Generalized Parton Distributions of the Photon

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- Generalized parton Distributions of the photon
- GPDs in position space
- GPDs when the photon helicity is flipped
- Summary and Discussions

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In collaboration with Sreeraj Nair and Vikash K. Ojha

Generalized parton distributions of the photon

- High energy scattering : parton content of the photon plays an important role: structure functions of the photon are well studied both theoretically and experimentally

- Generalized parton distributions (GPDs) of the nucleon : can be accessed in exclusive processes and are rich in content about informations on nucleon structure and spin

- Instead of a proton target, consider deeply virtual Compton scattering process on a photon $\gamma^*(Q)\gamma \to \gamma\gamma$; in the kinematic region of large center-of-mass energy, large virtuality (Q^2) but small squared momentum transfer $(-t)$

- At leading order in α and zeroth order in α_s the result for the amplitude was interpreted to be factorized and upto leading log terms was written in terms of the GPDs of the photon

S. Friot, B. Pire, L. Szymanowski, Phys. Lett. **B 645** 153 (2007)
R. Gabdrakhmanov, O.V. Teryaev, hep-ph/1204.6471

- Momentum transferred between the initial state and the final state photon is purely in the longitudinal direction

Generalized parton distributions of the photon

- Show logarithmic scale dependence already in parton model

- These are of particular interest as they can be calculated in perturbation theory and can act as theoretical tools to understand the basic properties of GPDs like polynomiality and positivity

- For a complete description, one would need to include the non-pointlike hadronic contribution

GPDs of the photon are defined as

$$F^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P') \mid \bar{\psi}(0)\gamma^+\psi(y^-) \mid \gamma(P)\rangle;$$

$$\tilde{F}^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P') \mid \bar{\psi}(0)\gamma^+\gamma^5\psi(y^-) \mid \gamma(P)\rangle.$$

Chosen light-front gauge $A^+ = 0$

$\tilde{F}^q$ can be calculated from terms of the form $\epsilon_\lambda^2 \epsilon_\lambda^{1*} - \epsilon_\lambda^1 \epsilon_\lambda^{2*}$ in the amplitude

Generalized parton distributions of the photon

We calculate the GPDs of the photon for non-zero $\Delta^\perp$ as well as skewness using overlaps of light-cone wave functions of the photon
Fock space expansion of the photon state can be written as

$$
\begin{aligned}
\mid \gamma(P)\rangle &= \sqrt{N}\Bigg[a^\dagger(P,\lambda)\mid 0\rangle + \sum_{\sigma_1,\sigma_2}\int\{dk_1\}\int\{dk_2\}\sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2) \\
&\qquad \phi_2(k_1,k_2,\sigma_1,\sigma_2)b^\dagger(k_1,\sigma_1)d^\dagger(k_2,\sigma_2)\mid 0\rangle\Bigg]
\end{aligned}
$$

where $\sqrt{N}$ is the overall normalization of the state; which in our calculation we can take as unity as any correction to it contributes at higher order in α
$\{dk\} = \int \frac{dk^+ d^2k^\perp}{\sqrt{2(2\Pi)^3 k^+}}$
Two particle boost invariant light front wave functions can be calculated analytically order by order in perturbation theory : these provide the leading contribution to the matrix element

Generalized parton distributions of the photon

Momenta of the initial and final photon are given by:

$$
\begin{aligned}
P &= \Big(P^+ \, , \, 0^\perp \, , \, 0 \Big) \, , \\
P' &= \left((1-\zeta)P^+ \, , \, -\Delta^\perp \, , \, \frac{\Delta^{\perp 2}}{(1-\zeta)P^+} \right) \, ,
\end{aligned}
$$

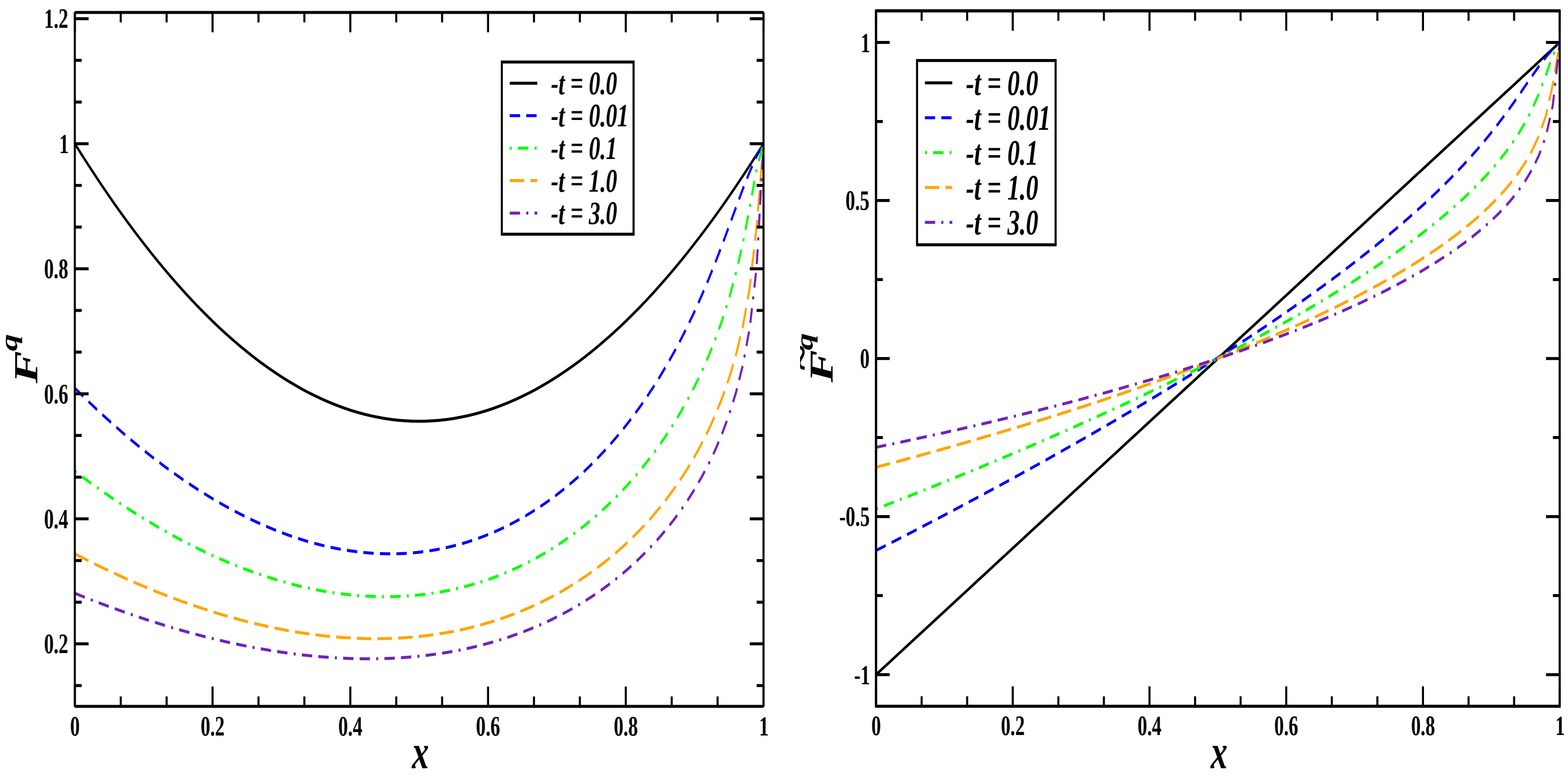
Four-momentum transfer from the target is

