

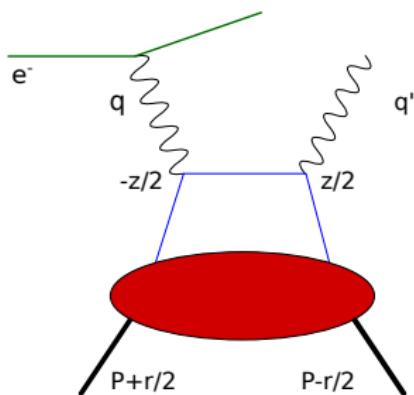
# *Parametrisation of the Generalized Parton Distribution $H$ with Double Distribution Models*

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# Deep Virtual Compton Scattering



DVCS can be factorised into:

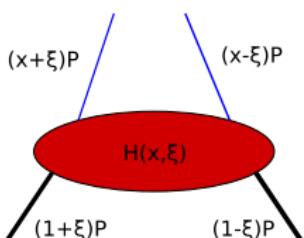
- a short range process → perturbative QFT.
- a long range process → non perturbative QCD.

One way to deal with npQCD :  
**Generalized Parton Distributions (GPD)**

Mueller et al. Fortsch. Phys. **42** (1994) 101  
 Ji Phys. Rev. Lett. **78** (1997) 610  
 Radyushkin Phys. Lett. B **380** (1996) 417

# Definition of the quark GPD $H$

$$\begin{aligned}
 & \frac{1}{2} \int \frac{e^{ixP^+z^-}}{2\pi} \langle P - \frac{r}{2} | \bar{q}(-\frac{z}{2}) \gamma^+ q(\frac{z}{2}) | P + \frac{r}{2} \rangle dz^-|_{z^+=0,z=0} \\
 = & \frac{1}{2P^+} [ H^q(x, \xi, t) \bar{u}(P - \frac{r}{2}) \gamma^+ u(P + \frac{r}{2}) \\
 & + E^q(x, \xi, t) \bar{u}(P - \frac{r}{2}) \frac{i\sigma^{+\alpha} r_\alpha}{2m} u(P + \frac{r}{2}) ]
 \end{aligned}$$



- $x$  : fraction of the longitudinal momentum carried by quark
- $\xi$  : kinematical parameter such that  $\xi \approx \frac{x_B}{2-x_B}$
- $t = r^2$  ( $t$  dependence will be omitted in what follows)

# Properties of the GPD $H$

GPDs are theoretically **constrained** to fulfill:

- **Polynomiality** in  $\xi$  of Mellin moments:

$$\int_{-1}^1 dx \ x^n H^q(x, \xi) = \sum_{i=0, \text{even}}^n (2\xi)^i A_{n+1,i}^q + \text{mod}(n, 2)(2\xi)^{n+1} C_{n+1}^q$$

- Support properties:  $|x| \leq 1$  and  $|\xi| \leq 1$
- **Forward limit**  $\rightarrow$  Reduction to PDF :  $H^q(x, 0) = f^q(x)$
- **H continuous** at point  $x = \xi$

Interests :

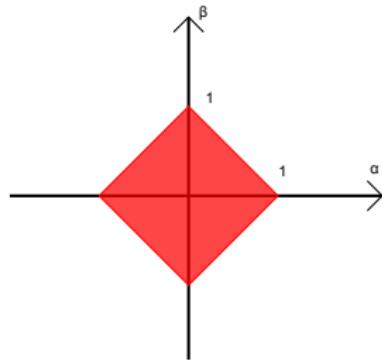
- Understand the **spin structure** of the proton
- Get the **3D imaging** of protons.

## Modeling GPD : Double Distribution

Double Distribution : popular way to model GPD.

Radyushkin Phys. Lett. B **380** (1996) 417

$$H(x, \xi) = \int_{\Omega} d\alpha d\beta (F(\beta, \alpha) + \xi G(\beta, \alpha)) \delta(x - \beta - \xi\alpha)$$



$$\Omega = \{(\alpha, \beta) | |\alpha| + |\beta| \leq 1\}$$

**Advantage :**

Easy way to respect the polynomiality in  $\xi$

$$\begin{aligned} & \int_{-1}^1 x^n H(x, \xi) dx \\ &= \int_{\Omega} (\beta + \xi\alpha)^n (F(\beta, \alpha) + \xi G(\beta, \alpha)) d\Omega \end{aligned}$$

