

# **NUCLEAR PARTON DISTRIBUTION FUNCTIONS**

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# OUTLINE

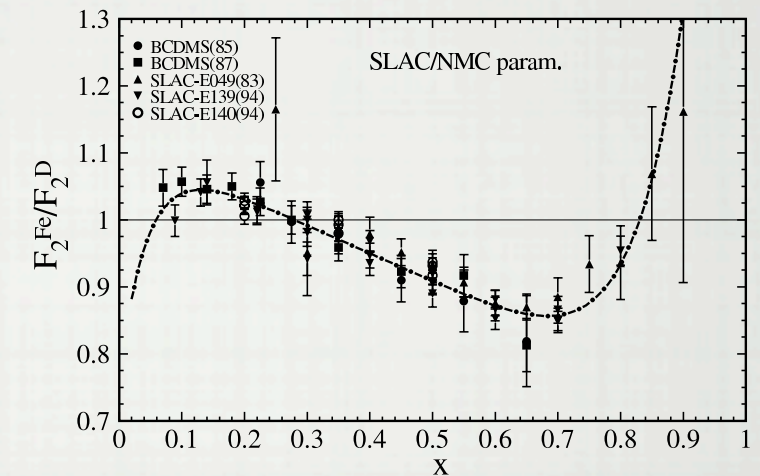
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1. Motivation for nuclear PDFs
2. Review of available nPDFs
3. Details of nuclear CTEQ analysis
4. Future experiments and nPDF needs



# MOTIVATION

- What are nuclear parton density functions (nPDF) ?
  - parton densities for partons in bound proton & neutron



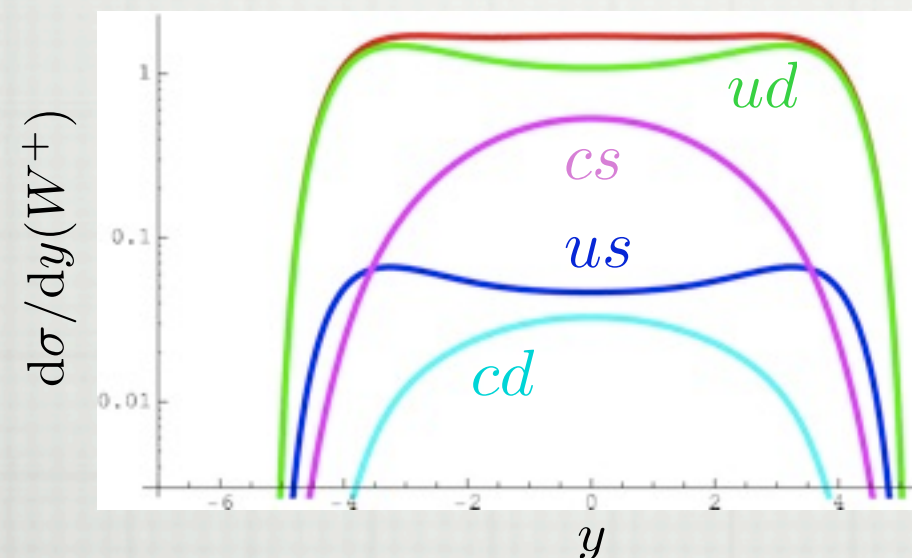
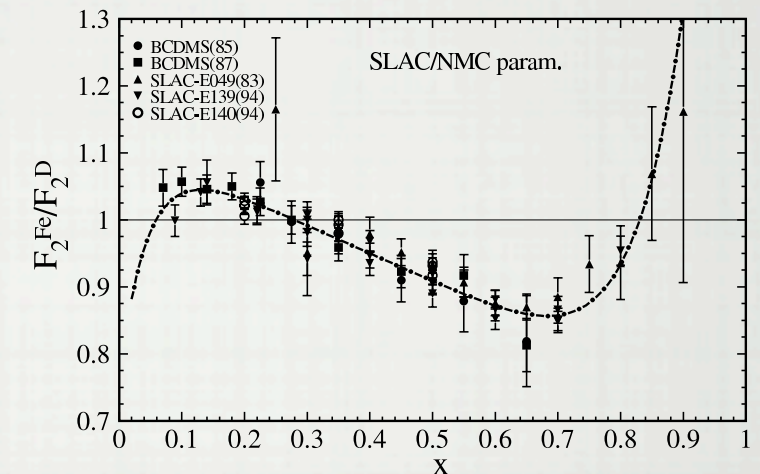
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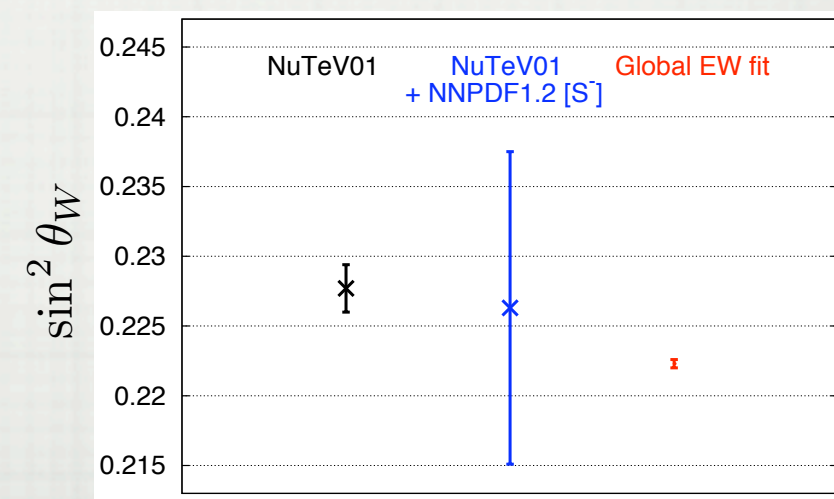
- Where are nuclear parton density functions useful ?

## I. Strange quark content of the proton

(anti-)strange PDF from (anti-)neutrino DIS with heavy nuclei - nuclear effects important



*W-boson production @ LHC*



*weak mixing angle from  
NuTeV experiment*



# MOTIVATION

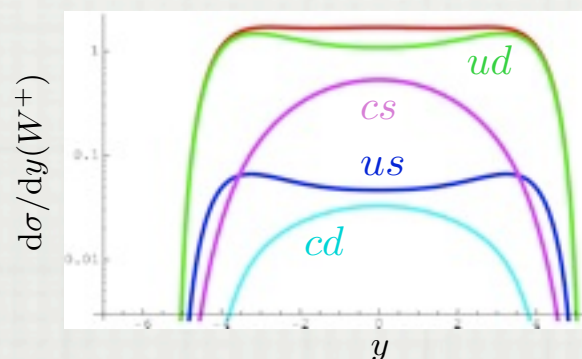
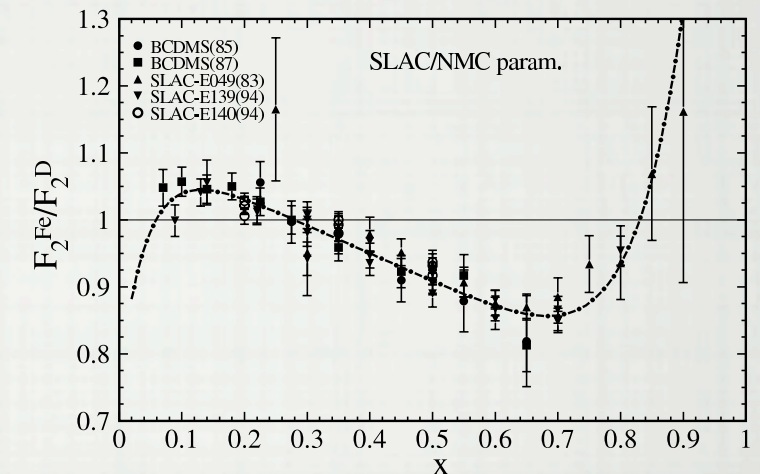
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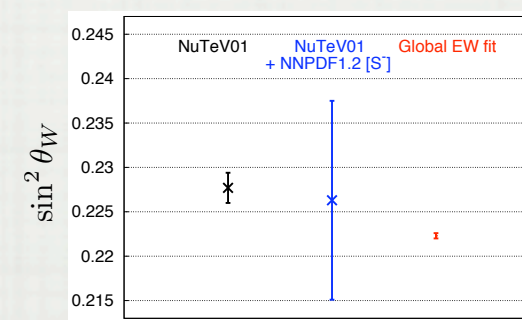
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*W-boson production @ LHC*



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NuTeV experiment*

## 2. Heavy ion collisions @ RHIC, LHC

lead & gold heavy nuclei - nuclear effects in gluon PDF substantial

# NPDF REVIEW

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- Review of existing global analyses of nuclear PDF



# NPDF REVIEW

- Review of existing global analyses of nuclear PDF

*I. Multiplicative nuclear correction factor*

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

free parton density

Hirai, Kumano, Nagai [[PRC76\(2007\)065207](#)] arXiv: 0709.0338

Eskola, Paukkunen, Salgado [[JHEP0904\(2009\)065](#)] arXiv: 0902.4154

de Florian, Sassot, Stratmann, Zurita [[PRD85\(2012\)074028](#)] arXiv: 1112.6324

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2. Convolution relation

$$f_i^A(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i(x_N/y, Q_0^2)$$

nucleon density in nucleus with  $y/A$  mom.fraction

de Florian, Sassot [PRD69(2004)074028] hep-ph/0311227



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## 3. Native nuclear PDF

$$f_i^A(x_N, Q_0^2) = f_i(x_N, A, Q_0^2)$$

bound parton density

$$f_i(x_N, Q_0^2) = f_i(x_N, A = 1, Q_0^2)$$

free parton density

nCTEQ [PRD80(2009)094004] arXiv: 0907.2357



# NPDF REVIEW

- Review of existing global analyses of nuclear PDF

**DE FLORIAN, SASSOT'04** [PRD69(2004)074028]  
LO, NLO

$$\chi^2_{\text{dof}} = 0.76$$

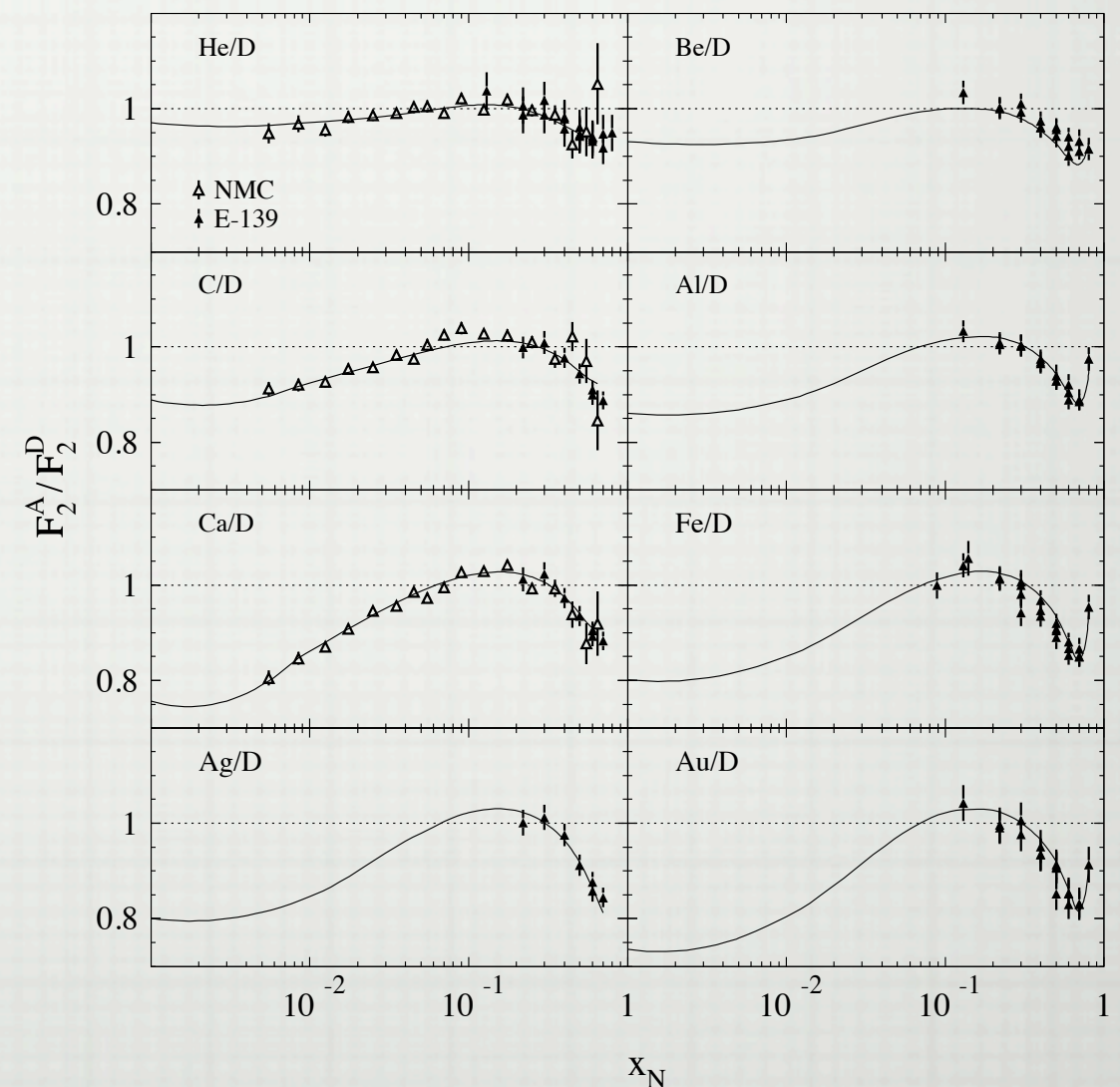
- first NLO analysis of nuclear data
- the only group using convolution relation

$$f_i^A(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i(x_N/y, Q_0^2)$$

- typical nucleon density for valence quarks

$$W_v(y, A, Z) = A[a_v \delta(1 - \epsilon_v - y) + (1 - a_v) \delta(1 - \epsilon_{v'} - y)] \\ + n_v \left(\frac{y}{A}\right)^{\alpha_v} \left(1 - \frac{y}{A}\right)^{\beta_v} + n_s \left(\frac{y}{A}\right)^{\alpha_s} \left(1 - \frac{y}{A}\right)^{\beta_s}$$

