$B \to K^* \ell\ell$

THE NEW FRONTIER

Of New Physics Searches in Flavor

Javier Virto

Universitat Autònoma de Barcelona

Based On…

THE NEW FRONTIER

arXiv:1202.4266[hep-ph]

arXiv:1207.2753[hep-ph]

arXiv:1303.xxxx[hep-ph]

In collaboration with…

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J. Matias   F. Mescia

M. Ramon

Roadmap

What do we Want?

- **Find** New Physics -- Establish deviations from the SM in flavor Physics
- **Measure** the New Physics
- **Identify** the New Physics -- Characterize its *fingerprints*

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- Establish **strong constraints** on NP...
- Model-independently
- or constrain/exclude NP models / paradigms

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Why flavor-changing (rare) Processes?

- Very **suppressed** in the SM -- through very particular mechanisms
- Very sensitive to NP -- Probe very high energy scales
- Complementary to Direct Searches -- and might provide a guideline

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Why $B \to K^*\ell\ell$?

- This is the object of this talk

Effective Operators for Flavor Physics

$$Q_1 = (\bar{s}_L\gamma_{\mu_1}T^a c_L)(\bar{c}_L\gamma^{\mu_1}T^a b_L),$$
$$Q_2 = (\bar{s}_L\gamma_{\mu_1}c_L)(\bar{c}_L\gamma^{\mu_1}b_L),$$
$$Q_3 = (\bar{s}_L\gamma_{\mu_1}b_L)\sum_q(\bar{q}\gamma^{\mu_1}q),$$
$$Q_4 = (\bar{s}_L\gamma_{\mu_1}T^a b_L)\sum_q(\bar{q}\gamma^{\mu_1}T^a q),$$
$$Q_5 = (\bar{s}_L\gamma_{\mu_1\mu_2\mu_3}b_L)\sum_q(\bar{q}\gamma^{\mu_1\mu_2\mu_3}q),$$
$$Q_6 = (\bar{s}_L\gamma_{\mu_1\mu_2\mu_3}T^a b_L)\sum_q(\bar{q}\gamma^{\mu_1\mu_2\mu_3}T^a q),$$

Non-Leptonic Decays

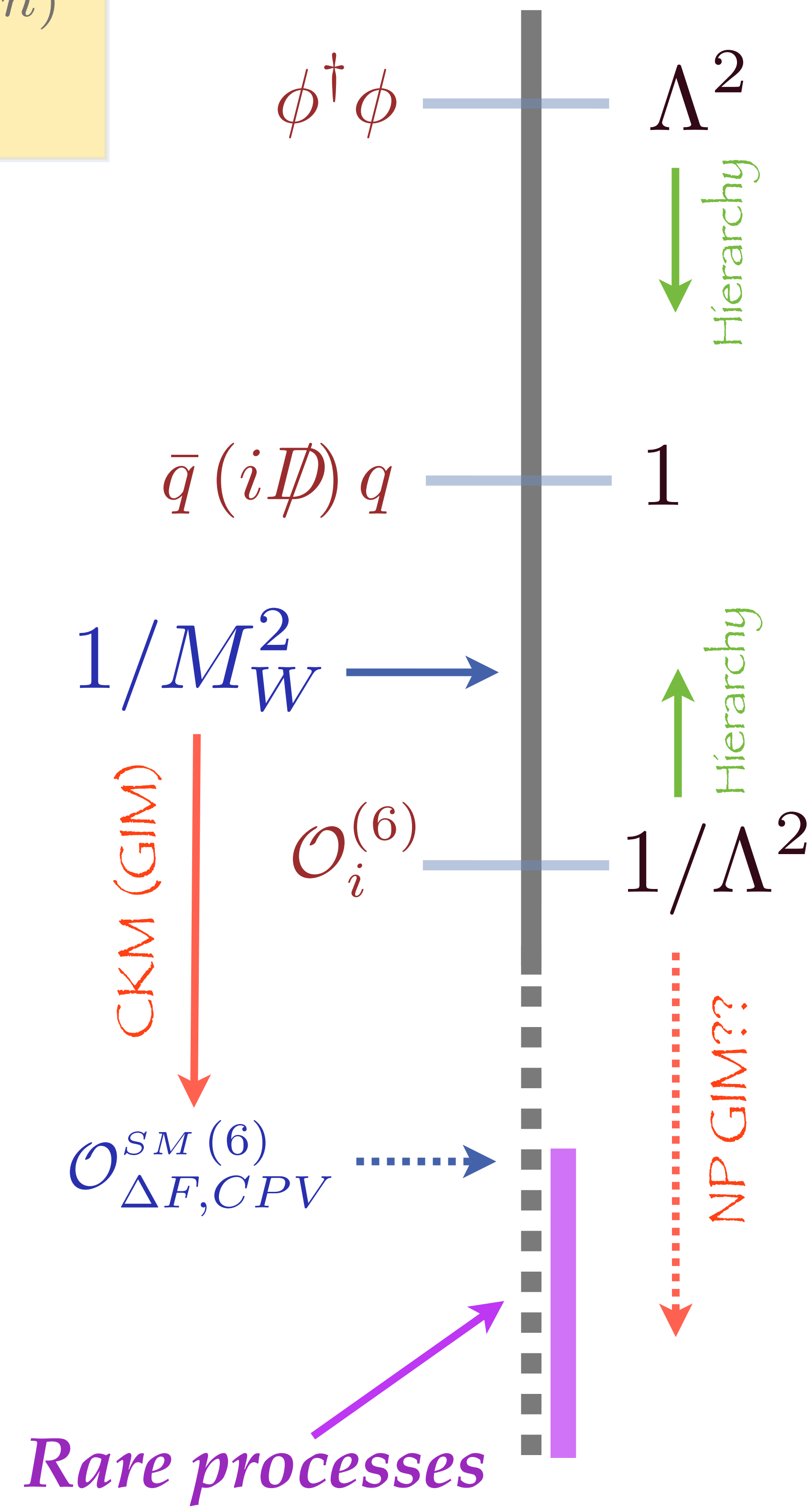
$$\mathcal{L} = \mathcal{L}^{SM} + \sum_n \frac{c_i}{\Lambda^n}\mathcal{O}_i^{(n)}$$

$$\begin{aligned}
\mathcal{Q}_1 &= \bar{d}_\alpha\gamma_\mu P_L s_\alpha\,\bar{d}_\beta\gamma^\mu P_L s_\beta\\
\mathcal{Q}_2 &= \bar{d}_\alpha P_L s_\alpha\,\bar{d}_\beta P_L s_\beta\\
\mathcal{Q}_3 &= \bar{d}_\alpha P_L s_\beta\,\bar{d}_\beta P_L s_\alpha\\
\mathcal{Q}_4 &= \bar{d}_\alpha P_L s_\alpha\,\bar{d}_\beta P_R s_\beta\\
\mathcal{Q}_5 &= \bar{d}_\alpha P_L s_\beta\,\bar{d}_\beta P_R s_\alpha
\end{aligned}$$

Neutral Meson Mixing

$$\mathcal{O}_7 = \frac{e}{16\pi^2}m_b(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}, \quad \mathcal{O}_9 = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l)$$
$$\mathcal{O}_7' = \frac{e}{16\pi^2}m_b(\bar{s}\sigma_{\mu\nu}P_L b)F^{\mu\nu}, \quad \mathcal{O}_9' = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu l)$$
$$\mathcal{O}_{10} = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu\gamma_5\ell), \quad \mathcal{O}_{10}' = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\gamma_5\ell),$$
$$\mathcal{O}_S = \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{\ell}\ell), \quad \mathcal{O}_S' = \frac{e^2}{16\pi^2}(\bar{s}P_L b)(\bar{\ell}\ell),$$
$$\mathcal{O}_P = \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{\ell}\gamma_5\ell), \quad \mathcal{O}_P' = \frac{e^2}{16\pi^2}(\bar{s}P_L b)(\bar{\ell}\gamma_5\ell),,$$

Radiative & Semileptonic Decays



RADIATIVE DECAYS vs SEMILEPTONIC

Constraints on **C7, C7'** (all other NP to zero).

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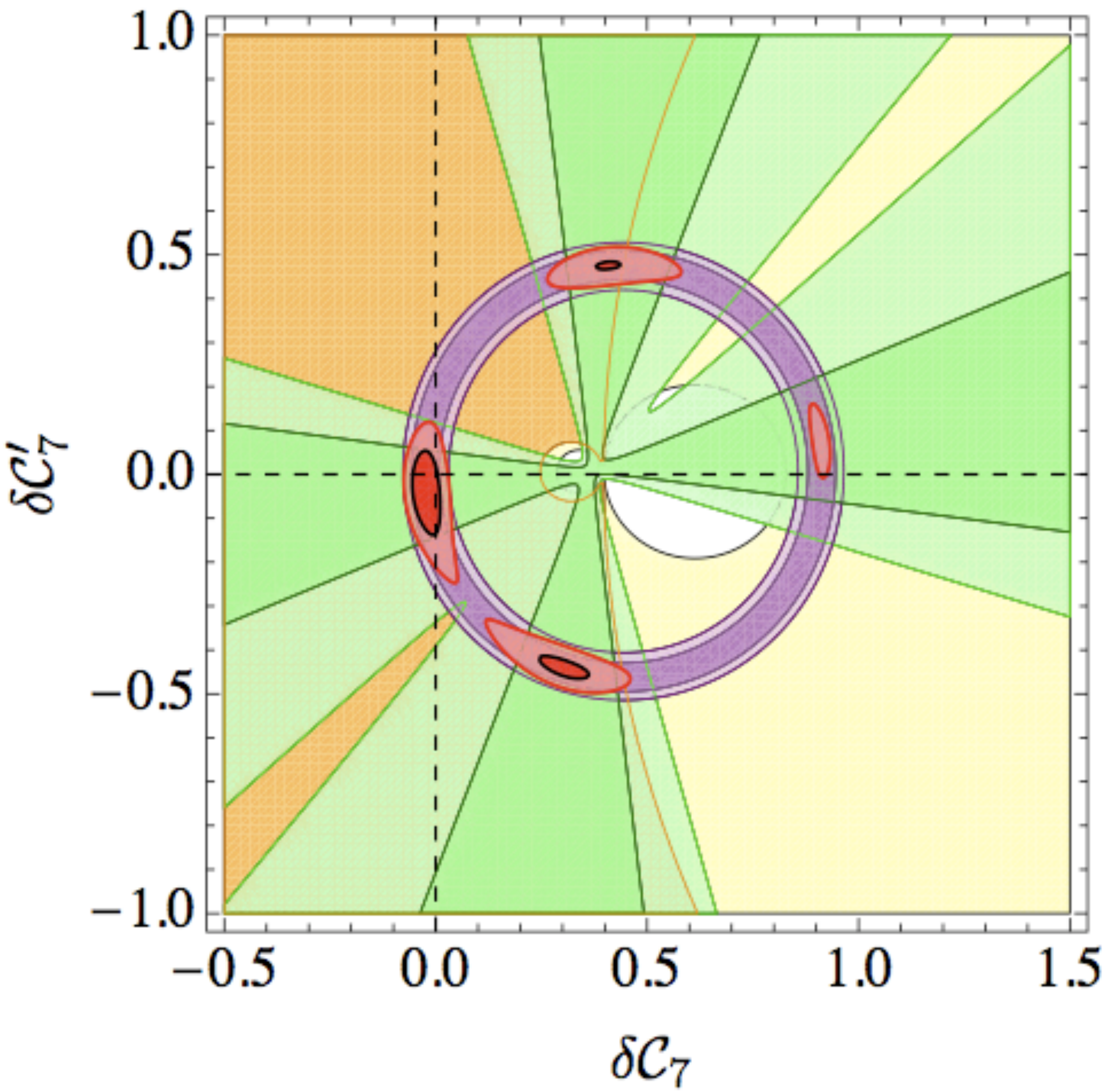