# Double Distribution Ambiguity

Teryaev Phys. Lett. B **510** (2001) 125

Tiburzi Phys. Rev. D **70** (2004) 057504

Rewrite the non forward matrix element in terms of DD :

$$\begin{aligned} & \langle P - \frac{r}{2} | \bar{\psi}(-\frac{z}{2}) \not{z} \psi(\frac{z}{2}) | P + \frac{r}{2} \rangle \\ &= \int_{\Omega} e^{-i\beta(Pz) - i\alpha \frac{(rz)}{2}} (2(Pz)F(\alpha, \beta, t) + (rz)G(\beta, \alpha)) d\alpha d\beta \end{aligned}$$

Matrix element **invariant** under the following transformation :

$$\begin{aligned} F(\beta, \alpha) &\rightarrow F(\beta, \alpha) + \frac{\partial \sigma}{\partial \alpha} \\ G(\beta, \alpha) &\rightarrow G(\beta, \alpha) - \frac{\partial \sigma}{\partial \beta} \\ \sigma(\beta, \alpha) &= -\sigma(\beta, -\alpha) \end{aligned}$$

This invariance allows for **different** methods to parametrize GPDs.

*DD+D versus one Component DD parametrizations*

*Two different parametrisations : DD+D and oCDD*

## *DD+D parametrisation*

- All the  $\beta$  dependence is included in  $F$  :

$$H(x, \xi) = \int_{\Omega} d\alpha d\beta (F_D(\beta, \alpha) + \xi \delta(\beta) D(\alpha)) \delta(x - \beta - \xi \alpha)$$

- $F_D(\beta, \alpha) \rightarrow$  Factorised Ansatz : Product of **PDF** and a **profile function**  $h(\beta, \alpha)$  :  $F_D(\beta, \alpha) = f(\beta)h(\beta, \alpha)$
- $D$  is mostly unknown.  
M. V. Polyakov and C. Weiss, Phys. Rev. D **60** (1999) 114017

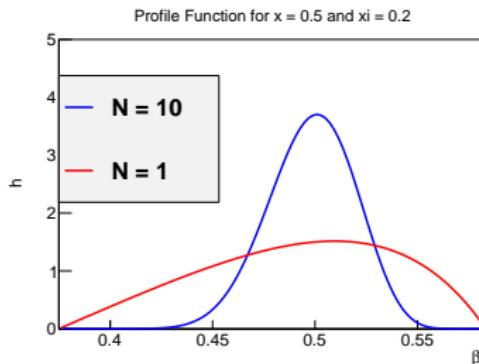
## Parametrisation of GPD

*DD+D versus one Component DD parametrizations*

*Two different parametrisations : DD+D and oCDD*

## Profile function $h_N$

$$h_N(\beta, \alpha) = \frac{\Gamma(2N+2)}{2^{2N+1}\Gamma^2(N+1)} \frac{[(1-\beta)^2 - \alpha^2]^N}{(1-\beta)^{2N+1}}$$



$$h_N(\beta, \frac{x-\beta}{\xi})$$

- Classical Ansatz
- Normalised such that  $\int_{-1+|\beta|}^{1-|\beta|} d\alpha h_N(\beta, \alpha) = 1$
- $\lim_{N \rightarrow +\infty} h_N(\beta, \alpha) = \delta(\alpha)$

*DD+D versus one Component DD parametrizations*

*Two different parametrisations : DD+D and oCDD*

## *oCDD parametrisation*

- Only a **one component** DD remains :

Belitsky et al. Phys. Rev. D **64**, 116002 (2001)

$$H(x, \xi) = \int_{\Omega} d\alpha d\beta (\beta + \xi\alpha) \delta(x - \beta - \xi\alpha) v(\beta, \alpha)$$

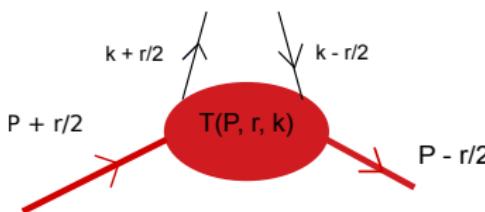
- $\beta v(\beta, \alpha) \rightarrow$  Factorised Ansatz : Product of **PDF** and a **profile function**  $h(\beta, \alpha)$  :  $\beta v(\beta, \alpha) = f(\beta)h(\beta, \alpha)$
- Advantage** : No more D-term !
- Drawback** : potentially more divergent ( $\frac{x}{\beta}$  term in the integral).