$$
\Delta = P - P' = \left(\zeta P^+ \, , \, \Delta^\perp \, , \, \frac{t + {\Delta^\perp}^2}{\zeta P^+} \right) \, ,
$$

where $t = \Delta^2$

$$
(1-\zeta)t = -{\Delta^\perp}^2
$$

GPDs of the photon



- Only the quark contribution is shown
- We took $Q = \Lambda = 20GeV$, $m = 3.3MeV$
- As $x \to 1$, most of the momentum is carried by the quark in the photon and the GPDs become independent of t

AM, S. Nair; PLB (2011)

GPDs of the photon in impact parameter space

Fourier transform with respect to the transverse momentum transfer $\Delta_\perp$ gives GPDs in impact parameter space

Burkardt (2000)

$$
\begin{aligned}
q(x,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} F^q(x,t) \\
&= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) F^q(x,t),
\end{aligned}
$$

$$
\begin{aligned}
\tilde{q}(x,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} \tilde{F}^q(x,t) \\
&= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) \tilde{F}^q(x,t),
\end{aligned}
$$

where $\Delta = |\Delta^\perp|$ and $b = |b^\perp|$

GPDs of the photon in impact parameter space

Introduced in analogy with the impact parameter dependent parton distribution of the proton

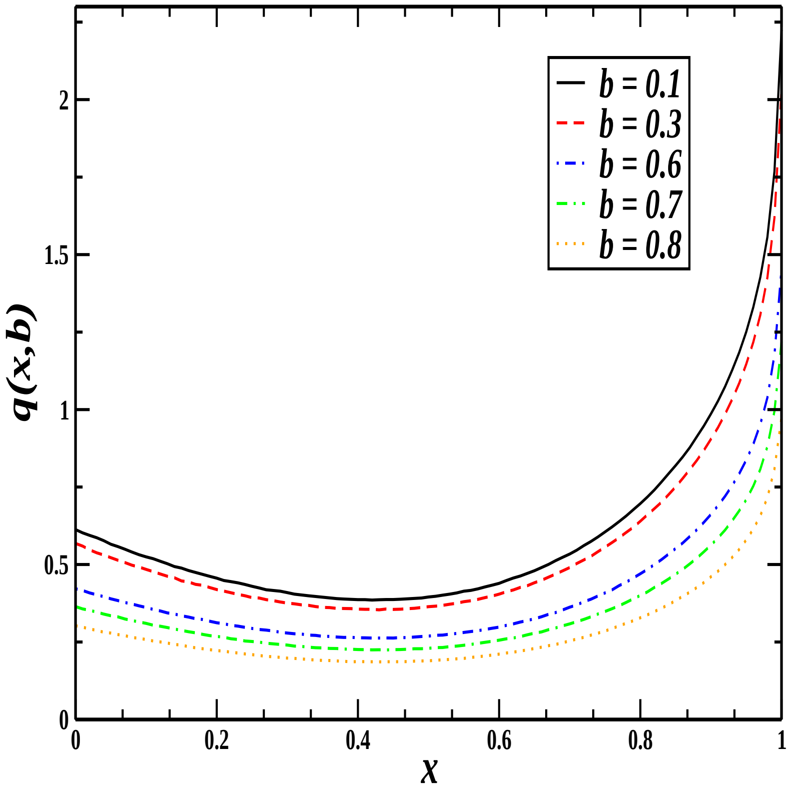
Have probabilistic interpretation : $q(x, b)$ is the probability of finding a quark of momentum fraction x and at transverse distance b from the center of the photon : parton distributions of the photon in the transverse plane

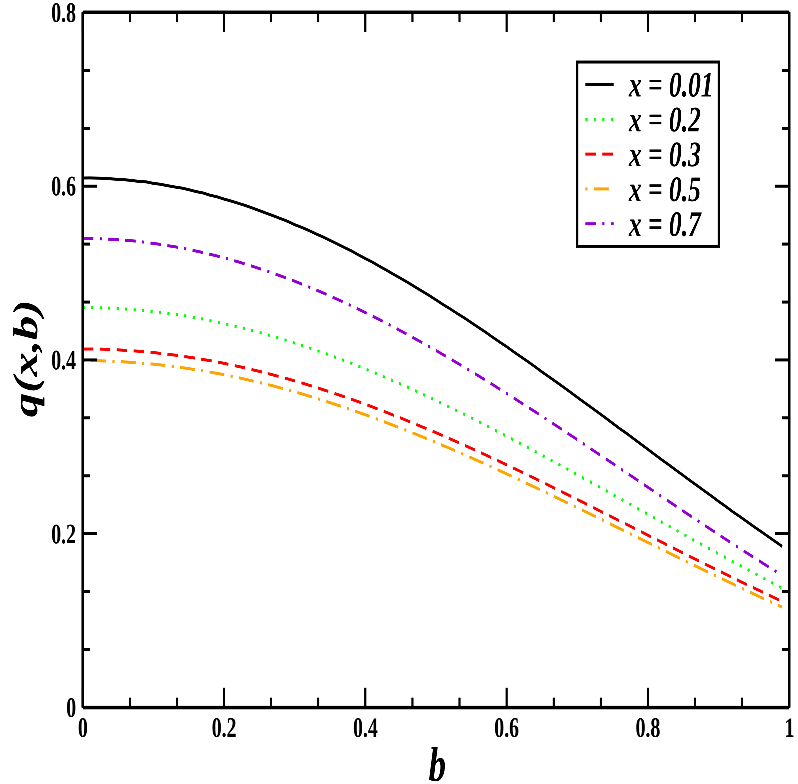
Impact parameter distribution for a polarized photon is given by $\tilde{q}(x, b^{\perp})$

