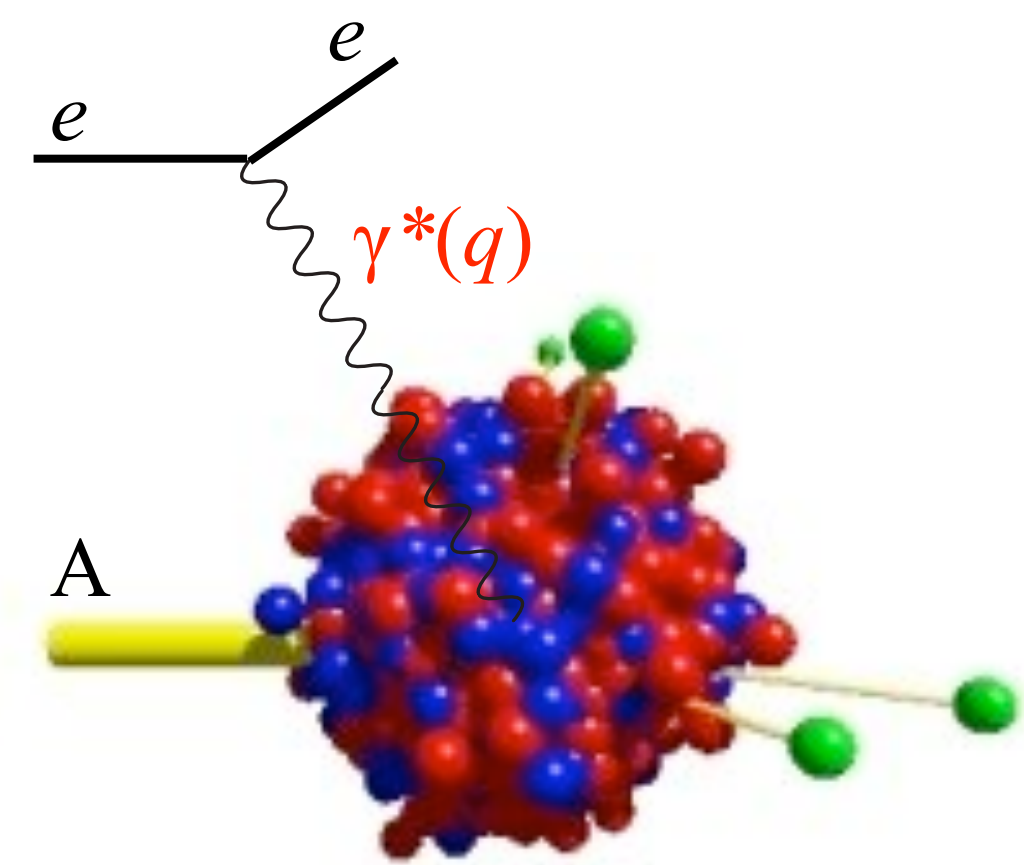
Measuring transverse size with virtual photons

Work done with Samu Kurki

arXiv:0911.3011
arXiv:1101.4810

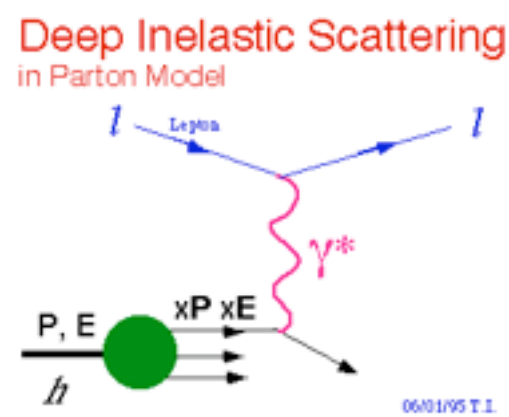
Photons are useful probes of strong dynamics at any Q^2

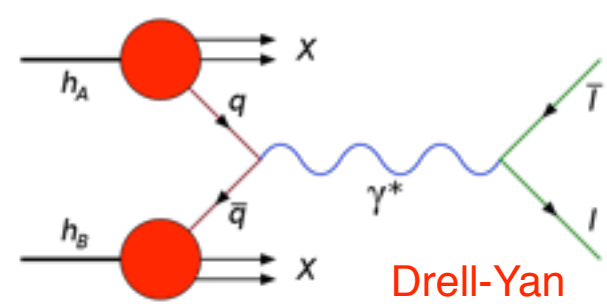
The q^2 dependence should reflect the effective size of the interaction region



A precise understanding existed only in the Bj limit

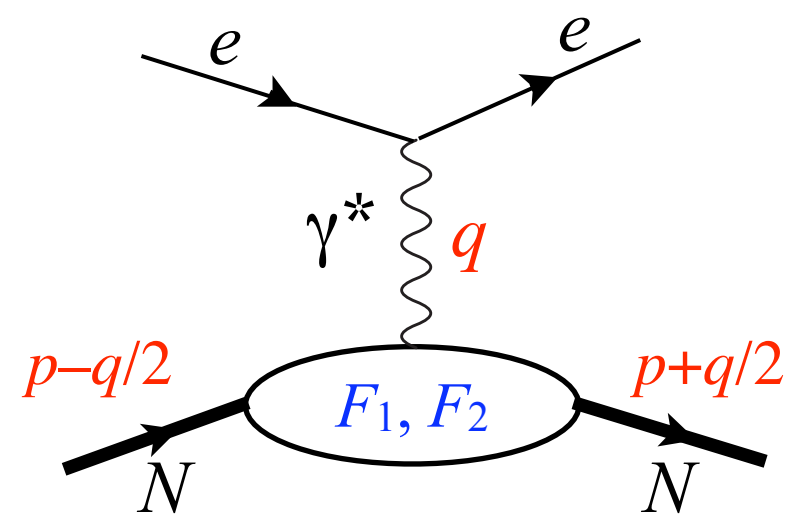
$q^2 \to \infty$





Nucleon Form Factors

Using Lorentz and gauge invariance, the scattering amplitude is expressed in terms of the Dirac F_1 and Pauli F_2 form factors, which depend on $Q^2 = -q^2$



