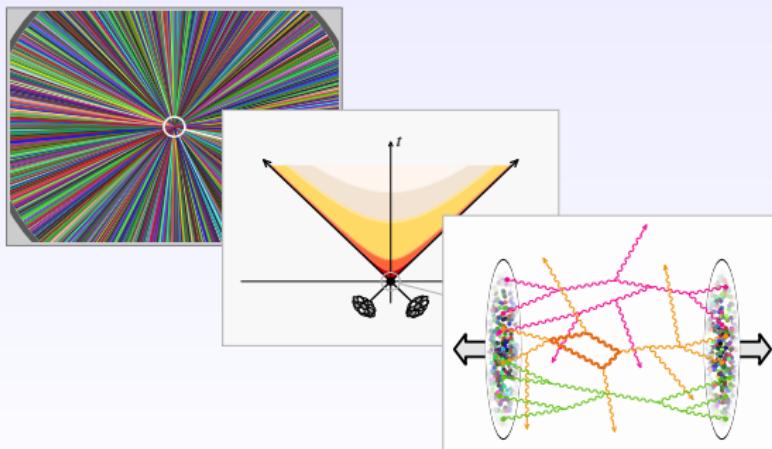


How quantum fields start to flow in Heavy Ion Collisions

GdR Physique Hadronique, Orsay, December 2012



François Gelis
IPHT, Saclay

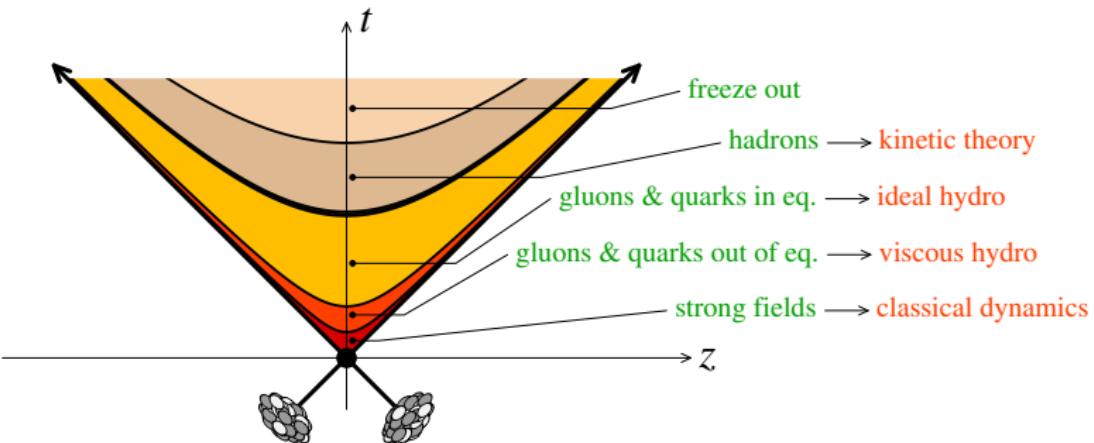
Outline

- ① Color Glass Condensate**
- ② Just after the collision**
- ③ Equation of state, Thermalization**
- ④ Bose-Einstein Condensation?**
- ⑤ Isotropization**

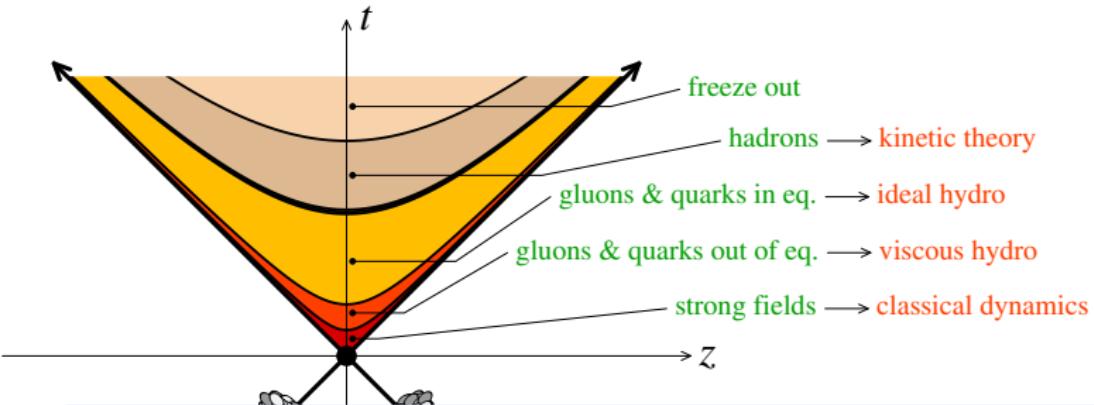
In collaboration with :

K. Dusling	(NCSU)
T. Epelbaum	(IPhT)
R. Venugopalan	(BNL)

Stages of a nucleus-nucleus collision



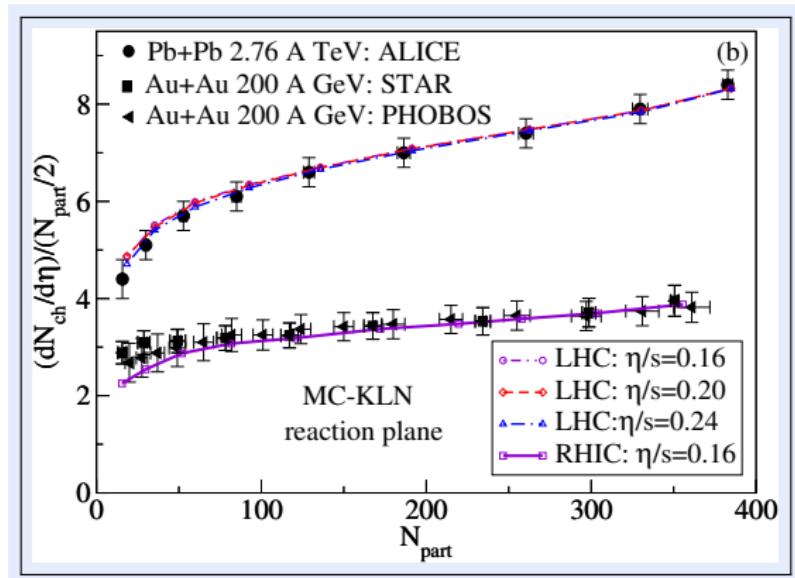
Stages of a nucleus-nucleus collision



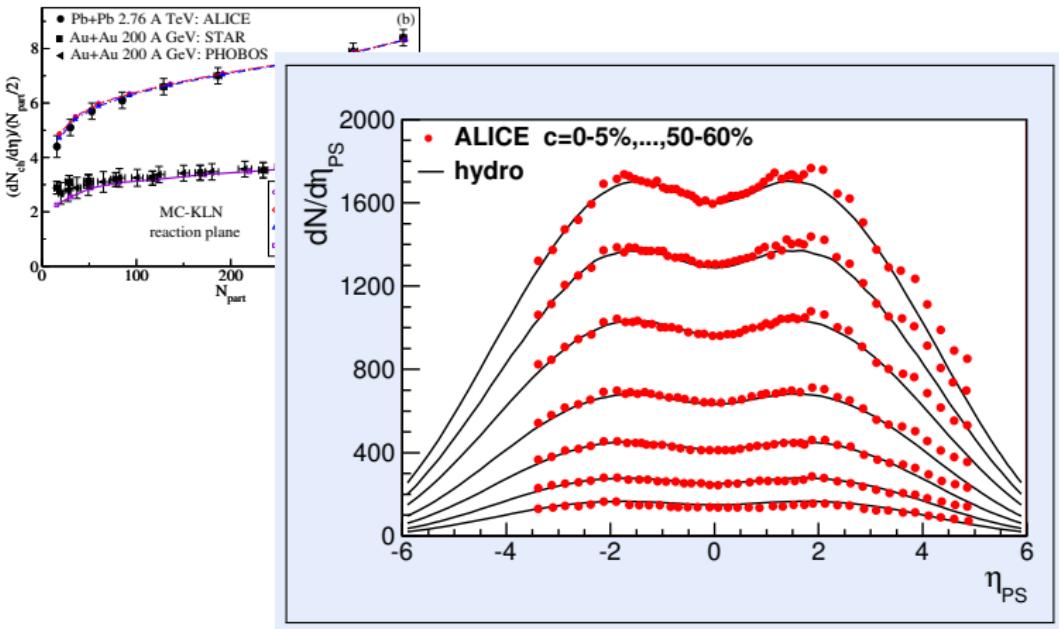
Goals :

- Understand why hydrodynamics works
- First principles description in QCD
- Weakly coupled (but strongly interacting...)