New insight to the transverse 'shape' of the photon

Used a cutoff on $\Delta^{\perp}$ integration satisfying $t << Q^2$; DVCS kinematics

GPDs of the photon in impact parameter space



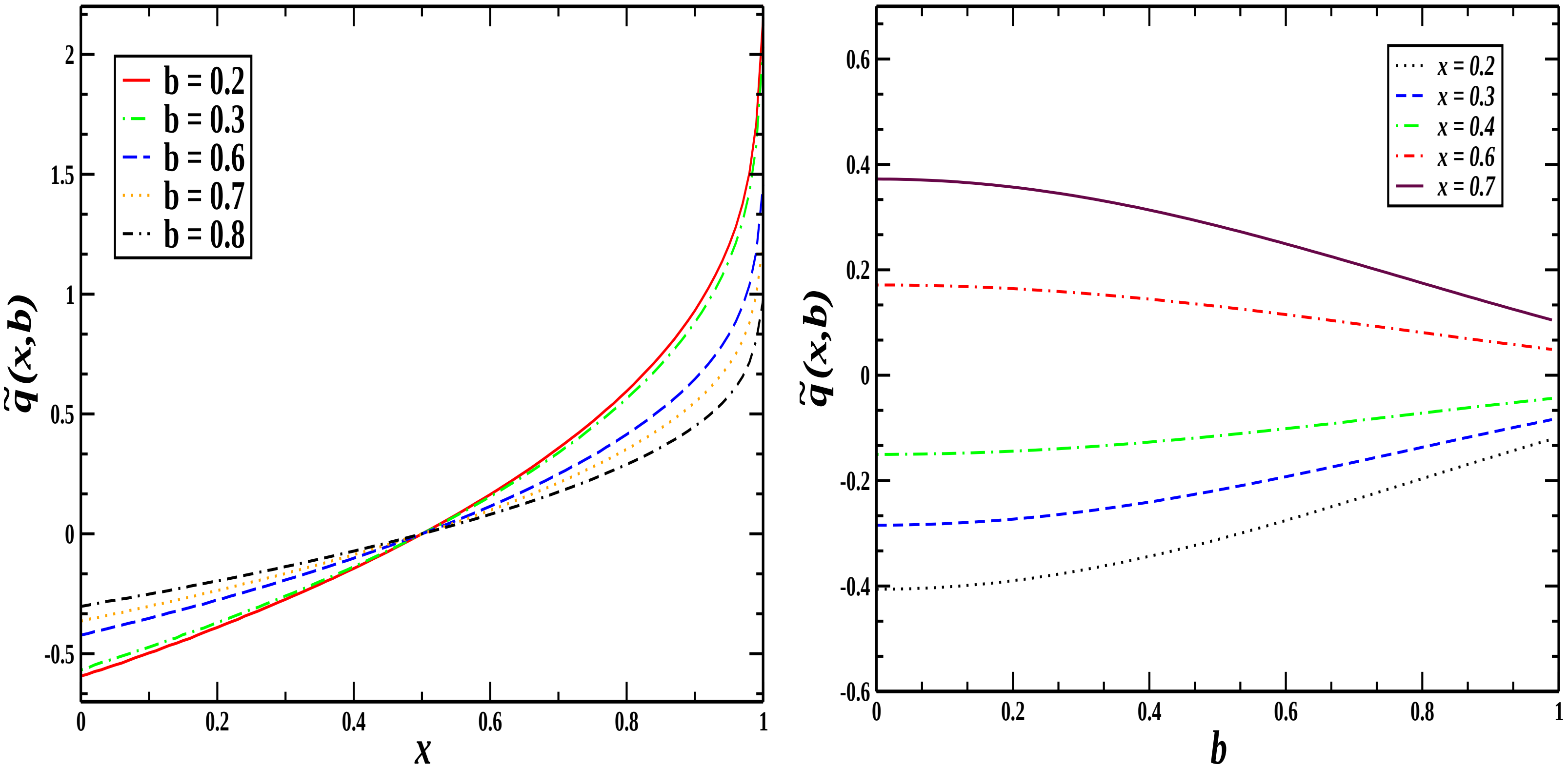


We have taken Λ = 20 GeV and Δ_{max}= 3 GeV where Δ_{max} is the upper limit in the Δ integration. b is in GeV^{-1} and $q(x, b)$ is in GeV^2

In the ideal definition the Fourier transform over Δ should be from 0 to ∞. In this case the $\Delta^{\perp}$ independent terms in F^q and $\tilde{F}^q$ would give $\delta^2(b^{\perp})$ in the impact parameter space

This means in the case of no transverse momentum transfer, the photon behaves like a point particle in transverse position space. The distribution in transverse space is a unique feature accessible only when there is non-zero momentum transfer in the transverse direction

GPDs of the photon in impact parameter space



The behavior in impact parameter space is qualitatively different from phenomenological models of proton GPDs

Parton distribution is more dispersed when the q and $\bar{q}$ share almost equal momenta

AM, S. Nair, PLB (2011)

Non-zero Skewness ζ

More general case : momentum transfer between the initial and final photon has both transverse and longitudinal components

$$q(x,\zeta,b) = \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} F^q(x,\zeta,t)$$
$$= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) F^q(x,\zeta,t),$$

$$\tilde{q}(x,\zeta,b) = \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} \tilde{F}^q(x,\zeta,t)$$
$$= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) \tilde{F}^q(x,\zeta,t),$$

We took the frame

$$P = \Big(P^+ \,,\, 0^\perp \,,\, 0 \Big) \,,$$
$$P' = \left((1-\zeta)P^+ \,,\, -\Delta^\perp \,,\, \frac{\Delta^{\perp 2}}{(1-\zeta)P^+} \right) \,,$$

