The Higgs boson mass and Standard Model up to the Planck scale

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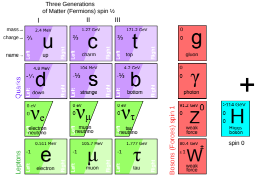
Outline

Standard Model – describes nearly everything



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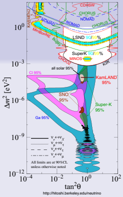
Einstein gravity

Describes

- all laboratory experiments – electromagnetism, nuclear processes, etc.
- all processes in the evolution of the Universe after the Big Bang Nucleosynthesis ($T < 1$ MeV, $t > 1$ sec)

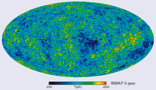
Experimental problems:

- Laboratory
  - ? Neutrino oscillations



- Cosmology
  - ? Baryon asymmetry of the Universe
  - ? Dark Matter

  - ? Inflation

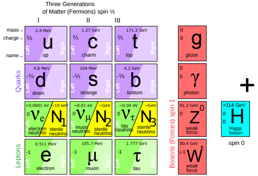


  - ? Dark Energy

Can we describe everything with as small extension as possible?

- Minimal number of new particles
- No new scales before inflation/gravity

νMSM+inflation – describes everything



+

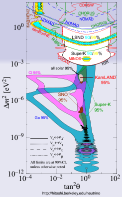
Einstein gravity

with νMSM

- Right handed neutrinos
  - see-saw generation of active neutrino masses
  - keV scale DM
  - Baryogenesys via leptogenesys

+ comological constant

Experimental problems:



- Laboratory
  - ✓ Neutrino oscillations
- Cosmology
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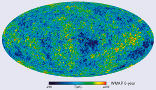
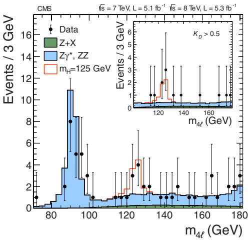
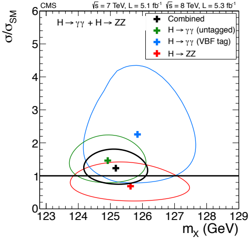
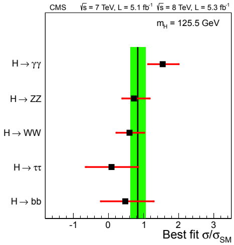
  - ? Inflation
  - ✓ Dark Energy



CMS “new boson” results







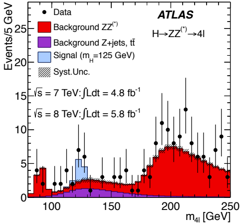
“New boson” mass

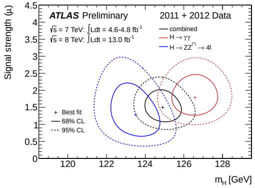
$$M_h = 125.3 \pm 0.4(\text{stat}) \pm 0.5(\text{syst})\,\text{GeV}$$

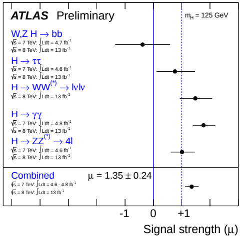
5.8σ for SM Higgs boson of this mass

[CMS’12]

ATLAS “new particle” results







“New particle” mass

$$M_h = 125.2 \pm 0.3(\text{stat}) \pm 0.6(\text{syst})\,\text{GeV}$$

[0] [ATLAS’12/12]

SM everywhere?

What happens if there is nothing else up to the Planck scales?
(or at least up to the scale of inflation)

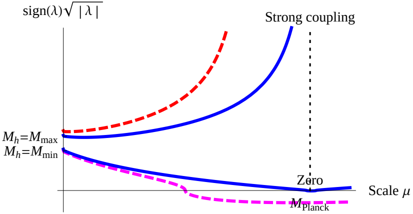
Renormalization evolution of the Higgs self coupling λ

$$(4\pi)^2\beta_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2+g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

- High M_h – strong coupling
- Low M_h – our (EW) vacuum is metastable.
- Boundary situation – $M_h = M_{\rm min}$

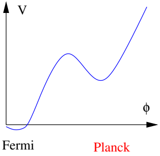
$$\lambda(\mu_0) = 0, \quad \beta_\lambda(\mu_0) \equiv \mu\frac{d\lambda}{d\mu} = 0$$

