

# Hard Exclusive Photoproduction of $\overline{D}^0$ -Mesons within the Generalized Parton Picture

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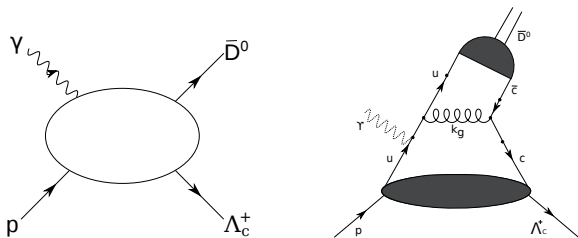
May 22, 2013

# Outline

- 1 Introduction
- 2 Reaction Mechanism of  $\gamma p \rightarrow \Lambda_c^+ \overline{D}^0$
- 3 Modeling and Results
- 4 Summary

# Motivation

- investigate the exclusive photoproduction  $\gamma p \rightarrow \Lambda_c^+ \bar{D}^0$  using a *handbag mechanism*



- provides information of the intrinsic charm of the proton
- exclusive charm photoproduction is a nearly unexplored field, theoretically as well as experimentally
- a further application of  $p \rightarrow \Lambda_c^+$  flavor changing GPDs, which were introduced by  
[\[Goritschnig A.T., Kroll P. and Schweiger W., Eur. Phys. J. A42 \(2009\)\]](#)

# The General Setting

## Our Framework

- pQCD:
  - describes (short distance) dynamics of quarks and gluons in a hadron (pQCD)
  - limited to the high energy regime  $\rightarrow$  non-perturbative methods for the low energy regime are needed
- use hadronic matrix elements to describe the (long distance) binding effects (**Generalized Parton Distributions (GPDs)** and **Distribution Amplitudes (DAs)**)

$\rightarrow$  **Factorization**

# A little bit of Kinematics. . . Symmetric CMS

## incoming momenta

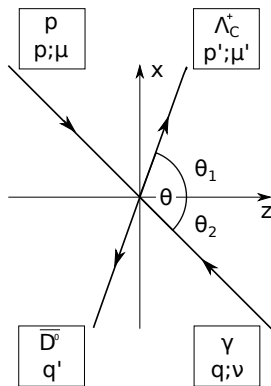
$$p = \left[ (1 + \xi)\bar{p}^+, p^-, -\frac{\Delta_{\perp}}{2} \right] \quad (p^2 = m^2)$$

$$q = \left[ q^+, q^-, +\frac{\Delta_{\perp}}{2} \right] \quad (q^2 = 0)$$

## outgoing momenta

$$p' = \left[ (1 - \xi)\bar{p}^+, p'^-, +\frac{\Delta_{\perp}}{2} \right] \quad (p'^2 = M^2)$$

$$q' = \left[ q'^+, q'^-, -\frac{\Delta_{\perp}}{2} \right] \quad (q'^2 = M_D^2)$$



- $\bar{p} := \frac{1}{2}(p + p')$
- $\xi := \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2\bar{p}^+}$
- $\Delta := p' - p = q - q'$

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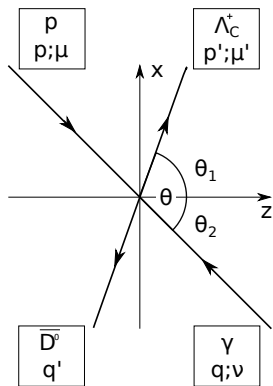
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## outgoing momenta

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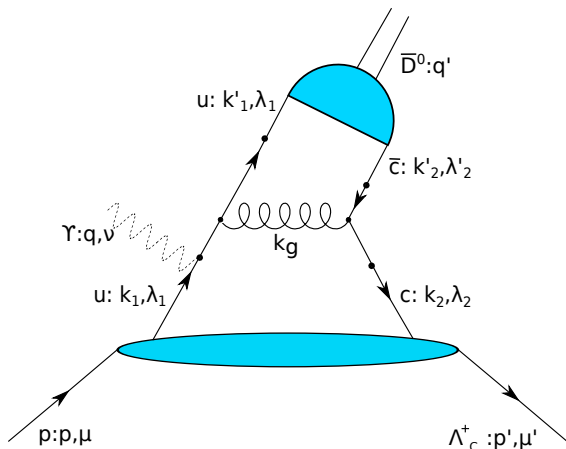


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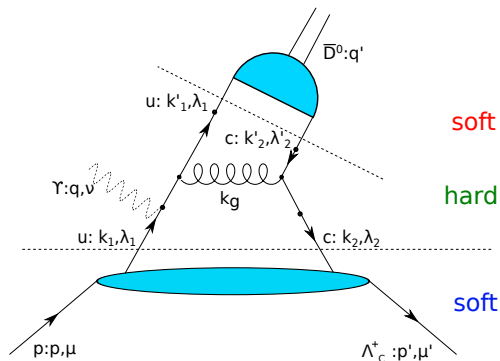
# Handbag Picture of the Process

$$s \geq (M + M_D)^2 \approx 17.23 \text{ GeV}^2$$

$$E_{\text{lab}}^\gamma \approx 8.7 \text{ GeV}$$



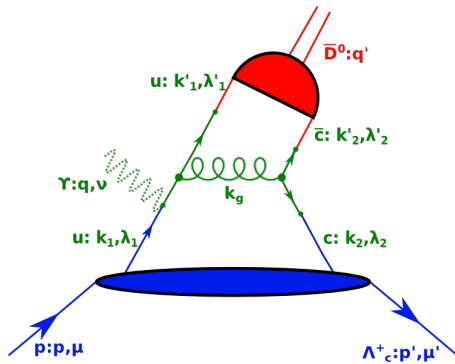
(possible) intrinsic charm content of the proton neglected



- **hard-scattering process:**  
described with 4 Feynman diagrams within pQCD  
(*natural hard scale*: heavy quark mass  $m_c$ )
- **long distance effects of  $p \rightarrow \Lambda_c^+$  transition:**  
parameterized by flavor changing GPDs
- **meson formation:**  
parameterized by DA



# Process Amplitude



$$\begin{aligned}
 \mathcal{M}_{\mu'0, \mu\nu} &\approx \int d^4 \bar{k} \int d^4 z_1 e^{i\bar{k}z_1} \int d^4 k'_1 \int d^4 z_2 e^{ik'_1 z_2} \\
 &\times \langle \Lambda_c^+ : p', \mu' | T \bar{\Psi}^c \left( -\frac{z_1}{2} \right) \Psi^u \left( +\frac{z_1}{2} \right) | p : p, \mu \rangle \tilde{H}^\nu (\bar{k}, k'_1, q) \\
 &\times \langle \bar{D}^0 : q' | T \bar{\Psi}^u (z_2) \Psi^c (0) | 0 \rangle
 \end{aligned}$$

$$(\bar{k} := \frac{1}{2} (k_1 + k_2))$$

# Factorization of Process Amplitude

## Goal

separate soft hadronic matrix elements from hard parton scattering

**Assumptions** (“Soft Physics Approach”):

- **parton virtualities** are smaller than a typical hadronic scale  $\Lambda \approx 1$  GeV:

$$k_u^{(\prime)2} \lesssim \Lambda^2 \quad \text{and} \quad |k_c^{(\prime)2} - m_c^2| \lesssim \Lambda^2$$

- **intrinsic transverse momenta** are restricted by:

$$\frac{\mathbf{k}_{i\perp}^{(\prime)2}}{x_i(z_i)} \lesssim \Lambda^2$$

- $\Lambda_c^+$ - and  $\bar{D}^0$ -wave function **peaked** at  $x_0 \approx \frac{m_c}{M}$  and  $z_0 \approx \frac{m_c}{M_D}$   
( $x_0 \rightarrow 1$  and  $z_0 \rightarrow 1$  for  $m_c \rightarrow \infty$ )