Non-zero Skewness ζ

Repeat the calculation of F^q and $\tilde{F}^q$ when both ζ and $\Delta^\perp$ are non-zero

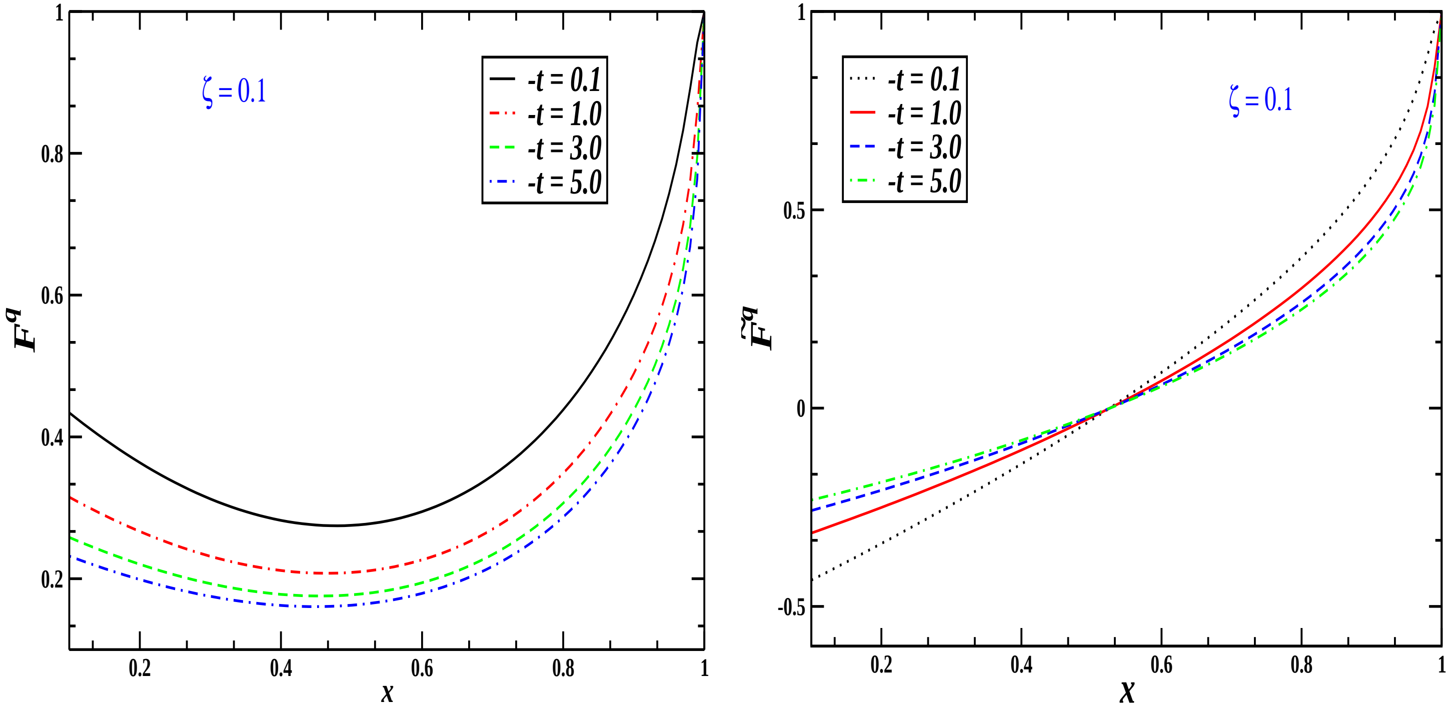
We limit ourselves to the kinematical region $1 > x > \zeta$ and $-1 < x < \zeta - 1$ where only the two-particle LFWFs contribute

When the skewness ζ is non-zero, GPDs in impact parameter space do not have a probabilistic interpretation

They are still interesting as they now probe the partons when the initial photon is displaced from the final photon in the transverse impact parameter space. This relative shift does not vanish when the GPDs are integrated over x in the amplitude

Diehl(2002)

