SM scalar potential and stability

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Talk at Moriond 2013

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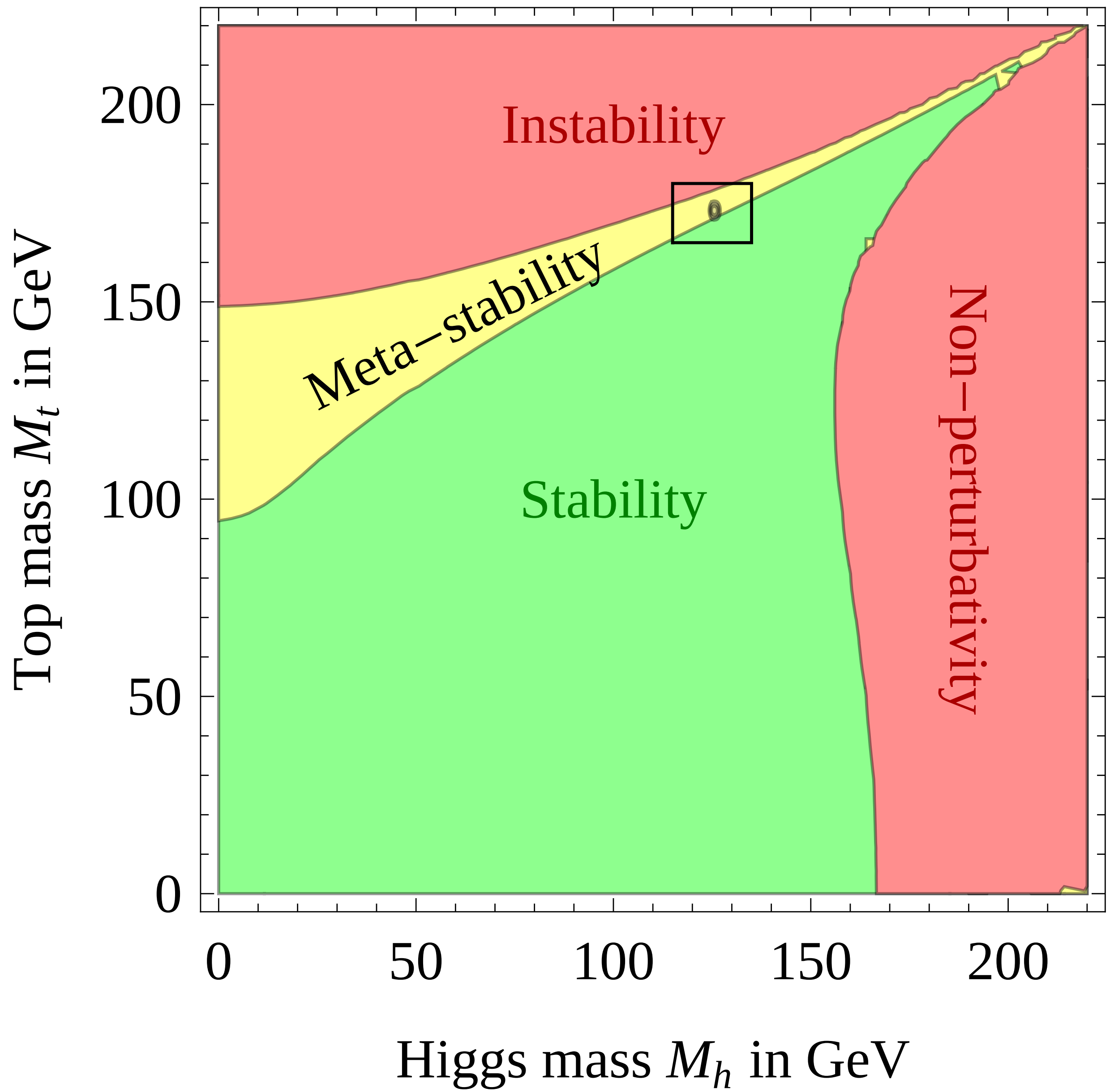
Trusting the SM up to the Planck scale



The SM scalar potential

Including quantum corrections, parameters get renormalised around the vev:

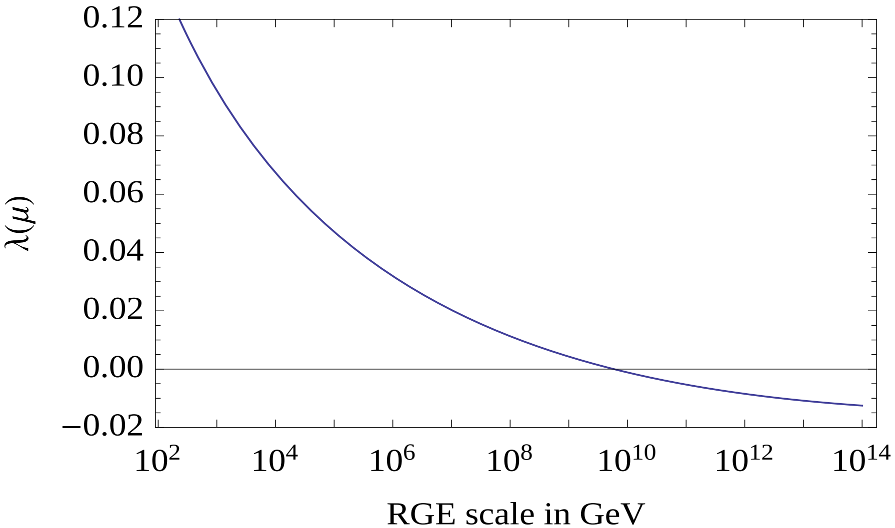
$$V(H) \approx \underbrace{-\frac{m^2(\mu\sim h)}{2}|H|^2}_{\text{negligible at } h\gg v} + \lambda(\mu\sim h)|H|^4 \qquad H = \begin{pmatrix} 0 \\ v + h/\sqrt{2} \end{pmatrix}$$