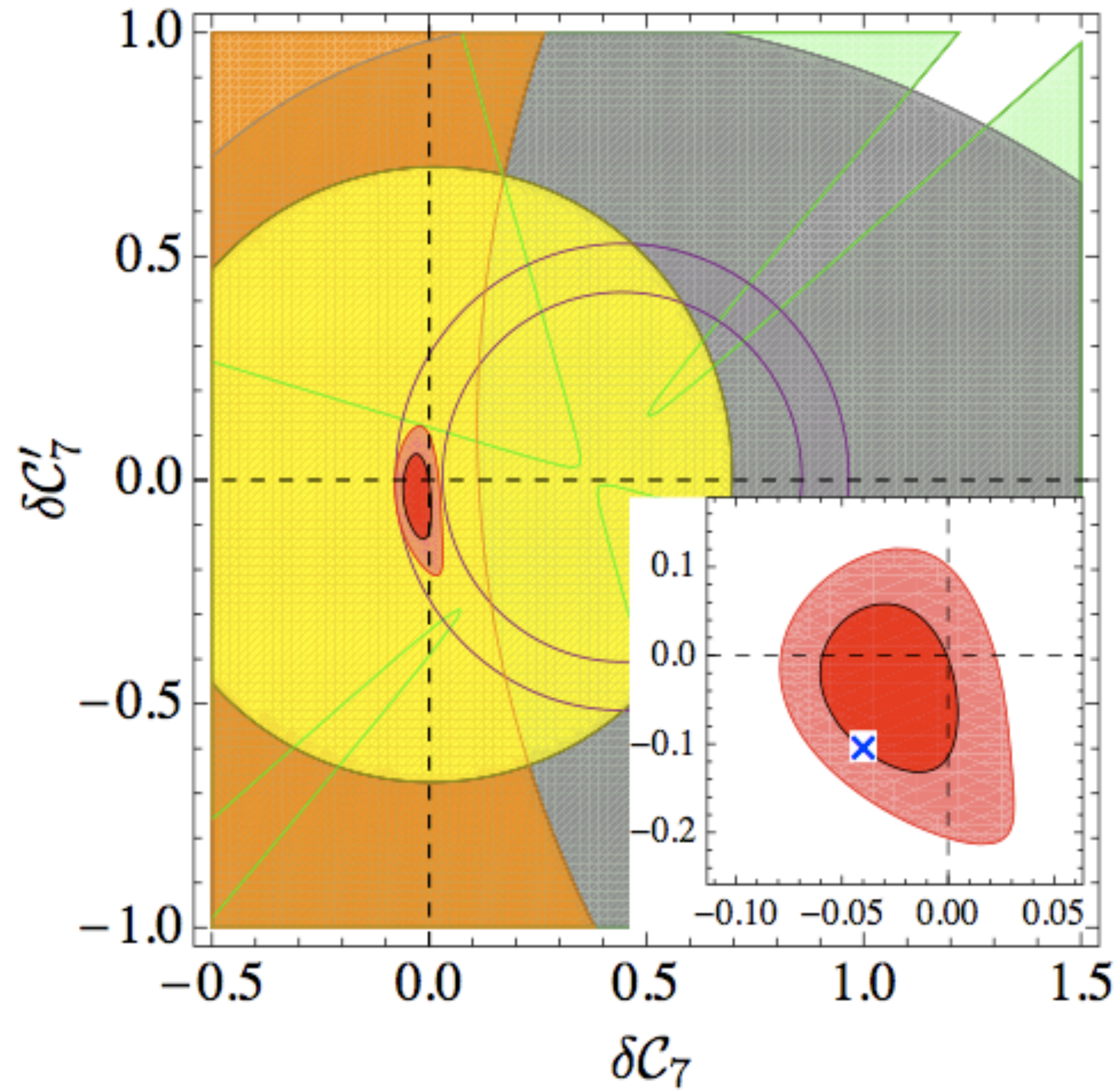

$BR(B \to X_s\gamma)$

$A_{CP}(B \to K^*\gamma)$

$A_I(B \to X_s\gamma)$

$\langle A_{\rm FB}(B \to K^*\mu^+\mu^-)\rangle_{[1,6]}$

$\langle F_L(B \to K^*\mu^+\mu^-)\rangle_{[1,6]}$

$BR(B \to X_s\mu^+\mu^-)$

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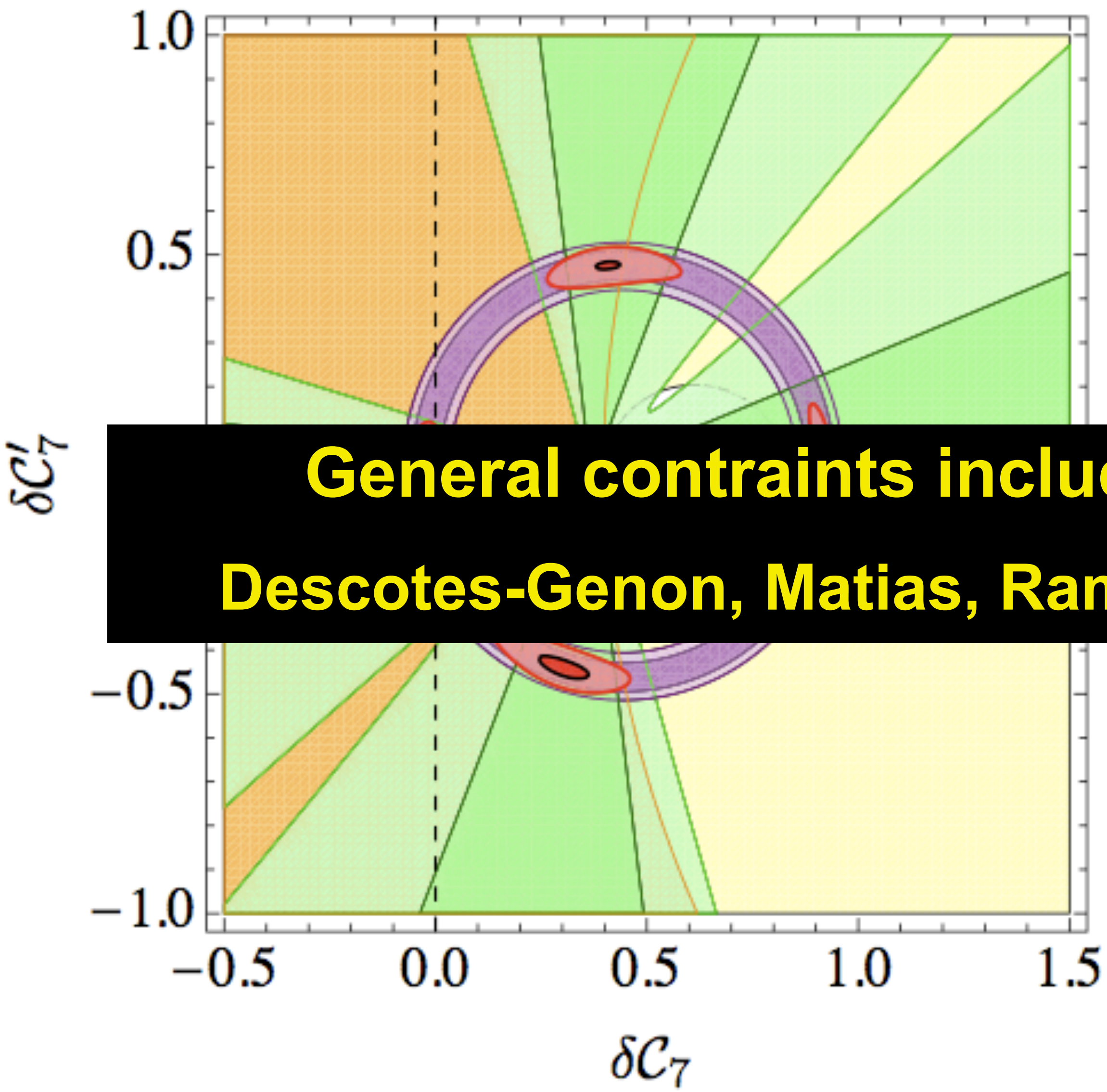

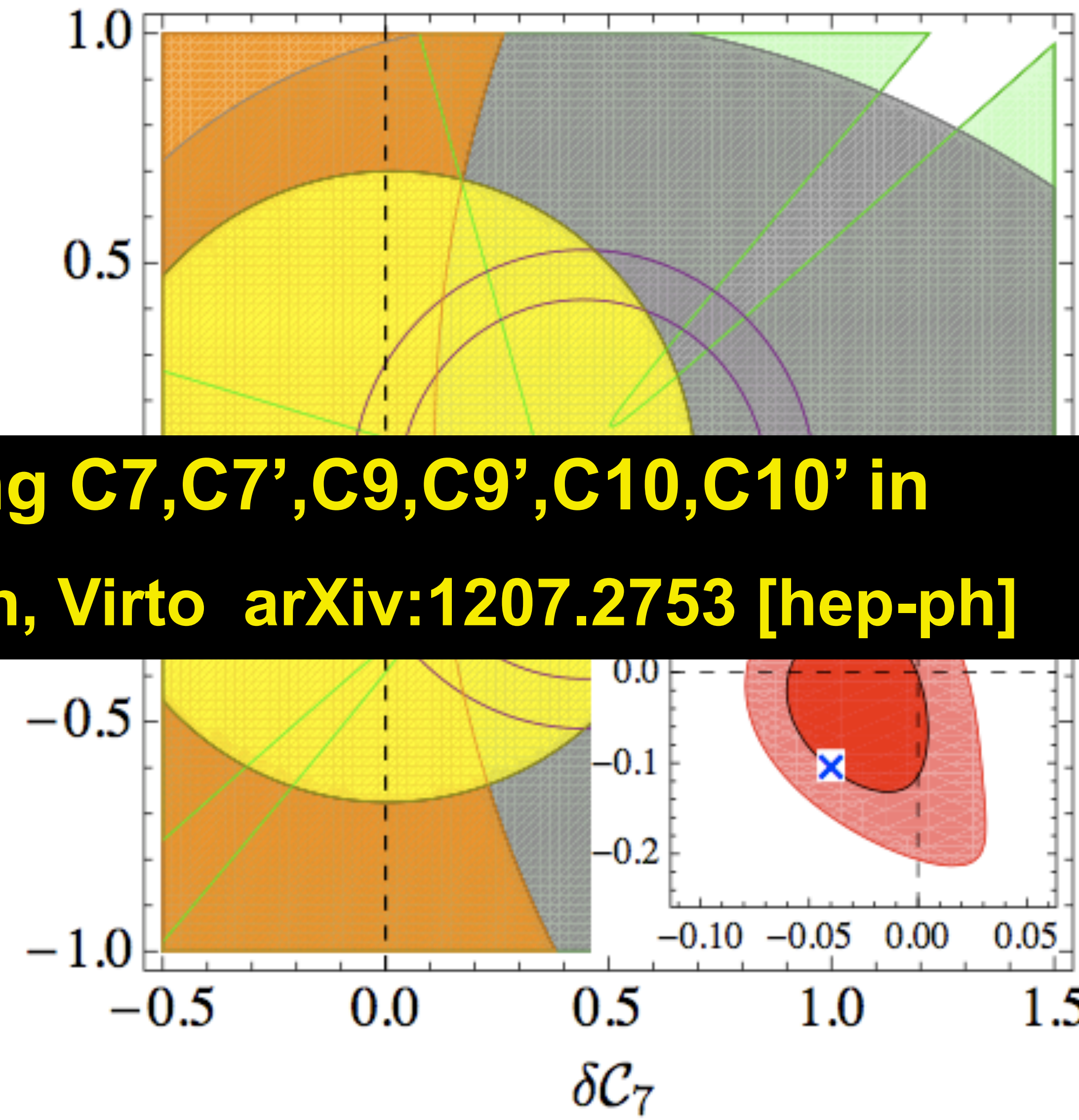

General contraints including C7,C7',C9,C9',C10,C10' in

Descotes-Genon, Matias, Ramon, Virto arXiv:1207.2753 [hep-ph]

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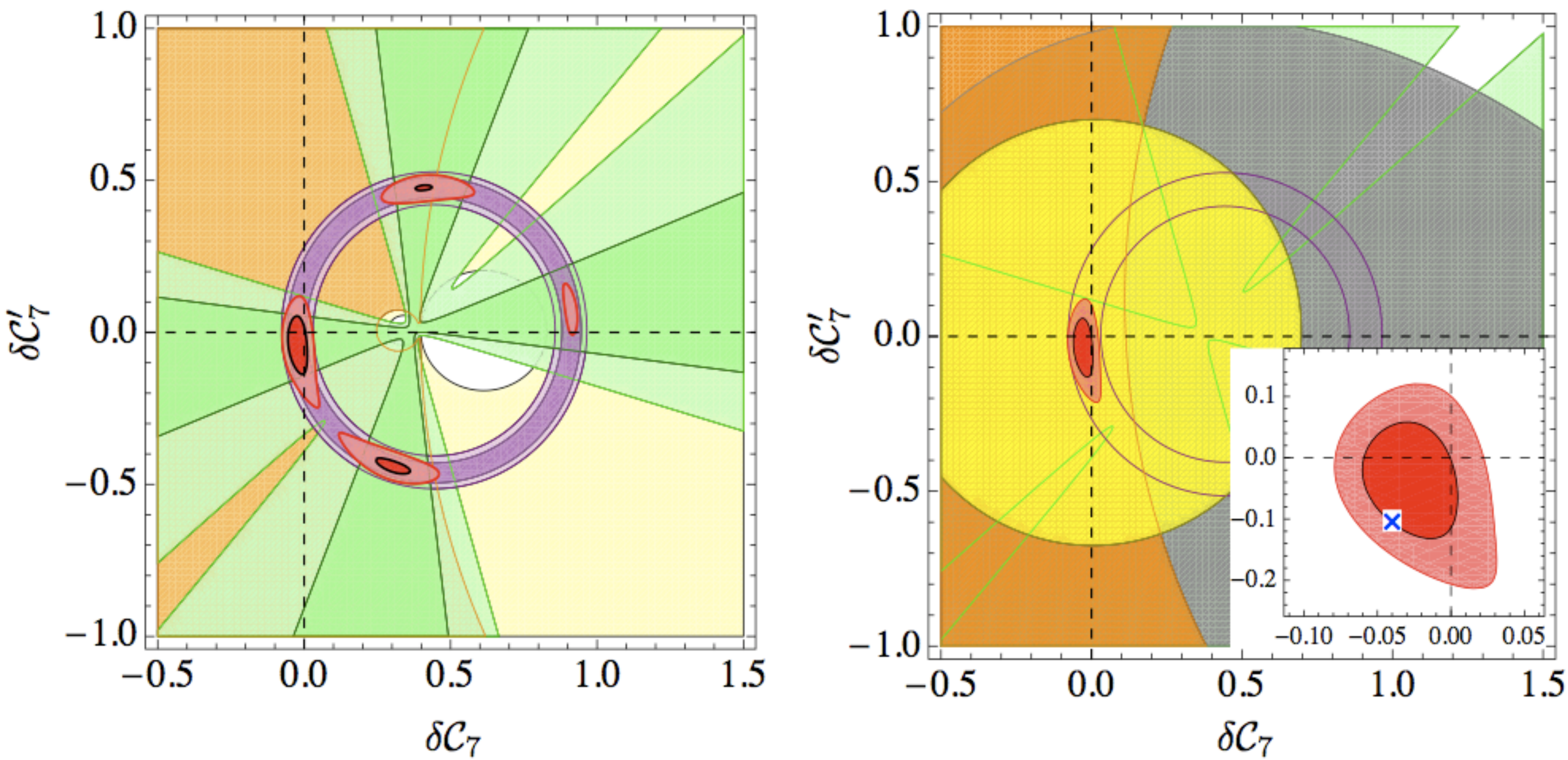

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What about ONLY $B \to K^*(\to K\pi)\ell^+\ell^-$?

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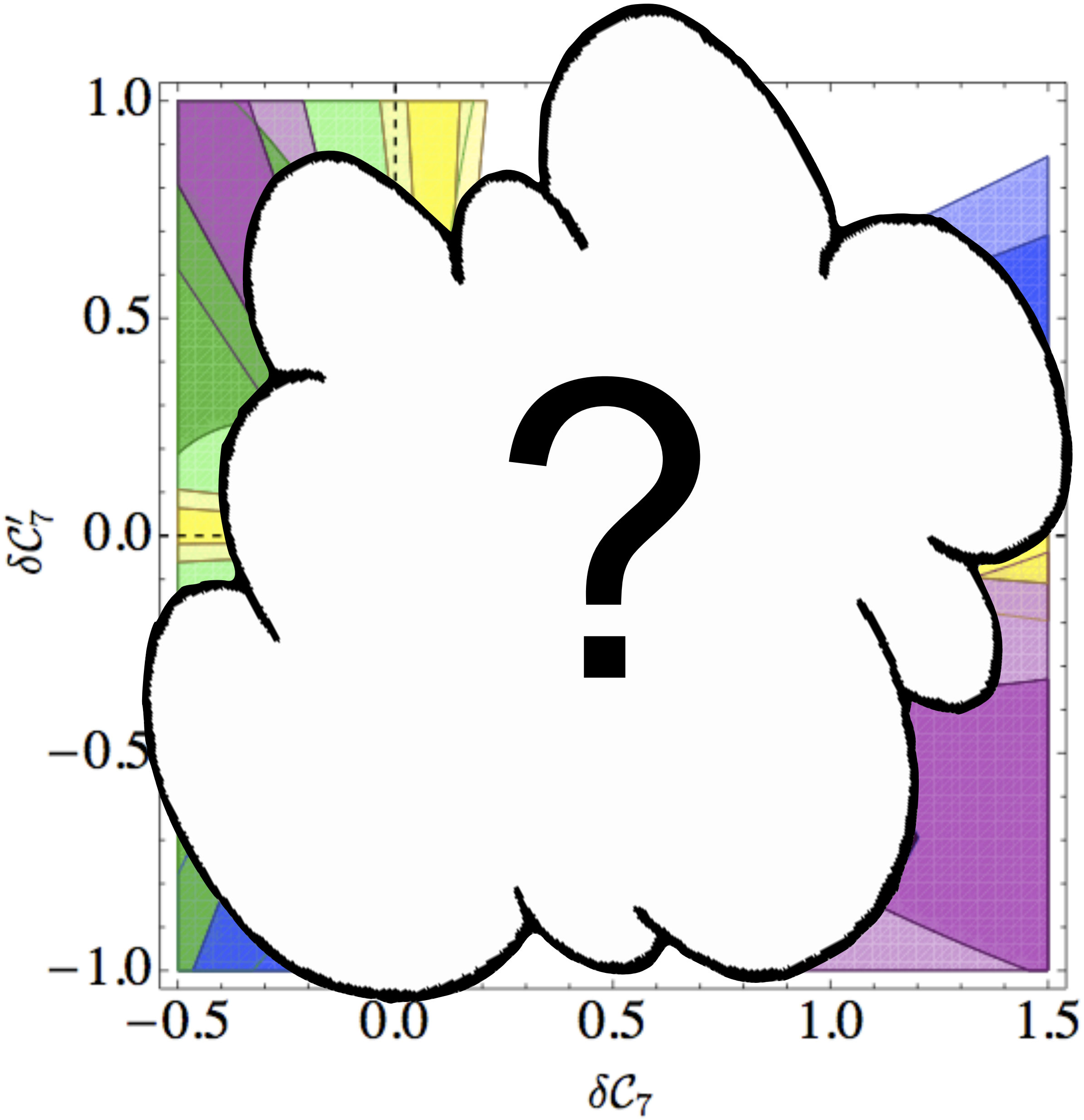

