Stabilization of the electroweak vacuum by a scalar threshold effect

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Recent next-to-next-to-leading-order studies have shown that, for the current experimental values of the Higgs mass, top quark mass and strong coupling, the Standard Model (SM) scalar potential has a second minimum, deeper than the electroweak one. I will review a simple and efficient mechanism to stabilise the electroweak vacuum. The mechanism involves an extra scalar singlet and can be easily at work in existing beyond the SM scenarios like the see-saw neutrino mechanism, invisible axion and unitarized Higgs inflation models.

1 Introduction

The Higgs sector of the Standard Model (SM) is the part of the theory that has been less confronted with the experiments. Interestingly, some of the most prominent theoretical problems arise from the Higgs sector. In particular, the measured mass $M_h \approx 125$ GeV of the Higgs particle recently discovered, by the experiments ATLAS [1] and CMS [2], is rather intriguing from the point of view of the stability of the electroweak (EW) vacuum of the Standard Model (SM). The stability of the EW vacuum is very sensitive to the Higgs mass m_h, the top mass M_t and the strong coupling α_s, in order of decreasing sensitivity.

The state-of-the-art analyses of the SM vacuum stability after the latest LHC data is done at the nex-to-next-to-leading order level [3]. The main result of such a study [3] is that for the current central experimental values of the relevant SM experimental inputs (m_h, M_t, gauge couplings and the Fermi constant G_F), the EW vacuum is unstable: there is a minimum deeper than the EW one at large field values $\phi \sim 10^{11}$ GeV.

Although the Higgs vev at the EW scale ($\Lambda_{EW} \approx 246$GeV) is sitting in an unstable minimum, this is not necessarily a problem; the reason is at least two-fold. Firstly, the SM is likely to be embedded in a more fundamental theory not far from the ∼TeV scale. The infamous EW scale fine-tuning problem is the main motivation for such new physics at the ∼TeV scale. And secondly, most probably the vacuum is not *very unstable*: the age of the universe Δt over the vacuum's life time τ is really small, $\Delta t/\tau \sim 10^{-90\pm 10}$. This is because the Higgs quartic coupling λ becomes negative along its Renormalization Group (RG) flow, but its absolute value remains small, $\lambda(\mu) \gtrsim -0.01$.

However, although there is not any physical problem with an unstable vacuum that is sufficiently long lived, we wold like to understand how it might be stabilised in different extensions of the SM which have different motivations than the stability issue. Furthermore, it can be desirable to cure the Higgs instability at large field values to avoid:

- cosmological constrains,

- an upper bound for the supersymmetric SM matching scale [3,4,5],
- constraints in the upper bound of see-saw neutrino partners [6,7],
- tension between models of Higgs inflation and the scale of inflation [8],

There are many *ad hoc* possibilities to modify the Higgs potential and rise the instability scale. For instance, additional bosonic fields change the behaviour of the Higgs quartic coupling in such a way that it tends to be greater when it is evolved from low to high energies (for recent analyses, see [9,10] and references therein). In reference [8] a simple and robust mechanism to stabilise the EW vacuum was proposed. It is based on a tree-level scalar threshold effect which can be easily at work in generalisations of the SM that have independent motivations from the instability of the EW vacuum.

In this note, we will review the main results presented in [8], that is: we will explain the aforementioned mechanism to stabilise the EW vacuum and provide examples of beyond the SM scenarios in which the mechanism can be operative.

2 Stabilising the Higgs by a scalar threshold effect

To explore the impact of an additional singlet scalar on the stability of the Higgs potential, we consider a tree-level scalar potential of the form

$$V_0 = \lambda_H \left(|H|^2 - v^2/2\right)^2 + \lambda_S \left(|S|^2 - w^2/2\right)^2 + \lambda_{HS} \left(|H|^2 - v^2/2\right) \left(|S|^2 - w^2/2\right) \ . \quad (1)$$

Here H is the Higgs doublet and S is a complex scalar field. V_0 is the most general dimension-four potential that respects a global Abelian symmetry under which only S is charged. Although we will consider here a single complex scalar, most of our conclusions remain valid also in the case of multi-Higgs doublets or real singlet fields (with a Z2 parity replacing the abelian symmetry).

The analysis will be performed at leading order: one-loop improved effective potential, i.e. tree-level potential with its couplings evaluated at $\mu \sim \phi$ by means of the one-loop beta functions, plus tree level matching conditions. By ϕ we denote the scalar field background, in some direction of the $H - S$ field space. Working at leading order is sufficient to show the points we are interested in and it has the advantage that we will be able to show analytical and simple expressions.