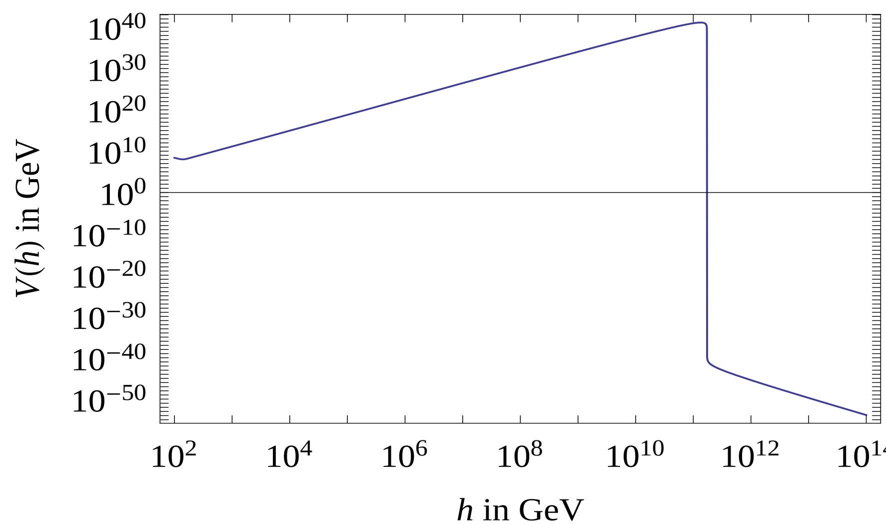
The RGE running of λ is computed solving:

$$(4\pi)^2 \frac{d\lambda}{d\ln\mu} = -6y_t^4 + \frac{9}{8}g_2^4 + \frac{27}{200}g_1^4 + \frac{9}{20}g_2^2 g_1^2 + \lambda(12y_t^2 - 9g_2^2 + \frac{9g_1^2}{5}) + 24\lambda^2 + \text{higher loops}$$

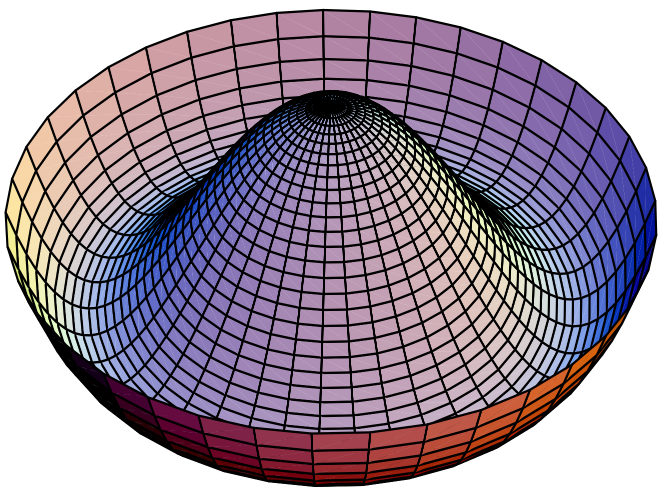
A too heavy Higgs makes $\lambda(\mu)$ non-perturbative at large energy

A too heavy top makes $\lambda(\mu)$ negative at large energy...
...so the SM potential falls down at large vev





Illustrative



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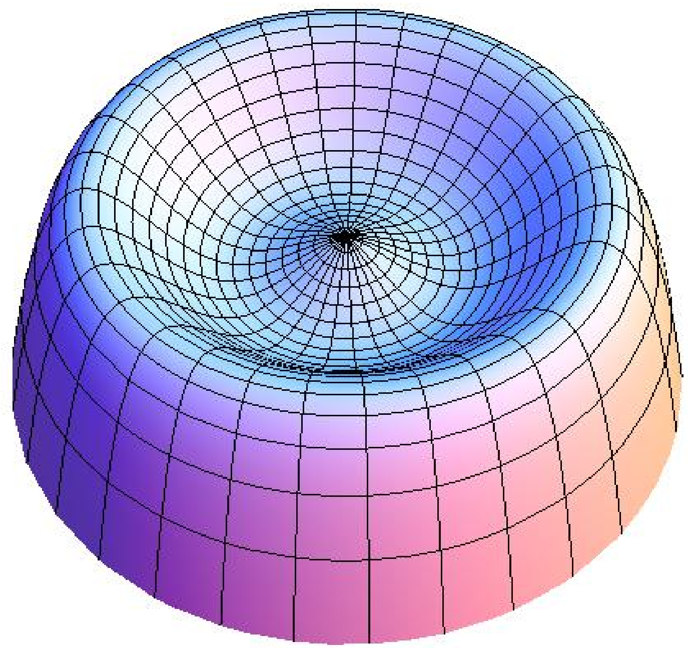
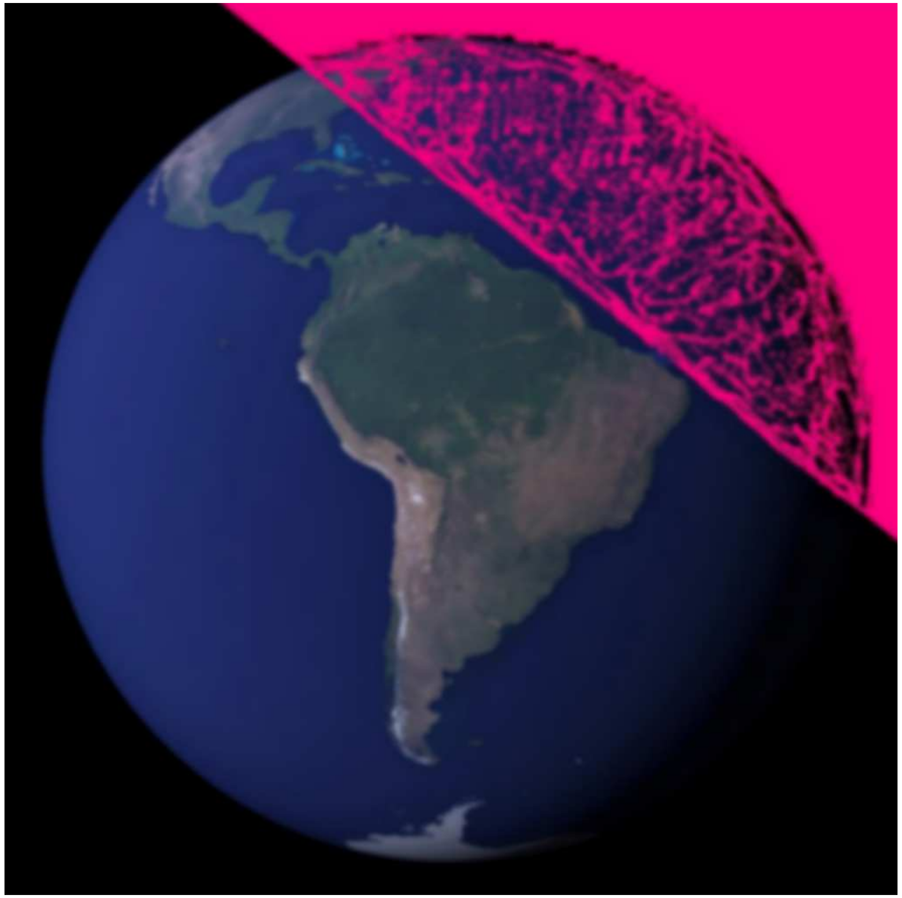