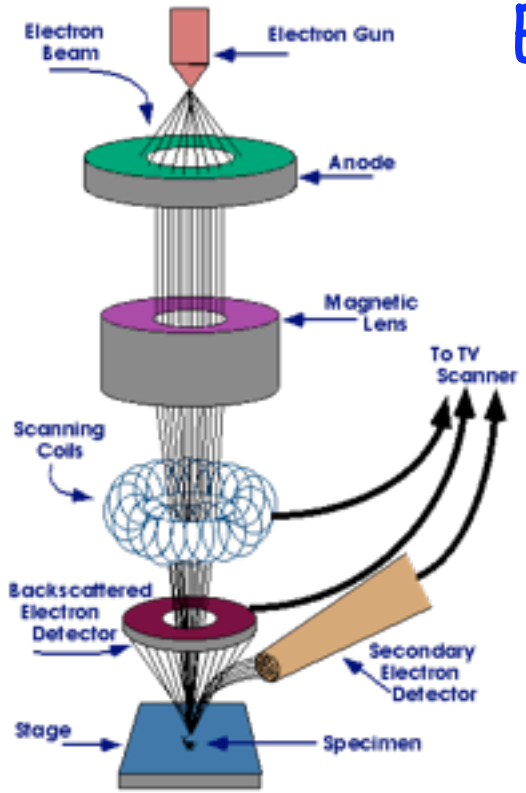
$$A^{\mu}_{\lambda\lambda'} = \langle p + \tfrac{1}{2}q, \lambda' | J^{\mu}(0) | p - \tfrac{1}{2}q, \lambda \rangle$$

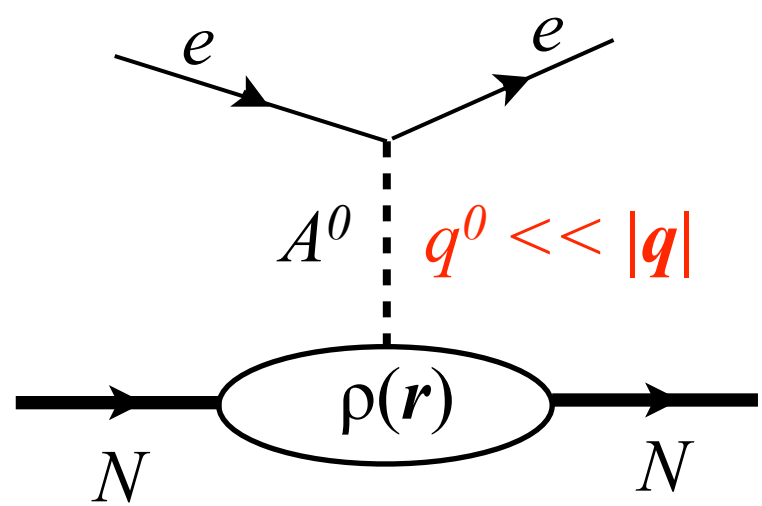
$$= \bar{u}(p + \tfrac{1}{2}q, \lambda') \left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2) \frac{i}{2m} \sigma^{\mu\nu} q_{\nu} \right] u(p - \tfrac{1}{2}q, \lambda)$$

$e\,p \to e\,p$ resembles electron microscopy of the proton.

Until recently, the spatial distributions were thought to be given by three-dimensional Fourier transforms of the form factors.

Electron microscopy





When the charge distribution in the target is static (m_q, $m_N >> Q$), instantaneous Coulomb exchange dominates (in the rest frame) and the 3-dim. Fourier transform of the form factors gives the spatial distribution of electric charge

$$F_1(Q^2) = \int d^3\boldsymbol{r}\, \rho(r) \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}) \qquad \langle r^2 \rangle = -6 \left. \frac{dF_1}{dQ^2} \right|_{Q^2=0}$$

This "electron microscopy" interpretation is invalid for relativistic motion: $q^0 \sim |\boldsymbol{q}|$ ⇒ the quarks move with the speed of the photon (in any frame).

Boosting to the Infinite Momentum Frame

The photon probes a hadron at a given $x^+ = t+z$, not at an instant in t

The Light Front ≈ Infinite Momentum Frame

Quark motion in the transverse direction slows down in the IMF: $v_\perp = \dfrac{p_\perp}{xE_h}$

A hadron state of momentum $P^+ = P^0 + P^3$ defined at given $x^+ = x^0 + x^3$ can be expanded in terms its quark and gluon Fock states as

$$|P^+, \boldsymbol{P}_\perp, \lambda\rangle_{x^+=0} = \sum_{n,\lambda_i} \prod_{i=1}^{n} \Big[\int_0^1 \frac{dx_i}{\sqrt{x_i}} \int \frac{d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

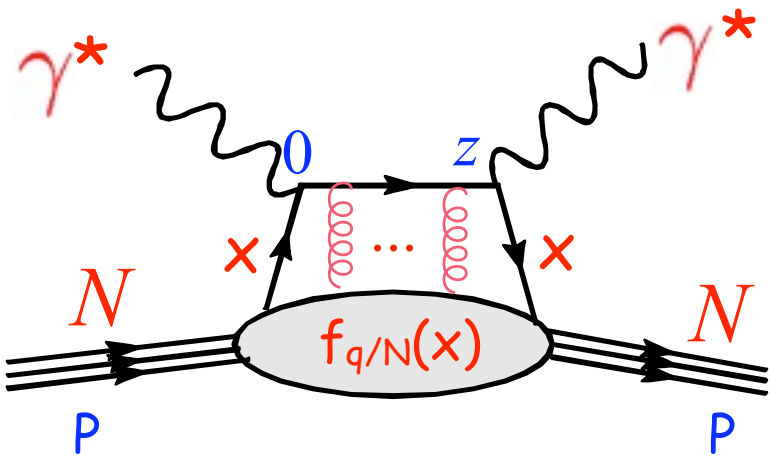
$$\times \psi_n(x_i, \boldsymbol{k}_i, \lambda_i)\, |n;\, x_i P^+, x_i \boldsymbol{P}_\perp + \boldsymbol{k}_i, \lambda_i\rangle_{x^+=0}$$

where the LF wave functions $\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)$ are independent of the hadron momentum P^+, $P_\perp$.

Note: The partons carry $x_i \boldsymbol{P}$ fractions of the total hadron momentum, like in non-relativistic physics with $m_i / M \rightarrow x_i$.

Inclusive Deep Inelastic Scattering (DIS)

In the DIS cross section the photon vertices of the amplitude and amplitude* are separated by a light-like distance z: z^+ , $z_\perp \to 0$; $z^- \sim 1/(2mx_{Bj})$. The parton distributions can be expressed in terms of LF wave functions:



$$f_{q/N}(x) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \Big[\int \frac{dx_i\, d^2\boldsymbol{k}_i}{16\pi^3} \Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \delta(x - x_k) |\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)|^2$$

Notes: – The parton distribution is obtained in the Bj limit ($Q^2 \to \infty$)

– The above expression is approximate, since rescattering of the struck parton (described by the Wilson line) is neglected.