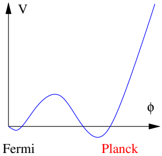
Coupling constant evolution:



Higgs effective potential

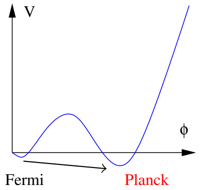
$$V(\varphi) \simeq \lambda(\varphi)\frac{\varphi^4}{4}$$

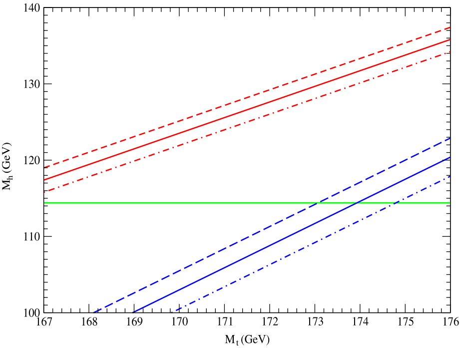




Even metastable EW vacuum overlives the Universe

Will the vacuum decay?





[Espinosa, Giudice, Riotto'07]

EW vacuum lifetime $> \tau_{\text{Universe}}$

$M_h > 111\,\text{GeV}$

Even metastable EW vacuum overlives the Universe

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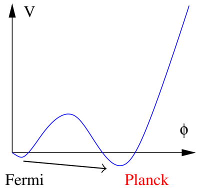

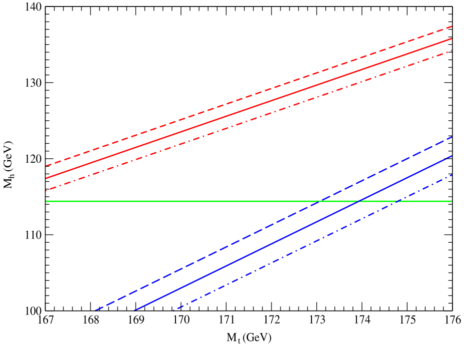

[Espinosa, Giudice, Riotto’07]

EW vacuum lifetime $> \tau_{\text{Universe}}$

$M_h > 111\,\text{GeV}$

Definitely not Standard Model if

Higgs mass is out of the window

$$111\,\mathrm{GeV} \lesssim M_h \lesssim 1\,\mathrm{TeV}$$

[0] Degrassi, et.al.'2012

Renormalization evolution of the Higgs self coupling λ

$$(4\pi)^2\beta_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

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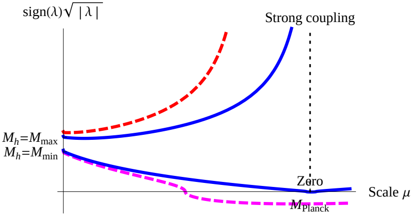

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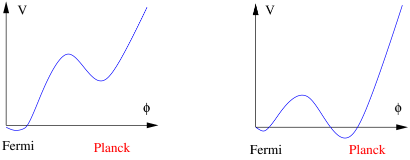

$$\lambda(\mu_0) = 0, \quad \beta_\lambda(\mu_0) \equiv \mu\frac{d\lambda}{d\mu} = 0$$

Higgs potential stability – which case is realized?

So, do we know – is Higgs light or heavy?

Calculation steps

Input: Pole masses M_t, M_h
(and other constants at scale $\mu = M_Z$)

- Convert to $\overline{\text{MS}}$ constants $\lambda(\mu)$, $y_t(\mu)$ at a scale μ between M_Z and M_t

$$
\begin{aligned}
y_t(\mu) &= 2^{3/4}\sqrt{G_F}M_t \times \left(1 + \delta y_t(M_t, \alpha_S, \alpha, s_W^2, M_Z; \mu)\right) \\
\lambda(\mu) &= \sqrt{2}G_F M_h^2 \times \left(1 + \delta\lambda(M_t, \alpha_S, \alpha, s_W^2, M_Z; \mu)\right)
\end{aligned}
$$

- Evolve with RG up to the Planck scale

$$
\mu\frac{d\lambda}{d\mu} = \beta_\lambda(\lambda, y_t, g_i), \quad \mu\frac{dy_t}{d\mu} = \beta_{y_t}(\lambda, y_t, g_i), \quad \ldots
$$

Output: $\lambda(\mu)$ in $\overline{\text{MS}}$

Finally: solve for $\lambda(\mu_0) = \lambda'(\mu_0) = 0$.