If your mexican hat turns out to be a dog bowl you have a problem...

Vacuum decay

If our vacuum is only a local minimum of the potential, at some point **quantum tunnelling towards the true minimum** will happen.
The process was studied by Coleman and is ‘similar’ to boiling of water (quantum field theory is formally similar to thermal field theory for matter...).
A bubble of negative-energy true vacuum can appear anywhere and anytime and start expanding at the speed of light.



The probability density of vacuum decay is $dp/dV\,dt = e^{-S}/R^4$, suppressed by the action S of the classical field configuration $h(r)$ that interpolates vacua

$h(\infty) =$ unstable vacuum $\qquad h(0) \approx$ other side of the potential barrier

Computing vacuum decay in the SM

The SM potential at tree level $V \simeq -|\lambda H|^4$ has neither the true vacuum nor the false vacuum nor the barrier!? Nevertheless the computation makes sense:

$$h(r) = \sqrt{\frac{2}{|\lambda(1/R)|}}\frac{2R}{r^2+R^2} \qquad r^2 = \vec{x}^2 - t^2.$$

$S > 0$ thanks to the kinetic term $|\partial_\mu H|^2$; due to scale invariance, the size R of the bubble is arbitrary so S is not fixed:

$$S = \int d^4x\, \mathscr{L}_{\rm SM}(h(r)) = \frac{8\pi^2}{3}\frac{1}{|\lambda(\mu=1/R)|} \ .$$

Our vacuum is **unstable** (decay rate faster than the age of the universe T_U) if

$$S \lesssim 500 \sim \frac{1}{4}\ln M_{\rm Pl}T_U \quad \Rightarrow \quad \lambda(\mu = \frac{1}{R}) \lesssim -0.05$$

and **meta-stable** if λ is negative but small (decay slower than T_U).

(Quantum corrections to the SM vacuum decay rate, computed in Isidori, Ridolfi, Strumia, Nucl. Phys. B609 (2001) 387, hep-ph/0104016, fix $R \sim 1/10^{17}$ GeV, where λ is minimal. Then gravitational corrections were computed in Isidori, Rychkov, Strumia, Tetradis, Phys.Rev. D77 (2008) 025034, 0712.0242, that also proposed inflation from $V_{\rm SM}$ with $M_h \approx 125$ GeV)

Living at the border

M_h and M_t lie around the meta-stability region so more precision is needed.

Extracting from data the fundamental SM parameters and extrapolating them to large energies is useful for other purposes.

Threshold corrections at the weak scale

for	1 loop	2 loop	3 loop
g_3	full	full	—
y_t	full	$\mathcal{O}(\alpha_3^2), \mathcal{O}(\alpha_3\alpha_{1,2})$	$\mathcal{O}(\alpha_3^3)$
$g_{1,2}$	full	?	—
λ	full	for $g_{1,2}=0$	—
m	full	for $g_{1,2}=0$	—

Renormalization Group Equations

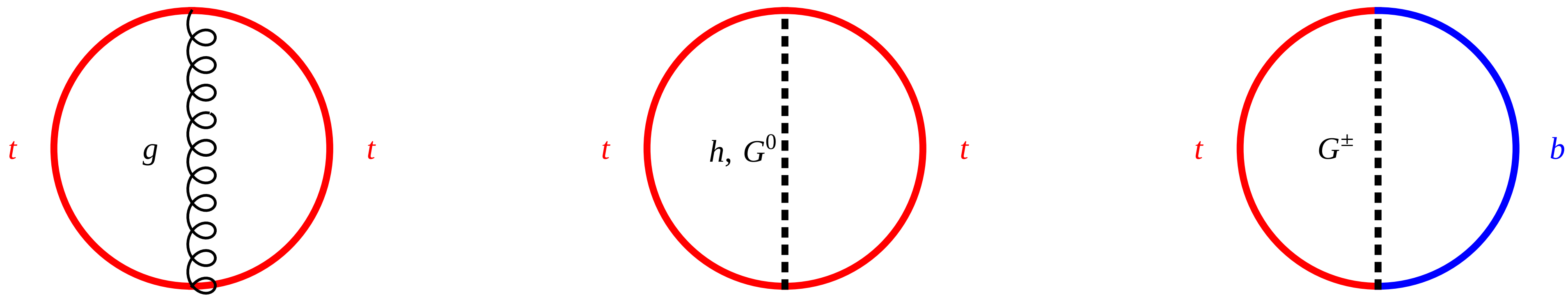
for	1 loop	2 loop	3 loop
$g_{1,2,3}$	full	full	full
y_t	full	full	full
λ	full	full	for $g_{1,2}=0$
m	full	full	—