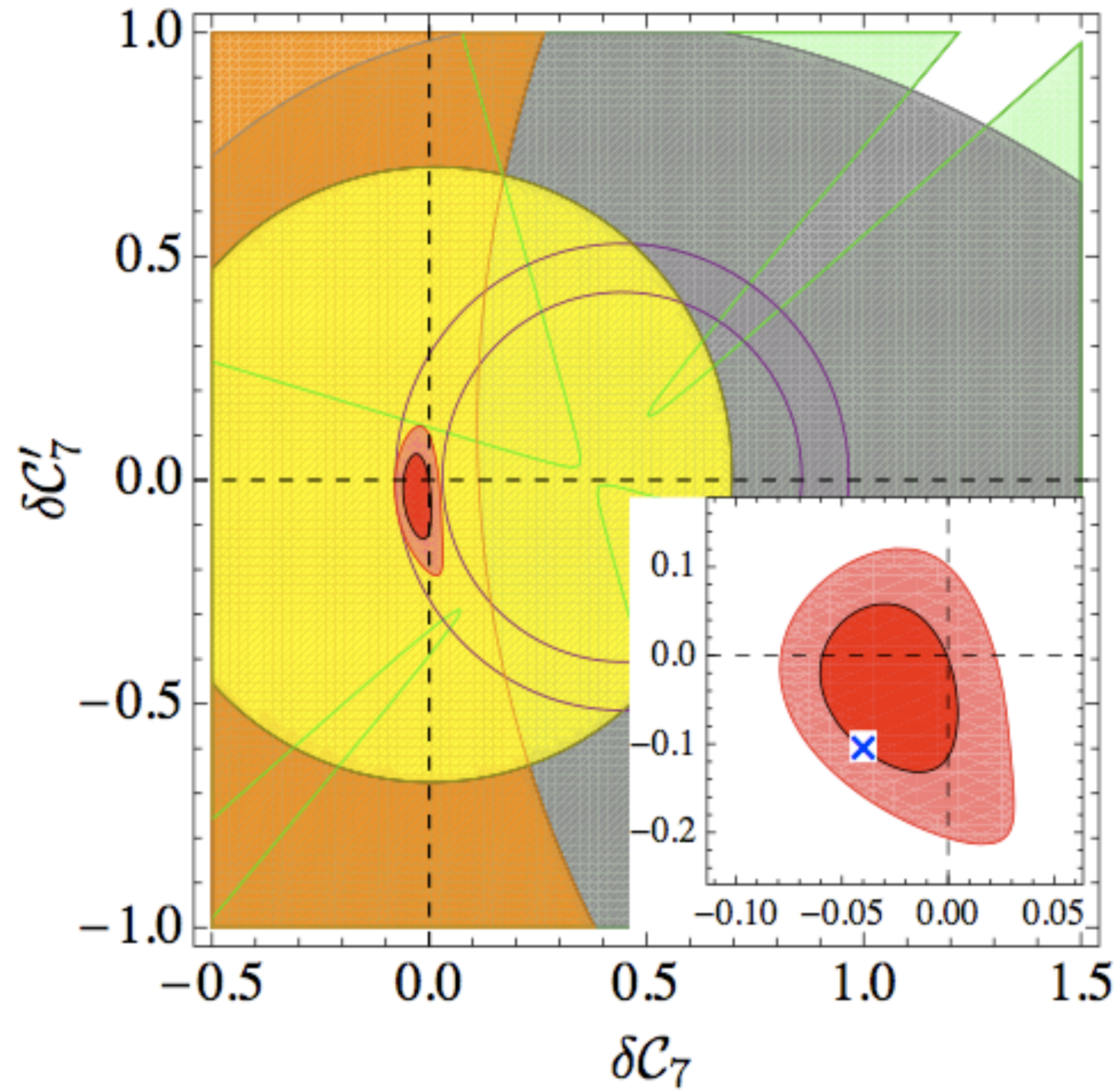

ONLY $B \to K^*(\to K\pi)\ell^+\ell^-$

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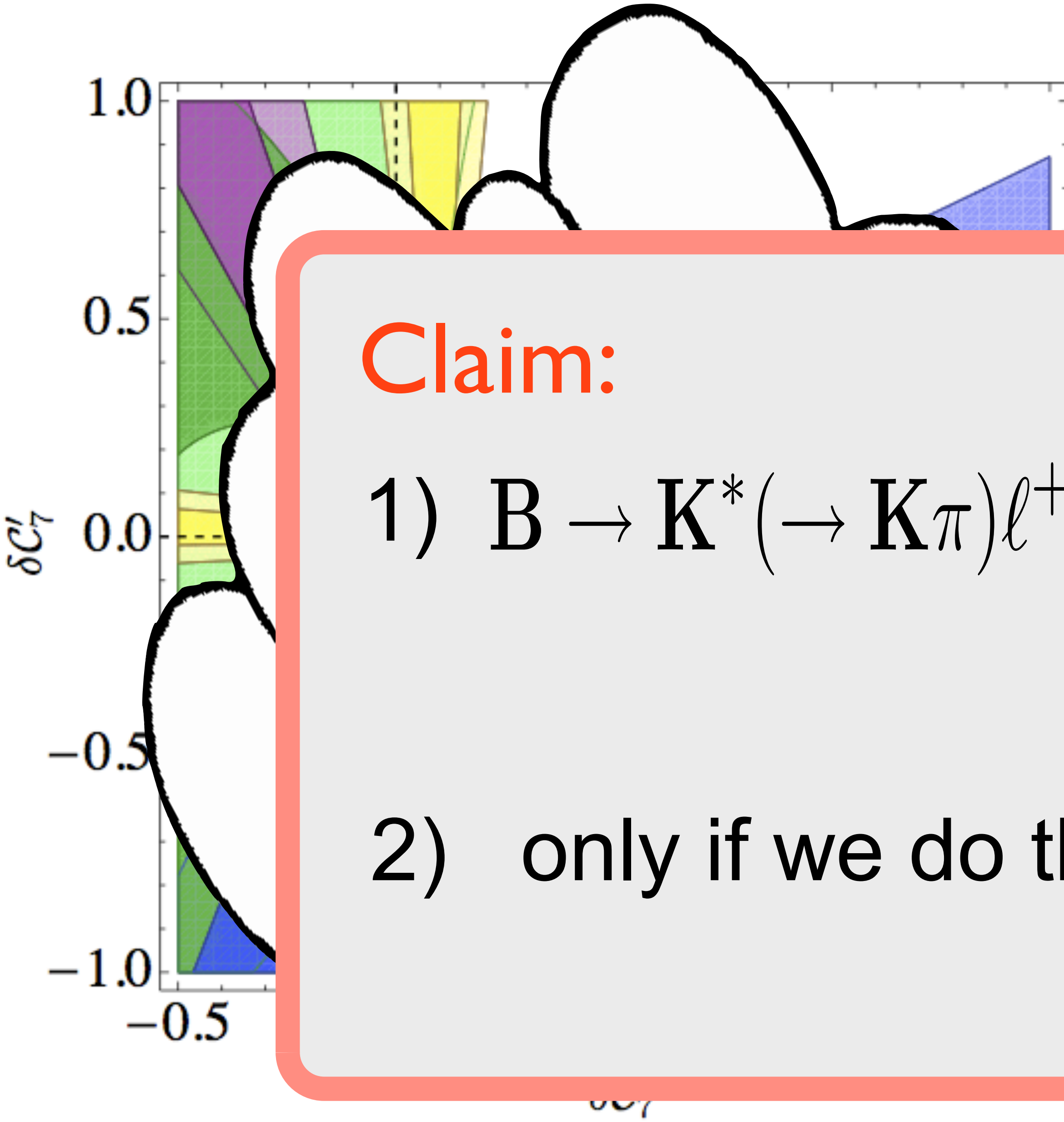

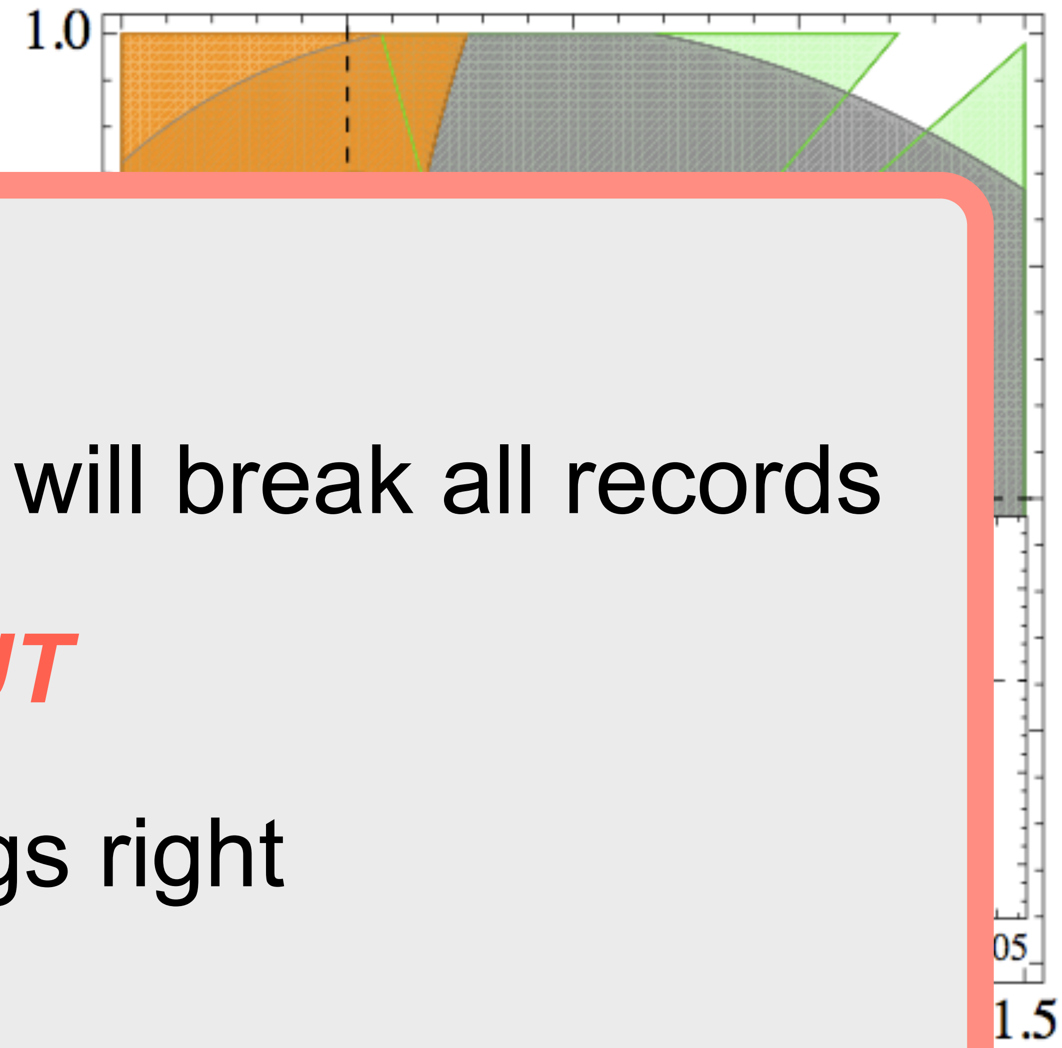

Claim:

1) $B \to K^*(\to K\pi)\ell^+\ell^-$ will break all records

BUT

2) only if we do things right

ONLY $B \to K^*(\to K\pi)\ell^+\ell^-$

$BR(B \to X_s\gamma)$ $\langle A_{FB}(B \to K^*\mu^+\mu^-)\rangle_{[1,6]}$

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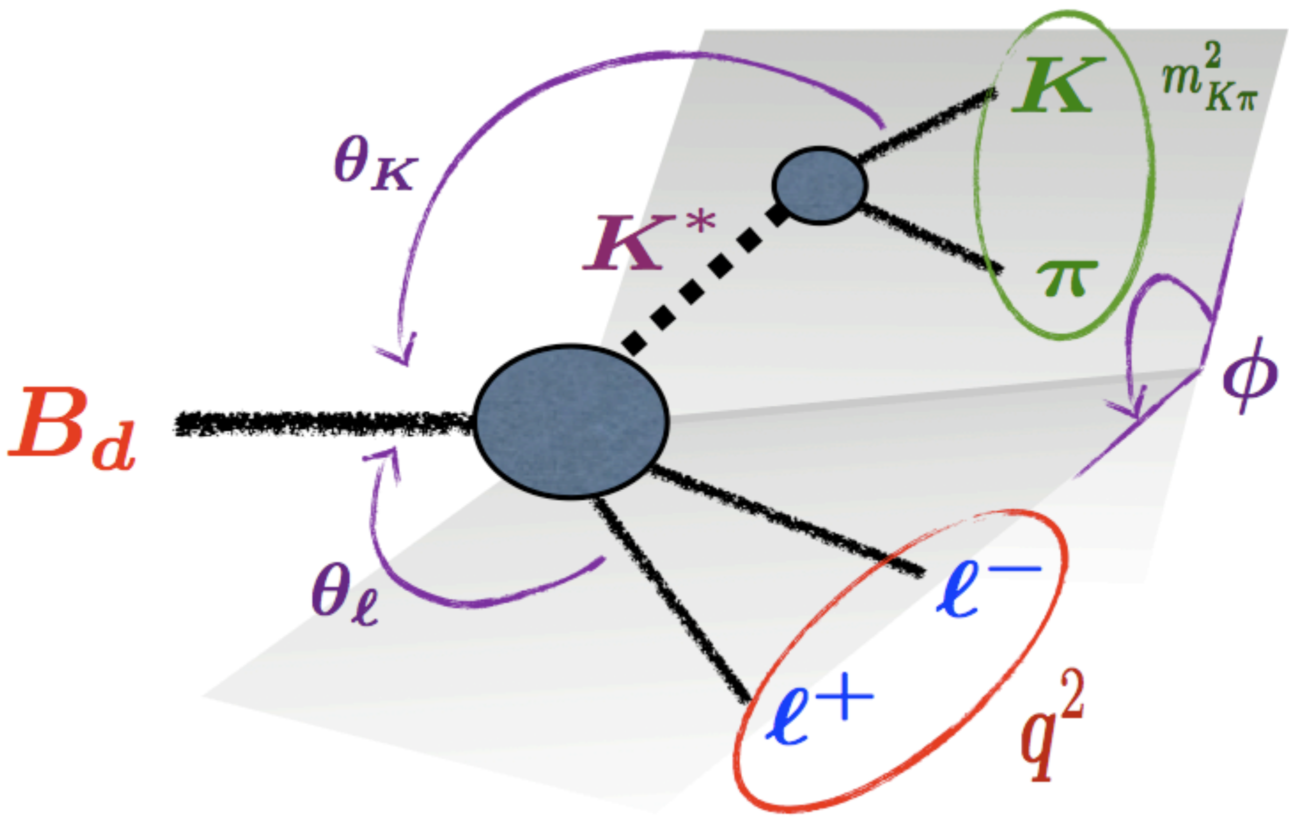
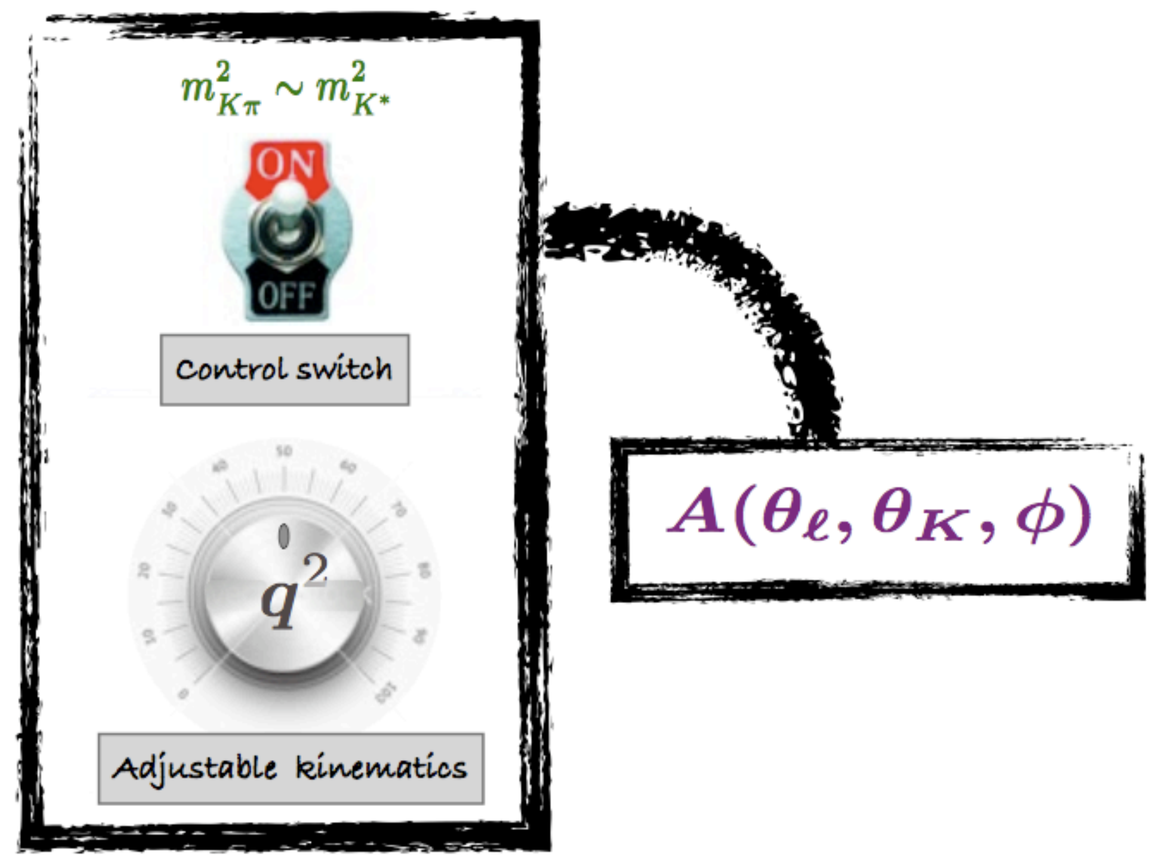
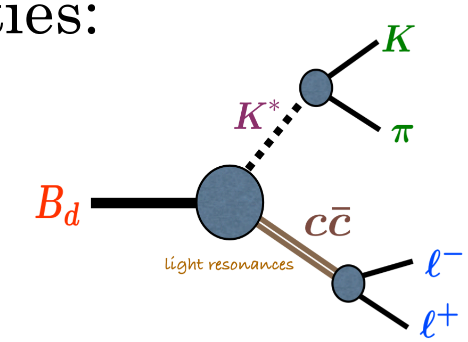
THE $B \to K^*(\to K\pi)\ell^+\ell^-$ DECAY