Calculation steps: state of the art

- Convert to $\overline{\text{MS}}$ constants $\lambda(\mu)$, $y_t(\mu)$ at a scale μ between M_Z and M_t

  δy_t Up to $O(\alpha_s^2)$, $O(\alpha)$

  $O(\alpha_s^3)$ [Chetyrkin, Steinhauser'99, Melnikov,Ritbergen'00]

  $O(\alpha\alpha_s)$ [FB, Kalmykov, Kniehl, Shaposhnikov'12]

  $\delta\lambda$ Up to $O(\alpha)$

  $O(\alpha\alpha_s)$ [FB, Kalmykov, Kniehl, Shaposhnikov'12]

  $O(y_t^4)$ (Yukawa part of $O(\alpha^2)$)

  [Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia'12]

- Evolve with RG up to Planck scales

  β_{g_i} two loops

  three loops [Mihaila, Salomon, Steinhauser'12]

  β_{y_t}, β_λ two loops

  three loops (no EW gauge contributions)

  [Chetyrkin, Zoller'12]

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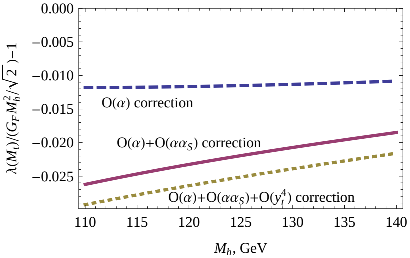
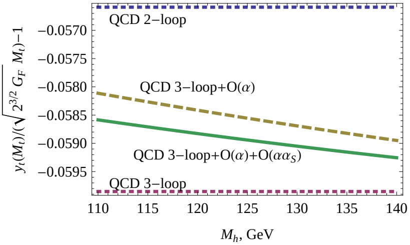
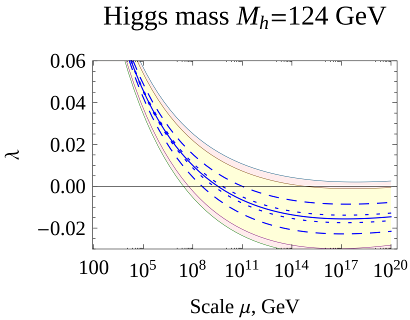
Size of new contributions to $M_{\min}$

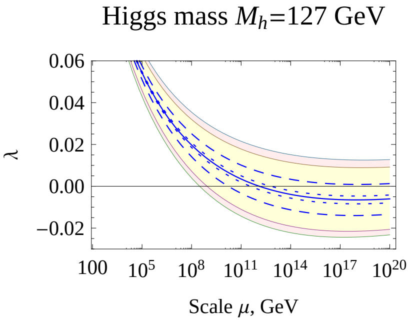
Contribution	$\Delta M_{\min}$, GeV
Three loop	
beta functions	-0.23
$\delta y_t \propto O(\alpha_s^3)$	-1.15
$\delta y_t \propto O(\alpha\alpha_s)$	-0.13
$\delta\lambda \propto O(\alpha\alpha_s)$	0.62
$\delta\lambda \propto O(y_t^4)$	0.2



