The Cosmological Constant Problem

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Outline

1- Introduction: the cosmological constant in the Einstein equations.

2- Observational constraints on the CC.

3- Regularization (or renormalization) of the vacuum energy density.

4- Possible loopholes in our approach to the CC problem.

5- General conclusions.

Based on

"Everything you always wanted to know about the Cosmological constant problem (but were afraid to ask)"

Comptes Rendus Physique 13 (2012) 566-665

arXiv:1205.3365

<u>See also:</u>

- S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)
- V. Sahni & A. Starobinsky, astro-ph/9904398
- T. Padmanabhan, hep-th/0212290
- J. Yokoyama, gr-qc/0305068
- J. Polchinsky, hep-th/0603249
- M. Li, X. Li, S. Wang & Y. Wang, arXiv:1103.5870

Historically introduced by Einstein to find a static cosmological solution in General Relativity (GR) [see N. Straumann, gr-qc/0208027]

4. ON AN ADDITIONAL TERM FOR THE FIELD EQUATIONS OF GRAVITATION

My proposed field equations of gravitation for any chosen system of coordinates run as follows: –

$$R_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right),$$

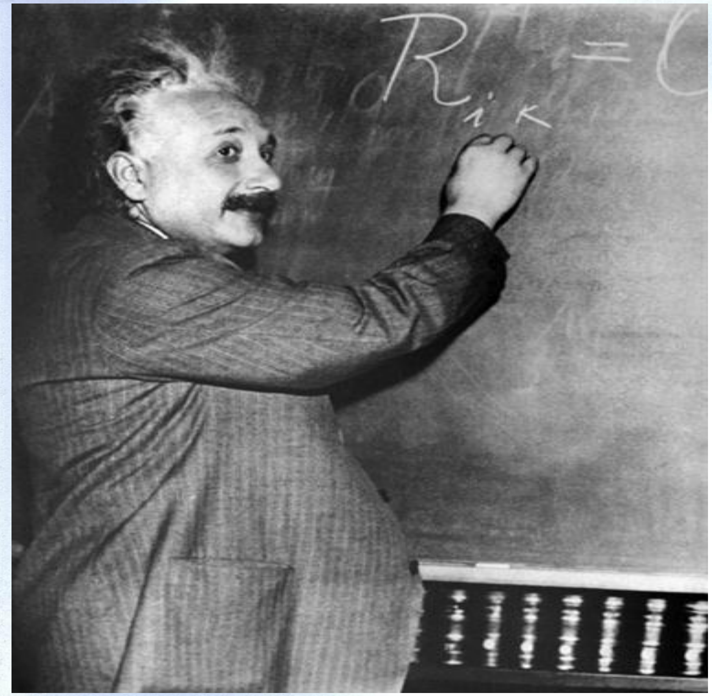
$$R_{\mu\nu} = -\frac{\partial}{\partial x_\alpha}\{\mu\nu,\alpha\} + \{\mu\alpha,\beta\}\{\nu\beta,\alpha\} + \frac{\partial^2 \log\sqrt{-g}}{\partial x_\mu \partial x_\nu} - \{\mu\nu,\alpha\}\frac{\partial \log\sqrt{-g}}{\partial x_\alpha} \qquad (13)$$

The system of equations (13) is by no means satisfied when we insert for the $g_{\mu\nu}$ the values given in (7), (8), and (12), and for the (contravariant) energy-tensor of matter the values indicated in (6). It will be shown in the next paragraph how this calculation may conveniently be made. So that, if it were certain that the field equations (13) which I have hitherto employed were the only ones compatible with the postulate of general relativity, we should probably have to conclude that the theory of relativity does not admit the hypothesis of a spatially finite universe.

However, the system of equations (14) allows a readily suggested extension which is compatible with the relativity postulate, and is perfectly analogous to the extension of Poisson's equation given by equation (2). For on the left-hand side of field equation (13) we may add the fundamental tensor $g_{\mu\nu}$, multiplied by a universal constant, $-\lambda$, at present unknown, without destroying the general covariance. In place of field equation (13) we write

$$R_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right) \qquad (13a)$$

This field equation, with λ sufficiently small, is in any case also compatible with the facts of experience derived from the solar system. It also satisfies laws of conservation of momentum and energy, because we arrive at (13a) in place of (13) by introducing into Hamilton's principle, instead of the scalar of Riemann's tensor, this scalar increased by a universal constant; and Hamilton's principle, of course, guarantees the validity of laws of conservation. It will be shown in §5 that field equation (13a) is compatible with our conjectures on field and matter.



$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2} \qquad (14)$$

are fulfilled.

Thus the newly introduced universal constant λ defines both the mean density of distribution ρ which can remain in equilibrium and also the radius R and the volume $2\pi^2 R^3$ of spherical space. The total mass M of the universe, according to our view, is finite, and is in fact

$$M = \rho \cdot 2\pi^2 R^3 = 4\pi^2 \frac{R}{\kappa} = \pi^2 (32/\kappa^3\rho)^{1/2} \qquad (15)$$

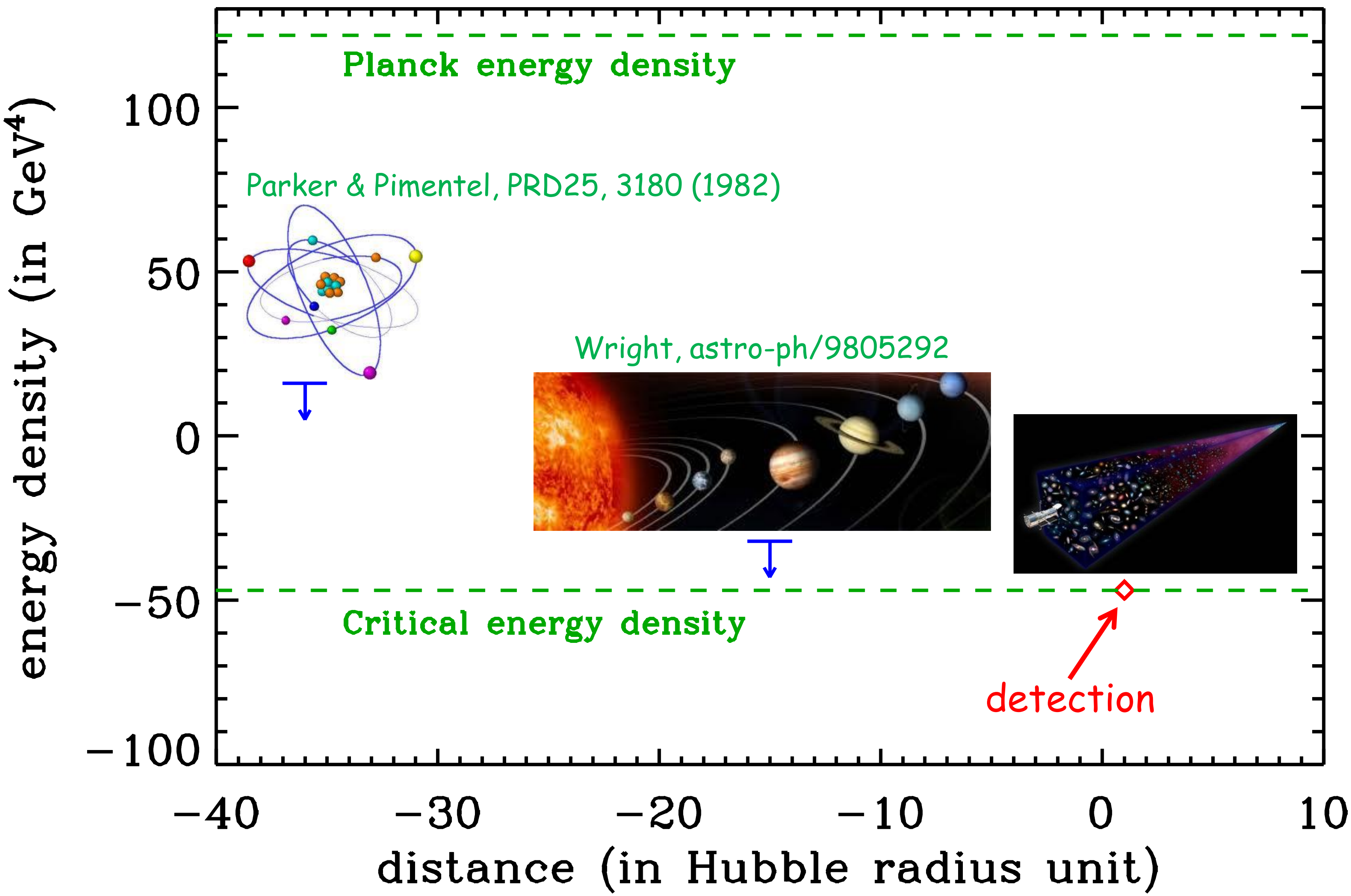
Thus the theoretical view of the actual universe, if it is in correspondence with our reasoning, is the following.

