# Neutrino and charged lepton flavour today 

(Alonso, Dhen, Hambye, B.G.)
(Alonso, D.Hernandez, Merlo, Rigolin, B.G.)

## Beyond Standard Model because

1) Experimental evidence for new particle physics:
```
*** Neutrino masses
*** Dark matter
** Matter-antimatter asymmetry
```

2) Uneasiness with SM fine-tunings, i.e. electroweak:
```
*** Hierarchy problem
*** Flavour puzzle
```


## FLAVOUR is the real issue in BSM electroweak

* The understanding of the physics behind is stalled since decades
* Precious data for the puzzle e.g.: B's, neutrinos


## Neutrino light on flavour?



## Neutrinos lighter because Majorana?



## Neutrino are optimal windows into the exotic -dark- sectors

* Can mix with new neutral fermions, heavy or light
* Interactions not obscured by strong and e.m. ones


## Dark portals

Only three singlet combinations in SM with $d<4$ :
$\mathrm{H}^{+} \mathrm{H}$
$\mathrm{B}_{\mathrm{\mu v}}$
$\overline{\mathrm{L}} \mathrm{H}$

Scalar

Vector

Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

## Dark portals

Only three singlet combinations in SM with $d<4$ :
$\mathrm{H}^{+} \mathrm{H}$ S
$B_{\mu \nu} \mathbf{V}_{\mu \nu}$
$\overline{\mathrm{L}} \mathrm{H} \Psi$

Scalar

Vector

Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

## Dark portals

Only three singlet combinations in SM with $d<4$ :
$\mathrm{H}^{+} \mathrm{H} \mathrm{S}^{2}$
$B_{\mu \nu} \mathbf{V}_{\mu \nu}$
$\overline{\mathrm{L}} \mathrm{H} \Psi$

Scalar

Vector

Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

## Dark portals

Only three singlet combinations in SM with $\mathrm{d}<4$ :
$\mathrm{H}^{+} \mathrm{H} \mathrm{S}^{2} \quad$ Scalar

$$
B_{\mu \nu} \mathbf{V}_{\mu \nu}
$$

Vector

Fermionic
fermion singlets $\Psi=$ "right-handed" neutrino

## DARK FLAVOURS ?


 $\begin{array}{lllllll}\overline{0} & \vdots & 0 & \pi & 3 & 0 & < \\ < & 0 & < & 0 & 0 & 0 \\ < & & < & <\end{array}$
.... they can be fermions

## DARK FLAVOURS ?

(large angle MSW)
$\mathrm{v}_{1} \longmapsto \mathrm{~V}_{2}-\mathrm{V}_{3}$




## DARK FLAVOURS ?

(large angle MSW)

$$
\mathrm{v}_{1} \longmapsto \mathrm{v}_{2}-\mathrm{v}_{3}
$$




| z | 三 | 0 | 자자N | ? | ๑ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ | Q | $<$ | , |  |  | 0 |
| $<$ | $<$ |  | $<$ | $\bigcirc$ | $<$ | $<$ |

## DARK FLAVOURS?




| F | 三 | $\bigcirc$ | 下 | ? | Q | $\bullet$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $\bigcirc$ | $<$ | $\cdots$ | 0 | 0 | $\bigcirc$ |
| $<$ | $<$ |  | 2 | e | $<$ | $<$ |

## DARK FLAVOURS?




| F | 三 | 0 | T | 3 | $\square$ | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $\stackrel{\square}{2}$ | $<$ |  | $\stackrel{\rightharpoonup}{2}$ | O | 0 |
| $<$ | $<$ |  | $\leqslant$ | $<$ | $<$ | $<$ |

Analysis of SM-DM with higer-dimensional ops. (d>=4) starting:

- with and withour flavour associated to DM:

$$
\frac{1}{\Lambda_{\mathrm{DM}^{2}}} \overline{\mathbf{Q}}_{\alpha} \gamma_{\mu} \mathbf{Q}_{\beta} \bar{\Psi}_{\mathrm{DM} \gamma} \gamma^{\mu} \Psi_{\mathrm{DM} \delta}
$$

Lepton Flavour violation (LFV) windows:

* Neutrino oscillations
<-neutral LFV

$$
\begin{aligned}
& * \mu-->e \gamma \\
& * \mu-->\mathbf{e ~ e ~ e ~}
\end{aligned}
$$

$$
\longleftarrow \text { charged LFV }
$$

* A fantastic experimental window being opened on lepton-flavour :
$\mu-e$ conversion in nuclei


## What is Muon to Electron Conversion?

1s state in a muonic atom

nuclear muon capture

$$
\mu^{-}+(A, Z) \rightarrow \nu_{\mu}+(A, Z-1)
$$

Neutrino-less muon nuclear capture

$$
\mu^{-}+(A, Z) \rightarrow e^{-}+(A, Z)
$$

Event Signature :
a single mono-energetic electron of 100 MeV Backgrounds:
(1) physics backgrounds ex. muon decay in orbit (DIO)
(2) beam-related backgrounds
ex. radiative pion capture, muon decay in flight,
(3) cosmic rays, false tracking

Consider together

## $\mu-->e$ conversion

$$
\mu-->e \gamma
$$

$$
\mu-->\mathbf{e} \mathbf{e}
$$

$$
\begin{aligned}
& \text { now } \\
& \mu-->e \text { conversion } \\
& R_{\mu \rightarrow e}^{T i}<4.3 \times 10^{-12} \quad---\cdots-\cdots----\gg 10^{-18} \\
& R_{\mu \rightarrow e}^{A u}<7 \times 10^{-13} \\
& R_{\mu \rightarrow e}^{P b}<4.6 \times 10^{-11} \\
& R_{\mu \rightarrow e}^{A l} \lesssim 10^{-16} \\
& \operatorname{Br}(\mu \rightarrow e \gamma)<2.4 \cdot 10^{-12}-------------><10^{-14} \\
& \mu-->e \text { e e } \\
& \operatorname{Br}(\mu \rightarrow e e e)<10^{-12} \\
& <10^{-16}
\end{aligned}
$$

$$
\begin{aligned}
& \text { now } \\
& \mu-->e \text { conversion } \\
& R_{\mu \rightarrow e}^{T i}<4.3 \times 10^{-12} \quad----------\gg 10^{-18} \\
& R_{\mu \rightarrow e}^{A u}<7 \times 10^{-13} \\
& R_{\mu \rightarrow e}^{P b}<4.6 \times 10^{-11} \\
& R_{\mu \rightarrow e}^{A l} \lesssim 10^{-16}
\end{aligned}
$$

## $\mu->e_{\gamma}$

\[

\]

Assume that singlet fermion(s) N exists in nature

?

What are the limits on their mass $\mathrm{m}_{\mathrm{N}}$ and mixings $\mathrm{U}_{\mathrm{IN}}$ ?
Can we observe them?

Assume that singlet fermion(s) N exists in nature

?

What are the limits on their mass $\mathrm{m}_{\mathrm{N}}$ and mixings UIN ? Can we observe them?

The paradigm model: Seesaw type-I $\quad \mathrm{N}_{\mathrm{R}}$

$$
\begin{gathered}
\mathcal{L}=\mathcal{L}_{S M}+i \overline{N_{R}} \not \partial N_{R}-\left[\overline{N_{R}} Y_{N} \tilde{\phi}^{\dagger} \ell_{L}+\frac{1}{2} \overline{N_{R}} M N_{R}^{c}+\text { h.c. }\right] \\
\mathbf{U}_{\mathbf{I N}} \sim \mathbf{Y} \mathbf{v} / \mathbf{M}
\end{gathered}
$$

Assume that singlet fermion(s) N exists in nature

?

What are the limits on their mass $\mathrm{m}_{\mathrm{N}}$ and mixings $\mathrm{U}_{\mathrm{IN}}$ ?
Can we observe them?

## $\mu$-->e conversion

mass eigenstates $n_{i}=v_{1}, v_{2}, v_{3}, N_{1} \ldots . N_{k}$


(d) Box Diagram

(e) Box Diagram

Figure 1: The five classes of diagrams contributing to $\mu$ to $e$ conversion in the type-I seesaw model.


Figure 1: The five classes of diagrams contributing to $\mu$ to $e$ conversion in the type-I seesaw model.

## They share just one form factor ("dipole")



Figure 1: The five classes of diagrams contributing to $\mu$ to $e$ conversion in the type-I seesaw model.

## Share all form factors, in different combinations

## $\mu-->e$ conversion

Many people before us computed it for singlet fermions:
De Gouvea
Mohapatra
Riazzudin+Marshak+Mohapatra 91,
Chang+Ng 94,
Ioannisian+Pilaftsis00,
Grimus + Lavoura
Pilaftsis and Underwood05,
Dennish + Kosmas + Valle06,
Ilakovac+Pilaftsis09ヶ We agree for
Deppish+P1lattsis11,
Dinh + Ibarra + Molinaro + Petcov12,
Aristizabal Sierra + Degee + Kamenik 12
Not two among those papers completely agree with each other, or they are not complete
typical applications assumed masses over 100 GeV or TeV

* we computed all contributions (logarithmic and constant)
* $\mu-->$ e conversion vanishes for masses in the 2-7 TeV mass regime

* we also considered the low mass region, sweeping over $\mathbf{e V}<\mathbf{m}_{\mathbf{N}}<$ thousands GeV


## In summary


(Alonso, Dhen, Hambye, B.G.)


## LHC is more competitive for concrete seesaw models:

## Low M, large Y is typical of seesaws with approximate Lepton Number conservation

## U(1) $\mathbf{L N}^{\mathbf{N}}$

(->~degenerate heavy neutrinos)
These models separate the flavor and the lepton number scale

Wyler+Wolfenstein 83, Mohapatra+Valle 86, Branco+Grimus+Lavoura 89, Gonzalez-Garcia+Valle 89, Ilakovac + Pilaftsis 95, Barbieri+Hambye + Romanino 03, Raidal+Strumia+Turzynski 05, Kersten+Smirnov 07, Abada+Biggio+Bonnet + Gavela + Hambye 07, Shaposhnikov 07, Asaka+Blanchet 08, Gavela+Hambye+D. Hernandez+ P. Hernandez 09

## seesaw I with Just TWO heavy neutrinos

$$
\mathcal{L}_{\mathcal{M}_{\nu}}=\left(\overline{\bar{L}}_{L}, \bar{N}^{c}, \bar{N}^{\prime c}\right)\left(\begin{array}{ccc}
0 & v Y & v Y^{\prime} \\
v Y^{\top} & 0 & \mathbf{M} \\
v Y^{\prime \top} & \mathbf{M} & 0
\end{array}\right)\left(\begin{array}{c}
\ell_{L}^{c} \\
N \\
N^{\prime}
\end{array}\right)
$$

Lepton number scale and flavour scale distinct

## Just TWO heavy neutrinos

$$
\begin{gathered}
\mathcal{L}_{\mathcal{M}_{\nu}}=\left(\bar{\ell}_{L}, \bar{N}^{c}, \bar{N}^{\prime c}\right)\left(\begin{array}{ccc}
0 & v Y & v Y^{\prime} \\
v Y^{T} & 0 & \mathbf{M} \\
v Y^{\prime T} & \mathbf{M} & 0
\end{array}\right)\left(\begin{array}{c}
\ell_{L}^{c} \\
N \\
N^{\prime}
\end{array}\right) \\
\mathrm{m}_{v}=\mathbf{Y} \frac{\mathrm{v}^{2} \mathbf{Y}^{\prime} \mathrm{T}}{\mathrm{M}} \quad \mathbf{U}_{\mathrm{IN}} \sim \frac{\mathbf{Y}}{\mathrm{M}} \\
\text {--> Lepton number conserved conserved if either } \mathrm{Y} \text { or } \mathrm{Y}^{\prime} \text { vanish: }
\end{gathered}
$$

Raidal, Strumia, Turszynski
Gavela, Hambye, Hernandez²

## Just TWO heavy neutrinos

$$
\mathcal{L}_{\mathcal{M}_{\nu}}=\left(\bar{e}_{L}, \bar{N}^{c}, \bar{N}^{\prime \prime}\right)\left(\begin{array}{ccc}
0 & v Y & v Y^{\prime} \\
v Y^{\top} & 0 & \mathbf{M} \\
v Y^{\prime T} & \mathbf{M} & 0
\end{array}\right)\left(\begin{array}{c}
\ell_{L} \\
N \\
N^{\prime}
\end{array}\right)
$$

--> One massless neutrino and only one Majorana phase $\boldsymbol{\alpha}$ the Yukawas are determined up to their overal magnitude

$$
\text { N.H. } \quad Y=\frac{y}{\sqrt{m_{\nu_{2}}+m_{\nu_{3}}}} U_{P M N S}\left(\begin{array}{c}
0 \\
-i \sqrt{m_{\nu_{2}}} e^{-i \alpha} \\
\sqrt{m_{\nu_{3}}} e^{-i \alpha}
\end{array}\right)
$$

Gavela, Hambye, Hernandez ${ }^{2}$
Raidal, Strumia, Turszynski

## In summary



## Varying the CP phases $\alpha$ and $\delta$, we get:



$$
\left|\mathbf{U}_{\mu \mathrm{N}} \mathbf{U}_{\mathrm{eN}} *\right| \quad \text { versus } \mathbf{m}_{\mathbf{N}}
$$

$\sim$ consistent with Cely et al. 12 , for $\alpha \sim 0, \delta \sim 0$

## Varying the CP phases $\alpha$ and $\delta$, we get:


-Dynamical Yukawas

## Yukawa couplings are the source of flavour in the SM



## Yukawa couplings are a source of flavour in the v-SM


(i.e. Seesaw type-I)


$$
\mathcal{L}=\mathcal{L}_{S M}+i \overline{N_{R}} \not \partial N_{R}-\left\lceil\overline{N_{R}} Y_{N} \tilde{\phi}^{\dagger} \ell_{L}+\frac{1}{2} \overline{N_{R}} M N_{R}^{c}+\text { h.c. }\right\rceil
$$

# May they correspond to dynamical fields <br> (e.g. vev of fields that carry flavor)? 

# In many BSM the Yukawas do not come from dynamical fields: 

## Some good ideas:

D.B. Kaplan-Georgi in the 80 's proposed a light SM scalar because being a (quasi) goldstone boson: composite Higgs
(D.B. Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison.......Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino... Frigerio, Pomarol, Riva, Urbano...)

