

Theoretical perspectives and future implications



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- **Introduction**
- **Observables and uncertainties**
 - focusing on rare decays
- **Model independent implications**
 - check the MFV hypothesis
- **Implications for supersymmetry**
 - interplay with direct searches



Interesting topics, but not covered in this talk:

- Many outstanding puzzles seem to be solved (a_{sl} , $\sin 2\beta$, ...)
- But others popped up ($B \rightarrow D^{(*)} \tau \nu_\tau$, $A_I(B \rightarrow K \mu^+ \mu^-)$, ...)
→ may disappear with more data!
- CP violation: opportunities from non-leptonic B and D decays
- Photon polarization
- ...

some already covered in today's talks

The focus of this talk will be the (near) future opportunities with rare decays



A multi-scale problem

- new physics: $1/\Lambda_{\text{NP}}$
- electroweak interactions: $1/M_W$
- hadronic effects: $1/m_b$
- QCD interactions: $1/\Lambda_{\text{QCD}}$

⇒ Effective field theory approach:

separation between low and high energies using Operator Product Expansion

- short distance: Wilson coefficients, computed perturbatively
- long distance: local operators

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=1 \dots 10, S, P} (C_i(\mu) \mathcal{O}_i(\mu) + C'_i(\mu) \mathcal{O}'_i(\mu)) \right)$$

New physics:

- Corrections to the Wilson coefficients: $C_i \rightarrow C_i + \Delta C_i^{\text{NP}}$
- Additional operators: $\sum_j C_j^{\text{NP}} \mathcal{O}_j^{\text{NP}}$



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Many flavour observables sensitive to new physics



Key decays:

- LHCb golden channels
 - $B_s \rightarrow \mu^+ \mu^-$
 - $B \rightarrow K^* \mu^+ \mu^-$
- Inclusive penguins
 - $B \rightarrow X_s \gamma$
 - $B \rightarrow X_s \mu^+ \mu^-$
- Tree level neutrino modes
 - $B \rightarrow \tau \nu_\tau$
 - $B \rightarrow D \tau \nu_\tau$
 - $D_s \rightarrow \tau \nu_\tau$
 - $K \rightarrow \mu \nu_\mu$

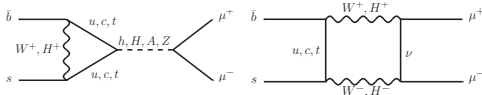


Relevant operators:

$$\mathcal{O}_{10} = \frac{e^2}{(4\pi)^2} (\bar{s} \gamma^\mu b_L) (\bar{\ell} \gamma_\mu \gamma_5 \ell)$$

$$\mathcal{O}_S = \frac{e^2}{16\pi^2} (\bar{s}_L^\alpha b_R^\alpha) (\bar{\ell} \ell)$$

$$\mathcal{O}_P = \frac{e^2}{16\pi^2} (\bar{s}_L^\alpha b_R^\alpha) (\bar{\ell} \gamma_5 \ell)$$



$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = \frac{G_F^2 \alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb} V_{ts}^*|^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \times \left\{ \left(1 - \frac{4m_\mu^2}{m_{B_s}^2}\right) |C_S - C_S'|^2 + \left| (C_P - C_P') + 2(C_{10} - C_{10}') \frac{m_\mu}{m_{B_s}} \right|^2 \right\}$$

Largest contributions in SM from a Z penguin top loop (75%) and a W box diagram (24%)

First experimental evidence:

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = (3.2_{-1.2}^{+1.4}(\text{stat})_{-0.3}^{+0.5}(\text{syst})) \times 10^{-9}$$

LHCb, Phys. Rev. Lett. 110 (2013) 021801

Previous limit: $\text{BR}(B_s \rightarrow \mu^+ \mu^-) < 4.2 \times 10^{-9}$ at 95% C.L.

ATLAS+CMS+LHCb combined value, LHCb-CONF-2012-017

→ Measurement consistent with the SM prediction!

→ Crucial to have a clear estimation of the uncertainties!

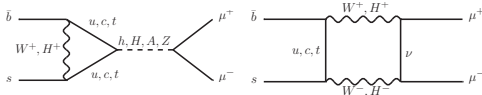


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Main source of uncertainty: f_{B_s}

- ETMC-11: 232 ± 10 MeV
- HPQCD-12: 227 ± 10 MeV
HPQCD NR-09: 231 ± 15 MeV
HPQCD HISQ-11: 225 ± 4 MeV
- Fermilab-MILC-11: 242 ± 9.5 MeV

Our choice: 234 ± 10 MeV

With the most up-to-date input parameters (PDG 2012), in particular $\tau_{B_s} = 1.497$ ps:

$$\text{SM prediction: } \text{BR}(B_s \rightarrow \mu^+ \mu^-) = (3.53 \pm 0.38) \times 10^{-9}$$

FM, S. Neshatpour, J. Orloff, JHEP 1208 (2012) 092

Most important sources of uncertainties:

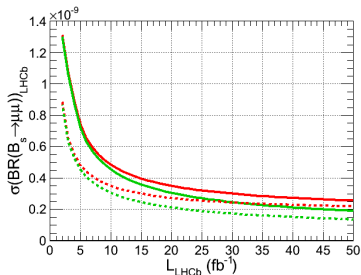
Uncertainty	f_{B_s}	EW cor.	scales	τ_{B_s}	V_{ts}	top mass	Overall
Present	8%	2%	2%	2%	5%	1.3%	10%
Future (~ 2020)	2%	< 1%	1%	0.5%	2.5%	0.5%	3.5%

Using $f_{B_s} = 227$ MeV and $\tau_{B_s} = 1.466$ ps, one gets: $\text{BR}(B_s \rightarrow \mu^+ \mu^-) = 3.25 \times 10^{-9}$

A. Buras et al. Eur.Phys.J. C72 (2012) 2172



Experimental expectations: uncertainty vs. luminosity



A. Arbey, M. Battaglia, FM, D. Martinez Santos, arXiv:1212.4887

Red line: systematic uncertainty of 5% for LHCb

Green line: ultimate systematic uncertainty of 1% for LHCb

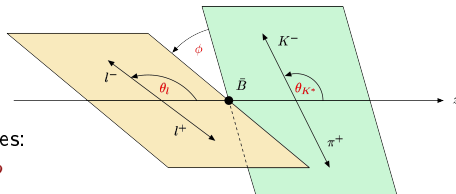
Dashed lines: LHC combinations

An ultimate uncertainty of $\sim 0.2 \times 10^{-9}$ can be expected after 50 fb^{-1} of data.



