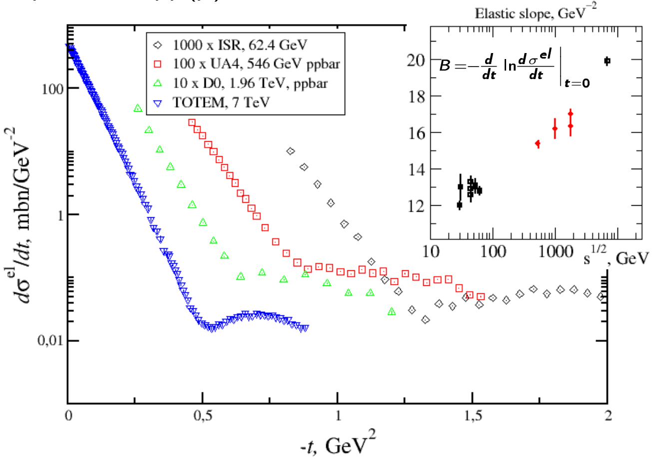
Elastic scattering – shrinkage of diffractive cone

A similar picture in $pp(\bar{p})$ elastic scattering (elastic diffraction):



... shrinkage of the diffractive cone and a displacement of the "dip".

Geometrical models

The *pp* elastic amplitude: $M(q) \simeq iD(q)\ (\Im M(q) \gg \Re M(q))$
Fourier transform: $f(Y,\mathbf{b}) = \frac{1}{(2\pi)^2}\int d^2q\, e^{-i\mathbf{qb}} D(Y,\mathbf{q})$.
$f(Y,\mathbf{b})$ is similar to the opacity in optics:

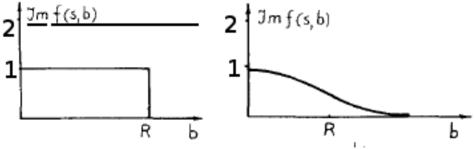
$$\sigma^{\rm el} = \int \frac{d^2q}{(2\pi)^2}\,|M(Y,\mathbf{q})|^2 = \int d^2b\,|f(Y,\mathbf{b})|^2.$$

Optical theorem: $\sigma^{\rm tot}(Y) = 2\,\Im M(Y,\mathbf{q}=0) = 2\int d^2b\, f(Y,\mathbf{b}),$

Definition $\sigma^{\rm inel}(b) \equiv 2f(\mathbf{b}) - |f(\mathbf{b})|^2$
Unitarity constraint: $0 < f(b) < 2 \quad \Rightarrow \quad 0 \leq \sigma^{\rm inel}(b) \leq 1$

Interpretation: $\sigma^{\rm inel}(b) \equiv$ **probability of inelastic interaction**

Geometrical models



Unitarity limit: $f(b) = 2\theta(R-b) \Rightarrow \sigma^{\text{inel}}(b) = 0$.
Black disk limit: $f(b) = \theta(R-b) \Rightarrow \sigma^{\text{inel}}(b) = \theta(R-b)$, $\sigma^{\text{el}} = 1/2\sigma^{\text{tot}}$.

The data suggest:

- Approx. constant opacity at small b (presence of dip)
- Spreading in b of constant opacity region with the growth of energy (shrinkage of diffracitve cone).
- The inelastic profile in the center is close to the upper limit (e.g. $\sigma^{\text{inel}}(b) = 0.94$ at $\sqrt{s} = 53$ GeV)

Inelastic diffraction – a special case of inelastic event

Example Event Displays from CDF Run II

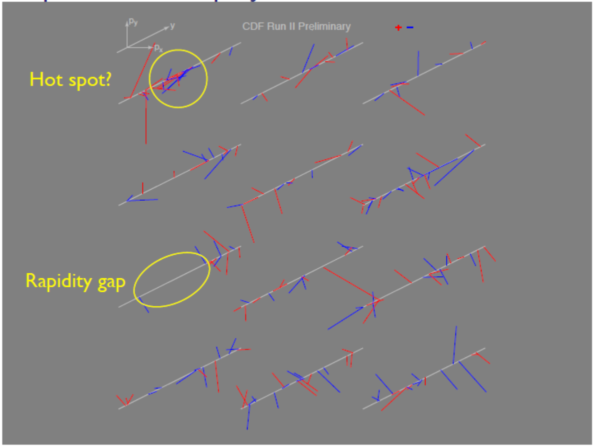
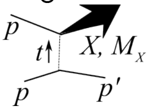