Photon GPDs for non-zero ζ

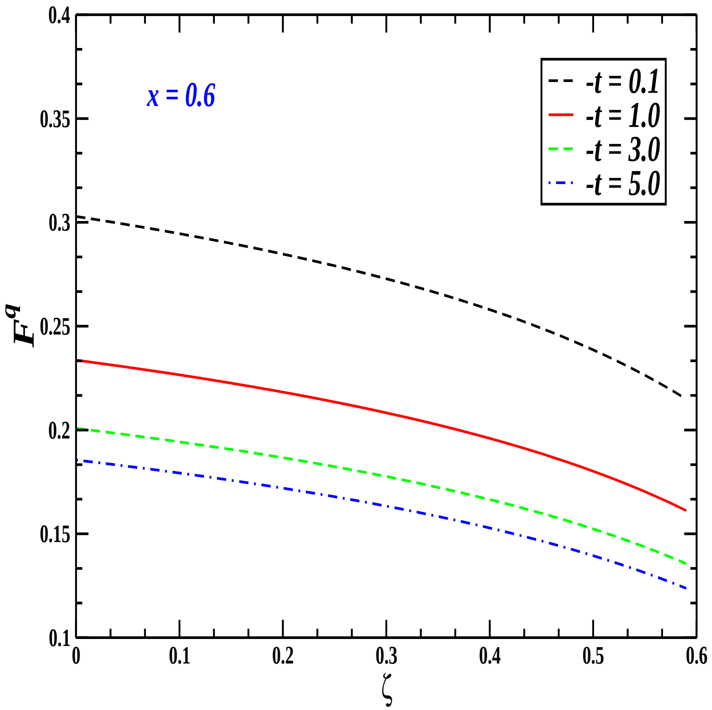


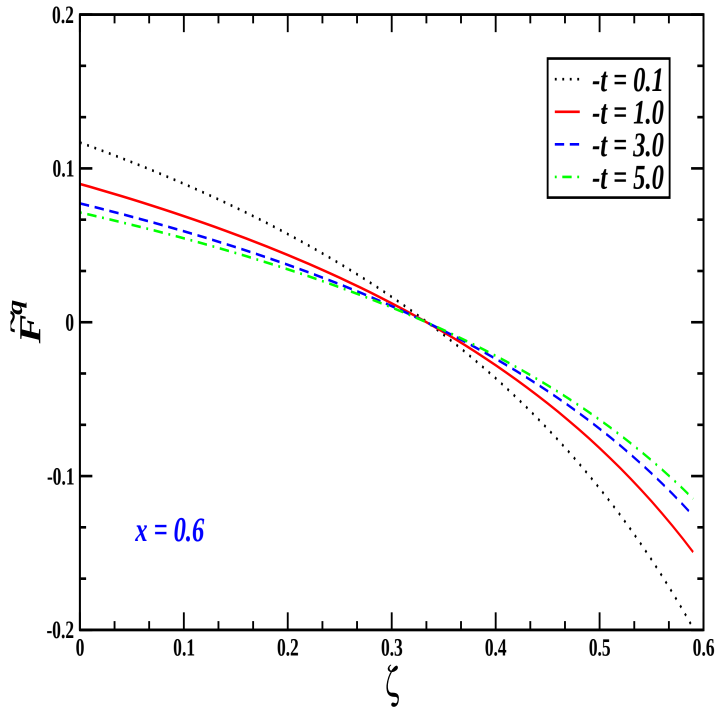
At leading order in α and zeroth order in α_s the results are logarithmically dependent on the scale

We took a fixed scale $\Lambda = 20\mathrm{GeV}$

AM, S. Nair, PLB (2012)

Photon GPDs for non-zero ζ



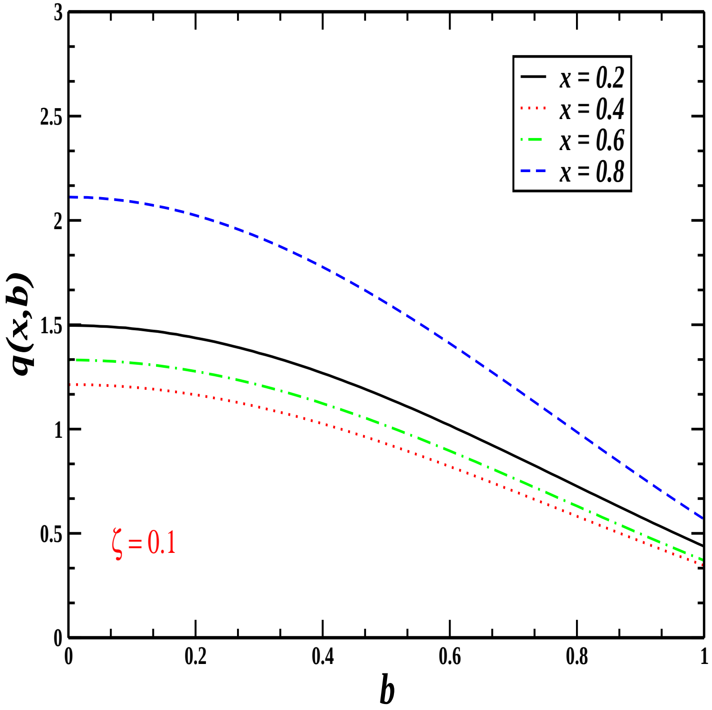


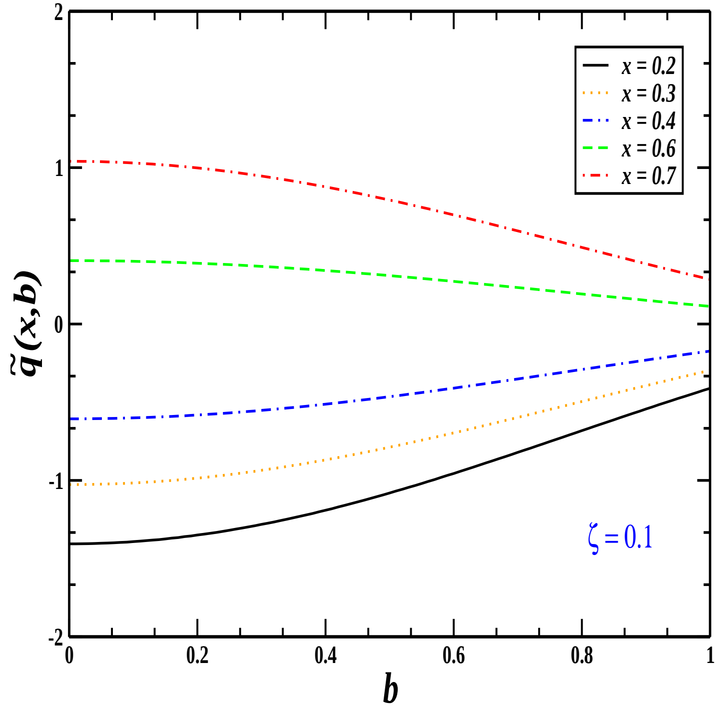
We took a fixed scale $\Lambda = 20\mathrm{GeV}$, non-zero quark mass $m = 3.3\mathrm{MeV}$

Helicity non-flip contributions; $x > \zeta$ region

AM, S. Nair (PLB, 2012)