## Some good ideas:

D.B. Kaplan-Georgi in the 80 's proposed a light SM scalar because being a (quasi) goldstone boson: composite Higgs

## Flavour "Partial compositeness" D.B Kaplan 91:

## A sort of "seesaw for quarks"

(nowadays sometimes justified from extra-dim physics )

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

## Some good ideas:

D.B. Kaplan-Georgi in the 80 's proposed a Higgs light because being a (quasi) goldstone boson: composite Higgs
"Partial compositeness":

## A sort of "seesaw for quarks"

(nowadays sometimes justified from extra-dim physics )

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

## Some good ideas:

D.B. Kaplan-Georgi in the 80 's proposed a Higgs light because being a (quasi) goldstone boson: composite Higgs
"Partial compositeness":

## A sort of "seesaw for quarks"

(nowadays sometimes justified from extra-dim physics )


Neutrino masses:

$\mathrm{d}=5$ Weinberg operator
(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

## Some good ideas:

D.B. Kaplan-Georgi in the 80 's proposed a Higgs light because being a (quasi) goldstone boson: composite Higgs
"Partial compositeness":

## A sort of "seesaw for quarks"

(nowadays sometimes justified from extra-dim physics )


Neutrino masses:


$$
\mathrm{m}_{v}=\mathrm{Y} \mathrm{v}^{2} / \mathrm{M} \mathrm{Y}^{\mathrm{T}}
$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

## Some good ideas:

D.B. Kaplan-Georgi in the 80 's proposed a Higgs light because being a (quasi) goldstone boson: composite Higgs
"Partial compositeness":

## A sort of "seesaw for quarks"

(nowadays sometimes justified from extra-dim physics )


Neutrino masses:


$$
\mathrm{m}_{v}=\mathrm{Y} \mu \mathrm{v}^{2} / \mathbf{M}^{2}
$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

# In other BSM Yukawas do correspond to dynamical fields: 

## Discrete symmetry ideas:

The Yukawas are indeed explained in terms of dynamical fields. And they do not need to worry about goldstone bosons.

In spite of $\theta_{13}$ not very small, some activity. For instance, combine generalized CP (Bernabeu, Branco, Gronau 80s) with discrete $Z_{2}$ groups in the neutrino sector : maximal $\theta_{23}$, strong constraints on values of CP phases
(Feruglio, Hagedorn and Ziegler 2013; Holthausen, Lindner and Schmidt 2013)

- Discrete approaches do not relate mixing to spectrum
- Difficulties to consider both quarks and leptons


## Some good ideas:



Frogatt-Nielsen '79: U(1)flavour symmetry

- Yukawa couplings are effective couplings,
- Fermions have U(1)flavour charges

$$
\left(\frac{\varphi}{\Lambda}\right)^{\mathrm{n}} \mathrm{QHq} \quad, \quad \mathrm{Y} \sim\left(\frac{\varphi}{\Lambda}\right)^{\mathrm{n}}
$$

e.g. $\mathrm{n}=0$ for the top, n large for light quarks, etc.

## Some good ideas:

## Minimal Flavour Violation:



- Use the flavour symmetry of the SM in the limit of massless fermions (Georgi+Chivukula)
quarks: $\quad G_{\text {flavour }}=\mathrm{U}(3)_{\mathrm{QL}} \times \mathrm{U}(3)_{\mathrm{UR}} \times \mathrm{U}(3)_{\mathrm{D}_{R}}$

The non-abelian part of the flavour symmetry of the SM:

$$
\mathrm{G}_{\mathrm{f}}=\mathrm{SU}(3)_{\mathrm{Q}_{\mathrm{L}}} \times \quad \mathrm{SU}(3)_{\mathrm{U}_{\mathrm{R}}} \times \quad \mathrm{SU}(3)_{\mathrm{D}_{\mathrm{R}}}
$$

broken by Yukawas:


## Some good ideas:



## Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula+ Georgi)

$$
\text { quarks: } \quad G_{\text {flavour }}=\mathrm{U}(3)_{\mathrm{QL}} \times \mathrm{U}(3)_{\mathrm{UR}} \times \mathrm{U}(3)_{\mathrm{DR}}
$$

- Assume that Yukawas are the only source of flavour in the SM and beyond

$$
\frac{1}{\Lambda_{\text {flavour }}{ }^{2}} \overline{\mathbf{Q}_{\boldsymbol{\alpha}}} \gamma_{\mu} \mathbf{Q}_{\beta} \overline{\mathbf{Q}_{\gamma}} \gamma^{\mu} \mathbf{Q}_{\delta}
$$

## Some good ideas:



## Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula + Georgi)

$$
\text { quarks: } \quad G_{\text {flavour }}=\mathrm{U}(3)_{\mathrm{QL}} \times \mathrm{U}(3)_{\mathrm{UR}} \times \mathrm{U}(3)_{\mathrm{DR}}
$$

- Assume that Yukawas are the only source of flavour in the SM and beyond

$$
\frac{\mathbf{Y}_{\alpha \beta}{ }^{+} \mathbf{Y}_{\delta \gamma}}{\boldsymbol{\Lambda}_{\text {flavour }}{ }^{2}} \overline{\mathbf{Q}_{\alpha}} \gamma_{\mu} \mathbf{Q}_{\beta} \overline{\mathbf{Q}_{\gamma}} \gamma^{\mu} \mathbf{Q}_{\delta}
$$

... agrees with flavour data being aligned with SM
... allows to bring down $\Lambda_{\text {flavour }}$--> TeV

> D'Ambrosio+Giudice+Isidori+Strumia;
> Cirigliano+Isidori+Grinstein+Wise

## Some good ideas:



## Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula Georgi)

$$
\text { quarks: } \quad G_{\text {flavour }}=\mathrm{U}(3)_{\mathrm{QL}} \times \mathrm{U}(3)_{\mathrm{UR}} \times \mathrm{U}(3)_{\mathrm{DR}}
$$

- Assume that Yukawas are the only source of flavour in the SM and beyond

$$
\frac{\mathbf{Y}_{\alpha \beta}{ }^{+} \mathbf{Y}_{\delta \gamma}}{\boldsymbol{\Lambda}_{\text {flavour }}{ }^{2}} \overline{\mathbf{Q}_{\alpha}} \gamma_{\mu} \mathbf{Q}_{\beta} \overline{\mathbf{Q}_{\gamma}} \gamma^{\mu} \mathbf{Q}_{\delta}
$$

... agrees with flavour data being aligned with SM
... allows to bring down $\Lambda_{\text {flavour }}$--> TeV
(Chivukula+Georgi 87; Hall+Randall; D’Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisntein
+Wise; Davidson+Pallorini; Kagan+G. Perez + Volanski+Zupan,... )
Lalak, Pokorski, Ross; Fitzpatrick, Perez, Randall; Grinstein, Redi, Villadoro

## Some good ideas:

## Related to MFV:



- Use the flavour symmetry of the SM in the limit of massless fermions quarks: $\quad G_{\text {flavour }}=\mathbf{U}(3)_{\mathrm{QL}} \times \mathbf{U}(3)_{\mathrm{UR}} \mathbf{X} \mathbf{U}(3)_{\mathrm{DR}}$


## Hybrid dynamical-non-dynamical Yukawas:

U(2) (Pomarol, Tomasini; Barbieri, Dvali, Hall, Romanino..)....

Sequential ideas (Feldman, Jung, Mannel; Berechiani-Nesti; Feretti e tal., Calibbi et al. ...)

## One step further

For this talk:
each Ysm -- >one single field $\mathcal{Y}$

$$
Y_{S M} \sim \frac{\langle Y\rangle}{\Lambda_{\mathrm{fl}}}
$$

## Can it shed light on why quark and neutrino mixings are so different?

Alonso, B.G., D. Hernandez, L. Merlo, Rigolin

For this talk:

## each Ysm $^{\text {-- }}$ >one single field $\mathcal{Y}$



## transforming under the SM flavour group

Anselm+Berezhiani 96; Berezhiani+Rossi 01; Alonso+Gavela+Merlo+Rigolin 11...

For this talk:

## each Ysm -- >one single field $\mathcal{Y}$

quarks:

$$
Y_{\mathrm{SM}} \sim \frac{\langle Y\rangle}{\Lambda_{\mathrm{fl}}}
$$




For this talk:

## each Ysm -- >one single field $\mathcal{Y}$

$$
Y_{\mathrm{SM}} \sim \frac{\langle Y\rangle}{\Lambda_{\mathrm{fl}}}
$$



¿ $V\left(y_{d}, y_{u}\right)$ ?

For this talk:

## each YSM -- >one single field $\mathcal{Y}$

$$
Y_{\mathrm{SM}} \sim \frac{\langle Y\rangle}{\Lambda_{\mathrm{fl}}}
$$



* Does the minimum of the scalar potential justify the observed masses and mixings?


## $v\left(y_{d}, y_{u}\right)$

* Invariant under the SM gauge symmetry
* Invariant under its global flavour symmetry Gflavour


## $\mathrm{G}_{\text {flavour }}=\mathrm{U}(3)_{\mathrm{Q}_{\mathrm{L}}} \times \mathrm{U}(3)_{\mathrm{UR}} \times \mathrm{U}(3)_{\mathrm{DR}}$

# The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings 

L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

## $\mathbf{v}\left(y_{d}, y_{u}\right)$

## Construction of the Potential

* 5 invariants at d=4 level:
(Feldman, Jung, Mannel)

$$
\begin{array}{cc}
\operatorname{Tr}\left(y_{\mathrm{u}}^{y_{\mathrm{u}}+}\right) & \operatorname{Tr}\left(y_{\mathrm{u}} y_{u^{+}}\right)^{2} \\
\operatorname{Tr}\left(y_{\mathrm{d}} y_{\mathrm{d}^{+}}\right) & \operatorname{Tr}\left(y_{\mathrm{d}} y_{\mathrm{d}^{+}}\right)^{2}
\end{array}
$$

$$
\operatorname{Tr}\left(y_{u} y_{u}^{+} y_{d} y_{d^{+}}\right)
$$

* results following general; for this talk we will illustrate in 2-generation


## $\mathbf{v}\left(y_{d}, y_{u}\right)$

## Construction of the Potential

* 5 invariants at d=4 level:
(Feldman, Jung, Mannel)

$$
\begin{array}{ll}
\operatorname{Tr}\left(y_{\mathrm{u}}^{y_{\mathrm{u}}+}\right) & \operatorname{Tr}\left(y_{\mathrm{u}} y_{\mathrm{u}^{+}}\right)^{2} \\
\operatorname{Tr}\left(y_{\mathrm{d}} y_{\mathrm{d}^{+}}\right) & \operatorname{Tr}\left(y_{\mathrm{d}} y_{\mathrm{d}^{+}}\right)^{2}
\end{array}
$$

$$
\operatorname{Tr}\left(\mathcal{Y}_{\mathrm{u}} \mathcal{Y}_{\mathrm{u}}^{+} \mathcal{Y}_{\mathrm{d}} \mathcal{Y}_{\mathrm{d}}^{+}\right)^{-- \text {mixing }}
$$

e.g. for the case of two families:
$\operatorname{Tr}\left(Y_{\mathrm{u}} \mathcal{Y}_{\mathrm{u}}{ }^{+} \mathcal{Y}_{\mathrm{d}} \mathcal{Y}_{\mathrm{d}}{ }^{+}\right) \propto\left(m_{c}^{2}-m_{u}^{2}\right)\left(m_{s}^{2}-m_{d}^{2}\right) \cos 2 \theta$
at the minimum: $\left(m_{c}^{2}-m_{u}^{2}\right)\left(m_{s}^{2}-m_{d}^{2}\right) \sin 2 \theta=0$


## -> NO MIXING

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

## And what happens for leptons?

## Any difference with Majorana neutrinos?

Alonso, B.G., D. Hernandez, Merlo, Rigolin.