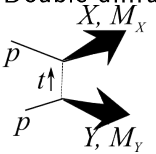
Illustration: talk by Chris Quigg at Spaatind'2012

Inelastic diffraction

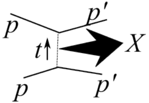
- Single diffraction

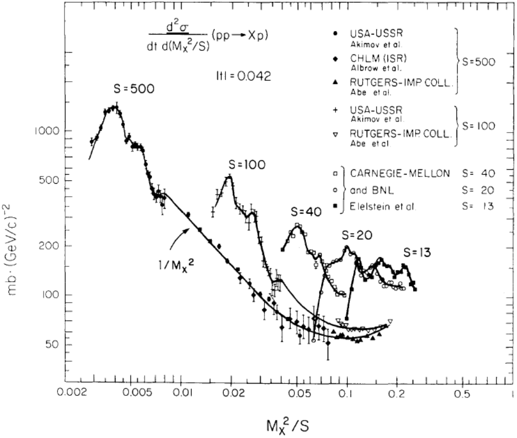


- Double diffraction



- Central diffraction





$$\Delta y_{\rm gap} = \ln s/M_X^2 \text{ -- rapidity gap}$$

s-channel view on small-M_X^2 diffraction

Amplitudes for scattering into elastic and diffractive channels can be organized into a matrix $||M_{ik}|| \simeq i||D_{ik}||$;
D_{11} – elastic amplitude; D_{1k} – dissociation to ch. k.

Orthogonal transformation: $D = QFQ^T; \quad F_{ij} = F_i\delta_{ij}, \; QQ^T = I.$

Interpretation (Good and Walker '60):

- $|p\rangle = \sum Q_{1k}|k\rangle$ – superposition of eigenstates with different scattering amplitudes;
- Eigenstates $|k\rangle$ undergo only elastic scattering.

Good–Walker formalism, example

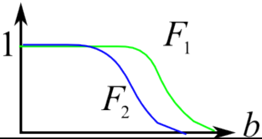
Example: 2 channels.

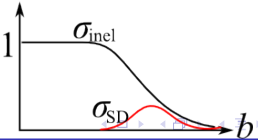
$$|p\rangle = \alpha_1|1\rangle + \alpha_2|2\rangle; \quad \alpha_1^2 + \alpha_2^2 = 1$$

$$D_{11} = \alpha_1^2 F_1 + \alpha_2^2 F_2; \quad D_{12} = \alpha_1\alpha_2(F_2 - F_1)$$

$$\sigma^{\text{tot}} = 2\int d^2b[\alpha_1^2 F_1(b) + \alpha_2^2 F_2(b)]; \quad \sigma^{\text{el}} = \int d^2b[\alpha_1^2 F_1(b) + \alpha_2^2 F_2(b)]^2$$

$$\sigma^{\text{SD}} = \int d^2b\,[\alpha_1\alpha_2(F_1(b) - F_2(b))]^2$$





Lessons from the example

Diffraction:

1: Has a peripheral nature

2: Sensitive to the shape of the edge

3: In case elastic amplitude saturates at black disc limit (growing disc) – $\ln s$ growth with c.m. energy (growing ring).

Lessons from the example

Diffraction:

1: Has a peripheral nature

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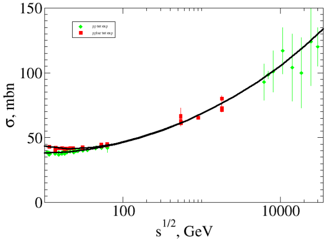
Now let us turn to high-M^2 diffraction...