# Factorization of Process Amplitude

## Consequence:

- $\mathbf{k}_{\perp(1,2)}$  and  $\mathbf{k}'_{\perp(1,2)}$  negligible as compared to  $k_{(1,2)}^+$  and  $k'_{(1,2)}^-$

$\Rightarrow$  partons are **on-mass-shell** and **collinear** to their **parent** hadron, i.e.

$$k_1 = x_1 p \quad (k_1^2 = 0) \quad k'_1 = (1-z)q' \quad (k_1'^2 = m_c^2)$$

$$k_2 = x_2 p' \quad (k_2^2 = 0) \quad k'_2 = zq' \quad (k_2'^2 = m_c^2)$$

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## Furthermore:

- $\tilde{H}^\nu(\bar{k}, k'_1, q) \approx \tilde{H}^\nu(\bar{k}^+, k_1'^-, q)$

$\Rightarrow \bar{k}^-, \bar{\mathbf{k}}_\perp, k_1'^+$  and  $\mathbf{k}'_{1\perp}$  integrations can be done analytically:

$$\int d^4 \bar{k} \rightarrow \int d\bar{k}^+ \quad \text{and} \quad \int d^4 k'_1 \rightarrow \int dk_1'^-$$

$\Rightarrow$  time ordering of field operators can be dropped

[Diehl M. and Gousset T., Phys. Lett. B248 (1998)]

# Simplified Process Amplitude

$$\begin{aligned}
 \mathcal{M}_{\mu'0,\mu\nu} &\approx \int d\bar{k}^+ \int dk_1'^- \tilde{H}^\nu(\bar{k}^+, k_1'^-, q) \\
 &\times \int dz_1^- e^{i\bar{k}^+ z_1^-} \langle \Lambda_c^+ : p', \mu' | \bar{\Psi}^c \left( -\frac{z_1^-}{2} \right) \Psi^u \left( \frac{z_1^-}{2} \right) | p : p, \mu \rangle \\
 &\times \int dz_2^+ e^{ik_1'^- z_2^+} \langle \bar{D}^0 : q' | \bar{\Psi}^u(z_2^+) \Psi^c(0) | 0 \rangle
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 \end{aligned}$$

$$\bar{x} = \frac{\bar{k}^+}{\bar{p}^+}$$

# Simplified Process Amplitude

$$\begin{aligned}
 \mathcal{M}_{\mu'0,\mu\nu} &\approx \int d\bar{x} \bar{p}^+ \int dk_1'^- \tilde{H}^\nu(\bar{x}, k_1'^-, q) \\
 &\times \int dz_1^- e^{i\bar{x}\bar{p}^+ z_1^-} \langle \Lambda_c^+ : p', \mu' | \bar{\Psi}^c \left(-\frac{z_1^-}{2}\right) \Psi^\mu \left(\frac{z_1^-}{2}\right) | p : p, \mu \rangle \\
 &\times \int dz_2^+ e^{ik_1'^- z_2^+} \langle \bar{D}^0 : q' | \bar{\Psi}^u(z_2^+) \Psi^c(0) | 0 \rangle
 \end{aligned}$$

# Hadronic Matrix Elements: $p \rightarrow \Lambda_c^+$ Transition

## Generalized Parton Distributions (light-cone gauge $A^+ = 0$ )

$\langle \Lambda_c^+ | \bar{\Psi}^c \Psi^u | p \rangle$  written in terms of:

[Goritschnig A.T., Kroll P. and Schweiger W., Eur. Phys. J. A42 (2009)]

$$\bar{p}^+ \int \frac{dz_1^-}{2\pi} e^{i\bar{x}\bar{p}^+ z_1^-} \times \dots$$

- $\dots \langle \Lambda_c^+ | \bar{\Psi}^c \gamma^+ \Psi^u | p \rangle =$

$$\bar{u}(p', \mu') \left[ H^{cu}(\bar{x}, \xi, t) \gamma^+ + E^{cu}(\bar{x}, \xi, t) \frac{i\sigma^{+\nu} \Delta_\nu}{M+m} \right] u(p, \mu)$$

- $\dots \langle \Lambda_c^+ | \bar{\Psi}^c \gamma^+ \gamma_5 \Psi^u | p \rangle =$

$$\bar{u}(p', \mu') \left[ \tilde{H}^{cu}(\bar{x}, \xi, t) \gamma^+ \gamma_5 + \tilde{E}^{cu}(\bar{x}, \xi, t) \frac{\Delta^+}{M+m} \gamma_5 \right] u(p, \mu)$$

- $\dots \langle \Lambda_c^+ | \bar{\Psi}^c i\sigma^{+j} \Psi^u | p \rangle =$

$$(\sigma^{\pm j} = i\gamma^{\pm} \gamma^j)$$

$$\bar{u}(p', \mu') \left[ H_T^{cu}(\bar{x}, \xi, t) i\sigma^{+j} + \tilde{H}_T^{cu}(\bar{x}, \xi, t) \frac{\bar{p}^+ \Delta^j - \Delta^+ \bar{p}^j}{Mm} \right. \\ \left. + E_T^{cu}(\bar{x}, \xi, t) \frac{\gamma^+ \Delta^j - \Delta^+ \gamma^j}{M+m} + \tilde{E}_T^{cu}(\bar{x}, \xi, t) \frac{\gamma^+ \bar{p}^j - \bar{p}^+ \gamma^j}{(M+m)/2} \right] u(p, \mu)$$



# Hadronic Matrix Elements: $p \rightarrow \Lambda_c^+$ Transition

## Generalized Parton Distributions (light-cone gauge $A^+ = 0$ )

valence Fock states of ground state hadrons are dominated by parton configurations with zero orbital angular momentum  $\rightarrow$  3 GPDs

$$\bar{p}^+ \int \frac{dz_1^-}{2\pi} e^{i\bar{x}\bar{p}^+ z_1^-} \times \dots$$

- $\dots \langle \Lambda_c^+ | \bar{\Psi}^c \gamma^+ \Psi^u | p \rangle =$

$$H^{cu}(\bar{x}, \xi, t) \bar{u}(p', \mu') \gamma^+ u(p, \mu)$$

- $\dots \langle \Lambda_c^+ | \bar{\Psi}^c \gamma^+ \gamma_5 \Psi^u | p \rangle =$

$$\tilde{H}^{cu}(\bar{x}, \xi, t) \bar{u}(p', \mu') \gamma^+ \gamma_5 u(p, \mu)$$

- $\dots \langle \Lambda_c^+ | \bar{\Psi}^c i\sigma^{+j} \Psi^u | p \rangle =$

$$H_T^{cu}(\bar{x}, \xi, t) \bar{u}(p', \mu') i\sigma^{+j} u(p, \mu)$$

# Hadronic Matrix Elements: $p \rightarrow \Lambda_c^+$ Transition

## Transition Form Factors

$u \rightarrow c$  transition GPDs have a **peak at  $\bar{x} = x_0 = \frac{m_c}{M}$**   $\Rightarrow$  in convolution integral they strongly weight  $\bar{x}$  regions close to  $x_0$

*peaking approximation*: in hard scattering amplitude  $\bar{x} \rightarrow x_0$

applying p.a. the hard-scattering amplitudes can be taken out of the  $\bar{x}$  integral

### Transition FFs

$$\text{FF}_i(\xi, t) = \int_{\xi}^1 \frac{d\bar{x}}{\sqrt{\bar{x}^2 - \xi^2}} \text{GPD}_i(\bar{x}, \xi, t)$$