The cosmological constant (CC): introduction

In presence of a Cosmological Constant, the Einstein field equations read

$$\underbrace{R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}}_{\text{geometry}} + \underbrace{\Lambda_{\mathrm{B}} g_{\mu\nu}}_{\text{CC}} = \underbrace{\kappa T_{\mu\nu}}_{\text{matter}}$$

- Preserves covariance
- Covariant derivative vanishes hence compatible with a conserved energy momentum tensor
- Dimension length^ (-2)
- The CC can always been seen as an extra source of matter: $T_{\mu\nu} = -\frac{\Lambda_{\mathrm{B}}}{\kappa} g_{\mu\nu}$
- The equation of state of the CC is: $w = \frac{p}{\rho} = -1$. The effective pressure is negative.



Planck energy density
Parker & Pimentel, PRD25, 3180 (1982)
Wright, astro-ph/9805292
Critical energy density
detection
100
50
0
−50
−100
−40
−30
−20
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0
10
energy density (in GeV⁴)
distance (in Hubble radius unit)

In 1998, two groups measure the expansion of the Universe and claim detection of a non-vanishing CC.

The Astronomical Journal, 116:1009–1038, 1998 September

OBSERVATIONAL EVIDENCE FROM SUPERNOVAE FOR AN ACCELERATING UNIVERSE AND A COSMOLOGICAL CONSTANT

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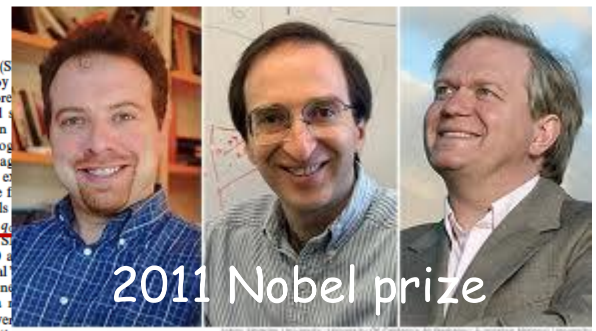
Received 1998 March 13; revised 1998 May 6

ABSTRACT

We present spectral and photometric observations of 10 Type Ia supernovae (S
range $0.16 \leq z \leq 0.62$. The luminosity distances of these objects are determined by
relations between SN Ia luminosity and light curve shape. Combined with pre
High-z Supernova Search Team and recent results by Riess et al., this expanded s
supernovae and a set of 34 nearby supernovae are used to place constraints on
logical parameters: the Hubble constant (H_0), the mass density (Ω_M), the cosmolog
vacuum energy density, Ω_Λ), the deceleration parameter (q_0), and the dynamical ag
The distances of the high-redshift SNe Ia are, on average, 10%–15% farther than ex
density ($\Omega_M = 0.2$) universe without a cosmological constant. Different light curve f
subsamples, and prior constraints unanimously favor eternally expanding models
logical constant (i.e., $\Omega_\Lambda > 0$) and a current acceleration of the expansion (i.e., q
constraint on mass density other than $\Omega_M \geq 0$, the spectroscopically confirmed S
consistent with $q_0 < 0$ at the 2.8 σ and 3.9 σ confidence levels, and with $\Omega_\Lambda > 0$ a
confidence levels, for two different fitting methods, respectively. Fixing a "minimal
0.2, results in the weakest detection, $\Omega_\Lambda > 0$ at the 3.0 σ confidence level from one
For a flat universe prior ($\Omega_M + \Omega_\Lambda = 1$), the spectroscopically confirmed SNe Ia r
and 9 σ formal statistical significance for the two different fitting methods. A univer
matter (i.e., $\Omega_M = 1$) is formally ruled out at the 7 σ to 8 σ confidence level for th
methods. We estimate the dynamical age of the universe to be 14.2 ± 1.7 Gyr includ
tainties in the current Cepheid distance scale. We estimate the likely effect of several sources of systematic error, including progenitor and metallicity evolution, extinction, sample selection bias, local perturbations in the expansion rate, gravitational lensing, and sample contamination. Presently, none of these effects appear to reconcile the data with $\Omega_\Lambda = 0$ and $q_0 \geq 0$.

Key words: cosmology: observations — supernovae: general



Johns Hopkins University; University Of California At Berkeley; Australian National University

From left, Adam Riess, Saul Perlmutter and Brian Schmidt shared the Nobel Prize in physics awarded Tuesday.

1. INTRODUCTION

This paper reports observations of 10 new high-redshift Type Ia supernovae (SNe Ia) and the values of the cosmological parameters derived from them. Together with the four high-redshift supernovae previously reported by our High-z Supernova Search Team (Schmidt et al. 1998; Garnavich et al. 1998a) and two others (Riess et al. 1998b), the sample of 16 is now large enough to yield interesting cosmological results of high statistical significance. Confidence in these results depends not on increasing the sample size but on improving our understanding of systematic uncertainties.

The time evolution of the cosmic scale factor depends on the composition of mass-energy in the universe. While the universe is known to contain a significant amount of ordinary matter, Ω_M, which decelerates the expansion, its dynamics may also be significantly affected by more exotic forms of energy. Preeminent among these is a possible energy of the vacuum (Ω_Λ), Einstein's "cosmological con-

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The Astrophysical Journal, 517:565–586, 1999 June 1

MEASUREMENTS OF Ω AND Λ FROM 42 HIGH-REDSHIFT SUPERNOVAE

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(The Supernova Cosmology Project)
Received 1998 September 8; accepted 1998 December 17

ABSTRACT

We report measurements of the mass density, Ω_M, and cosmological-constant energy density, Ω_Λ, of the universe based on the analysis of 42 type Ia supernovae discovered by the Supernova Cosmology Project. The magnitude-redshift data for these supernovae, at redshifts between 0.18 and 0.83, are fitted jointly with a set of supernovae from the Calán/Tololo Supernova Survey, at redshifts below 0.1, to yield values for the cosmological parameters. All supernova peak magnitudes are standardized using a SN Ia light-curve width-luminosity relation. The measurement yields a joint probability distribution of the cosmological parameters that is approximated by the relation $0.8\Omega_M - 0.6\Omega_\Lambda \approx -0.2 \pm 0.1$ in the region of interest ($\Omega_M \lesssim 1.5$). For a flat ($\Omega_M + \Omega_\Lambda = 1$) cosmology we find $\Omega_M^{\rm flat} = 0.28^{+0.09}_{-0.08}$ (1 σ statistical) $^{+0.05}_{-0.04}$ (identified systematics). The data are strongly inconsistent with a $\Lambda = 0$ flat cosmology, the simplest inflationary universe model. An open, $\Lambda = 0$ cosmology also does not fit the data well: the data indicate that the cosmological constant is nonzero and positive, with a confidence of $P(\Lambda > 0) = 99\%$, including the identified systematic uncertainties. The best-fit age of the universe relative to the Hubble time is $t_0^{\rm flat} = 14.9^{+1.4}_{-1.1}(0.63/h)$ Gyr for a flat cosmology. The size of our sample allows us to perform a variety of statistical tests to check for possible systematic errors and biases. We find no significant differences in either the host reddening distribution or Malmquist bias between the low-redshift Calán/Tololo sample and our high-redshift sample. Excluding those few supernovae that are outliers in color excess or fit residual does not significantly change the results. The conclusions are also robust whether or not a width-luminosity relation is used to standardize the supernova peak magnitudes. We discuss and constrain, where possible, hypothetical alternatives to a cosmological constant.