- the only framework using evolution in Mellin space & have PDFs also for  $x_N > 1$
- only standard DIS data sets (semi-global)
- no error analysis





# NPDF REVIEW

- Review of existing global analyses of nuclear PDF

**HIRAI, KUMANO, NAGAI'07** [PRC76(2007)065207]  
LO, NLO, ERROR PDFS

- uses multiplicative factor

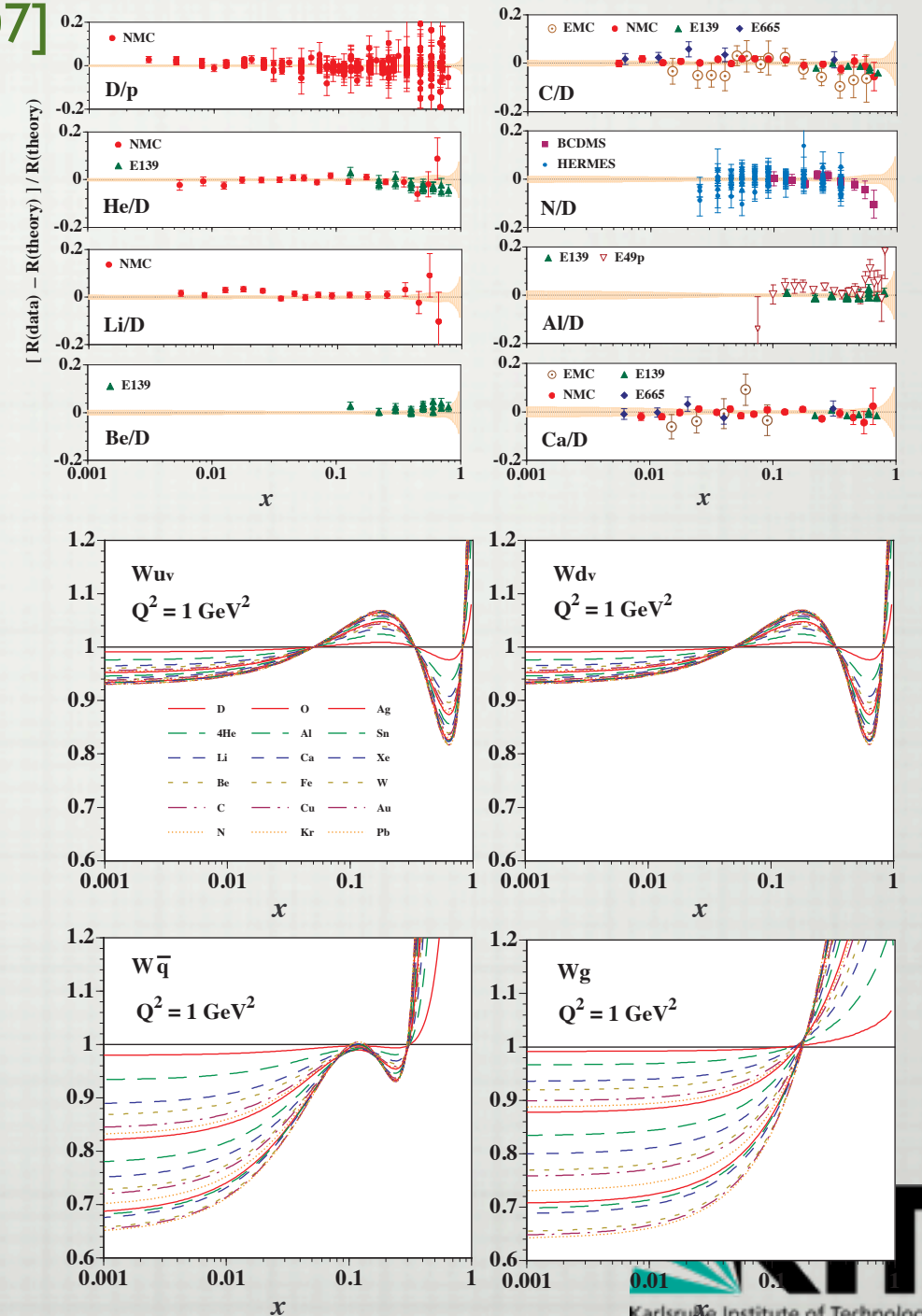
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in MRST 1998 and factor

$$R_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1 - x)^{\beta_i}}$$

- neglects region  $x > 1$
- includes all current DIS & DY data set (same as our analysis - discussed later)
- use Hessian method to produce error PDFs

$$\chi^2_{\text{dof}} = 1.2$$





# NPDF REVIEW

- Review of existing global analyses of nuclear PDF

**ESKOLA, PAUKKUNEN, SALGADO'09** [JHEP0904(2009)065]  
LO, NLO, ERROR PDFS

$$\chi^2/\text{dof} = 0.8$$

- uses multiplicative factor

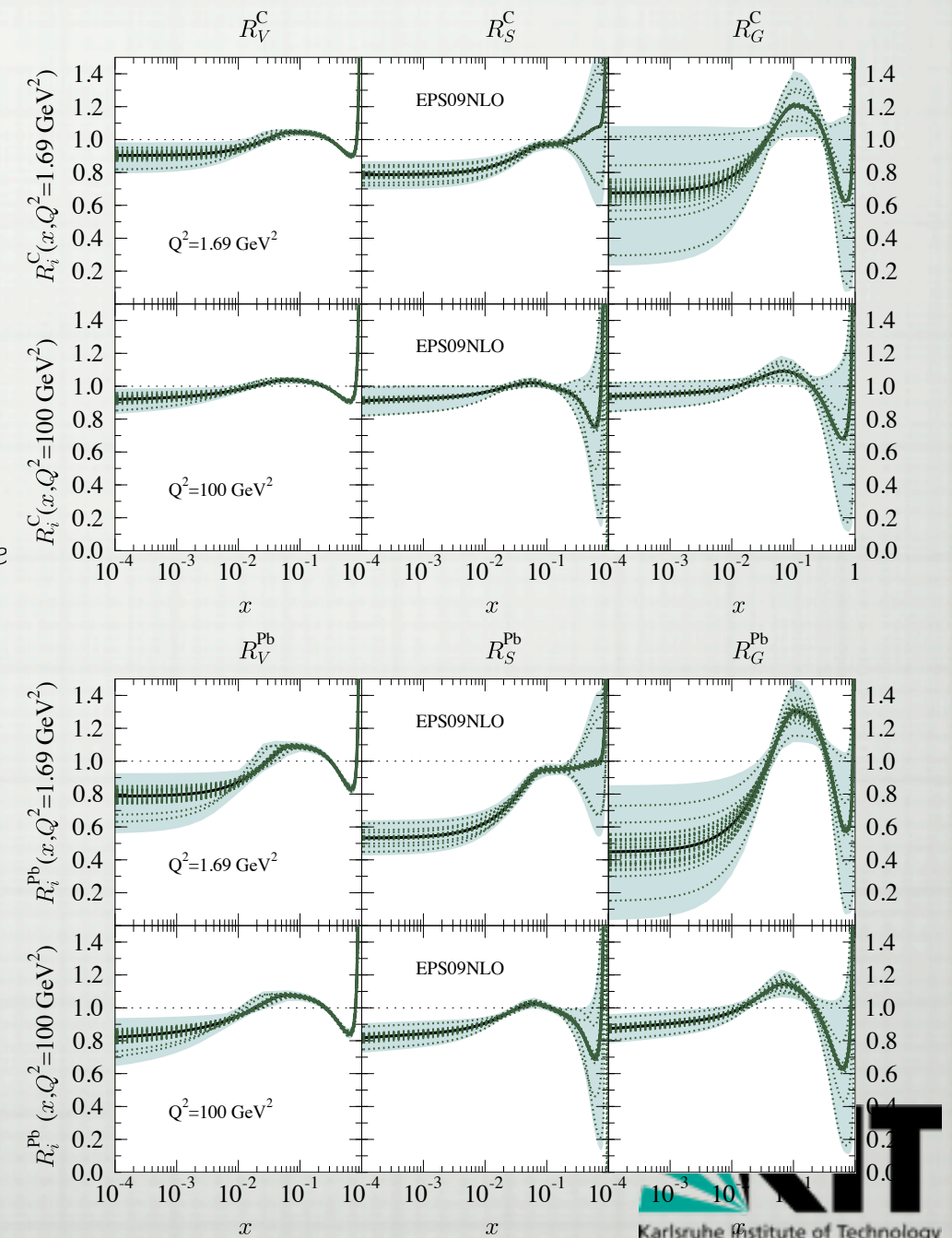
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in CTEQ6.1M and factor is a complicated piecewise defined function

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

with A-dependent parameters

- neglects region  $x > 1$
- includes all current DIS & DY data set &  $\pi^0$  RHIC data to constrain gluon
- use Hessian method to produce error PDFs





# NPDF REVIEW

- Review of existing global analyses of nuclear PDF

**DE FLORIAN, SASSOT, STRATMANN, ZURITA** [PRD85(2012)074028]  
LO, NLO, ERROR PDFS

- uses multiplicative factor

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

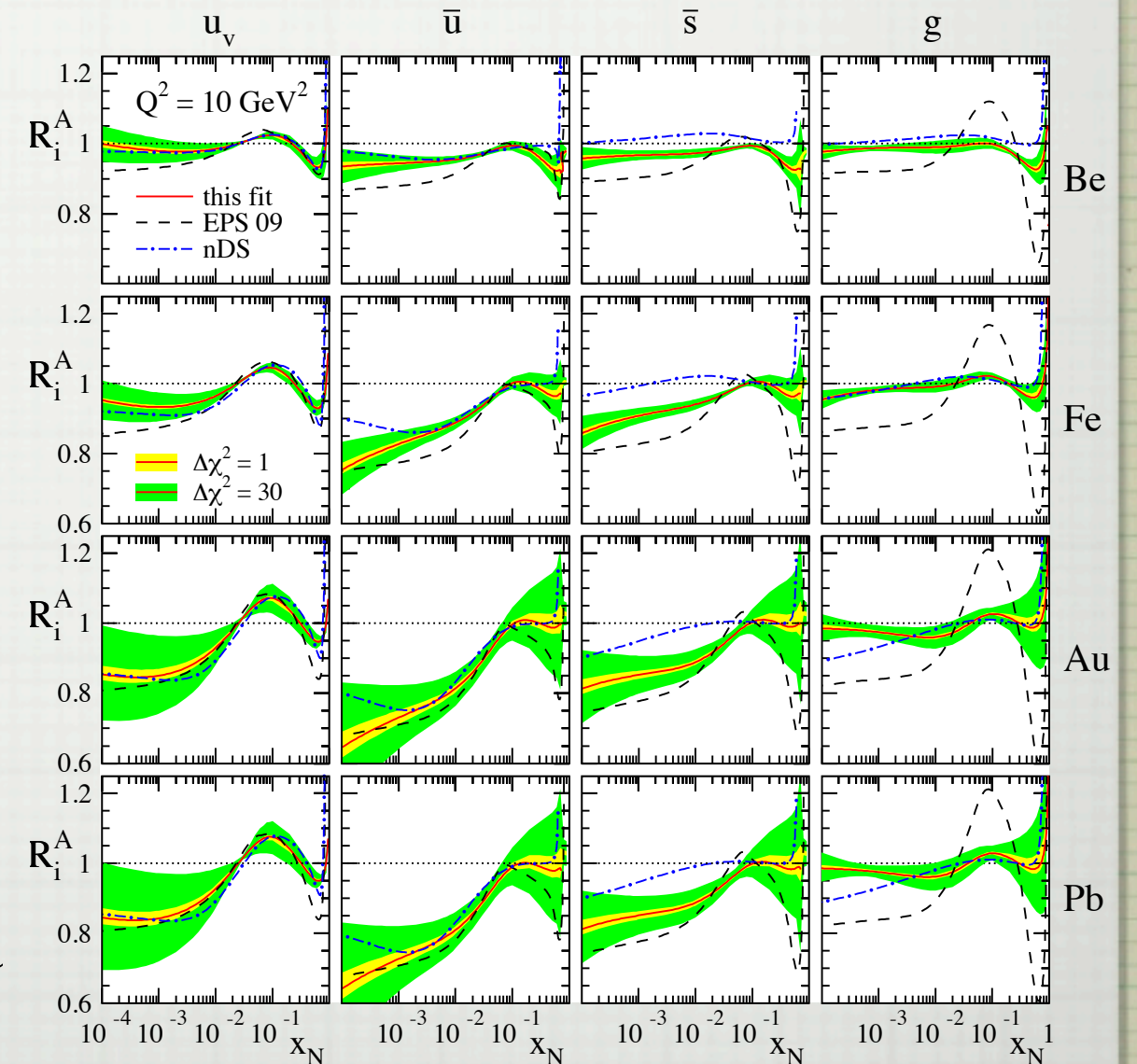
where proton PDF in MSTW08 and factor is a complicated function different for each flavour

$$R_v^A(x, Q_0^2) = \epsilon_1 x^{\alpha_v} (1-x)^{\beta_1} (1 + \epsilon_2 (1-x)^{\beta_2}) \times (1 + a_v (1-x)^{\beta_3})$$

$$R_s^A(x, Q_0^2) = R_v^A(x, Q_0^2) \frac{\epsilon_s}{\epsilon_1} \frac{1 + a_s x^{\alpha_s}}{a_s + 1}$$

$$R_g^A(x, Q_0^2) = R_v^A(x, Q_0^2) \frac{\epsilon_g}{\epsilon_1} \frac{1 + a_g x^{\alpha_g}}{a_g + 1}$$