Power-like contributions to the amplitude



PDG fit:

$$\sigma_{tot}^{pp(\bar{p})} = 18.3 s^{0.095} + 60.1 s^{-0.34} \pm 32.8 s^{-0.55}$$

Optical theorem:

$$\sigma_{tot} = \frac{1}{s} 2\Im A_{el}(q=0) \equiv 2\Im M_{el}(q=0)$$

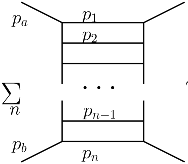
Indication: High energy elastic scattering goes via quasiparticle, "Reggeon", exchanges with powerlike asymptotic in c.m.energy. Leading contirbution – Pomeron, $M_{\mathbb{P}} \sim s^{\Delta}$, $\Delta > 0$.
Caveat: Single Pomeron exchange violates Froissart bound ($\sigma_{tot} \lesssim C \ln^2 s$)

s-channel ($s \to \infty$, $t = Q^2$ small) dominant contributions

Analiticity&unitarity:

- Power-like terms come from poles in the complex L plane of the t-channel amplitude, Pomeron = the rightmost singularity

Field theories (φ^3, QCD):

- $\sum_n$ 

$\sim \quad s^{\alpha(t)}$

$$p_1^+ \gg p_2^+ \gg \ldots \gg p_n^+$$
$$p_1^- \ll p_2^- \ll \ldots \ll p_n^-$$
$$p^\pm = p^0 \pm p^3$$

For phenomenological applications: $\mathbb{R}/\mathbb{P}$ = exchange of a "ladder" structure in the t-channel with ordering of the ladder rungs in rapidity $y = 1/2 \ln p_+/p_-$

The Pomeron

The 1-Pomeron exchange amplitude:

$$M_{1\mathbb{P}} \sim i\,\frac{\exp(\Delta y)\exp(-\frac{b^2}{4\alpha' y})}{4\pi\alpha' y}$$

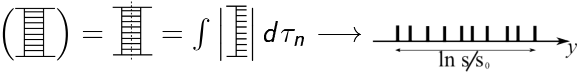
- Growing energy behaviour
  $\Rightarrow$ Ensures growth of the cross sections
- Diffusion in the transverse plane
  $\Rightarrow$ Ensures growth of the interaction radius
- Iteration of the $\mathbb{P}$ exchanges ensures the Froissart bound

Contributions to σ_{tot}

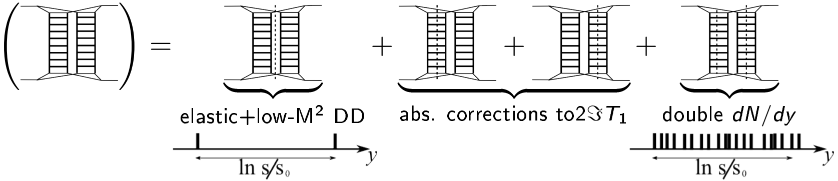
Contributions to imaginary part (Cutkosky rules):

- Cut the diagram for the elastic scattering amplitude
- Put cut lines on the mass shell, integrate over the phase space

Single "ladder" exchange – uniform rapidity distribution

$2\Im T_1 = 2\Im$ 

Double "ladder"

$2\Im$ 

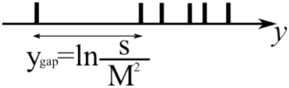
Iterating ladders slows the growth:

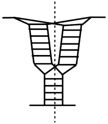
from $\sigma_{\rm tot} \sim s^{\Delta}$ down to $\sigma_{\rm tot} \sim \ln^2 s$.

Contributions to σ_{tot}

Rapidity gaps – splitting of the "ladder":

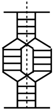
Single diffraction dissociation





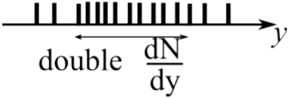
+ abs. corrections

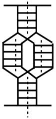
Double diffraction dissociation



+ abs. corrections

other cut of same graph





+ abs. corrections

RFT

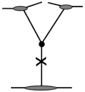
Reggeon Field Theory = the theory of the Pomeron (Reggeon) exchanges and interactions. The underlying principles of the RFT are analyticity and t-channel unitarity of the elastic amplitude.

- Attractive features from the phenomenological point of view:
  - Gives reliable quantitative predictions of hadronic X-sections
  - Different cuts of the RFT diagrams define X-sections of various inelastic processes via AGK rules
- Provides an intuitive understanding of HE interactions.
  - $\ln^2 s$ growth of the total cross sections due to diffusion of $\mathbb{P}$s in the transverse plane
  - Events with rapidity gaps correspond to certain cuts of the graphs with $\mathbb{R}/\mathbb{P}$ interactions (enhanced and loop graphs)

Enhanced and loop contributions become essential also for the elastic amplitude with growth of c.m. energies; untrivial task, under investigation by several groups (Ostapchenko, Khoze et al., Poghosyan; also Lund group non-RFT approach).