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### Transition FFs

$$\text{FF}_i(\xi, t) = \int_{\xi}^1 \frac{d\bar{x}}{\sqrt{\bar{x}^2 - \xi^2}} \text{GPD}_i(\bar{x}, \xi, t) \quad \bar{x} \geq \xi$$

( $\bar{x} \geq \xi$ ... kinematical requirement to produce  $c\bar{c}$ -pair)

# Hadronic Matrix Elements: $p \rightarrow \Lambda_c^+$ Transition

## Transition Form Factors

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### Transition FFs

$$\text{FF}_i(\xi, t) = \int_{\xi}^1 \frac{d\bar{x}}{\sqrt{\bar{x}^2 - \xi^2}} \text{GPD}_i(\bar{x}, \xi, t)$$

<b>GPD</b>	$\int_{\xi}^1 \frac{d\bar{x}}{\sqrt{\bar{x}^2 - \xi^2}} + \text{p.a.}$	<b>FF</b>
$H^{cu}$	$\longrightarrow$	$R_V$
$\tilde{H}^{cu}$		$R_A$
$H_T^{cu}$		$S_T$

form factor decomposition of the integrated  $p \rightarrow \Lambda_c^+$  transition matrix element

# Hadronic Matrix Elements: Meson

$$\int dk_1'^- \int \frac{dz_2^+}{(2\pi)} e^{iz_2^+ k_1'^-} \langle \bar{D}^0 : q' | \bar{\Psi}^u(z_2^+) \Psi^c(0) | 0 \rangle \approx$$

$$\frac{1}{2\sqrt{6}} f_D \left( \frac{\not{q}' - M_D}{\sqrt{2}} \right) \gamma^5 \int_0^1 dz \phi_D(z)$$

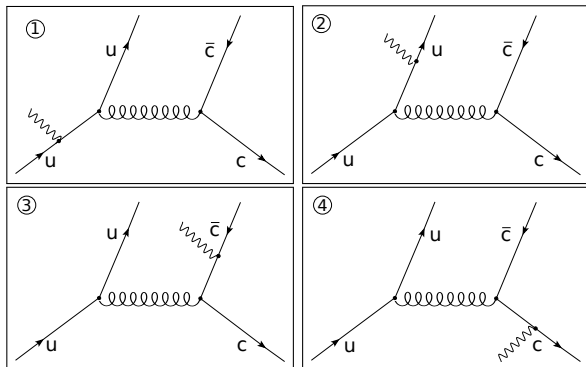
[Huang H. W. and Kroll P., Eur. Phys. C17 (2000)]

- $f_D$  ... decay constant
- massive pseudoscalar meson spin wave function  
[Beneke M. and Feldmann T., Nucl. Phys. B. 596 (2001)]
- $\phi_D$  ... leading twist meson distribution amplitude

$$\phi_D(z) = \int d^2\mathbf{k}_\perp \Psi_D(z, \mathbf{k}_\perp) \quad \int dz \phi_D(z) = 1$$

# Hard Process

- hard-scattering amplitude = sum of 4 Feynman diagrams



- for fwd. scattering gluon propagators (and quark propagators) are still highly virtual (i.e. their virtuality is greater than  $1 \text{ GeV}^2$ )

# Valence LCWF for Proton

- we choose  
[Bolz J. and Kroll P., Z. Phys. A 356 (1996)]

$$\begin{aligned}\Psi_P(x_1, x_2, x_3; \mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}, \mathbf{k}_{\perp 3}) \\ = N_p (1 + 3x_1) \exp \left[ -a_p^2 \sum \frac{\mathbf{k}_{\perp i}^2}{x_i} \right]\end{aligned}$$

$N_p$  and  $a_p$  determined by means of fits to

- $F_1^N(Q)$
- $J/\Psi \rightarrow p\bar{p}$  decay width
- $u_v(x)$ ,  $d_v(x)$

$$\left. \begin{aligned}N_p &= 160.93 \text{ GeV}^{-2} \\ a_p &= 0.75 \text{ GeV}^{-1}\end{aligned} \right\} \Rightarrow \begin{aligned}P_p &= 0.17 \\ \sqrt{\langle \mathbf{k}_{\perp}^2 \rangle} &= 411 \text{ MeV}\end{aligned}$$

# Valence LCWF for $\Lambda_c^+$

- use harmonic-oscillator type LCWF

$$\Psi_\Lambda(x_1, x_2, x_3; \mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}, \mathbf{k}_{\perp 3}) = N_\Lambda \exp[-f(x_1)] \exp\left[-a_\Lambda^2 \sum \frac{\mathbf{k}_{\perp i}^2}{x_i}\right]$$

for the mass exponential  $f(x_1)$  we use ideas of:

- *HQET* ...  $f_{KK}(x_1) = a_\Lambda^2 M^2 \frac{(x_1 - x_0)^2}{x_1(1-x_1)}$   
[Körner J.G. and Kroll P., Phys. Lett. B293 (1992)]
- *QCD sum rules* ...  $f_{BB}(x_1) = a_\Lambda M(1-x_1)$   
[Ball P., Braun V. M. and Gardi E., Phys. Lett. B665 (2008)]

$N_\Lambda$  and  $a_\Lambda$

chosen such that

$P_\Lambda$  and  $\sqrt{\langle \mathbf{k}_\perp^2 \rangle_c}$  are

$$\left. \begin{aligned} N_\Lambda &= 2117 \text{ GeV}^{-2}(\text{BB})/3477 \text{ GeV}^{-2}(\text{KK}) \\ a_\Lambda &= 0.75 \text{ GeV}^{-1} \end{aligned} \right\} \Rightarrow \begin{aligned} P_\Lambda &= 0.9 \\ \sqrt{\langle \mathbf{k}_\perp^2 \rangle_c} &\approx 450 \text{ MeV} \end{aligned}$$



# FFs at $s = 20 \text{ GeV}^2$ versus the CMS angle $\theta$

$\bar{x} \geq \xi$  DGLAP region  $\Rightarrow$  allows for overlap representation

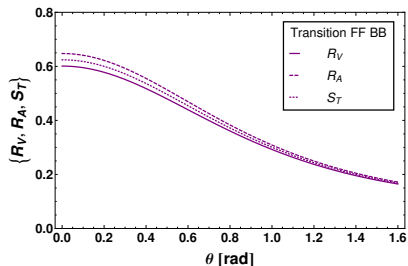
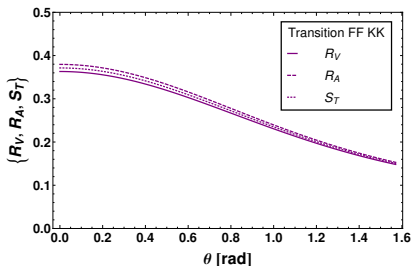
[Diehl M., Feldmann T., Jakob R. and Kroll P., Nucl.Phys. B596 (2001)]

$$\text{GPDs} \xrightarrow{\int d\bar{x}} \text{FF}$$

$$- R_V = S_T - \Delta R$$

$$- R_A = S_T + \Delta R$$

$\Delta R$  ... helicity of  $c$ -quark opposite to  $\Lambda_c^+$  (controlled by  $\rho$ )



small overlap of the  $\rho$  term with proton LCWF  $\Rightarrow R_V, R_A$  and  $S_T$  are almost identical

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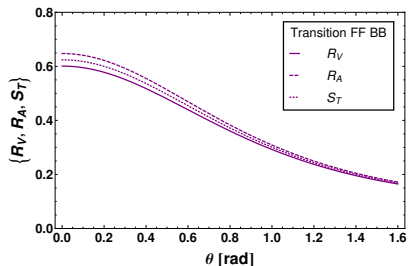
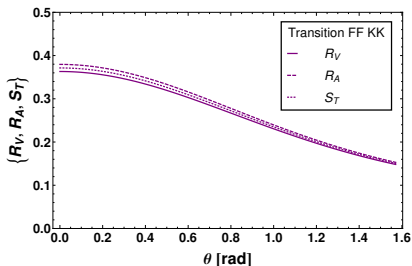
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# Meson DA (valence quark model)