Angular distributions

The full angular distribution of the decay $\bar{B}^0 \rightarrow \bar{K}^{*0} \ell^+ \ell^-$ ($\bar{K}^{*0} \rightarrow K^- \pi^+$) is completely described by four independent kinematic variables: q^2 (dilepton invariant mass squared), θ_ℓ , θ_{K^*} , ϕ



Differential decay distribution:

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} = \frac{9}{32\pi} J(q^2, \theta_\ell, \theta_{K^*}, \phi)$$

$J(q^2, \theta_\ell, \theta_{K^*}, \phi)$ are written in function of the angular coefficients $J_{1-9}^{s,c}$

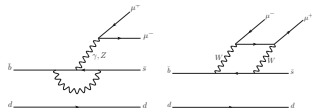
J_{1-9} : functions of the spin amplitudes A_0 , A_{\parallel} , A_{\perp} , A_t , and A_S

Spin amplitudes: functions of Wilson coefficients and form factors

Main operators:

$$\mathcal{O}_9 = \frac{e^2}{(4\pi)^2} (\bar{s} \gamma^\mu b_L) (\bar{\ell} \gamma_\mu \ell), \quad \mathcal{O}_{10} = \frac{e^2}{(4\pi)^2} (\bar{s} \gamma^\mu b_L) (\bar{\ell} \gamma_\mu \gamma_5 \ell)$$

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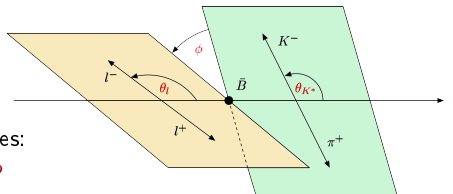
F. Kruger et al., Phys. Rev. D 61 (2000) 114028;

W. Altmannshofer et al., JHEP 0901 (2009) 019; U. Egede et al., JHEP 1010 (2010) 056



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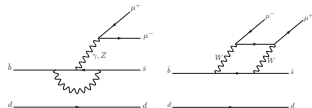
J_{1-9} : functions of the spin amplitudes A_0 , A_{\parallel} , A_{\perp} , A_t , and A_S

Spin amplitudes: functions of Wilson coefficients and form factors

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Dilepton invariant mass spectrum: $\frac{d\Gamma}{dq^2} = \frac{3}{4} \left(J_1 - \frac{J_2}{3} \right)$

Forward backward asymmetry:

$$A_{\text{FB}}(q^2) \equiv \left[\int_{-1}^0 - \int_0^1 \right] d \cos \theta_l \frac{d^2 \Gamma}{dq^2 d \cos \theta_l} \bigg/ \frac{d\Gamma}{dq^2} = \frac{3}{8} J_6 \bigg/ \frac{d\Gamma}{dq^2}$$

Forward backward asymmetry zero-crossing: $q_0^2 \simeq -2m_b m_B \frac{C_9^{\text{eff}}(q_0^2)}{C_7} + O(\alpha_s, \Lambda/m_b)$

→ fix the sign of C_9/C_7

Polarization fractions:

$$F_L(q^2) = \frac{|A_0|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}, \quad F_T(q^2) = 1 - F_L(q^2) = \frac{|A_{\perp}|^2 + |A_{\parallel}|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}$$

Transverse asymmetries:

$$A_T^{(1)}(q^2) = \frac{-2\Re(A_{\parallel} A_{\perp}^*)}{|A_{\perp}|^2 + |A_{\parallel}|^2} \qquad A_T^{(2)}(q^2) = \frac{|A_{\perp}|^2 - |A_{\parallel}|^2}{|A_{\perp}|^2 + |A_{\parallel}|^2}$$
$$A_T^{(3)}(q^2) = \frac{|A_{0L} A_{\parallel L}^* + A_{0R}^* A_{\parallel R}|}{\sqrt{|A_0|^2 |A_{\perp}|^2}} \qquad A_T^{(4)}(q^2) = \frac{|A_{0L} A_{\perp L}^* - A_{0R}^* A_{\perp R}|}{|A_{0L} A_{\parallel L}^* + A_{0R}^* A_{\parallel R}|}$$

→ Reduced form factor uncertainties



Two regions of interest:

- Low q^2 ($1 - 6 \text{ GeV}^2$)
 - small $1/m_b$ corrections
 - sensitivity to the interference of C_7 and C_9
 - high rate
 - long-distance effects not fully under control
 - non-negligible scale and m_c dependence
- High q^2 ($14.18 - 16 \text{ GeV}^2$)
 - negligible scale and m_c dependence due to the strong sensitivity to C_{10}
 - negligible long-distance effects of the type $B \rightarrow J/\Psi X_s \rightarrow X_s + X' \ell^+ \ell^-$
 - sizeable $1/m_b$ corrections
 - low rate



Main uncertainties from:

- form factors
- $1/m_b$ subleading corrections
- parametric uncertainties (m_b , m_c , m_t)
- CKM matrix elements
- scales