λ at NNLO in g_3, y_t

This was the main missing effect because g_3 and y_t get big at low E:

$$\lambda|_{\text{tree}} = \frac{M_h^2}{2V^2}, \qquad \lambda|_{1\text{ loop}} \sim -\frac{y_t^4(\bar\mu)}{(4\pi)^2} \qquad \lambda|_{2\text{ loop}} \sim \pm ? \frac{y_t^4}{(4\pi)^2}\frac{g_3^2+y_t^2}{(4\pi)^2}$$

The scale dependence of y_t was equivalent to a ± 3 GeV uncertainty on M_h. The result in the limit $M_h^2 \ll 4M_t^2$ was extracted from the known 2 loop potential



$$\lambda(\mu = M_t)|_{2\text{ loop}} = -\frac{G_\mu^2 M_t^4}{\pi^4} g_3^2 + \frac{8\sqrt{2}G_\mu^3 M_t^6}{(4\pi)^4}\left(30+\pi^2\right)$$

Negative. Like a -1 GeV shift in M_h towards instability

From `http://arxiv.org/abs/1205.6497` with Degrassi, di Vita, Miró, Espinosa, Giudice, Isidori

All SM couplings at full NNLO

λ at NLLO

Our *preliminary* result

$$\lambda(\mu = M_t)|_{\text{2 loop}} = -\frac{39.1 + 0.13\Delta r^{(2)}}{(4\pi)^4}$$

is again negative so again more instability

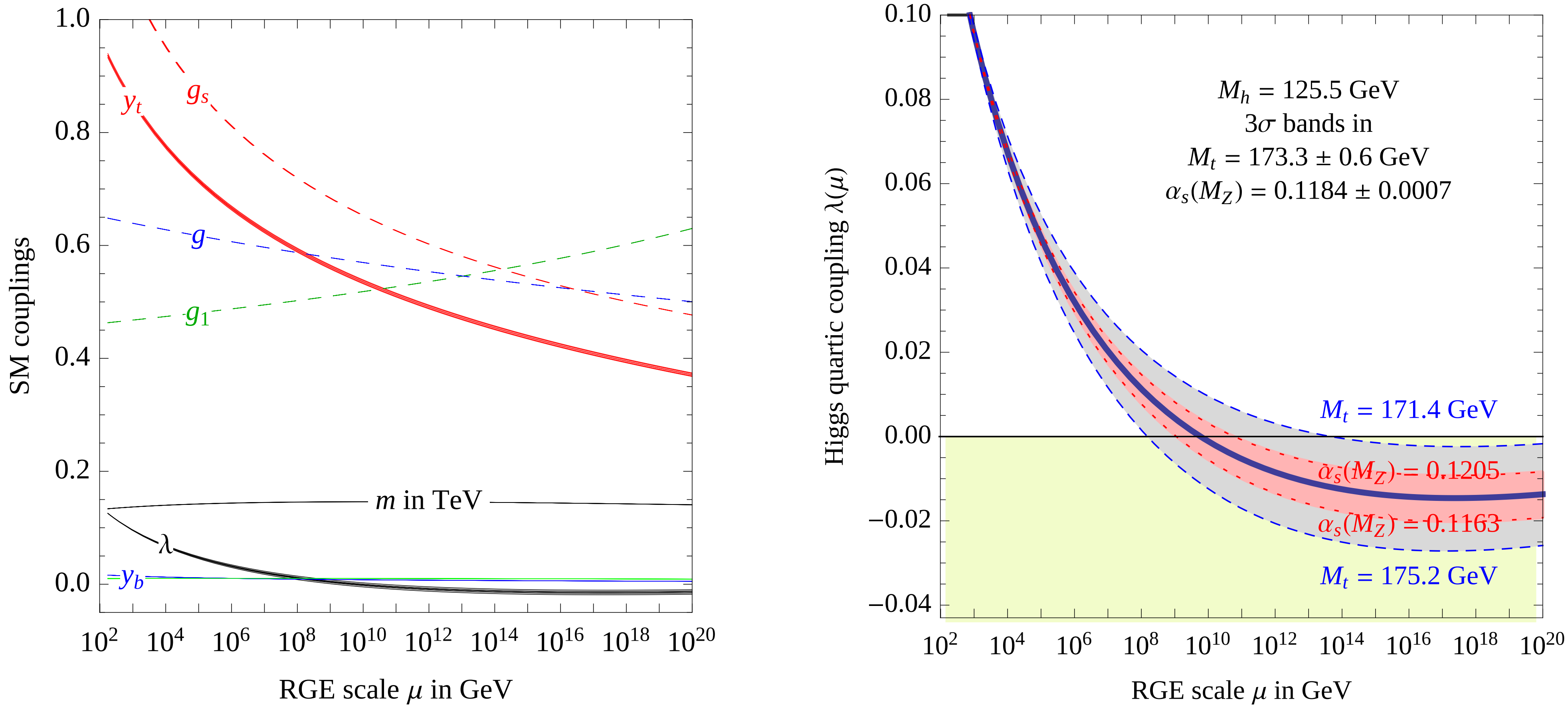
$$\lambda(\mu = M_t) = 0.12559 + 0.00205\left(\frac{M_h}{\text{GeV}} - 125\right) +$$

$$-0.00004\left(\frac{M_t}{\text{GeV}} - 173.15\right) \pm 0.00140_{\text{th}}$$

*!#**@$**!??