For λ_H, $\lambda_S > 0$ and $\lambda_{HS}^2 < \lambda_H \lambda_S$, the minimum of V_0 is at

$$\langle H^\dagger H \rangle = v^2/2 \ , \quad \langle S^\dagger S \rangle = w^2/2 \ , \quad (2)$$

and the potential is normalised such that at its minimum $V_0|_{min} = 0$. A nonzero vacuum expectation value (vev) of S, which is crucial for the mechanism to work, spontaneously breaks the global symmetry, giving rise to a potentially dangerous Goldstone boson. Gauging the symmetry of S or explicitly breaking it by small terms in V_0 can be used to evade these problems, but does not conceptually modify our results. For simplicity, we restrict our considerations to the potential in Eq. 1, but generalisations are straightforward.

At the minimum, the mass matrix is:

$$\mathcal{M}^2 = 2 \begin{pmatrix} \lambda_H v^2 & \lambda_{HS} v w \\ \lambda_{HS} v w & \lambda_S w^2 \end{pmatrix} \ ; \quad (3)$$

its eigenvalues are:

$$\begin{aligned} m_h^2 &= 2v^2 \left(\lambda_H - \frac{\lambda_{HS}^2}{\lambda_S}\right) + \mathcal{O}\left(\frac{v^4}{w^2}\right) , \\ M_s^2 &= 2\lambda_S w^2 + 2(\lambda_{HS}^2/\lambda_S) v^2 + \mathcal{O}\left(\frac{v^4}{w^2}\right) \ . \end{aligned} \quad (4)$$

We will consider $w^2 \gg v^2$ such that, at tree level, $m_h \sim v \sim \Lambda_{EW}$ and $M_s \sim w$.

The presence of the new scalar field S modifies the analysis of the stability conditions of the Higgs potential. As already mentioned in the introduction, the presence of new bosonic degrees of freedom change the running of the Higgs quartic coupling, above the mass scale of the new bosons, in such a way that the potential tends to be *more stable*. This is so because loops of scalars contribute positively to the beta function of the Higgs quartic coupling. These are the one-loop beta function of the quartic couplings of the Lagrangian defined in Eq. 1, above the mass scale of S:

$$\begin{aligned} 16\pi^2 \frac{\lambda_H}{d\ln\mu} &= \left(12y_t^2 - 3g'^2 - 9g^2\right)\lambda_H - 6y_t^4 + \frac{3}{8}\left[2g^4 + (g'^2+g^2)^2\right] + 24\lambda_H^2 + 4\lambda_{HS}^2 \ , \\ 16\pi^2 \frac{\lambda_S}{d\ln\mu} &= \frac{1}{2}\left(12y_t^2 - 3g'^2 - 9g^2\right)\lambda_{HS} + 4\lambda_{HS}\left(3\lambda_H + 2\lambda_S\right) + 8\lambda_{HS}^2 \ , \\ 16\pi^2 \frac{\lambda_{HS}}{d\ln\mu} &= 8\lambda_{HS}^2 + 20\lambda_S^2 \ . \end{aligned} \tag{5}$$

If the singlet mass M_S is below the SM instability scale Λ_I and $(\frac{\lambda_{HS}}{4\pi})^2 \ln(\Lambda_I/M_S)$ is large enough, the positive contribution of λ_{HS} to the RGE equation for λ_H can prevent it from becoming negative.

2.1 Threshold effect

Besides the loop contribution to the running of $\lambda(\mu)$ discussed above, there is a related tree-level effect through which the new singlet can affect the stability bound. Let us consider the limit in which M_S is much larger than the Higgs mass ($w^2 \gg v^2$). Then, we can integrate out the field S at the scale M_S. At the order of precision we are working this is equivalent to substituting S in Eq. 1 by its equations of motions. Making such substitution we obtain the effective potential, below the scale M_S,

$$V_{eff} = \lambda\left(|H|^2 - v^2/2\right) \tag{6}$$

and the matching condition, imposed at the scale M_S,

$$\lambda = \lambda_H - \frac{\lambda_{HS}^2}{\lambda_S} \ . \tag{7}$$

The effective potential Eq. 6 is nothing but the SM Higgs potential. As we will make clear in the following pages, the crucial point is that the matching condition of the Higgs quartic coupling corresponds to a positive shift as it is evolved from low to high energies. And, this effect does not decouple: it is independent of the mass M_S.