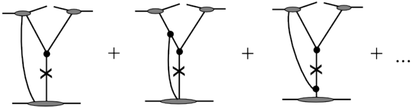
Contribution of diffractive cut

Lowest order contribution:



$$\frac{d^2\sigma_{SD}}{dt d(M^2/s)} \sim \left(\frac{M^2}{s}\right)^{-1-\Delta} s^{\Delta} \Rightarrow \sigma_{SD}(M^2/s < \alpha) \sim s^{\Delta}$$

Absorptive corrections:



Alternatives:

- Introduce reg. scale and compute order by order
- Use specific models with tuned $m\mathbb{P} \to n\mathbb{P}$ vertices $\to$ transforms power-like behaviour of Pomeron propagator to $\sim \ln^2$.
- Use effective approaches.

RFT

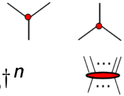
The elastic amplitude $T = A/(8\pi s)$ is factorized:

$$T = \sum_{n,m} V_n \otimes G_{nm} \otimes V_m$$

G_{mn} – process independent, obtained within 2D+1 field theory (only $\mathbb{P}$):

$$\mathcal{L} = \frac{1}{2}\phi^\dagger(\overleftarrow{\partial_y} - \overrightarrow{\partial_y})\phi - \alpha'(\nabla_\mathbf{b}\phi^\dagger)(\nabla_\mathbf{b}\phi) + \Delta\phi^\dagger\phi + \mathcal{L}_{int}.$$

Minimal choice (classic): $\mathcal{L}_{int} = i\, r_{3P}\phi^\dagger\phi(\phi^\dagger + \phi)$



Infinite $\sharp$ of vertices [KMR, Ostapchenko, MP+ABK]: $r_{mn}\phi^m{\phi^\dagger}^n$

Fine tuning of the vertices, some contributions neglected

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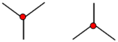
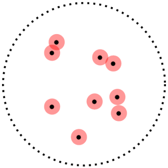


Infinite $\sharp$ of vertices [KMR, Ostapchenko, MP+ABK]: $r_{mn}\phi^m{\phi^\dagger}^n$

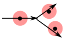
"Almost minimal": $i\, r_{3P}\phi^\dagger\phi(\phi^\dagger + \phi) + \chi{\phi^\dagger}^2\phi^2$

the reaction-diffusion approach is applicable for numerical computation of all-loop Green functions. [Grassberger'78; K.B.'01]

The reaction-diffusion (stochastic) approach.



Consider a system of classic "partons" in the transverse plane with:

- Diffusion (chaotical movement) D; 
- Splitting (λ – prob. per unit time) 
- Death (m_1) 
- Fusion ($\sigma_\nu \equiv \int d^2b\, p_\nu(b)$) 
- Annihilation ($\sigma_{m_2} \equiv \int d^2b\, p_{m_2}(b)$) 

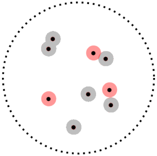
Parton number and positions are described in terms of

$$\boxed{\textbf{probability densities}\ \rho_N(y,\mathcal{B}_N)\ (N=0,1,...;\mathcal{B}_N \equiv \{b_1,\ldots,b_N\})}$$

with normalization $p_N(y) \equiv \frac{1}{N!}\int \rho_N(y,\mathcal{B}_N)\prod d\mathcal{B}_N;\quad \sum_0^\infty p_N = 1.$

Inclusive distributions

S-parton inclusive distributions:



$$f_s(y;\mathcal{Z}_s) = \sum_N \frac{1}{(N-s)!}\int d\mathcal{B}_N\ \rho_N(y;\mathcal{B}_N)\prod_{i=1}^{s}\delta(\mathbf{z}_i - \mathbf{b}_i);$$

$\int d\mathcal{Z}_s f_s(y;\mathcal{Z}_s) = \sum \frac{N!}{(N-s)!}\, p_N(y) \equiv \mu_s(y)$. – factorial moments.

Example: Start with a single parton with only diffusion and splitting allowed.

$$f_1^{1\ \text{parton}}(y,b) = \frac{\exp(\lambda y)\exp(-b^2/4Dy)}{4\pi D y}.$$

– the bare Pomeron propagator.