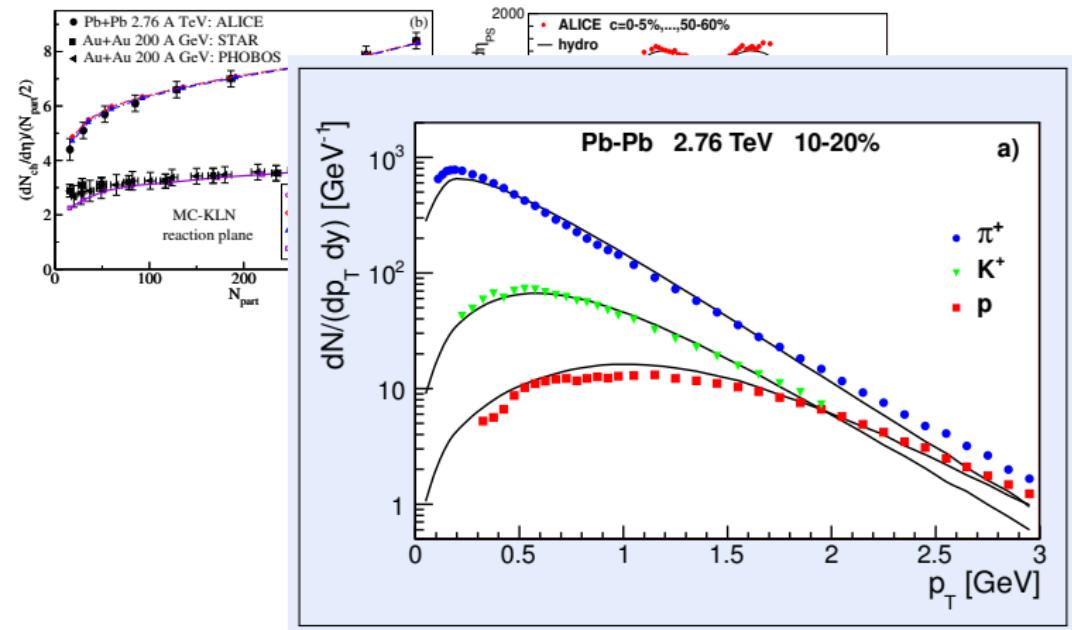
Hydrodynamics works!!



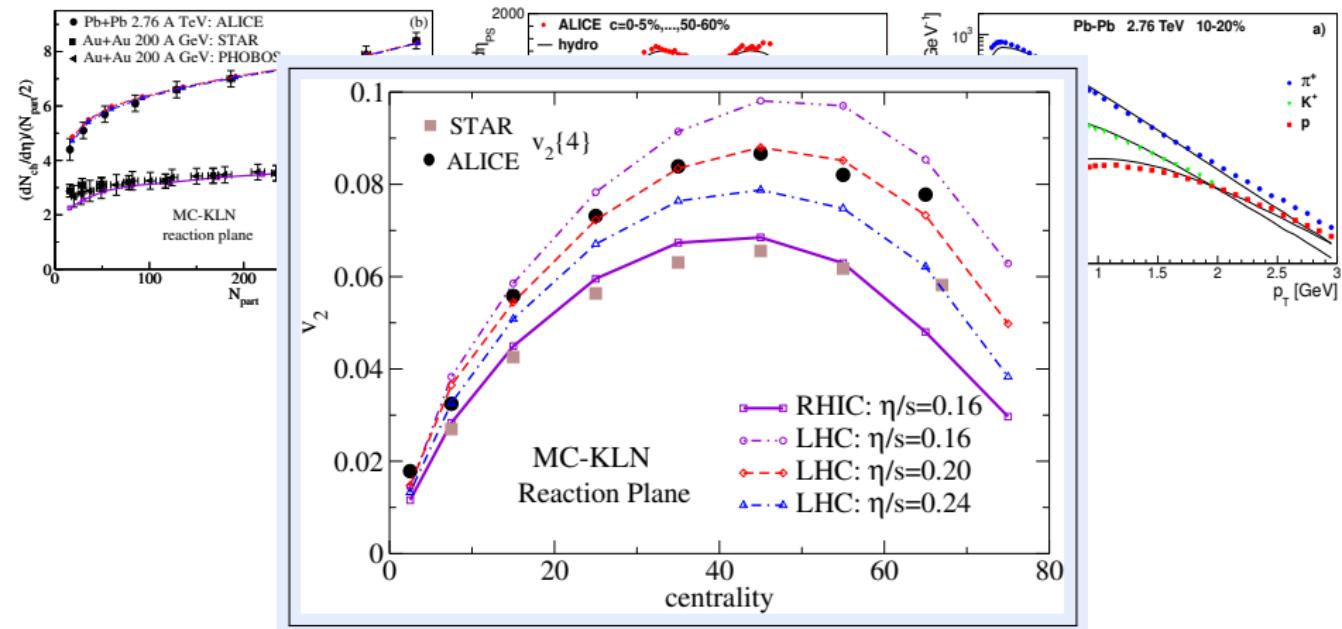
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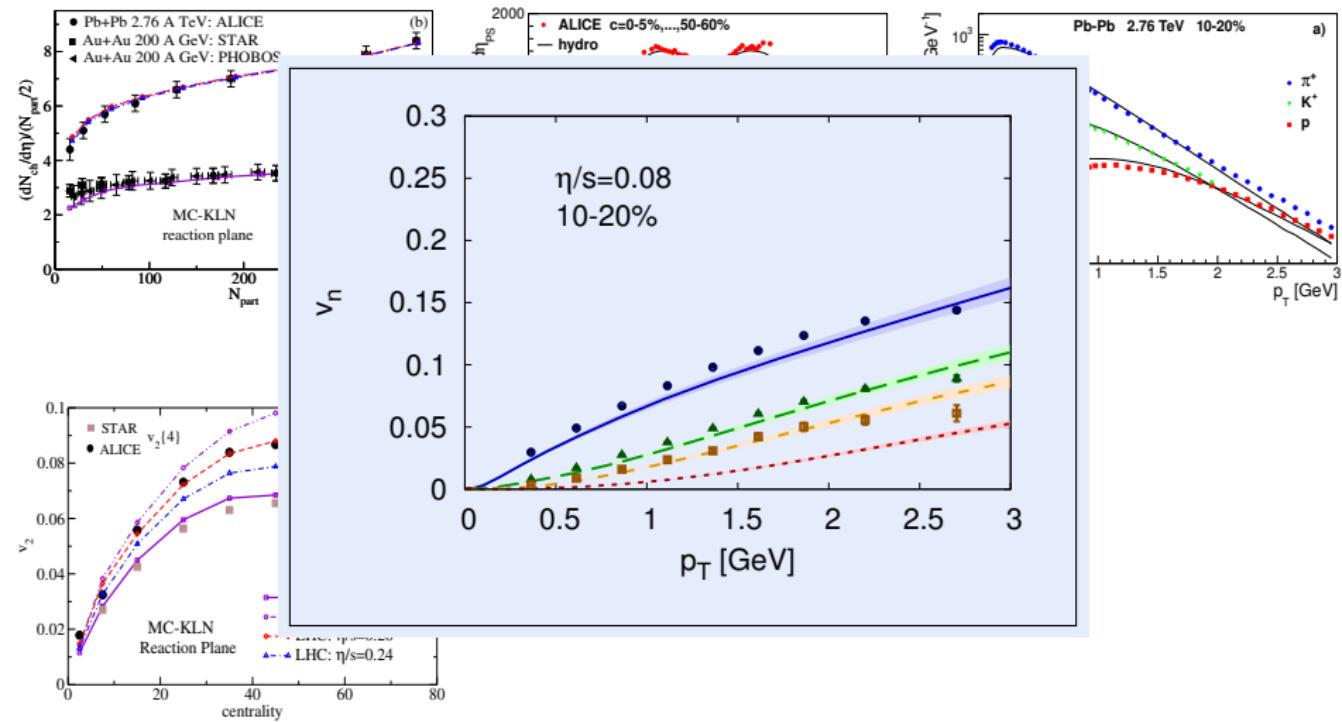
Hydrodynamics works!!