The Generalized Parton Distributions: GPD's

The GPD's are non-forward matrix elements of the PDF operator:

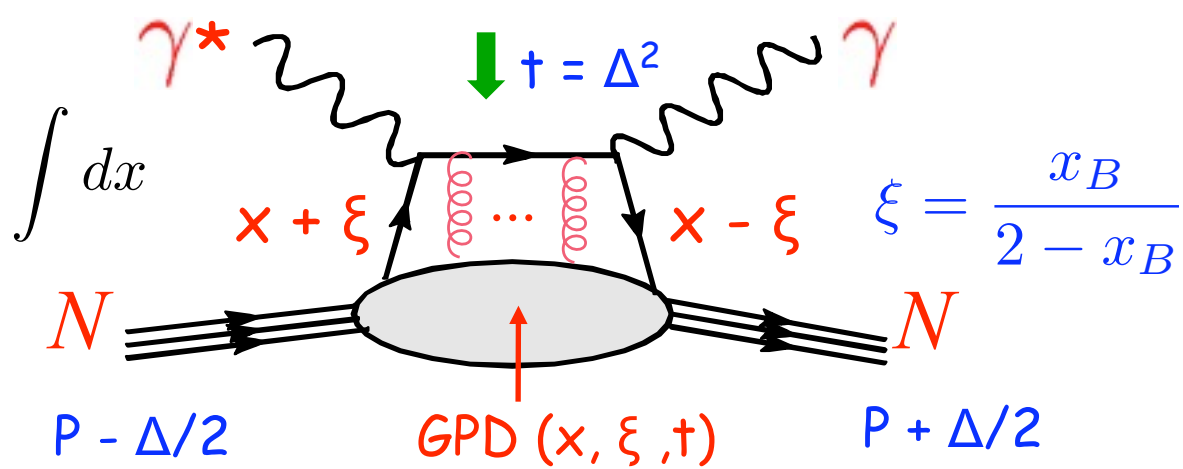
$$\frac{1}{8\pi}\int dr^- e^{imxr^-/2}\langle P+\tfrac{1}{2}\Delta|\bar{q}(-\tfrac{1}{2}r)\gamma^+ W[\tfrac{1}{2}r^-,-\tfrac{1}{2}r^-]q(\tfrac{1}{2}r)|P-\tfrac{1}{2}\Delta\rangle_{r^+=r_\perp=0}$$

$$=\frac{1}{2P^+}\bar{u}(P+\tfrac{1}{2}\Delta)\left[H(x,\xi,t)\gamma^+ + E(x,\xi,t)i\sigma^{+\nu}\frac{\Delta_\nu}{2m}\right]u(P-\tfrac{1}{2}\Delta)$$

The GPD amplitudes can be accessed experimentally through the Deeply Virtual Compton Scattering cross section at leading twist: $Q^2 \to \infty$.

DVCS: $e\,N \longrightarrow e' + \gamma + N$

Through $\Delta_\perp$, the GPD's contain information about the parton distributions in transverse space.



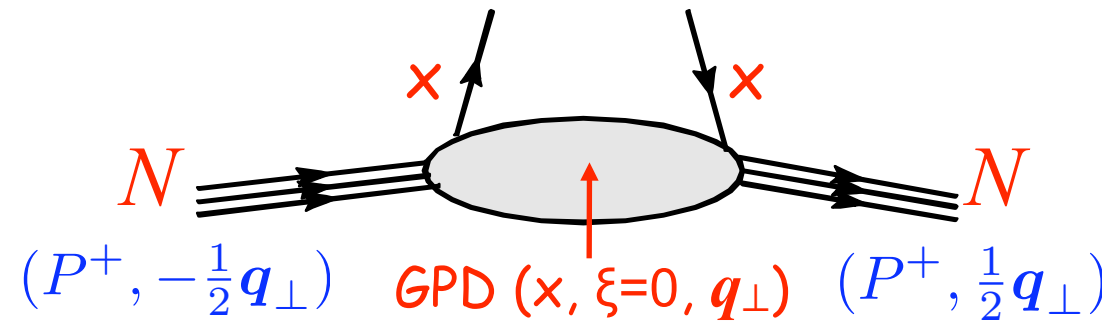
Impact parameter distributions via the GPD's

Extrapolating the GPD to $\xi = 0$ and Fourier transforming it *wrt.* $\boldsymbol{q}_\perp$

Soper (1977)
Burkardt (2000)
Diehl (2002)

$$f_{q/N}(x,\boldsymbol{b}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2}$$
$$\times \quad \langle P^+, \tfrac{1}{2}\boldsymbol{q}, \lambda | \bar{\mathrm{q}}(0^+, -\tfrac{1}{2}z^-, \boldsymbol{0}_\perp)\gamma^+ \mathrm{q}(0^+, \tfrac{1}{2}z^-, \boldsymbol{0}_\perp) | P^+, -\tfrac{1}{2}\boldsymbol{q}, \lambda\rangle$$

the GPD can be expressed in terms of LF wf's with the struck quark at transverse position $\boldsymbol{b}$ (still ignoring the Wilson line):



$$f_{q/N}(x,\boldsymbol{b}) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \left[\int dx_i \int 4\pi d^2\boldsymbol{b}_i\right] \delta\left(1 - \sum_i x_i\right) \frac{1}{4\pi} \delta^2\left(\sum_i x_i \boldsymbol{b}_i\right)$$
$$\times \quad \delta^{(2)}(\boldsymbol{b} - \boldsymbol{b}_k)\delta(x - x_k)\, |\psi_n^\lambda(x_i, \boldsymbol{b}_i, \lambda_i)|^2$$

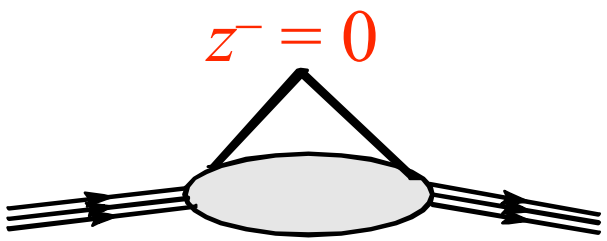
"Center of momentum" at the origin

Fourier-transformed wave function

Relation of GPD's to Form Factors

When GPD's are integrated over x the GPD reduces to a form factor, since

$$\int_{-\infty}^{\infty} dx \exp(ixP^+ z^-/2) \propto \delta(z^-)$$



ensures that the photon vertices coalesce.
The GPD's vanish for $|x| > 1$, hence the relations reduce to

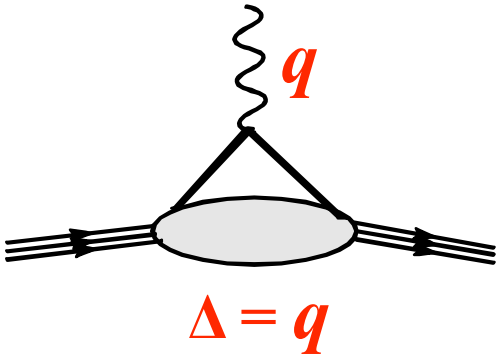
$$\int_{-1}^{1} dx H^q(x,\xi,t) = F_1^q(t) \qquad \text{Dirac}$$

$$\int_{-1}^{1} dx E^q(x,\xi,t) = F_2^q(t) \qquad \text{Pauli}$$

This gives constraints on GPD models and a great experimental simplification:
Form factors are easy to measure (compared to GPD's!).