Subject headings: cosmology: observations — distance scale — supernovae: general

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- If the Universe is homogeneous and isotropic and if gravity is described by GR and if there is no other exotic fluid then the CC is non-vanishing.

$$\begin{aligned}\rho_\Lambda &\sim 10^{-47}{\rm GeV}^4 \sim \rho_{\rm cri} \\ &\sim \left(10^{-3}{\rm eV}\right)^4\end{aligned}$$

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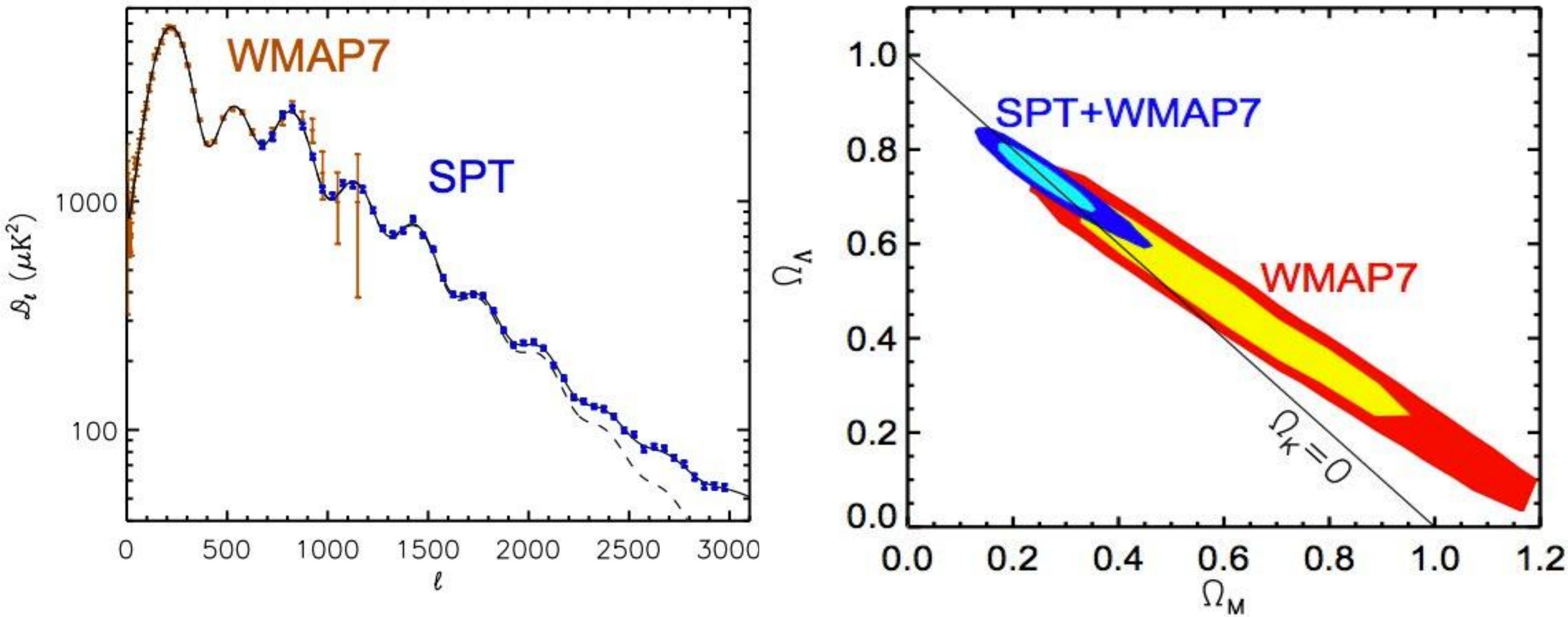
- 2012: there is now a bunch of different and independent measurements pointing towards this conclusion (age of the universe, SNIa, clusters abundance, lensing etc …)

Example: using the CMB only, a vanishing CC now seems to be ruled out at more than 5 sigma …



SPT data, arXiv:1210.7231

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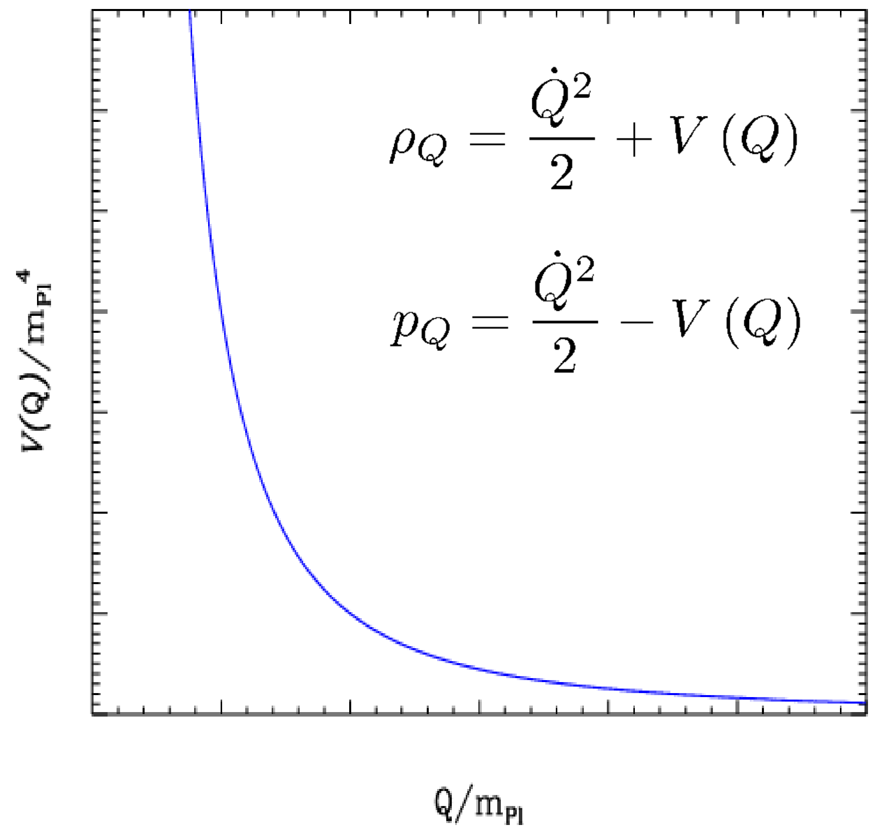
- The other alternatives (in-homogeneous universe, modified gravity, quintessence etc …) have their own problems.

A possible alternative is that there is no CC but a scalar field ("quintessence") playing the role of a "dark energy".

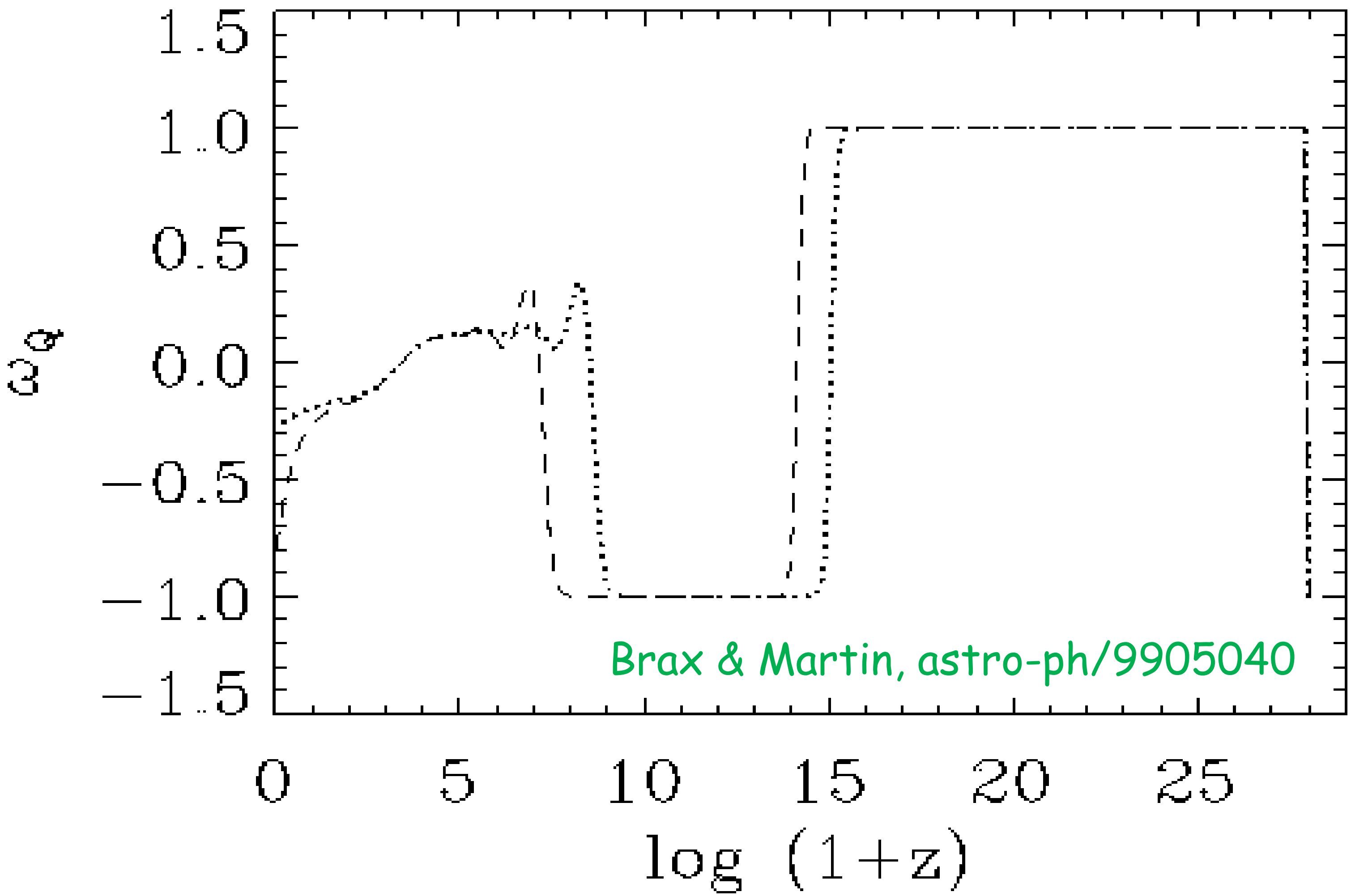
$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\rho_{\mathrm{CDM}} + \rho_{\mathrm{DE}}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\mathrm{Pl}}^2}\left(\rho_{\mathrm{CDM}} + \underbrace{\rho_{\mathrm{DE}} + 3p_{\mathrm{DE}}}_{\text{must be <0}}\right)$$