Photon GPDs for non-zero ζ

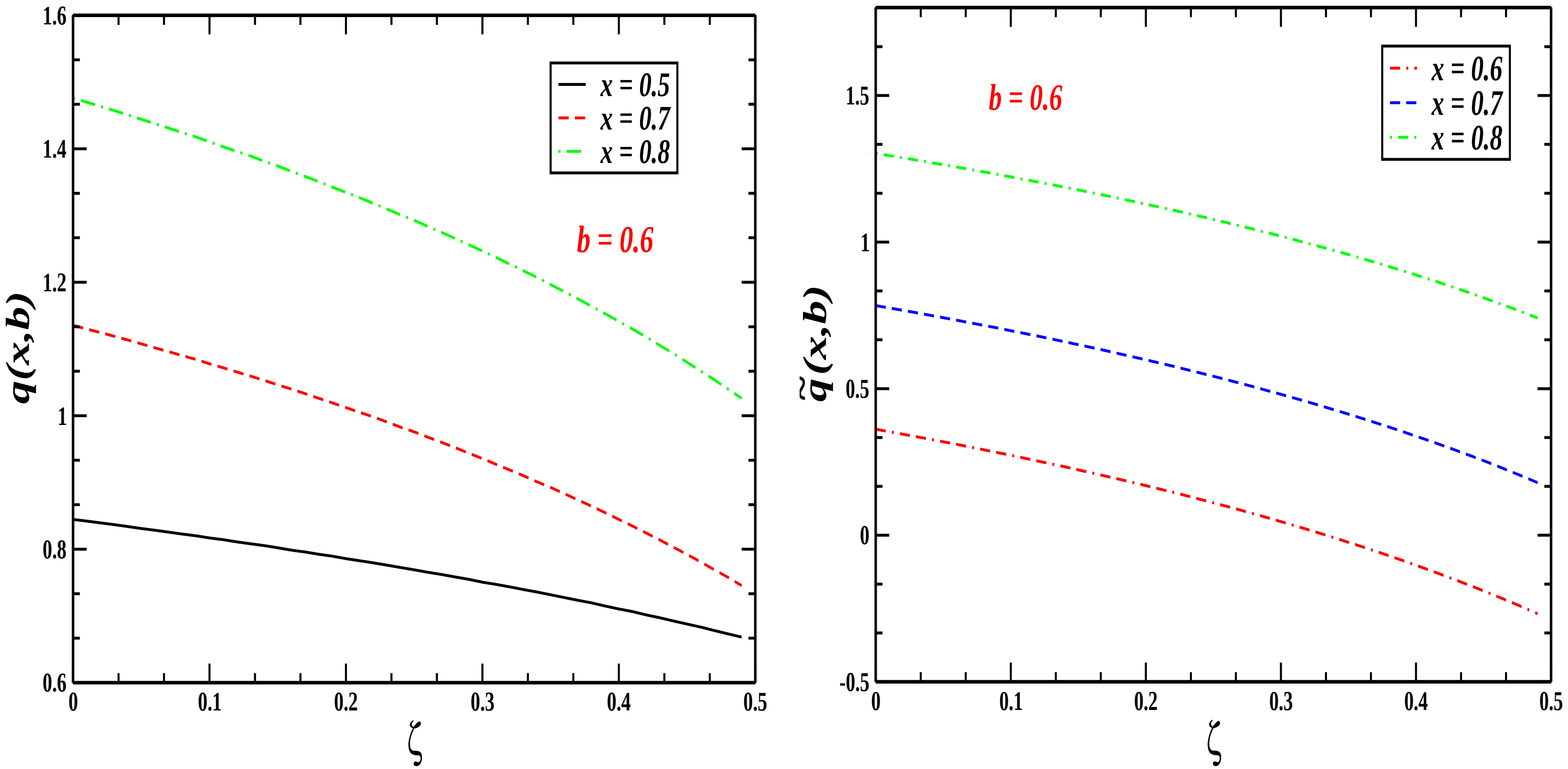




We have taken Λ = 20 GeV and Δ_{max}= 3GeV where Δ_{max} is the upper limit in the Δ integration. b is in GeV^{-1}

AM, S. Nair (PLB, 2012)

Photon GPDs for non-zero ζ



we have taken Λ = 20 GeV and Δ_{max}= 3GeV where Δ_{max} is the upper limit in the Δ integration. b is in GeV^{-1}

AM, S. Nair (PLB, 2012)

Photon GPDs with helicity flip

$$F^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P'), \lambda' \mid \bar{\psi}(0)\gamma^+\psi(y^-) \mid \gamma(P), \lambda \rangle$$

$$\tilde{F}^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P'), \lambda' \mid \bar{\psi}(0)\gamma^+\gamma^5\psi(y^-) \mid \gamma(P), \lambda \rangle$$

λ' and λ are not same, helicity of the target photon is flipped

Needs overlaps of LFWFS with non-zero orbital angular momentum (OAM) : similar to the proton GPD E

This needs non-zero transverse momentum transfer $\Delta^\perp$

Consider only $2 \to 2$ overlap when $\zeta = 0$

Can be calculated analytically, does not have log term; quadrupole structure

Photon GPDs with helicity flip : analytic form

Photon GPD with helicity flip can be calculated analytically using the two-particle light-front wave functions. In the limit when the momentum transfer is purely in the transverse direction, these can be written as

$$E_1 = \frac{\alpha e_q^2}{2\pi} x(1-x)^3 ((\Delta^1)^2 - (\Delta^2)^2) \Big[\int_0^1 \frac{dq}{B(q)} \;\; ((1-q)^2 - (1-q)) \Big].$$

