Status of SM calculation of lepton $g - 2$

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Photon 2013 Conference – Paris LPNHE, May 20 - 24, 2013

OUTLINE

- Introduction
- Status of QED calculations
- Contributions from strong interactions
- Contributions from weak interactions
- Summary and conclusions

Introduction

Response of a charged lepton to an external (static) electromagnetic field

$$\langle \ell; p'|\mathcal{J}_\rho(0)|\ell; p\rangle \equiv \bar{\mathsf{u}}(p')\Gamma_\rho(p', p)\mathsf{u}(p)$$

$$= \bar{\mathsf{u}}(p')\left[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\right]\mathsf{u}(p)$$

involving the relevant electromagnetic current $\mathcal{J}_\rho$ describing the coupling to the electromagnetic field:

$$\int d^4x\mathcal{L}_{\text{int}} = -\frac{e_\ell}{c}\int d^4x\,\mathcal{J}^\rho(x)\mathcal{A}_\rho(x)$$

$$F_1(k^2) \rightarrow \text{Dirac form factor}\,,\; F_1(0) = 1$$

$$F_2(k^2) \rightarrow \text{Pauli form factor} \rightarrow F_2(0) = a_\ell$$

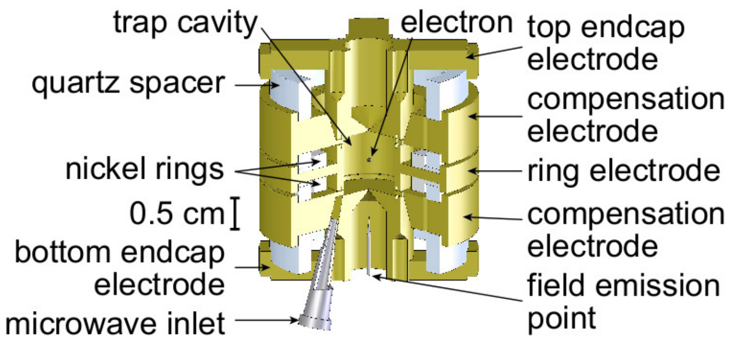
$$F_3(k^2) \rightarrow \cancel{P},\, \cancel{T}\,,\; \text{electric dipole moment} \rightarrow F_3(0) = d_\ell/e_\ell$$

$$F_4(k^2) \rightarrow \cancel{P}\,,\; \text{anapole moment}$$

$$a_\ell = \frac{1}{2}(g_\ell - 2)$$

Static quantity, $g_\ell > (<)2 \implies$ spin precession frequency larger (smaller) than cyclotron frequency

Experimentally measured to very high precision:



$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$$

$$\Delta a_e^{\text{exp}} = 2.8 \cdot 10^{-13} \text{ [0.24ppb]}$$

[D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)]

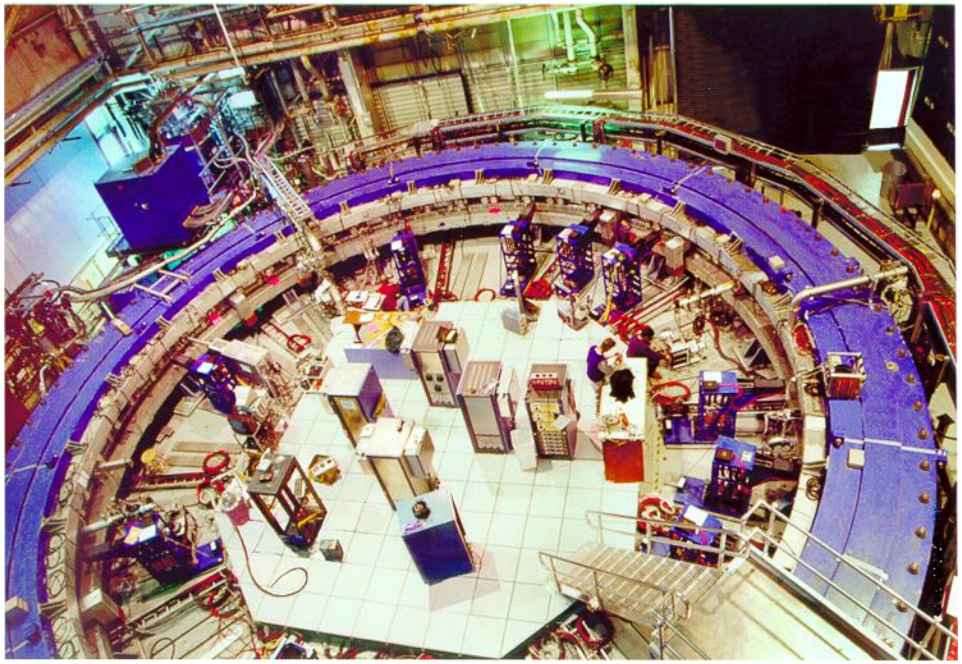
$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

