Brief Comments

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- Profile Likelihood
- Bayesian Tests

Profile Likelihood

Given a likelihood function

$$p(x, y \mid s, b) = \mathrm{Pois}(x \mid s + b)\,\mathrm{Pois}(y, cb)$$

replace b by an estimate

$$\hat{b}(s)$$

and use the **profile likelihood**

$$p_{PF}(x, y \mid s) = p(x, y \mid s, \hat{b}(s))$$

as if it were the likelihood function; for example, to compute central limits:

$$(1 - CL)/2 = \sum_{n=0}^{x} p_{PF}(n, y \mid \overline{s}(x)), \quad (1 - CL)/2 = \sum_{n=x}^{\infty} p_{PF}(n, y \mid \underline{s}(x))$$

$$\rightarrow [\underline{s}(x), \overline{s}(x)]$$

Bayesian Test

Compute **likelihoods**

$$p(x \mid s, b, H_1) = \mathrm{Pois}(x \mid s + b), \quad s = \varepsilon\sigma$$
$$p(x \mid b, H_0) \quad = \mathrm{Pois}(x \mid b)$$

Compute **priors**

$$p(s, b, H_1) = p(s \mid b, H_1)\, p(b \mid H_1)\, p(H_1)$$
$$= p(s \mid H_1)\, p(b)\, p(H_1)$$
$$p(b, H_0) \quad = p(b \mid H_0)\, p(H_0)$$

Bayesian Test

Compute **signal prior**

$$p(s\,|\,H_1) = \int d\varepsilon \int d\sigma \int dm\, \delta(s - \varepsilon\sigma)\, p(\varepsilon\,|\,m)\, p(\sigma\,|\,m)\, p(m)$$

$p(\sigma\,|\,m)$ SM prediction + theoretical uncertainty

$p(m)$ use latest Tevatron results

Compute **marginal likelihood**

$$p(x\,|\,H_1) = \int ds \int db\; p(x\,|\,s, b, H_1)\, p(s\,|\,H_1)\, p(b)$$

$$p(x\,|\,H_0) = \int db\; p(x\,|\,b, H_0)\, p(b)$$

Bayesian Test

Compute **probabilities of hypotheses**

$$p(H_1 \mid x) = p(x \mid H_1)\, p(H_1) / \, p(x)$$

$$p(H_0 \mid x) = p(x \mid H_0)\, p(H_0) / \, p(x)$$

or, the **Bayes factor**

$$B = p(x \mid H_1) / \, p(x \mid H_0)$$

or

$$S = \sqrt{2 \ln B}$$

If B (or S) > some threshold, claim a discovery!