## Leptons

## Just TWO heavy neutrinos

$$
\mathcal{L}_{\mathcal{M}_{\nu}}=\left(\bar{\ell}_{L}, \bar{N}^{c}, \bar{N}^{\prime c}\right)\left(\begin{array}{ccc}
0 & v Y & v Y^{\prime} \\
v Y^{\top} & 0 & \mathbf{M} \\
v Y^{\prime T} & \mathbf{M} & 0
\end{array}\right)\left(\begin{array}{c}
\ell_{L}^{c} \\
N \\
N^{\prime}
\end{array}\right)
$$

the Yukawas are determined up to their overal magnitude

$$
\text { N.H. } \quad Y=\frac{y}{\sqrt{m_{\nu_{2}}+m_{\nu_{3}}}} U_{P M N S}\left(\begin{array}{c}
0 \\
-i \sqrt{m_{\nu_{\nu}}} e^{-i \alpha} \\
\sqrt{m_{\nu_{3}}} e^{i \alpha}
\end{array}\right)
$$

## Leptons

## Just TWO heavy neutrinos

$$
\mathcal{L}_{\mathcal{M}_{\nu}}=\left(\bar{\ell}_{L}, \bar{N}^{c}, \bar{N}^{\prime c}\right)\left(\begin{array}{ccc}
0 & v Y & v Y^{\prime} \\
v Y^{\top} & 0 & \mathbf{M} \\
v Y^{\prime T} & \mathbf{M} & 0
\end{array}\right)\left(\begin{array}{c}
\ell_{L}^{c} \\
N \\
N^{\prime}
\end{array}\right)
$$

the Yukawas are determined up to their overal magnitude

$$
\text { N.H. } \quad Y=\frac{y}{\sqrt{m_{\nu_{2}}+m_{\nu_{3}}}} U_{P M N S}\left(\begin{array}{c}
0 \\
-i \sqrt{m_{\nu_{\nu}}} e^{-i \alpha} \\
\sqrt{m_{\nu_{3}}} e^{-i \alpha}
\end{array}\right)
$$

The flavour symmetry is $\mathbf{G}_{\mathbf{f}}=U(3)_{\ell_{L}} \times U(3)_{E_{R}} \times O(2)_{N}$

## Just TWO heavy neutrinos

$$
\begin{aligned}
& \mathcal{L}_{\mathcal{M}_{\nu}}=\left(\bar{\ell}_{L}, \bar{N}^{c}, \bar{N}^{\prime c}\right)\left(\begin{array}{ccc}
0 & v Y & v Y^{\prime} \\
v Y^{\top} & 0 & \mathbf{M} \\
v Y^{\prime T} & \mathbf{M} & 0
\end{array}\right)\left(\begin{array}{c}
\ell_{L}^{c} \\
N \\
N^{\prime}
\end{array}\right) \\
& \text { he Yukawas are determined up to their overal magnitude }
\end{aligned}
$$

$$
\text { N.H. } \quad Y=\frac{y}{\sqrt{m_{\nu_{2}}+m_{\nu_{3}}}} U_{P M N S}\left(\begin{array}{c}
0 \\
-i \sqrt{m_{\nu_{2}}} e^{-i \alpha} \\
\sqrt{m_{\nu_{3}}{ }^{2}} e^{i \alpha}
\end{array}\right)
$$

The flavour symmetry is $\mathbf{G}_{\mathbf{f}}=U(3)_{\ell_{L}} \times U(3)_{E_{R}} \times O(2)_{N}$

The flavour symmetry is $\mathbf{G}_{\mathbf{f}}=U(3)_{\ell_{L}} \times U(3)_{E_{R}} \times O(2)_{N}$ adds a new invariant for the lepton sector, in total:

$$
\begin{array}{ll}
\operatorname{Tr}\left(y_{\mathrm{E}} \mathcal{y}_{\mathrm{E}}^{+}\right) & \operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}}^{+}\right)^{2} \\
\operatorname{Tr}\left(y_{v} y_{v}^{+}\right) & \operatorname{Tr}\left(y_{v} y_{v}^{+}\right)^{2}
\end{array}
$$

$$
\begin{aligned}
& \operatorname{Tr}\left(y_{\mathrm{E}}{y_{\mathrm{E}}^{+}}^{y_{v}}{y_{v}^{+}}^{+} \longleftarrow{ }_{\mathrm{mixing}}^{\operatorname{Tr}\left(y_{v} \sigma_{2} y_{v}^{+}\right)^{2<--} \mathrm{O}(2)_{\mathrm{N}}}\right.
\end{aligned}
$$

$\mathrm{O}(2)_{\mathrm{N}}$ is simply associated to Lepton Number

## e.g., for 2 generations, the mixing terms in $\mathbf{V}\left(y_{E}, y_{v}\right)$ is :

## Leptons

$\operatorname{Tr}\left(y_{\mathrm{E}} \mathrm{y}_{\mathrm{E}^{+}} y_{v} y_{v^{+}}\right) \propto$ $\left(m_{\mu}^{2}-m_{e}^{2}\right)\left[\left(y^{2}+y^{\prime 2}\right)\left(m_{\nu_{2}}-m_{\nu_{1}}\right) \cos 2 \theta+\left(y^{2}-y^{\prime 2}\right) 2 \sqrt{m_{\nu_{2}} m_{\nu_{1}}} \sin 2 \alpha \sin 2 \theta\right]$

$$
\text { where } U_{\text {PMNS }}=\left(\begin{array}{cc}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{array}\right)\left(\begin{array}{cc}
\mathrm{e}^{-\mathrm{i} \alpha} & 0 \\
0 & \mathrm{e}^{\mathrm{i} \alpha} \alpha
\end{array}\right)
$$

Quarks

$$
\operatorname{Tr}\left(y_{u} y_{u}{ }^{+} y_{d} y_{d}^{+}\right) \propto\left(m_{c}^{2}-m_{u}^{2}\right)\left(m_{s}^{2}-m_{d}^{2}\right) \cos 2 \theta
$$

e.g., for 2 generations, the mixing terms in $\mathbf{V}\left(y_{E}, y_{v}\right)$ is :

Leptons
$\operatorname{Tr}\left(y_{\mathrm{E}} \mathrm{y}_{\mathrm{E}}{ }^{+} y_{v} y_{v^{+}}\right) \propto$
$\left(m_{\mu}^{2}-m_{e}^{2}\right)\left[\left(y^{2}+y^{\prime 2}\right)\left(m_{\nu_{2}}-m_{\nu_{1}}\right) \cos 2 \theta+\left(y^{2}-y^{\prime 2}\right)-\sqrt{m_{\nu_{2}} m_{\nu_{1}}} \sin 2 \alpha \sin 2 \theta\right.$
Mixing term unphysical if either
"up" or "down" fermions
degenerate

Mixing physical even with degenerate neutrino masses, if Majorana phase nontrivial

## Quarks

$$
\operatorname{Tr}\left(y_{\mathrm{u}} y_{\mathrm{u}}^{+} y_{\mathrm{d}} y_{\mathrm{d}}^{+}\right) \propto\left(m_{c}^{2}-m_{u}^{2}\right)\left(m_{s}^{2}-m_{d}^{2}\right) \cos 2 \theta
$$

e.g., for 2 generations, the mixing terms in $\mathbf{V}\left(\mathcal{Y}_{\mathrm{E}}, \mathcal{Y}_{v}\right)$ is :

Minimisation
$\operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}} y_{v} y_{v}{ }^{+}\right)$

* $\left(y^{2}-y^{\prime 2}\right) \sqrt{m_{\nu_{2}} m_{\nu_{1}}} \sin 2 \theta \cos 2 \alpha=0 \longrightarrow \quad \boldsymbol{\alpha}=\pi / 4$ or $3 \pi / 4$
* $\quad \operatorname{tg} 2 \theta=2 \frac{y^{2}-y^{\prime 2}}{y^{2}+y^{\prime 2}} \sin 2 \alpha \frac{\sqrt{m_{\nu_{2}} m_{\nu_{1}}}}{m_{\nu_{2}}-m_{\nu_{1}}}$

Large angles correlated with degenerate masses
Maximal Majorana phase

## What makes the difference?

- The Majorana character?
- The flavour group?
- The particular model?

Let us try to generalize to any model
-- for 2 families
-- for 3 families

## * Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.
diagonal
eigenvalues
Use Casas-Ibarra parametrization $\quad \mathbf{Y}_{\mathbf{v}}=\mathbf{U}_{\text {puss }} \mathbf{m}_{\mathbf{v}}{ }^{1 / 2} \mathbf{R} \mathbf{M}_{\mathbf{N}}{ }^{1 / 2}$

orthogonal
matrix

## * Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.
diagonal

$\operatorname{Tr}\left(Y_{\mathrm{E}} \mathcal{Y}_{\mathrm{E}}{ }^{+} \mathrm{Y}_{v} \mathcal{Y}_{v^{+}}\right)=\operatorname{Tr}\left(m_{i}^{1 / 2} U^{+} m_{i}^{2} \cup m_{i}^{1 / 2} R^{+} \mathrm{M}_{\mathrm{N}} R\right)$ orthogonal matrix

$$
\text { define } \mathrm{P}=\left(R^{+} \mathrm{M}_{\mathrm{N}} R\right) \xrightarrow[2 \text { fam. }]{ } \left\lvert\, \begin{aligned}
& * \sqrt{m_{1} m_{2}}\left|P_{12}\right| \sin \left[2 \alpha-\arg \left(P_{12}\right)\right]=0 \\
& * \operatorname{tg} 2 \theta=2\left|P_{12}\right| \sin 2 \alpha \frac{\sqrt{m_{\nu_{2}} m_{\nu_{1}}}}{m_{\nu_{1}} P_{11}-m_{\nu_{2}} P_{22}}
\end{aligned}\right.
$$

## * Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.
diagonal

$\operatorname{Tr}\left(Y_{\mathrm{E}} \mathcal{Y}_{\mathrm{E}}{ }^{+} \mathrm{Y}_{v} \mathrm{Y}_{v}{ }^{+}\right)=\operatorname{Tr}\left(m_{i}^{1 / 2} U^{+} m^{2} \cup m_{i}^{1 / 2} R^{+} \mathrm{M}_{\mathrm{N}} R\right)$ orthogonal matrix

$$
\begin{aligned}
& * \sqrt{m_{1} m_{2}}\left|P_{12}\right| \sin \left[2 \alpha-\arg \left(P_{12}\right)\right]=0 \\
& * \operatorname{tg} 2 \theta=2\left|P_{12}\right| \sin 2 \alpha \frac{\sqrt{m_{\nu_{2}} m_{\nu_{1}}}}{m_{\nu_{1}} P_{11}-m_{\nu_{2}} P_{22}}
\end{aligned}
$$

* In degenerate limit of heavy neutrinos $\mathbf{M}_{\mathbf{N} 1}=\mathbf{M}_{\mathbf{N} 2}=\mathbf{M}$

$$
\mathbf{R}=\left(\begin{array}{cc}
\operatorname{ch} \omega & -i \operatorname{sh} \omega \\
i \operatorname{sh} \omega & \operatorname{ch} \omega
\end{array}\right) \text { with } \omega \text { real }
$$

$$
\operatorname{tg} 2 \theta=\sin 2 \alpha \frac{2 \sqrt{m_{\nu_{2}} m_{\nu_{1}}}}{m_{\nu_{2}}-m_{\nu_{1}}} \operatorname{tgh} 2 \omega \quad \alpha=\pi / 4 \text { or } 3 \pi / 4
$$

## * Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.
diagonal

$\operatorname{Tr}\left(Y_{\mathrm{E}} \mathcal{Y}_{\mathrm{E}}{ }^{+} \mathrm{Y}_{v} \mathrm{Y}_{v}{ }^{+}\right)=\operatorname{Tr}\left(m_{i}^{1 / 2} U^{+} m^{2} \cup m_{i}^{1 / 2} R^{+} \mathrm{M}_{\mathrm{N}} R\right)$ orthogonal matrix

$$
\begin{aligned}
& * \sqrt{m_{1} m_{2}}\left|P_{12}\right| \sin \left[2 \alpha-\arg \left(P_{12}\right)\right]=0 \\
& * \operatorname{tg} 2 \theta=2\left|P_{12}\right| \sin 2 \alpha \frac{\sqrt{m_{\nu_{2}} m_{\nu_{1}}}}{m_{\nu_{1}} P_{11}-m_{\nu_{2}} P_{22}}
\end{aligned}
$$

* In degenerate limit of heavy neutrinos $\mathbf{M}_{\mathbf{N} 1}=\mathbf{M}_{\mathbf{N} 2}=\mathbf{M}$

$$
\mathbf{R}=\left(\begin{array}{cc}
\operatorname{ch} \omega & -i \operatorname{sh} \omega \\
i \operatorname{sh} \omega & \operatorname{ch} \omega
\end{array}\right) \text { with } \omega \text { real }
$$

e.g. in Previous model

$$
\operatorname{tg} 2 \theta=\sin 2 \alpha \frac{2 \sqrt{m_{\nu_{2}} m_{\nu_{1}}}}{m_{\nu_{2}}-m_{\nu_{1}}} \frac{y^{2}-y^{9}{ }^{2}}{y^{2}-y^{92}}
$$

$$
\boldsymbol{\alpha}=\pi / 4 \text { or } 3 \pi / 4
$$

* What is the role of the neutrino flavour group?