- includes all current DIS & DY data set &  $\pi^0$  RHIC data and  $F_2^{\nu A}$  from neutrino data
- use Hessian method to produce error PDFs





- CTEQ framework for nuclear PDF - based on CTEQ6M proton fit

- functional form for bound protons same as for free proton PDF (restrict  $x$  to  $0 < x < 1$ )

$$x f_k(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} + (1 + c_3 x)(1 - x)^{c_4}$$

- coefficients with A-dependance (reduces to proton for  $A=1$ )

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1} (1 - A^{-c_{k,2}}), \quad k = \{1, \dots, 5\}$$

- proton coefficients  $c_{k,0}$  fixed to special CTEQ6M fit without much of nuclear data

- PDF for a nucleus with A-nucleons out of which Z-protons

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

- Input scale and other input parameters as in CTEQ6M proton analysis

$$Q_0 = m_c = 1.3 \text{ GeV} \quad m_b = 4.5 \text{ GeV} \quad \alpha_s(m_Z) = 0.118$$

- Kinematic cuts on data

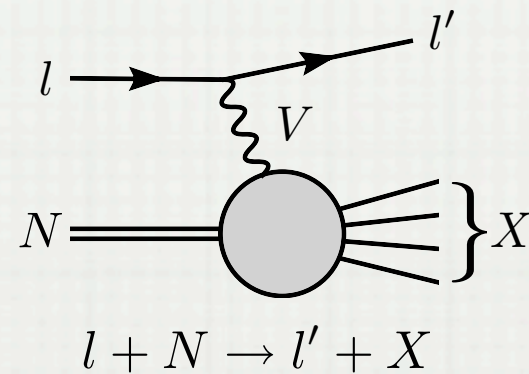
$$Q > 2 \text{ GeV} \quad W > 3.5 \text{ GeV}$$



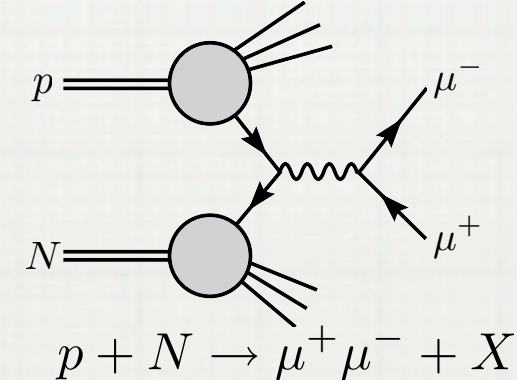
- Experiments included in the analysis

## Charged lepton

Deep Inelastic Scattering



Drell-Yan process



### CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

### FNAL E-665

$N = (\text{D, C, Ca, Pb, Xe})$

### DESY Hermes

$N = (\text{D, He, N, Kr})$

### SLAC E-139 & E-049

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

### FNAL E-772 & E-886

$N = (\text{D, C, Ca, Fe, W})$

1233 data points (708 after cuts)

- NPDF fit properties:

- we fit nuclear data with NLO QCD predictions & include heavy quark effects (ACOT)
- added nuclear observables to CTEQ fitting routines (need to treat 2 nuclei at once)

$$\text{DIS: } F_2^A / F_2^{A'}$$

$$\text{Drell-Yan: } \sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$$

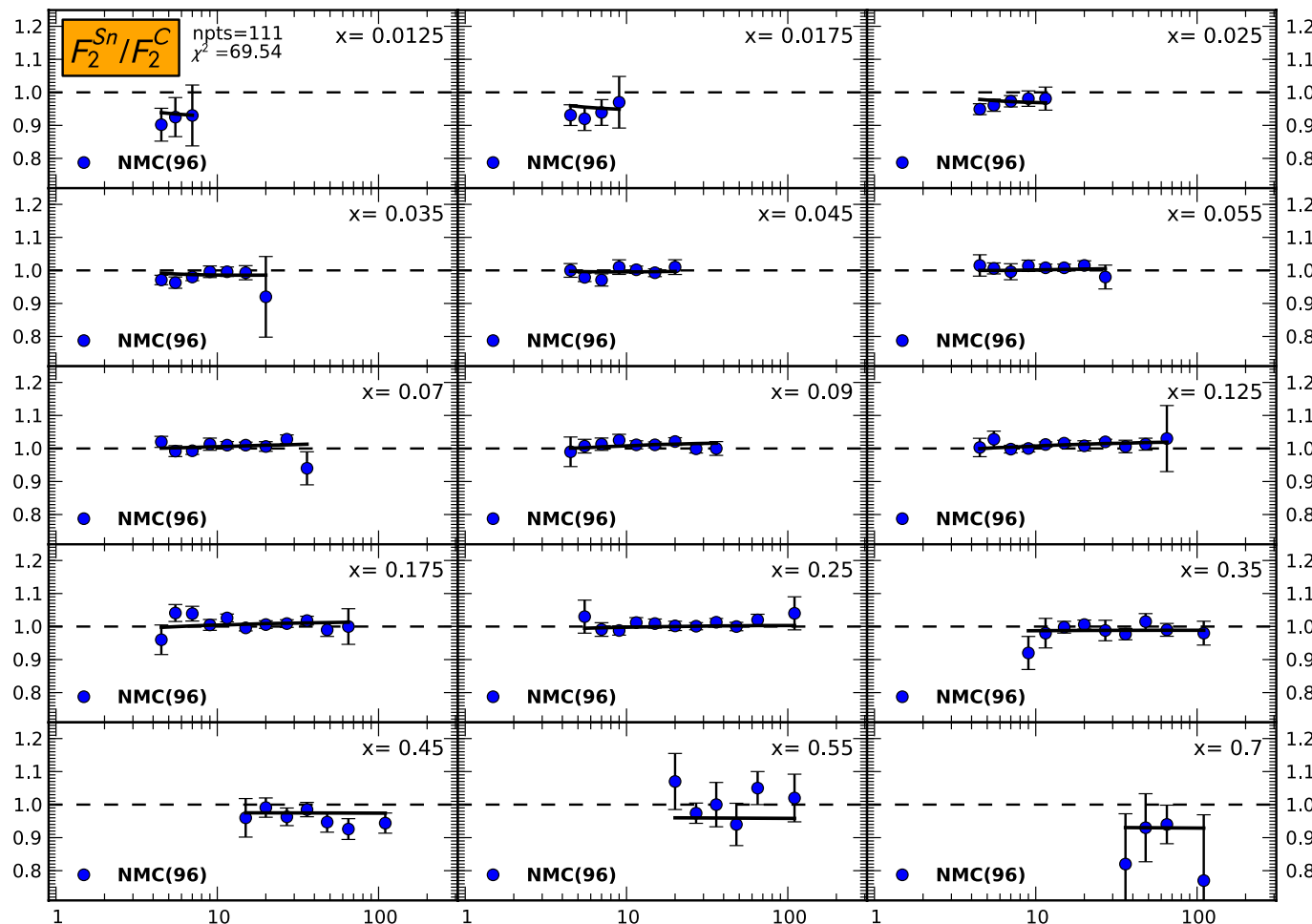
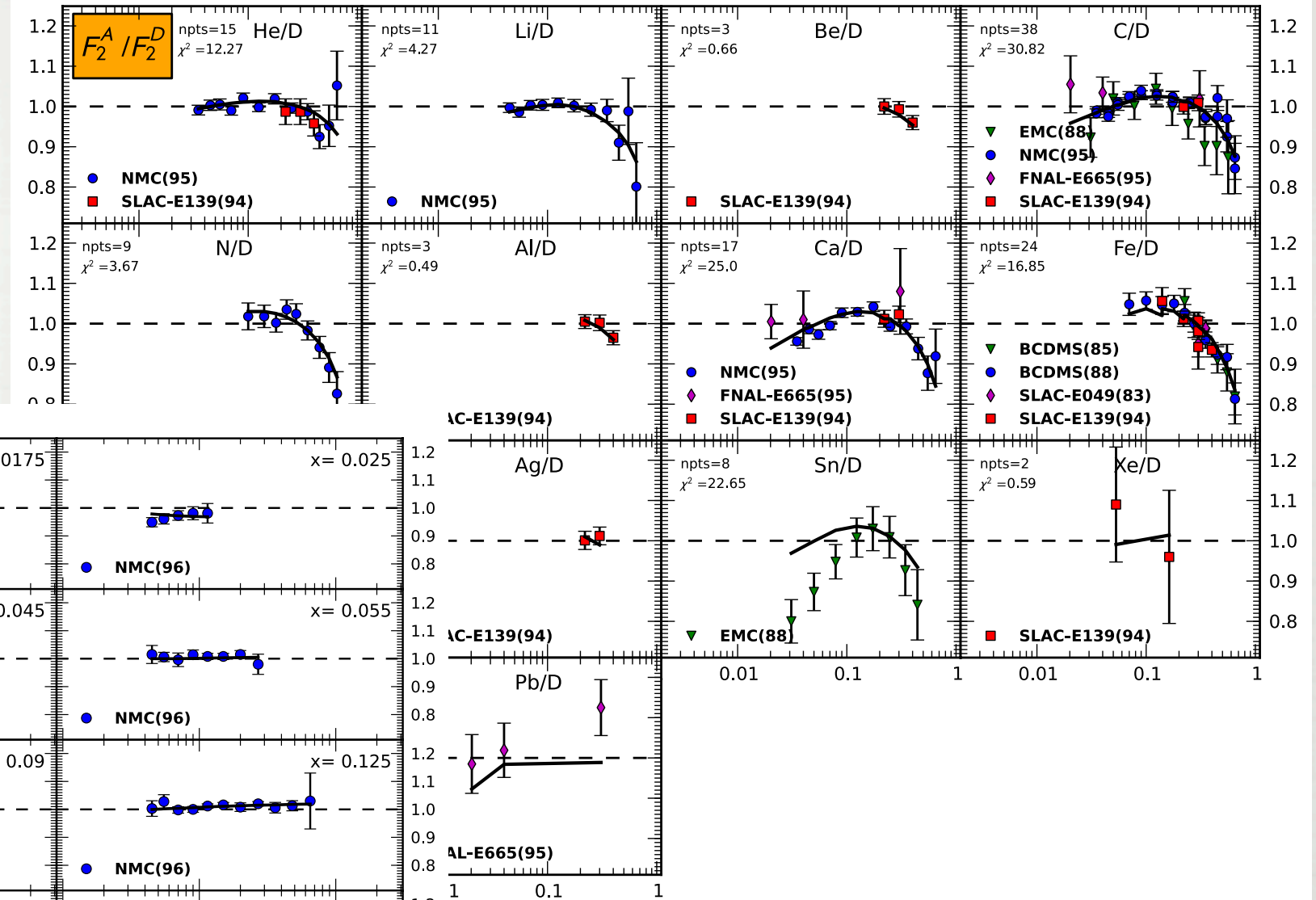
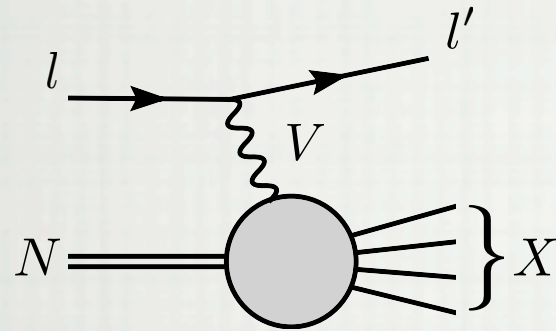
- applied standard CTEQ kinematical cuts  $Q > 2\text{GeV}$  &  $W > 3.5\text{GeV}$

- NPDF fit results:

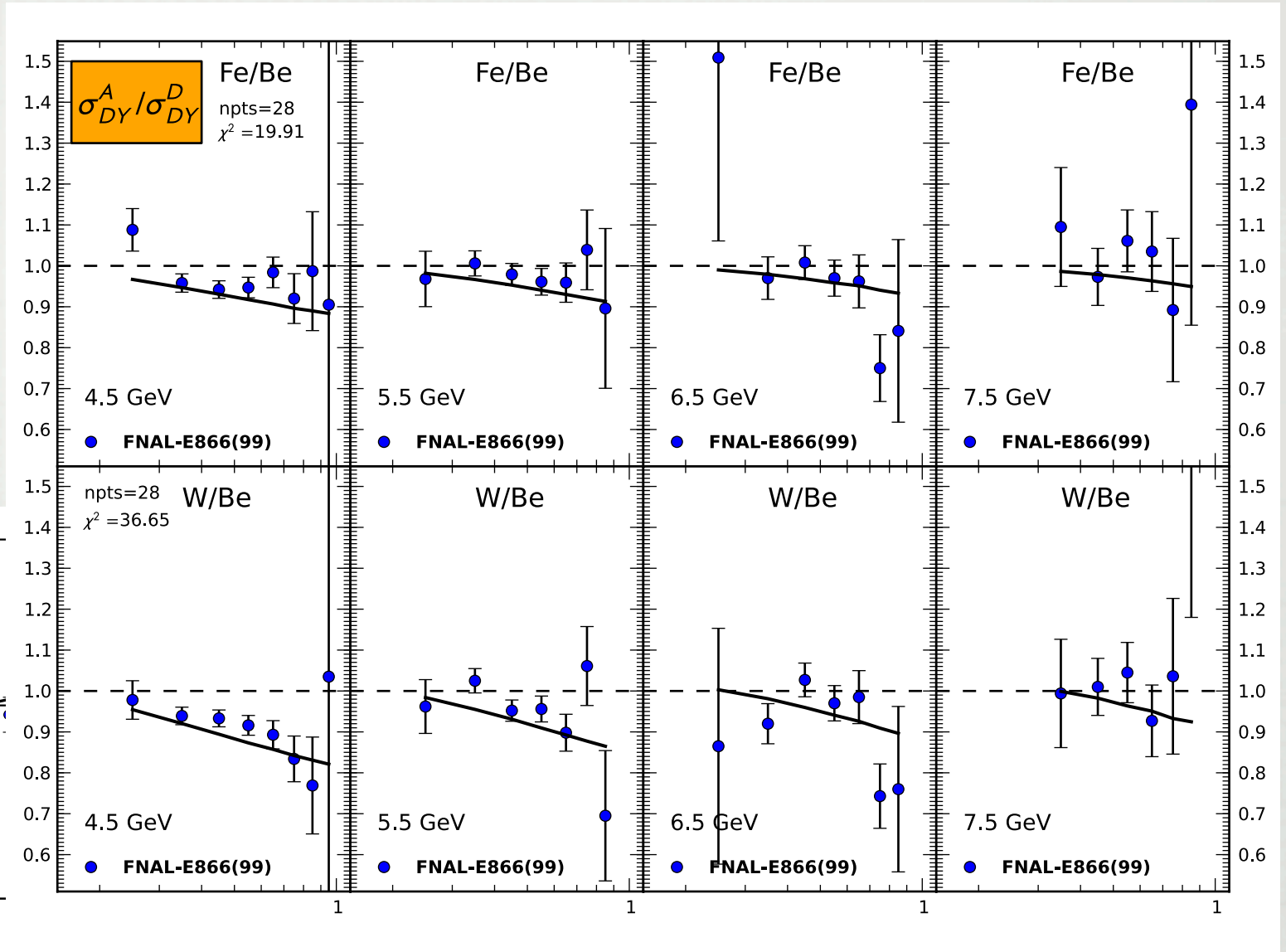
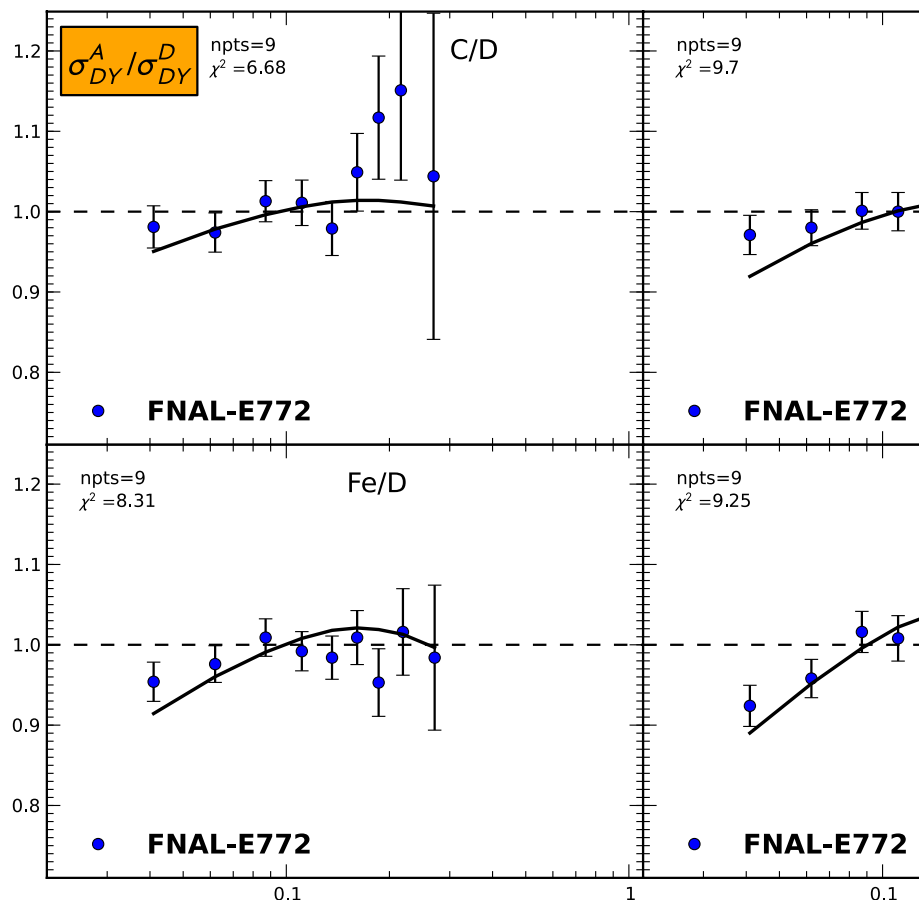
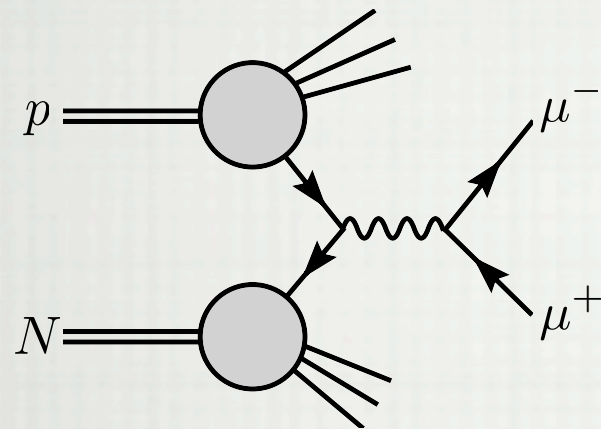
- 708 (1233) data points after (before) cuts
- 32 free parameters - 675 degrees of freedom
- overall  $\chi^2/\text{dof} = 0.95$
- individually for different data sets
  - for  $F_2^A / F_2^D$   $\chi^2/\text{pt} = 0.92$
  - for  $F_2^A / F_2^{A'}$   $\chi^2/\text{pt} = 0.69$
  - for  $\sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$   $\chi^2/\text{pt} = 1.08$



## Deep Inelastic Scattering



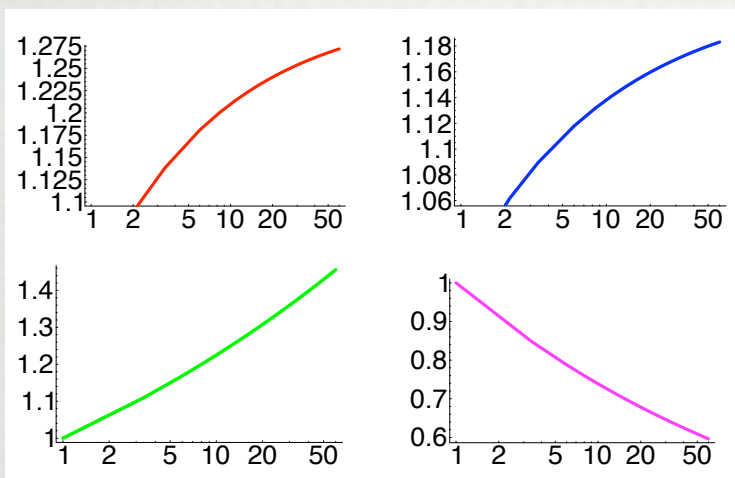
## Drell-Yan process



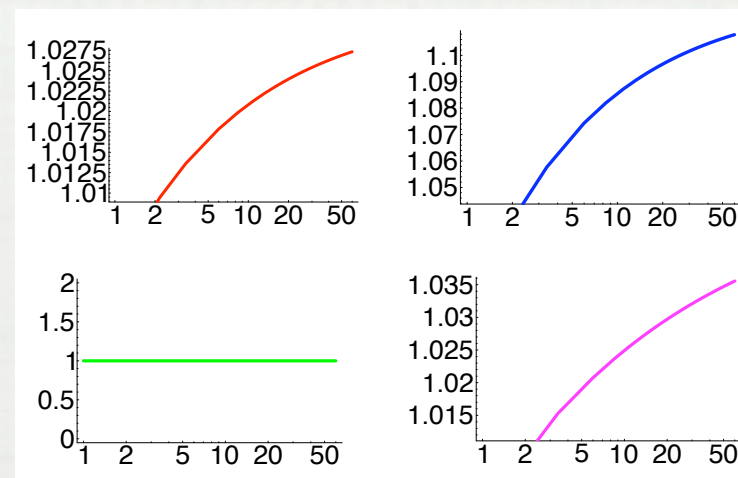


- CTEQ A-dependent parameter  $c_k(A)/c_{k,0}$  for parton distributions  $\begin{pmatrix} d_v & u_v \\ g & \bar{u} + \bar{d} \end{pmatrix}$