$$\gamma \sim 29.3, p \sim 3.094 \text{ GeV/c}$$

$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \text{ [0.54ppm]}$$

[G. W. Bennett et al, Phys Rev D 73, 072003 (2006)]



Are the SM calculations able to achieve a comparable level of accuracy ? General structure $a_\ell^{\text{SM}} = a_\ell^{\text{QED}} + a_\ell^{\text{had}} + a_\ell^{\text{weak}}$

The tau case

$$\tau_\tau = (290.6 \pm 1.1) \times 10^{-15}\,\mathrm{s}$$

- $e^+e^- \to \tau^+\tau^-\gamma$

$-0.052 < a_\tau^{exp} < +0.058$ (L3, 1998, 95% CL)

[Phys. Lett. B 434, 169 (1998)]

$-0.068 < a_\tau^{exp} < +0.065$ (OPAL, 1998, 95% CL)

[Phys. Lett. B 431, 188 (1998)]

- $e^+e^- \to e^+e^-\tau^+\tau^-$

$-0.052 < a_\tau^{exp} < +0.013$ (DELPHI, 2004, 95% CL)

[Eur. Phys. J. C 35, 159 (2004)]

- theory: $a_\tau = 117721(5) \cdot 10^{-8}$

[S. Eidelman, M. Passera, Mod. Phys. Lett. A 22, 159 (2007)]
[S. Narison, Phys Lett B 513 (2001); err. B 526, 414 (2002)]

Useful guidelines:

- Within the framework of a renormalizable quantum field theory, $F_2(k^2)$, $F_3(k^2)$ and $F_4(k^2)$ can only arise through loop corrections. These loop contributions have to be finite and calculable, since the possible counterterms correspond to non renormalizable interactions

- a_ℓ is dimensionless: the contributions from loops involving only photons and the lepton ℓ are mass independent and thus universal

- Massive degrees of freedom with $M \gg m_\ell$ contribute to a_ℓ through powers of m_ℓ^2/M^2 times logarithms (decoupling)

- Light degrees of freedom with $m \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m^2)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

QED calculations

- QED contributions : loops with only photons and leptons

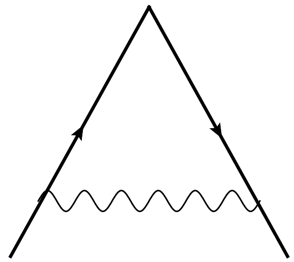
$$
\begin{aligned}
a_\ell^{QED} \quad &= \quad \sum_{n\geq 1} A_1^{(2n)} \left(\frac{\alpha}{\pi}\right)^n \\
&+ \sum_{n\geq 2} A_2^{(2n)}(m_\ell/m_{\ell'}) \left(\frac{\alpha}{\pi}\right)^n \\
&+ \sum_{n\geq 3} A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \left(\frac{\alpha}{\pi}\right)^n
\end{aligned}
$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