## Leptons: $\mathrm{G}_{\text {flavour }}=\mathbf{U}(2)_{\mathrm{L}} \times \mathrm{U}(2)$ ER $\times$ ?



Inmediate results using for both quark and leptons

$$
\mathrm{Y}=\mathrm{U}_{\mathrm{L}} \mathrm{y}^{\text {diag }} \mathrm{U}_{\mathrm{R}}
$$

* What is the role of the neutrino flavour group?


## U(n)

* What is the role of the neutrino flavour group?


## $\mathbf{U}(\mathbf{n})$

ie. $\mathbf{U}(3)_{\mathrm{L}} \times \mathbf{U}(3)_{\mathrm{En}} \times \mathbf{U}(2)_{\mathrm{Ne}}$ or: $\quad \mathbf{U}(3)_{\mathrm{L}} \times \mathrm{X}(3)_{\mathrm{Er}} \times \mathrm{U}(3)_{\mathrm{N} k}$

* What is the role of the neutrino flavour group?

$$
\text { e.g. } \mathbf{U}(\mathbf{n})_{\mathrm{NR}} \quad \text {... leptons }
$$

e.g. generic seesaw

$$
\begin{aligned}
\mathcal{L}= & \mathcal{L}_{S M}+i \overline{N_{R}} \not \partial N_{R}-\left[\overline{N_{R}} Y_{N} \tilde{\phi}^{\dagger} \ell_{L}+\frac{1}{2} \overline{N_{R}} \mathbf{M} N_{R}^{c}+\text { h.c. }\right] \\
& \text { with M carrying flavour } \longrightarrow \mathbf{M} \text { spurion }
\end{aligned}
$$

More invariants in this case:

$$
\begin{array}{ccc}
\operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}}\right) & \operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}}\right)^{2} & \operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}} y_{v} y_{v}^{+}\right) \\
\operatorname{Tr}\left(y_{v} y_{v}^{+}\right) & \operatorname{Tr}\left(y_{v} y_{v}^{+}\right)^{2} & \\
\operatorname{Tr}\left(M_{N} M_{N^{+}}\right) & \operatorname{Tr}\left(M_{N} M_{N^{+}}\right)^{2} & \operatorname{Tr}\left(M_{N} M_{N^{+}} y_{v}^{+} y_{v}\right)
\end{array}
$$

Result: no mixing for flavour groups $\mathrm{U}(\mathrm{n})$

O(n)

* What is the role of the neutrino flavour group?

$$
\mathrm{O}(2)_{\mathrm{NR}}
$$

e.g. two families


Generically, $O(2)$ allows :

- one mixing angle maximal
- one relative Majorana phase of $\pi / 2$
- two degenerate light neutrinos


## *3 families with $\mathrm{O}(2)_{\mathrm{NR}}$ :

- 3 light +2 heavy $N$ degenerate: bad $\theta_{12}$ quadrant. It cannot accomodate data!
- 3 light +3 heavy N : OK for $\boldsymbol{\theta}_{23}$ maximal and spectrum
experimentally $\sin ^{2} \theta_{23}=0.41+-0.03$ or $0.59+-0.02$
Gonzalez-Garcia, Maltoni, Salvado, Schwetz Sept. 2012
*What about the other angles?


## *3 families with $\mathrm{O}(2)_{\mathrm{NR}}$ :

- 3 light +2 heavy $N$ degenerate: bad $\theta_{12}$ quadrant. It cannot accomodate data!
- 3 light +3 heavy N : OK for $\boldsymbol{\theta}_{23}$ maximal and spectrum

Moriond this morning, T2K best fit point $\sin ^{2} 2 \theta_{23}=1.00-0.068$ 90\%CL

$$
\text { -> } 45^{\circ} \text { ! }
$$

*What about the other angles?

3 families: Leading order

* Quarks : no mixing
* Leptons (with 3 heavy N ), $\mathrm{O}(2)_{\mathrm{Nr}: ~}^{\text {: }}$ one $\sim$ maximal angle + one sizeable one + large Majorana phase


## *a good possibility for the other angles :

Yukawas --> add fields in the fundamental of the flavour group

1) Y -- > one single scalar $\quad Y \sim(3,1,3)$

2) $\mathrm{Y}-\mathrm{-}$ > two scalars $\chi \chi^{+} \sim(3,1,3)$

3) Y -- > two fermions $\bar{\Psi} \Psi \sim(3,1,3)$

4) Y -- > one single scalar $\quad Y \sim(3,1,3)$

5) $\mathrm{Y}-\mathrm{-}$ > two scalars $\chi \chi^{+} \sim(3,1,3)$

$$
\chi \sim(3,1,1)
$$


3) Y -- > two fermions $\bar{\Psi} \Psi \sim(3,1,3)$


1) Y -- > one single scalar $\quad \mathrm{V} \sim(3,1,3)$ $\mathrm{d}=5$ operator

2) Y -- > two scalars $\chi \chi^{+} \sim(3,1,3)$ $\mathrm{d}=6$ operator $\quad \chi \sim(3,1,1)$

3) Y -- > two fermions $\bar{\Psi} \Psi \sim(3,1,3)$ $\mathrm{d}=7$ operator

i.e. for quarks, a possible path:

* At leading (renormalizable) order:

$$
\begin{aligned}
& Y_{u} \equiv \frac{\left\langle y_{u}\right\rangle}{\Lambda_{f}}+\frac{\left\langle\chi_{u}^{L}\right\rangle\left\langle\chi_{u}^{R \dagger}\right\rangle}{\Lambda_{f}^{2}}=\left(\begin{array}{ccc}
0 & \sin \theta_{c} y_{c} & 0 \\
0 & \cos \theta_{c} y_{c} & 0 \\
0 & 0 & y_{t}
\end{array}\right), \\
& Y_{d} \equiv \frac{\left\langle y_{\mathrm{d}}\right\rangle}{\Lambda_{f}}+\frac{\left\langle\chi_{d}^{L}\right\rangle\left\langle\chi_{d}^{R \dagger}\right\rangle}{\Lambda_{f}^{2}}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & y_{s} & 0 \\
0 & 0 & y_{b}
\end{array}\right) .
\end{aligned}
$$

without unnatural fine-tunings

* The masses of the first family and the other angles from nonrenormalizable terms or other corrections or replicas ?


## Conclusions

* Exciting experimental windows ahead into neutrino(and/or DM) physics:
$\mu$-e conversion will test SM-singlet fermions in the $2 \mathrm{GeV}-6000 \mathrm{TeV}$ mass range!
* A dynamical origin for the Yukawa couplings, based on the continuous flavour symmetry of the SM allows to tackle flavour for both quarks and leptons, and for both masses and mixings

Simplest case explored here, with approximate $U(1)$ Ln seesaws:
-- Quarks: vanishing mixing at leading order
-- Majorana neutrinos with O(2) $\mathrm{N}_{\mathrm{r}}$ :
*one angle maximal (and another sizeable at leading order)
*large/small mixings <--> degenerate/hierarchical masses

## $\mathbf{O}(2)_{\mathrm{N}_{\mathrm{R}}}$ singled out

## Conclusions

* Exciting experimental windows ahead into neutrino(and/or DM) physics:
$\mu$-e conversion will test SM-singlet fermions in the $2 \mathrm{GeV}-6000 \mathrm{TeV}$ mass range!
* A dynamical origin forthervalinas based on the continuova a cavour for



## Back-up slides

## Higgs decay (LHC)

## e.g. $\quad \mathrm{H}-->v \mathrm{~N}$

Pilaftsis92....Chen et al.10, Dev+Franceschini+Mohapatra 12, Cely+Ibarra+Molinaro+Petcov
We get for the model-independent rate:

$$
\begin{gathered}
\operatorname{Br}(h \rightarrow \nu N)=\frac{\alpha_{W}}{8 M_{W}^{2} \Gamma_{h}^{t o t}} \sum_{i}^{k}\left(\left|U_{e N_{i}}\right|^{2}+\left|U_{\mu N_{i}}\right|^{2}+\left|U_{\tau N_{i}}\right|^{2}\right) m_{h} m_{N_{i}}^{2}\left(1-\frac{m_{N_{i}}^{2}}{m_{h}^{2}}\right)^{2} \\
\text { and using }\left|\Sigma_{i} U_{e N_{i}} U_{u N_{i}}^{*}\right|<\Sigma_{i, \alpha}\left|U_{\alpha N_{i}}\right|^{2}
\end{gathered}
$$

## COMET $\mu$-e conv. search

Phase-I phys run in 2017
Full COMET run in 2021-2022

- Search for cLFV mu-e conv.
$-10^{-16}$ sensitivity (Target S.E.S. $2.6 \times 10^{-17}$ )
Pion collection
- Improve $\mathbf{O}\left(10^{4}\right)$ than present upper bound such as SINDRUM-II BR[ $\mu^{-}+\mathrm{Au} \rightarrow$ $\left.\mathrm{e}^{-}+\mathrm{Au}\right]<7 \times 10^{-13}$
- Signature: 105 MeV monochromatic electron
- Beam requirement
- 8 GeV bunched slow extraction
$-1.6 \times 10^{21}$ pot needed to reach goal
$-7 \mathrm{uA}(56 \mathrm{~kW}) \times 4 \mathrm{SN}$ year $\left(4 \times 10^{7} \mathrm{sec}\right)$
- Extinction <10-9

courtesy of Yoshi Kuno


## $\mu-->e$ conversion

We performed an exact one-loop calculation, but for obvious approximations:
-- $\mathrm{m}_{\mathrm{e}}=0$ compared to $\mathrm{m}_{\mu}$
$-m_{v 1}=m_{v 2}=m_{v 3}=0$ compared to heavy neutrino masses (that is, assume $\mathrm{m}_{\mathrm{N}}>\mathrm{eV}$ )
-- higher orders in the external momentum neglected versus $\mathrm{M}_{\mathrm{w}}$, as usual

We did many checks to our results, e.g.:

* For "dipole" for factors .... check with b --> s l+ $\mathrm{l}^{-}$
* For the other form factors .... agreement with $\mu$--> eee form factors


## example of check: Decoupling limits

* Large mass $\mathrm{m}_{\mathrm{N}} \gg \mathrm{m}_{\mathbf{w}}$

In the seesaw, for $\mathrm{m}_{\mathrm{N}^{-}}>\infty$ the remaining theory is renormalizable (SM) --> rate must vanish then.
Our results do decouple for $x_{N}=m_{N}^{2} / M_{W}^{2} \gg 1$

$$
\begin{array}{ll}
\Gamma \sim\left(\log x_{N}\right)^{2} / x_{N}^{2}, & \text { for } \mu \rightarrow \mathrm{eee} \quad \text { and } \quad \mu \rightarrow \mathrm{e} \text { conversion } \\
\Gamma \sim 1 / x_{N}^{2}, & \text { for } \mu \rightarrow \mathrm{e} \gamma .
\end{array}
$$

* Low mass $\mathbf{m}_{\mathbf{N}} \ll \mathbf{m}_{\mathbf{W}}$
they also vanish for $\mathrm{m}_{\mathrm{N}}-->0$

$$
x_{N}=m_{N}^{2} / M_{W}^{2} \ll 1
$$

$$
\begin{array}{ll}
\Gamma \sim x_{N}^{2}\left(\log x_{N}\right)^{2}, & \text { for } \mu \rightarrow \mathrm{eee} \quad \text { and } \quad \mu \rightarrow \mathrm{e} \text { conversion; } \\
\Gamma \sim x_{N}^{2}, & \text { for } \mu \rightarrow \mathrm{e} \gamma .
\end{array}
$$

$\left|\mathbf{U}_{\mu \mathrm{N}} \mathbf{U}_{\mathrm{eN}}{ }^{*}\right|$ versus $\mathrm{m}_{\mathrm{N}}$


Sensitivity up to $\mathrm{m}_{\mathrm{N}} \sim \mathbf{6 0 0 0} \mathbf{~ T e V}$ for $\mathbf{T i}$

For the particular case of seesaw I : UIN $\sim \mathbf{Y} \mathbf{v} / \mathbf{M}$


## * Large mass $\mathrm{m}_{\mathrm{N}} \gg \mathrm{m}_{\mathbf{w}}$

When one $\mathrm{m}_{\mathrm{N}}$ scale dominates (e.g. degenerate heavy neutrinos or hierarchical) the ratio of any two $\mu$-e transitions only depends on $\mathbf{m}_{\mathbf{N}}$ (Chu, Dhen, Hambye 11)

Besides, $\mu$-e conversion vanishes at some large $\mathbf{m}_{N}$
(Dinh, Ibarra, Molinaro, Petcov 12)
For instance, we find that for light nuclei $(\alpha \mathrm{Z} \ll 1)$, it vanishes as

$$
\left.m_{N}^{2}\right|_{0}=M_{W}^{2} \exp \left(\frac{\frac{9}{8}(A-Z)+\left(\frac{9}{8}+\frac{31 s_{W}^{2}}{12}\right) Z}{\frac{3}{8}(A-Z)+\left(\frac{4 s_{W}^{2}}{3}-\frac{3}{8}\right) Z}\right)_{\text {(Alonso, Dhen, Hambye, B.G.) }}
$$

exponential sensitivity

## The ratios of two e- $\mu$ transitions....

we obtain:
$\mu$-e conversion
$\mu-->$ e $\gamma$

$\mu$-e conversion $\mu-->$ e e e

...typically vanishes for $\mathrm{m}_{\mathrm{N}}$ in 2-7 TeV range
(Alonso, Dhen, Hambye, B.G.)