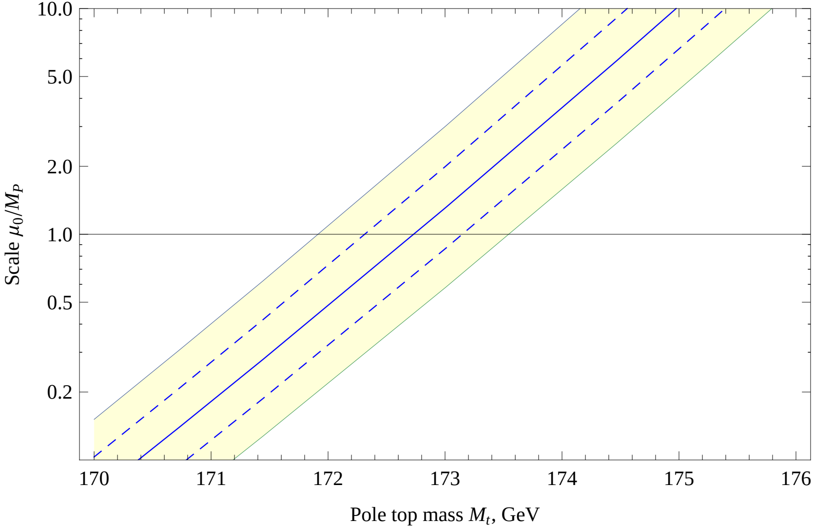


Scale for λ turning negative is high





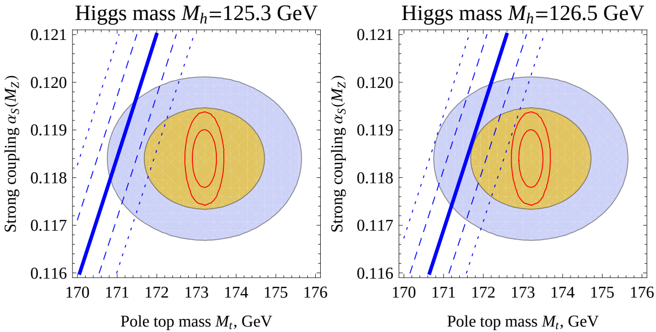
Scale for vanishing λ and β_λ



Critical Higgs mass is compatible with M_t and α_s

Tevatron value: $M_t = 173.2 \pm 0.6(\text{stat}) \pm 0.8(\text{syst})\,\text{GeV}$

$\alpha_s(M_Z) = 0.1184 \pm 0.0007$



$$M_{\text{min}} = \left[129.5 + \frac{M_t - 173.2\,\text{GeV}}{0.9\,\text{GeV}} \times 1.8 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.6 \pm 2\right] \text{GeV}$$

Part I Conclusions: Is there a coincidence?

- For the Higgs boson mass

$$M_{\text{min}} = \left[129.5 + \frac{M_t - 173.2\,\text{GeV}}{0.9\,\text{GeV}} \times 1.8 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.6 \pm 2\right] \text{GeV}$$

(that is somewhere between 125–134 GeV)
a coincidence takes place in the SM:

$$\lambda(\mu) = \beta_\lambda(\mu) = 0, \text{ for } \mu \simeq M_P$$

- To check this coincidence precise measurement of M_h *and* M_t is needed
  - Build a lepton collider at $\gtrsim$ 350 GeV!
  - Calculate of higher order relations between $\overline{\text{MS}}$ parameters and masses

Outline

Asymptotic safe model has a non-trivial UV fixed point

- Above Planck scale beta functions get additional terms

$$\beta_h^{\text{grav}} = \frac{a_h}{8\pi}\frac{\mu^2}{M_P^2(\mu)}h$$

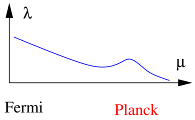
where $h \in \{g_1, g_2, g_3, \lambda, y_t\}$ – coupling constant and the running Planck mass is

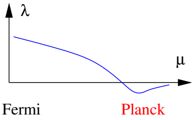
$$M_P^2(\mu) \simeq M_P^2 + 2\xi_0\mu^2$$

with $\xi_0 \simeq 0.024$

Asymptotic safety prediction of the Higgs mass

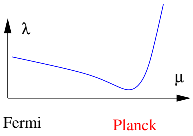
$a_\lambda < 0$ leads to the **bounds** $M_{\min} < M_h < M_{\max}$

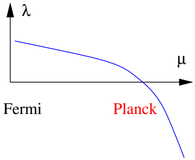


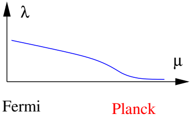


$a_\lambda > 0$ leads to the **prediction** $M_h = M_{\min}$

Up to a difference of 0.1–0.2 GeV







[Shaposhnikov, Wetterich'09]

There are other models predicting the same Higgs mass

- Forggart, Nielsen'96 – Multiple point principle.
  All the vacua should be degenerate – thus, the same prediction $M_h = M_{\rm min}$
- Masina, Notari'11 – inflation from the decay of the metastable Planck scale vacuum – $M_h \simeq M_{\rm min}$
- …

Inflation may change things

Adding inflation to the model – will it give bounds?