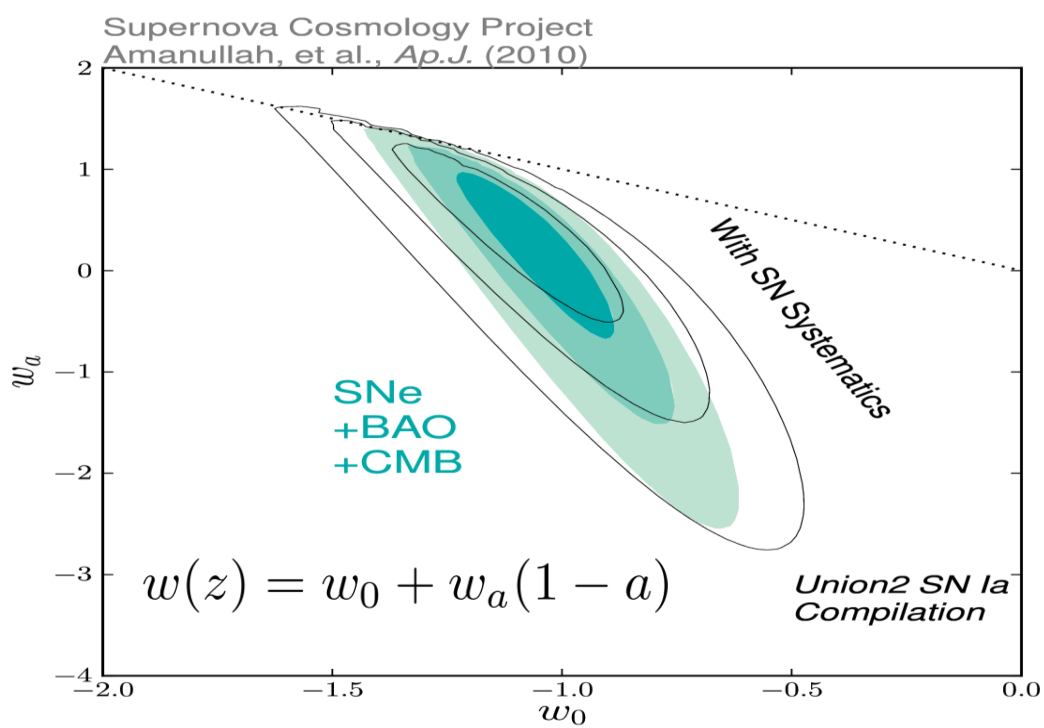


Ratra & Peebles, PRD37 3406 (1988)

In these models, dark energy is dynamical and the equation of state is a time-dependent quantity. Falsifiable since different from the CC



- Hard to find good models of particle physics which lead to the correct potentials
- Hard to control the interactions of quintessence with the other fields
- Hard not to destroy the flatness of the potential by quantum corrections
- Everything seems to indicate that w=-1 …



- The hard fact is that the following equation does not fit well the data

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\rho_{\rm CDM}$$

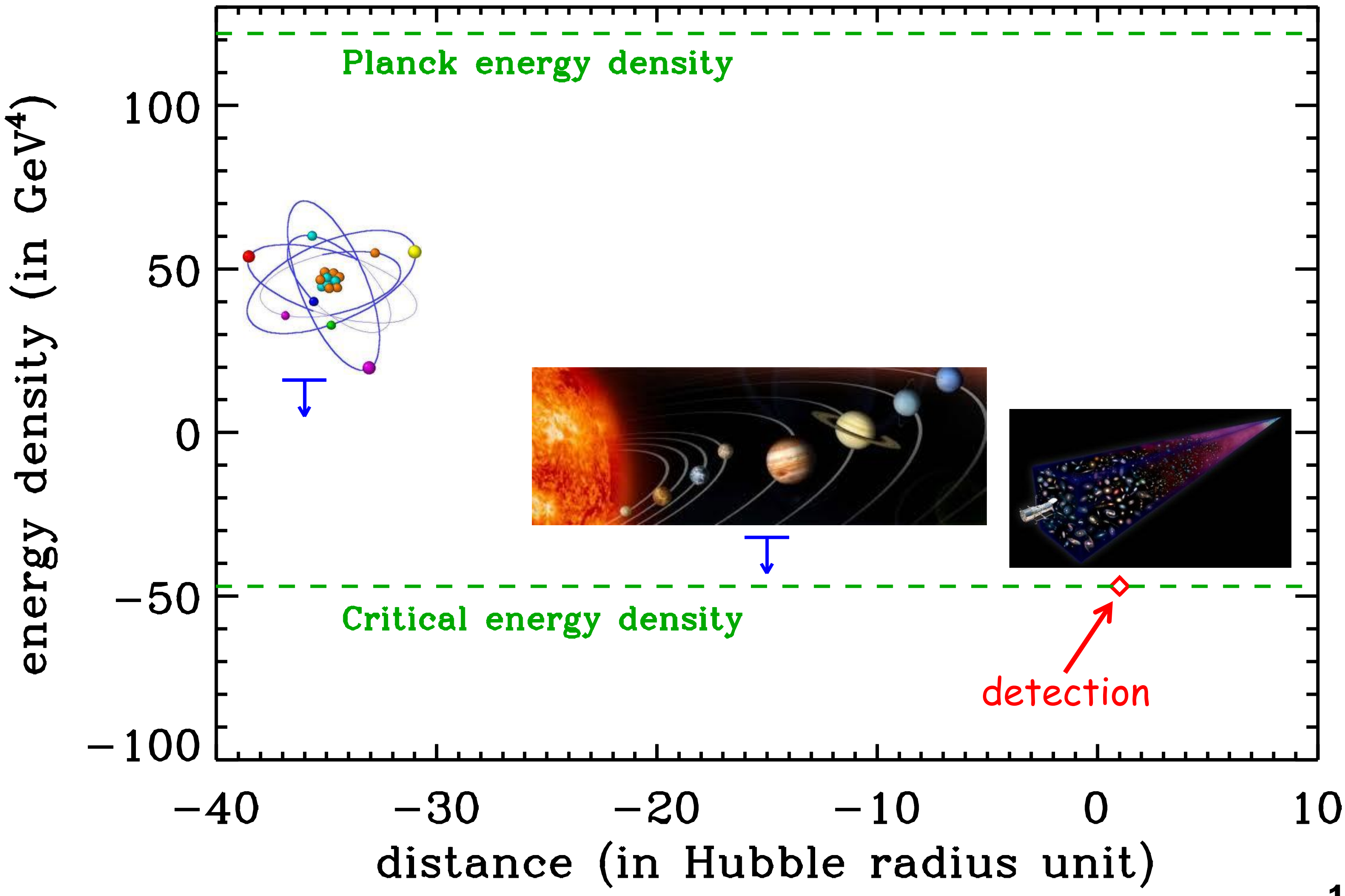
- If the Universe is homogeneous and isotropic and if gravity is described by GR and if there is no other exotic fluid then the CC is non-vanishing.