(with Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio)

From the EW scale to the Planck scale

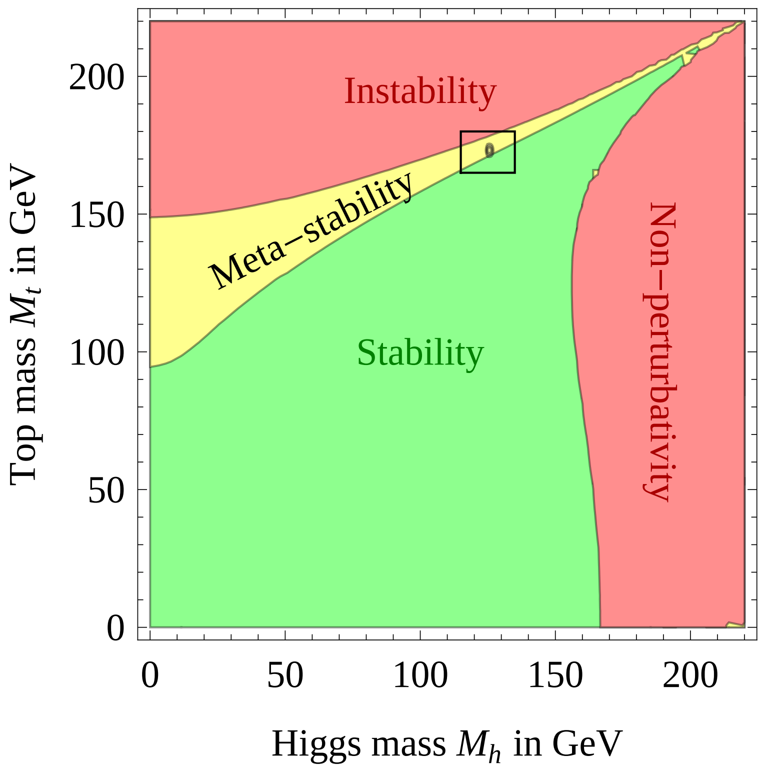


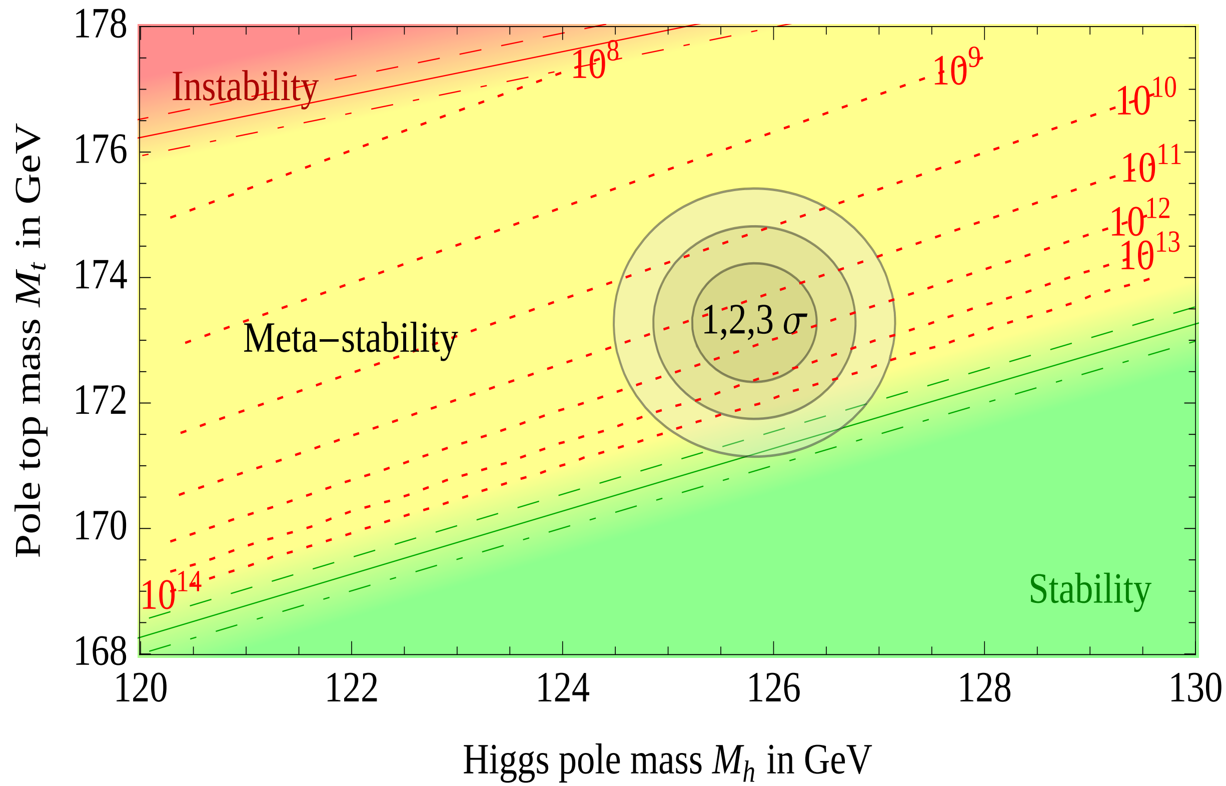
$$\lambda(\mu = M_{\rm Pl}) = -0.0144 + 0.0028\left(\frac{M_h}{\rm GeV} - 125\right) \pm 0.0047_{M_t} \pm 0.0018_{\alpha_s} \pm 0.0028_{\rm th}$$

For the measured masses both λ and its β-function vanish around $M_{\rm Pl}$!!?