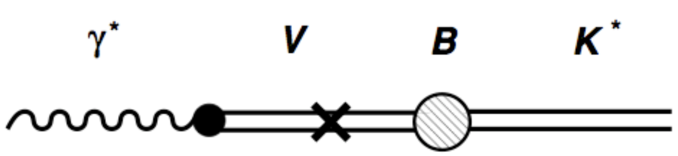


- It is a $b \to s$ penguin process: *Loop* + *Cabibbo suppressed* in the SM.
- Large number of angular observables available experimentally.
- Leptons can be *e*, *mu*, *tau*. Each has its own pheno.
- Also: CP Violation, Isospin asymmetry,... lepton polarization (future?)
- Semi-leptonic Meson Decay: Theory difficulties:
  - ☑ Form Factors
  - ☑ Non-factorizable contributions
  - ☑ Power corrections
  - ☑ Long-distance loops - resonances



- Other difficulties. E.g. S-wave pollution.

TWO different Kinematic Regimes *(or more?)*

q^2 = (invariant mass of $\ell^-\ell^+$)^2	Theoretical Framework
$q^2 \lesssim 7\,\mathrm{GeV}^2$ *Large Recoil*	SCET / QCDF / LEET [Beneke, Feldmann, Seidel,....]
$14\,\mathrm{GeV}^2 \lesssim q^2 \lesssim 20\,\mathrm{GeV}^2$ *Low Recoil*	HQET + OPE [Grinstein, Pirjol,...]
$q^2 \lesssim 1\,\mathrm{GeV}^2$ *(Very Large Recoil)*	 [Jäger, Camalich] [Khodjamirian, Mannel,Wang] Contribution from light vector resonances
$7\,\mathrm{GeV}^2 \lesssim q^2 \lesssim 14\,\mathrm{GeV}^2$	[Khodjamirian, Mannel,Pivovarov,Wang]

FF Relations

ANGULAR DISTRIBUTION

The differential angular decay rate distribution is

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l$$

$$+J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi$$

$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi$$

$$+J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$

[Kruger et.al. 2000]

- The coefficients *J(q^2)* are **observables**.
- The question is how well can we describe these observables **theoretically**.

The coefficients J_i can be written in terms of the Spin Amplitudes:

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\|^L|^2 + |A_\perp^R|^2 + |A_\|^R|^2\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\|^L A_\|^{R*}\right) ,$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2\,|A_S|^2 ,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\|^L|^2 + |A_\perp^R|^2 + |A_\|^R|^2\right] , \qquad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + |A_0^R|^2\right] ,$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\|^L|^2 + |A_\perp^R|^2 - |A_\|^R|^2\right] , \qquad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\|^{L*} + A_0^R A_\|^{R*})\right] ,$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\|^L A_S^* + A_\|^{R*} A_S)\right] ,$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\|^L A_\perp^{L*} - A_\|^R A_\perp^{R*})\right] , \qquad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_0^L A_S^* + A_0^{R*} A_S) ,$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\|^{L*} - A_0^R A_\|^{R*}) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* - A_\perp^{R*} A_S)\right] ,$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L*} + A_0^R A_\perp^{R*})\right] , \qquad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\|^{L*} A_\perp^L + A_\|^{R*} A_\perp^R)\right] ,$$

(or equivalently in terms of helicity amplitudes)

AMPLITUDES

At the Leading Order:

$$A_{\perp L,R} = N\sqrt{2}\lambda^{1/2}\left[\left[(C_9^{\rm eff} + C_9^{\rm eff\prime}) \mp (C_{10}^{\rm eff} + C_{10}^{\rm eff\prime})\right]\frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2}(C_7^{\rm eff} + C_7^{\rm eff\prime})T_1(q^2)\right]$$

$$A_{\parallel L,R} = -N\sqrt{2}(m_B^2 - m_{K^*}^2)\left[\left[(C_9^{\rm eff} - C_9^{\rm eff\prime}) \mp (C_{10}^{\rm eff} - C_{10}^{\rm eff\prime})\right]\frac{A_1(q^2)}{m_B - m_{K^*}}\right.$$
$$\left. + \frac{2m_b}{q^2}(C_7^{\rm eff} - C_7^{\rm eff\prime})T_2(q^2)\right],$$

$$A_{0L,R} = -\frac{N}{2m_{K^*}\sqrt{q^2}}\left\{\left[(C_9^{\rm eff} - C_9^{\rm eff\prime}) \mp (C_{10}^{\rm eff} - C_{10}^{\rm eff\prime})\right]\right.$$
$$\times\left[(m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*})A_1(q^2) - \lambda\frac{A_2(q^2)}{m_B + m_{K^*}}\right]$$
$$\left. + 2m_b(C_7^{\rm eff} - C_7^{\rm eff\prime})\left[(m_B^2 + 3m_{K^*}^2 - q^2)T_2(q^2) - \frac{\lambda}{m_B^2 - m_{K^*}^2}T_3(q^2)\right]\right\},$$

$$A_t = \frac{N}{\sqrt{q^2}}\lambda^{1/2}\left[2(C_{10}^{\rm eff} - C_{10}^{\rm eff\prime}) + \frac{q^2}{m_\mu}(C_P - C_P')\right]A_0(q^2)$$

$$A_S = -2N\lambda^{1/2}(C_S - C_S')A_0(q^2)$$

Spin Amplitudes in terms of **Wilson Coefficients** and **form factors**.

1 At the Leading Order:

$$A_\perp^{L,R} = \mathcal{N}_\perp \Big[\mathcal{C}^+_{9\mp10} V(q^2) + \mathcal{C}_7^+ T_1(q^2) \Big] + \mathcal{O}(\alpha_s, \cdots)$$

$$A_\parallel^{L,R} = \mathcal{N}_\parallel \Big[\mathcal{C}^-_{9\mp10} A_1(q^2) + \mathcal{C}_7^- T_2(q^2) \Big] + \mathcal{O}(\alpha_s, \cdots)$$

$$A_0^{L,R} = \mathcal{N}_0 \Big[\mathcal{C}^-_{9\mp10} A_{12}(q^2) + \mathcal{C}_7^- T_{23}(q^2) \Big] + \mathcal{O}(\alpha_s, \cdots)$$

“Clean” Observables - 5 easy pieces

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2 Define the FF ratios:

$$R_1 = T_1/V$$

$$R_2 = T_2/A_1$$

$$\tilde{R}_3 = T_{23}/A_{12}$$

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3 In **BOTH** limits:

At LARGE @ LOW recoil

(EFT Predictions)

$$R_{1,2} = 1 + \text{corrections} \;, \quad \tilde{R}_3 = \frac{q^2}{m_B^2} + \text{corrections}$$

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$$R_{1,2} = 1 + \text{corrections}\ , \quad \tilde{R}_3 = \frac{q^2}{m_B^2} + \text{corrections}$$

4 Plug it in

$$A_\perp^{L,R} = F_\perp^{L,R} V(q^2) + \mathcal{O}(\alpha_s, \cdots)$$

$$A_\parallel^{L,R} = F_\parallel^{L,R} A_1(q^2) + \mathcal{O}(\alpha_s, \cdots)$$

$$A_0^{L,R} = F_0^{L,R} A_{12}(q^2) + \mathcal{O}(\alpha_s, \cdots)$$

Short distance functions

“Clean” Observables - 5 easy pieces

1 At the Leading Order:

$$A_\perp^{L,R} = \mathcal{N}_\perp\left[\mathcal{C}^+_{9\mp10}V(q^2) + \mathcal{C}^+_7 T_1(q^2)\right] + \mathcal{O}(\alpha_s,\cdots)$$

$$A_\parallel^{L,R} = \mathcal{N}_\parallel\left[\mathcal{C}^-_{9\mp10}A_1(q^2) + \mathcal{C}^-_7 T_2(q^2)\right] + \mathcal{O}(\alpha_s,\cdots)$$

$$A_0^{L,R} = \mathcal{N}_0\left[\mathcal{C}^-_{9\mp10}A_{12}(q^2) + \mathcal{C}^-_7 T_{23}(q^2)\right] + \mathcal{O}(\alpha_s,\cdots)$$

2 Define the FF ratios:

$$R_1 = T_1/V$$

$$R_2 = T_2/A_1$$

$$\tilde{R}_3 = T_{23}/A_{12}$$

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At LARGE @ LOW recoil

(EFT Predictions)

$$R_{1,2} = 1 + \text{corrections}\ , \quad \tilde{R}_3 = \frac{q^2}{m_B^2} + \text{corrections}$$

4 Plug it in

$$A_\perp^{L,R} = F_\perp^{L,R}\, V(q^2) + \mathcal{O}(\alpha_s,\cdots)$$

$$A_\parallel^{L,R} = F_\parallel^{L,R}\, A_1(q^2) + \mathcal{O}(\alpha_s,\cdots)$$

$$A_0^{L,R} = F_0^{L,R}\, A_{12}(q^2) + \mathcal{O}(\alpha_s,\cdots)$$

Short distance functions

5 Make *suitable* **ratios**

E.g.

$$\frac{\mathrm{Re}(A_0^L A_\parallel^{L*} + A_0^R A_\parallel^{R*})}{\sqrt{(|A_\parallel^L|^2 + |A_\parallel^R|^2)(|A_0^L|^2 + |A_0^R|^2)}} = P_4$$

CAREFULL: Not guaranteed it is observable (symmetries!!!)

“Clean” Observables - Observations

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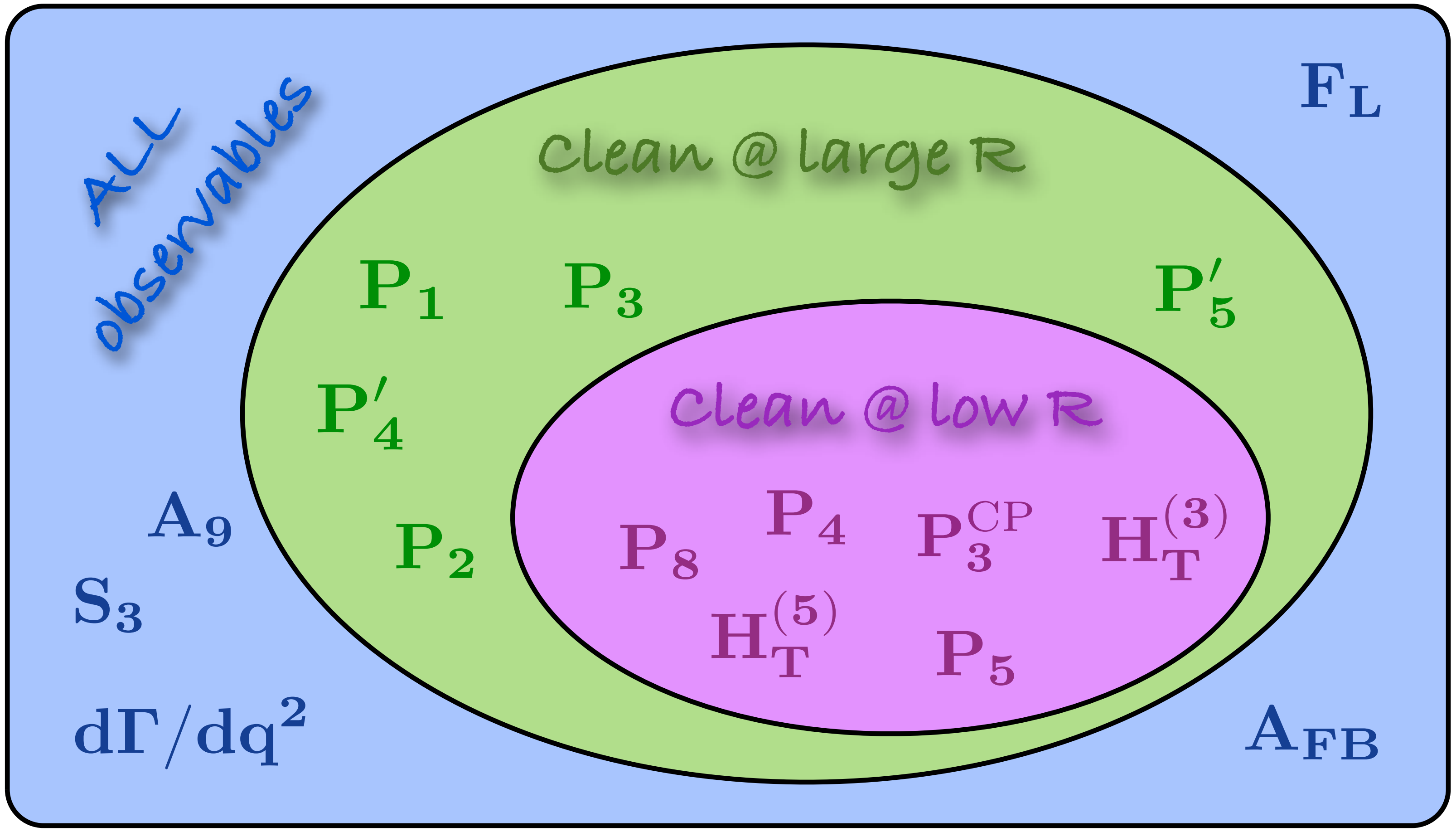