Impact parameter picture of GPD's inherited by FF's

Fourier transforms of form factors give
charge densities in impact parameter space:



$$\rho_0(\boldsymbol{b}) = \int_0^\infty \frac{dQ}{2\pi}\, Q\, J_0(b\,Q) F_1(Q^2)$$

$$= \sum_{n,\lambda_i,k} e_k \Big[\prod_{i=1}^{n} \int dx_i \int 4\pi d^2\boldsymbol{b}_i\Big] \delta(1-\sum_i x_i) \frac{1}{4\pi} \delta^{(2)}(\sum_i x_i \boldsymbol{b}_i)$$

$$\times\ \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\, |\psi_n^\lambda(x_i, \boldsymbol{b}_i, \lambda_i)|^2$$

No more Wilson line: Fock expansion is "exact"

No more "leading twist": Resolution in $b \sim 1/\boldsymbol{q}_{max}$

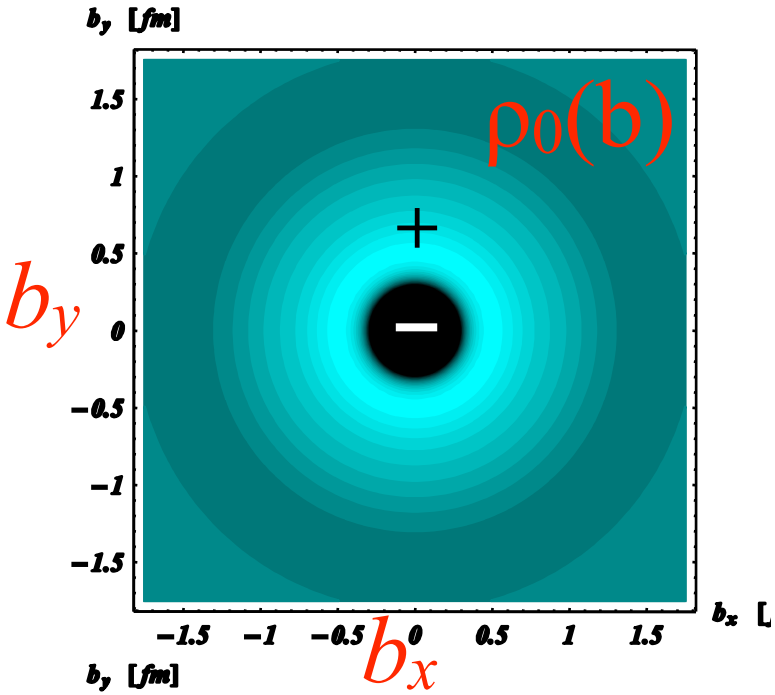
$$\rho(\boldsymbol{b}) = \int_0^\infty \frac{dq^2}{2\pi}\, K_0(\sqrt{q^2} b) \frac{\mathrm{Im} F_1(q^2 + i\varepsilon)}{\pi}$$

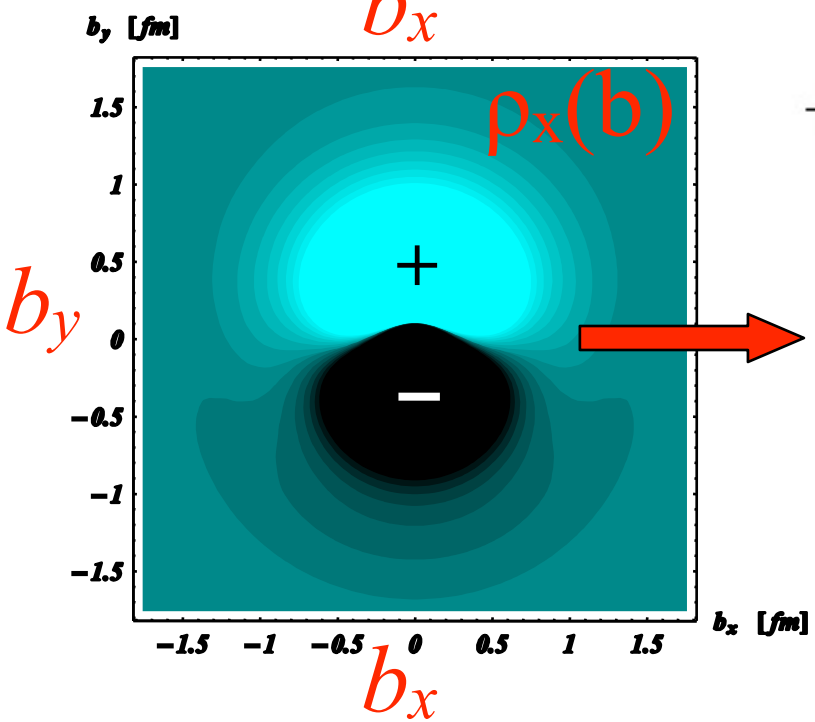
Dispersion relations connect to time-like photons ($q^2 > 0$)

Strikman and Weiss (2010)