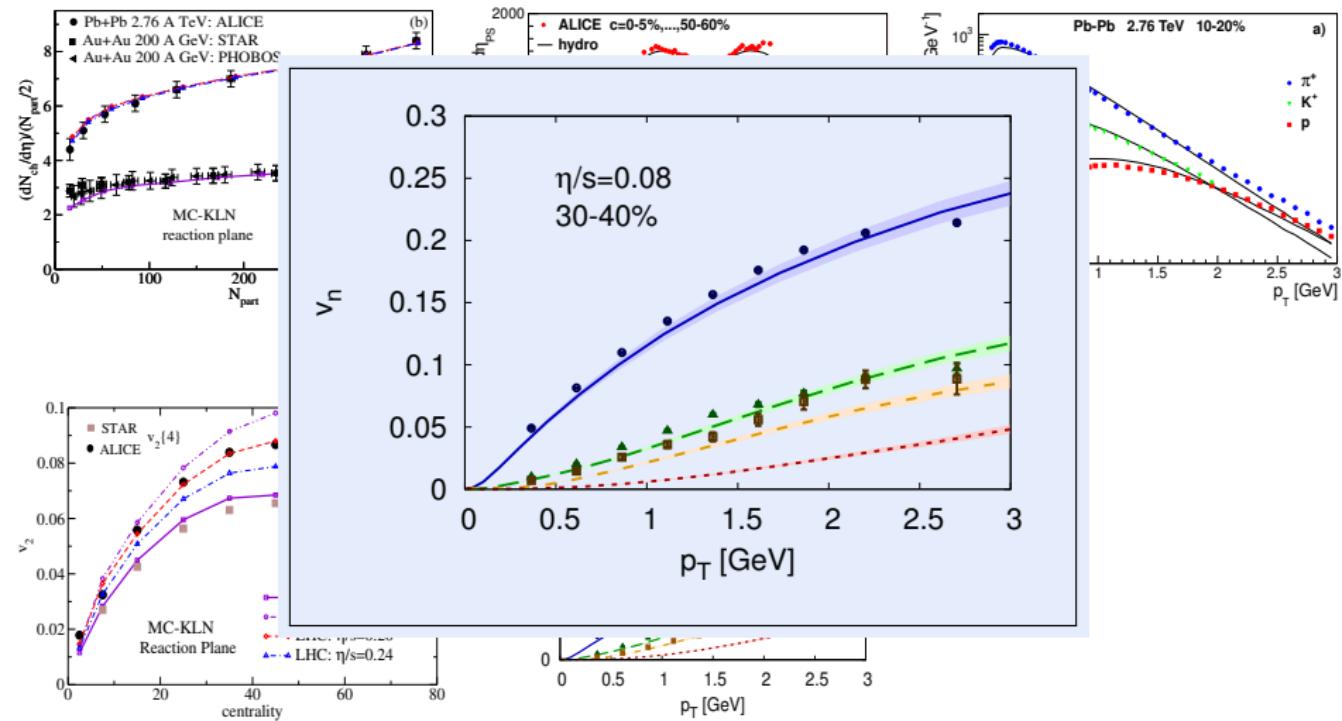
Hydrodynamics works!!



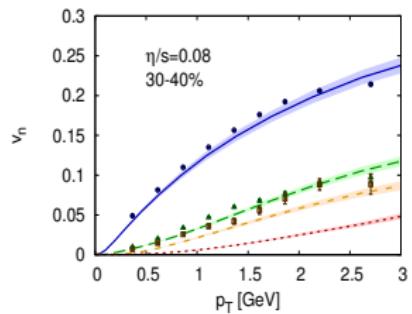
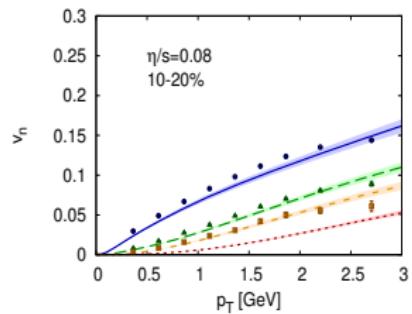
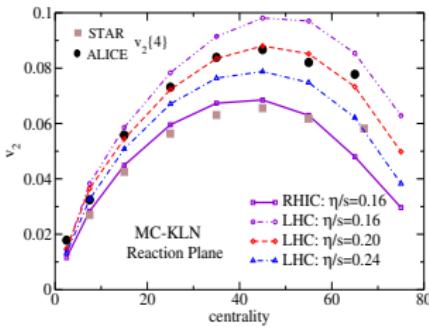
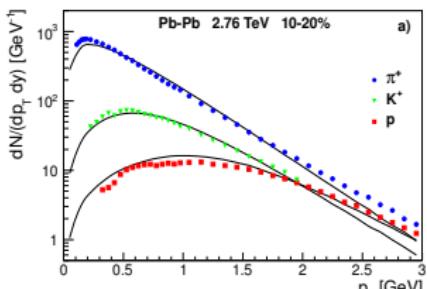
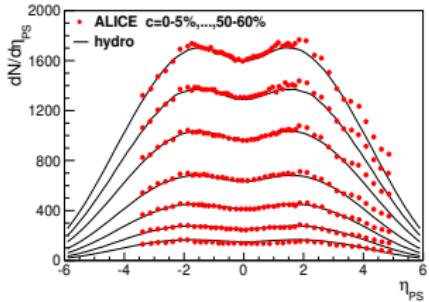
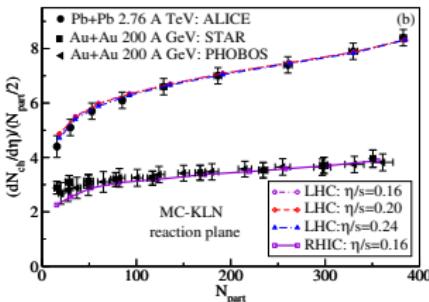
Hydrodynamics works!!



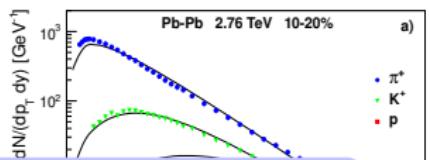
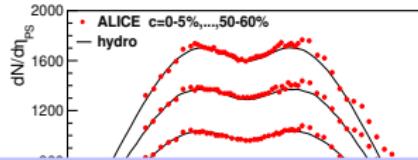
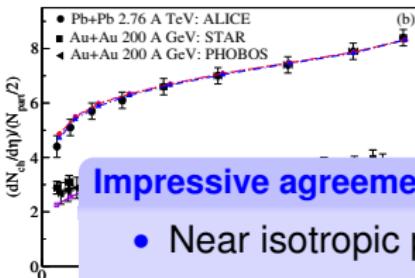
Hydrodynamics works!!



Hydrodynamics works!!



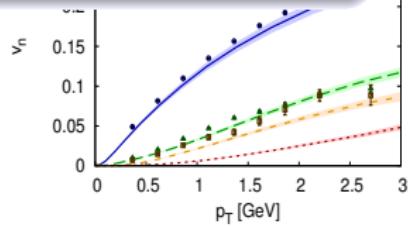
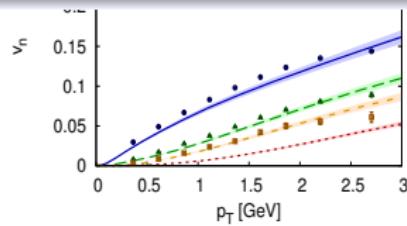
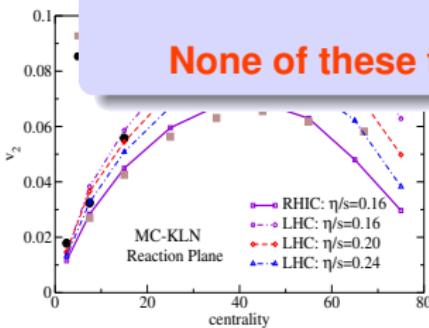
Hydrodynamics works!!

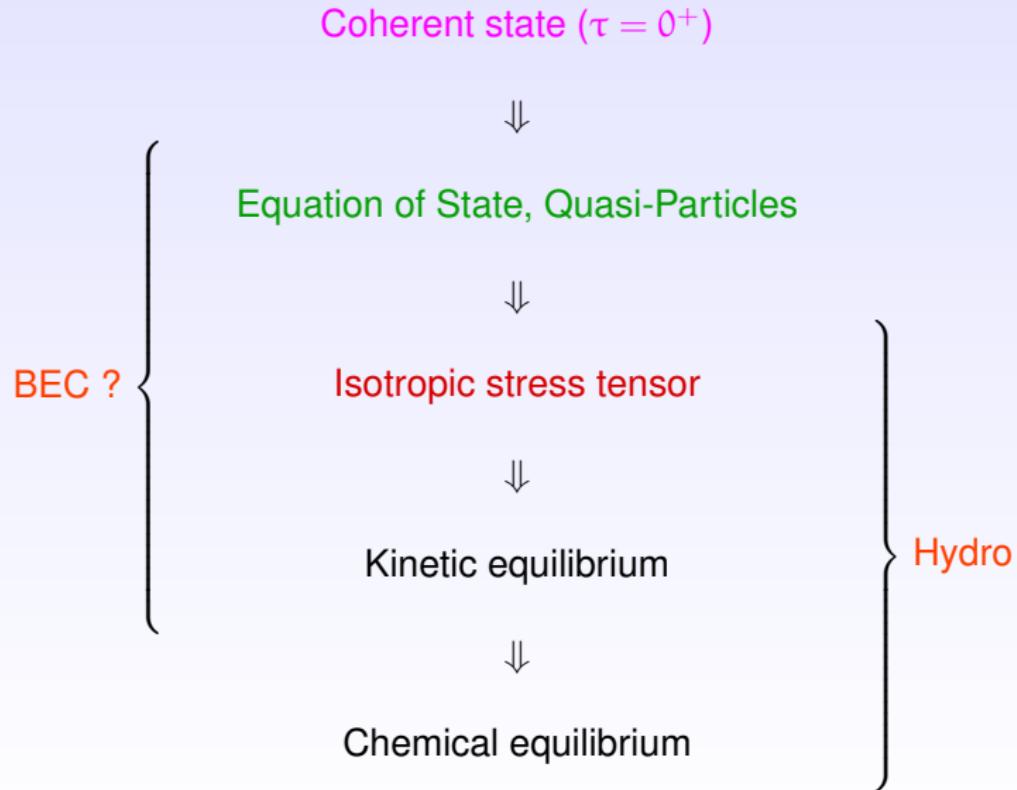


Impressive agreement, but: What makes hydrodynamics work so well?