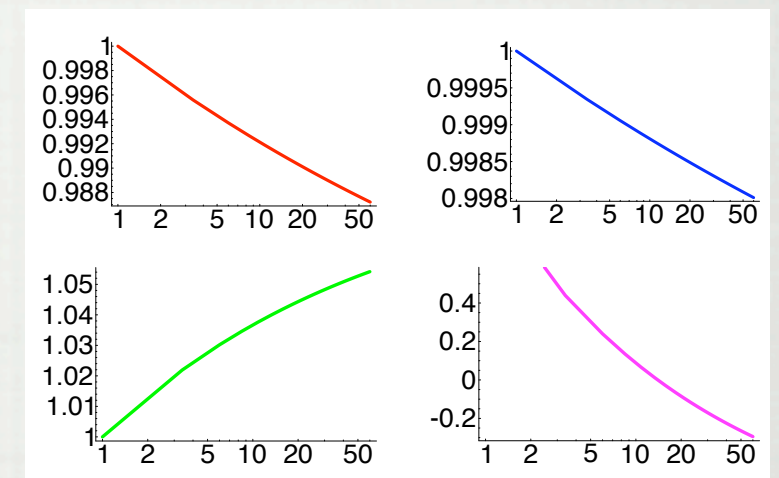
$c_1(A)/c_{k,0}$



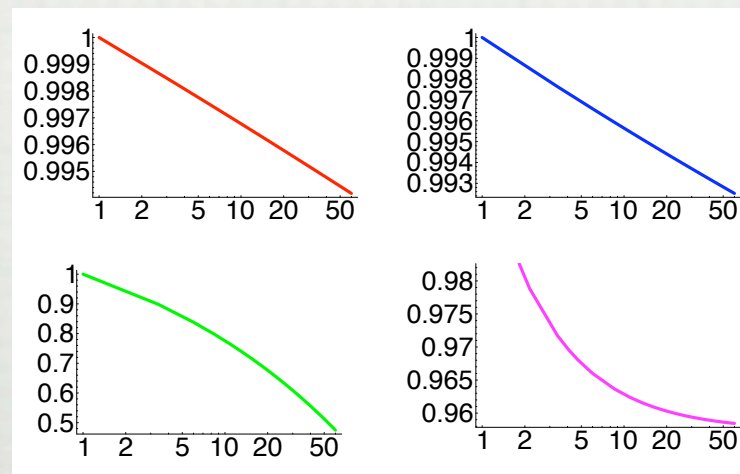
$c_2(A)/c_{k,0}$



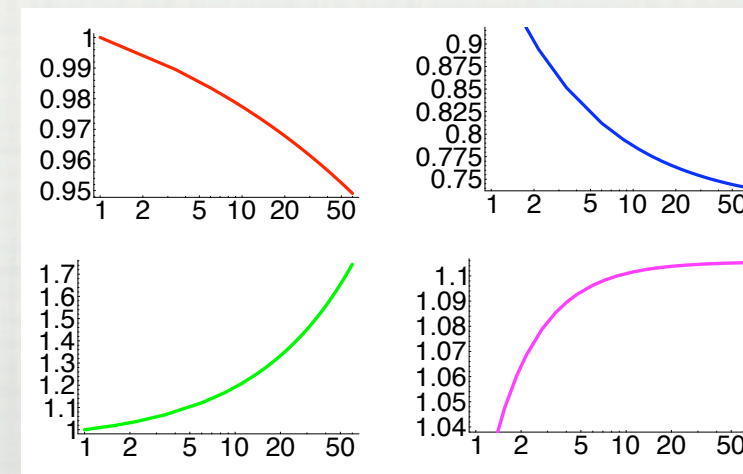
$c_3(A)/c_{k,0}$



$c_4(A)/c_{k,0}$

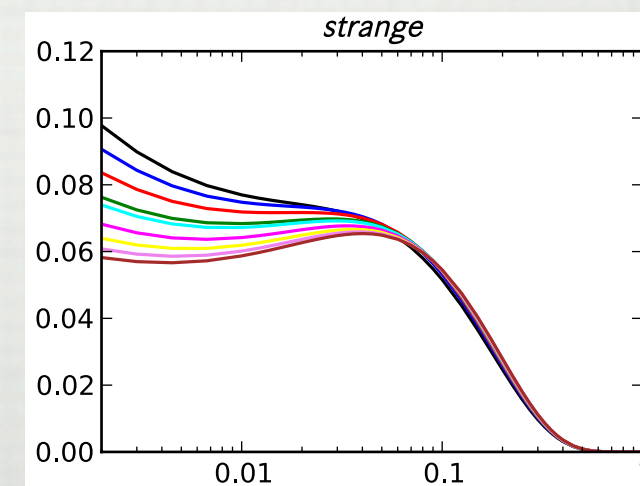
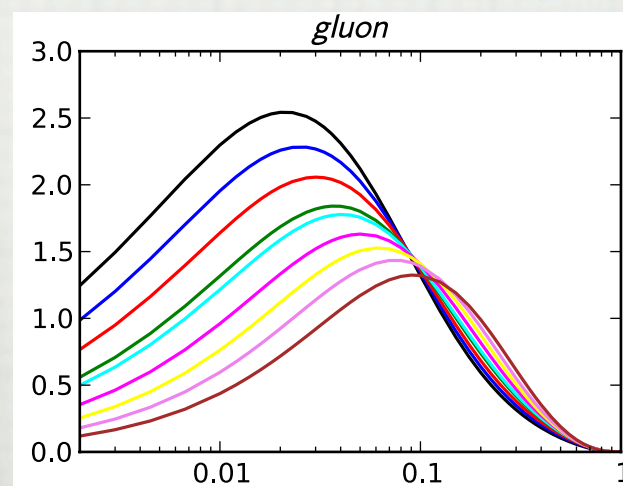
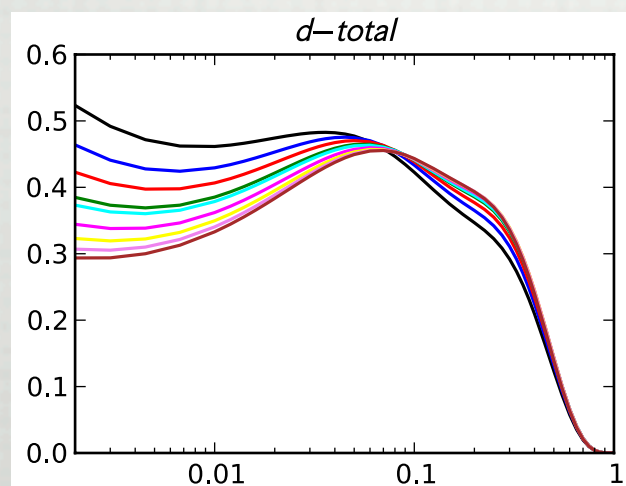
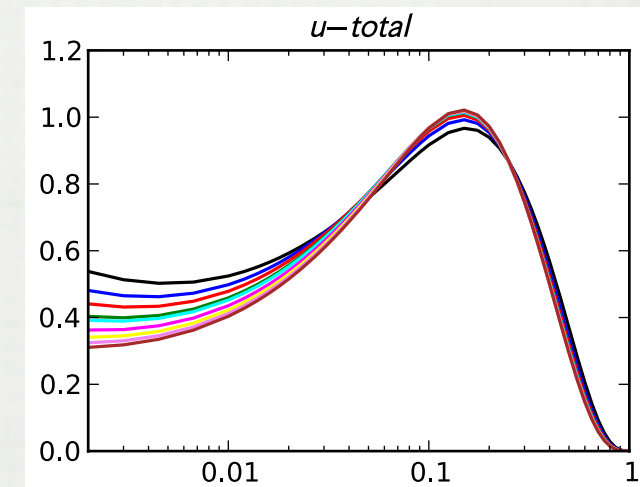
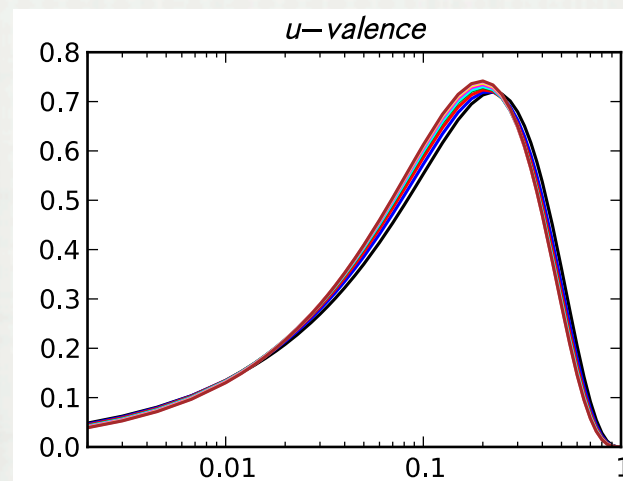
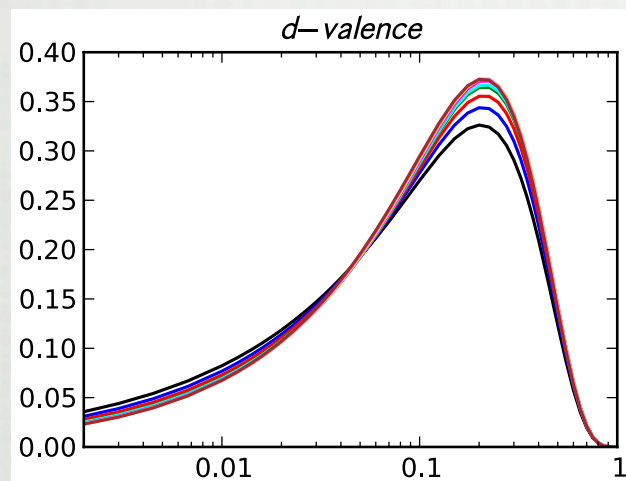


$c_5(A)/c_{k,0}$



- Parton density functions for bound partons as a function of  $x$

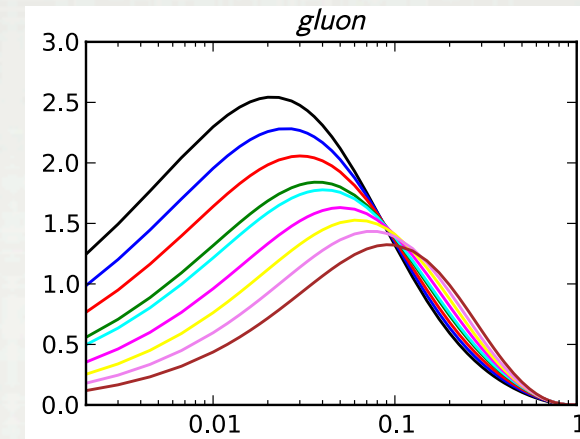
black      yellow      brown  
 $x f_k^A(x, Q)$  for  $A = (1, 2, 4, 9, 12, 27, 56, 108, 207)$   
 red      purple





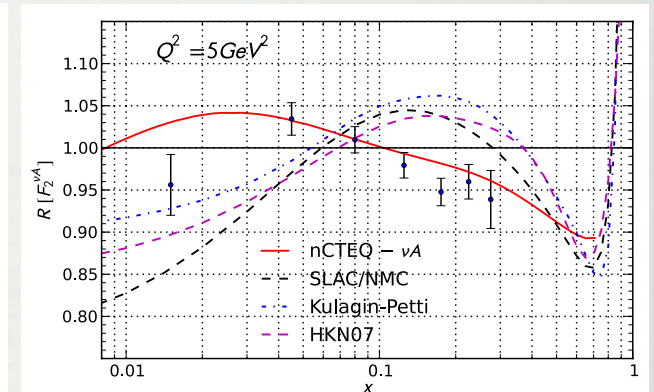
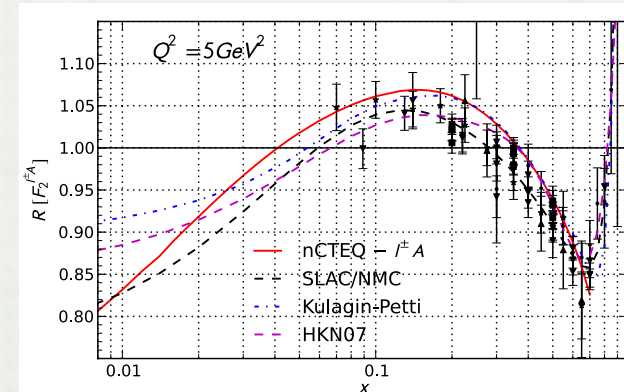
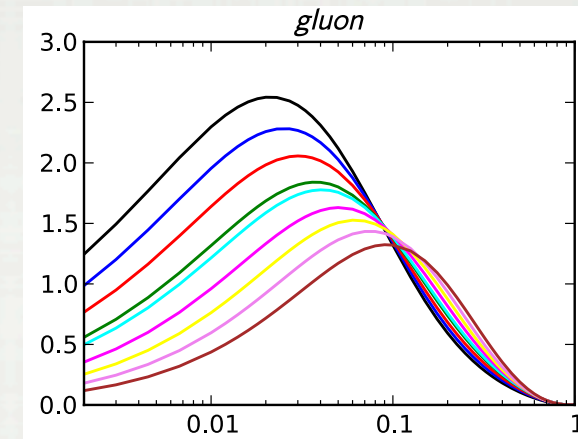
- Problems associated with nuclear PDF

- Problems associated with nuclear PDF
  - largely unconstrained nuclear gluon PDF
    - low- $x$  behavior, insufficient data



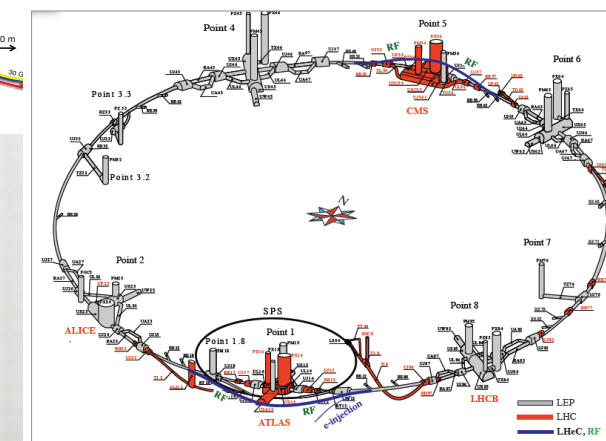
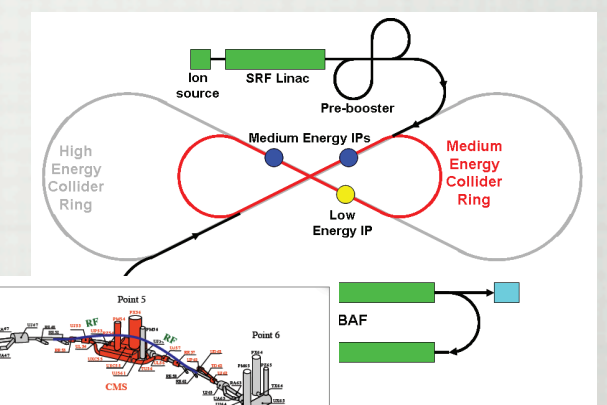
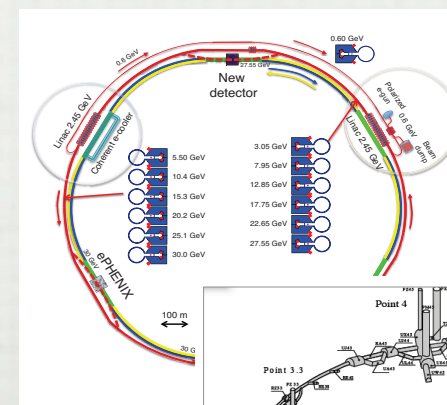
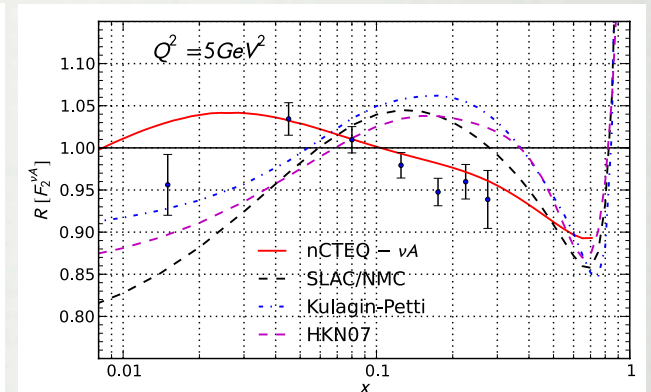
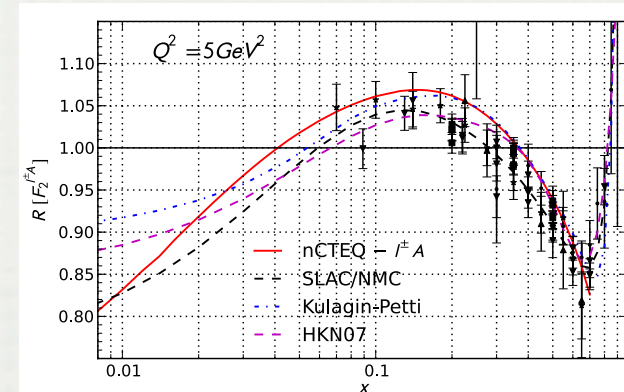
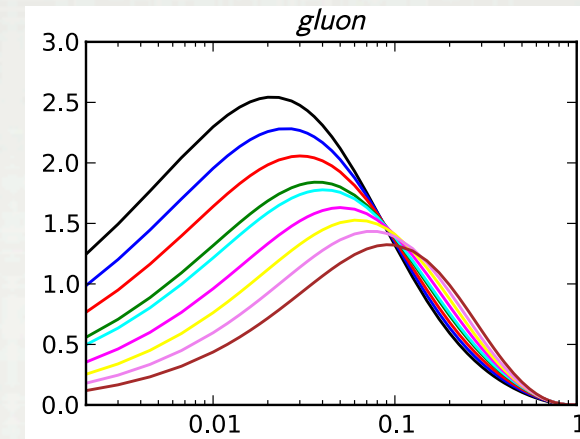


- Problems associated with nuclear PDF
  - largely unconstrained nuclear gluon PDF
    - low- $x$  behavior, insufficient data
  - problematic flavor separation for nuclear PDF
    - neutrino DIS data & nuclear strange quark



# NCTEQ

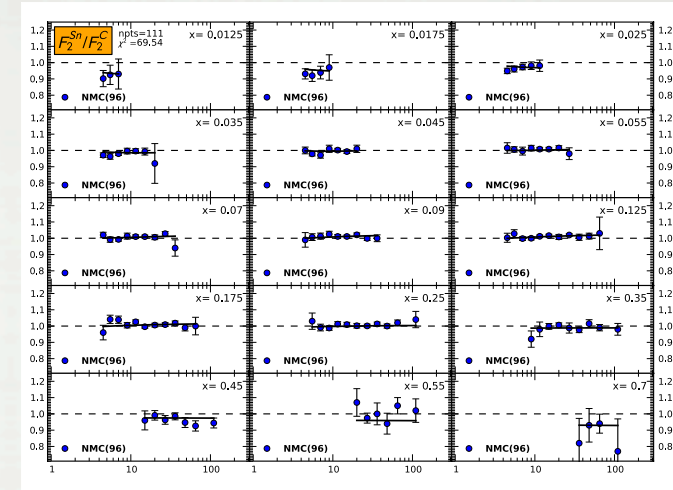
- Problems associated with nuclear PDF
  - largely unconstrained nuclear gluon PDF
    - low- $x$  behavior, insufficient data
  - problematic flavor separation for nuclear PDF
    - neutrino DIS data & nuclear strange quark
  - lacking data (4x less data than proton PDF)
    - need low- $x$  & precise data, for several nuclei