Non-minimal coupling of the Higgs gives inflation

Quite an old idea

Add $h^2 R$ term (required by renormalization) to the usual $M_P^2 R$ term in the gravitational action

- A.Zee'78, L.Smolin'79, B.Spokoiny'84
- D.Salopek J.Bond J.Bardeen'89

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \xi \frac{h^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\}$$

- h is the Higgs field; $M_P \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ GeV
- SM higgs vev $v \ll M_P/\sqrt{\xi}$

[FB, Shaposhnikov'08]

Conformal transformation – way to calculate

It is possible to get rid of the non-minimal coupling by the conformal transformation (change of variables)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \ , \qquad \Omega^2 \equiv 1 + \frac{\xi h^2}{M_P^2}$$

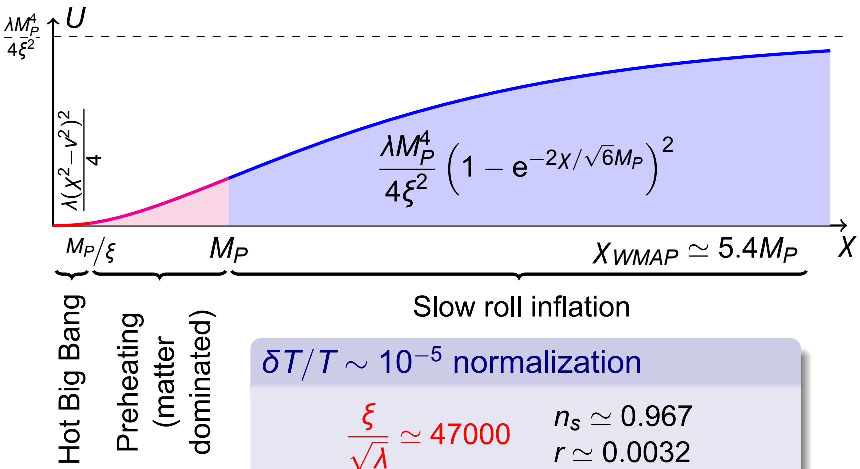
Redefinition of the Higgs field to get canonical kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \quad \begin{cases} h \simeq \chi & \text{for } h < M_P/\xi \\ \Omega^2 \simeq \exp\left(\frac{2\chi}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\xi \end{cases}$$

Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{\lambda}{4}\frac{h(\chi)^4}{\Omega(\chi)^4} \right\}$$

Potential – different stages of the Universe



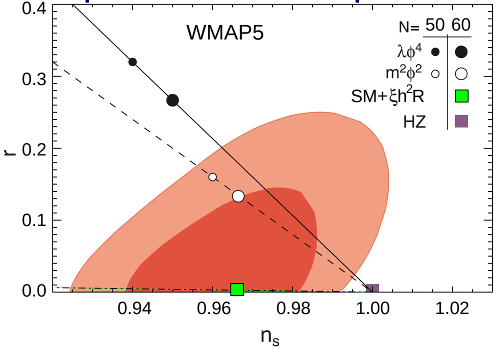
$\delta T/T \sim 10^{-5}$ normalization

$$\frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

$n_s \simeq 0.967$

$r \simeq 0.0032$

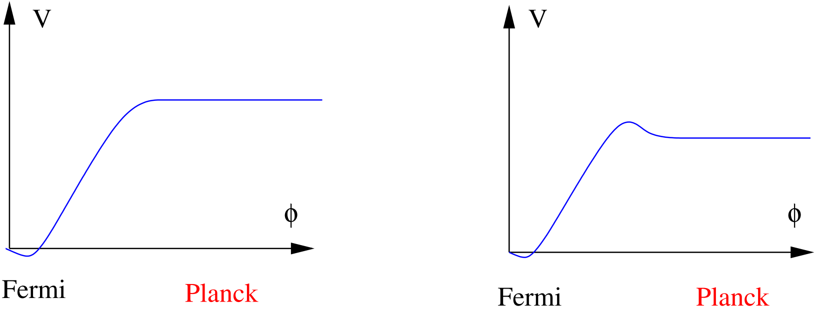
CMB parameters are predicted



spectral index	$n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2}$	$\simeq 0.97$
tensor/scalar ratio	$r \simeq \frac{192}{(4N+3)^2}$	$\simeq 0.0033$

$$\delta T/T \sim 10^{-5} \quad \Longrightarrow \quad \frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

No high energy minimum of potential should appear below inflation



In Higgs Inflation – Bound on the Higgs mass

$$M_h > M_{\text{min}}$$

Up to a difference of 0.1–0.2 GeV

[FB, Shaposhnikov'09]