- Near isotropic pressure tensor
- Not too far from equilibrium
- Low viscosity

None of these things is easy to get from QCD...

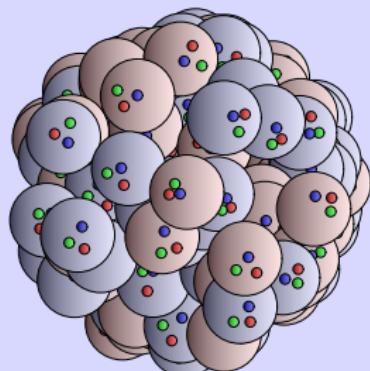




Color Glass Condensate

What do we need to know about nuclei?

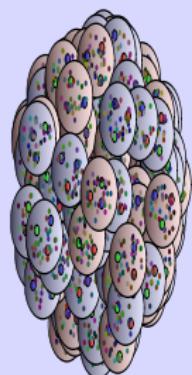
Nucleus at rest



- At low energy : valence quarks

What do we need to know about nuclei?

Slightly boosted nucleus



- At low energy : valence quarks
- At higher energy :
 - Lorenz contraction of longitudinal sizes
 - Time dilation ▷ slowing down of the internal dynamics
 - Gluons start becoming important

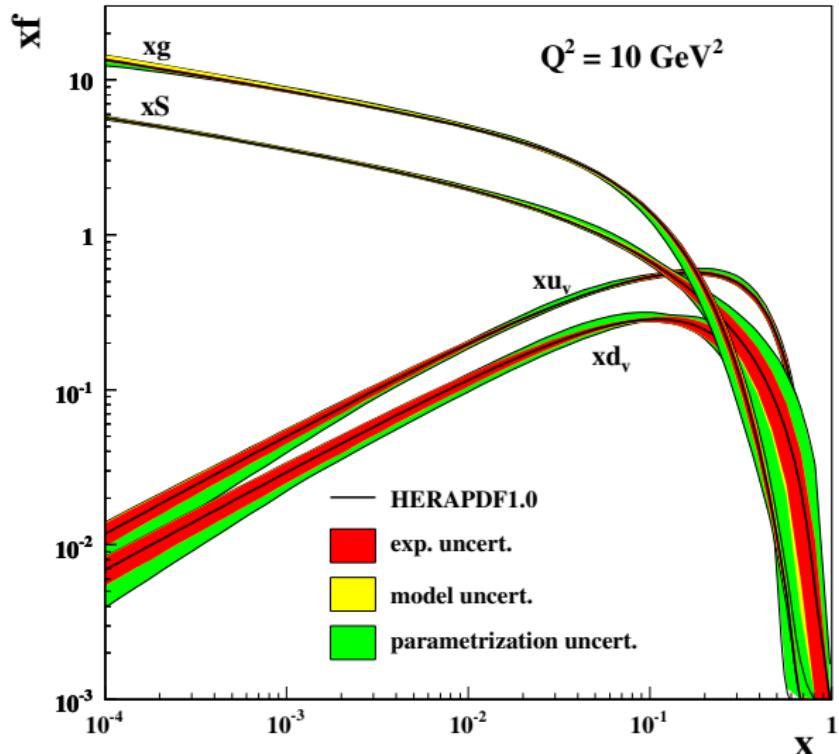
High energy nucleus



- At low energy : valence quarks
- At higher energy :
 - Lorenz contraction of longitudinal sizes
 - Time dilation ▷ slowing down of the internal dynamics
 - Gluons start becoming important
- At very high energy : gluons dominate

Nucleon parton distributions

H1 and ZEUS

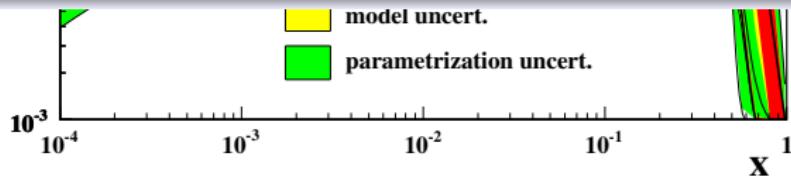
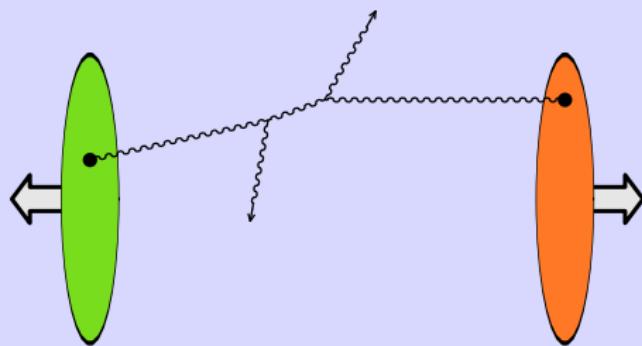


Nucleon parton distributions

H1 and ZEUS



Large x : dilute regime

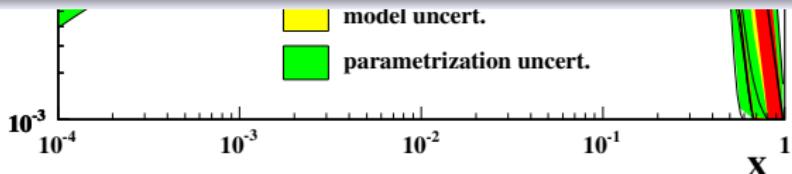
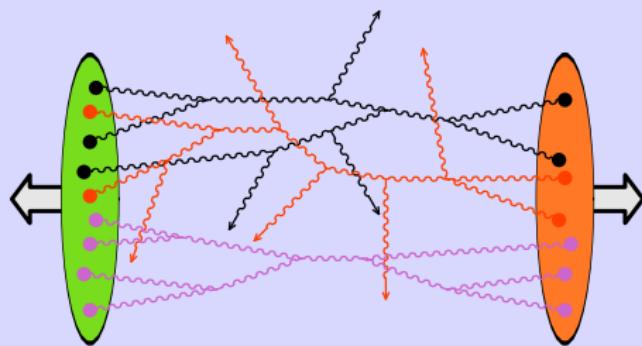


Nucleon parton distributions

H1 and ZEUS



Small x : dense regime, gluon saturation



Color Glass Condensate = effective theory of small x gluons

[McLerran, Venugopalan (1994), Jalilian-Marian, Kovner, Leonidov, Weigert (1997), Iancu, Leonidov, McLerran (2001)]

- The **fast partons** ($k^+ > \Lambda^+$) are frozen by time dilation
 - described as **static color sources** on the light-cone :

$$J^\mu = \delta^{\mu+} \rho(x^-, \vec{x}_\perp) \quad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are **random**, and described by a probability distribution $W_\Lambda[\rho]$
- Slow partons** ($k^+ < \Lambda^+$) may evolve during the collision
 - treated as standard gauge fields
 - eikonal coupling to the current J^μ : $J_\mu A^\mu$

$$\mathcal{S} = \underbrace{-\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu}}_{\mathcal{S}_{YM}} + \int \underbrace{J^\mu A_\mu}_{\text{fast partons}}$$

Leading Order

- All observables can be expressed in terms of classical solutions of Yang-Mills equations :

$$[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = J_1^\nu + J_2^\nu$$

- Boundary conditions for inclusive observables : retarded, with $\mathcal{A}(x) = 0$ at $x_0 = -\infty$

Example : inclusive spectrum at LO

$$\left. \frac{dN_1}{d^3 \bar{p}} \right|_{LO} \sim \int d^4x d^4y e^{i\bar{p} \cdot (x-y)} \square_x \square_y \mathcal{A}(x) \mathcal{A}(y)$$

Next to Leading Order

Relation between LO and NLO

$$\mathcal{O}_{\text{NLO}} = \left[\frac{1}{2} \int_{u,v} \int_k [a_k \mathbb{T}]_u [a_k^* \mathbb{T}]_v + \int_u [\alpha \mathbb{T}]_u \right] \mathcal{O}_{\text{LO}}$$

- \mathbb{T} is the generator of the shifts of the initial field :

$$\exp \left[\int_u [\alpha \mathbb{T}]_u \right] \mathcal{O} \left[\underbrace{\mathcal{A}_\tau(\mathcal{A}_{\text{init}})}_{\text{init. value}} \right] = \mathcal{O} \left[\mathcal{A}_\tau \left(\underbrace{\mathcal{A}_{\text{init}} + \alpha}_{\text{shifted init. value}} \right) \right]$$

- a_k, α are calculable analytically
- Valid for all inclusive observables,
e.g. spectra, the energy-momentum tensor, etc...