- In this framework, the Universe is accelerating.

- 2012: there is now a bunch of different and independent measurements pointing towards this conclusion. (age of the universe, SNIa, clusters abundance, lensing etc …)

- The other alternatives (in-homogeneous universe, modified gravity, quintessence etc …) have their own problems.

- Even if what we see in cosmology is not the CC, this implies a new upper limit on the CC energy density


Planck energy density
Critical energy density
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energy density (in GeV⁴)
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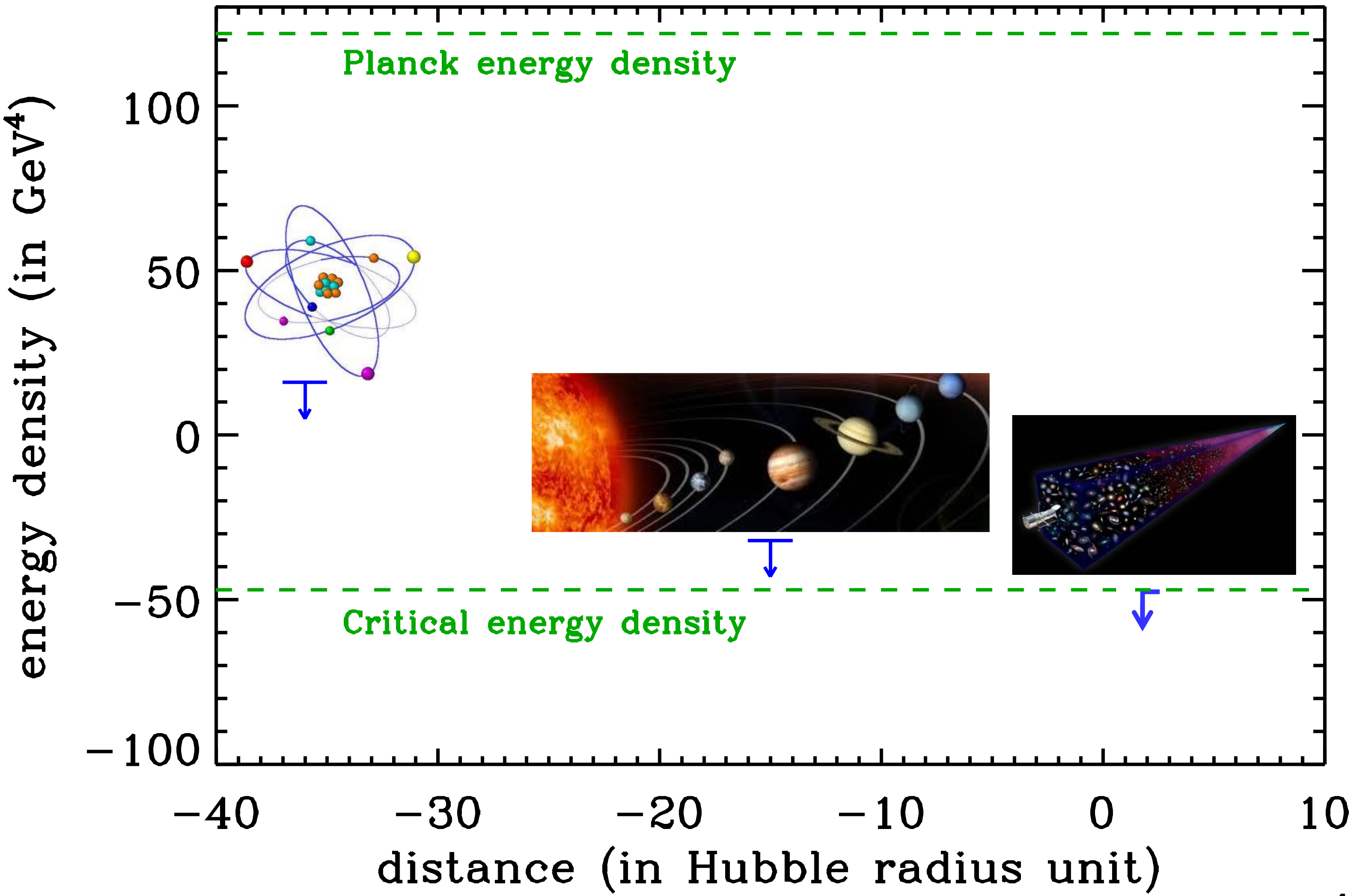
distance (in Hubble radius unit)


Planck energy density
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energy density (in GeV^4)
distance (in Hubble radius unit)

- Therefore, the CC remains the simplest explanation of the different cosmological measurements

- There is no sign in the observations that we need a dark energy different from the CC

- At this (classical) level, we have a theory with a new fundamental constant and its value has been determined by the measurements to be

$$\Lambda_{\rm B} \simeq 10^{-52} m^{-2}$$

- The CC is such that it is very difficult to check this value elsewhere than in cosmology … always a negligible effect.

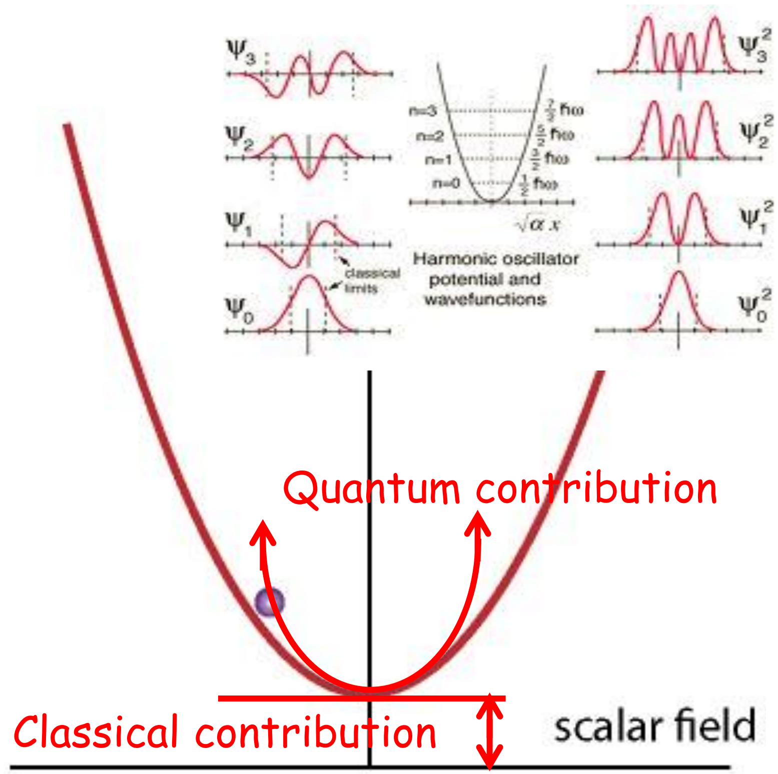
When QM and QFT are taken into account, the nature of the discussion is however drastically modified [A. Sakharov, Sov. Phys. Dokl. 12, 1040 (1968)]