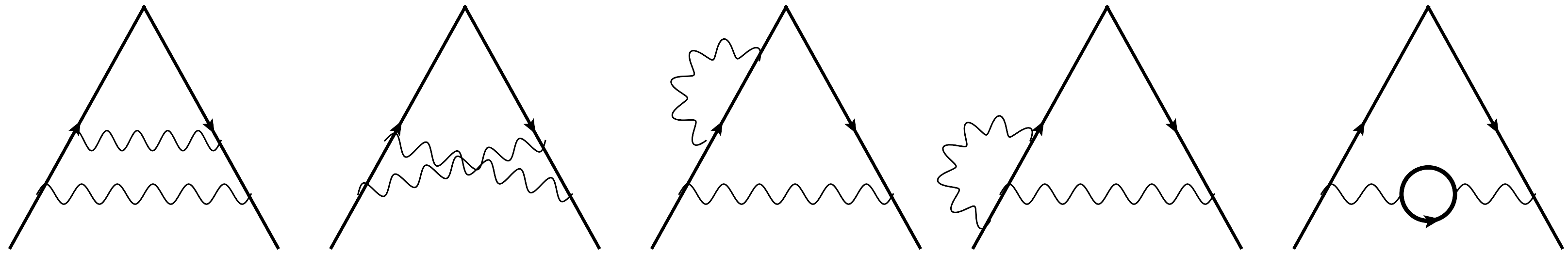
mass-dependent (non-universal) contributions

Analytic expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$, $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ known



$$A_1^{(2)} = \frac{1}{2}$$

[J. Schwinger, Phys. Rev. 73, 416L (1948)]



$$A_1^{(4)} = \frac{3}{4}\zeta(3) - \frac{\pi^2}{2}\ln 2 + \frac{\pi^2}{12} + \frac{197}{144} = -0.328\,478\,965\,579\,193\ldots$$

[C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)]

[A. Petermann, Helv. Phys. Acta 30, 407 (1957)]

$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} dt\,\sqrt{1-\frac{4m_{\ell'}^2}{t}}\,\frac{t+2m_{\ell'}^2}{t^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\frac{t}{m_\ell^2}}$$

[H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)]
[A. Petermann, Phys. Rev. 105, 1931 (1955)]
[H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)]
[M. Passera, Phys. Rev. D 75, 013002 (2007)]

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{3}\ln\left(\frac{m_\ell}{m_{\ell'}}\right) - \frac{25}{36} + \frac{\pi^2}{4}\frac{m_{\ell'}}{m_\ell} - 4\left(\frac{m_{\ell'}}{m_\ell}\right)^2 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &\quad +3\left(\frac{m_{\ell'}}{m_\ell}\right)^2 + \mathcal{O}\left[\left(\frac{m_{\ell'}}{m_\ell}\right)^3\right],\; m_\ell \gg m_{\ell'} \end{aligned}$$

[M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)]

$$A_2^{(4)}(m_\mu/m_e) = 1.094\,258\,312\,0(83)$$

$$m_\mu/m_e = 206.768\,2843(52)$$

P. J. Mohr, B. N. Taylor, D. B. Newell, CODATA 2010, arXiv:1203.5425v1[physics.atom-ph]

$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} dt\,\sqrt{1-\frac{4m_{\ell'}^2}{t}}\,\frac{t+2m_{\ell'}^2}{t^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\frac{t}{m_\ell^2}}$$

[H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)]
[A. Petermann, Phys. Rev. 105, 1931 (1955)]
[H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)]
[M. Passera, Phys. Rev. D 75, 013002 (2007)]

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{45}\left(\frac{m_\ell}{m_{\ell'}}\right)^2 + \frac{1}{70}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &\quad + \frac{9}{19600}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 + \mathcal{O}\left[\left(\frac{m_\ell}{m_{\ell'}}\right)^3 \ln\left(\frac{m_\ell}{m_{\ell'}}\right)\right], \quad m_{\ell'} \gg m_\ell \end{aligned}$$

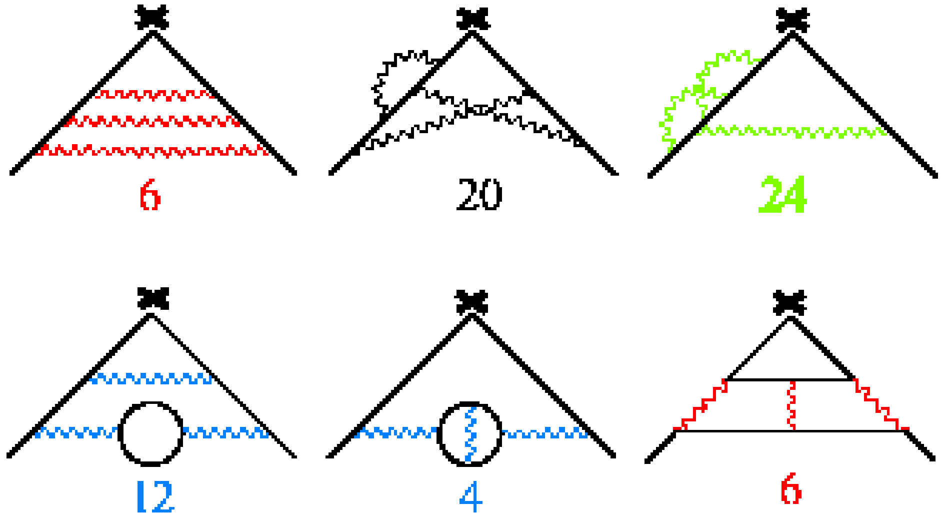
[B.E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1965)]
[M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)]