* Low mass regime $\mathbf{e V} \ll \mathrm{m}_{\mathbf{N}} \ll \mathrm{m}_{\mathbf{W}}$
. de Gouvea 05...


## * Low mass regime $\mathbf{e V} \ll \mathbf{m}_{\mathbf{N}} \ll \mathbf{m}_{\mathbf{W}}$

$\mu-->$ e conversion does not vanish for low mass

(Alonso, Dhen, Hambye, B.G.)

## * Low mass regime $\mathbf{e V} \ll \mathbf{m}_{\mathbf{N}} \ll \mathrm{m}_{\mathbf{w}}$



Peak decays + PS191+NuTev/CHARM + Delphi: Atre + Han + Pascoli + Zhang 09...... Richayskiy + Ivashko 12

Unitarity: Antusch+Biggio+Fdez-Martinez+Gavela+Lopez-Pavon 06; Antusch+Bauman+ Fedez-Martinez 09

## * Low mass regime $\mathbf{e V} \ll \mathbf{m}_{\mathbf{N}} \ll \mathrm{m}_{\mathbf{w}}$



## * Low mass regime $\mathbf{e V} \ll \mathbf{m}_{\mathbf{N}} \ll \mathrm{m}_{\mathbf{w}}$



BBN and SN: Kainulainen+Maalampi+Peltoniemi91, Kusenko+Pascoli+Semikoz 05, Mangano+Serpico 11, Ruchaysiliy +Ivashko 12,
Kufflick+McDermott+Zurek 12

## * Low mass regime $\mathbf{e V} \ll \mathbf{m}_{\mathbf{N}} \ll \mathrm{m}_{\mathbf{w}}$



This experiment (considered alone) will probe masses down to $\mathbf{m}_{\mathbf{N}}=2 \mathrm{MeV}$

## * Low mass regime $\mathbf{e V} \ll \mathbf{m}_{\mathbf{N}} \ll \mathrm{m}_{\mathbf{w}}$



Absolute bound from Higgs, from absence of


## at LHC:

Br $(\mathrm{H}->v \mathrm{~N})<0.4$
(Espinosa, Grojean, Muhlleitner, Trott, 12 Dev + Franceschini + Mohapatra12, Cely + Ibarra+Molinaro+Petcov 12)

## Varying the CP phases $\alpha$ and $\delta$, we get:


$\left|\mathbf{U}_{\mu \mathrm{N}} \mathrm{U}_{\mathrm{eN}}{ }^{*}\right|$ versus $\mathbf{m}_{\mathbf{N}}$

## Varying the CP phases $\alpha$ and $\delta$, we get:



## Orange and red model-dependent bounds limited by:

upper boundary: $(\alpha=\pi / 2, \delta=0)$
( $\alpha=\pi, \delta=3 \pi / 2$ )
lower boundary: $\sim(\alpha=-\pi / 2, \delta=0)$

$$
(\alpha=-\pi / 4, \delta=0)
$$

$\sim$ it could be consistent with Cely et al. 12, for $\alpha \sim 0, \delta \sim 0$

## Varying the CP phases $\alpha$ and $\delta$, we get:



For inverted hierarchy: some very low points for which $\mu$-->e very small, because the Yukawas involved ---> 0 for particular values of $\boldsymbol{\alpha}$ and $\boldsymbol{\delta} \quad$ (Alonso et al. 09, Alonso 08, Chu + Dhen+Hambye 11...)

For instance, in the minimal seesaw I, Lepton number scale and flavour scale linked:

$$
\begin{aligned}
& \mathscr{L}_{M_{\nu}}= {\left[\begin{array}{cc}
0 & \mathbf{Y}^{\mathrm{T}} \mathrm{v} \\
\mathbf{Y} \mathrm{v} & \mathrm{M}_{\mathrm{N}}
\end{array}\right] } \\
&-\mathcal{L}_{\text {seesaw } \mathrm{I}}=\bar{L} H Y_{E} E_{R}+\bar{L} \tilde{H} Y N+M \bar{N} N^{c}+h . c . \\
& \mathrm{m}_{\mathrm{v}}=\frac{\mathrm{Y}^{\mathrm{v}^{2}} \mathbf{Y}^{\mathrm{T}} \quad}{\mathrm{M}} \quad \mathbf{U}_{\mathbf{I N}} \sim \frac{\mathbf{Y}}{\mathrm{M}}
\end{aligned}
$$

# Why quark and neutrino mixings are so different? 

May dynamical Yukawas shed light on it?

## Assume that the Yukawa couplings have a dynamical origin at high energies

(L. Michel+Radicati 70, Cabibbo+Maiani71...Anselm+Berezhiani 96; Berezhiani+Rossi 01)

$$
\mathbf{Y}_{\mathrm{SM}} \sim<\varphi>\text { or } \mathbf{Y}_{\mathrm{SM}} \sim 1 /<\boldsymbol{1}>\text { or } \ldots \ldots .<(\varphi \chi)^{\mathrm{n}}>
$$



## Assume that the Yukawa couplings have a dynamical origin at high energies

(L. Michel+Radicati 70, Cabibbo+Maiani71...Anselm+Berezhiani 96; Berezhiani+Rossi 01)

$$
\mathbf{Y}_{\mathrm{SM}} \sim<\varphi>\text { or } \mathbf{Y}_{\mathrm{SM}} \sim 1 /<\varphi>\text { or } \ldots \ldots .<(\varphi \chi)^{\mathrm{n}}>
$$



$$
\mathcal{Y}_{d \sim(3, \overline{3}, 1)} \quad y_{u \sim(3,1, \overline{3})}
$$

$$
\begin{aligned}
& \left\langle y_{d}\right\rangle \\
& \Lambda_{f}
\end{aligned}=Y_{D}=V_{C K M}\left(\begin{array}{ccc}
y_{d} & 0 & 0 \\
0 & y_{s} & 0 \\
0 & 0 & y_{b}
\end{array}\right), \quad \frac{\left\langle y_{u}\right\rangle}{\Lambda_{f}}=Y_{U}=\left(\begin{array}{ccc}
y_{u} & 0 & 0 \\
0 & y_{c} & 0 \\
0 & 0 & y_{t}
\end{array}\right) .
$$

## * What is the role of the neutrino flavour group?

To analyze this in general, use common parametrization for quarks and leptons:

$$
Y=U_{L} \quad y^{\text {diag. }} U_{R}
$$

* Quarks, for instance: $\quad U_{R}$ unphysical, $\quad U_{L}-->U_{C K M}$

$$
\mathbf{Y}_{\mathbf{D}}=\mathbf{U}_{\text {CKM }} \operatorname{diag}\left(\mathrm{y}_{\mathrm{d}}, \mathrm{y}_{\mathrm{s}}, \mathrm{y}_{\mathrm{b}}\right) \quad ; \quad \mathbf{Y}_{\mathbf{U}}=\operatorname{diag}\left(\mathrm{y}_{\mathrm{u}}, \mathrm{y}_{\mathrm{c}}, \mathrm{y}_{\mathrm{t}}\right)
$$

* Leptons:

$$
Y_{E}=\operatorname{diag}\left(y_{e}, y_{\mu}, y_{\tau}\right) \quad ; \quad Y_{v}=U_{L} \quad y^{\text {diag. }} U_{R}
$$

UPMNS diagonalize $\quad m_{v} \sim Y_{v} \frac{v^{2}}{M} Y_{v}{ }^{T}=U_{L} y_{v}{ }^{\text {diag. }} \frac{U_{R} v^{2}}{M} U_{R}{ }^{T} y_{v}{ }^{\text {diag. }} U_{L}{ }^{T}$

## SU(n)

* What is the role of the neutrino flavour group?

$$
\text { e.g. } \mathrm{SU}(\mathbf{n})_{\mathrm{NR}} \quad \text {... leptons }
$$

e.g. generic seesaw

$$
\begin{aligned}
\mathcal{L}= & \mathcal{L}_{S M}+i \overline{N_{R}} \not N_{R}-\left[\overline{N_{R}} Y_{N} \bar{\phi}^{\dagger} \ell_{L}+\frac{1}{2} \overline{N_{R}} \mathbf{M} N_{R}^{c}+\text { h.c. }\right] \\
& \text { with } \mathbf{M} \text { carrying flavour } \longrightarrow \mathbf{M} \text { spurion }
\end{aligned}
$$

More invariants in this case:

$$
\begin{array}{ccc}
\operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}}\right) & \operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}}\right)^{2} & \operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}} y_{v} y_{v}^{+}\right) \\
\operatorname{Tr}\left(y_{v} y_{v}^{+}\right) & \operatorname{Tr}\left(y_{v} y_{v}^{+}\right)^{2} & \\
\operatorname{Tr}\left(M_{N} M_{N^{+}}\right) & \operatorname{Tr}\left(M_{N} M_{N^{+}}\right)^{2} & \operatorname{Tr}\left(M_{N} M_{N^{+}} y_{v}^{+} y_{v}\right)
\end{array}
$$

At the minimum:
${ }^{*} \operatorname{Tr}\left(y_{v} y_{v}{ }^{+} \mathrm{Y}_{\mathrm{E}} \mathrm{Y}_{\mathrm{E}}{ }^{+}\right)=\operatorname{Tr}\left(\mathrm{U}_{\mathrm{L}} \mathrm{y}_{\mathrm{v}}{ }^{\text {diag. } 2} \mathrm{U}_{\mathrm{L}}{ }^{+} \mathrm{y}_{1}\right.$ diag. $\left.{ }^{2}\right) \longrightarrow \mathrm{U}_{\mathrm{L}}=1$

* $\operatorname{Tr}\left(M_{N} M_{N^{+}} y_{v} y_{v}{ }^{+}\right)=\operatorname{Tr}\left(U_{\mathrm{R}} \mathrm{y}_{\mathrm{v}}{ }^{\text {diag. } 2} \mathrm{U}_{\mathrm{R}}{ }^{+} \mathrm{M}_{\mathrm{i}}{ }^{\text {diag. } 2}\right) \longrightarrow \mathrm{U}_{\mathrm{R}}=1$
same conclusion for $\mathbf{3}$ families of quarks:

$$
Y=U_{L} \quad y^{\text {diag. }} \mathrm{U}_{\mathrm{R}}
$$

* Quarks, for instance: $\quad U_{R}$ unphysical, $U_{L}-->U_{C K M}$

$$
\mathbf{Y}_{\mathrm{D}}=\mathbf{U}_{\text {CKM }} \operatorname{diag}\left(\mathrm{y}_{\mathrm{d}}, \mathrm{y}_{\mathrm{s}}, \mathrm{y}_{\mathrm{b}}\right) \quad ; \quad \mathbf{Y}_{\mathrm{U}}=\operatorname{diag}\left(\mathrm{y}_{\mathrm{u}}, \mathrm{y}_{\mathrm{c}}, \mathrm{y}_{\mathrm{t}}\right)
$$

$\operatorname{Tr}\left(y_{u} y_{u}{ }^{+} y_{d} y_{d}{ }^{+}\right)=\operatorname{Tr}\left(U_{L} y_{u}{ }^{\text {diag. } 2} U_{L}{ }^{+} y_{d}{ }^{\text {diag. } 2}\right)$
$\longrightarrow \mathrm{U}_{\mathrm{L}}=\mathrm{U}_{\mathrm{CKM}} \sim 1$ at the minimum

NO MIXING

## $\operatorname{Tr}\left(\mathcal{Y}_{\mathrm{u}} \mathcal{Y}_{\mathrm{u}}^{+} \mathcal{Y}_{\mathrm{d}} \mathscr{Y}_{\mathrm{d}^{+}}\right) \propto \sum_{i, j}\left|\mathbf{V}{ }_{C K M}^{i j}\right|^{2} m_{u_{i}}^{2} m_{d_{j}}^{2}$

e.g. for the case of two families:
$\operatorname{Tr}\left(y_{u} y_{u}{ }^{+} y_{d} y_{d}{ }^{+}\right) \propto\left(m_{c}^{2}-m_{u}^{2}\right)\left(m_{s}^{2}-m_{d}^{2}\right) \cos 2 \theta$