# Explicit model for oCDD parametrisation

Szczepaniak et al. Acta Phys. Polon. B **40** (2009) 2193

A. Radyushkin Phys. Rev. D **83** (2011) 076006

Dispersion relation  $T(P, r, k)$



- $T(P, r, k) = T((P - k)^2)$

- $T((P - k)^2) = T_0 + \int_0^{+\infty} d\sigma \rho(\sigma) \left[ \frac{1}{\sigma - (P - k)^2} - \frac{1}{\sigma} \right]$

# Explicit model for oCDD parametrisation

- Unknown function  $\rho(\sigma) \rightarrow$  trick : use the **forward limit!**
- At the end of the day :

$$\begin{aligned} \frac{H(x, \xi)}{x} &= \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha \frac{f(\beta)}{\beta} h_N(\beta, \alpha) \\ &\times \left[ \delta(x - \beta - \alpha\xi) - \delta\left(x - \frac{\alpha}{1-\beta}\xi\right) \frac{1}{(1-\beta)^2} \right] \end{aligned}$$

with

- $f(\beta) = \text{PDF}$
- $h_N(\beta, \alpha) = \frac{\Gamma(2N+2)}{2^{2N+1}\Gamma^2(N+1)} \frac{[(1-\beta)^2 - \alpha^2]^N}{(1-\beta)^{2N+1}}$

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- Play with two pieces :  $f(\beta)$  and  $N$

# The $T_0$ constant

Recalling that :

$$T((P - k)^2) = T_0 + \int_0^{+\infty} d\sigma \rho(\sigma) \left[ \frac{1}{\sigma - (P - k)^2} - \frac{1}{\sigma} \right]$$

$T_0 \rightarrow$  leads to an additional term :

$$D_0\left(\frac{x}{\xi}\right) = \frac{T_0}{2^{2N+1} N} \frac{x}{|\xi|} \left(1 - \left(\frac{x}{\xi}\right)^2\right) \theta\left(\left|\frac{x}{\xi}\right| < 1\right)$$

We set  $T_0 = 0$ .

N.B.: PDF behaviour allowed  $\approx \frac{1}{\beta^a}$  with  $a < 1$

## Available tools

- First idea → test both parametrisations with experimental data : JLab.  
C. M. Camacho *et al.* Phys. Rev. Lett. **97** (2006) 262002  
F. X. Girod *et al.* Phys. Rev. Lett. **100** (2008) 162002
- JLab advantages: precision, kinematics, valence sensitivity
- JLab data → mostly sensitive to  $H$  → use oCDD parametrisation only for  $H$ .
- Test model : Kroll-Goloskokov → compare the native GPD with the oCDD parametrisation for the same PDF  
S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **42**, **53** and **65**
- Leading order phenomenology

## Available tools

- Valence quarks → well treated by oCDD

Behaviour:  $\approx \frac{1}{\beta^{0.48}}$

- Sea quarks → need **additional** subtraction

Behaviour :  $\approx \frac{1}{\beta^{1.1+0.9t}} \rightarrow t_{crit} \approx -0.1 \text{GeV}^2$

Only valence quarks are parametrised thanks to oCDD  
→ JLab: **valence kinematics**

- No gluons

# Profile function

$$h_N(\beta, \alpha) = \frac{\Gamma(2N+2)}{2^{2N+1}\Gamma^2(N+1)} \frac{[(1-\beta)^2 - \alpha^2]^N}{(1-\beta)^{2N+1}}$$

- Usual choice for valence quarks  $N_{val} = 1$  is chosen  
→ This is an asymptotic behaviour.
- What does asymptotic mean?  
→ JLab :  $1\text{GeV}^2 < Q^2 < 4\text{GeV}^2$   
→ Take the freedom to fit  $N_{val}$

Limit  $N \rightarrow \infty$

- GK and oCDD have common limit when  $N \rightarrow \infty$
- This limit called Forward Parton Distribution (FPD) is independent of  $\xi$
- For the GPD H :

$$H_{FPD}(x) = f(x)$$

# Compton Form Factor (CFF)

$$\mathcal{H} = \int_{-1}^{+1} dx H(x, \xi, t) \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