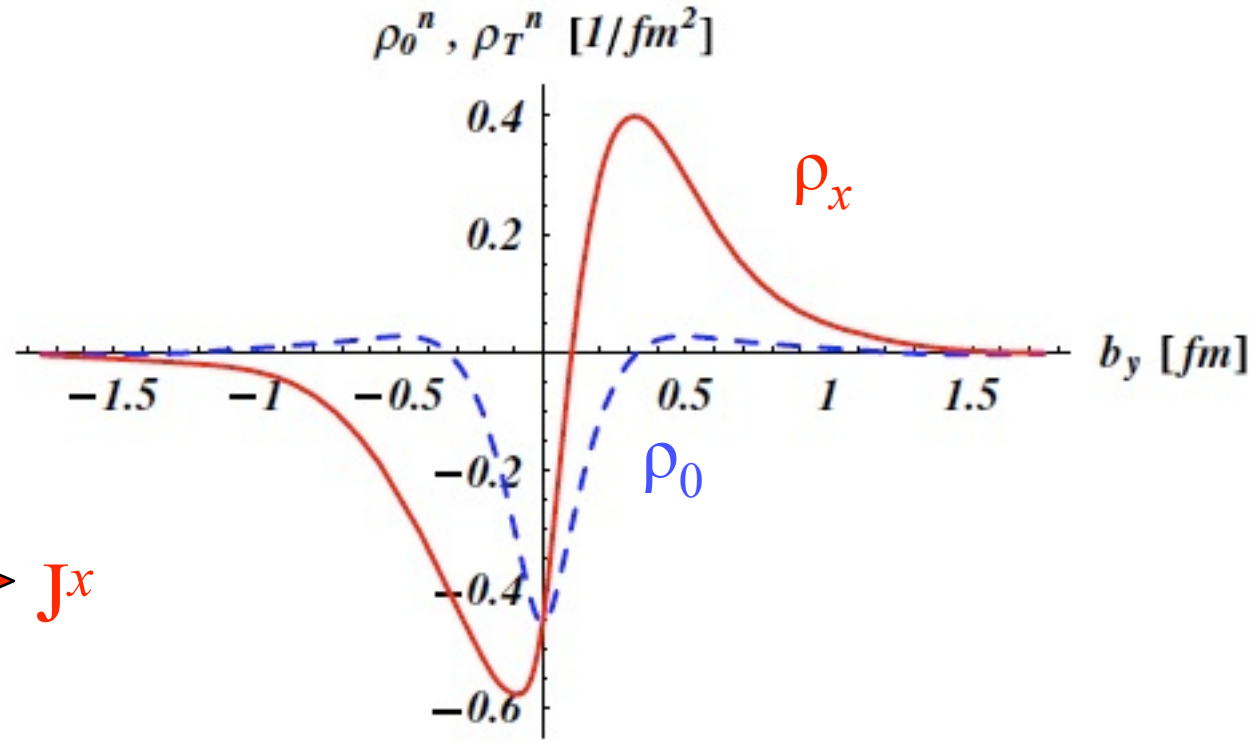
Using measured form factors, find the

empirical quark transverse densities in neutron





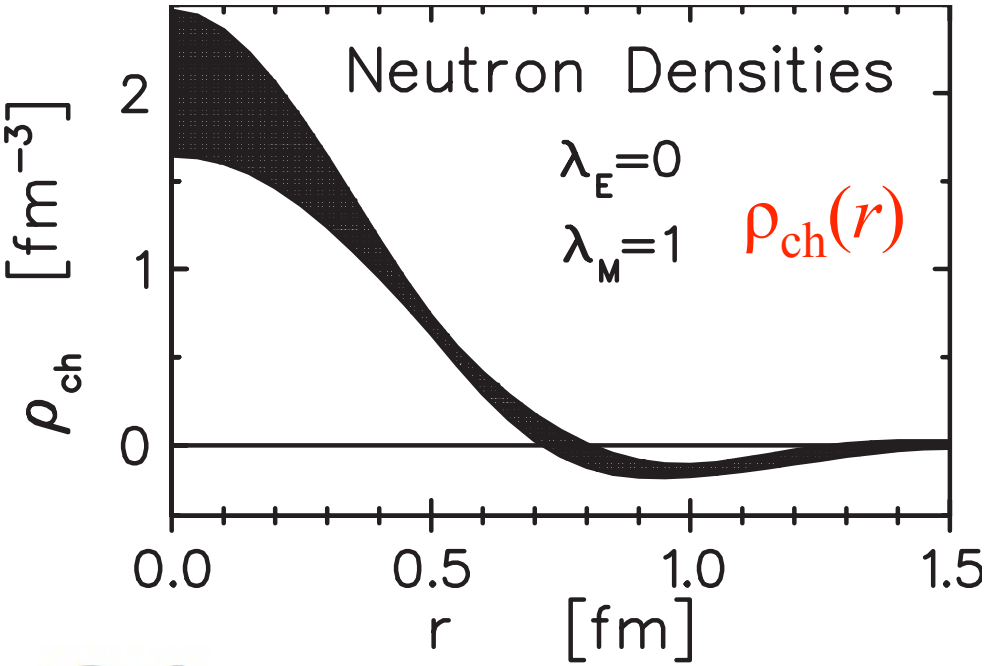
J^x



Miller (2007)
Carlson and Vanderhaeghen (2008)

data : Bradford, Bodek, Budd, Arrington (2006)

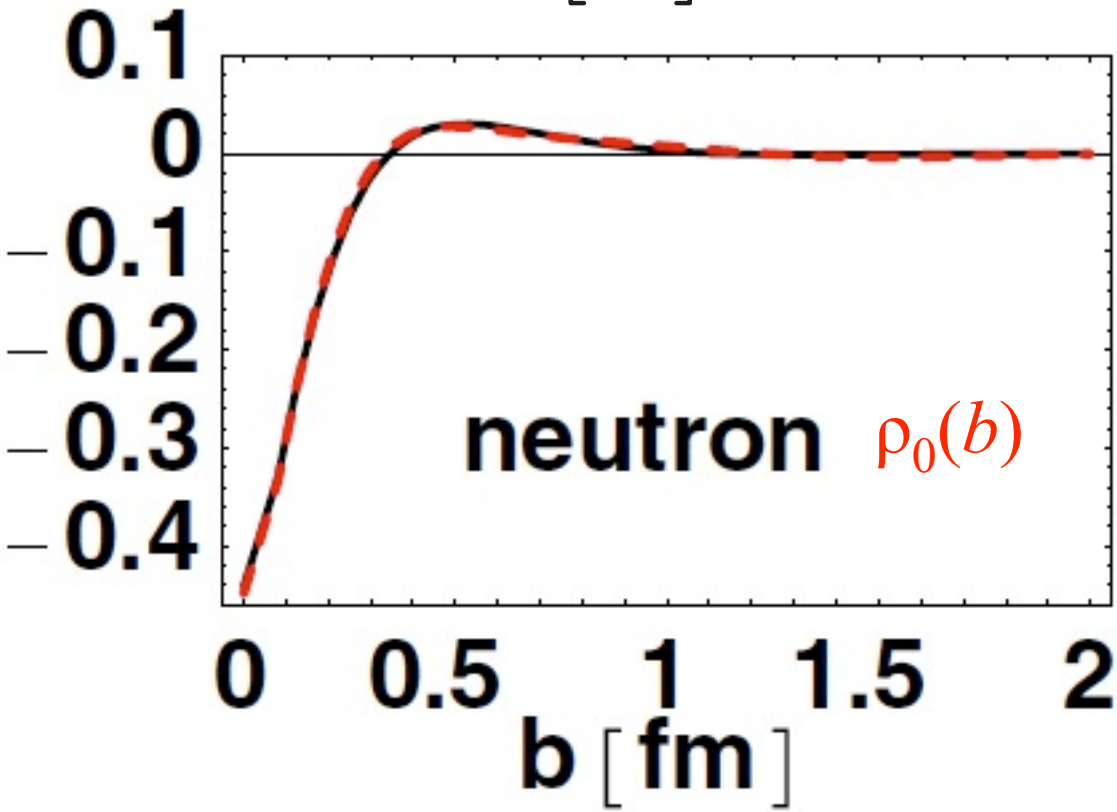
Qualitative change in central neutron charge density