LCWF for the  $\bar{D}^0$ :

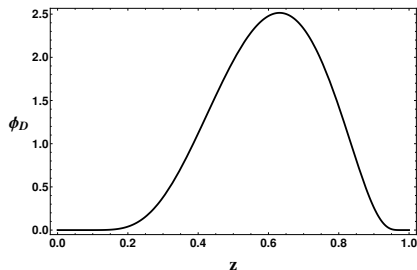
[Körner, J.G. and Kroll, P., Phys. Lett. B293 (1992)]

$$\Psi_D(z, \mathbf{k}_\perp) = N_D \exp \left[ -a_D^2 M_D \frac{(z - z_0)^2}{z(1-z)} \right] \exp \left[ -a_D^2 \frac{\mathbf{k}_\perp^2}{z(1-z)} \right]$$

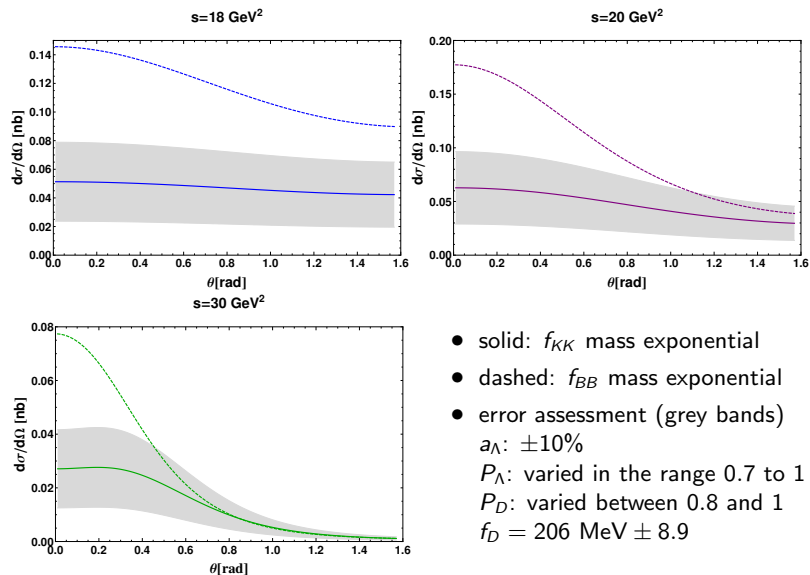
$N_D$  and  $a_D$  chosen such that  $P_D$  and  $f_D$  are

$$\left. \begin{aligned} N_D &= 55.18 \text{ GeV}^{-2} \\ a_D &= 0.86 \text{ GeV}^{-1} \end{aligned} \right\} \Rightarrow \begin{aligned} P_D &= 0.9 \\ f_D &= 206 \text{ MeV} \end{aligned}$$

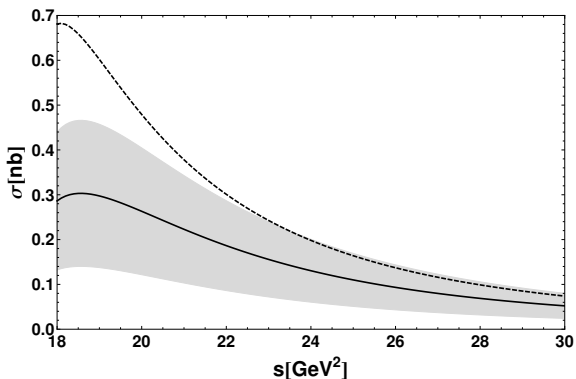
corresponding DA looks like



# Estimation for Differential Cross Section

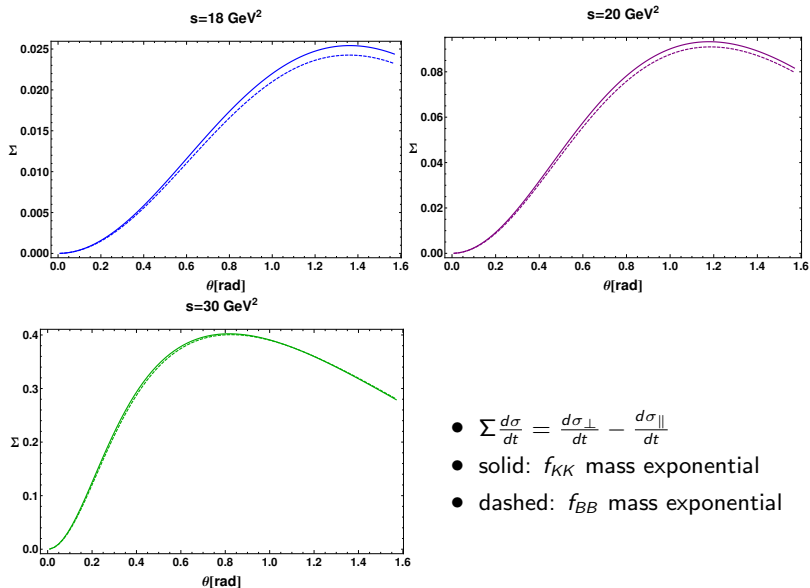


# Estimation for Integrated Cross Section



- solid:  $f_{KK}$                   dashed:  $f_{BB}$
- grey error band
- experimental finding of much larger cross sections could be an indication for a non-negligible charm-quark content of the proton

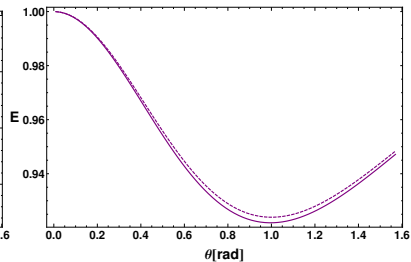
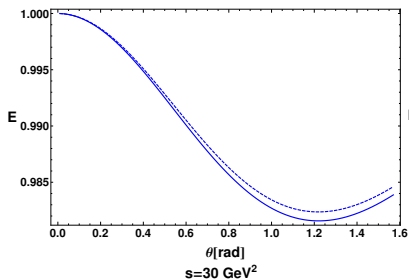
# Estimation for $\Sigma$



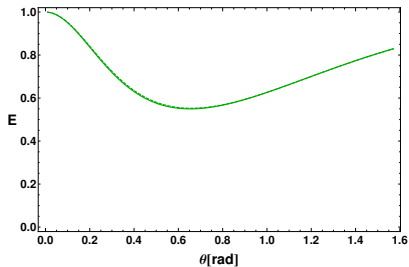
# Estimation for $E$

$s=18 \text{ GeV}^2$

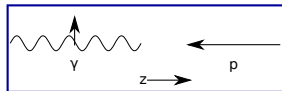
$s=20 \text{ GeV}^2$



$s=30 \text{ GeV}^2$



- solid:  $f_{KK}$  mass exponential
- dashed:  $f_{BB}$  mass exponential



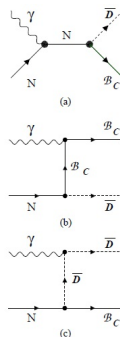
# Exclusive Charm Photoproduction in the Literature

only references found

[Rekalo, P.M. and Tomasi-Gustafsson, Phys. Lett. B 500 (2001)]

[Rekalo, P.M. and Tomasi-Gustafsson, Phys. Rev. D 69 (2004)]