(This would be the main message bla bla quantum gravity bla bla)

Meta-stability favoured at about 2σ





$$M_h\ [\mathrm{GeV}] > 129.8 + 1.4\left(\frac{M_t\ [\mathrm{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\mathrm{th}}\ .$$

$M_h = 125.8 \pm 0.4\,\mathrm{GeV}$ (naive average of latest results)

The main uncertainty is M_t

Can we trust present and future measurements at the quoted level of precision?

$$M_t = \begin{cases} 173.2 \pm 0.9 \text{ GeV} & \text{Tevatron} \\ 173.2 \pm 0.94 \text{ GeV} & \text{CMS} \\ 174.5 \pm 2.4 \text{ GeV} & \text{ATLAS} \end{cases}$$

Better not to know how sausages are made. Monte Carlo are used to reconstruct the pole t mass from its decay products, that contain j, ν and initial state radiation. Reconstructing the pig mass from the sausages is not easy.

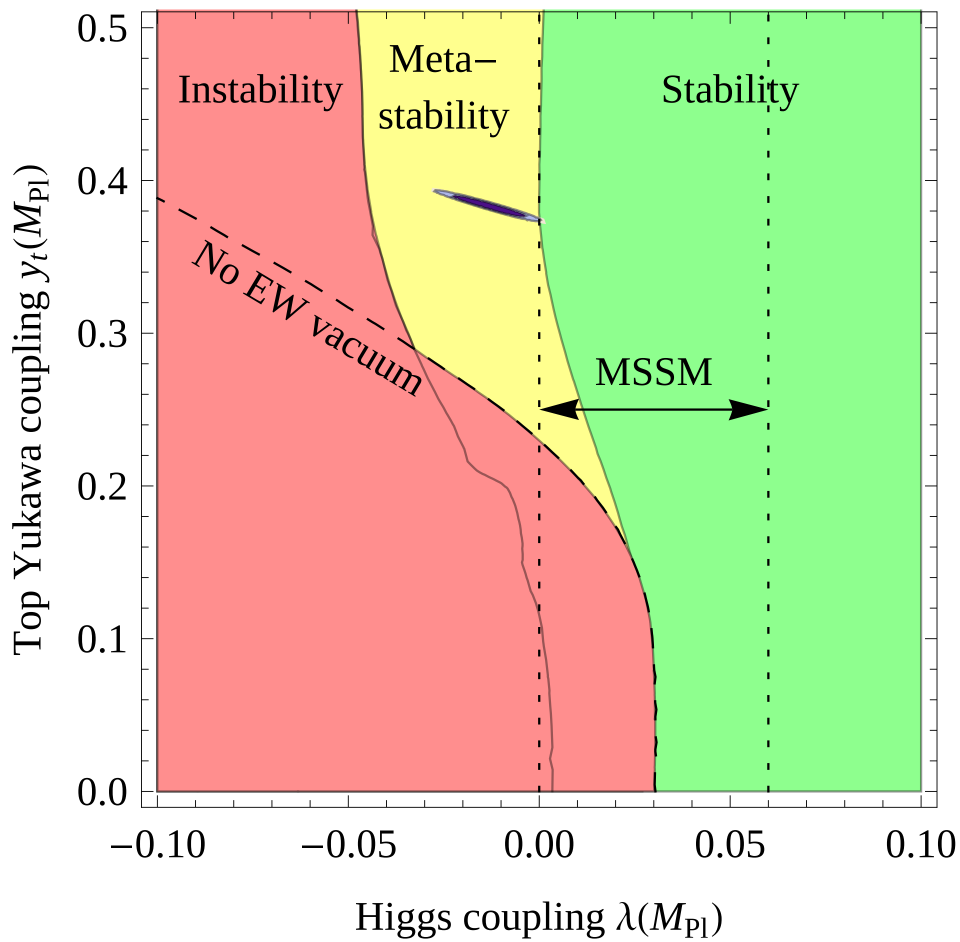
Furthermore, quarks are not free particles, so the top pole mass does not exist. In practice this means an uncertainty of $\pm\Lambda_{\text{QCD}} \sim 0.3\,\text{GeV}$. And $\Gamma_t = 1.2\,\text{GeV}$..

m_t can be extracted from the cross-sections, but with $\pm$few GeV uncertainty.

A linear collider can do better, if it exists and reaches the $t\bar{t}$ threshold.