Initial state logarithms

- Upper cutoff on the loop momentum : $k^\pm < \Lambda$,
to avoid double counting with the sources $J_{1,2}^\gamma$
 - ▷ logarithms of the cutoff

Central result for factorization

$$\begin{aligned} \frac{1}{2} \int\limits_{\mathbf{u}, \mathbf{v}} \int\limits_{\mathbf{k}} [\alpha_{\mathbf{k}} \mathbb{T}]_{\mathbf{u}} [\alpha_{\mathbf{k}}^* \mathbb{T}]_{\mathbf{v}} + \int\limits_{\mathbf{u}} [\alpha \mathbb{T}]_{\mathbf{u}} = \\ = \log(\Lambda^+) \mathcal{H}_1 + \log(\Lambda^-) \mathcal{H}_2 + \text{terms w/o logs} \end{aligned}$$

$\mathcal{H}_{1,2}$ = JIMWLK Hamiltonians of the two nuclei

Factorization of the logarithms

Inclusive observables at Leading Log accuracy

$$\langle \mathcal{O} \rangle_{\text{Leading Log}} = \int [D\rho_1 D\rho_2] W_1[\rho_1] W_2[\rho_2] \underbrace{\mathcal{O}_{\text{LO}}[\rho_1, \rho_2]}_{\text{fixed } \rho_{1,2}}$$

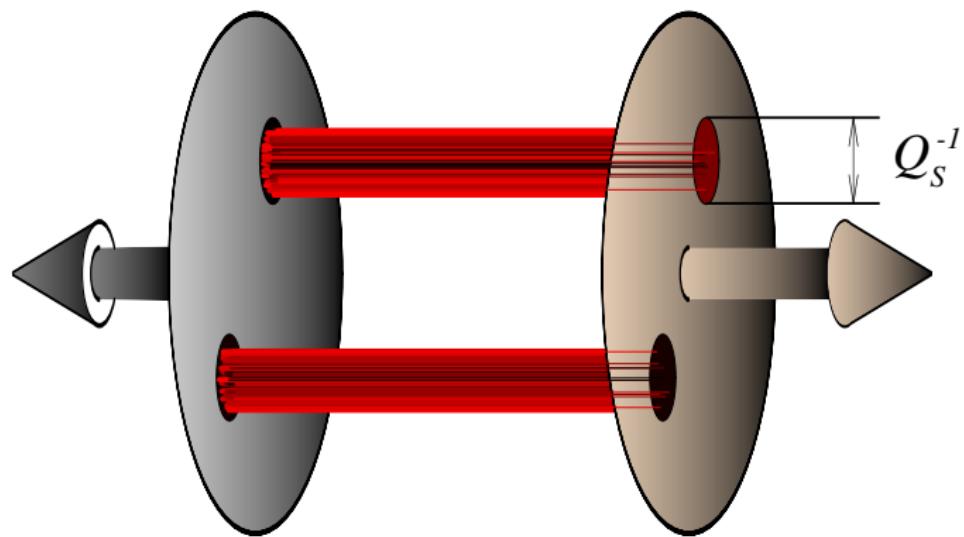
- Logs absorbed into the scale evolution of $W_{1,2}$

$$\Lambda \frac{\partial W}{\partial \Lambda} = \mathcal{H}W \quad (\text{JIMWLK equation})$$

- Universality** : same W 's for all inclusive observables

**Just after
the collision**

Energy momentum tensor of the Glasma fields



Energy momentum tensor of the Glasma fields



$T^{\mu\nu}$ for longitudinal \vec{E} and \vec{B}

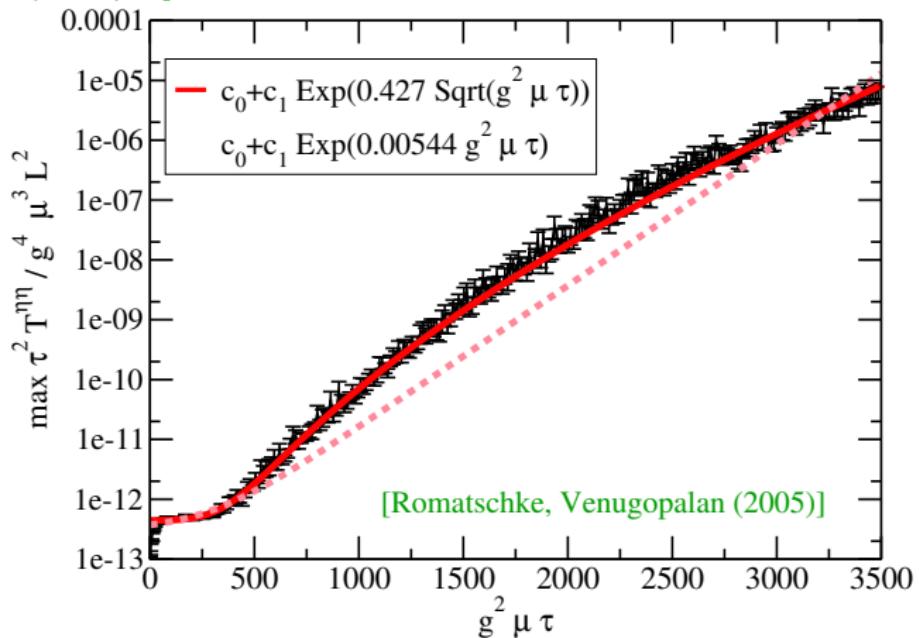
$$T_{\text{LO}}^{\mu\nu}(\tau = 0^+) = \text{diag}(\epsilon, \epsilon, \epsilon, -\epsilon)$$

▷ far from ideal hydrodynamics



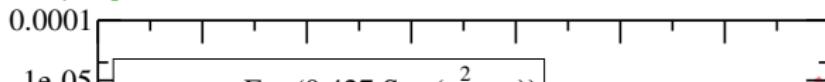
Weibel instabilities for small perturbations

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007),...]



Weibel instabilities for small perturbations

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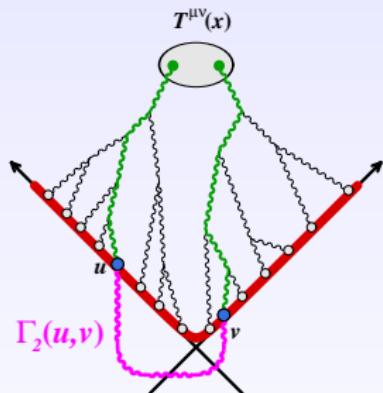
- For some k 's, the field fluctuations α_k diverge like $\exp \sqrt{\mu \tau}$ when $\tau \rightarrow +\infty$
- Some components of $T^{\mu\nu}$ have secular divergences when evaluated at fixed loop order
- When $\alpha_k \sim \mathcal{A} \sim g^{-1}$, the power counting breaks down and additional contributions must be resummed :

$$g e^{\sqrt{\mu \tau}} \sim 1 \quad \text{at} \quad \tau_{\max} \sim \mu^{-1} \log^2(g^{-1})$$

$$\frac{g^2}{\mu} \tau$$

Improved power counting

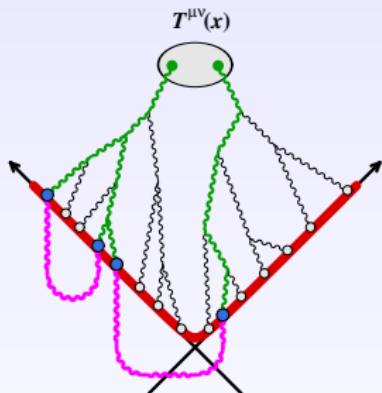
$$\text{Loop} \sim g^2 \quad , \quad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- **1 loop** : $(ge^{\sqrt{\mu\tau}})^2$

Improved power counting

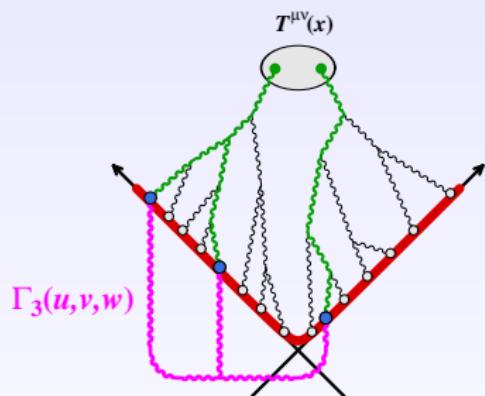
$$\text{Loop} \sim g^2 \quad , \quad T \sim e^{\sqrt{\mu\tau}}$$