- hadronic approach
- $\sigma_{\text{tot}} \approx 10 - 100 \text{ nb}$  at  $E_{\text{lab}}^\gamma = 11 \text{ GeV}$   
depends mainly on coupling  $g_{N\Delta}$   
but:  $g_{N\Delta}$  unknown  
→ rely on SU(4)-symmetry
- different beam asymmetry behavior



**Experimental data needed to decipher between different dynamical models!**



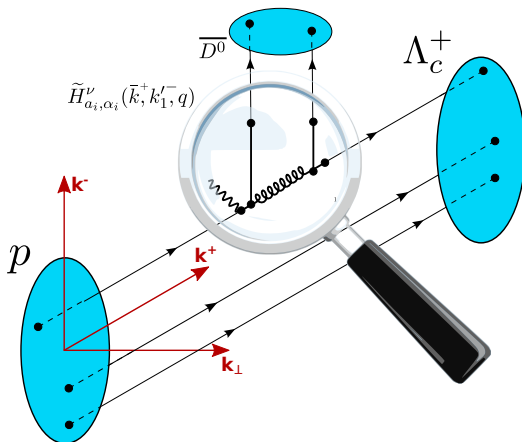
# Summary

- presented a “*QCD inspired*” **model** for  $\gamma p \rightarrow \Lambda_c^+ \overline{D^0}$
- under plausible physical restrictions on parton virtualities and intrinsic transverse momenta, one can factorize this process into a hard partonic subprocess and soft hadronic matrix elements
- partonic subprocess treated by means of pQCD
- hadronic matrix elements parameterized by 8 GPDs and a meson DA
  - overlap representation for the  $p \rightarrow \Lambda_c^+$  transition GPDs: with s-wave LCWFs  $\rightarrow$  3 GPDs contribute
  - meson DA from a simple Gaussian model for the LCWF of the  $D$ -meson valence Fock state
- estimation for (unpolarized) differential and integrated cross sections
- estimation for non-vanishing spin observables  $\Sigma$  and  $E$
- “... seems possible” @ JLAB12GeV (talk given by E. Chudakov)  
[<http://hallaweb.jlab.org/12GeV/SuperBigBite/meetings/01/talk/chudakov-jpsi.pdf>]

**Thank you very much for your attention.**

# A Picture is worth a Thousand Words

$$\int dk_1^- \int \frac{dz_2^+}{(2\pi)} e^{ik_1^- z_2^+} \langle \bar{D}^0 : q' | \bar{\Psi}_{a_3, \alpha_4}^u(\bar{z}_2^+) \Psi_{a_4, \alpha_4}^c(0) | 0 \rangle$$



$$\int d\bar{x} \bar{p}^+ \int \frac{dz_1^-}{(2\pi)} e^{i\bar{x}\bar{p}^+ z_1^-} \langle \Lambda_c^+ : p', \mu' | \bar{\Psi}_{a_1, \alpha_1}^c\left(-\frac{\bar{z}_1^-}{2}\right) \Psi_{a_2, \alpha_2}^u\left(\frac{\bar{z}_1^-}{2}\right) | p : p, \mu \rangle$$

# Hadronic Matrix Elements: $p \rightarrow \Lambda_c^+$ Transition

## Outline of the Calculation

[Goritschnig A.T., Kroll P. and Schweiger W., Eur. Phys. J. A42 (2009)]

$$\bar{p}^+ \int \frac{dz_1^-}{(2\pi)} e^{i\bar{x}\bar{p}^+z^-} \langle \Lambda_c^+ : p', \mu' | \bar{\Psi}^c \left( -\frac{z_1^-}{2} \right) \Psi^u \left( \frac{z_1^-}{2} \right) | p : p, \mu \rangle$$

- algebraic manipulations of **quark field operator product** (projector techniques, inserting spinor products which are  $\sim 1$  and using helicity projectors)  
 $\Rightarrow$  different **Dirac structures** come into the game

$\bar{\Psi}^c \Psi^u$  in terms of ("leading twist" contributions)

$$\bar{\Psi}^c \gamma^+ \Psi^u, \quad \bar{\Psi}^c \gamma^+ \gamma_5 \Psi^u \quad \text{and} \quad \bar{\Psi}^c i\sigma^{+j} \Psi^u \quad (\sigma^{\pm j} = i\gamma^{\pm} \gamma^j)$$

- decompose Dirac structures into Lorentz covariants:  
 introduce **Generalized Parton Distributions (GPDs)**

# Valence-Quark-Model for Proton and $\Lambda_c^+$

- valence Fock state of the proton

[Sotiropoulos M. G. and Sterman G. F., Nucl. Phys. B425, 489 (1994)]

$$|p, +\rangle = \int [dx]_3 [d^2\mathbf{k}_\perp]_3 \left\{ (\Psi_{123} \mathcal{M}_{+-+}^u + \Psi_{213} \mathcal{M}_{-++}^u) - (\Psi_{132} + \Psi_{231}) \mathcal{M}_{++-}^u \right\}$$

- valence Fock state of the  $\Lambda_c^+$

[Goritschnig A.T., Kroll P. and Schweiger W., Eur. Phys. J. A42, 43 (2009)]

$$|\Lambda_c^+, +\rangle = \int [dx]_3 [d^2\mathbf{k}_\perp]_3 \left\{ (\mathcal{M}_{++-}^c - \mathcal{M}_{+-+}^c) + \rho(x_2 - x_3) \mathcal{M}_{-++}^c \right\} \Psi_\Lambda(x_i, \mathbf{k}_{\perp i})$$

- 3-quark states given by

$$\mathcal{M}_{\lambda_1 \lambda_2 \lambda_3}^q := \frac{1}{\sqrt{x_1 x_2 x_3}} |q : x_1, \mathbf{k}_{\perp 1}, \lambda_1\rangle |u : x_2, \mathbf{k}_{\perp 2}, \lambda_2\rangle |d : x_3, \mathbf{k}_{\perp 3}, \lambda_3\rangle$$

# Final Expression for Process Amplitude

$$\begin{aligned}
 \mathcal{M}_{\frac{1}{2}0, \frac{1}{2}\nu} &= \frac{1}{4\sqrt{6}} f_D \sqrt{1 - \xi^2} \\
 &\quad \left[ (R_V + R_A) \int dz \phi_D(z) H_{\frac{1}{2}, \frac{1}{2}}^\nu + (R_V - R_A) \int dz \phi_D(z) H_{-\frac{1}{2}, -\frac{1}{2}}^\nu \right] \\
 \mathcal{M}_{-\frac{1}{2}0, -\frac{1}{2}\nu} &= \frac{1}{4\sqrt{6}} f_D \sqrt{1 - \xi^2} \\
 &\quad \left[ (R_V - R_A) \int dz \phi_D(z) H_{\frac{1}{2}, \frac{1}{2}}^\nu + (R_V + R_A) \int dz \phi_D(z) H_{-\frac{1}{2}, -\frac{1}{2}}^\nu \right] \\
 \mathcal{M}_{\frac{1}{2}0, -\frac{1}{2}\nu} &= \frac{1}{2\sqrt{6}} f_D \sqrt{1 - \xi^2} S_T \int dz \phi_D(z) H_{\frac{1}{2}, -\frac{1}{2}}^\nu \\
 \mathcal{M}_{-\frac{1}{2}0, \frac{1}{2}\nu} &= \frac{1}{2\sqrt{6}} f_D \sqrt{1 - \xi^2} S_T \int dz \phi_D(z) H_{-\frac{1}{2}, \frac{1}{2}}^\nu
 \end{aligned}$$