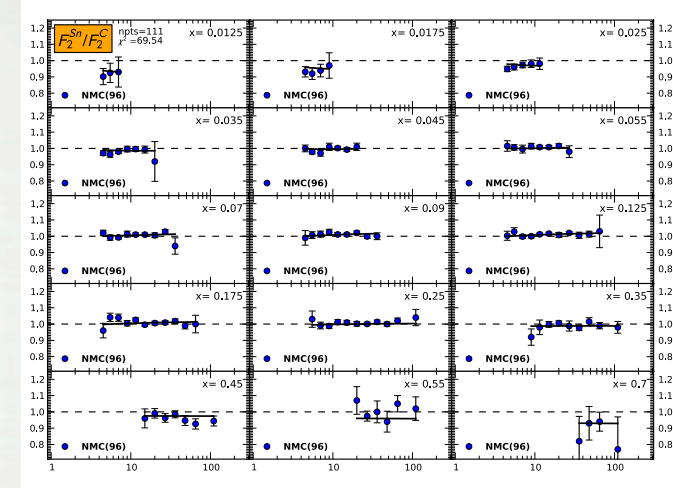
# GLUON NPDF

- Nuclear modifications in the gluon PDF
  - important for predictions @ LHC for AA or pA collisions
  - only data 'directly' constraining gluon Sn/C from NMC
  - data from RHIC have to include fragmentation functions

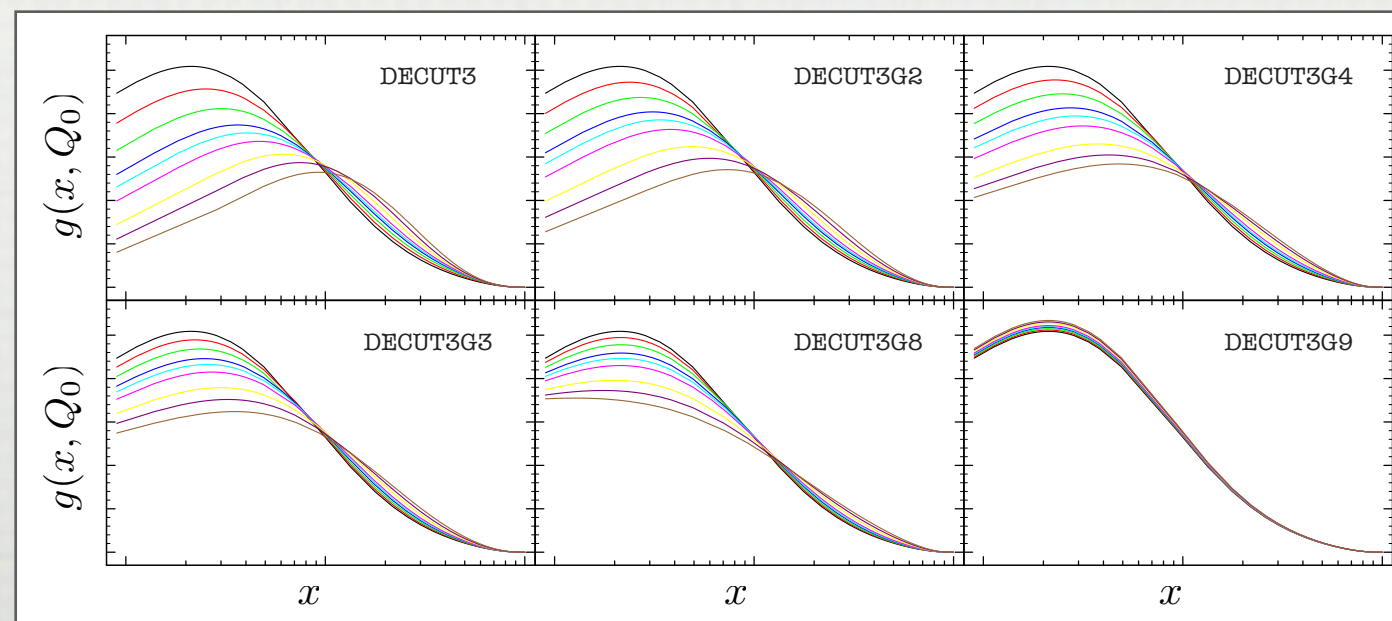


# GLUON NPDF

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  - important for predictions @ LHC for AA or pA collisions
  - only data 'directly' constraining gluon Sn/C from NMC
  - data from RHIC have to include fragmentation functions
- nCTEQ estimate of gluon nPDF uncertainty
  - vary gluon nPDF assumptions & parameterizations
  - large uncertainty for low  $x < 0.1$  in nCTEQ framework
  - need further data to constrain gluon nPDF



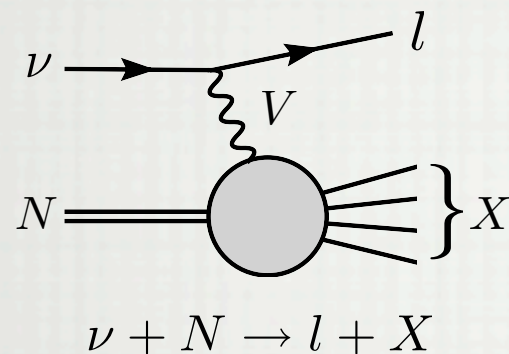
Stavreva, Schienbein, Arleo, KK, Olness, Yu, Owens  
 [JHEP 1101 (2011) 152] arXiv: 1012.1178





# NEUTRINO DIS

- Neutrino DIS cross-section data



NuTeV & di-muon  
 $N = \text{Fe}$

→ 2310 data points

CHORUS  
 $N = \text{Pb}$

→ 824 data points

All charged lepton  
DIS & Drell-Yan data  
→ 708 data points

- Challenges in combining the neutrino & charged lepton data

- deal with the disparity of number of data points - assigning weights to neutrino data
- neutrino DIS data only with 2 heavy nuclei - insufficient to get a reliable  $A$ -dependence
- do all neutrino data show the different behavior or only NuTeV?

- Different neutrino observables

$$\frac{d\sigma^{\nu A}}{dx dQ^2} \quad \& \quad \frac{d\sigma^{\bar{\nu} A}}{dx dQ^2} \quad \text{vs.} \quad F_2^{\nu+\bar{\nu}}(x, Q^2) \quad \& \quad xF_3^{\nu+\bar{\nu}}(x, Q^2)$$

needs theory assumptions to extract

- Nuclear correction factors

- we show correction factors defined e.g. as  $R[F_2^{\nu}] = F_2^{\nu A} / F_2^{\nu A, \text{free}}$

from free proton PDF

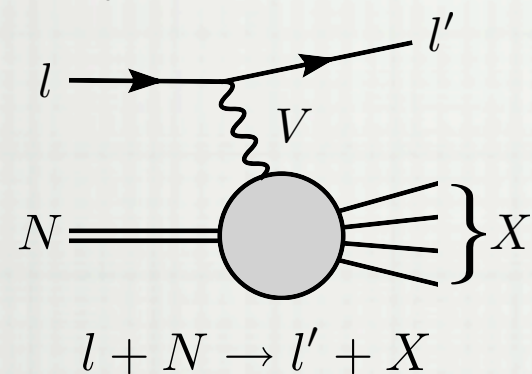


# NEUTRINO DIS

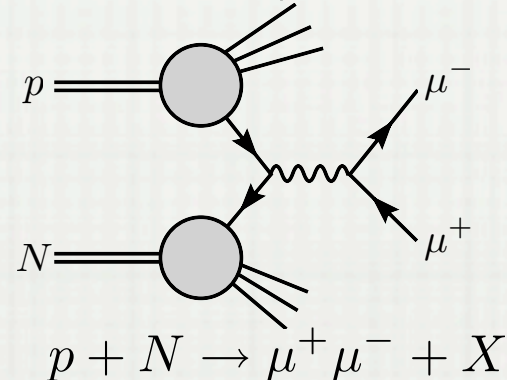
- Experiments included in the analysis

## *Charged lepton*

Deep Inelastic Scattering

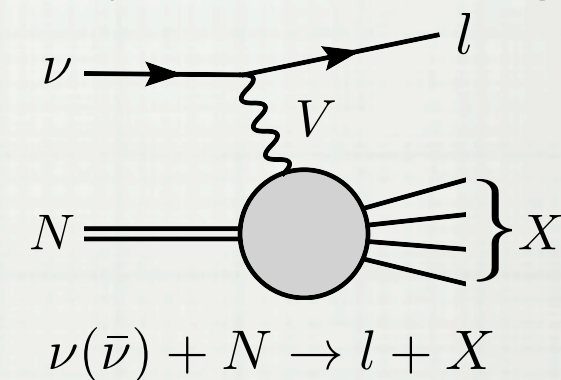


Drell-Yan process



## *Neutrino*

Deep Inelastic Scattering



### **CERN BCDMS & EMC & NMC**

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

### **FNAL E-665**

$N = (\text{D, C, Ca, Pb, Xe})$

### **DESY Hermes**

$N = (\text{D, He, N, Kr})$

### **SLAC E-139 & E-049**

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

### **FNAL E-772 & E-886**

$N = (\text{D, C, Ca, Fe, W})$

### **CHORUS**

$N = \text{Pb}$

### **CCFR & NuTeV**

$N = \text{Fe}$

*1233 data points (708 after cuts)*

*3832 data points (3134 after cuts)*



# NEUTRINO DIS

- Comparison of charged lepton and neutrino fits

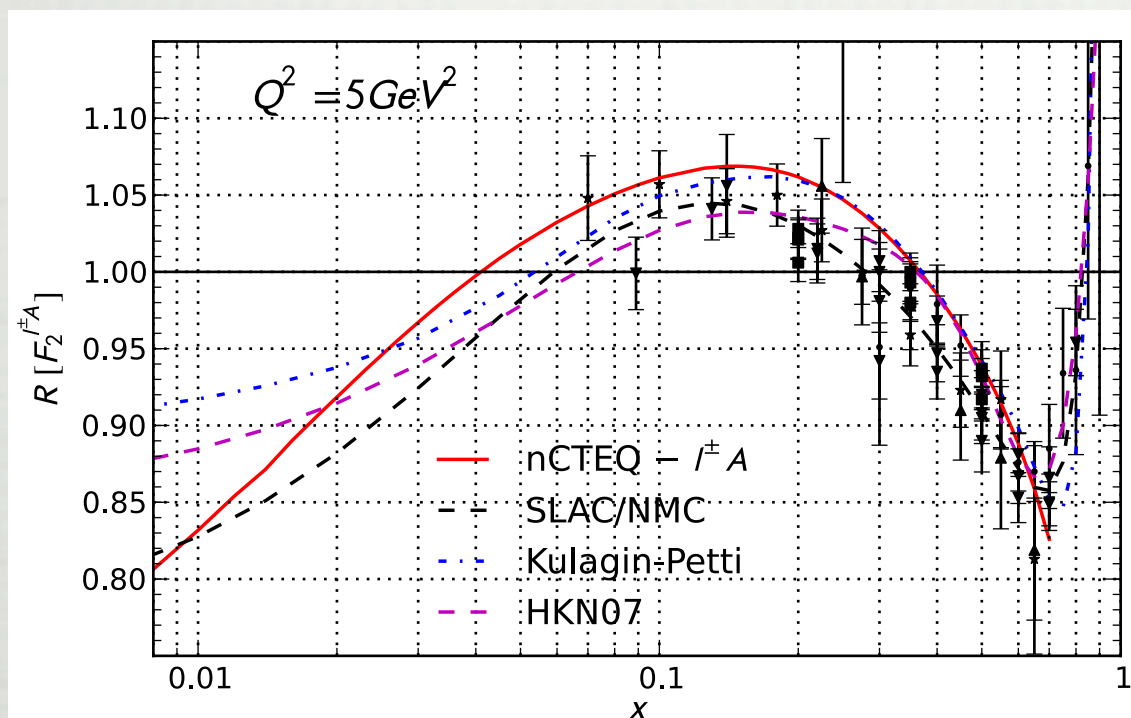
KK et al.