- **1 loop** : $(ge^{\sqrt{\mu\tau}})^2$
- **2 disconnected loops** : $(ge^{\sqrt{\mu\tau}})^4$

Improved power counting

$$\text{Loop} \sim g^2 \quad , \quad T \sim e^{\sqrt{\mu\tau}}$$



- **1 loop** : $(ge^{\sqrt{\mu\tau}})^2$
- **2 disconnected loops** : $(ge^{\sqrt{\mu\tau}})^4$
- **2 nested loops** : $g(ge^{\sqrt{\mu\tau}})^3$
▷ subleading

Leading terms at τ_{\max}

- All disconnected loops to all orders
▷ exponentiation of the 1-loop result

Resummation of the leading secular terms

$$\begin{aligned} T_{\text{resummed}}^{\mu\nu} &= \exp \left[\frac{1}{2} \int \underbrace{\int_k [a_k T]_u [a_k^* T]_v}_{S(u,v)} + \int [\alpha T]_u \right] T_{\text{LO}}^{\mu\nu} [\mathcal{A}_{\text{init}}] \\ &= \underbrace{T_{\text{LO}}^{\mu\nu} + T_{\text{NLO}}^{\mu\nu}}_{\text{in full}} + \underbrace{T_{\text{NNLO}}^{\mu\nu} + \dots}_{\text{partially}} \end{aligned}$$

Resummation of the leading secular terms

$$\begin{aligned}
 T_{\text{resummed}}^{\mu\nu} &= \exp \left[\frac{1}{2} \int_{\mathbf{u}, \mathbf{v}} \underbrace{\int_{\mathbf{k}} [\mathbf{a}_{\mathbf{k}} \mathbb{T}]_{\mathbf{u}} [\mathbf{a}_{\mathbf{k}}^* \mathbb{T}]_{\mathbf{v}}}_{\mathcal{G}(\mathbf{u}, \mathbf{v})} + \int_{\mathbf{u}} [\boldsymbol{\alpha} \mathbb{T}]_{\mathbf{u}} \right] T_{\text{LO}}^{\mu\nu} [\mathcal{A}_{\text{init}}] \\
 &= \int [D\chi] \exp \left[-\frac{1}{2} \int_{\mathbf{u}, \mathbf{v}} \chi(\mathbf{u}) \mathcal{G}^{-1}(\mathbf{u}, \mathbf{v}) \chi(\mathbf{v}) \right] T_{\text{LO}}^{\mu\nu} [\mathcal{A}_{\text{init}} + \chi + \boldsymbol{\alpha}]
 \end{aligned}$$

- The evolution remains classical, but we must average over a Gaussian ensemble of initial conditions
- The 2-point correlation $\mathcal{G}(\mathbf{u}, \mathbf{v})$ of the fluctuations is known
- Fluctuations \Leftrightarrow 1/2 quantum per mode

- The Gaussian distribution of initial fields is the Wigner representation of a **coherent state** $|\mathcal{A}\rangle$ (i.e. an eigenstate of the annihilation operators)

$$a_k |\mathcal{A}\rangle = \alpha_k |\mathcal{A}\rangle$$

Coherent states are the “most classical quantum states” (their wavefunction has the minimal extent permitted by the uncertainty principle, shared equally in X and P)

- The initial density operator is $\rho_0 = |\mathcal{A}\rangle\langle\mathcal{A}|$ (pure state)

Contrast this with a thermal state, $\rho_{\text{thermal}} = \sum_n e^{-E_n/T} |\Psi_n\rangle\langle\Psi_n|$

- $|\mathcal{A}\rangle$ is not an eigenstate of the Hamiltonian
 - ▷ decoherence via interactions

Analogous (but simpler) scalar toy model

ϕ^4 field theory coupled to a source

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \phi)^2 - \frac{g^2}{4!}\phi^4 + J\phi$$

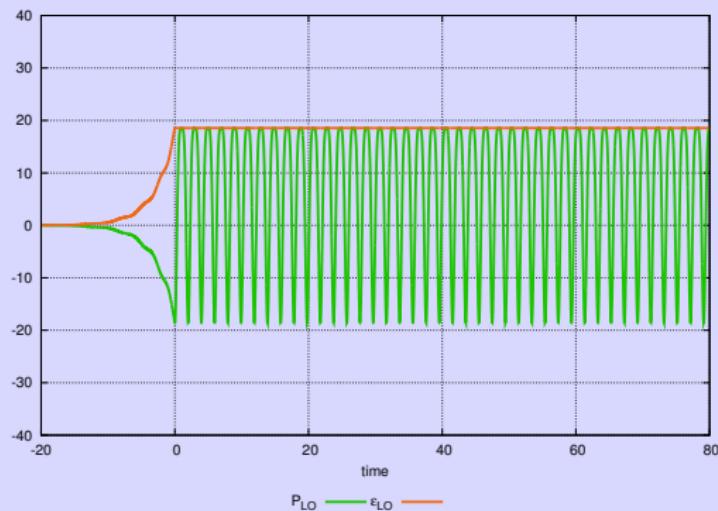
Strong external source: $J \propto \frac{Q^3}{g}$

- In 3+1-dim, g is dimensionless, and the only scale is Q
- This theory has unstable modes (parametric resonance)
- Two setups have been studied :
 - Fixed volume system (equation of state, thermalization)
 - Longitudinally expanding system (isotropization)

Equation of State Thermalization

Pathologies in fixed order calculations

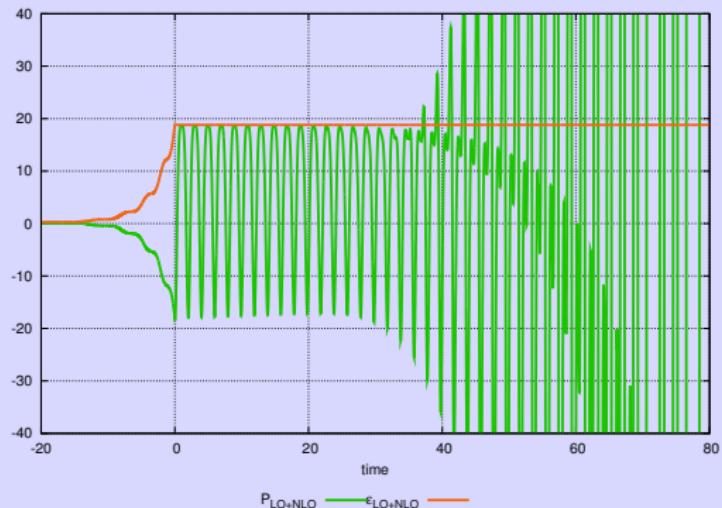
Tree



- Oscillating pressure at LO : no equation of state

Pathologies in fixed order calculations

Tree + 1-loop



- Oscillating pressure at LO : no equation of state
- Small NLO correction to the energy density (protected by energy conservation)
- Secular divergence in the NLO correction to the pressure

Spectrum of fluctuations

Initial conditions

$$\phi(t_0, \mathbf{x}) = \underbrace{\varphi(t_0, \mathbf{x})}_{\text{class. field}} + \underbrace{\int \frac{d^3 k}{(2\pi)^3 2k} \left[c_k e^{i \lambda_k t_0} V_k(\mathbf{x}) + \text{c.c.} \right]}_{\text{random Gaussian fluctuations}}$$

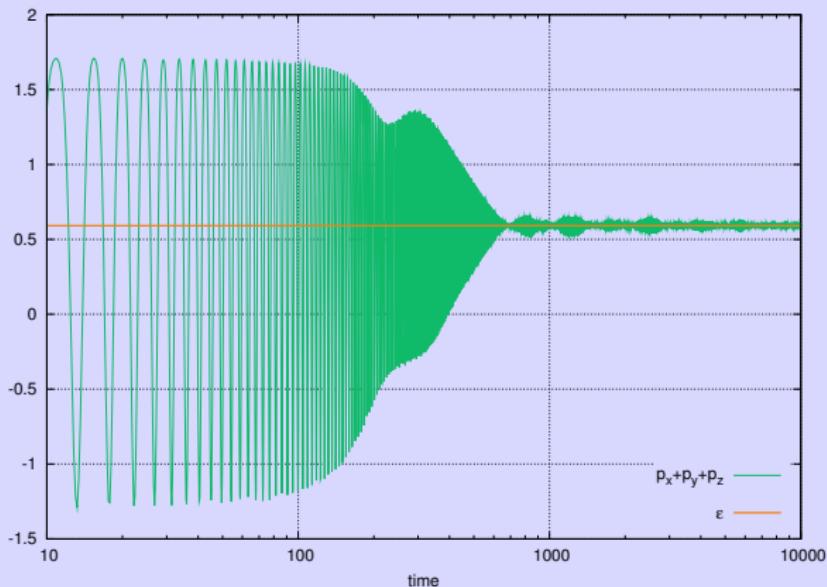
- c_k = complex random Gaussian number:

$$\langle c_k \rangle = 0, \quad \langle c_k c_l^* \rangle = (2\pi)^3 k \delta(\mathbf{k} - \mathbf{l})$$