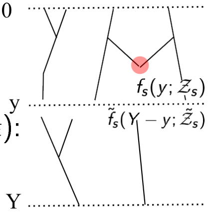
The set of evolution equations for $f_s(\mathcal{Z}_s)$, $(s = 1,\ldots)$ coincides with the set of equations for the Green functions of the RFT.

The amplitude.

Green functions:

$f_s(y;\mathcal{Z}_s) \propto \sum_m \int d\mathcal{X}_m\, V_m(\mathcal{X}_m) G_{mn}(0;\mathcal{X}_m|y;\mathcal{Z}_n)$;

$f_m(y = 0, \mathcal{X}_m) \propto V_m(\mathcal{X}_m)$ – particle–mPomeron vertices



The amplitude ($g(b)$ assumed narrow; $\int g(b) d^2b \equiv \epsilon$):
$T(Y) = \langle A|T|\tilde{A}\rangle =$

$$= \sum_{s=1}^{\infty} \frac{(-1)^{s-1}}{s!} \int d\mathcal{Z}_s d\tilde{\mathcal{Z}}_s f_s(y;\mathcal{Z}_s)\tilde{f}_s(Y-y;\tilde{\mathcal{Z}}_s) \prod_{i=1}^{s} g(\mathbf{z}_i - \tilde{\mathbf{z}}_i - \mathbf{b}).$$

It does not depend on the linkage point y ("boost invariance") **if**

$$\lambda \int g(b) d^2b = \int p_{m_2}(b) d^2b + \frac{1}{2} \int p_\nu(b) d^2b \ ,$$

Correspondence RFT–Stochastic model

We use the simplest form of $g(b)$, $p_{m_2}(b)$ and $p_\nu(b)$:

$$p_{m_2}(\mathbf{b}) = m_2\ \theta(a - |\mathbf{b}|); \quad p_\nu(\mathbf{b}) = \nu\ \theta(a - |\mathbf{b}|);$$

$$g(\mathbf{b}) = \theta(a - |\mathbf{b}|);.$$

with a – some small scale; $\epsilon \equiv \pi a^2$.

RFT	stochastic model
Rapidity y	Evolution time y
Slope α'	Diffusion coefficient D
$\Delta = \alpha(0) - 1$	$\lambda - m_1$
Splitting vertex r_{3P}	$\lambda\sqrt{\epsilon}$
Fusion vertex r_{3P}	$(m_2 + \frac{1}{2}\nu)\sqrt{\epsilon}$
Quartic coupling χ	$\frac{1}{2}(m_2 + \nu)\epsilon$

Few things to note:

Boost invariance ($\lambda = m_2 + \frac{\nu}{2}$) $\Leftrightarrow$ equality of fusion and splitting vertices

The $2 \to 2$ vertex cannot be set to zero ($m_2, \nu > 0$).

Summary of the stochastic approach I

Peculiarities of the approach:

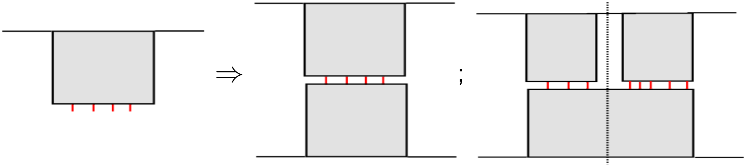
- Presence of the triple and $2 \to 2$ couplings
- Regularization scale (equivalient to the cutoff or the Pomeron size) enters via parton interaction distance $(g(b), p_{m_2}(b), p_\nu)$.
- $\mathbb{P}$ exchanges only
- Neglect of the real part of the $\mathbb{P}$ exchange amplitude.

Summary of the stochastic approach II

In theory: One could compute numerically the whole set of the RFT Green functions and use them for constructing amplitude and all possible cuts. However, this is practically impossible – too expencive numerically.

In practice: It is possible to compute numerically certain convolututions of RFT Green function which correspond to:

- the elastic scattering amplitude
- the single diffractive cut of the amplitude.



For calculation of the SD cut we rely on the AGK result for the lower block: its independence on the position of the cut.

Calculation method – the amplitude I

Key: compute the amplitudes of interest event-by-event (not f_s).