$$\begin{aligned} A_2^{(4)}(m_e/m_\mu) &= 5.197\,386\,67(26)\cdot 10^{-7} \\ A_2^{(4)}(m_e/m_\tau) &= 1.837\,98(34)\cdot 10^{-9} \\ A_2^{(4)}(m_\mu/m_\tau) &= 7.8079(15)\cdot 10^{-5} \end{aligned}$$

$$m_\mu/m_\tau = 5.946\,49(54)\cdot 10^{-2} \qquad m_e/m_\tau = 2.875\,92(26)\cdot 10^{-4}$$

P. J. Mohr, B. N. Taylor, D. B. Newell, CODATA 2010, arXiv:1203.5425v1[physics.atom-ph]

order $(\alpha/\pi)^3$: 72 diagrams



$$A_1^{(6)} = \frac{87}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[\left(a_4 + \frac{1}{24}\ln^4 2\right) - \frac{1}{24}\pi^2\ln^2 2\right] - \frac{239}{2160}\pi^4$$
$$+\frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} \qquad \left[a_p = \sum_1^\infty 1/(2^n n^p)\right]$$

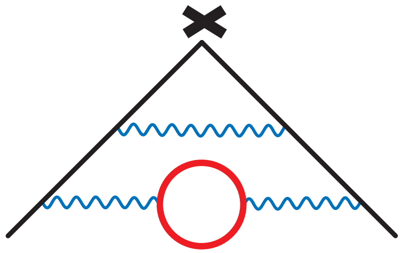
[S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)]
[S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)]

$$A_1^{(6)} = 1.181\,241\,456\ldots$$

numerical evaluations: $A_1^{(6)}(\text{num}) = 1.181\,259(40)\ldots$

[T. Kinoshita, Phys. Rev. Lett. 75, 4728 (1995)]

$$A_2^{(6)}(m_\ell/m_{\ell'}) \;=\; A_2^{(6;\mathrm{VP})}(m_\ell/m_{\ell'}) \;+\; A_2^{(6;\mathrm{LxL})}(m_\ell/m_{\ell'})$$



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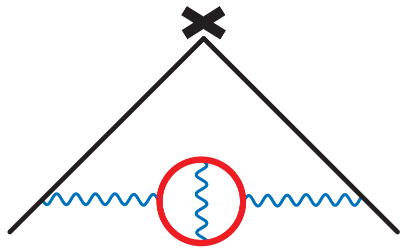

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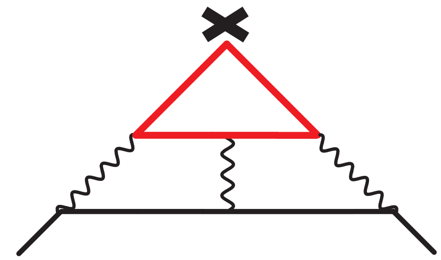

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[S. Laporta, Nuovo Cim 106A, 675 (1993)]

[S. Laporta and E. Remiddi, Phys. Lett. B301, 440 (1993)]

$$A_2^{(6)}(m_e/m_\mu) \;=\; -7.373\,941\,55(27)\cdot 10^{-6}$$

$$A_2^{(6)}(m_e/m_\tau) \;=\; -6.5830(11)\cdot 10^{-8}$$

$$A_2^{(6)}(m_\mu/m_e) \;=\; 22.868\,380\,04(23)$$

$$A_2^{(6)}(m_\mu/m_\tau) \;=\; 36.070(13)\cdot 10^{-5}$$

$$A_3^{(6)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) \;=\; 0.190\,9(1)\cdot 10^{-12}$$

$$A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) \;=\; 5.277\,6(11)\cdot 10^{-4}$$