Modifying the gravity action gives inflation

Another way to get inflation in the SM

The first working inflationary model

[Starobinsky’80]

The gravity action gets higher derivative terms

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R + \frac{\zeta^2}{4}R^2\right\} + S_{SM}$$

Conformal transformation

conformal transformation (change of variables)

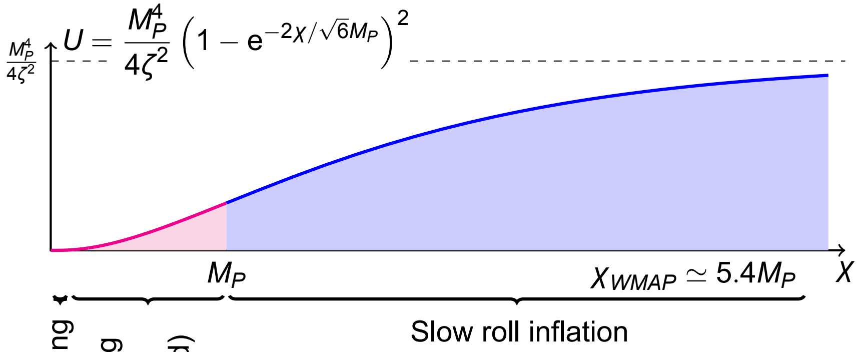
$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \,, \qquad \Omega^2 \equiv \exp\left(\frac{\chi(x)}{\sqrt{6}M_P}\right)$$

$\chi(x)$ – new field (d.o.f.) "scalaron"

Resulting action (Einstein frame action)

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{-\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - \frac{M_P^4}{4\zeta^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2\right\}$$

Inflationary potential

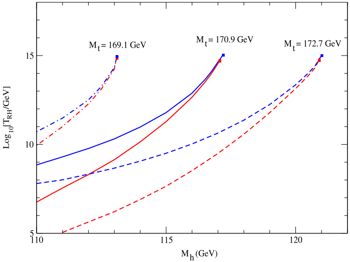


$\delta T/T \sim 10^{-5}$ normalization

$\zeta \simeq 47000$ $\quad n_s \simeq 0.965$, $\quad r \simeq 0.0036$

The SM vacuum should not decay at hot stage after inflation

The electroweak vacuum may decay at high temperature



[Espinosa, Giudice, Riotto'07]

Reheating is due to M_P suppressed operators $\Rightarrow$ temperature is low $T_r \sim 10^7 - 10^9$ GeV

Higgs mass bounds in R^2 is weak

$$m_H > 116\,\text{GeV}$$

(superseded by LEP/LHC)

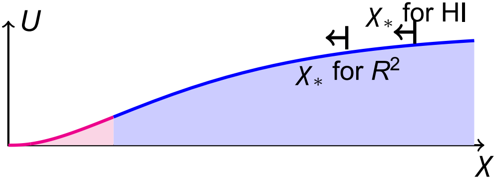
Different T_r means different field at horizon exit

- Hubble at the Horizon exit $H_* = \frac{k}{a_0}\frac{a_0}{a_r}\frac{a_r}{a_e}e^{N_*}$

$$\frac{a_r}{a_0} = \left(\frac{g_0}{g_r}\right)^{1/3}\frac{T_0}{T_r}, \qquad \frac{a_r}{a_e} = \left(\frac{V_e}{g_r\frac{\pi^2}{30}T_r^4}\right)^{1/3}$$

- E-folding number of the hirizon exit

$$N_* \simeq 57 - \frac{1}{3}\log\frac{10^{13}\,\mathrm{GeV}}{T_r} \quad\Rightarrow\quad N_{HI} = 57.7, \qquad N_{R^2} = 54.4$$



[0] Bezrukov, Gorbunov'12

Different predictions for CMB observables

Higgs inflation: $n_s = 0.967, \quad r = 0.0032$
R^2 inflation: $n_s = 0.965, \quad r = 0.0036$

- Planck $\Delta n_s \sim 0.0045$ — not there, but not too far away
- CMBPol $\Delta n_s \sim 0.0016$, $\delta r \sim 10^-3$

Summary

Coincidence in pure SM

- $\lambda(M_P) = \left.\frac{d\lambda}{d\mu}\right|_{\mu=M_P} = 0$
- for $M_h = M_{\text{min}} =$

$$\left[128.9 + \frac{M_t - 172.9\,\text{GeV}}{1.1\,\text{GeV}} \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.6 \pm 2\right]\,\text{GeV}$$