The mixing term in $\mathbf{V}$ is now:
$\operatorname{Tr}\left(y_{\mathrm{E}} \mathcal{Y}_{\mathrm{E}}{ }^{+} \mathrm{y}_{v} \mathrm{y}_{v^{+}}\right)=$
$\left(y^{2}+y^{\prime 2}\right) \sum_{l, i}\left|U_{P M N S}^{l i}\right|^{2} m_{l}^{2} m_{\nu_{i}}+\left(y^{2}-y^{\prime 2}\right)\left[\quad \mathrm{i} \sum_{l, i<j}\left(U_{P M N S}^{l i}\right)^{*} U^{l j} m_{l}^{2} \sqrt{m_{\nu_{i}} m_{\nu_{j}}}+c . c\right.$.
while for quarks it was:

$$
\operatorname{Tr}\left(y_{\mathrm{u}} y_{\mathrm{u}}^{+} y_{\mathrm{d}} y_{\mathrm{d}}^{+}\right) \propto \sum_{i, j}\left|U_{C K M}^{i j}\right|^{2} m_{u_{i}}^{2} m_{d_{j}}^{2}
$$

The mixing term in $\mathbf{V}$ is now:
$\operatorname{Tr}\left(y_{E} y_{E^{+}} y_{v} y_{v}{ }^{+}\right) \propto$
$\left(y^{2}+y^{\prime 2}\right) \sum_{l, i}\left|U_{P M N S}^{l i}\right|^{2} m_{l}^{2} m_{\nu_{i}}+\underbrace{\left(y^{2}-y^{\prime 2}\right)\left[\quad \mathrm{i} \sum_{l, i<j}\left(U_{P M N S}^{l i}\right)^{*} U^{l j} m_{l}^{2} \sqrt{m_{\nu_{i}} m_{\nu_{j}}}+\text { c.c.c. }\right.}$
extra term because of Majorana character
while for quarks it was:

$$
\operatorname{Tr}\left(y_{\mathrm{u}} y_{\mathrm{u}}^{+} y_{\mathrm{d}} y_{\mathrm{d}}^{+}\right) \propto \sum_{i, j}\left|U_{C K M}^{i j}\right|^{2} m_{u_{i}}^{2} m_{d_{j}}^{2}
$$

* What is the role of the neutrino flavour group?

$$
\text { e.g. } \mathrm{SU}(\mathbf{n})_{\mathrm{NR}} \quad \text {... leptons }
$$

e.g. generic seesaw

$$
\begin{aligned}
\mathcal{L}= & \mathcal{L}_{S M}+i \overline{N_{R}} \not N_{R}-\left[\overline{N_{R}} Y_{N} \bar{\phi}^{\dagger} \ell_{L}+\frac{1}{2} \overline{N_{R}} \mathbf{M} N_{R}^{c}+\text { h.c. }\right] \\
& \text { with } \mathbf{M} \text { carrying flavour } \longrightarrow \mathbf{M} \text { spurion }
\end{aligned}
$$

More invariants in this case:

$$
\begin{array}{ccc}
\operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}}\right) & \operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}}\right)^{2} & \operatorname{Tr}\left(y_{\mathrm{E}} y_{\mathrm{E}^{+}} y_{v} y_{v}^{+}\right) \\
\operatorname{Tr}\left(y_{v} y_{v}^{+}\right) & \operatorname{Tr}\left(y_{v} y_{v}^{+}\right)^{2} & \\
\operatorname{Tr}\left(M_{N} M_{N^{+}}\right) & \operatorname{Tr}\left(M_{N} M_{N^{+}}\right)^{2} & \operatorname{Tr}\left(M_{N} M_{N^{+}} y_{v}^{+} y_{v}\right)
\end{array}
$$

At the minimum:
${ }^{*} \operatorname{Tr}\left(y_{v} y_{v}{ }^{+} \mathrm{Y}_{\mathrm{E}} \mathrm{Y}_{\mathrm{E}}{ }^{+}\right)=\operatorname{Tr}\left(\mathrm{U}_{\mathrm{L}} \mathrm{y}_{\mathrm{v}}{ }^{\text {diag. } 2} \mathrm{U}_{\mathrm{L}}{ }^{+} \mathrm{y}_{1}\right.$ diag. $\left.{ }^{2}\right) \longrightarrow \mathrm{U}_{\mathrm{L}}=1$

* $\operatorname{Tr}\left(M_{N} M_{N^{+}} y_{v} y_{v}{ }^{+}\right)=\operatorname{Tr}\left(U_{\mathrm{R}} \mathrm{y}_{\mathrm{v}}{ }^{\text {diag. } 2} \mathrm{U}_{\mathrm{R}}{ }^{+} \mathrm{M}_{\mathrm{i}}{ }^{\text {diag. } 2}\right) \longrightarrow \mathrm{U}_{\mathrm{R}}=1$

$$
\mathrm{G}_{\mathrm{f}}=\mathrm{U}(3)_{\mathrm{Q}} \quad \mathrm{x} \quad \mathrm{U}(3)_{\mathrm{U}} \quad \mathrm{x} \quad \mathrm{U}(3)_{\mathrm{D}}
$$

$$
\begin{aligned}
& V\left(y_{u}, y_{u}\right)=\sum_{i}\left[-\mu_{i^{2}}^{2} \operatorname{Tr}\left(y_{i} y_{i}^{+}\right)-\lambda_{i} \operatorname{Tr}\left(y_{i} y_{i}^{+}\right)^{2}\right] \\
& +\sum_{i \not i j}\left[\lambda_{i j} \operatorname{Tr}\left(y_{i} y_{i}^{+} y_{j} y_{j}^{+}\right)\right]+\ldots
\end{aligned}
$$

it only relies on $\mathrm{G}_{\mathrm{f}}$ symmetry and SM gauge symmetry
It allows for either (too) hierarchical or degenerate spectrum

Use the flavour symmetry of the SM with masless fermions:

$$
\mathrm{G}_{\mathrm{f}}=\mathrm{U}(3)_{\mathrm{Q}_{L}} \mathrm{x} \quad \mathrm{U}(3)_{U_{R}} \quad x \quad U(3)_{D_{R}}
$$

replace Yukawas by fields:


Spontaneous breaking of flavour symmetry dangerous

```
--> i.e. gauge it (Grinstein, Redi, Villadoro, 2010)
        (Feldman, 2010)
        (Guadagnoli, Mohapatra, Sung, 2010)
```

$$
\mathrm{G}_{\mathrm{f}}=\mathrm{U}(3)_{\mathrm{Q}} \quad \mathrm{x} \quad \mathrm{U}(3)_{\mathrm{U}} \quad x \quad \mathrm{U}(3)_{\mathrm{D}}
$$

$$
\begin{aligned}
& V\left(y_{u}, y_{u}\right)=\sum_{i}\left[-\mu_{i^{2}}^{2} \operatorname{Tr}\left(y_{i} y_{i}^{+}\right)-\lambda_{i} \operatorname{Tr}\left(y_{i} y_{i}^{+}\right)^{2}\right] \\
& +\sum_{i f j}\left[\lambda_{i j} \operatorname{Tr}\left(y_{i} y_{i}^{+} y_{j} y_{j}^{+}\right)\right]+\ldots
\end{aligned}
$$

it only relies on $\mathrm{G}_{\mathrm{f}}$ symmetry and SM gauge symmetry

## and analyzed its minima

# Can its minimum correspond naturally to the observed masses and mixings? 

i.e. with all dimensionless $\lambda$ 's $\sim 1$
and dimensionful $\mu^{\prime} \mathrm{s} \leqq \Lambda_{\mathrm{f}}$

Spectrum for flavons $\Sigma$ in the bifundamental:

* 3 generations: for the largest fraction of the parameter space, the stable solution is a degenerate spectrum

$$
\left(\begin{array}{lll}
\mathrm{y}_{\mathrm{u}} & & \\
& \mathrm{yc}_{\mathrm{c}} & \\
& & \mathrm{yt}_{\mathrm{t}}
\end{array}\right) \sim\left(\begin{array}{lll}
\mathrm{y} & & \\
& \mathrm{y} & \\
& & \mathrm{y}
\end{array}\right)
$$

instead of the observed hierarchical spectrum, i.e.

$$
\left(\begin{array}{lll}
\mathrm{yu}_{\mathrm{u}} & & \\
& \mathrm{yc}_{\mathrm{c}} & \\
& & \mathrm{yt}_{\mathrm{t}}
\end{array}\right) \sim\left(\begin{array}{lll}
0 & & \\
& 0 & \\
& & \mathrm{y}
\end{array}\right)
$$

(at leading order)

Spectrum: the hierarchical solution is unstable in most of the

$$
\begin{aligned}
& \text { parameter space. } \quad \text { Stability: } \frac{\tilde{\mu}^{2}}{\mu^{2}}<\frac{2 \lambda^{\prime 2}}{\lambda} \\
& \qquad V^{(4)}=\sum_{i=u, d}\left(-\mu_{i}^{2} A_{i}+\tilde{\mu}_{i} B_{i}+\lambda_{i} A_{i}^{2}+\lambda_{i}^{\prime} A_{i i}\right)+g_{u d} A_{u} A_{d}+\lambda_{u d} A_{u d} . \\
& \text { ie, the u-part: } \quad V^{(4)}=-\mu_{u}^{2} A_{u}+\tilde{\mu}_{u} B_{u}+\lambda_{u} A_{u}^{2}+\lambda_{u}^{\prime} A_{u u}
\end{aligned} .
$$



Spectrum: the hierarchical solution is unstable in most of the parameter space.

$$
\begin{aligned}
& \text { Stability: } \frac{\tilde{\mu}^{2}}{\mu^{2}}<\frac{2 \lambda^{\prime 2}}{\lambda} \\
& \left.A_{i}+\tilde{\mu}_{i} B_{i}+\lambda_{i} A_{i}^{2}+\lambda_{i}^{\prime} A_{i i}\right)+g_{u d} A_{u} A_{d}+\lambda_{u d} A_{u d} .
\end{aligned}
$$

ie, the u-part:

$$
V^{(4)}=-\mu_{u}^{2} A_{u}+\tilde{\mu}_{u} B_{u}+\lambda_{u} A_{u}^{2}+\lambda_{u}^{\prime} A_{u u}
$$



Nardi emphasized this solution (and extended the analysis to include also $U(1)$ factors)

## Normal hierarchy:

Up to terms of $\mathcal{O}\left(\sqrt{r}, s_{13}\right)$, we find

$$
r=\frac{\left|\Delta m_{12}^{2}\right|}{\left|\Delta m_{13}^{2}\right|}
$$

$$
Y_{N}^{T} \simeq y\left(\begin{array}{c}
e^{i \delta} s_{13}+e^{-i \alpha} s_{12} r^{1 / 4} \\
s_{23}\left(1-\frac{\sqrt{r}}{2}\right)+e^{-i \alpha} r^{1 / 4} c_{12} c_{23} \\
c_{23}\left(1-\frac{\sqrt{r}}{2}\right)-e^{-i \alpha} r^{1 / 4} c_{12} s_{23}
\end{array}\right)
$$

## Inverted hierarchy:

$$
Y_{N}^{T} \simeq \frac{y}{\sqrt{2}}\left(\begin{array}{c}
c_{12} e^{i \alpha}+s_{12} e^{-i \alpha} \\
c_{12}\left(c_{23} e^{-i \alpha}-s_{23} s_{13} e^{i(\alpha-\delta)}\right)-s_{12}\left(c_{23} e^{i \alpha}+s_{23} s_{13} e^{-i(\alpha+\delta)}\right) \\
-c_{12}\left(s_{23} e^{-i \alpha}+c_{23} s_{13} e^{i(\alpha-\delta)}\right)+s_{12}\left(s_{23} e^{i \alpha}-c_{23} s_{13} e^{-i(\alpha+\delta)}\right)
\end{array}\right)
$$

## The invariants can be written in terms of masses and mixing

* two families:

$$
\begin{gathered}
<\Sigma_{\mathrm{d}}>=\Lambda_{\mathrm{f}} . \operatorname{diag}\left(\mathrm{y}_{\mathrm{d}}\right) ; \quad<\Sigma_{\mathrm{u}}>=\Lambda_{\mathrm{f}} . V_{\text {Cabibbo }} \operatorname{diag}\left(\mathrm{y}_{\mathrm{u}}\right) \\
Y_{D}=\left(\begin{array}{cc}
y_{d} & 0 \\
0 & y_{s}
\end{array}\right), \quad Y_{U}=\nu_{C}^{\dagger}\left(\begin{array}{cc}
y_{u} & 0 \\
0 & y_{c}
\end{array}\right) \quad V_{\text {Cabibbo }}=\left(\begin{array}{cc}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{array}\right)
\end{gathered}
$$

$$
<\operatorname{Tr}\left(\Sigma_{\mathrm{u}} \Sigma_{\mathrm{u}}^{+}\right)>=\Lambda_{\mathrm{f}}^{2}\left(\mathrm{yu}^{2}+\mathrm{y}_{\mathrm{c}}^{2}\right) ;<\operatorname{det}\left(\Sigma_{\mathrm{u}}\right)>=\Lambda_{\mathrm{f}}^{2} \mathrm{y}_{\mathrm{u}} \mathrm{y}_{\mathrm{c}}
$$

$$
<\operatorname{Tr}\left(\Sigma_{\mathrm{u}} \Sigma_{\mathrm{u}}^{+} \Sigma_{\mathrm{d}} \Sigma_{\mathrm{d}}^{+}\right)>=\Lambda_{\mathrm{f}}^{4}\left[\left(\mathrm{yc}^{2}-\mathrm{yu}^{2}\right)\left(\mathrm{ys}^{2}-\mathrm{y}_{\mathrm{d}}^{2}\right) \cos 2 \theta+\ldots\right] / 2
$$

## Minimum of the Potential

## Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$
\frac{\partial V}{\partial y_{i}}=0 \quad \frac{\partial V}{\partial \theta_{i}}=0
$$

Take the angle for example:

$$
\frac{\partial V}{\partial \theta_{c}} \propto\left(y_{c}^{2}-y_{u}^{2}\right)\left(y_{s}^{2}-y_{d}^{2}\right) \sin 2 \theta_{c}=0
$$



Non-degenerate masses $\longrightarrow \sin 2 \theta_{c}=0 \quad$ No mixing!