- The vacuum state has the following stress-energy tensor

$$\langle T_{\mu\nu}\rangle = -\left[V\left(\phi_{\rm min}\right)+\rho_{\rm vac}\right]g_{\mu\nu}$$

$$\downarrow$$

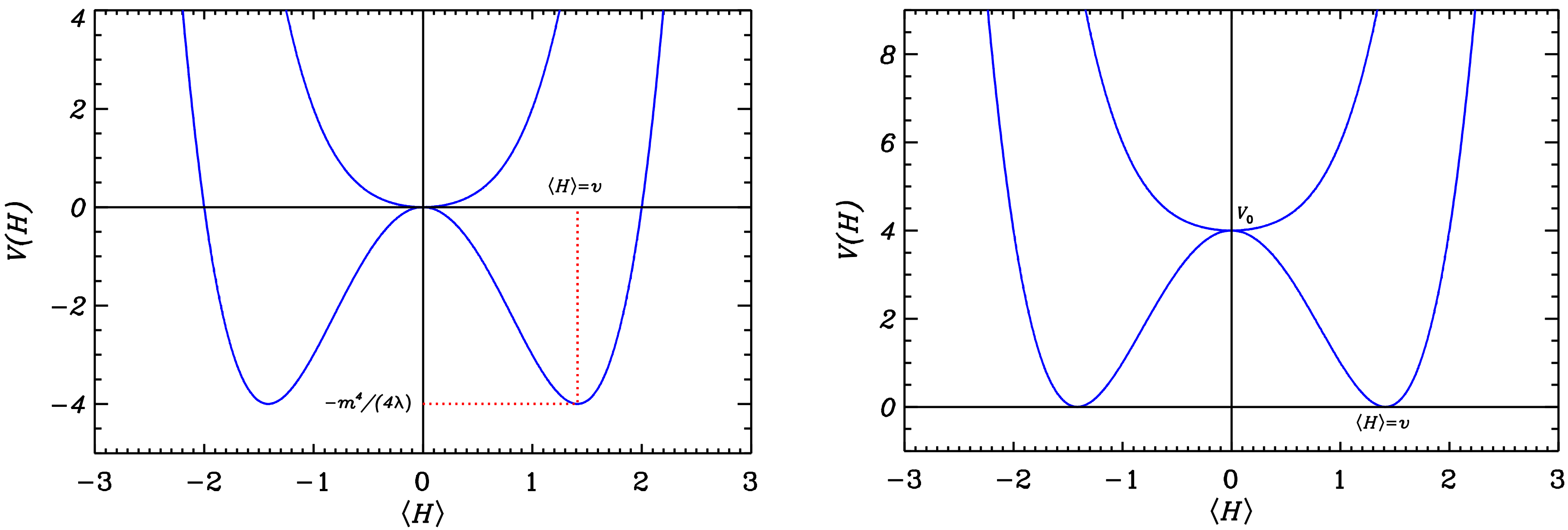
$$\Lambda_{\rm eff} = \Lambda_{\rm B} + \frac{1}{M_{\rm Pl}^2}\left[V\left(\phi_{\rm min}\right)+\rho_{\rm vac}\right]$$



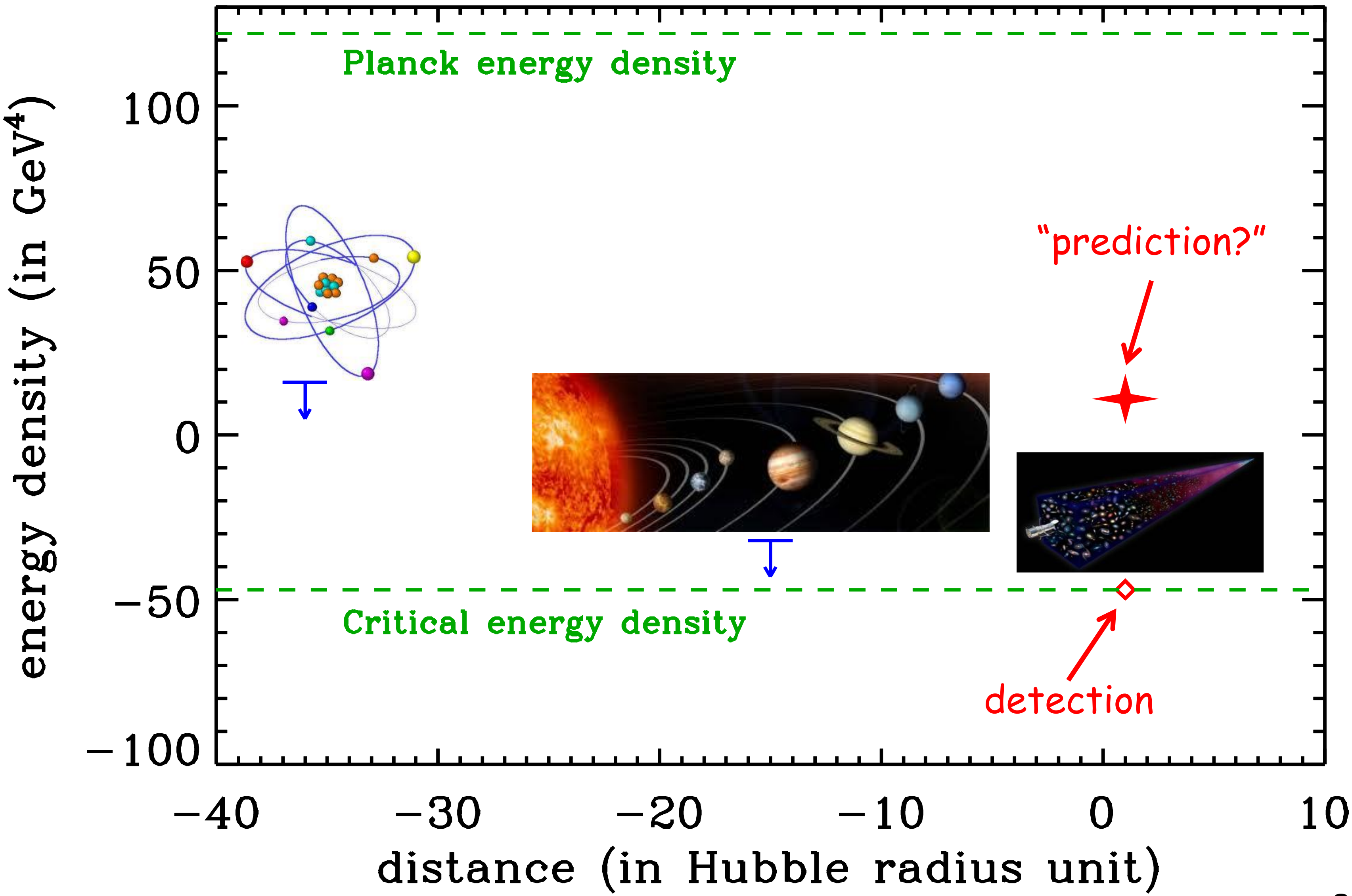
- In flat spacetime, only differences of energy are measurable so not important … In curved spacetime, the absolute value is important.

- A priori, the vacuum fluctuations gravitate as any other form of energy

An example is the Electro-Weak transition



$$V\left(\phi_{\min}\right) = -\frac{1}{4} m_{_\mathrm{H}}^2 v^2 = -\frac{\sqrt{2}}{16}\frac{m_{_\mathrm{H}}^2}{G_{_\mathrm{F}}^2} \simeq -1.2\times 10^8\,\mathrm{GeV}^4$$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"prediction?"
Critical energy density
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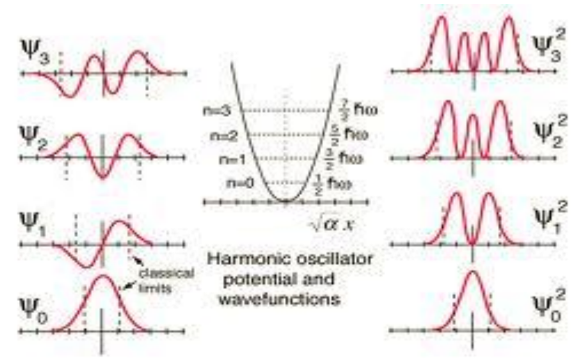
10
distance (in Hubble radius unit)

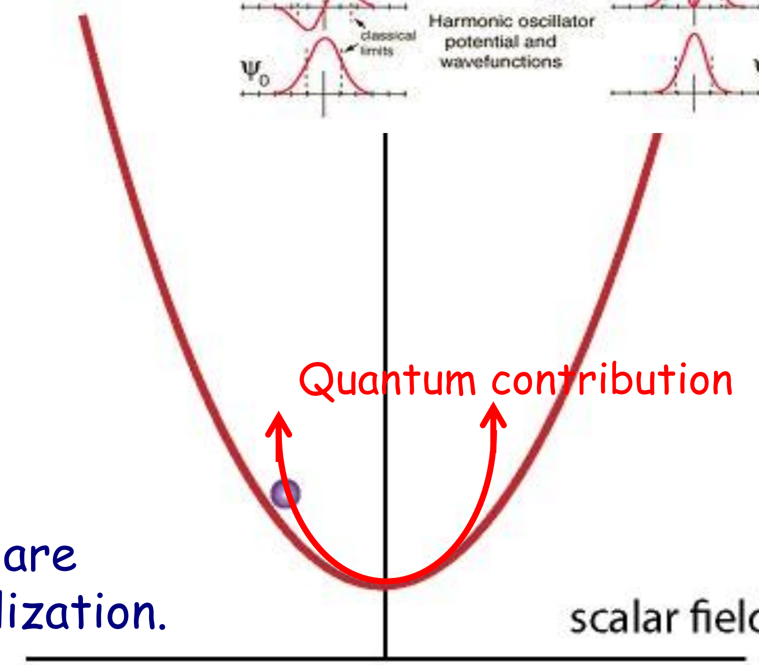
- Because of Heisenberg principle the position and the velocity of a quantum harmonic oscillator cannot vanish at the same time

$$\langle H \rangle = \frac{\hbar\omega}{2}$$



- A quantum field=infinite collections of quantum oscillators

$$\langle H \rangle = \sum \frac{\hbar\omega}{2} = \infty$$



- This should not cause any panic since we are used to tame infinities in QFT: renormalization.