3-dimensional Fourier transform with phenomenological factors (2001)

J. J. Kelly, hep-ph/0111251

$\rho_{ch}(r=0) > 0$



$\rho_0(b=0) < 0$

Transverse Fourier transform (2007)

G. Miller, PRL 99 (2007) 112001

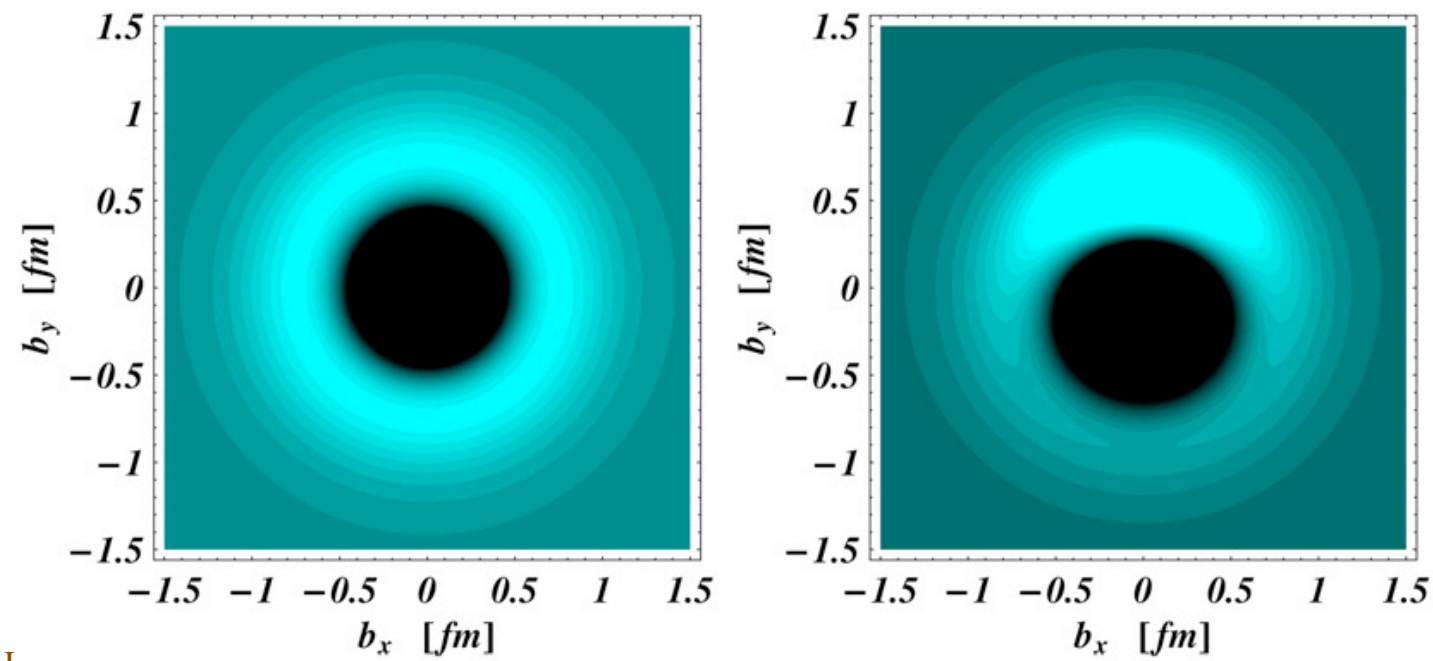
Generalization to transition Form Factors

In the case of transition form factors, the density is no longer positive definite but the charge distribution is still interesting:

"It is found that the transition from the proton to its first radially excited state is dominated by up quarks in a central region of around 0.5 fm and by down quarks in an outer band which extends up to about 1 fm."

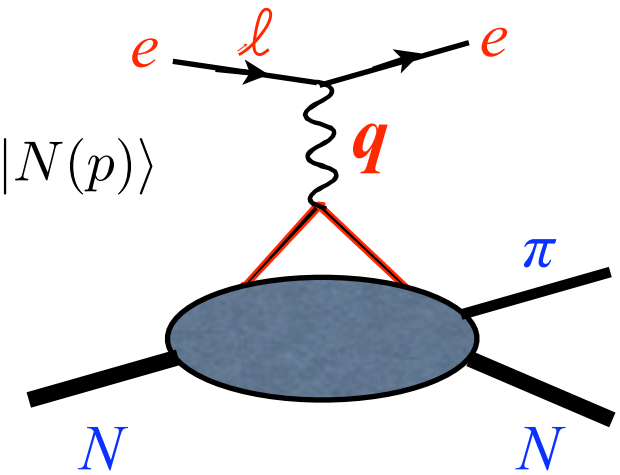
Tiator and Vanderhaeghen (2009)

$$\gamma^* N \rightarrow P_{11}(1440)$$



Generalization to any γ* transition

PH and S. Kurki
arXiv:1101.4810



$$\mathcal{M}(\ell N \to \ell' f) = -e^2 \bar{u}(\ell')\gamma_\mu u(\ell)\frac{1}{q^2}\int d^4x e^{-iq\cdot x}\langle f|J^\mu(x)|N(p)\rangle$$

Need to identify J^+ current contribution for

LF Fock expansion: E.g.: $\ell^- \longrightarrow \infty$ at fixed q

$$J^+(x) = e_q\,\bar{\mathrm{q}}(x)\gamma^+\mathrm{q}(x) = 2e_q\,\mathrm{q}_+^\dagger(x)\mathrm{q}_+(x)$$

$\mathrm{q}_+(x) = ¼\,\gamma^-\gamma^+\,\mathrm{q}(x)$

$$\mathrm{q}_+(0^+, x^-, \boldsymbol{x}) = \int \frac{dk^+}{k^+}\theta(k^+)\Big[b(k^+,\boldsymbol{x})u_+(k^+)e^{-i\frac{1}{2}k^+x^-} + d^\dagger(k^+,\boldsymbol{x})v_+(k^+)e^{i\frac{1}{2}k^+x^-}\Big]$$

where the LF spinors satisfy: $u_+^\dagger(k^+,\lambda')u_+(k^+,\lambda) = k^+\delta_{\lambda'\lambda}$

Fourier transform to impact parameter: $|p^+, \boldsymbol{p}\rangle = 4\pi\int d^2\boldsymbol{b}\, e^{i\boldsymbol{p}\cdot\boldsymbol{b}}|p^+, \boldsymbol{b}\rangle$