In the low energy theory (Eq. 6), below M_S, the condition to ensure stability of the EW vacuum is the same as in the SM. The instability in the SM appears for $\Lambda_I^2 \gg v^2$. Therefore, at the level of precision we are working the stability condition takes the simple form $\lambda(\phi) > 0$, for $v \ll \phi < M_S$.

Naively, as the tree-level shift $\delta\lambda$ corresponds to a larger Higgs quartic coupling above M_S, the chances of keeping it positive seem improved. However, as we show below, the tree-level conditions for stability change from $\lambda > 0$ in the effective theory below M_S to $\lambda_H > \delta\lambda$ in the full theory above M_S. Thus, it appears that the threshold correction $\delta\lambda$ does not help stability at all. To understand what happens, one has to reexamine the stability conditions more carefully. First of all, recall that the tree-level potential V_0 in Eq. 1 is a good approximation to the full potential if we evaluate couplings and masses (collectively denoted by λ_i below) at a renormalization scale of the order of the field values of interest. Once we express the scalar potential as $V_0[\lambda_i(\mu = \varphi), \varphi]$, potentially large logarithms of the form $\ln m_i(\varphi)/\mu$ (where

$m_i(\varphi) \sim \varphi$ is a typical field-dependent mass) are kept small. Roughly speaking, this means that V_0 with a fixed μ_c will be reliable as long as one examines $\varphi \sim \mu_c$ and restricts field excursions to $|\varphi - \mu_c| < \mu_c e^{8\pi^2\lambda_0/\lambda_1^2}$ (where λ_0 denotes a coupling in the tree-level potential and λ_1 a coupling affecting the radiative corrections, e.g. the top Yukawa coupling squared). By adjusting $\mu \sim \varphi$ one can evaluate reliably the potential at all field values, but the previous estimate tells us when we can use $V_0[\lambda_i(\mu_c), \varphi]$, which has a simpler field dependence.

In the full theory, Eq. 1, above the M_S scale, the first two obvious stability conditions are

$$\lambda_H > 0 \quad \text{and} \quad \lambda_S > 0, \tag{8}$$

to avoid unbounded from below directions for large field values, along the H and S directions. Then, for the mixing term, we can distinguish two cases: $\lambda_{HS} > 0$ or $\lambda_{HS} < 0$. Now we discuss each case in turn [a].

- **Case $\lambda_{\mathbf{HS}} > 0$**

  In this case, V_0 can become negative only when $|S| < w/\sqrt{2}$ (neglecting corrections proportional to v). In this situation, the most dangerous field configuration is well approximated by setting $S = 0$ in Eq. 1, such that

$$V_0(H, 0) \approx \lambda_H |H|^4 - \frac{\lambda_{HS}}{2\lambda_S} M_S^2 |H|^2 + \frac{M_S^4}{16\lambda_S} \ . \tag{9}$$

  The extra stability condition ($V_0 > 0$) is then

$$\lambda_{HS}^2(\mu) < \lambda_H(\mu)\lambda_S(\mu) \ . \tag{10}$$

  Note that this can be rewritten as $\lambda_H > \delta\lambda = \lambda_{HS}^2/\lambda_S$ and ensures that the light scalar state does not become tachyonic, see Eq. 4. If this condition were violated at some scale μ_*, it would lead to an instability for field configurations with

$$|S| < \frac{M_S}{2\sqrt{\lambda_S}}, \quad \mu_- < |H| < \mu_+, \quad \mu_\pm^2 = \frac{M_S^2 \lambda_{HS}}{4\lambda_H\lambda_S}\left(1 \pm \sqrt{1 - \frac{\lambda_H\lambda_S}{\lambda_{HS}^2}}\right)\Bigg|_{\mu_*} , \tag{11}$$

  which could be trusted provided $\mu_- < \mu_* < \mu_+$. Note that, if $\mu_* \gg \mu_\pm$, this would not mean that there is an instability to worry about, as it would be located outside the range of validity of the tree-level approximation $V_0(\lambda_i(\mu_*), \varphi)$. Thus, as long as the condition in Eq. 10 is satisfied for renormalization scales within a relatively narrow range of energies around M_S (which fixes the mass scale of $\mu_\pm$), there is no instability even if this condition were eventually violated at higher scales. Only if parameters happen to lie near a critical point in which at least one of conditions (Eq. 10 or 11) is barely satisfied, radiative corrections can become important and invalidate the stability analysis performed with the tree-level potential. In this case one should resort to the one-loop approximation of the potential; otherwise, our analysis is reliable.