In terms of Planck-scale parameters

There is a new red region, with $\lambda < 0$ at the weak scale

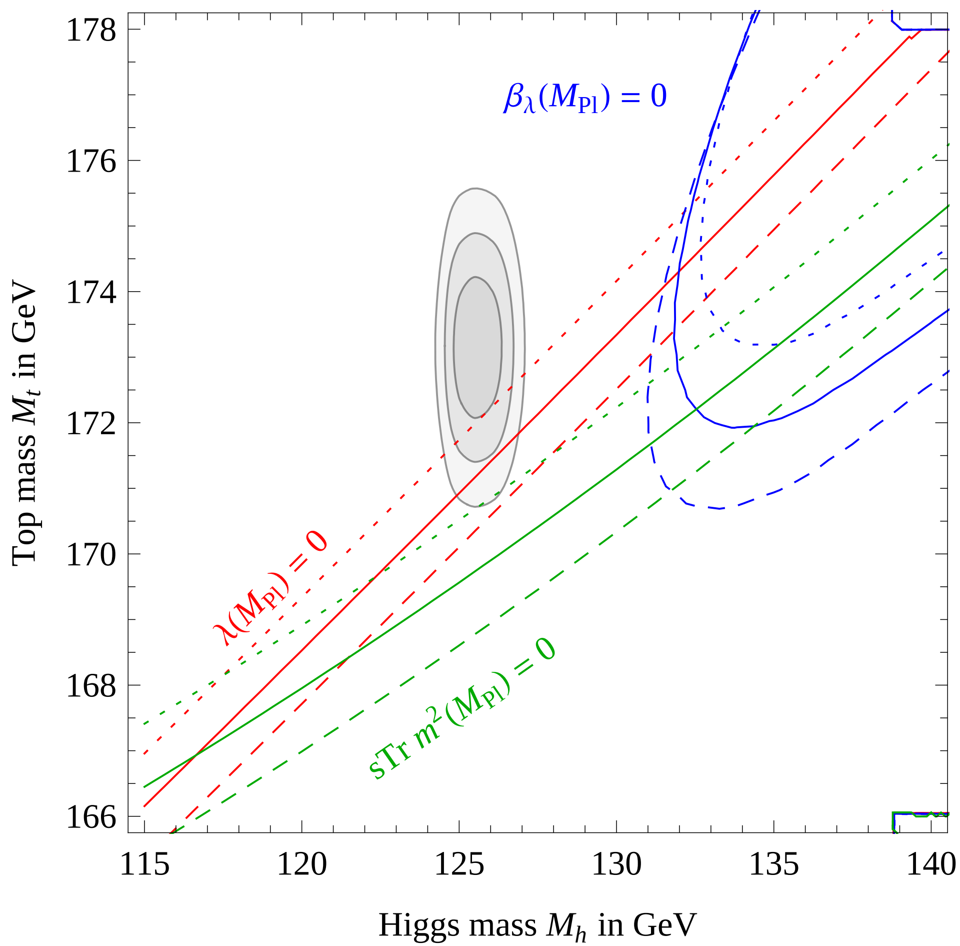


Like in a Rorschach test, different physicists see different things...

If $\lambda(M_{\rm Pl}) = 0$, then M_t has the minimal value needed for stability

Indeed smaller M_t leads to negative $\beta_\lambda(M_{\rm Pl})$ such that λ transists negative

Veltman throat at the Planck scale?