- However, this particular type of infinity is usually not renormalized but ignored on the basis that, in flat spacetime, only differences of energies are measurable.

The first attempt to estimate the gravitational impact of vacuum fluctuations was done by W. Pauli [see "Die allgemeinen Principein des Wellenmechanik"]

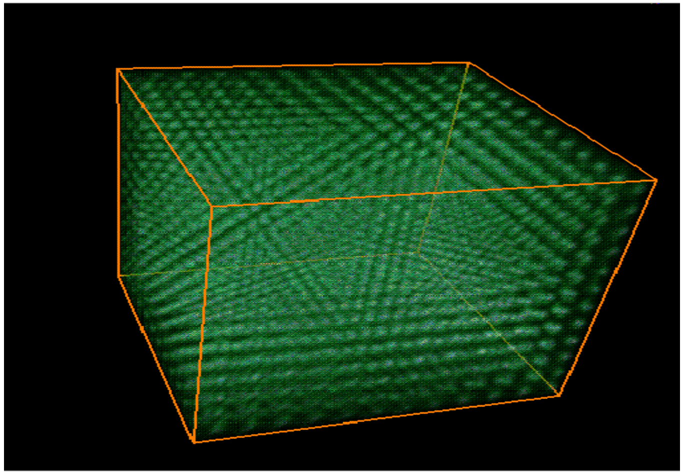
$$H^2 + \frac{k}{a^2} = \frac{1}{3M_{\rm Pl}^2}\rho + \frac{\Lambda}{3}$$

$$-\left(2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2}\right) = \frac{1}{M_{\rm Pl}^2}p - \Lambda$$

Einstein static universe

$$a^2 = \frac{2M_{\rm Pl}^2}{\rho} \qquad a \simeq 31 \text{ kms}$$

$$\rho = \frac{1}{8\pi^2}\omega_{\rm max}^4$$



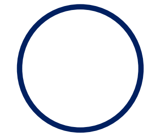
Radiation field in a box

"it could not even reach to the moon"

The cosmological constant & QFT

In a modern language, the main issue is how to renormalize the vacuum energy density

$$\Lambda_{\rm eff} = \Lambda_{\rm B} + \frac{1}{M_{\rm Pl}^2}\frac{1}{(2\pi)^3}\int {\rm d}^3\mathbf{k}\frac{1}{2}\omega(\mathbf{k}) = \Lambda_{\rm B} - \frac{m^2}{4M_{\rm Pl}^2}$$



- The vacuum contribution is expressed in terms of Feynman bubble diagrams, ie diagrams with no external leg.
- These diagrams have bad convergence properties, worst than ordinary loop diagrams: they remain infinite even in the QM limit.
- In non-gravitational physics, these graphs always cancel out.
- When gravity is taken into account, one must regularize them.

Regularizing the bubble graphs …

$$\langle \rho \rangle = \frac{1}{2}\frac{1}{(2\pi)^3}\int \mathrm{d}^3\mathbf{k}\,\omega(\mathbf{k}) = \frac{1}{4\pi^2}\int_0^{\infty}\mathrm{d}k\,k^2\sqrt{k^2+m^2}$$

$$\langle p \rangle = \frac{1}{6}\frac{1}{(2\pi)^3}\int \mathrm{d}^3\mathbf{k}\frac{k^2}{\omega(\mathbf{k})} = \frac{1}{3}\frac{1}{4\pi^2}\int_0^{+\infty}\mathrm{d}k\frac{k^4}{\sqrt{k^2+m^2}}$$

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$$\langle p\rangle = \frac{1}{6}\frac{1}{(2\pi)^3}\int \mathrm{d}^3\mathbf{k}\frac{k^2}{\omega(\mathbf{k})} = \frac{1}{3}\frac{1}{4\pi^2}\int_0^{+\infty}\mathrm{d}k\frac{k^4}{\sqrt{k^2+m^2}}$$

Introducing a cut-off breaks Lorentz invariance and leads to a wrong equation of state

$$\left.\begin{aligned}\langle\rho\rangle &= \frac{1}{4\pi^2}\int_0^{\mathbf{M}}\mathrm{d}k\,k^2\sqrt{k^2+m^2} \simeq \frac{M^4}{16\pi^2}\\ \langle p\rangle &= \frac{1}{3}\frac{1}{4\pi^2}\int_0^{\mathbf{M}}\mathrm{d}k\frac{k^4}{\sqrt{k^2+m^2}} \simeq \frac{1}{3}\frac{M^4}{16\pi^2}\end{aligned}\right\}\qquad p=\rho/3$$

Regularizing the bubble graphs

$$\langle\rho\rangle = \frac{1}{2}\frac{1}{(2\pi)^3}\int \mathrm{d}^3\mathbf{k}\,\omega(\mathbf{k}) = \frac{1}{4\pi^2}\int_0^{\infty}\mathrm{d}k\,k^2\sqrt{k^2+m^2}$$

$$\langle p\rangle = \frac{1}{6}\frac{1}{(2\pi)^3}\int \mathrm{d}^3\mathbf{k}\frac{k^2}{\omega(\mathbf{k})} = \frac{1}{3}\frac{1}{4\pi^2}\int_0^{+\infty}\mathrm{d}k\frac{k^4}{\sqrt{k^2+m^2}}$$

Lorentz invariant methods (i.e. dimensional regularization) leads to the correction equation of state and

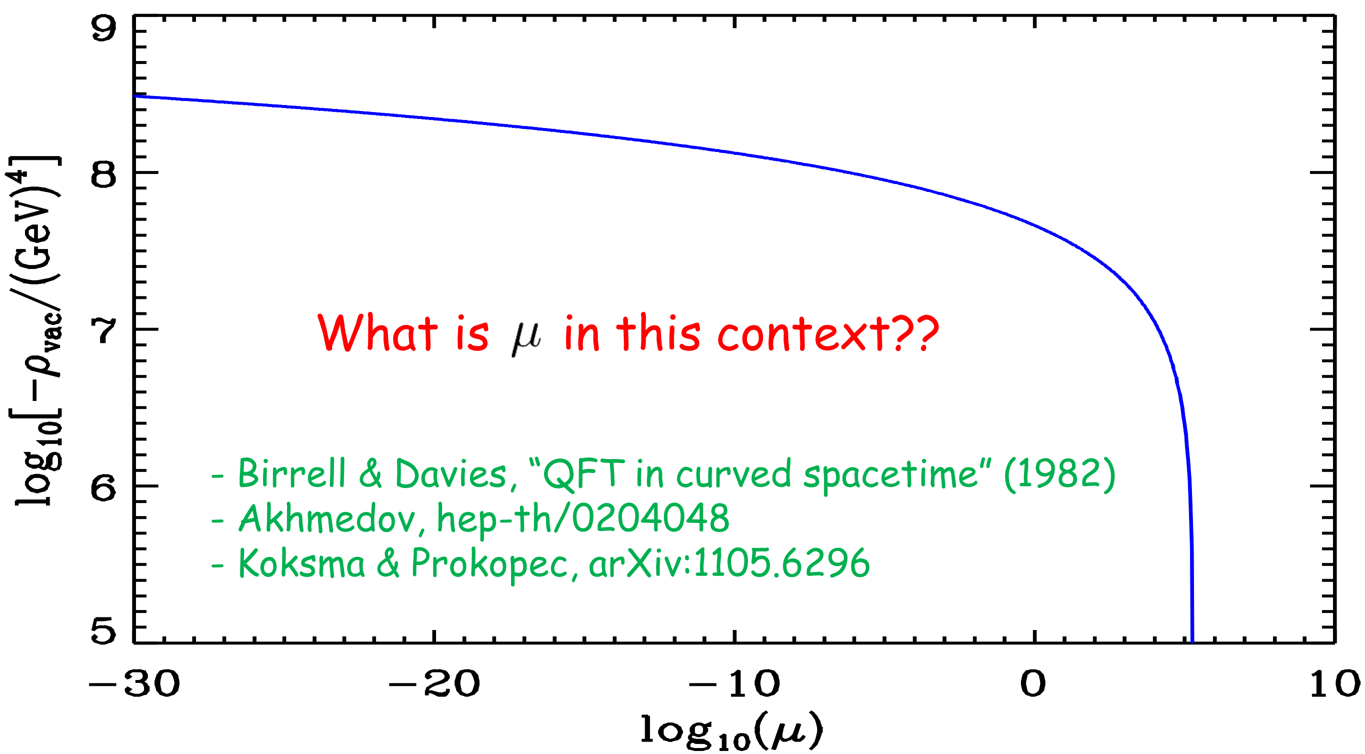
$$\langle\rho\rangle = \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{\mu^2}\right)$$