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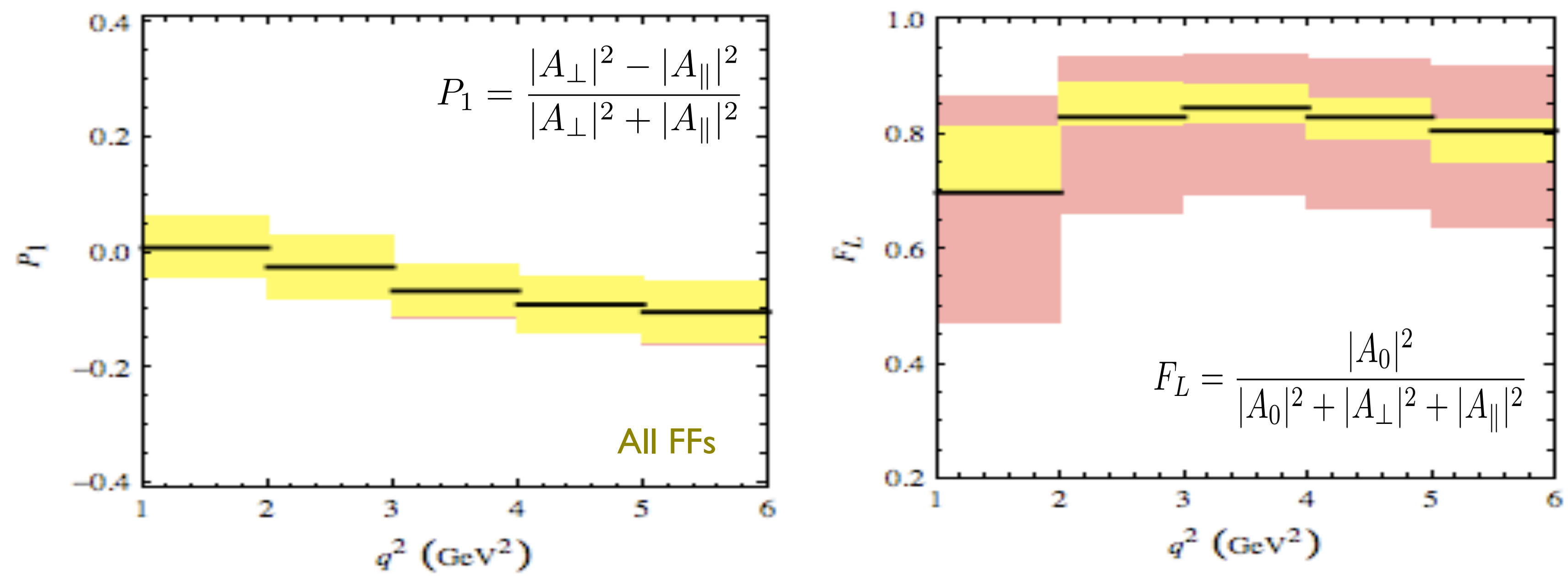

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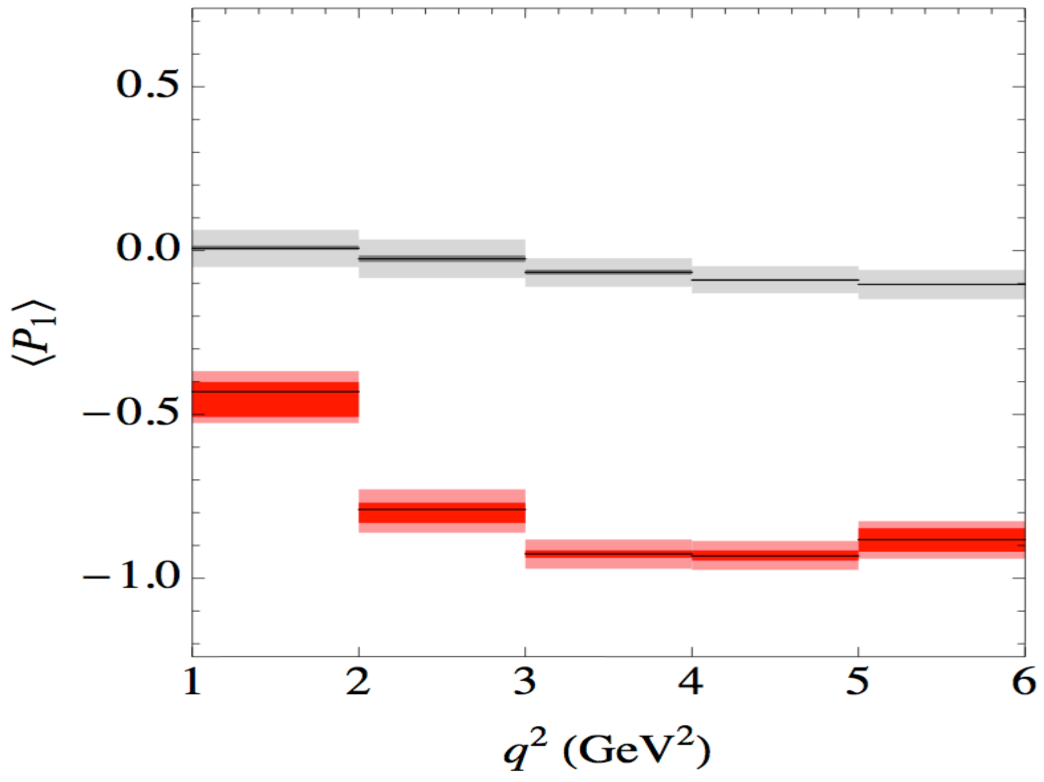
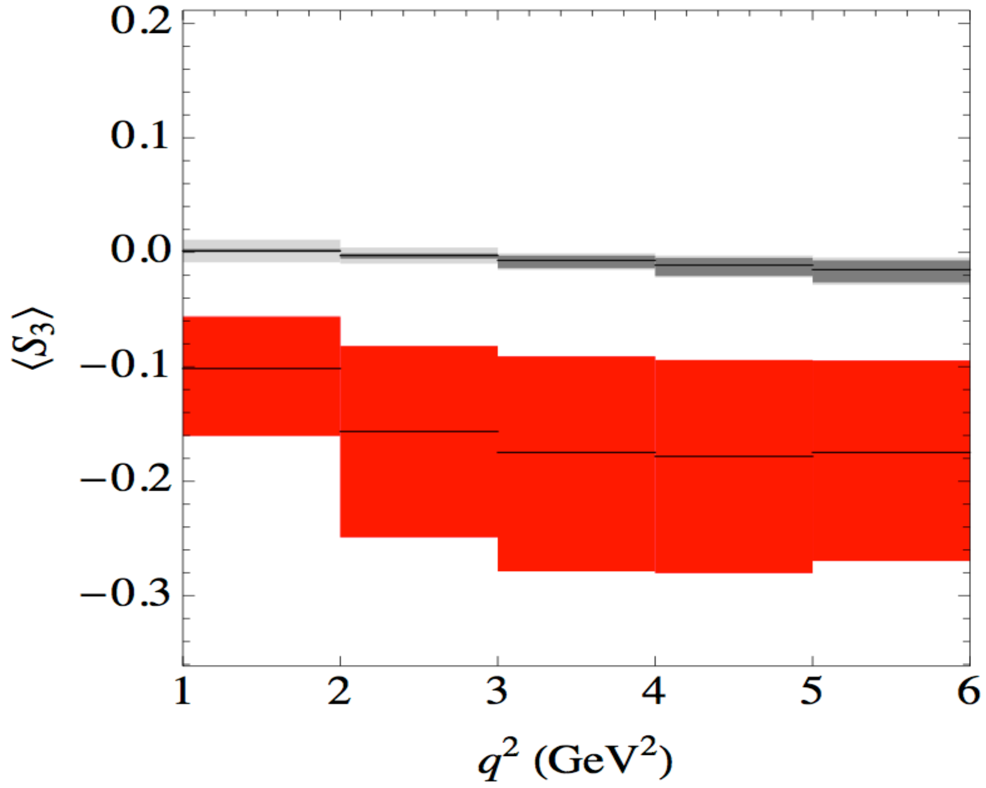
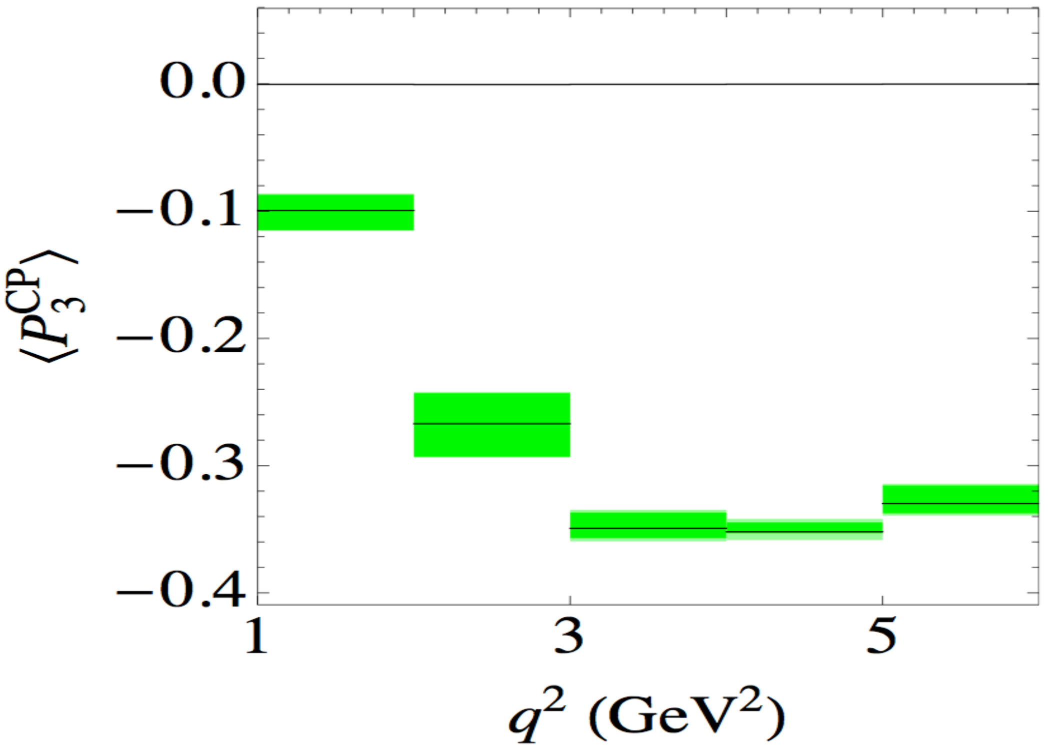
6. **CP Violation:** All the formalism can be repeated for CP averaged + CP violating observ.

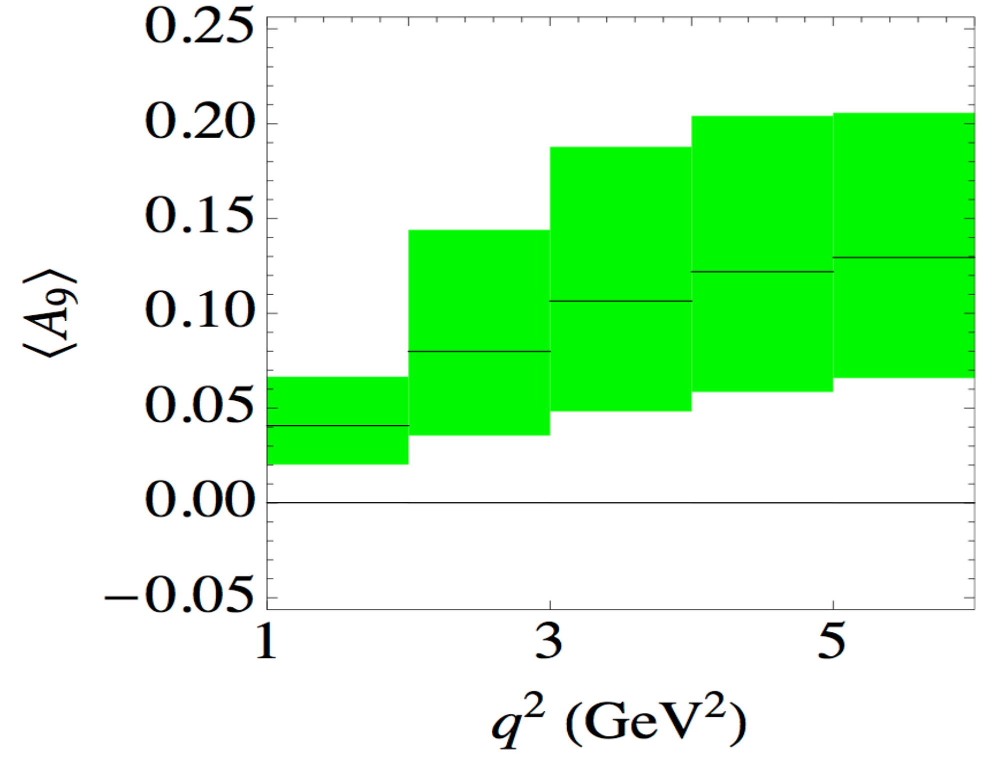
$$\langle \mathbf{P_i} \rangle \qquad \langle \mathbf{P_i^{CP}} \rangle$$

Clean vs. Unclean









Optimal sets of Observables (Bases)

1. Best compromise: Theoretically clean vs. Clean experimental extraction. **Short term**.

$$\{d\Gamma/dq^2, A_{FB}, P_1, P_2, P_3, P_4', P_5', P_6'\}$$

2. CP-violating basis:

$$\{A_{\rm CP}, A_{FB}^{\rm CP}, P_1^{\rm CP}, P_2^{\rm CP}, P_3^{\rm CP}, P_4'^{\rm CP}, P_5'^{\rm CP}, P_6'^{\rm CP}\}$$

3. Compromise LOW+LARGE recoil (future):

$$\{d\Gamma/dq^2, A_{FB}, P_1, P_4 = H_T^{(1)}, P_5 = H_T^{(2)}, P_8 = H_T^{(4)}, H_T^{(3)}, H_T^{(5)}\}$$

Relationships between different Clean Observables:

$P_1 = A_T^{(2)}$	$2P_2 = A_T^{(\rm re)}$	$2P_3 = -A_T^{(im)}$	$P_{4,5,8} = H_T^{(1,2,4)}$
$H_T^{(3)} = \frac{2P_2}{\sqrt{1-P_1^2}}$	$H_T^{(5)} = \frac{2P_3}{\sqrt{1-P_1^2}}$		