Observable	SM value	(FF)	(SL)	(QM)	(CKM)	(Scale)	Total
$10^7 \times BR(B \rightarrow K^* \mu^+ \mu^-)_{[1,6]}$	2.32	± 1.34	± 0.04	$^{+0.04}_{-0.03}$	$^{+0.08}_{-0.13}$	$^{+0.09}_{-0.05}$	± 1.35
$\langle A_{FB}(B \rightarrow K^* \mu^+ \mu^-) \rangle_{[1,6]}$	-0.06	± 0.04	± 0.02	± 0.01	—	—	± 0.05
$\langle F_L(B \rightarrow K^* \mu^+ \mu^-) \rangle_{[1,6]}$	0.71	± 0.13	± 0.01	± 0.01	—	—	± 0.13
$q_0^2(B \rightarrow K^* \mu^+ \mu^-)/\text{GeV}^2$	4.26	± 0.30	± 0.15	$^{+0.14}_{-0.04}$	—	$^{+0.02}_{-0.04}$	± 0.35

FM, S. Neshatpour, J. Orloff, JHEP 1208 (2012) 092

Forecast (for ~ 2020):

Observable	SM value	(FF)	(SL)	(QM)	(CKM)	(Scale)	Total
$10^7 \times BR(B \rightarrow K^* \mu^+ \mu^-)_{[1,6]}$	2.32	± 0.70	± 0.04	—	± 0.05	$^{+0.09}_{-0.05}$	± 0.71
$\langle A_{FB}(B \rightarrow K^* \mu^+ \mu^-) \rangle_{[1,6]}$	-0.06	± 0.03	± 0.02	—	—	—	± 0.04
$\langle F_L(B \rightarrow K^* \mu^+ \mu^-) \rangle_{[1,6]}$	0.71	± 0.10	± 0.01	—	—	—	± 0.10
$q_0^2(B \rightarrow K^* \mu^+ \mu^-)/\text{GeV}^2$	4.26	± 0.20	± 0.15	—	—	$^{+0.02}_{-0.04}$	± 0.25

At high q^2 : relative uncertainty of 25% (15%) for BR and 23% (10%) for A_{FB} .



Opportunities to improve the situation by defining **clean observables**!

Strategy: Find observables with limited sensitivity to soft form factors:

- suitable ratios of the spin amplitudes
- where soft form factors cancel at LO
- which show good sensitivity to NP

Example:

$$P_4 = \frac{\Re(A_{0L}A_{\parallel L}^* + A_{0R}A_{\parallel R}^*)}{\sqrt{(|A_{0L}|^2 + |A_{0R}|^2)(|A_{\parallel L}|^2 + |A_{\parallel R}|^2)}}$$

The $P_i^{(\prime)}$ observables could be measured soon at LHCb.

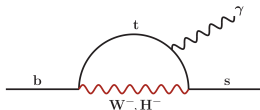
J. Matias, F. Mescia, M. Ramon, J. Virto, JHEP 1204 (2012) 104

S. Descotes-Genon, J. Matias, M. Ramon, J. Virto, JHEP 1301 (2013) 048



Inclusive branching ratio of $B \rightarrow X_s \gamma$

Contributing loops:



Main operator: \mathcal{O}_7

but higher order contributions from $\mathcal{O}_1, \dots, \mathcal{O}_8$.

NNLO calculations available for the SM

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \rightarrow X_c e \bar{\nu})_{\text{exp}} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\text{em}}}{\pi C} [P(E_0) + N(E_0)]$$

$$\begin{aligned} P(E_0) &= P^{(0)}(\mu_b) + \alpha_s(\mu_b) \left[P_1^{(1)}(\mu_b) + P_2^{(1)}(E_0, \mu_b) \right] \\ &+ \alpha_s^2(\mu_b) \left[P_1^{(2)}(\mu_b) + P_2^{(2)}(E_0, \mu_b) + P_3^{(2)}(E_0, \mu_b) \right] + \mathcal{O}(\alpha_s^3(\mu_b)) \end{aligned}$$

$$\begin{cases} P^{(0)}(\mu_b) &= \left(C_7^{(0)\text{eff}}(\mu_b) \right)^2 \\ P_1^{(1)}(\mu_b) &= 2C_7^{(0)\text{eff}}(\mu_b) C_7^{(1)\text{eff}}(\mu_b) \\ P_1^{(2)}(\mu_b) &= \left(C_7^{(1)\text{eff}}(\mu_b) \right)^2 + 2C_7^{(0)\text{eff}}(\mu_b) C_7^{(2)\text{eff}}(\mu_b) \end{cases}$$



- SM contributions known to NNLO accuracy

M. Misiak et al., Phys. Rev. Lett. 98 (2007) 022002

- THDM contributions known to NNLO accuracy

T. Hermann, M. Misiak, M. Steinhauser, JHEP 1211 (2012) 036

- SUSY contributions known partially to NNLO accuracy

C. Greub et al., Nucl. Phys. B 853 (2011) 240

SM prediction: $\text{BR}(\bar{B} \rightarrow X_s \gamma) = (3.08 \pm 0.24) \times 10^{-4}$

SuperIso v3.4

Most important sources of uncertainties:

Uncertainty	parametric	higher order	m_c interpol.	non-perturb.	Overall
Present	3%	3%	3%	5%	8%
Future (~ 2020)	1%	3%	<1%	2%	4%

Experimental values (HFAG 2012): $\text{BR}(\bar{B} \rightarrow X_s \gamma) = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$

Expected Belle II and Babar experimental accuracy: 5%



NNLL QCD calculation + electromagnetic corrections

$$\frac{d\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{d\hat{s}} = \frac{G_F m_b |V_{ts}^* V_{tb}|^2}{48\pi^3} \left(\frac{\alpha_{em}}{4\pi} \right) (1 - \hat{s})^2 \times \left\{ (1 + 2\hat{s}) |\tilde{C}_9^{eff}|^2 + |\tilde{C}_{10}^{eff}|^2 + 4 \left(1 + \frac{2}{\hat{s}} |\tilde{C}_7^{eff}|^2 + 12 \tilde{C}_7^{eff}(\hat{s}) \text{Re}(\tilde{C}_9^{eff}(\hat{s})) \right) \right\} + \dots$$

A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Nucl. Phys. B685 (2004) 351