# Spin Observables

- for spin observables we need “usual” helicity amplitude  $\phi_{\mu'0,\mu\nu}$ :  
 $\phi_{\mu'0,\mu\nu}$  related to LC-helicity amplitude  $\mathcal{M}_{\mu'0,\mu\nu}$  by a unitary transformation
- non-vanishing **single-polarization observable** (photon asymmetry) :

$$\Sigma \frac{d\sigma}{dt} = \frac{d\sigma_{\perp}}{dt} - \frac{d\sigma_{\parallel}}{dt} = \frac{1}{16\pi(s-m^2)^2} \text{Re}(S_1^* S_2 - ND^*)$$

- non-vanishing **spin correlation**:

$$E \frac{d\sigma}{dt} = \frac{1}{32\pi(s-m^2)^2} (|N|^2 - |S_1|^2 + |S_2|^2 - |D|^2)$$

$$N = \phi_{-\frac{1}{2}0, \frac{1}{2}1} \quad S_1 = \phi_{-\frac{1}{2}0, -\frac{1}{2}1}$$

$$D = \phi_{\frac{1}{2}0, -\frac{1}{2}1} \quad S_2 = \phi_{\frac{1}{2}0, \frac{1}{2}1}$$

[Barker I.S, Donnachie A. and Storrow, J.K., Nucl. Phys. B95 (1975)]

## $p \rightarrow \Lambda_c^+$ transition: Quark Field Operators

write  $\Psi^q = 1\Psi^q = (\mathcal{P}_+ + \mathcal{P}_-) \Psi^q$ ,  $q = u, c$ :

$$\Psi^q = \frac{1}{2} (\gamma^- \gamma^+ + \gamma^+ \gamma^-) \Psi^q$$

### Light-Quark Field $\Psi^u$

$$k_1 = \left[ k_1^+, \frac{x_1^2 \Delta_\perp^2 / 4}{2k_1^+}, -x_1 \frac{\Delta_\perp}{2} \right] \xrightarrow{\text{transv. b.}} \tilde{k}_1 = [k_1^+, 0, \mathbf{0}]$$

$$(k_1^+ = x_1 p^+)$$

eliminate  $\gamma^-$  using the energy projector

$$\begin{aligned} \sum_{\lambda_1} u(\tilde{k}_1, \lambda_1) \bar{u}(\tilde{k}_1, \lambda_1) &= \tilde{k}_1^+ \gamma^- + \tilde{k}_1^- \gamma^+ - \tilde{\mathbf{k}}_1 \gamma_\perp \\ &= k_1^+ \gamma^- \end{aligned}$$

$p \rightarrow \Lambda_c^+$  transition: Quark Field Operators  
**Light-Quark Field  $\Psi^u$**

$$\Psi^u = \frac{1}{2k_1^+} \sum_{\lambda_1} \left[ u(\tilde{k}_1, \lambda_1) (\bar{u}(\tilde{k}_1, \lambda_1) \gamma^+ \Psi^u) \right. \\ \left. + \gamma^+ u(\tilde{k}_1, \lambda_1) (\bar{u}(\tilde{k}_1, \lambda_1) \Psi^u) \right]$$



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$p \rightarrow \Lambda_c^+$  transition: Quark Field Operators

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go back to the CMS

$$\Psi^u(z_1/2) = \frac{1}{2k_1^+} \sum_{\lambda_1} u(k_1, \lambda_1) (\bar{u}(k_1, \lambda_1) \gamma^+ \Psi^u(z_1/2))$$

$p \rightarrow \Lambda_c^+$  transition: Quark Field Operators

write  $\Psi^q = 1\Psi^q = (\mathcal{P}_+ + \mathcal{P}_-) \Psi^q$ ,  $q = u, c$ :

$$\Psi^q = \frac{1}{2} (\gamma^- \gamma^+ + \gamma^+ \gamma^-) \Psi^q$$

**Heavy-Quark Field  $\Psi^c$**

$$k_2 = \left[ k_2^+, \frac{m_c^2 + x_2^2 \Delta_\perp^2 / 4}{2k_2^+}, x_2 \frac{\Delta_\perp}{2} \right] \xrightarrow{\text{transv. b.}} \hat{k}_2 = \left[ k_2^+, \frac{m_c^2}{2k_2^+}, \mathbf{0} \right]$$

$$(k_2^+ = x_2 p'^+)$$

eliminate  $\gamma^-$  using the energy projector

$$\begin{aligned} \sum_{\lambda_2} u(\hat{k}_2, \lambda_2) \bar{u}(\hat{k}_2, \lambda_2) &= \hat{k}_2^+ \gamma^- + \hat{k}_1^- \gamma^+ - \hat{\mathbf{k}}_2 \gamma_\perp + m_c \\ &= k_2^+ \gamma^- + \frac{m_c^2}{2k_2^+} \gamma^+ + m_c \end{aligned}$$

# $p \rightarrow \Lambda_c^+$ transition: Quark Field Operators

## Heavy-Quark Field $\Psi^c$

$$\begin{aligned} \Psi^c = & \frac{1}{2k_2^+} \sum_{\lambda_2} \left\{ u(\hat{k}_2, \lambda_2) \left( \bar{u}(\hat{k}_2, \lambda_2) \gamma^+ \Psi^c \right) \right. \\ & \left. + \gamma^+ \left[ u(\hat{k}_2, \lambda_2) \left( \bar{u}(\hat{k}_2, \lambda_2) \Psi^c \right) - 2m_c \Psi^c \right] \right\} \end{aligned}$$

# $p \rightarrow \Lambda_c^+$ transition: Quark Field Operators

## Heavy-Quark Field $\Psi^c$

$$\Psi^c = \frac{1}{2k_2^+} \sum_{\lambda_2} \left\{ u(\hat{k}_2, \lambda_2) \left( \bar{u}(\hat{k}_2, \lambda_2) \gamma^+ \Psi^c \right) + \gamma^+ \left[ u(\hat{k}_2, \lambda_2) \left( \bar{u}(\hat{k}_2, \lambda_2) \Psi^c \right) - 2m_c \Psi^c \right] \right\}$$

# $p \rightarrow \Lambda_c^+$ transition: Quark Field Operators

## Heavy-Quark Field $\Psi^c$

$$\Psi^c = \frac{1}{2k_2^+} \sum_{\lambda_2} \left\{ u(\hat{k}_2, \lambda_2) (\bar{u}(\hat{k}_2, \lambda_2) \gamma^+ \Psi^c) + \gamma^+ \left[ u(\hat{k}_2, \lambda_2) (\bar{u}(\hat{k}_2, \lambda_2) \Psi^c) - 2m_c \Psi^c \right] \right\}$$

go back to the CMS

$$\Psi^c(-z_1/2) = \frac{1}{2k_2^+} \sum_{\lambda_2} u(k_2, \lambda_2) (\bar{u}(k_2, \lambda_2) \gamma^+ \Psi^c(-z_1/2))$$

## $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

we have to deal with  $\bar{\Psi}_{\alpha_1}^c(-z_1/2) \Psi_{\alpha_2}^u(+z_1/2) =$

$$= \frac{1}{4k_1^+ k_2^+} \sum_{\lambda_1} \left[ \begin{aligned} & \left( \bar{\Psi}^c(-z_1/2) \gamma^+ u(k_2, \lambda_1) \right) \left( \bar{u}(k_1, \lambda_1) \gamma^+ \Psi^u(z_1/2) \right) \bar{u}_{\alpha_1}(k_2, \lambda_1) u_{\alpha_2}(k_1, \lambda_1) \\ & + \left( \bar{\Psi}^c(-z_1/2) \gamma^+ u(k_2, -\lambda_1) \right) \left( \bar{u}(k_1, \lambda_1) \gamma^+ \Psi^u(z_1/2) \right) \bar{u}_{\alpha_1}(k_2, -\lambda_1) u_{\alpha_2}(k_1, \lambda_1) \end{aligned} \right]$$

$$(\lambda_2 = \pm \lambda_1)$$

## $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

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## $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