[Phys.Rev.Lett. 106(2011) 122301] arXiv: 1012.1178

*Fit to charged lepton data*

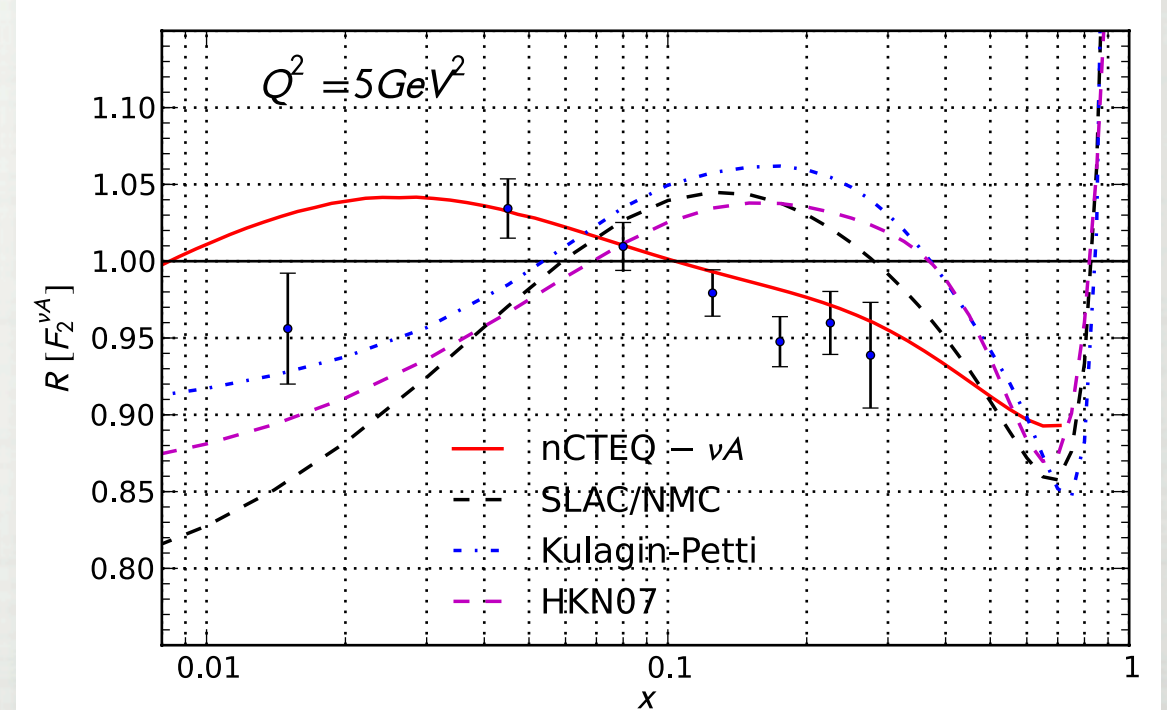
*DIS & DY*

$$\chi^2/\text{d.o.f} = 0.89$$



*Fit to only neutrino DIS*

$$\chi^2/\text{d.o.f} = 1.33$$



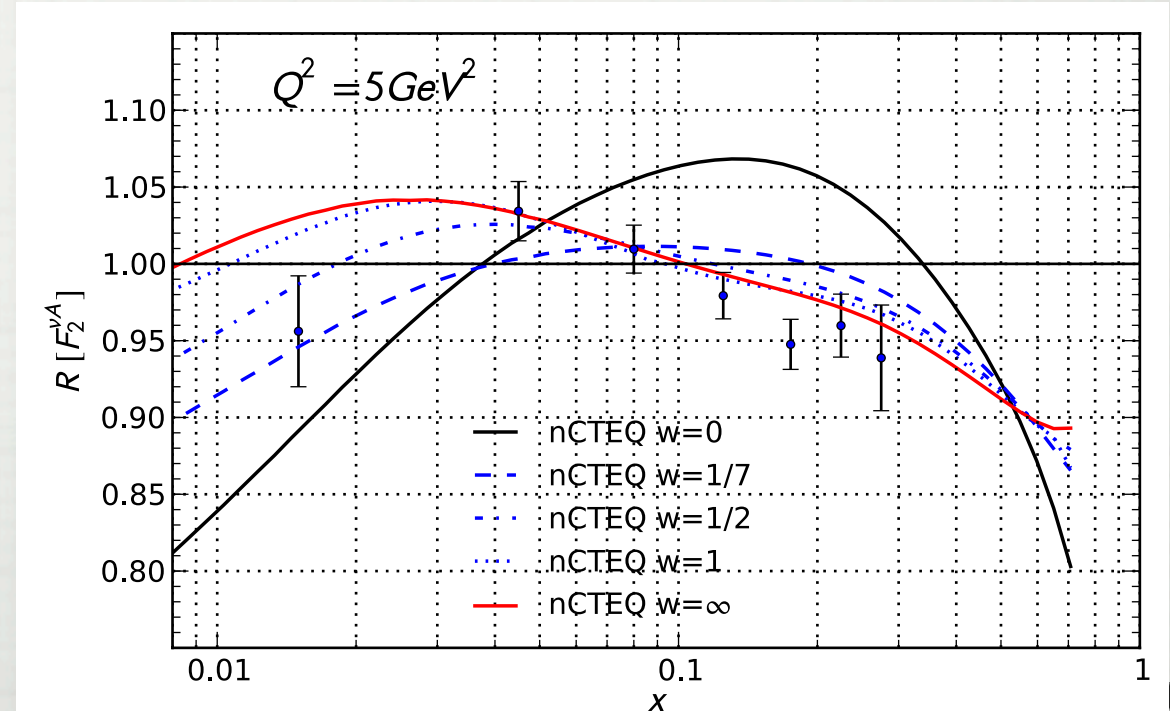
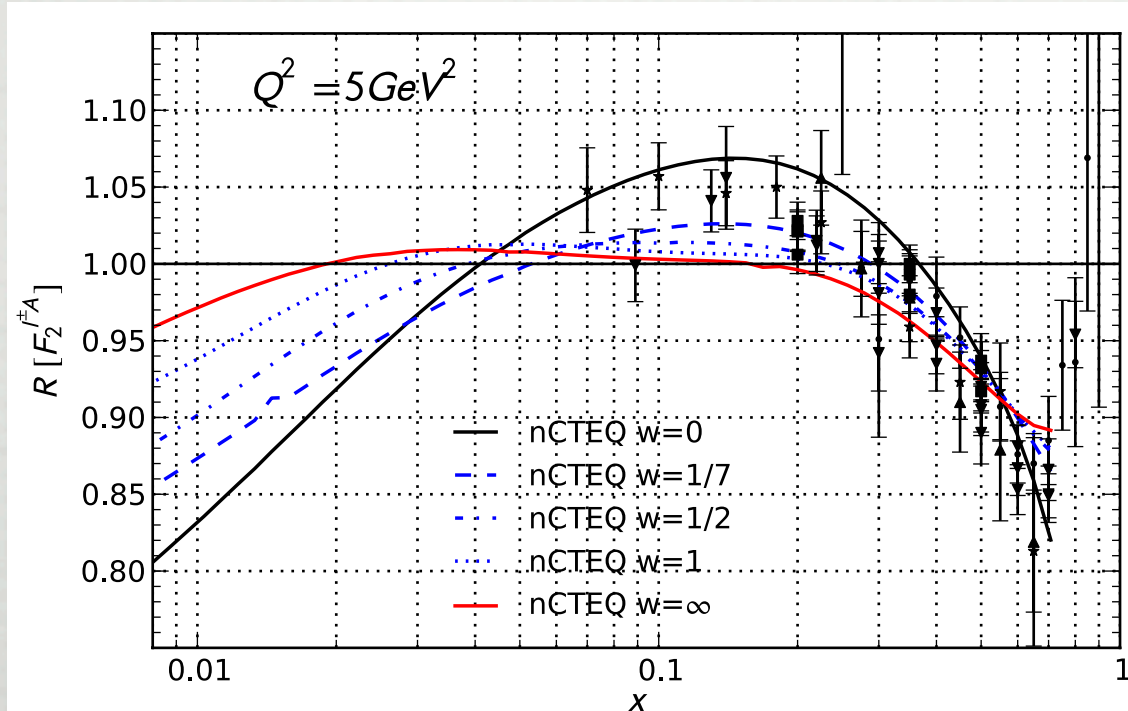
- can we explain the difference and fit all data together in a global fit ?

# NEUTRINO DIS

- Analysis of fits with different weights of neutrino DIS (correlated errors)

$w$	$l^\pm A$	$\chi^2$ (/pt)	$\nu A$	$\chi^2$ (/pt)	total $\chi^2$ (/pt)
0	708	630 (0.89)	-	-	$630 \pm 58$
1/7	708	645 (0.91)	3134	4681 (1.50)	$5326 \pm 203$
1/2	708	680 (0.96)	3134	4375 (1.40)	$5055 \pm 192$
1	708	736 (1.04)	3134	4246 (1.36)	$4983 \pm 190$
$\infty$	-	-	3134	4167 (1.33)	$4167 \pm 176$

$$P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$



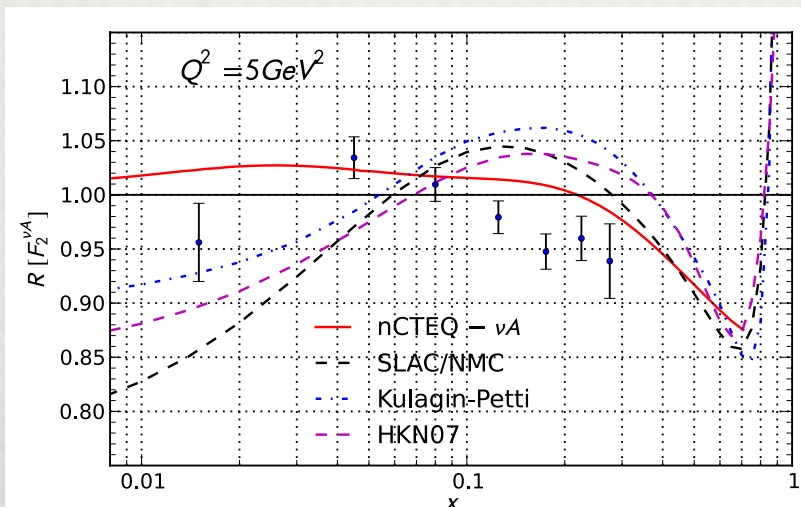
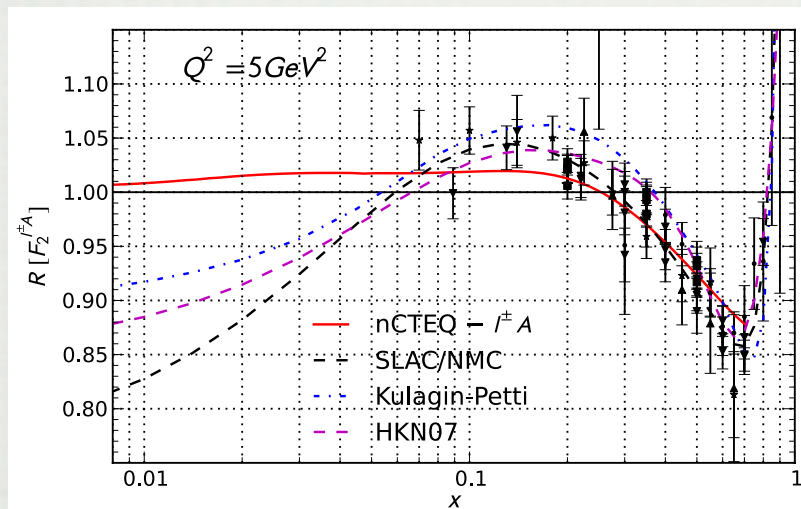


# NEUTRINO DIS

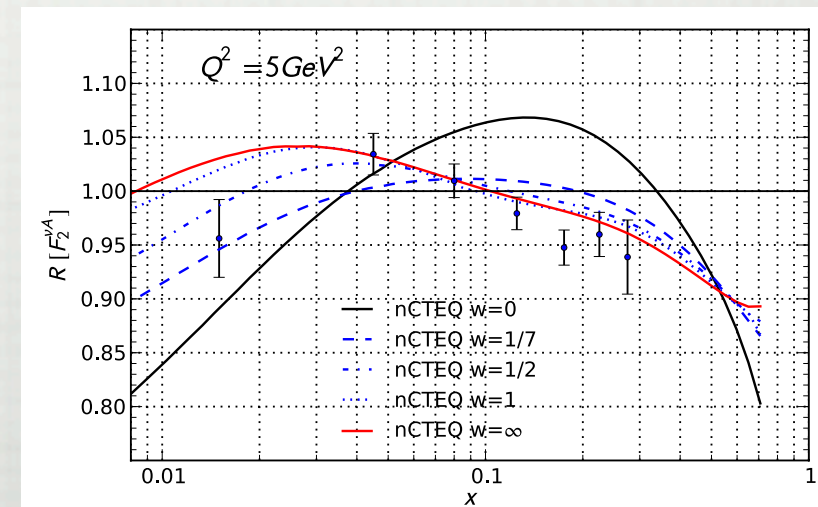
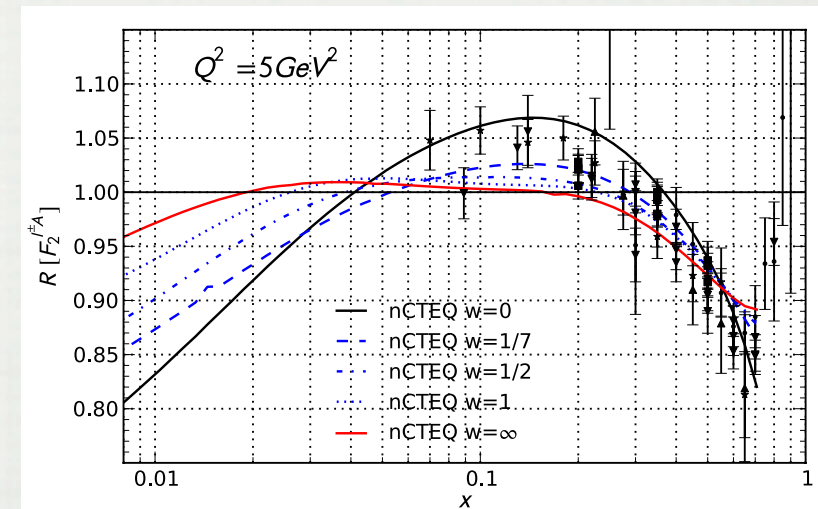
- Analysis of fits with neutrino DIS (uncorrelated errors)