Kruger, Matias 2005

Bobeth, Hiller, van Dyk 2010, 2011, 2012

Becirevic, Schneider 2011

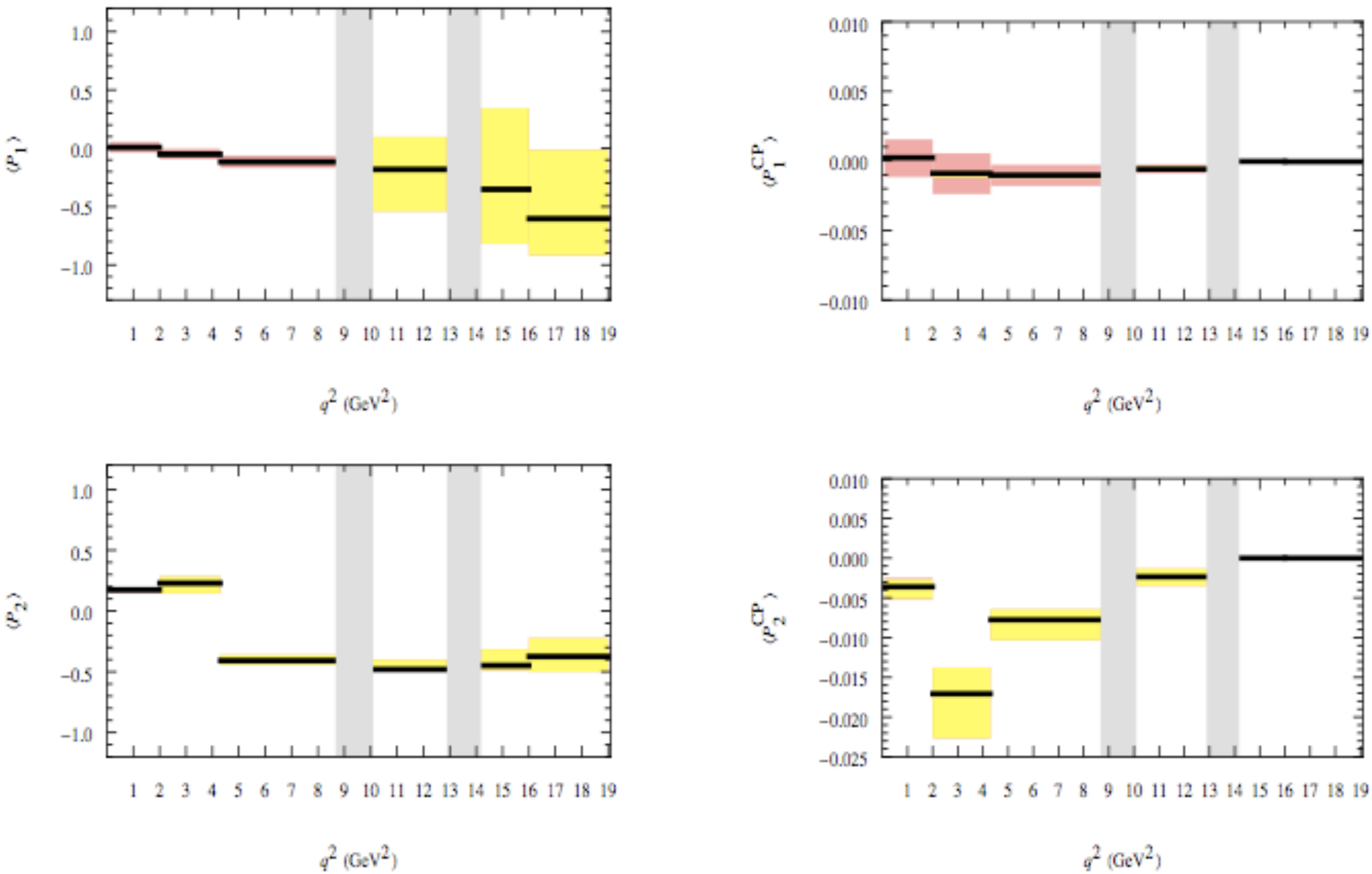
Matias, Mescia, Ramon, Virto 2012

Descotes-Genon, Matias, Ramon, Virto 2012

Descotes-Genon, Hurth, Matias, Virto 2013

SM Predictions in All q^2

Descotes-Genon, Hurth, Matias, Virto 2013



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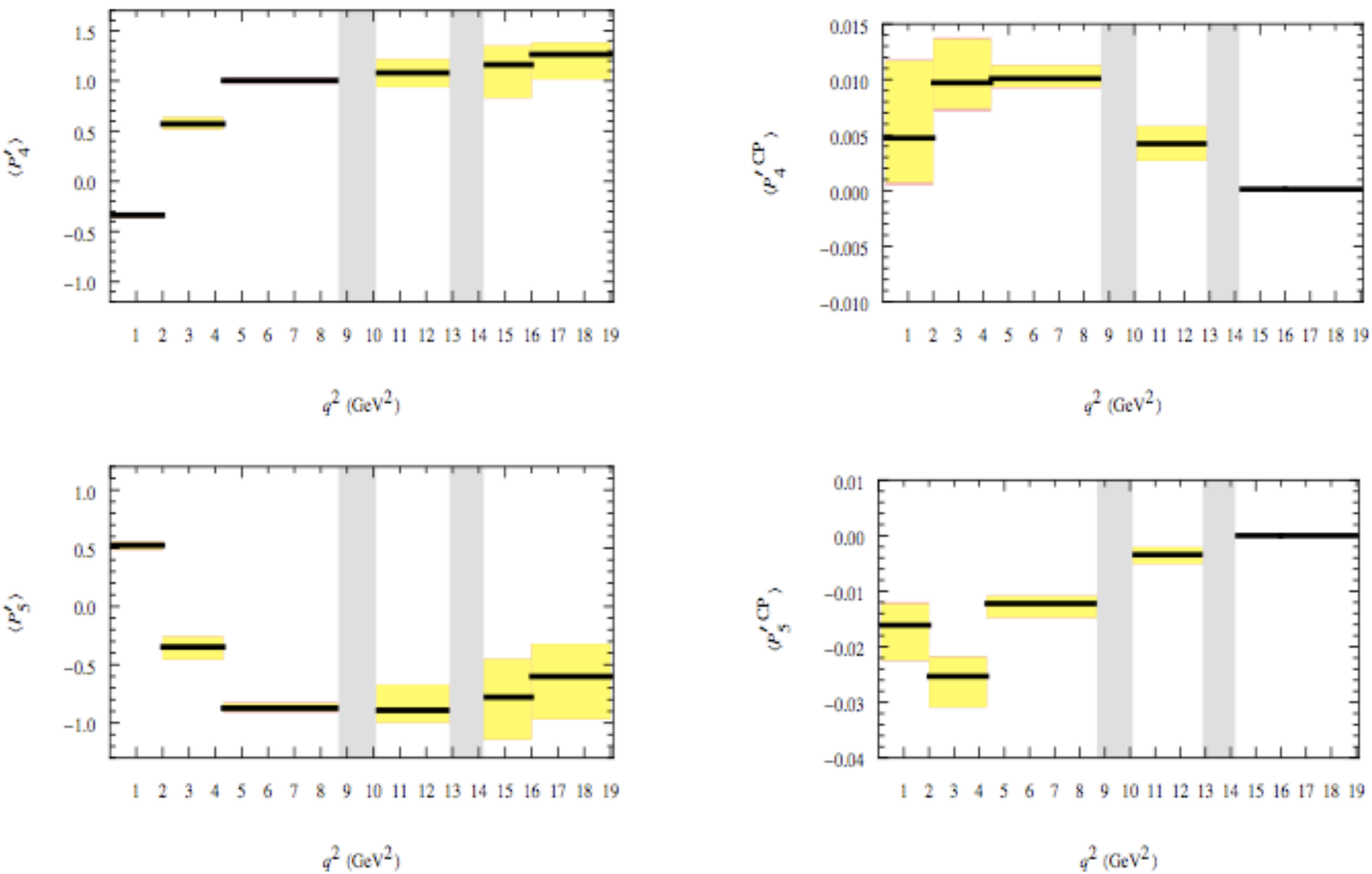

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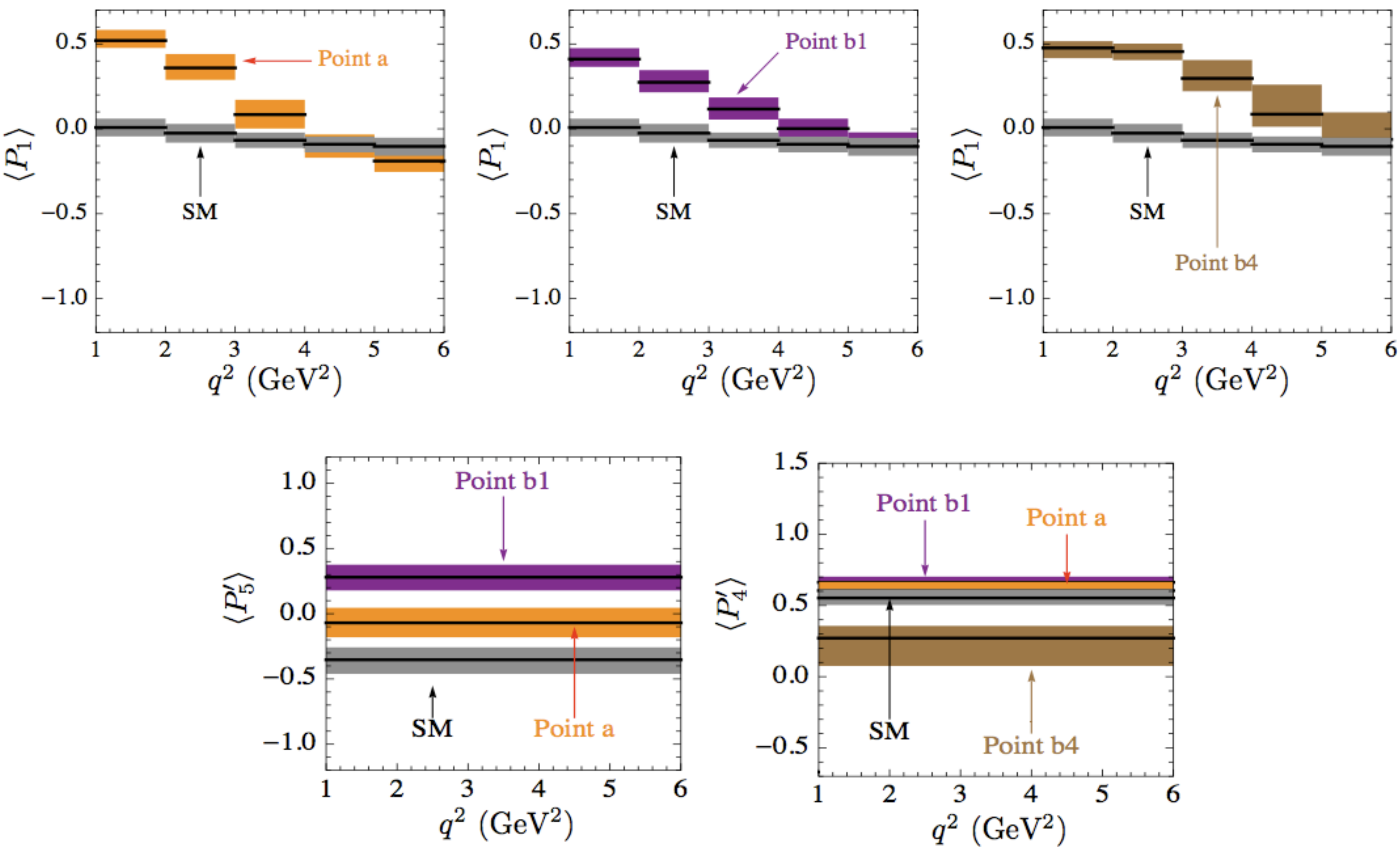
Soon to test:

Bin (GeV^2)	$\langle P_1\rangle = \langle A_T^{(2)}\rangle$	$\langle P_2\rangle = \frac{1}{2}\langle A_T^{(\mathrm{re})}\rangle$	$\langle P_3\rangle = -\frac{1}{2}\langle A_T^{(\mathrm{im})}\rangle$
$[1\,,2]$	$0.007^{+0.009+0.055}_{-0.005-0.057}$	$0.402^{+0.021+0.007}_{-0.021-0.009}$	$-0.003^{+0.001+0.026}_{-0.002-0.026}$
$[0.1\,,2]$	$0.007^{+0.008+0.043}_{-0.004-0.046}$	$0.173^{+0.008+0.020}_{-0.008-0.020}$	$-0.002^{+0.001+0.021}_{-0.001-0.021}$
$[2\,,4.3]$	$-0.052^{+0.009+0.045}_{-0.009-0.048}$	$0.228^{+0.055+0.016}_{-0.084-0.017}$	$-0.004^{+0.001+0.023}_{-0.003-0.022}$
$[4.3\,,8.68]$	$-0.117^{+0.002+0.049}_{-0.002-0.048}$	$-0.408^{+0.047+0.009}_{-0.036-0.006}$	$-0.001^{+0.000+0.027}_{-0.001-0.028}$
$[10.09\,,12.89]$	$-0.181^{+0.278+0.028}_{-0.360-0.028}$	$-0.481^{+0.080+0.003}_{-0.005-0.002}$	$0.003^{+0.000+0.015}_{-0.001-0.015}$
$[14.18\,,16]$	$-0.352^{+0.696+0.014}_{-0.467-0.015}$	$-0.449^{+0.136+0.004}_{-0.041-0.004}$	$0.004^{+0.000+0.002}_{-0.001-0.002}$
$[16\,,19]$	$-0.603^{+0.589+0.009}_{-0.315-0.010}$	$-0.374^{+0.151+0.004}_{-0.126-0.004}$	$0.003^{+0.001+0.002}_{-0.001-0.001}$
$[1\,,6]$	$-0.056^{+0.009+0.041}_{-0.008-0.042}$	$0.080^{+0.055+0.020}_{-0.075-0.020}$	$-0.003^{+0.001+0.021}_{-0.002-0.021}$