- **Case $\lambda_{\mathbf{HS}} < 0$**

  A similar analysis can be performed for this case. Provided Eq. 8 is fulfilled, the potential can only be negative at field values $|S| > w/\sqrt{2}$. The stability condition is:

$$-\lambda_{HS}(\mu) < \sqrt{\lambda_H(\mu)\lambda_S(\mu)} \ . \tag{12}$$

[a] This separation makes sense because λ_{HS} renormalizes multiplicatively, since it is the only coupling connecting both sectors: the new singlet and the SM Higgs sector. Therefore λ_{HS} does not flip sign along the running.

If this condition is not fulfilled at some scale μ_* an instability occurs with

$$|S| > \frac{M_S}{2\sqrt{\lambda_S}}, \quad c_- < \frac{|H|}{|S|} < c_+, \quad \mu_\pm^2 = \frac{-\lambda_{HS}}{\lambda_H}\left(1 \pm \sqrt{1 - \frac{\lambda_H \lambda_S}{\lambda_{HS}^2}}\right)\Bigg|_{\mu_*}. \tag{13}$$

Eq. 13 determines a direction in field space along which the potential becomes unbounded from bellow. Therefore, the condition of Eq. 12 has to be fulfilled for arbitrarily high scales.

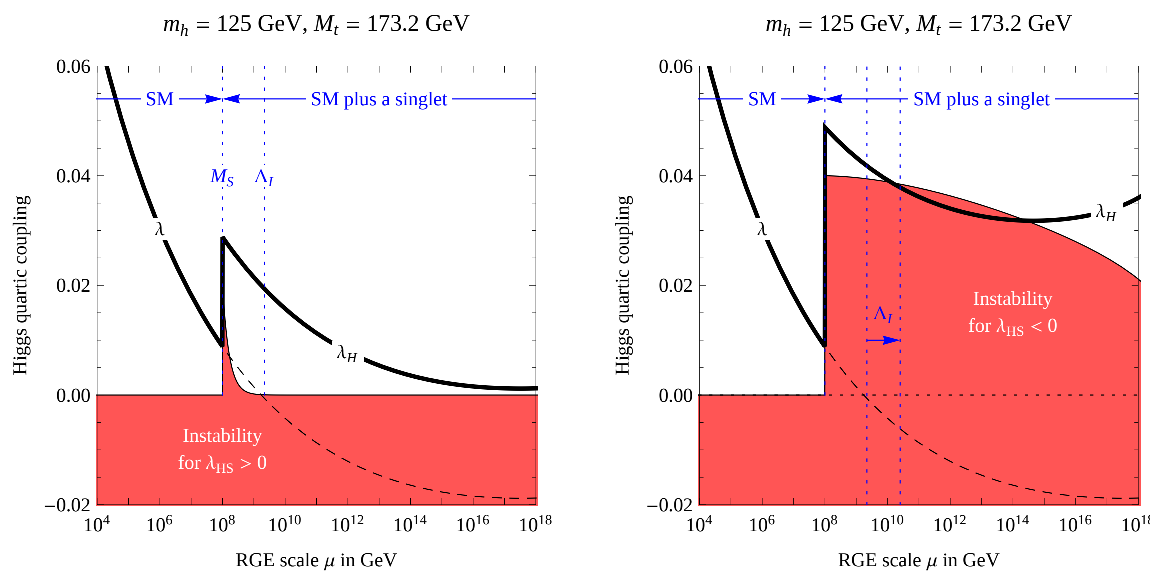

Figure 1: *Running of the Higgs quartic coupling in the SM and in the model with a scalar singlet, here assumed to have the mass* $M_S = 10^8$ GeV. *Left: if* $\lambda_{HS} > 0$, *thanks to the tree level shift at the singlet mass, the coupling never enters into the instability region, even assuming that singlet contributions to the RG equations are negligible. Right: if* $\lambda_{HS} < 0$ *the instability can be shifted away or avoided only by singlet contributions to the RG equations.*

In Fig. 1 we show two realistic plots of the running of the Higgs quartic coupling as a function of the renormalization scale for both cases $\lambda_{HS} > 0$ (left) and $\lambda_{HS} < 0$ (right). The red region is the instability region. If the quartic coupling enters in that region for some field value, there is a minimum deeper than the EW vacuum at such field value. The left plot shows that the mechanism just described is potentially very effective in stabilising the SM vacuum. As it was mentioned before, apart from the matching condition there is the related RGE effect, which also tends to increase the value of λ_H.