T. Huber, T. Hurth and E. Lunghi, Nucl. Phys. B802 (2008) 40

$$\text{BR}(B \rightarrow X_s \ell^+ \ell^-)_{\text{low}} = (1.79 \pm 0.15) \times 10^{-6}$$

$$\text{BR}(B \rightarrow X_s \ell^+ \ell^-)_{\text{high}} = (0.22 \pm 0.06) \times 10^{-6}$$

Theory error at low q^2 : 7% + 5% non perturbative \rightarrow 8.5% overall

Future: 3% + 3% \rightarrow 4% overall

Theory error at high q^2 : 25% overall

(can be reduced to 15% by normalizing to $B \rightarrow X_u \ell \nu$)

Future: 10% overall

Average of Belle (latest results in 2005) and Babar (latest in 2004):

$$\text{BR}(B \rightarrow X_s \ell \ell)_{\text{low}} = (1.60 \pm 0.68) \times 10^{-6}$$

$$\text{BR}(B \rightarrow X_s \ell \ell)_{\text{high}} = (0.42 \pm 0.13) \times 10^{-6}$$

Expected final Belle and Babar experimental accuracy: 15%



Tree level process, mediated by W^+ and H^+ , higher order corrections from sparticles



$$\text{BR}(B \rightarrow \tau \nu) = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_\tau^2 f_B^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \left|1 - \left(\frac{m_B^2}{m_{H^+}^2}\right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta}\right|^2$$

$$\epsilon_0 = -\frac{2\alpha_s}{3\pi} \frac{\mu}{m_{\tilde{g}}} H_2\left(\frac{m_Q^2}{m_{\tilde{g}}^2}, \frac{m_D^2}{m_{\tilde{g}}^2}\right), \quad H_2(x, y) = \frac{x \ln x}{(1-x)(x-y)} + \frac{y \ln y}{(1-y)(y-x)}$$



Large uncertainty from V_{ub} and f_B

Also used:

$$R_{\tau \nu_\tau}^{\text{MSSM}} = \frac{\text{BR}(B_u \rightarrow \tau \nu_\tau)_{\text{MSSM}}}{\text{BR}(B_u \rightarrow \tau \nu_\tau)_{\text{SM}}} = \left[1 - \left(\frac{m_B^2}{m_{H^+}^2}\right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta}\right]^2$$

Similar processes: $B \rightarrow D \tau \nu_\tau$, $D_s \rightarrow \ell \nu_\ell$, $D \rightarrow \mu \nu_\mu$, $K \rightarrow \mu \nu_\mu$, ...



Tree level process, mediated by W^\pm and H^\pm , higher order corrections from sparticles



$$\text{BR}(B \rightarrow \tau \nu) = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_\tau^2 f_B^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \left|1 - \left(\frac{m_B^2}{m_{H^\pm}^2}\right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta}\right|^2$$

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Similar processes: $B \rightarrow D \tau \nu_\tau$, $D_s \rightarrow \ell \nu_\ell$, $D \rightarrow \mu \nu_\mu$, $K \rightarrow \mu \nu_\mu$, ...



Tree level process, mediated by W^\pm and H^\pm , higher order corrections from sparticles



$$\text{BR}(B \rightarrow \tau \nu) = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_\tau^2 f_B^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \left|1 - \left(\frac{m_B^2}{m_{H^\pm}^2}\right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta}\right|^2$$

$$\epsilon_0 = -\frac{2\alpha_s}{3\pi} \frac{\mu}{m_{\tilde{g}}} H_2\left(\frac{m_Q^2}{m_{\tilde{g}}^2}, \frac{m_D^2}{m_{\tilde{g}}^2}\right), \quad H_2(x, y) = \frac{x \ln x}{(1-x)(x-y)} + \frac{y \ln y}{(1-y)(y-x)}$$

⚠ Large uncertainty from V_{ub} and f_B

Also used:

$$R_{\tau\nu\tau}^{\text{MSSM}} = \frac{\text{BR}(B_u \rightarrow \tau\nu_\tau)_{\text{MSSM}}}{\text{BR}(B_u \rightarrow \tau\nu_\tau)_{\text{SM}}} = \left[1 - \left(\frac{m_B^2}{m_{H^\pm}^2}\right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta}\right]^2$$

Similar processes: $B \rightarrow D\tau\nu_\tau$, $D_s \rightarrow \ell\nu_\ell$, $D \rightarrow \mu\nu_\mu$, $K \rightarrow \mu\nu_\mu$, ...



Main source of uncertainties: V_{ub} and f_B

V_{ub} from inclusive decays (PDG2012): $|V_{ub}| = (4.41 \pm 0.15 \pm 0.16) \times 10^{-3}$

V_{ub} from exclusive decays (PDG2012): $|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$

Average V_{ub} (PDG2012): $|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$

Average of f_B (ETMC, HPQCD, Fermilab-MILC): 194 ± 10 MeV

$\text{BR}(B \rightarrow \tau \nu)_{\text{SM}} = (1.15 \pm 0.29) \times 10^{-4}$

Current theoretical uncertainty on $\text{BR}(B \rightarrow \tau \nu)$: 25%

Expected uncertainty on $|V_{ub}|f_B$: 3%

Future theoretical uncertainty on $\text{BR}(B \rightarrow \tau \nu)$: 7%

Experimental average (ICHEP 2012): $\text{BR}(B \rightarrow \tau \nu) = (1.14 \pm 0.23) \times 10^{-4}$

Expected experimental precision (Belle II): 5-6%



Building precision observables

Example of double ratio of leptonic decays:

$$R = \left(\frac{\text{BR}(B_s \rightarrow \mu^+ \mu^-)}{\text{BR}(B_u \rightarrow \tau \nu)} \right) / \left(\frac{\text{BR}(D_s \rightarrow \tau \nu)}{\text{BR}(D \rightarrow \mu \nu)} \right)$$

From the form factor point of view:

$$R \propto \left(\frac{f_{B_s}}{f_B} \right)^2 / \left(\frac{f_{D_s}}{f_D} \right)^2 \approx 1$$

R has no dependence on the form factors, contrary to each decay taken individually!