- Future accelerator needed to clear up the situation – Higgs and top factory – e^+e^- collider up to $\sim 350\,\text{GeV}$

- Possible consequences for SM
  - In some models (i.e. asymptotic safety) – $M_h = M_{\text{min}}$ is the prediction
  - In some models (i.e. Higgs inflation) – $M_h > M_{\text{min}}$
  - In some models (R^2 inflation) – no problem with light M_h

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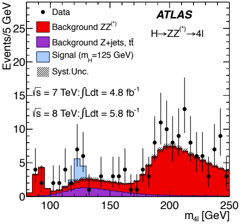
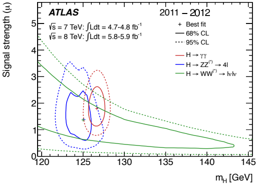
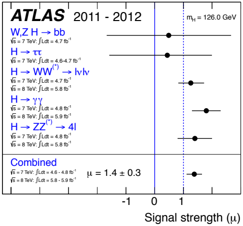
- CMS Collaboration, [arXiv:1207.7235 [hep-ex]]
- ATLAS Collaboration, [arXiv:1207.7214 [hep-ex]]
- ATLAS Collaboration, ATLAS-CONF-2012-170

ATLAS “new particle” results (July)







“New particle” mass

$$M_h = 126.0 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})\,\text{GeV}$$

5.9σ for SM Higgs boson of this mass

[ATLAS’12]

Exact effective potetnial definition

$$V(\varphi) \propto \lambda(\varphi)\varphi^4 \left[1 + O\left(\frac{\alpha}{4\pi}\log(M_i/M_j)\right)\right],$$

Corrections to the potential

1-loop effective potential

$$\Delta U(\chi) \sim \sum_{\text{particles}} \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2} \quad \Big| \quad \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2/\Omega^2(\chi)}$$

In Einstein frame: $m^2(\chi) \sim g^2 h^2(\chi)/\Omega^2(\chi)$

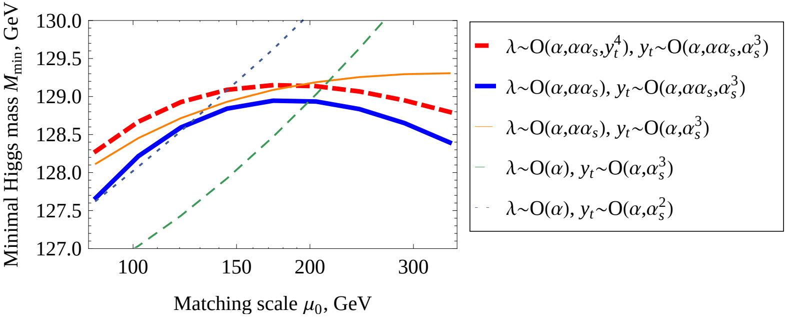
- Correct by RG running
- Ambiguity in the theory definition in UV

Cutoff frame dependence and choice

	choice I	choice II
Jordan frame	$M_P^2 + \xi h^2$	M_P^2
Einstein frame	M_P^2	$\frac{M_P^4}{M_P^2 + \xi h^2}$

FB, Magnin, Shaposhnikov'09

RG scale dependence



Error budget

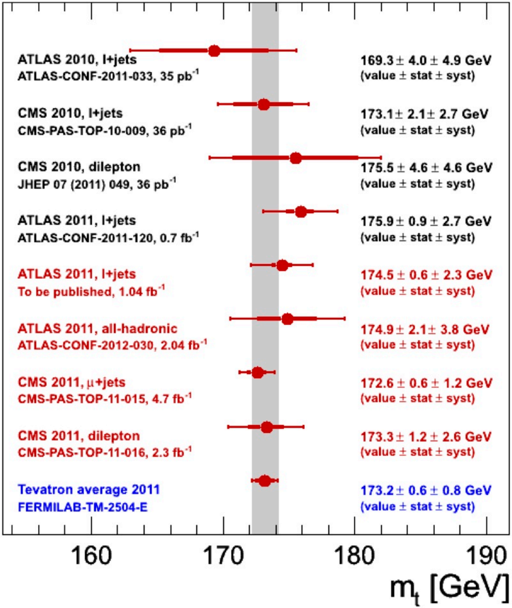
Theoretical

Source of uncertainty	Nature of estimate	$\Delta_{theor} M_{min}$, GeV
3-loop matching λ	Sensitivity to μ	1.0
3-loop matching y_t	Sensitivity to μ	0.2
4-loop α_s to y_t	educated guess	0.4
confinement, y_t	educated guess	0.5
4-loop RG $M_W \to M_P$	educated guess	< 0.2
total uncertainty	sum of squares	1.2
total uncertainty	linear sum	2.3