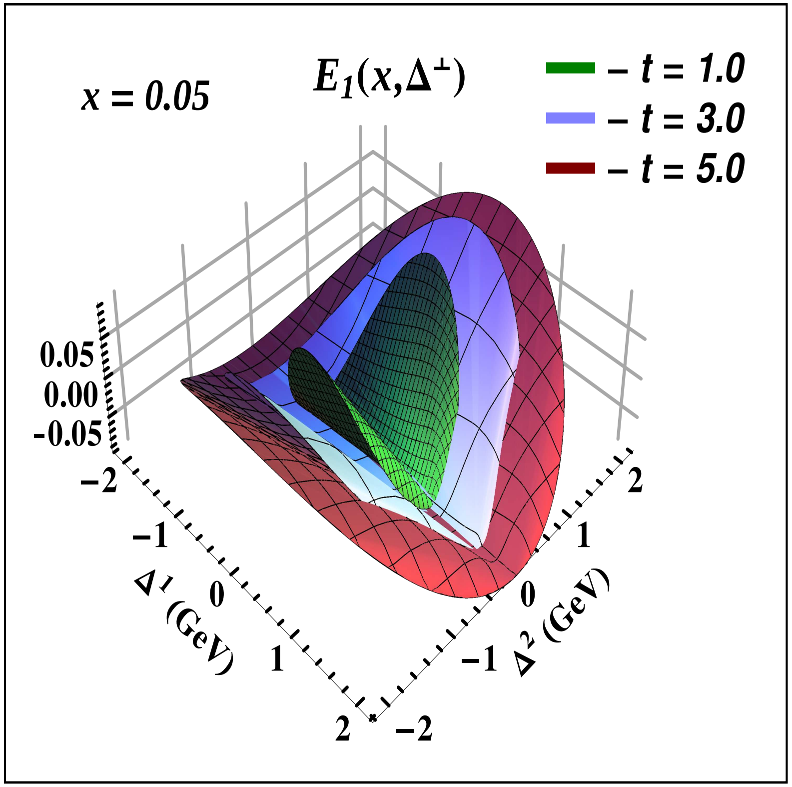
where

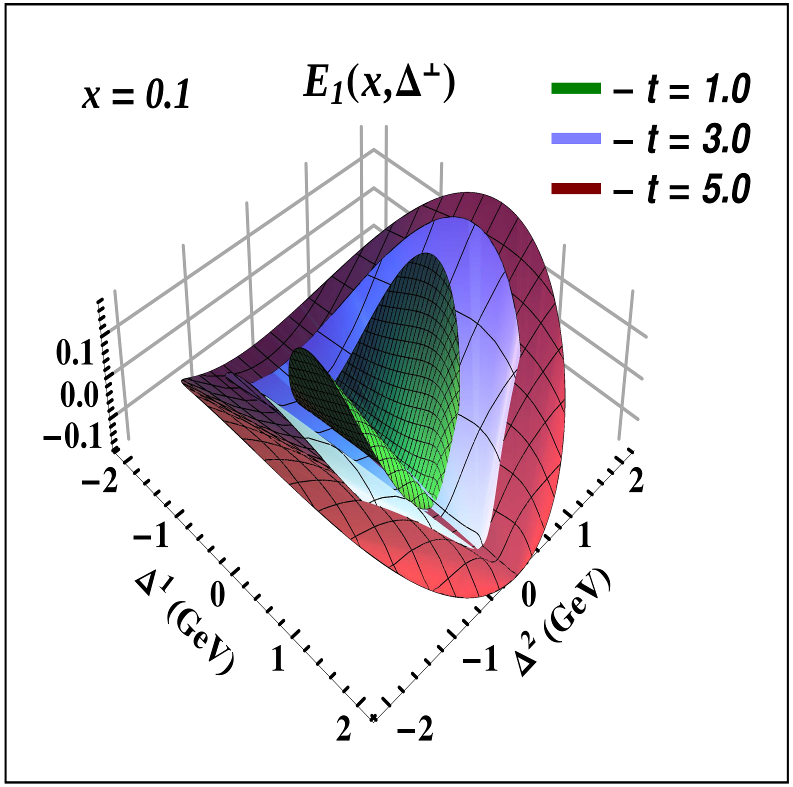
$$B(q) = m^2 \Big(1 - x(1-x) \Big) + q(1-q)(1-x)^2 (\Delta^\perp)^2.$$

Expected quadrupole structure : as the photon is a spin one particle, in order to flip its helicity, the overlapping light-front wave functions should have a difference of orbital angular momentum of two units

Photon GPD $\tilde{F}^q$ when the helicity is flipped is zero in this kinematics

Photon GPDs with helicity flip





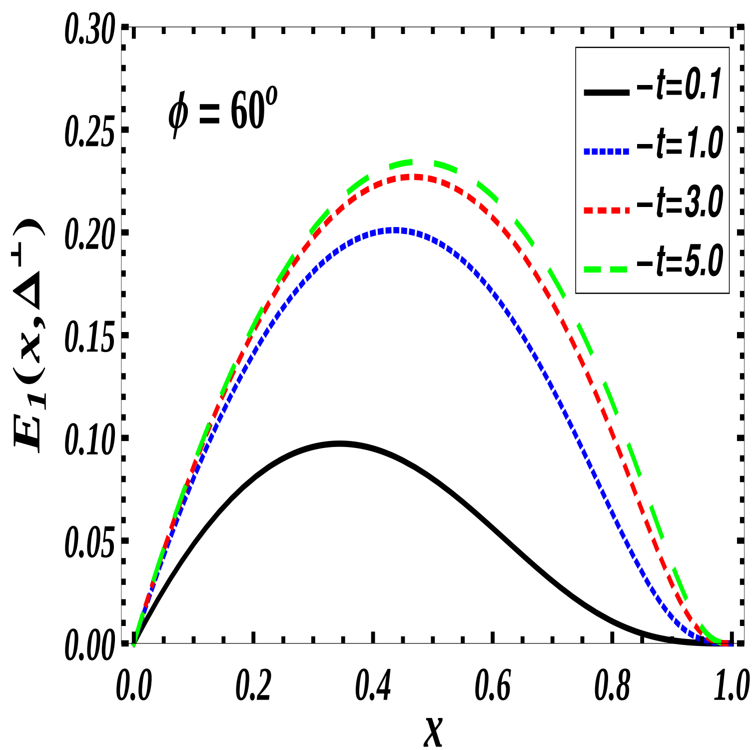
Plots of the helicity-flip GPD $E_1(x, \Delta^\perp)$ vs Δ^1, Δ^2 for different values of x. We have taken $\zeta = 0$

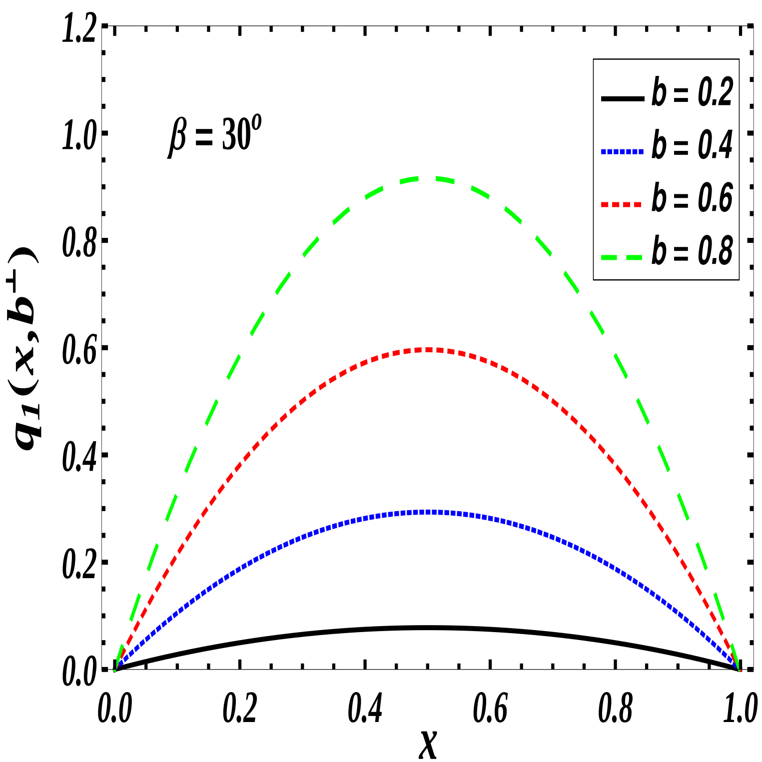
Shown only the quark contribution, quark mass $m = 3.3\mathrm{MeV}$