Notice also that $\frac{\partial V^{(4)}}{\partial \theta} \sim \sqrt{J} \quad$ (Jarlskog determinant)

## Minimum of the Potential

## Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$
\frac{\partial V}{\partial y_{i}}=0 \quad \frac{\partial V}{\partial \theta_{i}}=0
$$

Take the angle for example:

$$
\frac{\partial V}{\partial \theta_{c}} \propto\left(y_{c}^{2}-y_{u}^{2}\right)\left(y_{s}^{2}-y_{d}^{2}\right) \sin 2 \theta_{c}=0
$$



Non-degenerate masses $\longrightarrow \sin 2 \theta_{c}=0$ No mixing!
Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...


## Minimum of the Potential

## Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$
\frac{\partial V}{\partial y_{i}}=0 \quad \frac{\partial V}{\partial \theta_{i}}=0
$$

Take the angle for example:

$$
\frac{\partial V}{\partial \theta_{c}} \propto\left(y_{c}^{2}-y_{u}^{2}\right)\left(y_{s}^{2}-y_{d}^{2}\right) \sin 2 \theta_{c}=0
$$



$$
\text { Non-degenerate masses } \quad \sin 2 \theta_{c}=0 \quad \text { No mixing ! }
$$

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...

* Without fine-tuning, for two families the spectrum is degenerate
* To accomodate realistic mixing one must introduce wild fine tunnings of $\mathrm{O}\left(10^{-10}\right)$ and nonrenormalizable terms of dimension 8
* at renormalizable level: 7 invariants instead of the 5 for two families

$$
\begin{array}{ll}
\operatorname{Tr}\left(\Sigma_{u} \Sigma_{u}^{\dagger}\right) \stackrel{\text { vev }}{=} \Lambda_{f}^{2}\left(y_{t}^{2}+y_{c}^{2}+y_{u}^{2}\right), & \operatorname{Det}\left(\Sigma_{u}\right) \stackrel{\text { vev }}{=} \Lambda_{f}^{3} y_{u} y_{c} y_{t}, \\
\operatorname{Tr}\left(\Sigma_{d} \Sigma_{d}^{\dagger}\right) \stackrel{\text { vev }}{=} \Lambda_{f}^{2}\left(y_{b}^{2}+y_{s}^{2}+y_{d}^{2}\right), & \operatorname{Det}\left(\Sigma_{d}\right) \stackrel{\text { vev }}{=} \Lambda_{f}^{3} y_{d} y_{s} y_{b}, \\
=\operatorname{Tr}\left(\Sigma_{u} \Sigma_{u}^{\dagger} \Sigma_{u} \Sigma_{u}^{\dagger}\right) \stackrel{\text { vev }}{=} \Lambda_{f}^{4}\left(y_{t}^{4}+y_{c}^{4}+y_{u}^{4}\right), & \\
=\operatorname{Tr}\left(\Sigma_{d} \Sigma_{d}^{\dagger} \Sigma_{d} \Sigma_{d}^{\dagger}\right) \stackrel{\text { vev }}{=} \Lambda_{f}^{4}\left(y_{b}^{4}+y_{s}^{4}+y_{d}^{4}\right), & \\
=\operatorname{Tr}\left(\Sigma_{u} \Sigma_{u}^{\dagger} \Sigma_{d} \Sigma_{d}^{\dagger}\right) \stackrel{\text { vev }}{=} \Lambda_{f}^{4}\left(P_{0}+P_{\text {int }}\right), &
\end{array}
$$

Interesting angular dependence: $P_{0} \equiv-\sum_{i<j}\left(y_{u_{i}}^{2}-y_{u_{j}}^{2}\right)\left(y_{d_{i}}^{2}-y_{d_{j}}^{2}\right) \sin ^{2} \theta_{i j}$,

$$
\begin{aligned}
P_{i n t} \equiv & \sum_{i<j, k}\left(y_{d_{i}}^{2}-y_{d_{k}}^{2}\right)\left(y_{u_{j}}^{2}-y_{u_{k}}^{2}\right) \sin ^{2} \theta_{i k} \sin ^{2} \theta_{j k}+ \\
& -\left(y_{d}^{2}-y_{s}^{2}\right)\left(y_{c}^{2}-y_{t}^{2}\right) \sin ^{2} \theta_{12} \sin ^{2} \theta_{13} \sin ^{2} \theta_{23}+ \\
& +\frac{1}{2}\left(y_{d}^{2}-y_{s}^{2}\right)\left(y_{c}^{2}-y_{t}^{2}\right) \cos \delta \sin 2 \theta_{12} \sin 2 \theta_{23} \sin \theta_{13},
\end{aligned}
$$

## The real, unavoidable, problem is again mixing:

* Just one source:

$$
\operatorname{Tr}\left(\Sigma_{\mathrm{u}} \Sigma_{\mathrm{u}}{ }^{+} \Sigma_{\mathrm{d}} \Sigma_{\mathrm{d}}{ }^{+}\right)=\Lambda_{\mathrm{f}}^{4}\left(\mathrm{P}_{0}+\mathrm{P}_{\mathrm{int}}\right)
$$

$P_{0}$ and $P_{\text {int }}$ encode the angular dependence,

$$
\begin{aligned}
P_{0} \equiv & -\sum_{i<j}\left(y_{u_{i}}^{2}-y_{u_{j}}^{2}\right)\left(y_{d_{i}}^{2}-y_{d_{j}}^{2}\right) \sin ^{2} \theta_{i j}, \\
P_{\text {int }} \equiv & \sum_{i<j, k}\left(y_{d_{i}}^{2}-y_{d_{k}}^{2}\right)\left(y_{u_{j}}^{2}-y_{u_{k}}^{2}\right) \sin ^{2} \theta_{i k} \sin ^{2} \theta_{j k}+ \\
& -\left(y_{d}^{2}-y_{s}^{2}\right)\left(y_{c}^{2}-y_{t}^{2}\right) \sin ^{2} \theta_{12} \sin ^{2} \theta_{13} \sin ^{2} \theta_{23}+ \\
& +\frac{1}{2}\left(y_{d}^{2}-y_{s}^{2}\right)\left(y_{c}^{2}-y_{t}^{2}\right) \cos \delta \sin 2 \theta_{12} \sin 2 \theta_{23} \sin \theta_{13},
\end{aligned}
$$

Sad conclusions as for 2 families:
needs non-renormalizable + super fine-tuning

## Y --> quadratic in fields $\chi$

$$
\mathbf{Y} \sim \frac{\left\langle\chi \chi^{\dagger}\right\rangle}{\Lambda_{f}^{2}}
$$


$\longrightarrow$ Automatic strong mass hierarchy and one mixing angle already at the renormalizable level

Holds for $\mathbf{2}$ and $\mathbf{3}$ families !

## 2) $Y$--> quadratic in fields $\chi$ <br> $$
\mathbf{Y} \sim \frac{\left\langle\chi \chi^{\dagger}\right\rangle}{\Lambda_{f}^{2}}
$$ <br> 

$$
\text { i.e. } \mathrm{Y}_{\mathrm{D}} \sim \frac{\chi^{\mathrm{L}} \mathrm{~d}\left(\chi^{\mathrm{R}} \mathrm{~d}\right)^{+} \sim(3,1,1)(1,1, \overline{3}) \sim(3,1, \overline{3})}{\Lambda_{\mathrm{f}}{ }^{2}}
$$

## Y $\rightarrow$ quadratic in fields $\chi$

It is very simple:

- a square matrix built out of 2 vectors

$$
\left(\begin{array}{c}
\mathrm{d} \\
\mathrm{e} \\
\mathrm{f} \\
\vdots
\end{array}\right)(\mathrm{a}, \mathrm{~b}, \mathrm{c} \ldots \ldots . .)
$$

has only one non-vanishing eigenvalue
strong mass hierarchy at leading order:
-- only 1 heavy "up" quark
-- only 1 heavy "down" quark

## only $|\chi|$ 's relevant for scale

## Minimum of the Potential

## Dimension 6 Yukawa Operator

The invariants are:

$$
\begin{array}{cc}
\chi_{u}^{L \dagger} \chi_{u}^{L}, & \chi_{u}^{R \dagger} \chi_{u}^{R}, \quad \chi_{d}^{L \dagger} \chi_{d}^{L}, \\
\chi_{d}^{R \dagger} \chi_{d}^{R}, & \chi_{u}^{L \dagger} \chi_{d}^{L}=\left|\chi_{u}^{L}\right|\left|\chi_{d}^{L}\right| \cos \theta_{c} .
\end{array}
$$


$\boldsymbol{\theta}_{\mathrm{c}}$ is the angle between up and down L vectors

## Minimum of the Potential

## Dimension 6 Yukawa Operator

The invariants are:

$$
\begin{array}{cc}
\chi_{u}^{L \dagger} \chi_{u}^{L}, & \chi_{u}^{R \dagger} \chi_{u}^{R}, \quad \chi_{d}^{L \dagger} \chi_{d}^{L} \\
\chi_{d}^{R \dagger} \chi_{d}^{R}, & \chi_{u}^{L \dagger} \chi_{d}^{L}=\left|\chi_{u}^{L}\right|\left|\chi_{d}^{L}\right| \cos \theta_{c}
\end{array}
$$

We can fit the angle and the masses in the Potential; as an example:


$$
\begin{gathered}
V^{\prime}=\lambda_{u}\left(\chi_{u}^{L \dagger} \chi_{u}^{L}-\frac{\mu_{u}^{2}}{2 \lambda_{u}}\right)^{2}+\lambda_{d}\left(\chi_{d}^{L \dagger} \chi_{d}^{L}-\frac{\mu_{d}^{2}}{2 \lambda_{d}}\right)^{2} \\
+\lambda_{u d}\left(\chi_{u}^{L \dagger} \chi_{d}^{L}-\frac{\mu_{u d}^{2}}{2 \lambda_{u d}}\right)^{2}+\cdots
\end{gathered}
$$

Whose minimum sets (2 generations):

$$
y_{c}^{2}=\frac{\mu_{u}^{2}}{2 \lambda_{u} \Lambda_{\mathrm{f}}^{2}} \quad y_{s}^{2}=\frac{\mu_{d}^{2}}{2 \lambda_{d} \Lambda_{\mathrm{f}}^{2}} \quad \cos \theta=\frac{\mu_{u d}^{2} \sqrt{\lambda_{u} \lambda_{d}}}{\mu_{u} \mu_{d} \lambda_{u d}}
$$

Towards a realistic 3 family spectrum
e.g. replicas of $\chi^{L}, \chi_{u}^{R}, \chi_{d}^{R}$
???

Suggests sequential breaking:

$$
\begin{gathered}
\mathbf{S U ( 3 )}{ }^{\mathbf{3}} \underset{\mathrm{mt}, \mathrm{mb}}{\mathbf{S U}(2)^{\mathbf{3}}} \xrightarrow[\mathbf{m c}, \mathbf{m s}, \boldsymbol{\theta} \mathbf{C}]{\cdots} \cdots \cdots \\
Y_{u} \equiv \frac{\left\langle\chi^{L}\right\rangle\left\langle\chi_{u}^{R \dagger}\right\rangle}{\Lambda_{f}^{2}}+\frac{\left\langle\chi_{u}^{\prime L}\right\rangle\left\langle\chi_{u}^{\prime R \dagger}\right\rangle}{\Lambda_{f}^{2}}=\left(\begin{array}{ccc}
0 & \sin \theta y_{c} & 0 \\
0 & \cos \theta y_{c} & 0 \\
0 & 0 & y_{t}
\end{array}\right) \\
Y_{d} \equiv \frac{\left\langle\chi^{L}\right\rangle\left\langle\chi_{d}^{R \dagger}\right\rangle}{\Lambda_{f}^{2}}+\frac{\left\langle\chi_{d}^{\prime L}\right\rangle\left\langle\chi_{d}^{\prime R \dagger}\right\rangle}{\Lambda_{f}^{2}}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & y_{s} & 0 \\
0 & 0 & y_{b}
\end{array}\right)
\end{gathered}
$$

## Towards a realistic 3 family spectrum

e.g. replicas of $\chi^{L}, \chi_{u}^{R}, \chi_{d}^{R}$
???