- The value of the CC is the observed value at the renormalization point

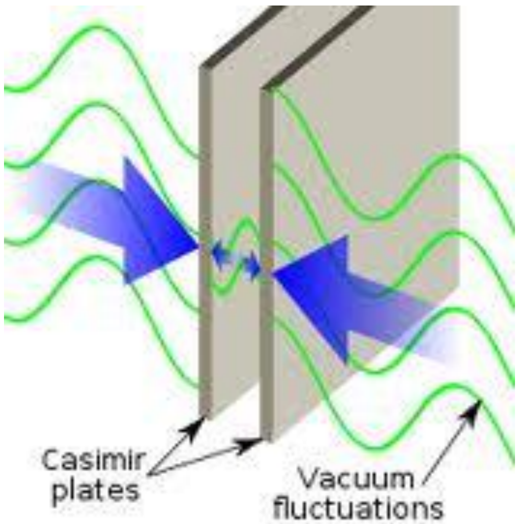
$$\Lambda_{\rm eff} = \Lambda_{\rm obs} \qquad \text{at} \qquad \mu = \mu_{\rm obs}$$

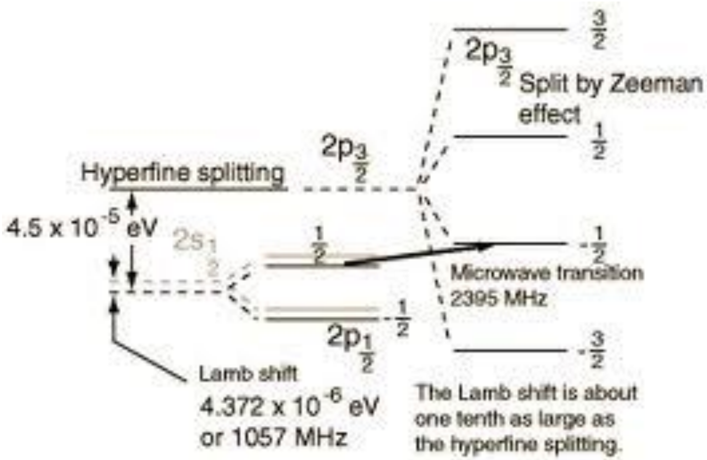
- Then, the CC becomes a “running quantity” with

$$\rho_{\rm vac} = \rho_{\rm B} + \rho_{\rm vac}^{\rm EW} + \rho_{\rm vac}^{\rm QCD} + \sum_i n_i \frac{m_i^4}{64\pi^2} \ln\left(\frac{m_i^2}{\mu^2}\right)$$



- A possible loophole is that vacuum fluctuations are just an artifact of QFT. However, we observe their influence in the Casimir effect or in the Lamb shift effect.





The cosmological constant: possible loopholes

- Maybe vacuum fluctuations have abnormal gravitational properties?? But vacuum fluctuations participate for a non-negligible amount to the mass of nuclei … and they are observed to obey the UFF (WEP).

$$m_{\rm i} = \sum_X m_{\rm i}^{(X)} + \frac{1}{c^2} \sum_{X \neq Y} E^{(X)} + \frac{1}{c^2} E^{(Y)}$$

Lamb shift in the nucleus

$$E^{(Y)} = E^{(Y)}_{\text{"normal"}} + \eta_Y E^{(Y)}_{\text{"normal"}}$$

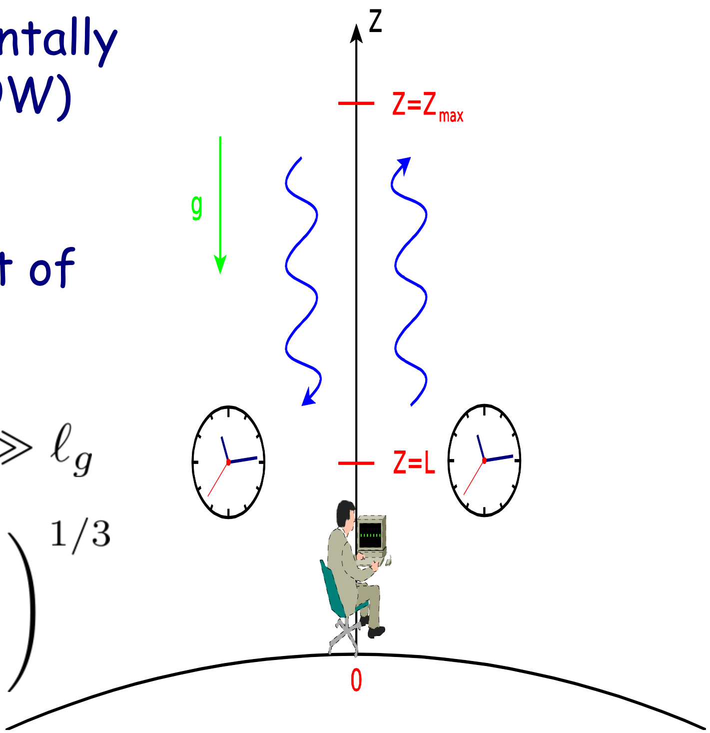
- This implies a violation of UFF which is not seen ….

$$m_{\rm g} = m_{\rm i} - \eta_Y E^{(Y)}_{\text{"normal}}$$

The UFF in QM is described by the following Schrodinger equation

$$i\hbar\frac{\partial\Psi(t,z)}{\partial t} = -\frac{\hbar^2}{2m_{\rm ini}}\frac{\partial^2\Psi(t,z)}{\partial z^2} + m_{\rm grav} g z \Psi(t,z)$$

- The validity of this equation has been experimentally checked by the Collela Overhausser Werner (COW) experiment and by atomic interferometry.
- UFF can be checked by measuring times of flight of quantum particles.
- The classical result is recovered if $|L - z_{\rm max}| \gg \ell_g$



$$\ell_g = \left(\frac{\hbar^2}{2m_{\rm ini}m_{\rm grav}g}\right)^{1/3} = \left(\frac{\hbar^2 r_\oplus^2}{2m_{\rm ini}m_{\rm grav}GM_\oplus}\right)^{1/3}$$

One gram particle: $\ell_g = 10^{-10}\,\mathrm{m}$

Neutron: $\ell_g = 1.5\,\mathrm{mm}$

P. Davies, CQG 21 5677 (2004)

Conclusions:

- The cosmological constant problem is the impossibility to reconcile the renormalized value of vacuum energy with its observed value in cosmology and/or with the upper contraints obtained in others experimental situations.

- It is then natural to question the assumptions made to arrive at this result: failure of our renormalization technique, vacuum fluctuations=fake , abnormal gravitational properties of the vacuum etc …

- However, investigating these issues does not seem to reveal any inconsistencies (at the theoretical/observational level).

- It is frustrating that cosmology be the only situation where one can measure (and not only constrain) the CC!

- The CC problem is a deep problem since it lies at the crossroads between gravity and QM. In brief, the question is: what are the gravitational properties of the quantum vacuum?