A. Czarnecki, M. Skrzypek, Phys Lett B 449, 354 (1999) order $(\alpha/\pi)^4$: 891 diagrams

only a few diagrams are known analytically $\longrightarrow$
numerical evaluation of Feynman-parametrized loop integrals

$$
\begin{aligned}
A_1^{(8)} &= -1.434(138) && \text{[Kinoshita and Lindquist (1990)]} \\
&= -1.557(70) && \text{[Kinoshita (1995)]} \\
&= -1.4092(384) && \text{[Kinoshita (1997)]} \\
&= -1.5098(384) && \text{[Kinoshita (2001)]} \\
&= -1.7366(60) && \text{[Kinoshita (2005)]} \\
&= -1.7260(50) && \text{[Kinoshita (2005)]} \\
&= -1.7283(35) && \text{[Kinoshita and Nio, Phys. Rev. D 73, 013003(2006)]} \\
&= -1.9144(35) && \text{[Aoyama et al., Phys. Rev. Lett. 99, 110406 (2007)]} \longleftarrow \\
&= -1.9106(20) && \text{[Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]}
\end{aligned}
$$

order $(\alpha/\pi)^4$: 891 diagrams

$$
\begin{aligned}
A_2^{(8)}(m_e/m_\mu) &= 9.222(66) \cdot 10^{-4} \\
A_2^{(8)}(m_e/m_\tau) &= 8.24(12) \cdot 10^{-6} \\
A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) &= 7.465(18) \cdot 10^{-7}
\end{aligned}
$$

(not significant at present)

$$
\begin{aligned}
A_2^{(8)}(m_\mu/m_e) &= 132.685\,2(60) \\
A_2^{(8)}(m_\mu/m_\tau) &= 0.042\,34(12) \\
A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) &= 0.062\,72(4)
\end{aligned}
$$

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

Automated generation of diagrams and systematic numerical evaluation of Feynman-parametrized loop integrals

Complete results have been published

[T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)]

Five of these subsets are known analytically

[S. Laporta, Phys. Lett. B 328, 522 (1994)]

[J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)]

$$A_1^{(10)} = 9.168(571)$$

$$A_2^{(10)}(m_e/m_\mu) = -0.003\,82(39)$$

$$A_2^{(10)}(m_\mu/m_e) = 742.18(87)$$

$$A_2^{(10)}(m_\mu/m_\tau) = -0.068(5)$$

$$A_3^{(10)}(m_\mu/m_e, m_\mu/m_\tau) = 2.011(10)$$

[T. Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)]

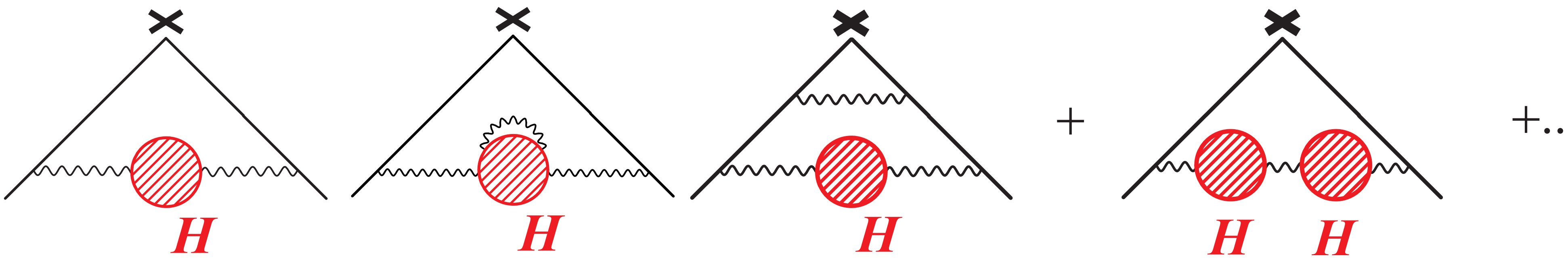
$$a_\ell^{\mathsf{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