3 Examples

In this section we discuss a situation of physical interest where this mechanism can naturally operate. This is the see-saw mechanism for neutrino masses. This is the simplest version of the see-saw mechanism:

$$\delta\mathcal{L} = i\bar{N}\gamma\partial N + y_\nu LNH + \frac{M_N}{2}N^2 + \text{h.c.} \tag{14}$$

In the above equation, we have coupled a very massive Majorana fermion N, of mass M_N, to the SM Higgs H and lepton doublet L. Then, when the Higgs takes vev, the interaction term mixes the mass terms of the SM neutrinos and N. Upon diagonalizing, the SM neutrinos get mass

$$m_\nu = \frac{y_\nu^2 v^2}{M_N} \ , \tag{15}$$

which is small provided $M_N \gg v$.

The impact of the see-saw mechanism on the stability of the EW vacuum has been discussed in the literature [6]. The new neutrinos tend to make the potential more unstable, because they change the RGE flow of the quartic coupling above their mass scale. Their impact depends on the value of the Yukawa couplings, which increase as $y_\nu \propto \sqrt{M_N}$, and becomes sizeable for $M_N \gtrsim 10^{13}$ GeV.

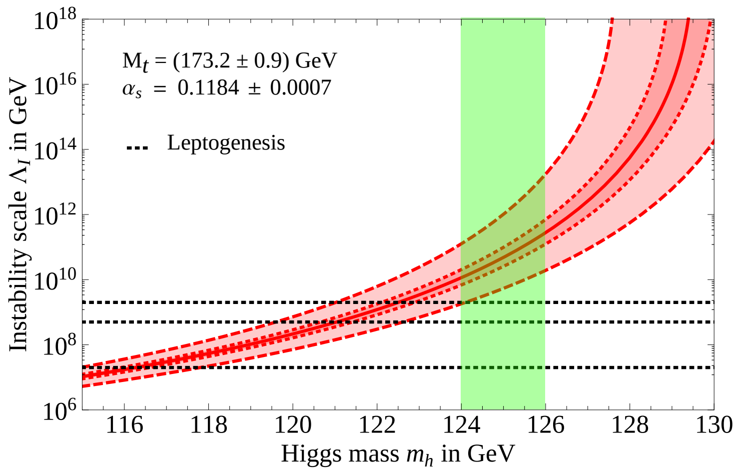

Figure 2: *In red, the SM instability scale Λ_I as a function of the Higgs mass. The central curve correspond to $M_t = 173.2$ GeV and $\alpha_s(M_Z) = 0.1184$ and the side-bands to 1-sigma deviations (with larger deviation for the top mass). In black, the three dashed lines correspond to the lower limits of the mass of the lightest right-handed neutrino coming from thermal leptogenesis. The green band is the experimentally favoured Higgs mass.*

We do not know what originated the large M_N mass, but an appealing way of giving mass to N is through a Higgs-like mechanism with a scalar S which gets a large vev. This scalar, apart from giving mass to the right-handed neutrinos, is coupled to the Higgs in a way that it can stabilise the SM vacuum by the threshold effect we described in the preceding section. Obviously, if the new scalar is to stabilise the Higgs potential, its mass scale should be smaller than the instability scale $\Lambda_I \sim 10^{11}$ GeV. Interestingly, there are lower bounds coming from leptogenesis [11] (if this explains the baryon asymmetry and for hierarchical right handed neutrinos), which are compatible with the upper bound.

In Fig. 2 we show a plot of the SM instability scale as a function of the Higgs mass. The dotted lines are the different lower bounds on the mass of the right handed neutrinos, the scale where approximately the threshold effect operates. The red band is the upper bound for the singlet mass, if the singlet is to stabilise the vacuum. Therefore, for the experimentally favoured value of $m_h \sim 125$ GeV, there is room for the mechanism to be naturally at work, that is: to give mass to N and stabilise the EW vacuum.

Lastly, we want to mention that in reference [8], other beyond the SM scenarios where the mechanism can naturally operate have been identified: invisible axion models and unitarized Higgs inflation. See reference [8] for further details.

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