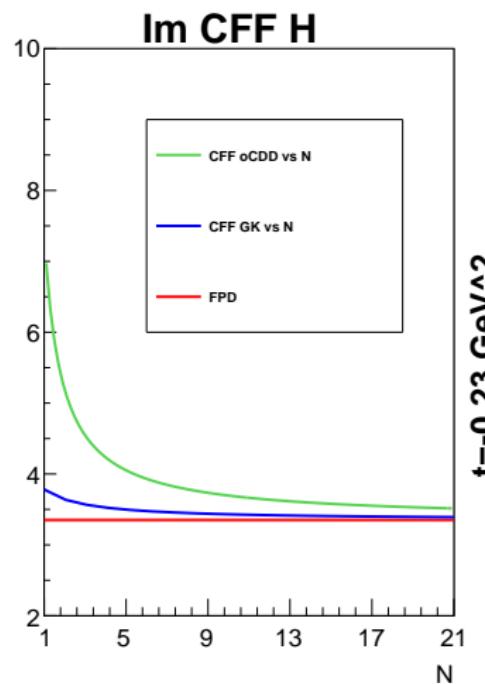
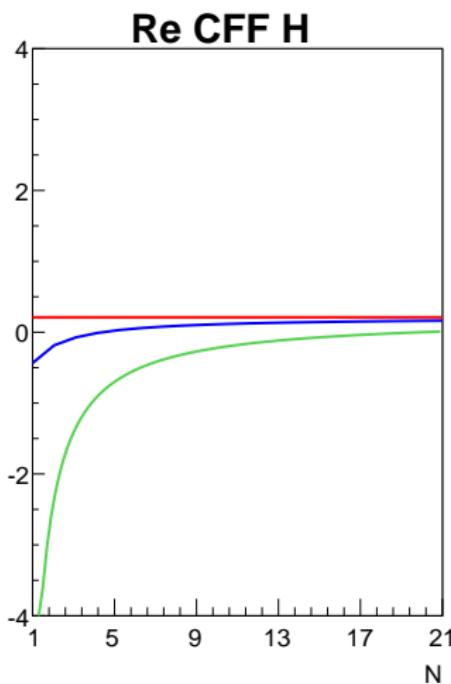
Integration yields **real** and **imaginary** parts to  $\mathcal{H}$  :

## Compton Form Factor at Leading Order

$$\Re(\mathcal{H}) = \mathcal{P} \int_{-1}^{+1} dx H(x, \xi, t) \left( \frac{1}{\xi - x} - \frac{1}{\xi + x} \right)$$

$$\Im(\mathcal{H}) = \pi \left( H(\xi, \xi, t) - H(-\xi, \xi, t) \right)$$

# Profile function (valence dependence with $N$ )



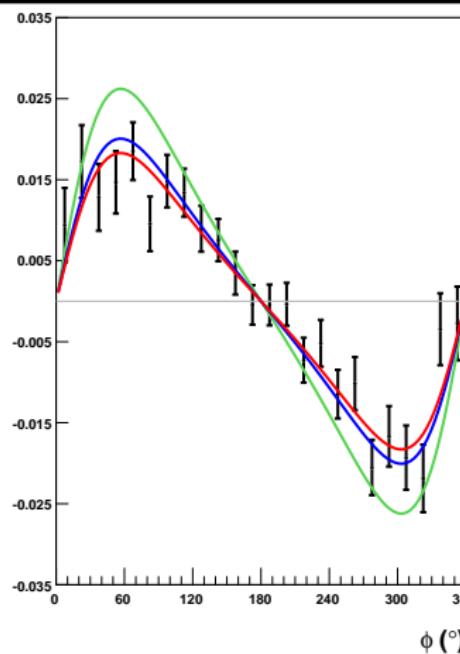
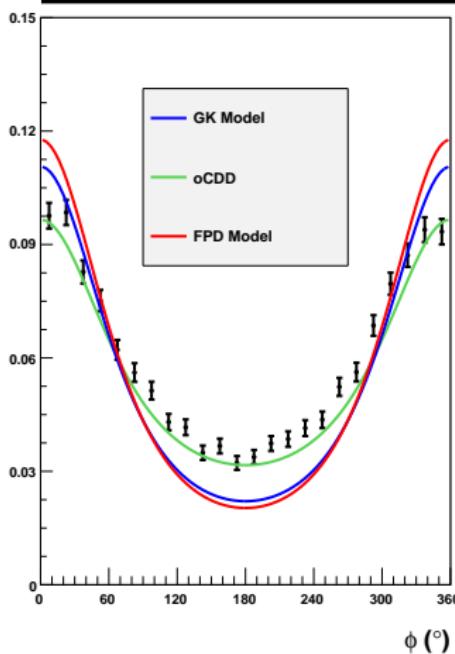
## Parametrisation of GPD