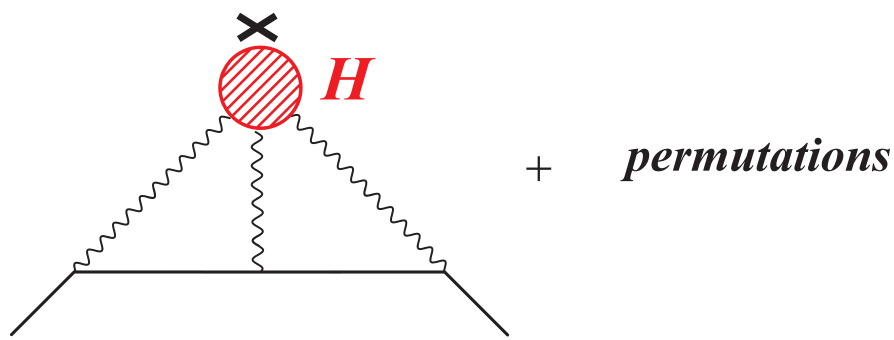
	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.9144(35)$	$130.879\,6(63)$
$C_\ell^{(10)}$	$9.16(58)$	$753.29(1.04)$

Contributions from strong interactions

- Hadronic contributions : quark loops

$$a_\ell^{\text{had}} = a_\ell^{\text{HVP-LO}} + a_\ell^{\text{HVP-HO}} + a_\ell^{\text{HLxL}}$$





$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t)$$

$$K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

[M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)]

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates
- $a_\ell^{\text{HVP-LO}}$ is related to an experimental quantity

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t)$$

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- $a_\ell^{\text{HVP-LO}}$ is related to an experimental quantity

"at this stage theoreticians have finished their job, and experimentalists take over"

[M. Davier, LPNHE workshop, Feb. 2010]

$\longrightarrow$ *talks by A. Passeri, B. Malaescu, S. Eidelman, Z. Zhang, M. Benayoun, H. Spiesberger, B. Moussallam*

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t)$$

$$K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

[M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)]

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates
- $a_\mu^{\text{HVP-LO}}$ is related to an experimental quantity

Lattice QCD $\longrightarrow$ e.g. [P. A. Boyle et al, Phys. Rev. D 85, 074504 (2011)]

Latest results

$$a_\mu^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$$ [M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)]

$$a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$$ [K. Hagiwara et al., J. Phys. G 38, 085003 (2011)]

$$a_e^{\text{HVP-LO}} = 1.866(11) \cdot 10^{-12}$$ [D. Nomura, T. Teubner, Nucl. Phys. B 867, 236 (2013)]

$$a_\mu^{\text{HVP-HO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^3 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K^{(2)}(t) R^{\text{had}}(t)$$

[J. Calmet, S. Narison, M. Perrottet, E. de Rafael, Phys. Lett. B 61, 283 (1976)]

[B. Krause, Phys. Lett. B 390, 392 (1997)]

$$a_\mu^{\text{HVP-HO}} = -9.84 \pm 0.07 \cdot 10^{-10}$$ [K. Hagiwara et al., J. Phys. G 38, 085003 (2011)]

$$a_e^{\text{HVP-LO}} = -0.2234(14) \cdot 10^{-12}$$ [D. Nomura, T. Teubner, Nucl. Phys. B 867, 236 (2013

a_μ^{HLxL}

not related to an experimental observable...

available non-perturbative tools in QCD:

- large-N_C limit
- low-energy effective theory (ChPT) at long distances

[E. de Rafael, Phys. Lett. B 322, 239 (1994)]