New Physics Complementarity: An Example

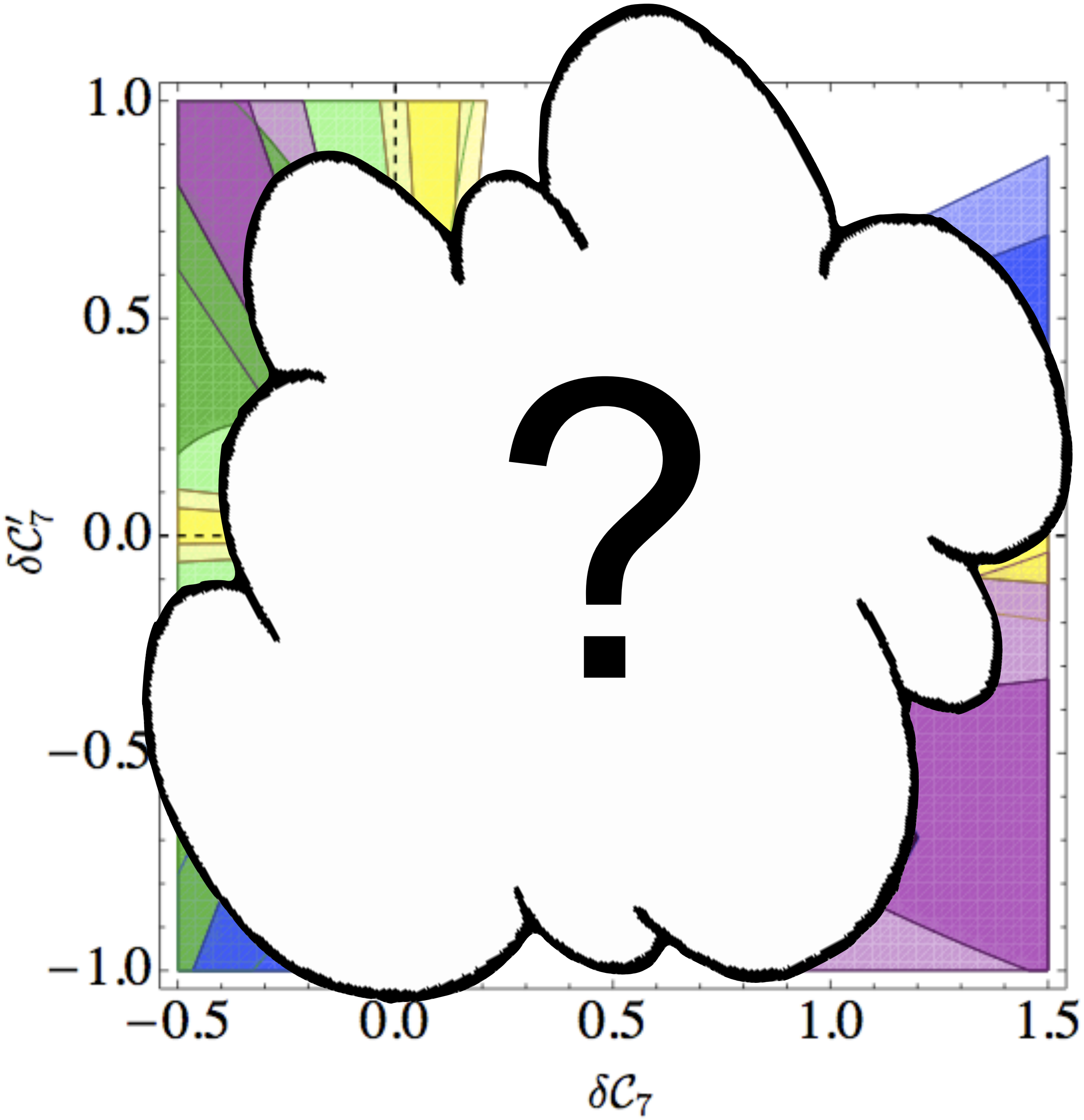


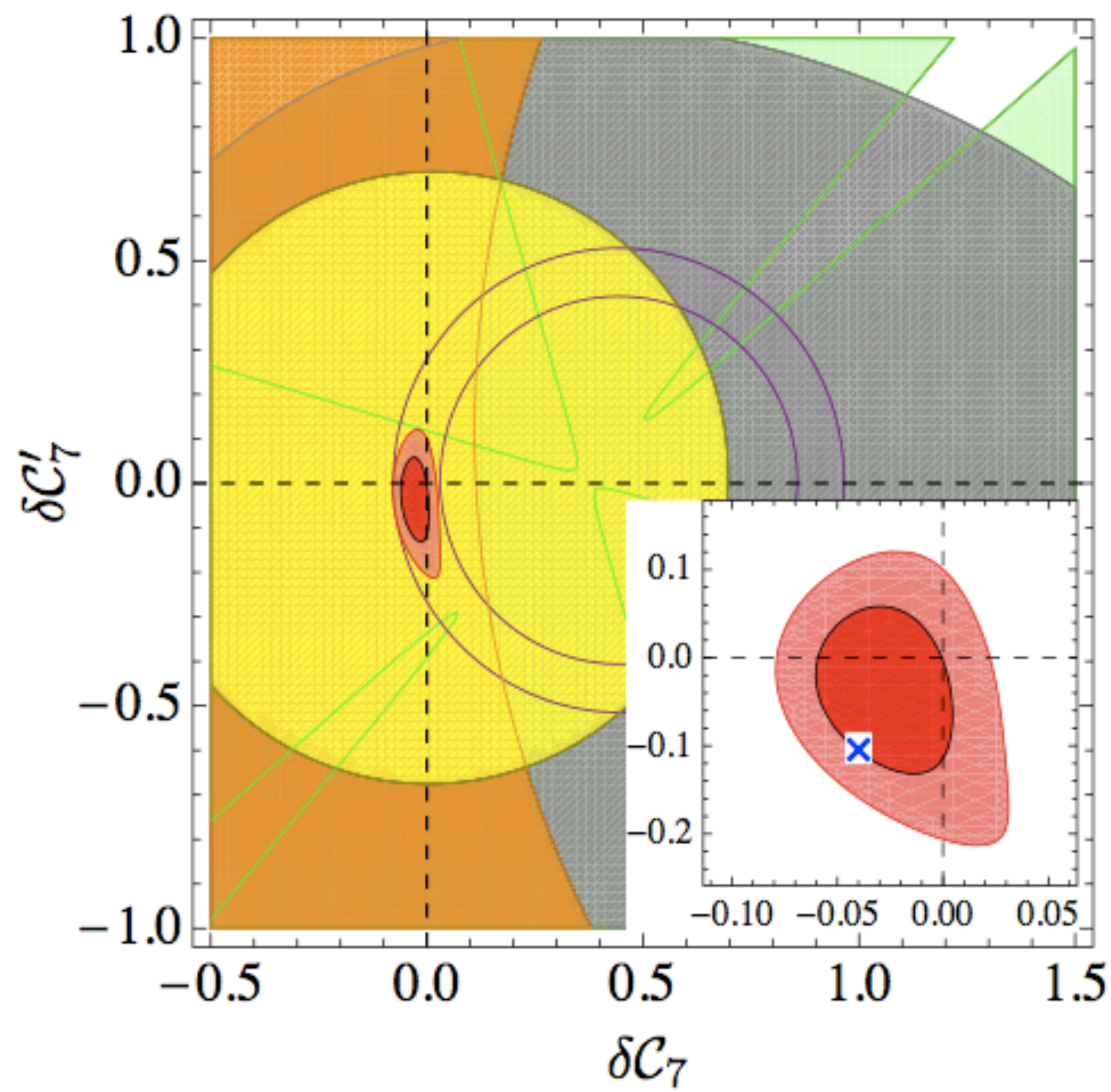
Descotes-Genon, Matias, Ramon, Virto 1207.2753

Un-blinding $B \to K^*(\to K\pi)\ell^+\ell^-$

Descotes-Genon, Matias, Ramon, Virto 1207.2753

1. Constraints on **C7, C7'** (all other NP to zero).





ONLY $B \to K^*(\to K\pi)\ell^+\ell^-$

- Central values equal to SM predictions.
- Errors = 0.10 (similar to present exp. errors).

$BR(B \to X_s\gamma)$ $\langle A_{FB}(B \to K^*\mu^+\mu^-)\rangle_{[1,6]}$

$A_{CP}(B \to K^*\gamma)$ $\langle F_L(B \to K^*\mu^+\mu^-)\rangle_{[1,6]}$

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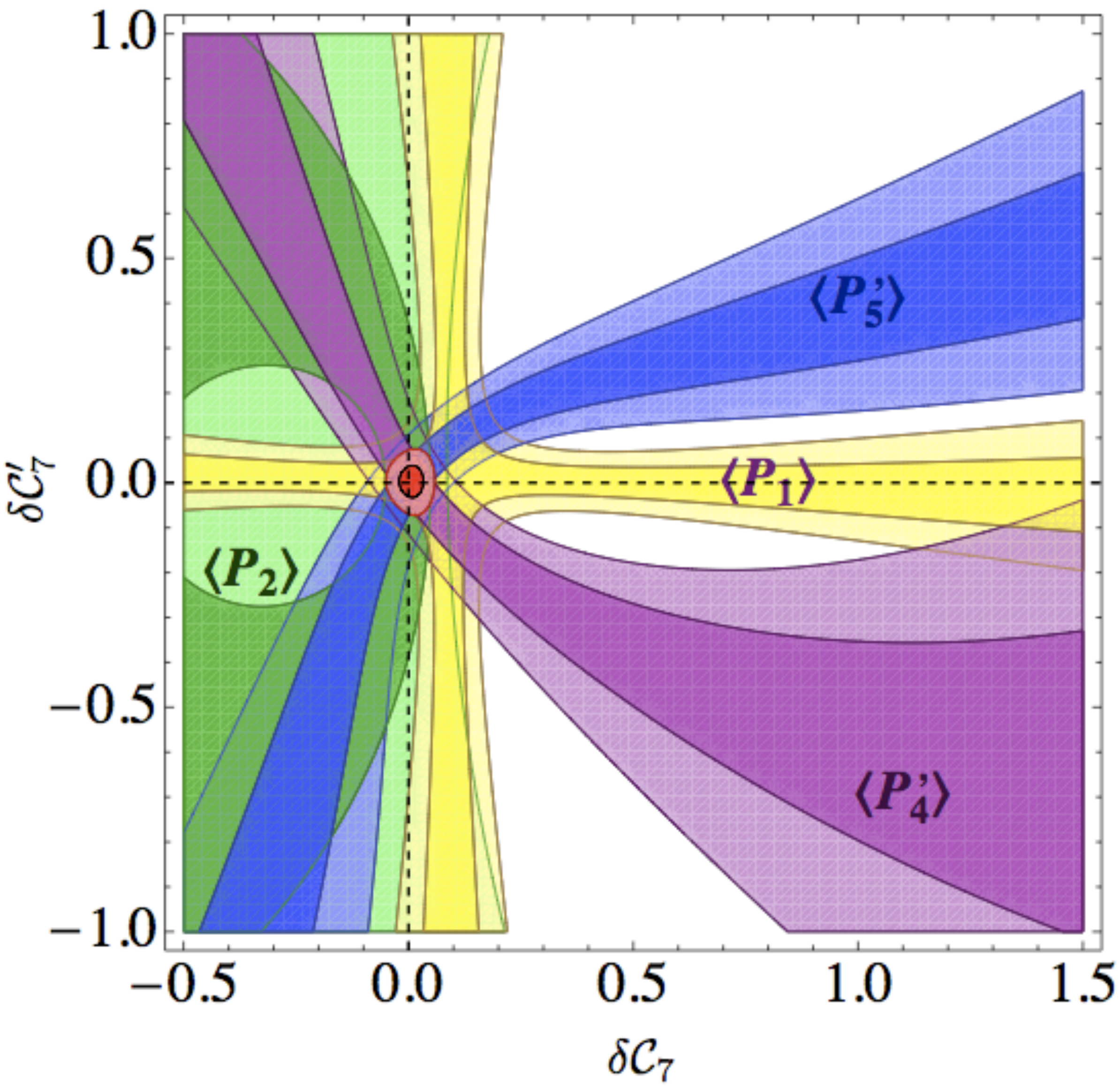

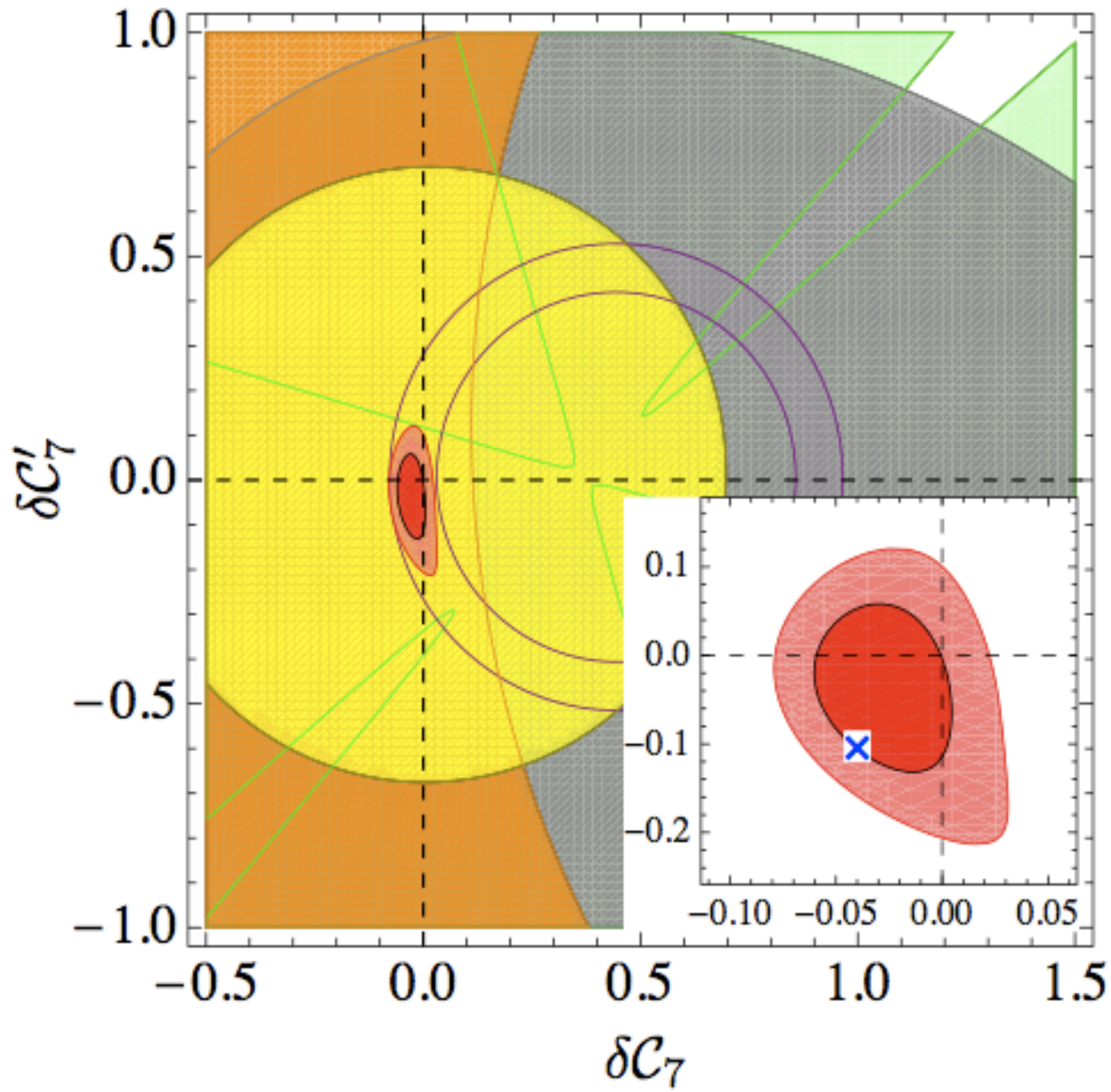

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SUMMARY

- $\mathrm{B} \to \mathrm{K}^*(\to \mathrm{K}\pi)\ell^+\ell^-$ will provide the **strongest** constraints on radiative and semileptonic operators.

- *However*: It is important to consider theoretically **CLEAN** observables

$$\{d\Gamma/dq^2, A_{FB}, P_1, P_2, P_3, P_4', P_5', P_6'\}$$

- P_1, P_2, P_3 can be *already* extracted from experimental measurements, and already impose interesting constraints on C7 and C7'.

- We must pay *close attention* to developments in this topic in next months!!!

BACK UP

- Many people have worked on these ideas regarding $B \to K^* \mu^+ \mu^-$

Experiments:

Belle Collaboration 0904.0770[hep-ex]
CDF Collaboration 1108.0695[hep-ex]
BaBar Collaboration 1204.3933[hep-ex]
LHCb Collaboration LHCb-CONF-2012-008
LHCb Collaboration 1205.3422[hep-ex]
.....

SM & Angular Observables

Beneke, Feldmann, Seidel 0106067, 0412400
Kruger, Matias 0502060
Bobeth, Hiller, Piranishvili 0805.2525
Egede, Hurth, Matias, Ramon, Reece 0807.2589, 1005.0571
Altmannshofer, Ball, Bharucha, Buras, Staub, Wick 0811.1214
Bobeth, Hiller, van Dyk 1006.5013 + 2011, 2012
Matias, Mescia, Ramon, Virto 1202.4266
Camalich, Jaegger 2013
Descotes-Genon, Hurth, Matias, Virto 2013
.....