- λ_k, V_k determined by:

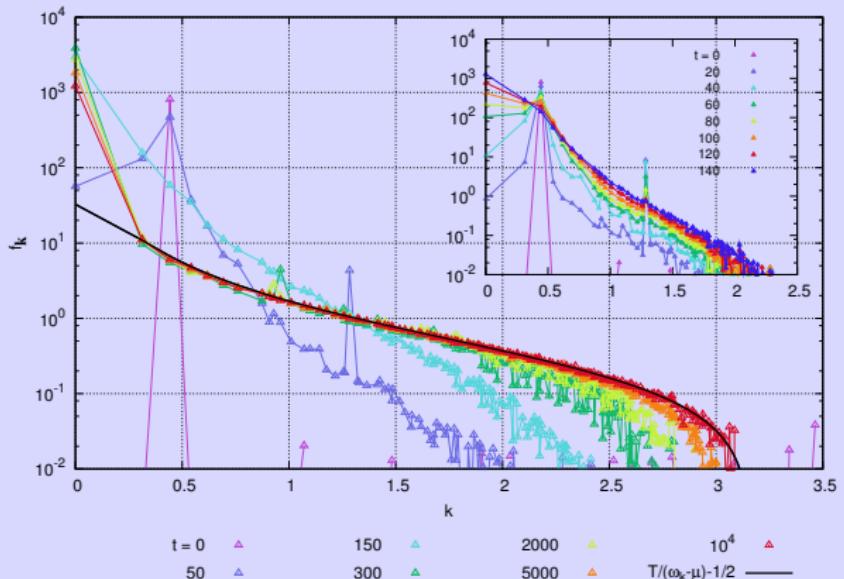
$$\left[-\nabla^2 + \frac{1}{2} g^2 \varphi^2(t_0, \mathbf{x}) \right] V_k(\mathbf{x}) = \lambda_k^2 V_k(\mathbf{x})$$

Resummed energy momentum tensor



- No secular divergence in the resummed pressure
- The pressure relaxes to the equilibrium equation of state

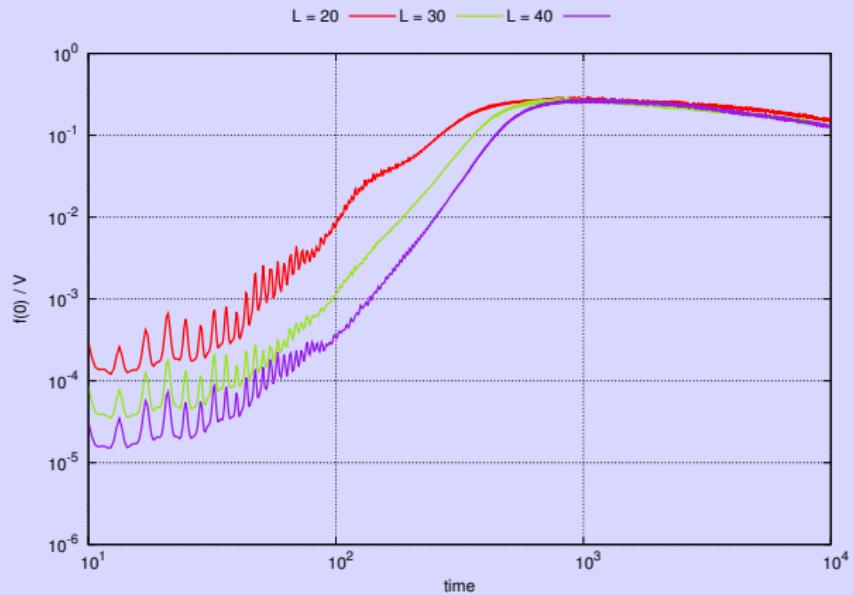
Occupation number



- Resonant peak at early times
- Late times : classical equilibrium with a chemical potential
- $\mu \approx m +$ excess at $k = 0$: Bose-Einstein condensation?

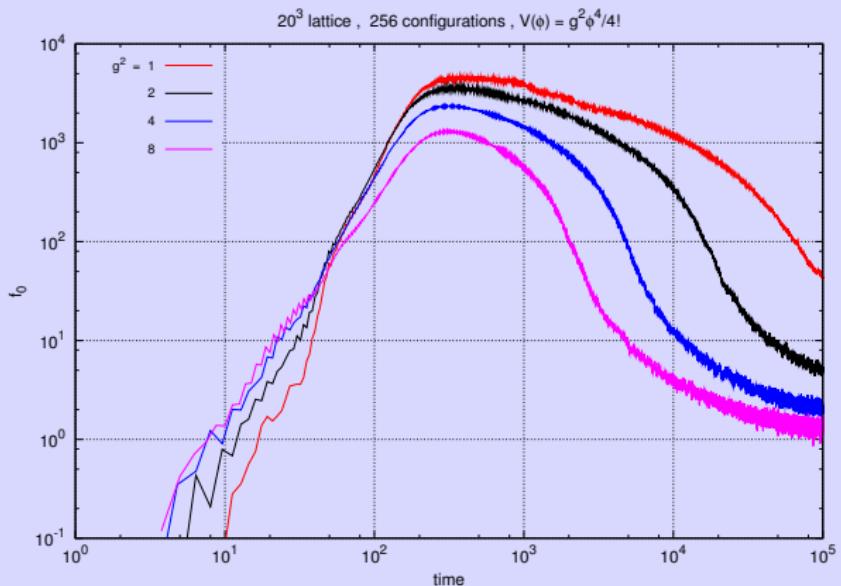
Bose-Einstein Condensation?

Volume dependence of the zero mode



$$f(\mathbf{k}) = \frac{1}{e^{\beta(\omega_{\mathbf{k}} - \mu)} - 1} + n_0 \delta(\mathbf{k}) \implies f(0) \propto V = L^3$$

Evolution of the condensate

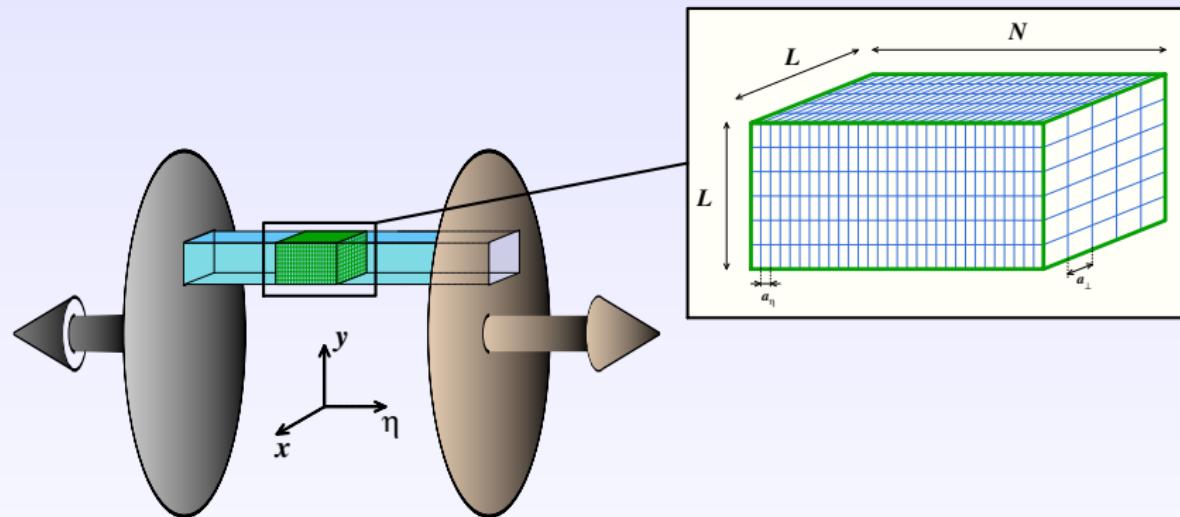


- Formation time almost independent of the coupling
- Condensate lifetime much longer than its formation time
- Smaller amplitude and faster decay at large coupling

Isotropization

(in an expanding system)

Discretization of the expanding volume



- Comoving coordinates : τ, η, x_\perp
- Only a small volume is simulated + periodic boundary conditions
- $L^2 \times N$ lattice with $L \sim 30 - 50$, $N \sim 300 - 600$

Spectrum of fluctuations

Initial conditions

$$\phi(\tau_0, \eta, \mathbf{x}_\perp) = \underbrace{\varphi(\tau_0, \mathbf{x}_\perp)}_{\text{class. field}} + \underbrace{\int \frac{d^2 \mathbf{k}_\perp d\nu}{(2\pi)^3} \left[c_{\nu\mathbf{k}} H_{\nu\mathbf{k}}^{(2)}(\lambda_\mathbf{k}\tau_0) V_\mathbf{k}(\mathbf{x}_\perp) e^{-i\nu\eta} + \text{c.c.} \right]}_{\text{random Gaussian fluctuations}}$$