Comparison to JLab hall A data

Fit of  $N_{val}$  parameterJLab Hall A data at  $t = -0.23 \text{ GeV}^2$ 

$$\frac{1}{2}(\sigma(h_e = +1) + \sigma(h_e = -1)) \text{ (nb/GeV}^4)$$

$$\frac{1}{2}(\sigma(h_e = +1) - \sigma(h_e = -1)) \text{ (nb/GeV}^4)$$

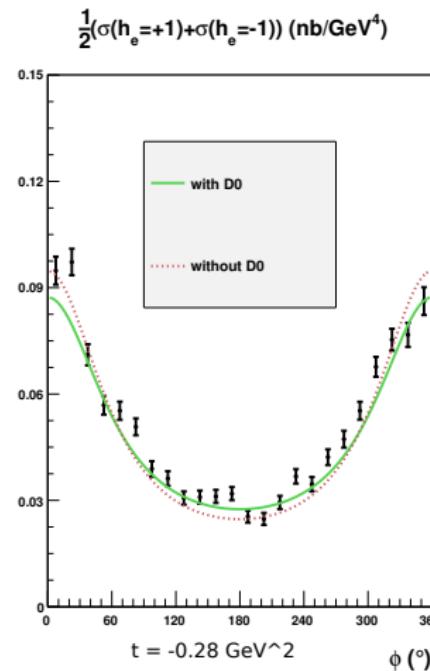
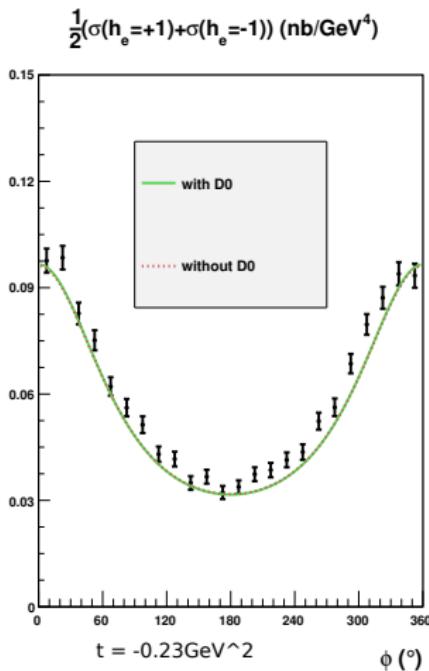


JLab Hall A data :  $\chi^2$

Model	Normalised $\chi^2$ on previous plots	Global normalised $\chi^2$
oCDD	3.7	3.8
GK	11.0	6.1
FPD	14.3	7.3

- At  $t = -0.23 \text{ GeV}^2$  oCDD clearly better
- Improvement of GK and FPD on the other kinematics of Hall A:  $\rightarrow$  3 times more kinematics of differences of cross section than sum
- Good improvement but not a good agreement
- What about the  $T_0$  parameter?

# Impact of $D_0$ term



## Impact of $D_0$ term

- $D_0$  effect is clearly **dependent** of  $t$
- However → data comparisons are **not** strongly affected
- Main part of the improvement comes from DD
- $T_0 \rightarrow$  still not enough to get a good agreement

## *Summary*

- Start : 2 **equivalent** parametrisations
- Hypothesis : factorised GPD into a PDF and a profile function  
→ equivalence is broken → 2 distinct models
- Basic phenomenological test of these 2 models
- Comparison of the results

## *Conclusion*

- Can get GK by modifying  $N_{val}$  → you **don't lose anything**
- Still, it is **more flexible**
- Data comparison is better
- More elegant

**It is worth it!**

## *Outlook*

Still, several points need further investigation :

- Applying oCDD formalism to GPD  $E$ ,  $\tilde{H}$  and  $\tilde{E}$
- Regularise the model for the sea quarks
- Evolution?
- Next to Leading Order?

Thank you

## *Collaborators*

Many thanks to

- H. Moutarde
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- J. Ball
- M. Garçon
- M. Defurne

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# Appendices

# Comparison to Hall B data