Suggests sequential breaking:

$$
\begin{aligned}
& \mathrm{SU}(\mathbf{3})^{\mathbf{3}} \xrightarrow[\mathrm{mt}, \mathrm{mb}]{ } \mathrm{SU}(\mathbf{2})^{\mathbf{3}} \xrightarrow[\mathrm{mc}, \mathrm{~ms}, \theta_{\mathrm{C}}]{ }{ }^{\ldots . . . . . . . . .}
\end{aligned}
$$

$$
\begin{aligned}
& Y_{d} \equiv \frac{\left\langle\chi^{L}\right\rangle\left\langle\chi_{d}^{R \dagger}\right\rangle}{\Lambda_{f}^{2}}+\frac{\left\langle\chi_{d}^{\prime L}\right\rangle\left\langle\chi_{d}^{\prime R \dagger}\right\rangle}{\Lambda_{f}^{2}}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & y_{s} & 0 \\
0 & 0 & y_{b}
\end{array}\right) .
\end{aligned}
$$

* From bifundamentals: $\left\langle y_{\mathrm{u}}\right\rangle=\left(\begin{array}{ccc}0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_{t}\end{array}\right)$

$$
\left\langle\mathcal{Y}_{\mathrm{d}}\right\rangle=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & y_{b}
\end{array}\right)
$$

* From fundamentals $\chi$ : $y_{c}, y_{s}$ and $\theta_{C}$

Towards a realistic 3 family spectrum

## Combining fundamentals and bi-fundamentals

i.e. combining $d=5$ and $d=6$ Yukawa operators

$$
\begin{gathered}
\Sigma_{u} \sim(3, \overline{3}, 1), \quad \Sigma_{d} \sim(3,1, \overline{3}), \quad \Sigma_{R} \sim(1,3, \overline{3}), \\
\chi_{u}^{L} \in(3,1,1), \quad \chi_{u}^{R} \in(1,3,1), \quad \chi_{d}^{L} \in(3,1,1), \quad \chi_{d}^{R} \in(1,1,3) .
\end{gathered}
$$

The Yukawa Lagrangian up to the second order in $1 / \Lambda_{f}$ is given by:

$$
\mathscr{L}_{Y}=\bar{Q}_{L}\left[\frac{\Sigma_{d}}{\Lambda_{f}}+\frac{\chi_{d}^{L} \chi_{d}^{R \dagger}}{\Lambda_{f}^{2}}\right] D_{R} H+\bar{Q}_{L}\left[\frac{\Sigma_{u}}{\Lambda_{f}}+\frac{\chi_{u}^{L} \chi_{u}^{R \dagger}}{\Lambda_{f}^{2}}\right] U_{R} \tilde{H}+\text { h.c. },
$$

* What is the role of the neutrino flavour group?

$$
\text { e.g. } O(2)_{\mathrm{NR}} \quad \text {... leptons }
$$

e.g. seesaw with approximately conserved lepton number

$$
\mathcal{L}_{\mathcal{M}_{\nu}}=\left(\bar{\ell}_{L}, \bar{N}^{c}, \bar{N}^{\prime c}\right)\left(\begin{array}{ccc}
0 & v Y & v Y^{\prime} \\
v Y^{\top} & 0 & \mathbf{M}^{\mathrm{T}} \\
v Y^{\prime T} & \mathbf{M} & 0
\end{array}\right)\left(\begin{array}{c}
\ell_{L}^{c} \\
N \\
N^{\prime}
\end{array}\right)
$$

* What is the role of the neutrino flavour group?

$$
\text { e.g. } O(2)_{\mathrm{NR}} \quad \text {... leptons }
$$

e.g. seesaw with approximately conserved lepton number

$$
\mathcal{L}_{\text {mass }}=\bar{\ell}_{L} \phi Y_{E} E_{R}+\bar{\ell}_{L} \tilde{\phi} \tilde{Y}_{\nu}\left(N_{1}, N_{2}\right)^{T}+M\left(\bar{N}_{1} N_{1}^{c}+\bar{N}_{2} N_{2}^{c}\right)+\text { h.c. }
$$

$$
\tilde{Y}_{\nu}=\frac{1}{\sqrt{2}} U_{P M N S} f_{m_{\nu}}\left(\begin{array}{cc}
y+y^{\prime} & -i\left(y-y^{\prime}\right) \\
i\left(y-y^{\prime}\right) & y+y^{\prime}
\end{array}\right)
$$

$$
\begin{gathered}
U(3)_{\ell_{L}} \times \cdot U(3)_{E_{R}} \times O(2)_{N} \\
Y_{E}=\frac{\left.y_{E}\right\rangle}{\Lambda_{f}} \sim(3, \overline{3}, 1) ; \quad\left(Y, Y^{\prime}\right)=\frac{<y_{v}>}{\Lambda} \sim(3,1,2) \\
<y_{E}>\propto\left(\begin{array}{ccc}
\mathrm{m}_{\mathrm{e}} & 0 & 0 \\
0 & \mathrm{~m}_{\mu} & 0 \\
0 & 0 & \mathrm{~m}_{\tau}
\end{array}\right)<y_{v}>\propto U_{P M N S}\left(\begin{array}{cc}
0 & 0 \\
\sqrt{m_{\nu_{2}}} & 0 \\
0 & \sqrt{m_{\nu_{3}}}
\end{array}\right)\left(\begin{array}{cc}
-i y & i y^{\prime} \\
y & y^{\prime}
\end{array}\right)
\end{gathered}
$$

* e- $\mu, \mu-\tau$ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez ${ }^{2}$;

$$
\operatorname{Br}(\mu \rightarrow e \gamma) / \operatorname{Br}(\tau \rightarrow e \gamma) \quad \operatorname{Br}(\mu \rightarrow e \gamma) / \operatorname{Br}(\tau \rightarrow \mu \gamma)
$$




IH



## Gavela, Hambye, Hernandez²;

Degeneracy in the Majorana phase $\alpha$



Figure 3: Left: Ratio $B_{e \mu} / B_{e \tau}$ for the normal hierarchy (solid) and the inverse hierarchy (dashed) as a function of $\alpha$ for $\left(\delta, s_{13}\right)=(0,0.2)$. Right: the same for the ratio $B_{c \mu} / B_{\mu \tau}$.


Figure 5: $m_{e e}$ as a function of $\alpha$ for the normal (solid) and inverted (dashed) hierarchies, for $\left(\delta, s_{13}\right)=(0,0.2)$.

Gavela, Hambye, Hernandez ${ }^{2}$;


* Alonso + Li, 2010, MINSIS report: possible suppresion of $\mu$-e transitions for large $\theta_{13}$
* e- $\mu, \mu-\tau$ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez² 09 ; .....

* Alonso $+\mathrm{Li}, 2010:$ possible suppresion of $\mu$-e transitions ->important impact of $\nu_{\mu}-v_{\tau}$ at a near detectors

$$
B_{\mu \rightarrow e \gamma} \propto\left|Y_{N_{e}} Y_{N_{\mu}}\right|^{2}
$$

i.e. $\quad Y_{N}^{T} \simeq y\left(\begin{array}{c}e^{i \delta} s_{13}+e^{-i \alpha} s_{12} r^{1 / 4} \\ s_{23}\left(1-\frac{\sqrt{r}}{2}\right)+e^{-i \alpha} r^{1 / 4} c_{12} c_{23} \\ c_{23}\left(1-\frac{\sqrt{r}}{2}\right)-e^{-i \alpha} r^{1 / 4} c_{12} s_{23}\end{array}\right) \quad r=\frac{\left|\Delta m_{12}^{2}\right|}{\left|\Delta m_{13}^{2}\right|}$ Normal hierarchy,$~ . ~$

* e- $\mu, \mu-\tau$ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez² 09;

* Alonso +Li , 2010: possible suppresion of $\mu$-e transitions ->important impact of $\nu_{\mu}-v_{\tau}$ at a near detectors


We find that there are regions where an experiment as MINSIS
would improve the present bounds on our Model


- Three types of models yield the Weinberg operator at tree level


Type I

$$
\mathrm{m}_{\mathrm{v}} \sim \mathrm{v}^{2} \mathrm{Y}_{\mathrm{N}}^{\top} \frac{1}{\mathrm{M}_{\mathrm{N}}} \mathrm{Y}_{\mathrm{N}}
$$



Type II

$$
m_{v} \sim v^{2} Y_{\Delta} \frac{\mu}{M_{\Delta}{ }^{2}}
$$



Type III

$$
\mathrm{m}_{\mathrm{v}} \sim \mathrm{v}^{2} \mathrm{Y}_{\Sigma}^{\top}{\frac{1}{\mathrm{M}_{\Sigma}}}_{\Sigma}
$$

Use the flavour symmetry of the SM with masless fermions:

$$
\mathrm{G}_{\mathrm{f}}=\mathrm{SU}(3)_{\mathrm{Q}_{\mathrm{L}}} \times \mathrm{SU}(3)_{\mathrm{U}_{\mathrm{R}}} \times \quad \mathrm{SU}(3)_{\mathrm{D}_{\mathrm{R}}} \times \mathrm{U}(1) \mathrm{s}
$$

which is broken by Yukawas:

*In the $O(2)$ model used before: $\operatorname{tgh} 2 \omega=\frac{y^{2}-y^{\prime 2}}{y^{2}-y^{\prime 2}}$ and

$$
\operatorname{tg} 2 \theta=\sin 2 \alpha \frac{2 \sqrt{m_{\nu_{2}} m_{\nu_{1}}}}{m_{\nu_{2}}-m_{\nu_{1}}} \frac{\mathrm{y}^{2}-\mathrm{y}^{\prime 2}}{\mathrm{y}^{2}-\mathrm{y}^{\prime 2}}
$$

$$
\alpha=\pi / 4 \text { or } 3 \pi / 4
$$

*If we had used instead a flavor $\operatorname{SU}(2)$ model $\sinh 2 \omega=0-->$ NO MIXING

## Some good ideas:

"Partial compositeness":
D.B. Kaplan-Georgi in the 80s proposed a composite Higgs:

* Higgs light because the whole Higgs doublet is multiplet of goldstone bosons

They explored SU(5)--> SO(5).
$\underset{\text { (Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison) }}{\text { Explicit breaking of } \operatorname{SU}(2) x U(1) \text { symmetry via external gauged } U(1) ~}$
(Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison)
Nowadays SO(5)--> SO(4) and explicit breaking via SM weak interaction (Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino...)

SO(6) --> SO(5) to get also DM (Frigerio, Pomarol, Riva, Urbano)

Anarchy: alive with not so small $\theta_{13}$ and not $\theta_{23}$ not maximal
no symmetry in the lepton sector, just random numbers


- Does not relate mixing to spectrum
- Does not address both quarks and leptons


## Seesaw models


$\mathrm{M} \sim 1 \mathrm{TeV}$ is suggested by electroweak hierarchy problem



$$
\begin{aligned}
\delta m_{H}^{2}= & -3 \frac{\lambda_{3}}{16 \pi^{2}}\left[\Lambda^{2}+M_{\Delta}^{2}\left(\log \frac{M_{\Delta}^{2}}{\Lambda^{2}}-1\right)\right] \\
& -\frac{\mu_{\Delta}^{2}}{2 \pi^{2}} \log \left(\left|\frac{M_{\Delta}^{2}-\Lambda^{2}}{M_{\Delta}^{2}}\right|\right)
\end{aligned}
$$


(Abada, Biggio, Bonnet, Hambye, M.B.G.)

$$
\delta m_{H}^{2}=-3 \frac{Y_{\Sigma}^{\dagger} Y_{\Sigma}}{16 \pi^{2}}\left[2 \Lambda^{2}+2 M_{\Sigma}^{2} \log \frac{M_{\Sigma}^{2}}{\Lambda^{2}}\right]
$$

Ratios of two e- $\mu$ transitions may depend only on $\mathbf{m}_{\mathbf{N}}$ (Chu, Dhen, Hambye 11) and $e-\mu$ conversion may vanish ${ }^{(\text {Dinh et al 12 }}$ 12)
we obtain:

$$
\left.m_{N}^{2}\right|_{0}=M_{W}^{2} \exp \left(\frac{\frac{9}{8}(A-Z)+\left(\frac{9}{8}+\frac{310_{2}^{2}}{12}\right) Z}{\frac{3}{8}(A-Z)+\left(\frac{4_{2}^{2}}{3}-\frac{\frac{3}{8}}{8}\right) Z}\right) \quad(\alpha Z \ll 1)
$$

$\mu$-e conversion
$\mu-->$ e $\gamma$

$\mu$-e conversion

$$
\mu-->\text { e e e }
$$


...typically vanishes for $m_{N}$ in $2-7 \mathrm{TeV}$ range
(Alonso, Dhen, Hambye, B.G.)

## *3 families with $\mathrm{O}(2)_{\mathrm{NR}}$ :

- 3 light +2 heavy $N$ degenerate: bad $\theta_{12}$ quadrant. It cannot accomodate data!
- 3 light +3 heavy N : OK for $\boldsymbol{\theta}_{23}$ maximal and spectrum
experimentally $\sin 2 \theta_{23}=0.41+-0.03$ or $0.59+-0.02$
Gonzalez-Garcia, Maltoni, Salvado, Schwetz Sept. 2012
*What about the other angles?

$$
\left(\begin{array}{cc}
(\mathrm{O}(2) \\
0 & 0
\end{array}()\right)_{3 \times 3}
$$