- No dependence on lattice quantities
- Interesting for V_{ub} determination
- Interesting for probing new physics
- Promising experimental situation

B. Grinstein, Phys. Rev. Lett. 71 (1993)

A.G. Akeroyd, FM, JHEP 1010 (2010)



Implications



Minimal Flavour Violation (MFV): Flavour and CP symmetries are broken as in the SM
→ all flavour- and CP-violating interactions linked to the known structure of Yukawa couplings

Assuming MFV, what are the presently allowed ranges of the Wilson coefficients?

T. Hurth, FM, Nucl. Phys. B865 (2012) 461

Relevant Operators:

$$\mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9, \mathcal{O}_{10} \quad \text{and} \quad \mathcal{O}_{S-P} \propto (\bar{s}P_R b)(\bar{\mu}P_L \mu) \equiv \mathcal{O}_0^I$$

NP manifests itself in the shifts of the individual coefficients with respect to the SM values:

$$C_i(\mu) = C_i^{\text{SM}}(\mu) + \delta C_i$$

- Scans over the values of $\delta C_7, \delta C_8, \delta C_9, \delta C_{10}, \delta C_0^I$
- Calculation of flavour observables
- Comparison with experimental results
- Constraints on the Wilson coefficients C_i
- Prediction of flavour observables

Allows to test the MFV hypothesis!

see also: Hurth, Isidori, Kamenik, Mescia, Nucl.Phys. B808 (2009) 326



→ Global fits of the $\Delta F = 1$ observables obtained by minimization of

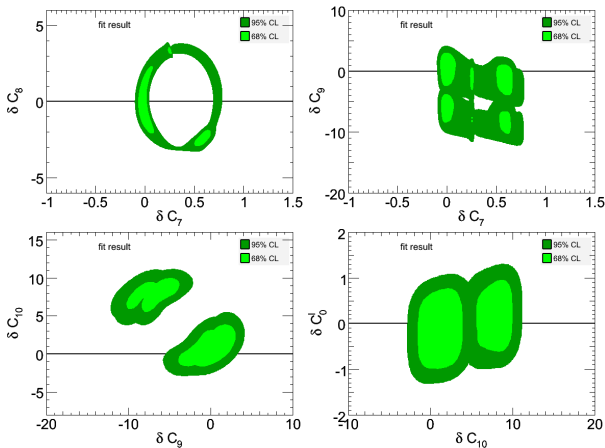
$$\chi^2 = \sum_i \frac{(O_i^{\text{exp}} - O_i^{\text{th}})^2}{(\sigma_i^{\text{exp}})^2 + (\sigma_i^{\text{th}})^2}$$

Observables:

- $\text{BR}(B \rightarrow X_s \gamma)$
- $\text{BR}(B \rightarrow X_d \gamma)$
- $\Delta_0(B \rightarrow K^* \gamma)$
- $\text{BR}^{\text{low}}(B \rightarrow X_s \mu^+ \mu^-)$
- $\text{BR}^{\text{high}}(B \rightarrow X_s \mu^+ \mu^-)$
- $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$
- $\text{BR}^{\text{low}}(B \rightarrow K^* \mu^+ \mu^-)$
- $\text{BR}^{\text{high}}(B \rightarrow K^* \mu^+ \mu^-)$
- $A_{FB}^{\text{low}}(B \rightarrow K^* \mu^+ \mu^-)$
- $A_{FB}^{\text{high}}(B \rightarrow K^* \mu^+ \mu^-)$
- $q_0^2(A_{FB}(B \rightarrow K^* \mu^+ \mu^-))$
- $F_L^{\text{low}}(B \rightarrow K^* \mu^+ \mu^-)$



Before LHCb:

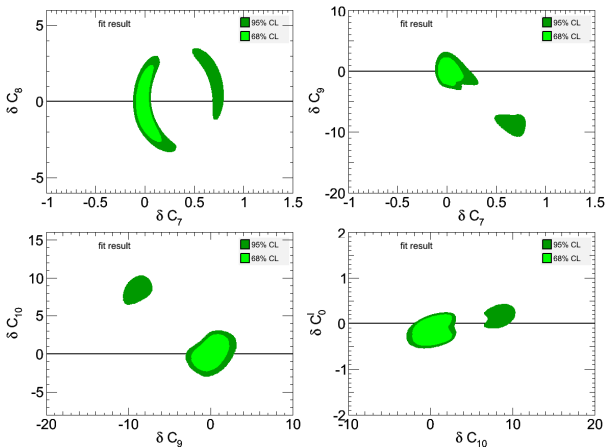


T. Hurth, FM, Nucl. Phys. B865 (2012) 461

Use these results to make predictions for new observables!
Check consistencies!



After LHCb:

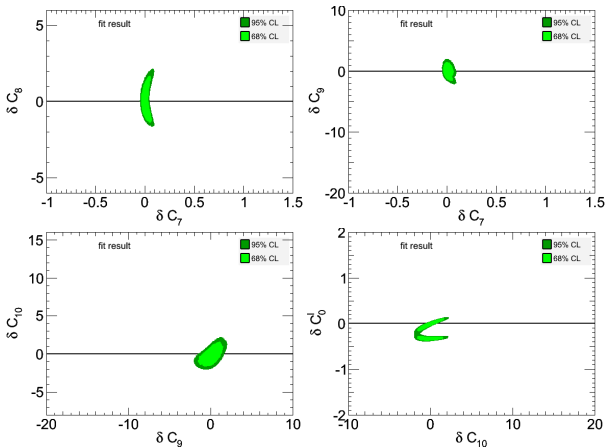


T. Hurth, FM, to appear in *Rev. Mod. Phys.*

Use these results to make predictions for new observables!
Check consistencies!



Ultimate precision:



Use these results to make predictions for new observables!
Check consistencies!