- Boost invariance $\Rightarrow \varphi(\tau_0, \mathbf{x}_\perp)$ independent of η

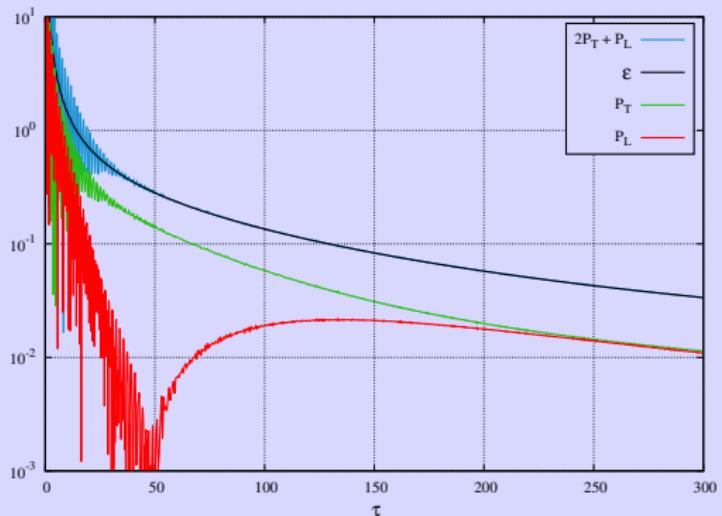
- $c_{\nu\mathbf{k}}$ = complex random Gaussian number:

$$\langle c_{\nu\mathbf{k}} \rangle = 0, \quad \langle c_{\nu\mathbf{k}} c_{\sigma\mathbf{l}}^* \rangle = (2\pi)^3 \delta(\mathbf{k}_\perp - \mathbf{l}_\perp) \delta(\nu - \sigma)$$

- $\lambda_\mathbf{k}, V_\mathbf{k}$ determined by:

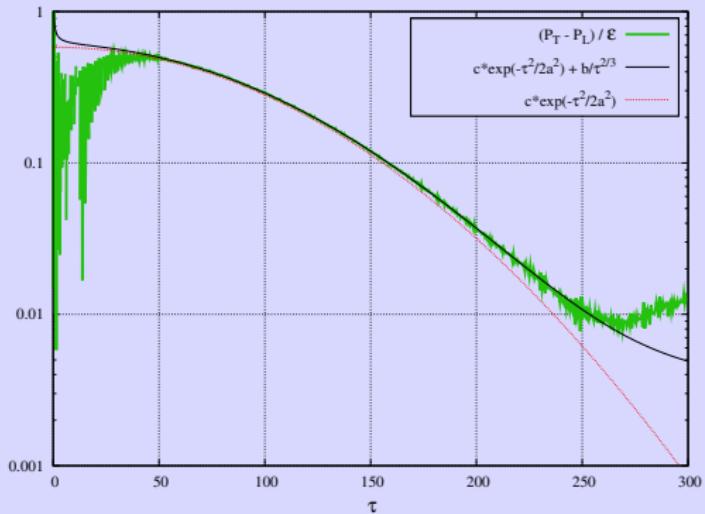
$$\left[-\nabla_\perp^2 + \frac{1}{2} g^2 \varphi^2(\tau_0, \mathbf{x}_\perp) \right] V_\mathbf{k}(\mathbf{x}_\perp) = \lambda_\mathbf{k}^2 V_\mathbf{k}(\mathbf{x}_\perp)$$

Isotropization



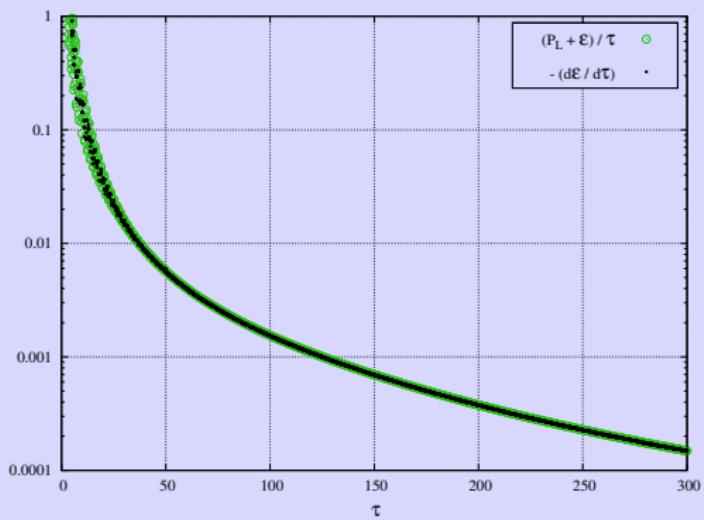
- At early times, P_L drops much faster than P_T (redshifting of the longitudinal momenta due to the expansion)
- Drastic change of behavior when the expansion rate becomes smaller than the growth rate of the instability
- Eventually, isotropic pressure tensor : $P_L \approx P_T$

Isotropization



- Fast approach to isotropy
- Lattice artifacts at late times

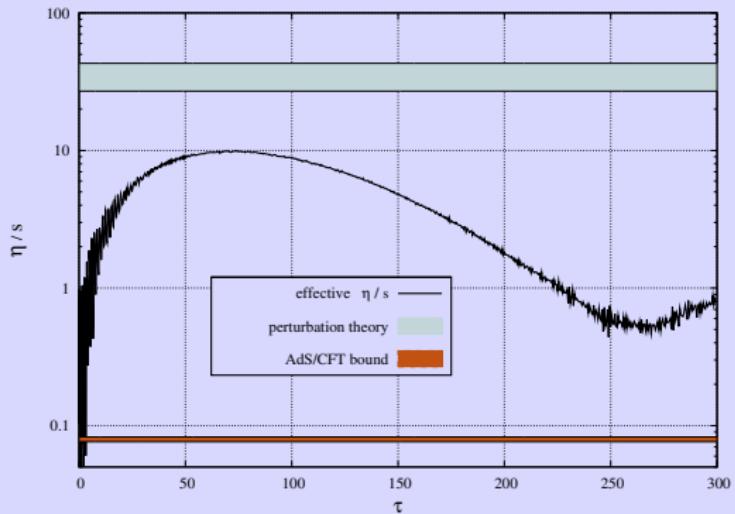
Note: Energy and Momentum are conserved



$$\frac{\partial \epsilon}{\partial \tau} + \frac{\epsilon + P_L}{\tau} = 0$$

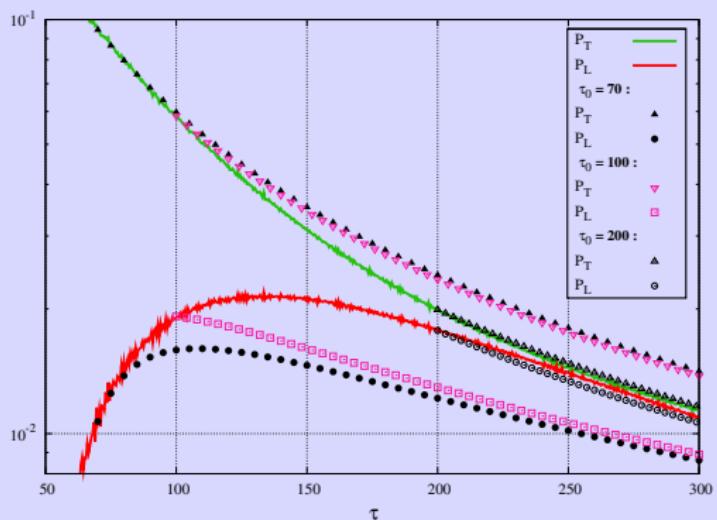
Note : this should be satisfied regardless of whether hydro works or not..

Effective shear viscosity



$$P_T = \frac{\epsilon}{3} + \frac{2}{3} \frac{\eta}{s} \frac{s}{\tau} \quad , \quad P_L = \frac{\epsilon}{3} - \frac{4}{3} \frac{\eta}{s} \frac{s}{\tau} \quad , \quad s \approx \epsilon^{3/4}$$

Comparison with 1st order hydrodynamics



- Faster relaxation than in hydrodynamics
- Hydrodynamics works well once $P_L \approx P_T$

Summary and Outlook

- Initial state = Coherent state (very far from equilibrium)
- Decoherence \Rightarrow Equation of State
- Bose-Einstein Condensation of the particles in excess?
- Fast isotropization driven by instabilities,
Effective viscosity much lower than in perturbation theory
- Thermalization
(so far, all numerical studies done for a toy scalar model)

What's next? **QCD**

- generalizable to Yang-Mills theory
- gauge invariant formulation
- seamless integration with the CGC description of AA collisions
- computationally expensive (\sim [scalar case] $\times 3 \times (N_c^2 - 1)$)