- N-channel eikonal vertices $\Rightarrow$
  - $\Rightarrow$ Superposition of N Poissons in parton $\sharp$ distribution
- MC evolution upto the given rapidity $\Rightarrow$
  - $\Rightarrow$ A sample of partons at certain positions

$$f_s^{\text{sample}}(\mathcal{Z}_s) = \sum_{\{\hat{\mathbf{x}}_{i_1},\ldots,\hat{\mathbf{x}}_{i_s}\}\in\hat{\mathcal{X}}_N} \delta(\mathbf{z}_1 - \hat{\mathbf{x}}_{i_1})\ldots\delta(\mathbf{z}_s - \hat{\mathbf{x}}_{i_s})$$

Calculation method – the amplitude I

Key: compute the amplitudes of interest event-by-event (not f_s).

- N-channel eikonal vertices $\Rightarrow$
  - $\Rightarrow$ Superposition of N Poissons in parton $\sharp$ distribution
- MC evolution upto the given rapidity $\Rightarrow$
  - $\Rightarrow$ A sample of partons at certain positions

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Instead of doing this ...

$$T^{el} = \sum_{n,s,k} \frac{(-1)^{s-1}}{s!} \underbrace{P_n(\mathcal{X}) \otimes f_{ns}(\mathcal{X}|\mathcal{Z})}_{f_s(y,\mathcal{Z})} \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{Z}}) \otimes \underbrace{\tilde{f}_{ks}(\tilde{\mathcal{X}}|\tilde{\mathcal{Z}}) \otimes \tilde{P}_k(\tilde{\mathcal{X}})}_{\tilde{f}_s(Y-y,\tilde{\mathcal{Z}})}$$

Calculation method – the amplitude I

Key: compute the amplitudes of interest event-by-event (not f_s).

- N-channel eikonal vertices $\Rightarrow$
  - $\Rightarrow$ Superposition of N Poissons in parton $\sharp$ distribution
- MC evolution upto the given rapidity $\Rightarrow$
  - $\Rightarrow$ A sample of partons at certain positions

$$f_s^{\mathrm{sample}}(\mathcal{Z}_s) = \sum_{\{\hat{\mathbf{x}}_{i_1},\ldots,\hat{\mathbf{x}}_{i_s}\}\in\hat{\mathcal{X}}_N} \delta(\mathbf{z}_1 - \hat{\mathbf{x}}_{i_1})\ldots\delta(\mathbf{z}_s - \hat{\mathbf{x}}_{i_s})$$

... we do this:

$$T^{el} = \sum_{n,k} P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z}-\tilde{\mathcal{Z}}) \otimes \tilde{f}_{ks}(\tilde{\mathcal{X}}|\tilde{\mathcal{Z}})}_{T_{sample}} \otimes \tilde{P}_k(\tilde{\mathcal{X}}).$$

$$T^{el}_{\mathrm{sample}} = \sum_{s=1}^{N_{min}} (-1)^{s-1} \sum_{i_1<i_2\ldots<i_s} \sum_{j_1<\ldots<j_s} g_{i_1 j_1}\ldots g_{i_s j_s}.$$

Calculation method – the amplitude II

Setting the linkage point to full rapidity interval $y = Y$ simplifies the calculation: $\tilde{f}_s(y = 0, \mathcal{Z}_s) = N_s(\mathcal{Z}_s)/\epsilon^{s/2}$ and the MC average involves evolution from only one side:

$$T^{el} = \sum_n P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{X}}) \otimes \tilde{P}_s(\tilde{\mathcal{X}}).}_{T_{sample}}$$

$$T^{el}_{\text{sample}} = \sum_{s=1}^{N} (-1)^{s-1} \tilde{\mu}_s \epsilon^s \sum_{i_1 < i_2 \ldots < i_s} \tilde{p}_s(\hat{\mathbf{x}}_{i_1} - \mathbf{b}, \ldots, \hat{\mathbf{x}}_{i_s} - \mathbf{b}).$$

Calculation method – the SD cut

For the SD cut substituting "event-by-event Green functions" gives

$$T^{SD}_{\mathrm{sample}} = 2\,T^{el}_{\mathrm{sample}} - T'_{\mathrm{sample}}$$

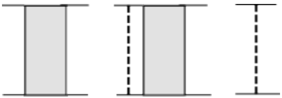
T'_{sample} is computed the same way as T^{el}_{sample} with two distinctions:

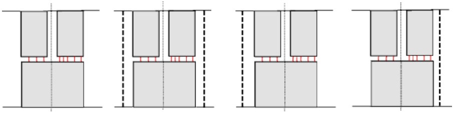
- Not one, but two sets from the projectile side
- which are evolved independently until the Δy_{gap} and then combined into a single one

Resumé: The elastic scattering amplitude and its SD cut are computed within the same numerical framework.