$E_1(x, \Delta^\perp)$ is zero when $\Delta^1 = \Delta^2$. The curvature is sharper as $\mid t \mid$ decreases

AM, S. Nair, V. Ojha (PLB, 2013)

Photon GPDs with helicity flip



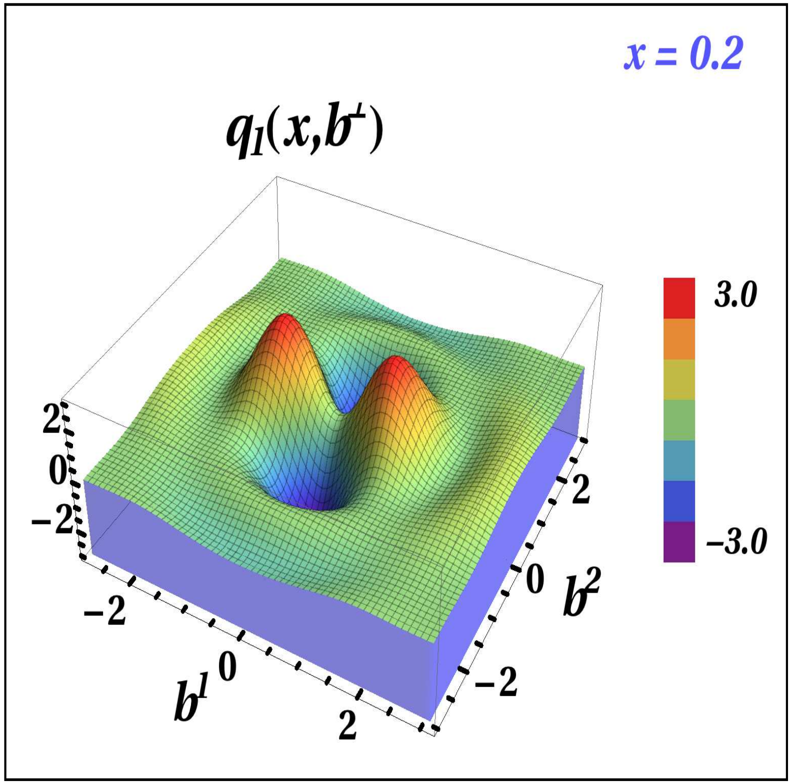


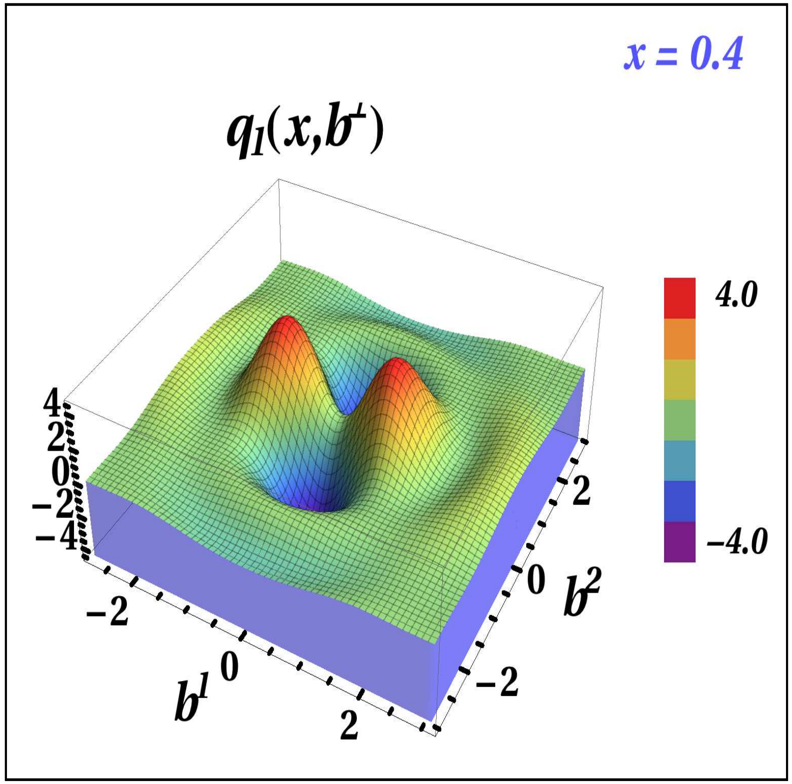
GPD is zero both at $x = 0$ and $x = 1$ and $\Delta^\perp = 0$

In order to flip the helicity one needs non-zero OAM in the two-particle LFWFs and the OAM is zero when there is no momentum transfer in the transverse direction. At $x = 0$ and $x = 1$ all momenta are carried by either the quark or the antiquark in the photon. Then there is no relative motion and no OAM contribution

AM, S. Nair, V. Ojha (PLB, 2013)

Photon GPDs with helicity flip





Quadrupole structure

From the analytic expression it can be seen that the helicty-flip GPD gives a distortion in $b^\perp$ space

AM, S. Nair, V. Ojha (PLB, 2013)

Summary and Discussions

•Presented a calculation of the generalized parton distributions of the photon, both polarized and unpolarized, when the momentum transfer in the longitudinal and transverse direction are non-zero; at zeroth order in α_s and leading order in α

•We kept the mass terms at the vertex. The GPDs when the helicity of the photon is not flipped are logarithmically dependent on the scale; we calculated at leading logarithmic order

• GPDs probe the two-particle $q\bar{q}$ structure of the photon

• Taking a Fourier transform (FT) with respect to $\Delta_\perp$ we obtain impact parameter dependent parton distribution of the photon

•The GPD involving the photon helicity flip has a quadrupole structure and gets contribution from the non-zero orbital angular momentum of the photon light-front wave function

•It is to be noted that a complete understanding of the photon GPDs beyond leading logs would require also the non-pointlike hadronic contributions which will be model dependent. However, the GPDs of the photon calculated here may act as interesting tools to understand the partonic substructure of the photon. Accessing them in experiment is a challenge