$w$	$l^\pm A$	$\chi^2$ (/pt)	$\nu A$	$\chi^2$ (/pt)	total $\chi^2$ (/pt)
1-corr	708	736 (1.04)	3134	4246 (1.36)	4983 (1.30)
1-uncorr	708	809 (1.14)	3110	3115 (1.00)	3924 (1.02)

uncorrelated errors

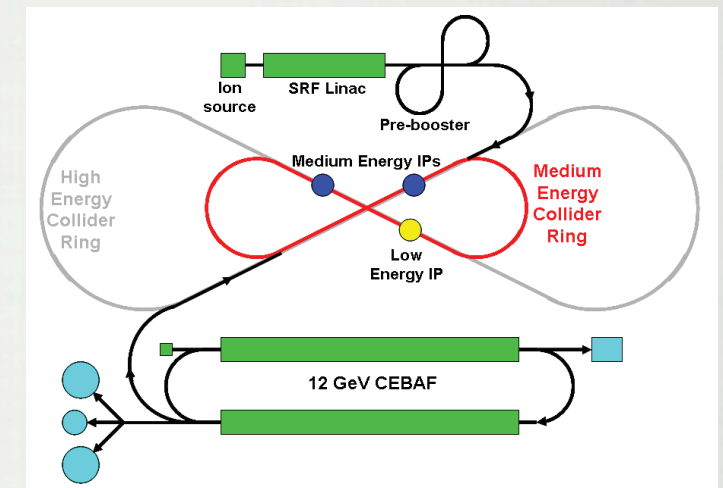
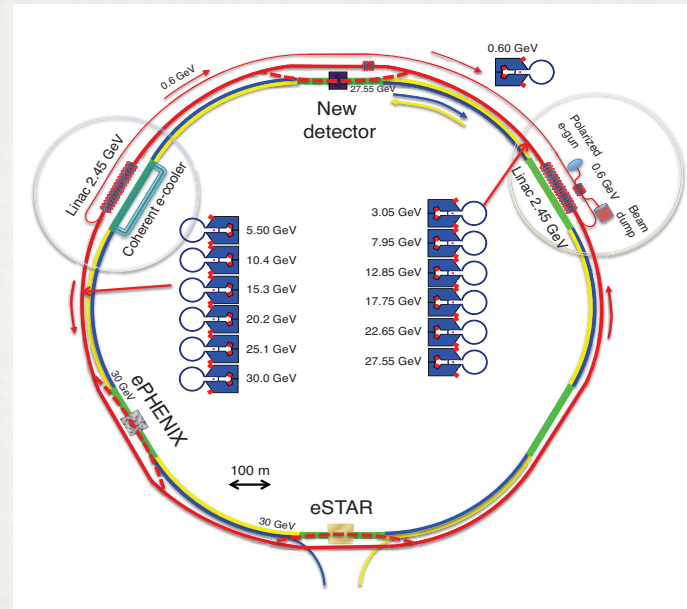


correlated errors



# FUTURE EXPERIMENTS

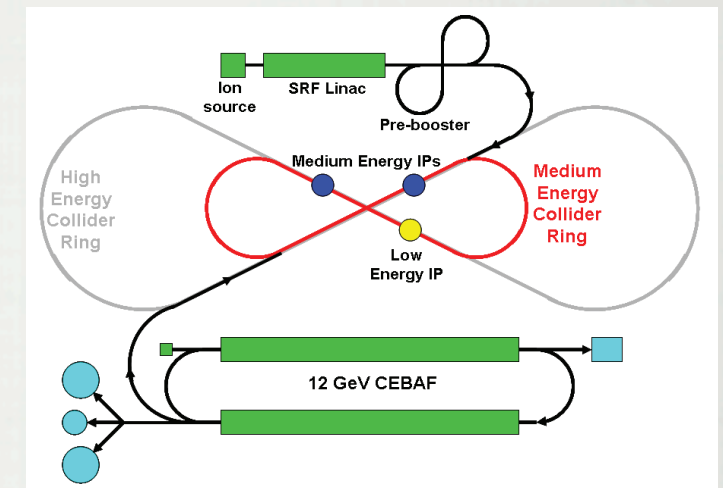
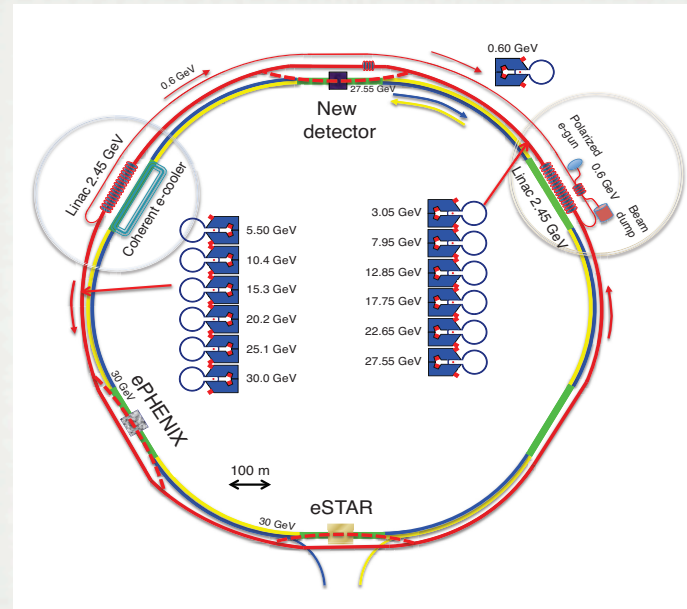
- Electron-Ion-Collider (EIC)
  - 2 different proposals - JLab & RHIC
  - multiple nuclear targets
  - reach to small-x



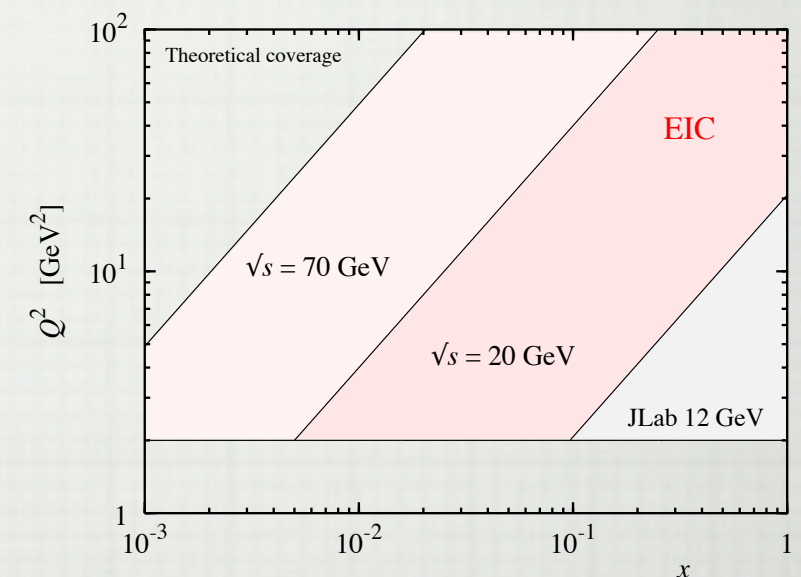
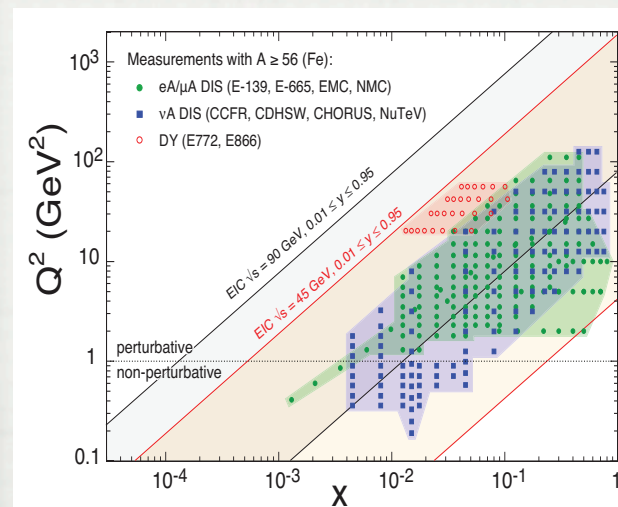


# FUTURE EXPERIMENTS

- Electron-Ion-Collider (EIC)
  - 2 different proposals - JLab & RHIC
  - multiple nuclear targets
  - reach to small-x



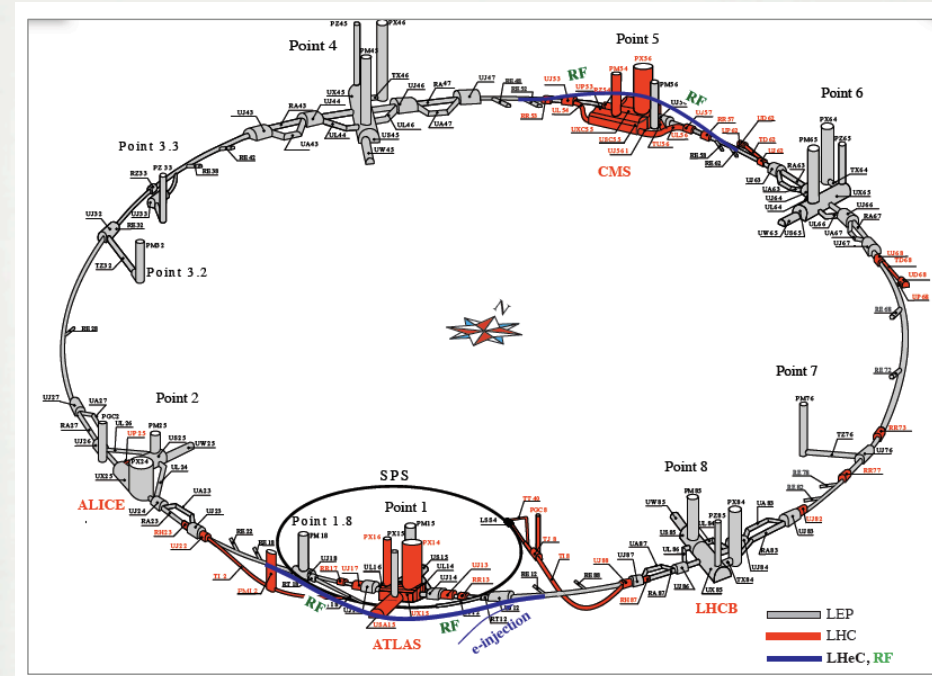
- nPDF requirements on EIC
  - coverage in  $x$ - $Q^2$  plane (small- $x$ )
  - precision (e.g. for gluon PDF)
  - # nuclei



# FUTURE EXPERIMENTS

## ● LHeC

- 2 different proposals - ring-ring and linac-LHC
- only Pb (possibly Ca) targets
- reach to very small-x





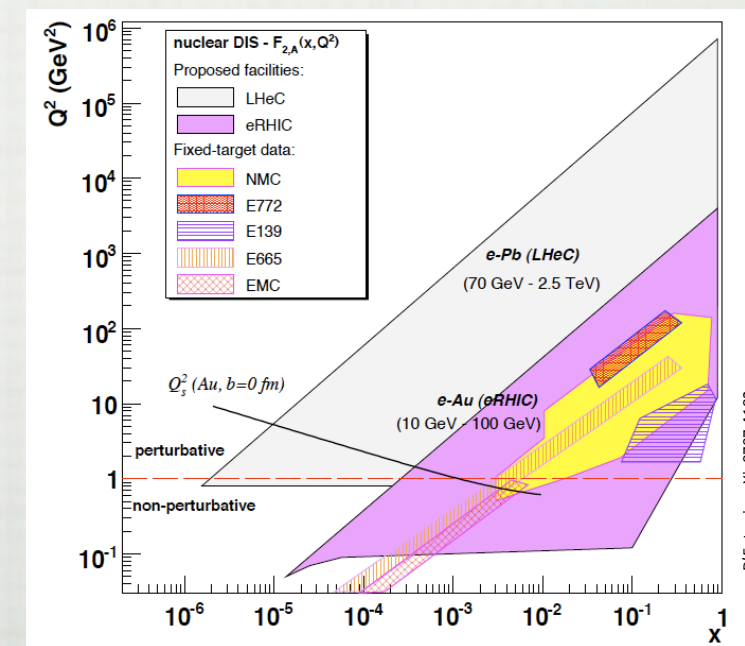
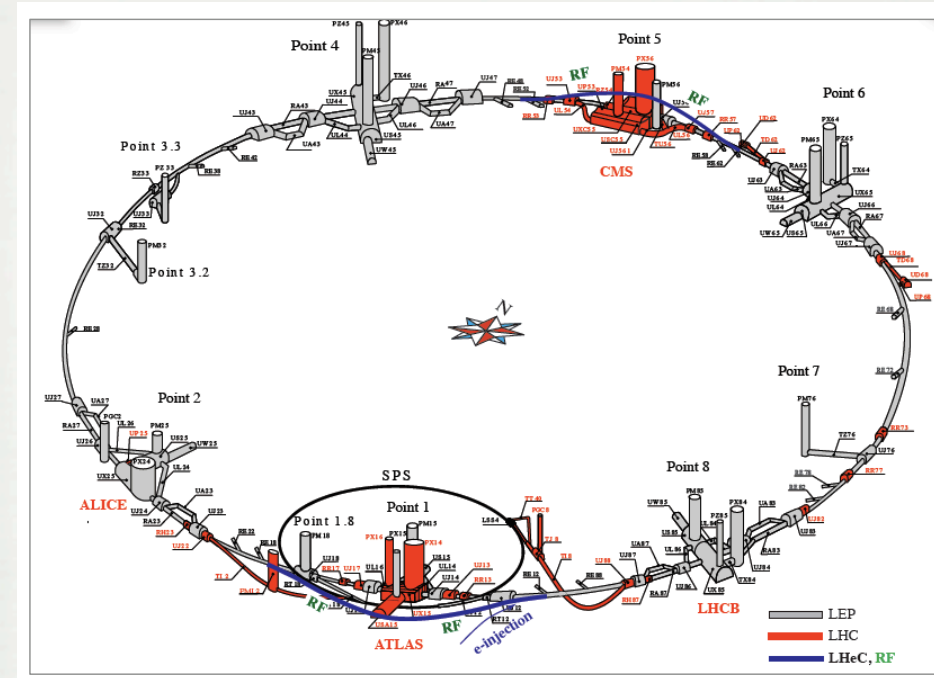
# FUTURE EXPERIMENTS

## ● LHeC

- 2 different proposals - ring-ring and linac-LHC
- only Pb (possibly Ca) targets
- reach to very small-x

## ● nPDF requirements on LHeC

- coverage in  $x$ - $Q^2$  plane (small- $x$ )
- precision (e.g. for gluon PDF)
- # nuclei



# CONCLUSIONS & OUTLOOK

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- nPDF analysis put to test @ LHC in pA & AA collisions
- Some important open questions remain (all can be solved by having more data)
  - uncertainty in nuclear gluon PDF at small- $x$ 
    - need HERA-like measurements for many nuclear targets covering also small- $x$  (EIC & LHeC)
  - neutrino DIS on nuclei
    - at the moment NuTeV incompatible with the rest of charged lepton data
    - proton strange quark information from LHC vital
    - new(old) data would solve the problem - NOMAD or NuSonG



**THANK YOU**