Model parameters

- Two-channel eikonal p–$n\mathbb{P}$ vertices to incorporate low-M^2 diffraction
- Account the secondary Reggeons contribution to the lowest order
- Neglect the real part of the Pomeron exchange amplitude (keeping it for the secondary Reggeons)
- Neglect central diffraction in calculation of SD cross sections (CD contribution is accounted twice in calculation of 2-side SD, the extra contribution should have been subtracted).



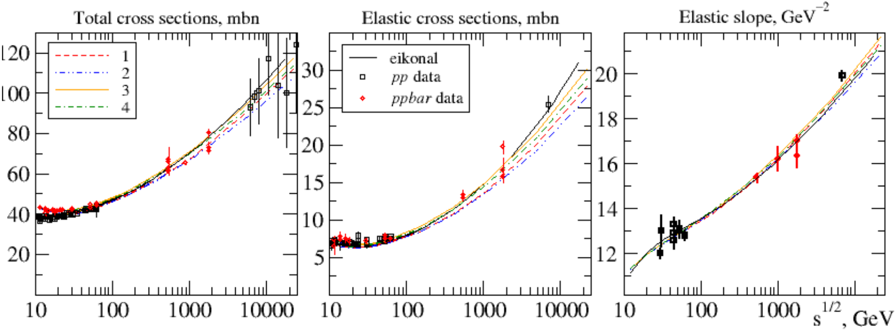


Model parameters

$r_{3\mathbb{P}}$ – fixed [Kaidalov'79]
a – regularization scale
$1 + \Delta$ – bare Pomeron intercept
α' – Pomeron slope
$|p\rangle = \beta_1|1\rangle + \beta_2|2\rangle; \quad |\beta_1|^2 \equiv C_1; \ |\beta_2|^2 \equiv C_2 = 1 - C_1$.
$\mathbb{P}$ couplings to $|1\rangle$ and $|2\rangle$: $g_{1/2} = g_0(1 \pm \eta)$
R – size of the p–$\mathbb{P}$ vertex (Gaussian)
Strategy:

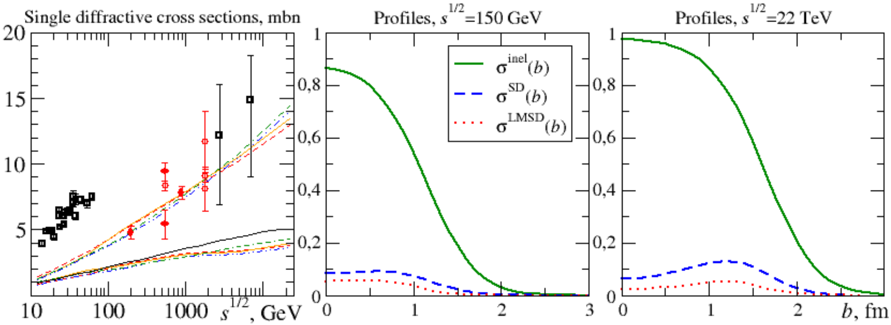
1. Eikonal fit to σ_{tot}, σ_{el}, B and low energy low-M^2 σ_{SD}
2. All-loop fit to σ_{tot}, σ_{el}, B starting with parameter set from [1]
3. Calculation of diffractive cross sections with parameters obtained at [2]

Results on X-sections and slope $\left(B = \frac{d}{dt} \ln \frac{d\sigma_{el}}{dt}\Big|_{t=0}\right)$



$\chi_3 > \chi_1 = \chi_4 > \chi_2$; $a_1 = a_2 = 0.018$ fm; $a_3 = a_4 = 0.036$ fm. $C_1 = C_2 = 0.5$, $\eta = 0.55$.
$\Delta = 0.195$; $\alpha' = 0.154$ GeV^{-2}; $R^2 = 3.62$ GeV^{-2}; $g_0 = 4.7$ GeV^{-1}; $r_{3P} = 0.087$ GeV^{-1} [Kaidalov'79].