Experimental

Source of uncertainty		$\Delta_{exp} M_{min}$, GeV
M_t		~ 2
α_s		~ 0.6
total uncertainty	sum of squares	2.1

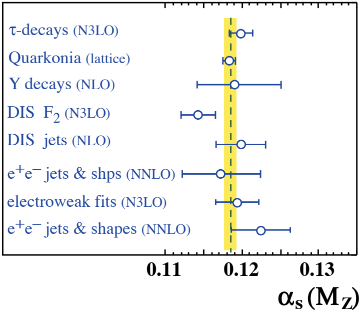
Top mass determination

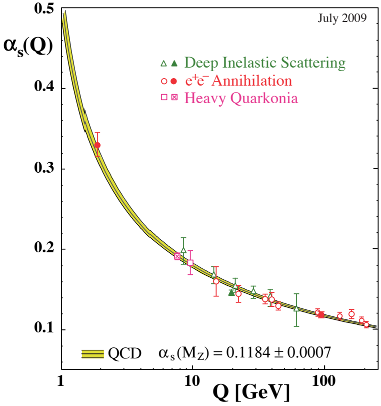


In addition:

- Problems with relation of M_{Pythia} and M_{pole} – up to $\sim 1\,\text{GeV}$

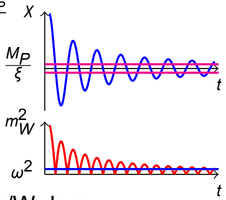
α_s determination





Preheating

- Background evolution after inflation $\chi < M_P$ $(h < M_P/\sqrt{\xi})$
    - Quadratic potential $U \simeq \frac{\mu^2}{2}\chi^2$ with $\mu = \sqrt{\frac{\lambda}{3}}\frac{M_P}{\xi}$
    - Matter dominated stage $a \propto t^{2/3}$



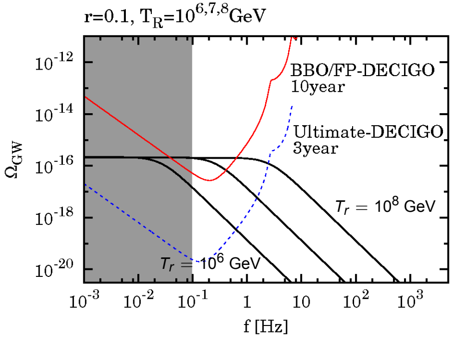
- Stohastic resonance
    - Particle masses $m_W^2(\chi) \sim g^2 \frac{M_P|\chi|}{\xi}$
    - W bosons are created (non-relativistic)
        - $\sqrt{\langle\chi^2\rangle} \gtrsim 23\left(\frac{\lambda}{0.25}\right)\frac{M_P}{\xi}$: non-resonant creation/W decay
        - $\sqrt{\langle\chi^2\rangle} \lesssim 23\left(\frac{\lambda}{0.25}\right)\frac{M_P}{\xi}$: resonant creation/W annihilation
    - Higgs creation – relativistic, less efficient
      $\sqrt{\langle\chi^2\rangle} \sim 2.6\left(\frac{\lambda}{0.25}\right)^{1/2}\frac{M_P}{\xi}$

Reheating at

$$T_r \gtrsim 3.4 \times 10^{13}\ \mathrm{GeV}$$

Bezrukov et.al'2008, Garcia-Bellido'2008

Features in tensor perturbations for gravity wave detectors



[?]

Gravity waves at matter dominated stage

- Primordial density of scalar perturbations $\delta\rho/\rho \sim 10^{-5}$
- Grow $\propto$ scalefactor at matter domination
- Can reach $\delta\rho/\rho \sim 1$ for long matter domination and small scales, generating scalaron (inflaton) “clumps”
- Gravity waves can be generated
  - collapse of scalaron perturbations
  - merging of clumps
  - evaporation of clumps at reheating

For R^2 inflation can be in DECIGO reach [?]