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$$\frac{\bar{u}(k_2, \lambda_1) \gamma^+ u(k_1, \lambda_1)}{2\sqrt{k_1^+ k_2^+}} = \mathbf{1} \quad \frac{\mathbf{1}}{i(2\lambda_1 i)^j} \frac{\bar{u}(k_2, -\lambda_1) i\sigma^{\pm j} u(k_1, \lambda_1)}{2\sqrt{k_1^+ k_2^+}} = \mathbf{1}$$

$$(\sigma^{\pm j} = i\gamma^{\pm} \gamma^j)$$

## $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

we have to deal with  $\bar{\Psi}_{\alpha_1}^c(-z_1/2) \Psi_{\alpha_2}^u(+z_1/2) =$

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$$\frac{\bar{u}(k_2, \lambda_1) \gamma^+ u(k_1, \lambda_1)}{2\sqrt{k_1^+ k_2^+}} = \mathbf{1} \quad \frac{1}{i(2\lambda_1 i)^j} \frac{\bar{u}(k_2, -\lambda_1) i\sigma^{+j} u(k_1, \lambda_1)}{2\sqrt{k_1^+ k_2^+}} = \mathbf{1}$$

$$u(k_1, \lambda_1) \bar{u}(k_1, \lambda_1) = k_1 \cdot \gamma \frac{1 - 2\lambda_1 \gamma_5}{2} \quad u(k_2, \lambda_1) \bar{u}(k_2, \lambda_1) = \left( k_2 \cdot \gamma + m_c \right) \frac{1 + \gamma_5 \mathbf{S}_1 \cdot \gamma}{2}$$

## $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

$$\begin{aligned} \bar{\Psi}_{\alpha_1}^c(-z_1/2)\Psi_{\alpha_2}^u(z_1/2) = & \\ \frac{1}{2\sqrt{k_1^+k_2^+}} \sum_{\lambda_1} \left\{ \bar{\Psi}^c(-z_1/2)\gamma^+ \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2)\bar{u}_{\alpha_1}(k_2, \lambda_1)u_{\alpha_2}(k_1, \lambda_1) \right. & \\ \left. - i(-2\lambda_1 i)^j \bar{\Psi}^c(-z_1/2)i\sigma^{+j} \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2)\bar{u}_{\alpha_1}(k_2, -\lambda_1)u_{\alpha_2}(k_1, \lambda_1) \right\}. & \end{aligned}$$

define **vector**, **axial-vector** and **tensor** currents

$$\begin{aligned} V^\mu(-z/2, z/2) & := \bar{\Psi}^c(-z/2) \gamma^\mu \Psi^u(z/2) & -\bar{\Psi}^u(z/2) \gamma^\mu \Psi^c(-z/2) \\ A^\mu(-z/2, z/2) & := \bar{\Psi}^c(-z/2) \gamma^\mu \gamma_5 \Psi^u(z/2) & -\bar{\Psi}^u(z/2) \gamma^\mu \gamma_5 \Psi^c(-z/2) \\ T^{\mu\nu}(-z/2, z/2) & := \bar{\Psi}^c(-z/2) i\sigma^{\mu\nu} \Psi^u(z/2) & -\bar{\Psi}^u(z/2) i\sigma^{\mu\nu} \Psi^c(-z/2) \\ T_5^{\mu\nu}(-z/2, z/2) & := \bar{\Psi}^c(-z/2) i\sigma^{\mu\nu} \gamma_5 \Psi^u(z/2) & -\bar{\Psi}^u(z/2) i\sigma^{\mu\nu} \gamma_5 \Psi^c(-z/2) \end{aligned}$$

added **antiparticle contribution** to have manifest charge symmetry

# $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

$$\begin{aligned} \bar{\Psi}_{\alpha_1}^c(-z_1/2)\Psi_{\alpha_2}^u(z_1/2) = & \\ \frac{1}{2\sqrt{k_1^+k_2^+}} \sum_{\lambda_1} \left\{ \bar{\Psi}^c(-z_1/2)\gamma^+ \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2)\bar{u}_{\alpha_1}(k_2, \lambda_1)u_{\alpha_2}(k_1, \lambda_1) \right. & \\ \left. - i(-2\lambda_1 i)^j \bar{\Psi}^c(-z_1/2)i\sigma^{+j} \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2)\bar{u}_{\alpha_1}(k_2, -\lambda_1)u_{\alpha_2}(k_1, \lambda_1) \right\}. & \end{aligned}$$

define vector, axial-vector and tensor currents

$$\begin{aligned} V^+(-z/2, z/2) & := \bar{\Psi}^c(-z/2) \gamma^+ \Psi^u(z/2) & -\bar{\Psi}^u(z/2) \gamma^+ u \Psi^c(-z/2) \\ A^+(-z/2, z/2) & := \bar{\Psi}^c(-z/2)\gamma^+\gamma_5\Psi^u(z/2) & -\bar{\Psi}^u(z/2)\gamma^+\gamma_5\Psi^c(-z/2) \\ T^{+j}(-z/2, z/2) & := \bar{\Psi}^c(-z/2)i\sigma^{+j}\Psi^u(z/2) & -\bar{\Psi}^u(z/2)i\sigma^{+j}\Psi^c(-z/2) \\ T_5^{+j}(-z/2, z/2) & := \bar{\Psi}^c(-z/2)i\sigma^{+j}\gamma_5\Psi^u(z/2) & -\bar{\Psi}^u(z/2)i\sigma^{+j}\gamma_5\Psi^c(-z/2) \end{aligned}$$

# $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

$$\bar{\Psi}_{\alpha_1}^c(-z_1/2)\Psi_{\alpha_2}^u(z_1/2) = \frac{1}{2\sqrt{k_1^+k_2^+}} \sum_{\lambda_1} \left\{ \bar{\Psi}^c(-z_1/2)\gamma^+ \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2) \bar{u}_{\alpha_1}(k_2, \lambda_1) u_{\alpha_2}(k_1, \lambda_1) - i(-2\lambda_1 i)^j \bar{\Psi}^c(-z_1/2) i\sigma^{+j} \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2) \bar{u}_{\alpha_1}(k_2, -\lambda_1) u_{\alpha_2}(k_1, \lambda_1) \right\}.$$

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## $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

$$\begin{aligned} \bar{\Psi}_{\alpha_1}^c(-z_1/2)\Psi_{\alpha_2}^u(z_1/2) = \\ \frac{1}{2\sqrt{k_1^+k_2^+}} \sum_{\lambda_1} \left\{ \bar{\Psi}^c(-z_1/2)\gamma^+ \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2) \bar{u}_{\alpha_1}(k_2, \lambda_1) u_{\alpha_2}(k_1, \lambda_1) \right. \\ \left. - i(-2\lambda_1 i)^j \bar{\Psi}^c(-z_1/2) i\sigma^{+j} \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2) \bar{u}_{\alpha_1}(k_2, -\lambda_1) u_{\alpha_2}(k_1, \lambda_1) \right\}. \end{aligned}$$

define vector, **axial-vector** and tensor currents

$$\begin{aligned} V^+(-z/2, z/2) &:= \bar{\Psi}^c(-z/2) \gamma^+ \Psi^u(z/2) & -\bar{\Psi}^u(z/2) \gamma^+ \Psi^c(-z/2) \\ \mathbf{A}^+(-z/2, z/2) &:= \bar{\Psi}^c(-z/2) \gamma^+ \gamma_5 \Psi^u(z/2) & -\bar{\Psi}^u(z/2) \gamma^+ \gamma_5 \Psi^c(-z/2) \\ T^{+j}(-z/2, z/2) &:= \bar{\Psi}^c(-z/2) i\sigma^{+j} \Psi^u(z/2) & -\bar{\Psi}^u(z/2) i\sigma^{+j} \Psi^c(-z/2) \\ T_5^{+j}(-z/2, z/2) &:= \bar{\Psi}^c(-z/2) i\sigma^{+j} \gamma_5 \Psi^u(z/2) & -\bar{\Psi}^u(z/2) i\sigma^{+j} \gamma_5 \Psi^c(-z/2) \end{aligned}$$