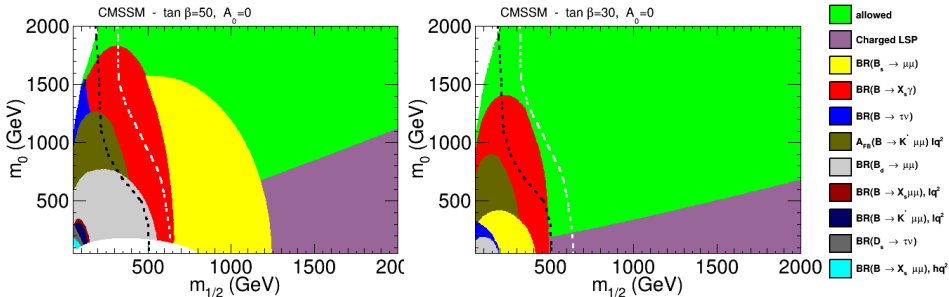


Constraints on CMSSM

Constrained MSSM (CMSSM): University assumptions at the GUT scale

Parameters: m_0 , $m_{1/2}$, A_0 , $\tan\beta$ and sign of μ

Present situation (using the latest results):



Dashed black line: CMS exclusion limit with 1.1 fb⁻¹ data

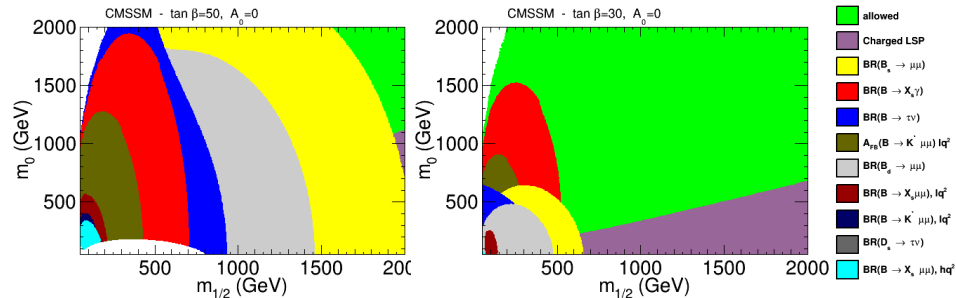
Dashed white line: CMS exclusion limit with 4.4 fb⁻¹ data



Constrained MSSM (CMSSM): University assumptions at the GUT scale

Parameters: m_0 , $m_{1/2}$, A_0 , $\tan \beta$ and sign of μ

Ultimate precision:

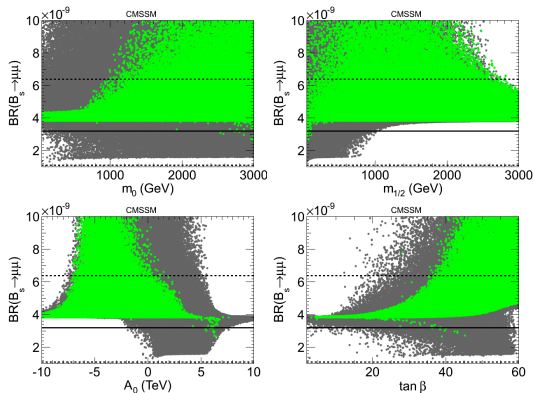


Dashed black line: CMS exclusion limit with 1.1 fb^{-1} data

Dashed white line: CMS exclusion limit with 4.4 fb^{-1} data



Flat scans on the CMSSM parameters with $\mu > 0$



A. Arbey, M. Battaglia, FM, D. Martinez Santos, arXiv:1212.4887

Solid line: central value of the $BR(B_s \rightarrow \mu^+ \mu^-)$ measurement

Dashed lines: 2σ experimental deviations

Gray points: all valid points

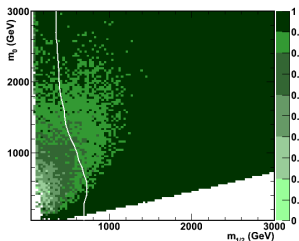
Green points: points in agreement with the Higgs mass constraint

$BR(B_s \rightarrow \mu^+ \mu^-)$ smaller than SM and the Higgs mass constraint cannot be satisfied simultaneously!!

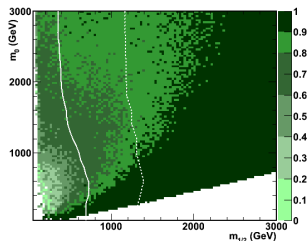


Fraction of CMSSM points compatible with the LHCb 95% C.L. constraints on $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$

Current constraints



Ultimate constraints

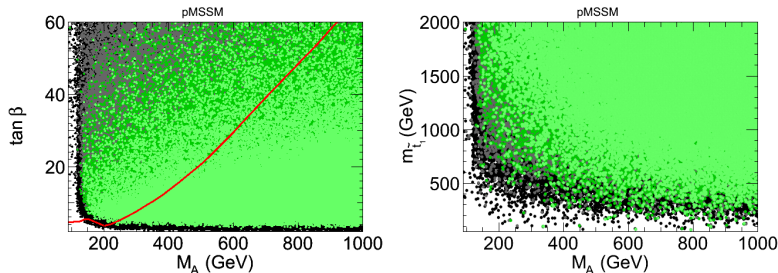


A. Arbey, M. Battaglia, F.M. D. Martinez Santos, [arXiv:1212.4887](https://arxiv.org/abs/1212.4887)

Continuous line: ATLAS SUSY searches at 8 TeV with 5.8 fb^{-1} of data
Dotted line: reach estimated by CMS for searches at 14 TeV with 300 fb^{-1}



Phenomenological MSSM (pMSSM): No universality assumptions, 19 free parameters



A. Arbey, M. Battaglia, FM, D. Martinez Santos, arXiv:1212.4887

Black points: all the valid pMSSM points

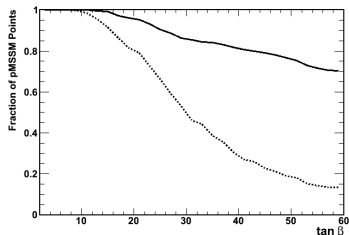
Gray points: $123 < M_h < 129$ GeV

Dark green points: in agreement with the latest $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$