Model-Independent Constraints

Descotes-Genon, Gosh, Matias, Ramon 1104.3342
Bobeth, Hiller, van Dyk 1105.0376
Altmannshofer, Paradisi, Straub 1111.1257
Bobeth, Hiller, van Dyk, Wacker 1111.2558
Beaujean, Bobeth, van Dyk, Wacker 1205.1838
Altmannshofer, Straub 1206.0273
Becirevic, Kou, Le Yaouanc, Tayduganov 1206.1502
Mahmoudi, Hurth 2012
Descotes-Genon, Matias, Ramon, Virto 1207.2753
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FORM FACTOR issues

- We use the FF's computed from LCSR's with B-meson DA's

  and SE parametrization for q^2 dependence:

Khodjamirian, Mannel, Pivovarov, Wang, 2010

Form factor	$F(0)$	b_F	m_F (GeV)
$V(q^2)$	$0.36^{+0.23}_{-0.12}$	$-4.8^{+0.8}_{-0.4}$	5.412
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$$\tau_\pm = (m_B \pm m_{K*})^2,\ \tau_0 = \tau_+ - \sqrt{\tau_+ - \tau_-}\sqrt{\tau_+}.$$

Much more conservative errors than e.g. Ball-Zwicky 2004

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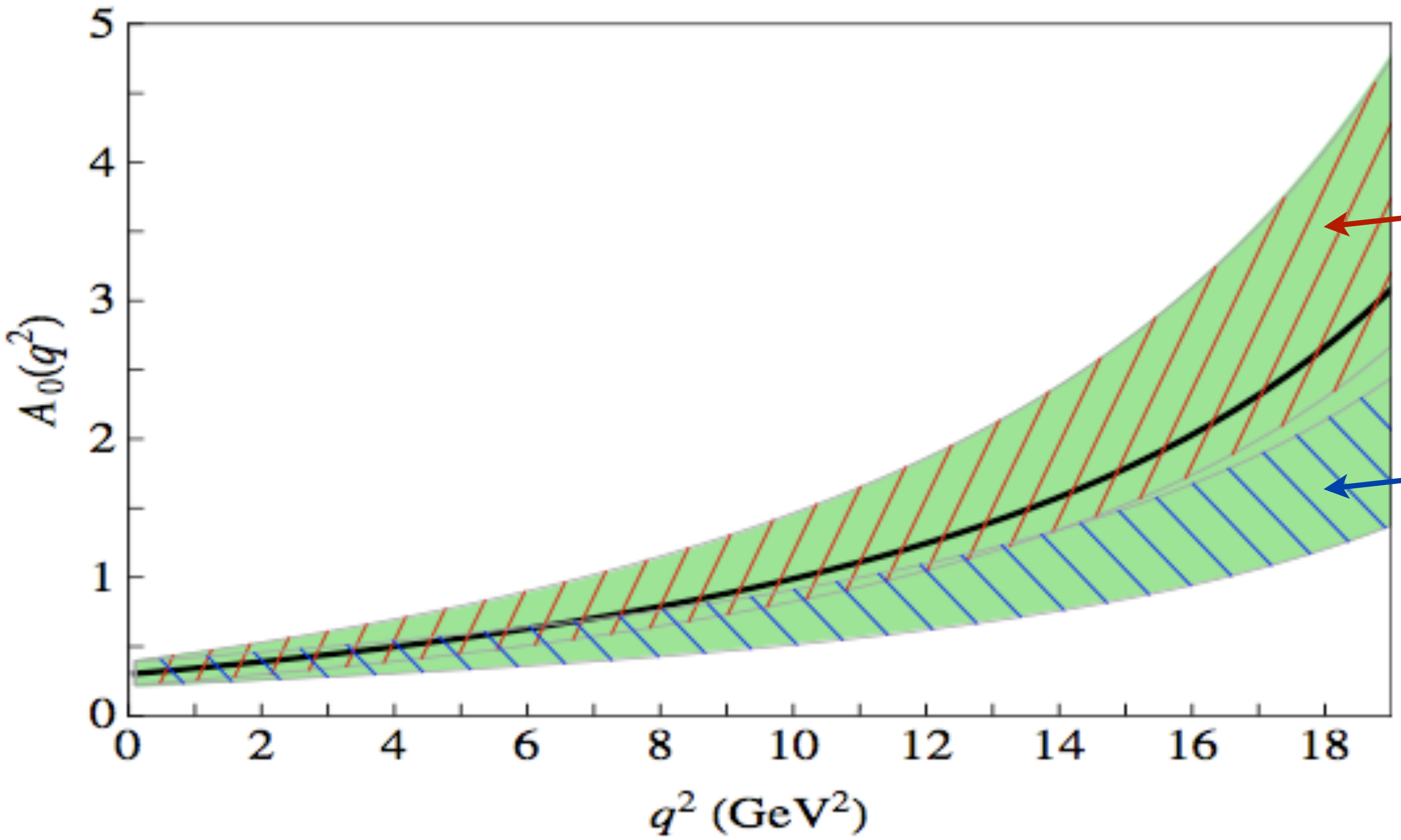
(only enters in At amplitude, suppressed by m^2/s)

All consistent (LCSR/LEET)

FORM FACTOR issues

- Treatment for A0(q^2): Very conservative



LCSR's (KMPW)

$$A_0(q^2) = \frac{E}{m_{K^*}} \frac{\xi_{\|}(q^2)}{\Delta_{\|}(q^2)} + \mathcal{O}(\Lambda/m_b)$$

FORM FACTOR issues

- At LOW recoil: we have the FF ratios:

$$R_1 = \frac{T_1(q^2)}{V(q^2)}, \qquad R_2 = \frac{T_2(q^2)}{A_1(q^2)} \qquad R_3 = \frac{q^2}{m_B^2}\frac{T_3(q^2)}{A_2(q^2)}$$

Bobeth, Hiller, Van Dyk, 2010

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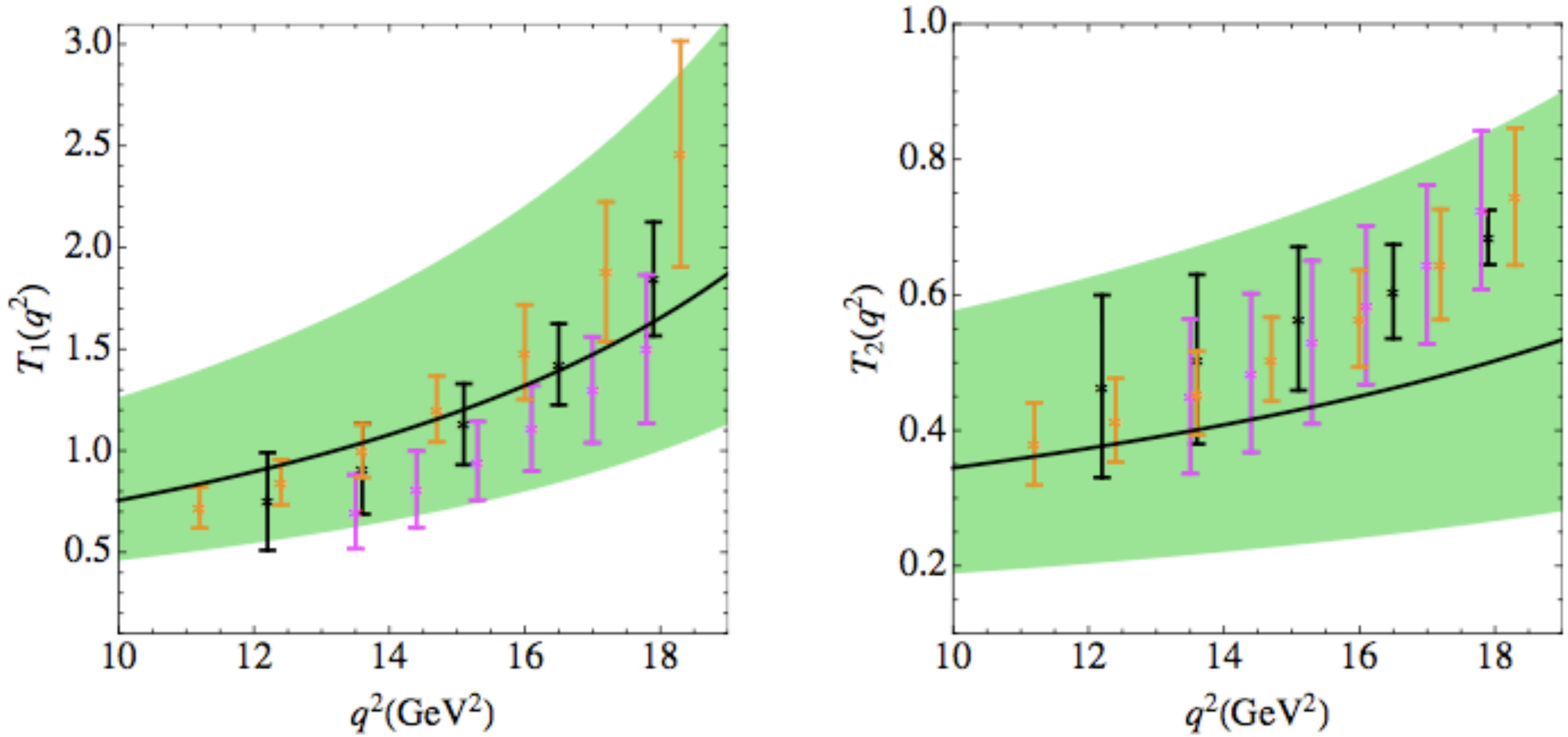

Lattice: Becirevic, Lubicz, Mescia 2007

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~0.4!!!

Bobeth, Hiller, Van Dyk, 2010

Grinstein, Pirjol, 2004

The 1/mb scaling does not seem to be consistent with LCSRs

$$\hat{R}_3 = \frac{q^2}{m_B^2}\frac{T_3}{2\frac{m_V}{m_B}A_0(q^2) - \left(1+\frac{m_V}{m_B}\right)A_1(q^2) + \left(1-\frac{m_V}{m_B}\right)A_2(q^2)}$$

FORM FACTOR issues

- At LOW recoil: we have the FF ratios:

$$R_1 = \frac{T_1(q^2)}{V(q^2)}, \checkmark \quad R_2 = \frac{T_2(q^2)}{A_1(q^2)} \checkmark \quad R_3 = \frac{q^2}{m_B^2}\frac{T_3(q^2)}{A_2(q^2)} \;\text{X}$$

Bobeth, Hiller, Van Dyk, 2010

~0.4!!!

Grinstein, Pirjol, 2004

The 1/mb scaling does not seem to be consistent with LCSRs

$$\hat{R}_3 = \frac{q^2}{m_B^2}\frac{T_3}{2\frac{m_V}{m_B}A_0(q^2) - \left(1+\frac{m_V}{m_B}\right)A_1(q^2) + \left(1-\frac{m_V}{m_B}\right)A_2(q^2)}$$

Same order!!!!

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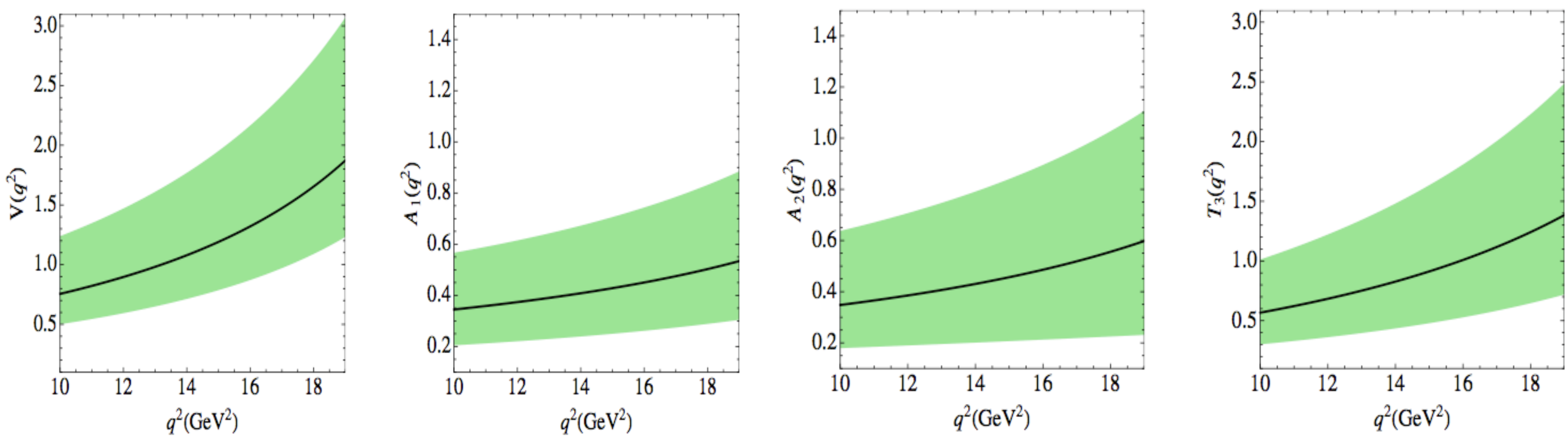
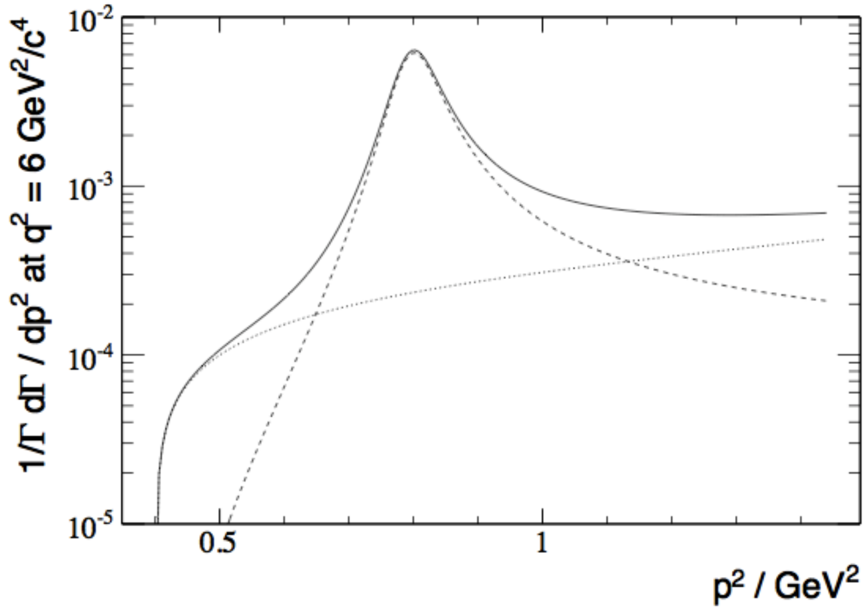
- We use R1 and R2 with 20% 1/mb correction, *BUT NOT R3*



S-wave contribution

S-Wave:

[Becirevic-Tayduganov, Matias, Blake-Egede-Shires]



$$\frac{1}{\Gamma_{\text{full}}}\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_\ell\, d\hat{\phi}} = \frac{9}{16\pi}\Bigg[\left(\frac{2}{3}F_S + \frac{4}{3}A_S\cos\theta_K\right)\sin^2\theta_\ell$$

$$+(1-F_S)\Big[2F_L\cos^2\theta_K\sin^2\theta_\ell + \frac{1}{2}F_T\sin^2\theta_K(1+\cos^2\theta_\ell)$$

$$+\frac{1}{2}F_T P_1\sin^2\theta_K\sin^2\theta_\ell\cos 2\hat{\phi} + 2F_T P_2\sin^2\theta_K\cos\theta_\ell$$

$$-F_T P_3\sin^2\theta_K\sin^2\theta_\ell\sin^2\hat{\phi}\Big]$$

In principle a fit to the whole (folded) distribution can disentangle the S-wave contribution

Also, model-independent bounds can be set on the interference terms:

$A_S \leq 2\sqrt{3}\sqrt{F_S(1-F_S)F_L} \longrightarrow \frac{3}{16\pi}A_S \leq 0.044$ for $F_S \sim 7\%$

Similar bounds for other interference coefficients of the order of few per mille