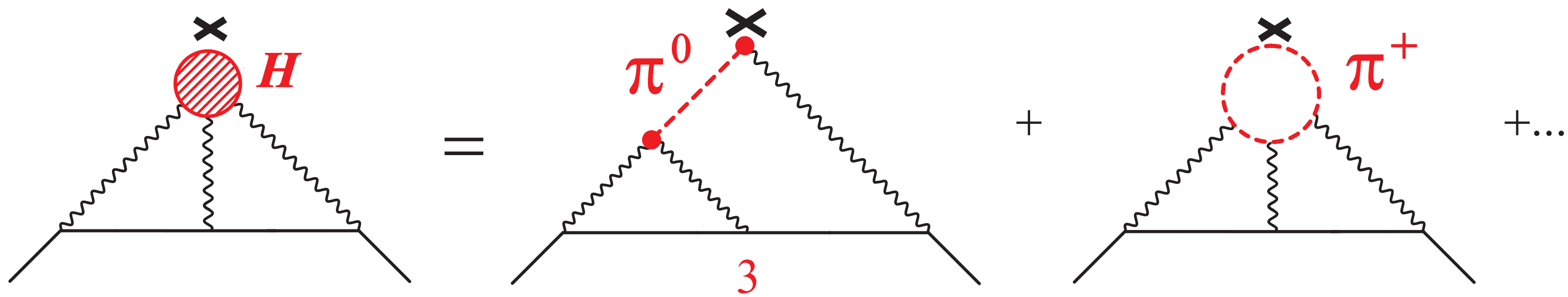
- OPE at short distances

[K. Melnikov, A. Vainshtein, Phys. Rev. D 70, 113006 (2004)]

- (lattice simulations) [S. Chowdury et al, PoS (LATTICE 2008) 251]

a_μ^{HLxL}

many identifiable contributions...



- single meson exchanges $\mathcal{O}(N_C)$
- π^0 (η, η') exchange $\mathcal{O}(N_C \times p^6)$
  $\longrightarrow$ different definitions...
- mesonic loops $\mathcal{O}(N_C^0)$
- $\pi^\pm$ and $K^\pm$ loops $\mathcal{O}(N_C^0 \times p^4)$
  finite for point-like $\pi^\pm$, result varies a lot according to form factor model used...
- constituent quark loop loop $\mathcal{O}(N_C)$
  matching of short and long distances, double counting...
- OPE: systematic analysis remains to be done

a_μ^{HLxL}

- only two "complete", but model-dependent calculations...

$$
\begin{aligned}
a_\mu^{\text{HLxL}} &= +(8.3 \pm 3.2) \cdot 10^{-10} && \text{[Bijnens et al, Nucl. Phys. B 474 (1999); ibid-err B 626 (2002)]} \\
a_\mu^{\text{HLxL}} &= +(89.6 \pm 15.4) \cdot 10^{-11} && \text{[Hayakawa et al, Phys. Rev. D 57 (1998); ibid-err D 66 (2002)]}
\end{aligned}
$$

...after the sign change

[M.K. and A. Nyffeler, Phys. Rev. D 65, 073034 (2002)]

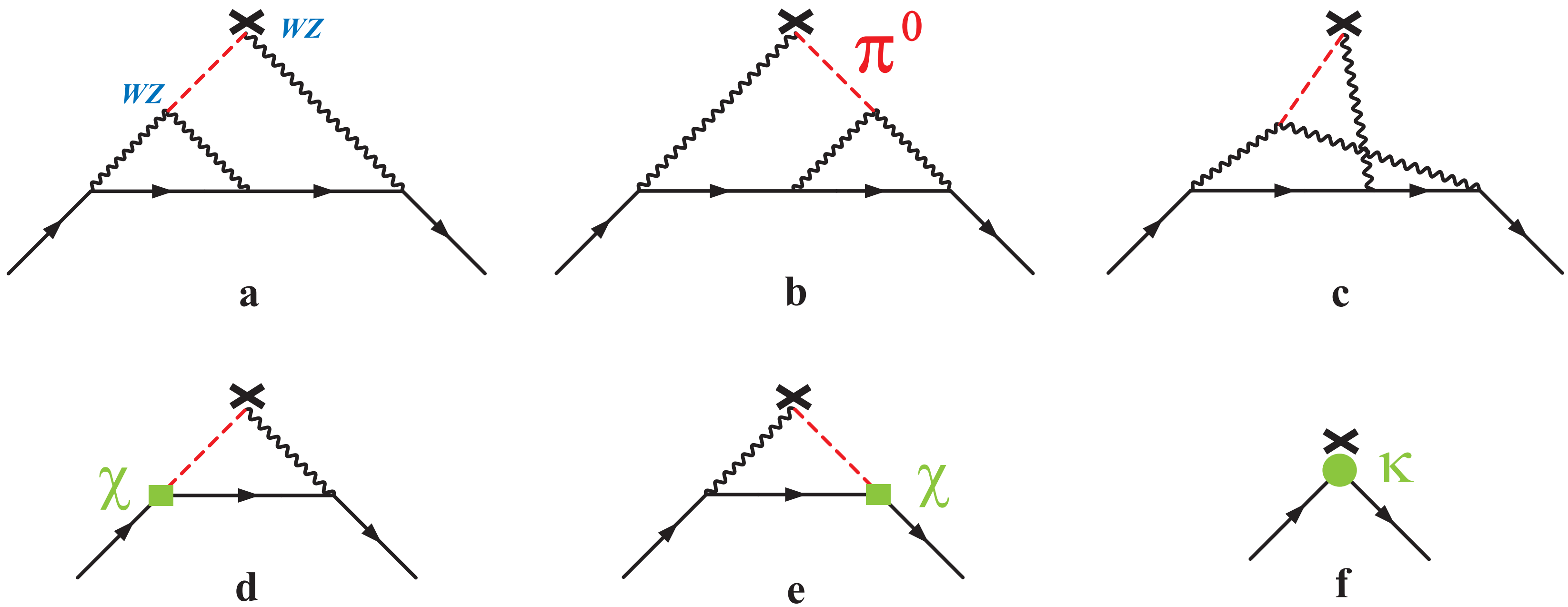
low-energy region does not provide the most important contribution to a_μ^{HLxL}