Light green points: in agreement with the ultimate LHCb $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ measurement

Red line: excluded at 95% C.L. by the latest CMS $A/H \rightarrow \tau^+ \tau^-$ searches





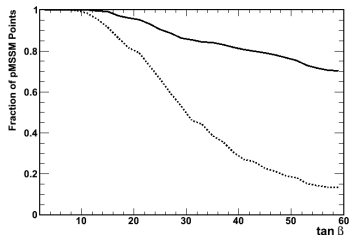
A. Arbey, M. Battaglia, FM, D. Martinez Santos, arXiv:1212.4887

Continuous line: in agreement with the latest $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ measurement

Dotted line: in agreement with the ultimate LHCb $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ measurement

Fraction of points	Current bounds	Projected bounds
All pMSSM points	95.3%	67.8%
Accepted pMSSM points	97.7%	78.1%
Points not excluded by LHC searches	95.1%	63.3%
Points compatible at 90% C.L. with Higgs results	97.2%	70.0%





A. Arbey, M. Battaglia, FM, D. Martinez Santos, arXiv:1212.4887

Continuous line: in agreement with the latest $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ measurement

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- Simplest NP scenarios already ruled out...
- NP should be subtle!
- Flavour physics can help guiding direct searches
- Theory uncertainties are well under control for most of the decays
- Improvements in lattice evaluations are necessary
- Important to define clean observables
- Exploit the complementarity between the different observables and check consistencies



- If all the future key measurements happen to be SM like:
 - Keep testing!
 - Many of the NP scenarios would still be very difficult to exclude...
 - impossible to exclude supersymmetry!
 - Define clean observables and try to establish tensions...
- If an excess/deficit in a flavour observable:
 - very important to check the manifestations elsewhere
 - check consistencies!
 - guide direct searches!
- If a new particle is discovered in high p_T experiments:
 - Flavour data can help discriminating between different models/hypotheses!



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Backup



Going beyond constrained scenarios

- CMSSM useful for benchmarking, model discrimination,...
- However the mass patterns could be more complicated

Phenomenological MSSM (pMSSM)

- The most general CP/R parity-conserving MSSM
- Minimal Flavour Violation at the TeV scale
- The first two sfermion generations are degenerate
- The three trilinear couplings are general for the 3 generations

→ 19 free parameters

10 sfermion masses, 3 gaugino masses, 3 trilinear couplings, 3 Higgs/Higgsino

A. Djouadi, J.-L. Kneur, G. Moultaka, [hep-ph/0211331](#)

→ Interplay between low energy observables and high p_T results

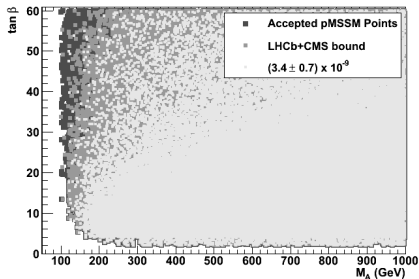
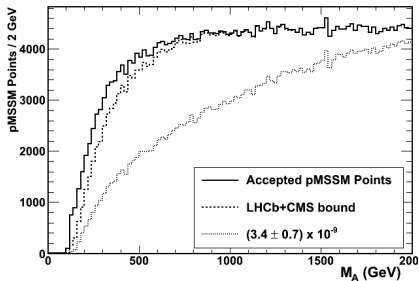


Considering 2 scenarios:

- 2011 bound from LHCb+CMS + estimated th syst:

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) < 1.26 \times 10^{-8}$$

- SM like branching ratio with estimated 20% total uncertainty



Light M_A strongly constrained!

A. Arbey, M. Battaglia, F.M., Eur.Phys.J. C72 (2012) 1847

A. Arbey, M. Battaglia, F.M., Eur.Phys.J. C72 (2012) 1906



In the large $\tan \beta$ region, the largest contribution to C_{Q_1} and C_{Q_2} comes from the chargino-stop loops:

$$C_{Q_1} \approx -C_{Q_2} \approx -\mu A_t \frac{\tan^3 \beta}{(1 + \epsilon_b \tan \beta)^2} \frac{m_t^2}{m_{\tilde{t}}^2} \frac{m_b m_\mu}{4 \sin^2 \theta_W M_W^2 M_A^2} f(x_{\tilde{t}\mu})$$

where

$$x_{\tilde{t}\mu} = m_{\tilde{t}}^2 / \mu^2$$

$m_{\tilde{t}}$: geometric average of the two stop masses

$$f(x) = -\frac{x}{1-x} - \frac{x}{(1-x)^2} \ln x$$

Since $f(x) > 0$ the sign of C_{Q_1} is opposite to that of the μA_t term



Theory prediction: CP-averaged quantities, effect of $B_s - \bar{B}_s$ oscillations disregarded

Experimental measurement: untagged branching fraction

K. De Bruyn et al., Phys. Rev. D86, 014027; Phys. Rev. Lett. 109, 041801 (2012)

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{untag}} = \left(\frac{1 + \mathcal{A}_{\Delta\Gamma} y_s}{1 - y_s^2} \right) \text{BR}(B_s \rightarrow \mu^+ \mu^-)$$

with

$$y_s \equiv \frac{1}{2} \tau_{B_s} \Delta\Gamma_s = 0.088 \pm 0.014$$

$$\mathcal{A}_{\Delta\Gamma} = \frac{|P|^2 \cos(2\varphi_P) - |S|^2 \cos(2\varphi_S)}{|P|^2 + |S|^2}$$

S and P are related to the Wilson coefficients by:

$$S = \sqrt{1 - 4 \frac{m_\mu^2}{M_{B_s}^2} \frac{M_{B_s}^2}{2m_\mu}} \frac{1}{m_b + m_s} \frac{C_{Q_1} - C'_{Q_1}}{C_{10}^{SM}}, \quad P = \frac{C_{10}}{C_{10}^{SM}} + \frac{M_{B_s}^2}{2m_\mu} \frac{1}{m_b + m_s} \frac{C_{Q_2} - C'_{Q_1}}{C_{10}^{SM}}$$