Inelastic and diffractive profiles



Conclusions

- Total, elastic and single diffractive cross sections are computed in RFT within the same numerical framework to all orders in the number of loops;
- A satisfactory description on total and elastic cross sections is obtained within the all-loop framework;
- The single diffractive cross sections energy behaviour is compatible with logarithmic growth.

Backup – cross sections definitions

$$\sigma^{\rm tot}(Y) = 2\,\Im M(Y,\mathbf{q}=0), \quad \sigma^{\rm el} = \int \frac{d^2q}{(2\pi)^2}\,|M(Y,\mathbf{q})|^2\ ,$$

$$f(Y,\mathbf{b}) = \frac{1}{(2\pi)^2}\int d^2q\, e^{-i\mathbf{qb}} M(Y,\mathbf{q})\ .$$

$$\sigma^{\rm tot}(Y) = 2\int d^2b\,\Im f(Y,\mathbf{b})\ , \quad \sigma^{\rm el} = \int d^2b\,|f(Y,\mathbf{b})|^2.$$

$$f(Y,\mathbf{b}) \simeq iT(Y,\mathbf{b}), \quad T \equiv \Im f$$

$$B = -\frac{d}{dt}\ln\frac{d\sigma^{el}}{dt}\Big|_{t=0} = \frac{\int b^2\Im A(b)d^2b\int\Im A(b)d^2b+\int b^2\Re A(b)d^2b\int\Re A(b)d^2b}{2\left((\int\Im A(b)d^2b)^2+(\int\Re A(b)d^2b)^2\right)}$$

Backup – secondary trajectories

pp: $\Im f_{pp}(b) = \Im A_P(b) + [\Im A_+(b) + \Im A_-(b)]\,[1 - \Im A_P(b)]$
$\Re f_{pp}(b) = \left[\Re A_{R_+} + ReA_{R_-}\right][1 - \Im A_P(b)]$

pp: $\Im f_{pp}(b) = \Im A_P(b) + [\Im A_+(b) - \Im A_-(b)]\,[1 - \Im A_P(b)]$
$\Re f_{pp}(b) = \left[\Re A_{R_+} - ReA_{R_-}\right][1 - \Im A_P(b)]$

pp SD:

$$f_{pp}^{\text{Diff}}(b) = f_{pp}^{\text{Diff}}(b)\Big|_{\mathbb{P}\text{only}} \left[1 + |A_{R_+}(b) + A_{R_-}(b)|^2 - 2\Im(A_{R_+}(b) + A_{R_-}(b))\right]$$

$$A_\pm(y,b) = \eta_\pm \beta_\pm^2 \frac{\exp(\Delta_\pm y)}{2\alpha'_\pm y + 2R_\pm^2} \exp\left(-\frac{b^2}{4(\alpha'_\pm y + R_\pm^2)}\right)$$

$$\eta_\pm = \pm i - \frac{1 \pm \cos \pi\alpha_\pm(0)}{\sin \pi\alpha_\pm(0)}$$

Backup – parameters of the fit

$C_1 = C_2 = 0.5$; $\eta = 0.55$; $r_{3P} = 0.087$ GeV^{-1};
$\chi_1 = \chi_4 = 0.0005569$ fm$^2 = 0.01435$ GeV^{-2},
$\chi_2 = 0.0002785$ fm$^2 = 0.00717$ GeV^{-2},
$\chi_3 = 0.0011134$ fm$^2 = 0.0287$ GeV^{-2}.

Trajectory	$\mathbb{P}$	R_+	R_-
$\alpha(0) - 1$	0.195	-0.34	-0.55
α', GeV^{-2}	0.154	0.70	1.0
R^2, GeV^{-2}	3.62	3.0	5.2
$\beta_{0/+/-}$, GeV^{-1}	4.7	4.05	2.59

$\Delta_{eikonal} = 0.14$.

In terms of the stochastic approach:

	a, fm	λ	m_1	m_2	ν	$\bar{N}$	D, fm^2	R_P, fm
1	0.018	0.54722	0.35222	0	1.09488	29	0.0065	0.375
2	0.018	0.54722	0.35222	0.54722	0	29	0.0065	0.375
3	0.036	0.27361	0.07861	0	0.54722	14.5	0.0065	0.375
4	0.036	0.27361	0.07861	0.27361	0	14.5	0.0065	0.375