## $p \rightarrow \Lambda_c^+$ transition: Quark Field Product

$$\begin{aligned} \bar{\Psi}_{\alpha_1}^c(-z_1/2)\Psi_{\alpha_2}^u(z_1/2) = & \\ \frac{1}{2\sqrt{k_1^+k_2^+}} \sum_{\lambda_1} \left\{ \bar{\Psi}^c(-z_1/2)\gamma^+ \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2)\bar{u}_{\alpha_1}(k_2, \lambda_1)u_{\alpha_2}(k_1, \lambda_1) \right. & \\ \left. - i(-2\lambda_1 i)^j \bar{\Psi}^c(-z_1/2)i\sigma^{+j} \frac{1+2\lambda_1\gamma_5}{2} \Psi^u(z_1/2)\bar{u}_{\alpha_1}(k_2, -\lambda_1)u_{\alpha_2}(k_1, \lambda_1) \right\}. & \end{aligned}$$

define vector, axial-vector and **tensor** currents

$$\begin{aligned} V^+(-z/2, z/2) & := \bar{\Psi}^c(-z/2) \gamma^+ \Psi^u(z/2) & -\bar{\Psi}^u(z/2) \gamma^+ \Psi^c(-z/2) \\ A^+(-z/2, z/2) & := \bar{\Psi}^c(-z/2) \gamma^+ \gamma_5 \Psi^u(z/2) & -\bar{\Psi}^u(z/2) \gamma^+ \gamma_5 \Psi^c(-z/2) \\ T^{+j}(-z/2, z/2) & := \bar{\Psi}^c(-z/2) i\sigma^{+j} \Psi^u(z/2) & -\bar{\Psi}^u(z/2) i\sigma^{+j} \Psi^c(-z/2) \\ T_5^{+j}(-z/2, z/2) & := \bar{\Psi}^c(-z/2) i\sigma^{+j} \gamma_5 \Psi^u(z/2) & -\bar{\Psi}^u(z/2) i\sigma^{+j} \gamma_5 \Psi^c(-z/2) \end{aligned}$$

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$T_5^{+j}$  reproduces  $T^{+j}$ :  $T_5^j = (-1)^j T^{+k}$ ,  $k \neq j$  and  $k, j = 1, 2$

# $p \rightarrow \Lambda_c^+$ transition: Hadronic Matrix Element

finally we obtain with the following notation

$$\mathcal{H}_{\mu'\mu}^{cu} := \bar{p}^+ \int \frac{dz_1^-}{2\pi} e^{i\bar{x}\bar{p}^+z_1^-} \langle \Lambda_c^+ : p', \mu' | V^+(-z_1^-/2, z_1^-/2) | p : p, \mu \rangle$$

$$\widetilde{\mathcal{H}}_{\mu'\mu}^{cu} := \bar{p}^+ \int \frac{dz_1^-}{2\pi} e^{i\bar{x}\bar{p}^+z_1^-} \langle \Lambda_c^+ : p', \mu' | A^+(-z_1^-/2, z_1^-/2) | p : p, \mu \rangle$$

$$\mathcal{H}_{j\mu'\mu}^{Tcu} := \bar{p}^+ \int \frac{dz_1^-}{2\pi} e^{i\bar{x}\bar{p}^+z_1^-} \langle \Lambda_c^+ : p', \mu' | T^{+j}(-z_1^-/2, z_1^-/2) | p : p, \mu \rangle$$

our final result for the  $p \rightarrow \Lambda_c^+$  transition matrix element

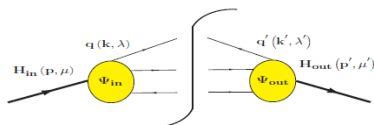
$$\begin{aligned} & \bar{p}^+ \int \frac{dz_1^-}{(2\pi)} e^{i\bar{x}\bar{p}^+z_1^-} \langle \Lambda_c^+ : p', \mu' | \bar{\Psi}_{\alpha_1}^c \left( -\frac{z_1^-}{2} \right) \Psi_{\alpha_2}^u \left( \frac{z_1^-}{2} \right) | p : p, \mu \rangle = \\ & \frac{1}{4\sqrt{k_1^+k_2^+}} \sum_{\lambda_1} \left\{ \left[ \mathcal{H}_{\mu'\mu}^{cu} + 2\lambda_1 \widetilde{\mathcal{H}}_{\mu'\mu}^{cu} \right] \bar{u}_{\alpha_1}(k_2, \lambda_1) u_{\alpha_2}(k_1, \lambda_1) \right. \\ & \left. - 2\lambda_1 \left[ \mathcal{H}_{1\mu'\mu}^{Tcu} - 2\lambda_1 i \mathcal{H}_{2\mu'\mu}^{Tcu} \right] \bar{u}_{\alpha_1}(k_2, -\lambda_1) u_{\alpha_2}(k_1, \lambda_1) \right\}. \end{aligned}$$

# Modelling GPDs: Overlap Representation of GPDs

$\bar{x} \geq \xi$  DGLAP region  $\Rightarrow$  allows for overlap representation

[Diehl M.,Feldmann T. Jakob. R. and Kroll P., Nucl.Phys. B596 (2001)]

$$\mathcal{H}_{\mu'\mu}^{cu} \sim \sum_{N,\beta} \int [d\bar{x}]_N [d^2\bar{\mathbf{k}}_\perp]_N \psi_{N,\beta',\text{out}}^{\mu'*}(\{x', \mathbf{k}'_\perp\}) \psi_{N,\beta,\text{in}}^\mu(\{x, \mathbf{k}_\perp\})$$



$$|H : p, \mu\rangle = \sum_{N,\beta} \int [dx]_N [d^2\hat{\mathbf{k}}_\perp]_N \psi_{N,\beta}^\mu(x_i, \hat{\mathbf{k}}_{i\perp}) |N, \beta : x_1, \dots, x_N; \hat{\mathbf{k}}_1, \dots, \hat{\mathbf{k}}_N\rangle$$

## Hard-Scattering Amplitude

transform  $H_{\lambda_2, \lambda_1}^{hel, \nu}$  to the LC with the help of a **unitary transformation**

$$H_{\lambda_2, \lambda_1}^{\nu}(\bar{x}, zq', q) = \bar{u}(k_2, \lambda_2) \tilde{H}_D^{\nu}(\bar{k}, k'_1, q) u(k_1, \lambda_1)$$

$$\begin{pmatrix} u_H(\lambda = +1/2) \\ u_H(\lambda = -1/2) \end{pmatrix} = U \cdot \begin{pmatrix} u_{LC}(\lambda = +1/2) \\ u_{LC}(\lambda = -1/2) \end{pmatrix}$$

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$$H_{\frac{1}{2}, \frac{1}{2}}^1 = \frac{1}{\sqrt{1 + \beta'^2}} \left[ H_{\frac{1}{2}, \frac{1}{2}}^{hel, 1} - \beta' H_{-\frac{1}{2}, \frac{1}{2}}^{hel, 1} \right]$$

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$$\beta' := \frac{M}{p^{r0} + |\mathbf{p}'|} \tan \frac{\theta_1}{2}$$