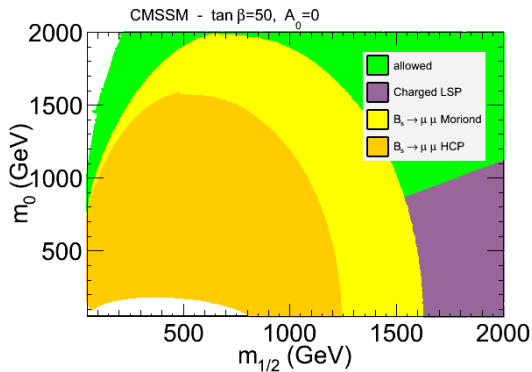
$$\varphi_S = \arg(S), \quad \varphi_P = \arg(P)$$

The SM expectation for this corrected branching fraction is:

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{untag}} = (3.87 \pm 0.46) \times 10^{-9}$$



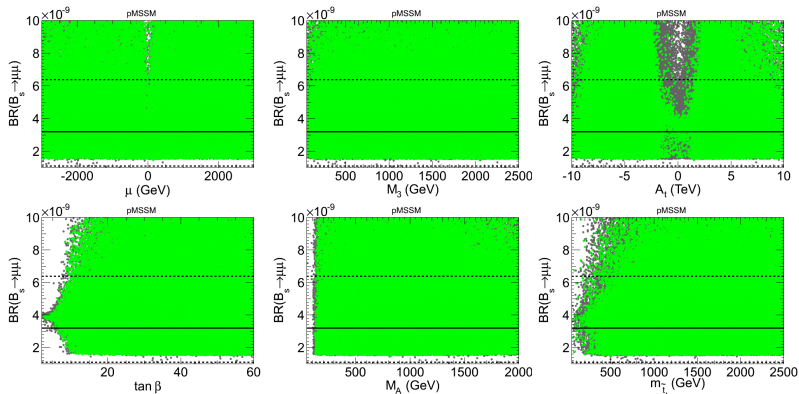
$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ Moriond limit vs. HCP 2012 measurement



SuperIso v3.3



Constraints on pMSSM



A. Arbey, M. Battaglia, FM, D. Martinez Santos, arXiv:1212.4887

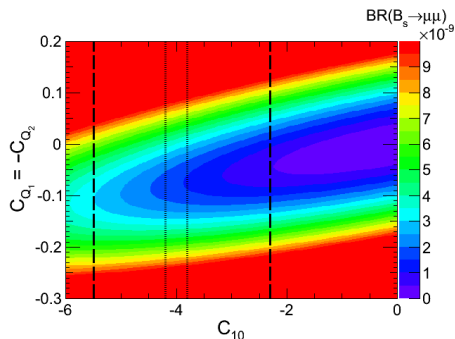
Solid line: central value of the $BR(B_s \rightarrow \mu^+ \mu^-)$ measurement

Dashed lines: 2σ experimental deviations

Gray points: all valid points

Green points: points in agreement with the Higgs mass constraint





A. Arbey, M. Battaglia, FM, D. Martinez Santos, arXiv:1212.4887

Dotted vertical lines: delimit the range of C_{10} in the CMSSM

Dashed lines: delimit the range of C_{10} in the pMSSM.



Tree level process similar to $B \rightarrow \tau \nu$

$$\mathcal{B}(D_s \rightarrow \ell \nu) = \frac{G_F^2}{8\pi} |V_{cs}|^2 f_{D_s}^2 m_\ell^2 M_{D_s} \tau_{D_s} \left(1 - \frac{m_\ell^2}{M_{D_s}^2}\right)^2 \\ \times \left[1 + \left(\frac{1}{m_c + m_s}\right) \left(\frac{M_{D_s}}{m_{H^+}}\right)^2 \left(m_c - \frac{m_s \tan^2 \beta}{1 + \epsilon_0 \tan \beta}\right)\right]^2 \text{ for } \ell = \mu, \tau$$

- Competitive with and complementary to analogous observables
- Dependence on only one lattice QCD quantity
- Interesting if lattice calculations eventually prefer $f_{D_s} < 250$ MeV
- Promising experimental situation (BES-III)



Sensitive to f_{D_s} and m_s/m_c



Use the allowed ranges for the Wilson coefficients to make predictions for the observables which are not yet measured

In particular:

- $\text{BR}(B_d \rightarrow \mu^+ \mu^-) < 0.38 \times 10^{-9}$
Current LHCb limit: $\text{BR}(B_d \rightarrow \mu^+ \mu^-) < 1.0 \times 10^{-9}$
- $10^{-7} < \text{BR}(\bar{B} \rightarrow X_s \tau^+ \tau^-)_{q^2 > 14.4 \text{ GeV}^2} < 3.7 \times 10^{-7}$
- $q_0^2(A_{FB}(B \rightarrow X_s \mu^+ \mu^-)) > 1.94 \text{ GeV}^2$
- $B \rightarrow K^* \mu^+ \mu^-$ transverse asymmetries:
 - $A_T^{(2)} \in [-0.065, -0.022]$
 - $A_T^{(3)} \in [0.34, 0.99]$
 - $A_T^{(4)} \in [0.19, 1.27]$
 - $A_T^{(5)} \in [0.15, 0.49]$

→ **A measurement beyond these results would indicate a new flavour structure!**



- public C program
- dedicated to the flavour physics observable calculations
- various models implemented
- interfaced to several spectrum calculators
- modular program with a well-defined structure
- complete reference manuals available

<http://superiso.in2p3.fr>

FM, Comput. Phys. Commun. 178 (2008) 745

FM, Comput. Phys. Commun. 180 (2009) 1579

FM, Comput. Phys. Commun. 180 (2009) 1718