Expand into LF Fock states:

$$|p^+, \boldsymbol{b}\rangle = \frac{1}{4\pi}\sum_n \Big[\prod_{i=1}^n \int_0^1 \frac{dx_i}{\sqrt{x_i}} \int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\boldsymbol{b}-\sum_i x_i\boldsymbol{b}_i)$$

$$\times\ \psi_n(x_i, \boldsymbol{b}_i - \boldsymbol{b})\prod^n b^\dagger(x_i p^+, \boldsymbol{b}_i) d^\dagger(\ )a^\dagger(\ )|0\rangle$$

$\Rightarrow$

$$\frac{1}{2p^+}\langle f(p^+, \boldsymbol{b}_f)|J^+(0)|N(p^+, \boldsymbol{b}_N)\rangle \equiv \frac{1}{(4\pi)^2}\delta^2(\boldsymbol{b}_f - \boldsymbol{b}_N)\mathcal{A}_{fN}(-\boldsymbol{b}_N)$$

where

$$\mathcal{A}_{fN}(\boldsymbol{b}) = \frac{1}{4\pi}\sum_n \Big[\prod_{i=1}^n \int_0^1 dx_i \int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\sum_i x_i\boldsymbol{b}_i)$$

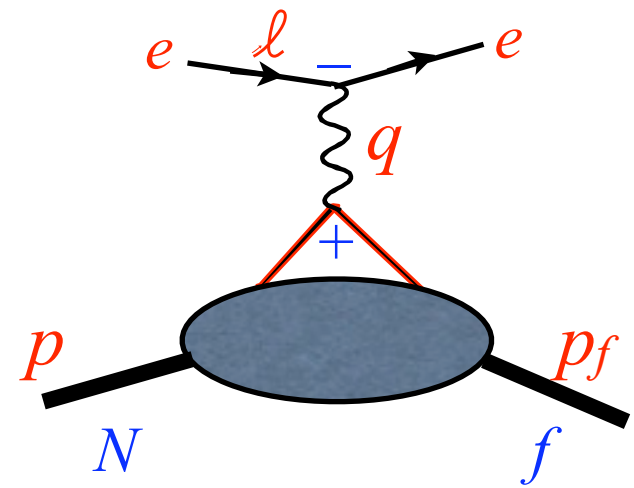
$$\times\ \psi_n^{f\,*}(x_i, \boldsymbol{b}_i)\psi_n^N(x_i, \boldsymbol{b}_i)\sum_k e_k\delta^2(\boldsymbol{b}_k - \boldsymbol{b})$$

is diagonal in Fock states n in frames where $q^+ = 0$ ($\Rightarrow$ no pair production)

FT of γ* matrix element in momentum space

In the frame:

$$
\begin{aligned}
p &= (p^+, p^-, -\tfrac{1}{2}\boldsymbol{q}) \\
q &= (0^+, q^-, \boldsymbol{q}) \\
p_f &= (p^+, p^- + q^-, \tfrac{1}{2}\boldsymbol{q})
\end{aligned}
$$



we have

$$
\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle = \mathcal{A}_{fN}(\boldsymbol{b})
$$

where $\mathcal{A}_{fN}(\boldsymbol{b})$ is given by the previous overlap of Fock amplitudes, which are universal features of N and f.

The $\boldsymbol{b}$-distribution may be studied as a function of the final state f, providing information about the transverse size of the intermediate Fock states.

When f consists of several hadrons their relative momenta must be consistent with the LF Fock expansion at all $p_f = q + p$

Example: f = π (p1) N(p2)

In order to conform with the Lorentz covariance of LF states, at any p_f :

$$|\pi N(p_f^+, \boldsymbol{p}_f; \Psi^f)\rangle \equiv \int_0^1 \frac{dx}{\sqrt{x(1-x)}} \int \frac{d^2\boldsymbol{k}}{16\pi^3} \Psi^f(x, \boldsymbol{k}) |\pi(p_1) N(p_2)\rangle$$

where $\Psi^f(x,\boldsymbol{k})$ is a freely chosen function of the relative variables x, $\boldsymbol{k}$:

$$p_1^+ = x p_f^+ \qquad \boldsymbol{p}_1 = x\boldsymbol{p}_f + \boldsymbol{k}$$

$$p_2^+ = (1-x) p_f^+ \qquad \boldsymbol{p}_2 = (1-x)\boldsymbol{p}_f - \boldsymbol{k}$$

With x, $\boldsymbol{k}$ being independent of p_f, this defines the pion and nucleon momenta p_1, p_2 at all photon momenta q.

The $|\pi N(p_f^+, \boldsymbol{p}_f; \Psi^f)\rangle$ state has an LF Fock expansion of standard form, in terms of the pion and nucleon Fock amplitudes.

Illustration (1): $\gamma^* + \mu \rightarrow \mu + \gamma$

The QED matrix element $\mathcal{A}^{\mu\gamma}_{\lambda_1,\lambda_2} = \frac{1}{2p^+}\langle\mu(p_1,\lambda_1)\gamma(p_2,\lambda_2)|J^+(0)|\mu(p,\lambda=\frac{1}{2})\rangle$

expressed in terms of the relative variables *x*, ***k*** is:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{q};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2} - \frac{\boldsymbol{e}_-\cdot(\boldsymbol{k}-(1-x)\boldsymbol{q})}{(1-x)^2m^2+(\boldsymbol{k}-(1-x)\boldsymbol{q})^2}\right]$$

where $\boldsymbol{e}_\lambda\cdot\boldsymbol{k} = -\lambda e^{i\lambda\phi_k}|\boldsymbol{k}|/\sqrt{2}$. The Fourier transform gives:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{b};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2}\delta^2(\boldsymbol{b}) - \frac{i}{2\sqrt{2}\pi}\frac{m\,e^{-i\phi_b}}{1-x}K_1(mb)\exp\left(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\right)\right]$$

In the first term the γ^* interacts with the initial muon, which by definition is at b = 0. The second term reflects the distribution of the final muon in transverse space.

This expression agrees exactly with the wave function overlap formula.

Illustration (2): $\gamma^* + \mu \rightarrow \mu + \gamma$

Choosing $\Psi(x', \boldsymbol{k}) = \delta(x' - x)\sqrt{x(1-x)}\,\exp\Big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}'_\mu}{1-x}\Big)$

corresponds to fixing the impact parameter $b_\mu{}'$ of the final muon. Then

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{b}; x, \boldsymbol{b}'_\mu) = \sqrt{x(1-x)}\,\psi^{\uparrow}_{+\frac{1}{2}+1}(x, \boldsymbol{b}'_\mu)\Big[-\delta^{(2)}(\boldsymbol{b}) + \delta^{(2)}(\boldsymbol{b} - \boldsymbol{b}'_\mu)\Big]$$

which again conforms with the general overlap expression of LF Fock state wave functions.