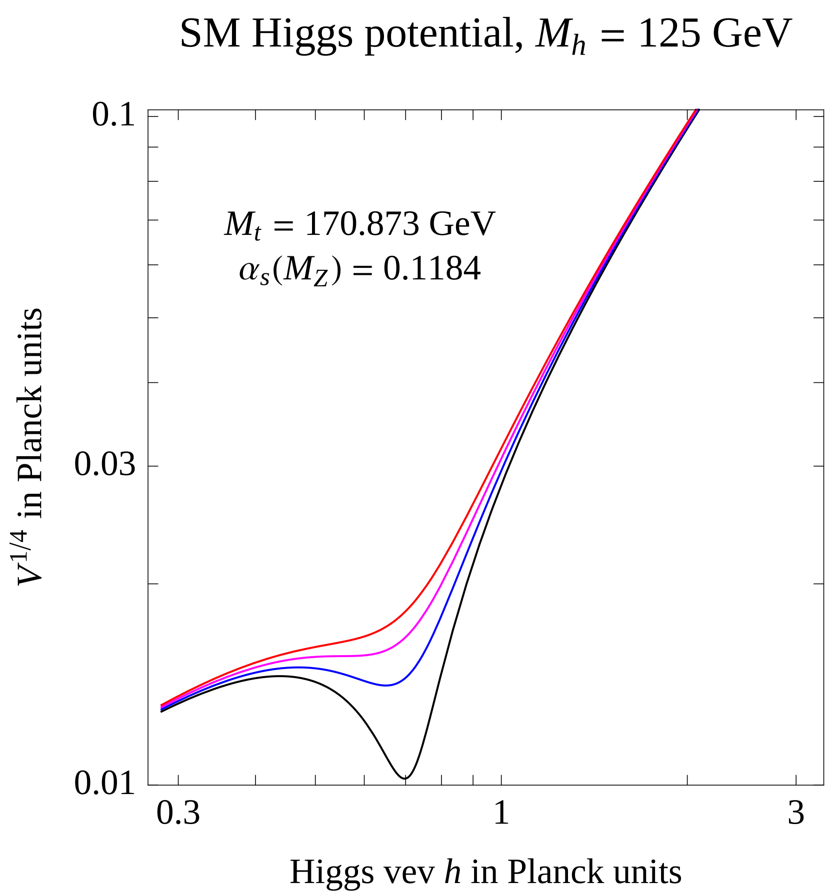


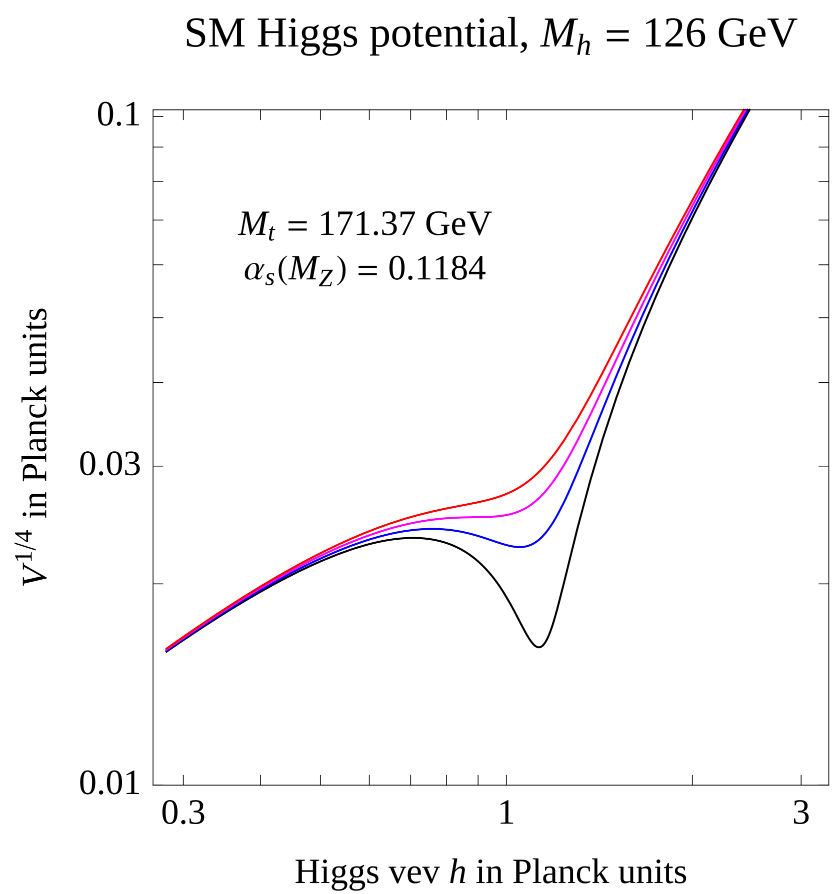
Excluded

Cut-off for $y_t^2\Lambda^2$ must be lower than for $g^2\Lambda^2$

Implications: Higgs inflation?

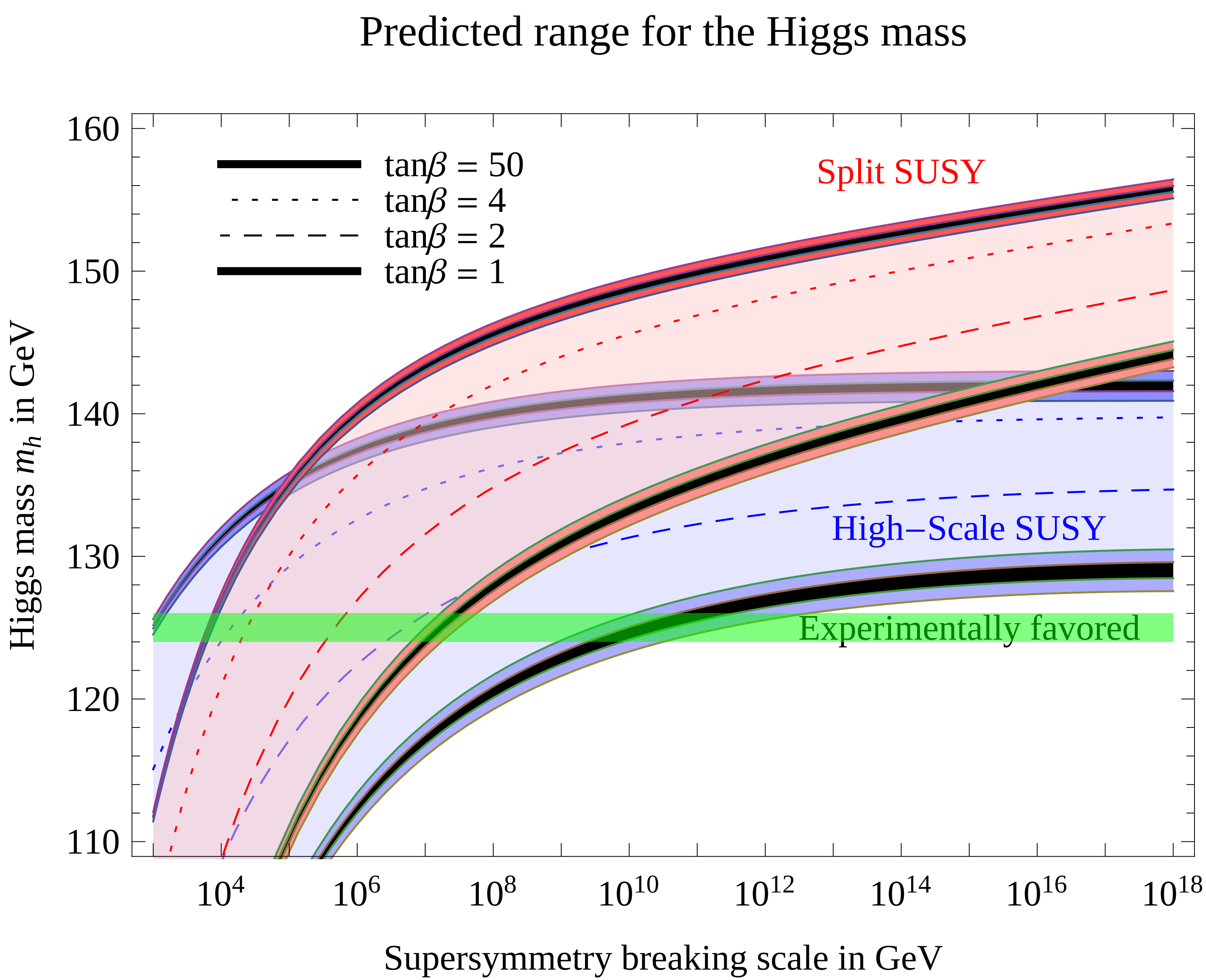
A) Criticality allows inflation with a plateau or a second minimum. Needs adjustments. In practice it predicts $\lambda = \beta_\lambda = 0$ and so...





B) Inflation with a non-minimal coupling to gravity, $|H|^2 R$. Maybe it allows inflation or maybe the theory is uncontrollable. In practice it predicts $\lambda > 0$.

Getting the SUSY scale from M_h



Thickness is $\pm 1\sigma$ on α_3 and on M_t. Theory error is now ± 1 GeV. Extra uncertainties coming from unknown SUSY thresholds are not in the figure.

Conclusions

The main result of LHC might be M_h. At the stability/meta-stability border.

Precision computations tell that we live in the meta-stability region.
Important to compute higher orders and be sure that $M_t > 171.5\,\text{GeV}$.

Maybe $\lambda(M_{\rm Pl}) \sim \beta(\lambda(M_{\rm Pl})) \sim 0$ has some deep meaning...

Extrapolating the SM up to the Planck scale might be too much.
New physics can give absolute stability, see next talk.