Fourier transform of the cross section

The $\gamma^*+N \to f$ amplitudes have dynamical phases (resonances,...).
$\Rightarrow$ Calculating their Fourier transforms requires a partial wave analysis.

However, one can Fourier transform the measured cross section itself.

Then the ***b***-distribution reflects the difference between the impact parameters of the photon vertex in the amplitude and its complex conjugate:

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left| \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle \right|^2 = \int d^2\boldsymbol{b}_q \, \mathcal{A}_{fN}(\boldsymbol{b}_q) \, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b})$$

For $|f\rangle = |\pi(p_1)N(p_2)\rangle$, parametrized with the relative variables x and $\boldsymbol{k}$,

$$\mathcal{S}_{fN}(\boldsymbol{b}; x, \boldsymbol{k}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \, \boldsymbol{q}^4 \, \frac{d\sigma(\ell N \to \ell' \pi N)}{d^2\boldsymbol{q} \, dx \, d^2\boldsymbol{k}}$$

$$= \frac{\alpha^2}{4\pi^3} \frac{1}{x(1-x)} \int d^2\boldsymbol{b}_q \, \mathcal{A}_{fN}(\boldsymbol{b}_q; x, \boldsymbol{k}) \, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b}; x, \boldsymbol{k})$$

Illustration (3): $\sigma(\gamma^* + \mu \rightarrow \mu + \gamma)$

For the QED example considered above the Fourier transform of the cross section can be done analytically:

$$\mathcal{S}^{\mu\gamma}(\boldsymbol{b}; x, \boldsymbol{k}) = 4e^2 x \Bigg\{ \frac{\boldsymbol{k}^2/2}{[(1-x)^2 m^2 + \boldsymbol{k}^2]^2} \delta^{(2)}(\boldsymbol{b}) - \frac{|\boldsymbol{k}| \cos(\phi_b - \phi_k)}{(1-x)^2 m^2 + \boldsymbol{k}^2} \frac{im}{2\pi} \frac{\exp\left(-i \frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\right)}{1-x} K_1(mb)$$

$$+ \frac{1}{4\pi} \frac{\exp\left(-i \frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\right)}{(1-x)^2} \left[K_0(mb) - \tfrac{1}{2} mb\, K_1(mb) \right] \Bigg\}$$

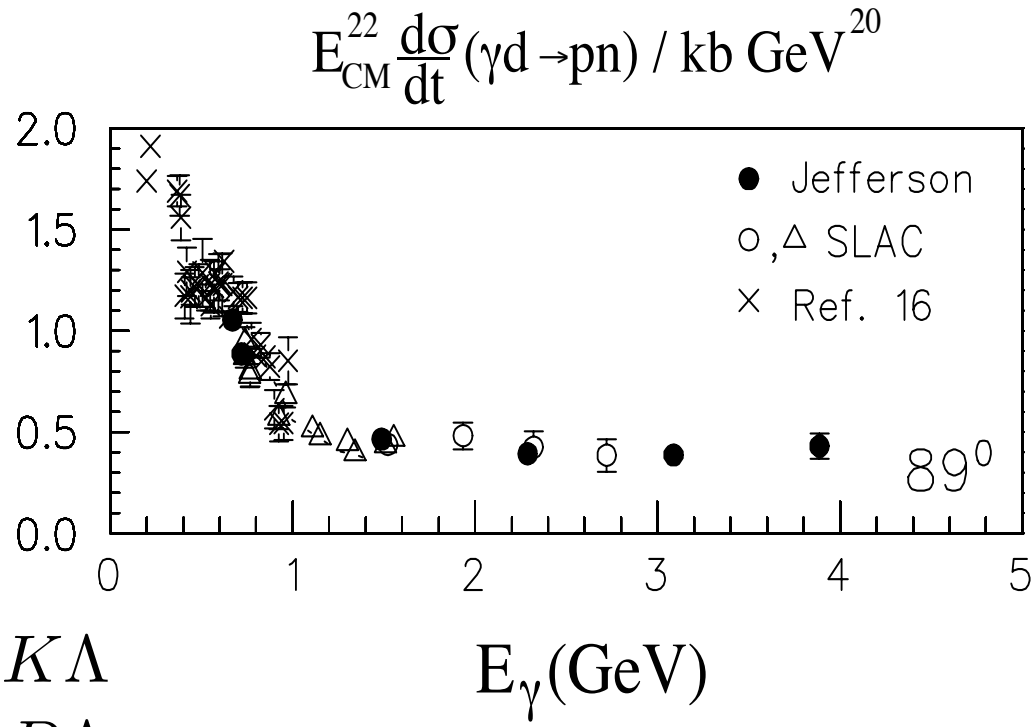
The 3 terms correspond to 2, 1 and 0 of the γ^* interactions occurring on the initial muon.

The imaginary part arises from an angular correlation between $\boldsymbol{b}$ and $\boldsymbol{k}$.

Remarks

In $\gamma^* N \to \pi N$, expect the b-distribution to narrow with the relative transverse momentum k between the π and the N.

$\sigma(\gamma D \to pn) \propto E^{-22}$ at large angles, suggesting compact states. A measurement of the q^2-dependence would allow a direct measurement of the transverse size.



In heavy quark production: $\gamma^* N \to K\Lambda$, $\gamma^* N \to D\Lambda_c$

the b-distribution should narrow with the quark mass if the photon couples directly to the heavy quarks.

One may compare the b-distribution in ordinary and diffractive events.

Generalization to $q^2 > 0$

Summary (1)

Intuitively, the $\boldsymbol{q}$ -dependence of a virtual photon interaction gives information about the charge distribution in space.

The target is illuminated "instantaneously" only when the charge carriers are non-relativistic. This is the case in electron microscopy.

Quarks move inside hadrons with ≈ velocity of light.

The photon phase is constant at fixed Light-Front time $x^+ = t + z$

In the IMF ≈ LF formulation, transverse quark velocities are non-relativistic

2-dim. FT's of form factors describe charge densities in transverse space

Unlike pdf's, no "leading twist" limit is implied.

The resolution in impact parameter is expected to be $\Delta b \sim 1/Q_{max}$

Summary (2)

The formulation can be generalized to transition form factors $\gamma^* N \to N^*$ and to any (multi-hadron) final (and initial) state: $\gamma^* \mathrm{A} \to f$

FT of the cross section $\sigma(\gamma^* N \to f)$ gives the distribution in the transverse distance $\boldsymbol{b}$ between the photon vertex in $\mathrm{T}(\gamma^* N \to f)$ and $[\mathrm{T}(\gamma^* N \to f)]^*$