$\longrightarrow$ computing HLxL to higher orders in ChPT will not lead to significant improvement in our understanding of a_μ^{HLxL}

Try to get further insight or to test new ideas by studying simple models

cf. D. Greynat, E. de Rafael, JHEP1207, 020 (2012) based on a version of the chiral constituent quark model that is renormalizable in the large-N_C limit [S. Weinberg, Phys. Rev. Lett. 105, 261601 (2010)] (see also E. de Rafael, Phys. Lett. B 703, 60 (2011))

- pion exchange contribution dominates (large-N_C effective theory)



$$a_\mu^{\text{HLxL}} = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \underbrace{\mathcal{C}\left[\ln^2\frac{\mu_0}{m_\ell} + c_1(\mu_0)\ln\frac{\mu_0}{m_\ell} + c_0(\mu_0)\right]}_{\mathcal{O}(N_C \times p^6),\ \pi^0,\eta,\eta'\text{exch.}} + \underbrace{\mathcal{F}\left(\frac{M_\pi}{m_\ell},\frac{M_K}{m_\ell}\right)}_{\mathcal{O}(N_C^0 \times p^4)\ \pi^\pm, K^\pm \text{loops}\cdots} \right\}$$

$$\mathcal{C} = \frac{N_C^2}{48\pi^2}\cdot\frac{m_\ell^2}{F_\pi^2}$$

[M.K., A. Nyffeler, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002)]

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K l. + subl. in Nc	–	–	–	0 ± 10	–	–	–
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3	21 ± 3
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

BPP: J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75 (1995) 1447 [Erratum-ibid. 75 (1995) 3781]; Nucl. Phys. B 474 (1996) 379; [Erratum-ibid. 626 (2002) 410]

HKS: M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75 (1995) 790; Phys. Rev. D 54 (1996) 3137

KN: M. Knecht, A. Nyffeler, Phys. Rev. D 65 (2002) 073034

MV: K. Melnikov, A. Vainshtein, Phys. Rev. D 70 (2004) 113006

BP: J. Bijnens, J. Prades, Acta Phys. Polon. B 38 (2007) 2819; J. Prades, Nucl. Phys. Proc. Suppl. 181-182 (2008) 15; J. Bijnens, J. Prades, Mod. Phys. Lett. A 22 (2007) 767

BdRV: J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306 [hep-ph]

N/NJ: A. Nyffeler, Phys. Rev. D 79, 073012 (2009); F. Jegerlehner, A. Nyffeler, Phys. Rep. (2009)

Recent (partial) reevaluations

$$a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$$ [J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306]

"best estimate"

$$a_\mu^{\text{HLxL}} = (11.5 \pm 4.0) \cdot 10^{-10}$$ [A. Nyffeler, Phys. Rev. D 79, 073012 (2009)]

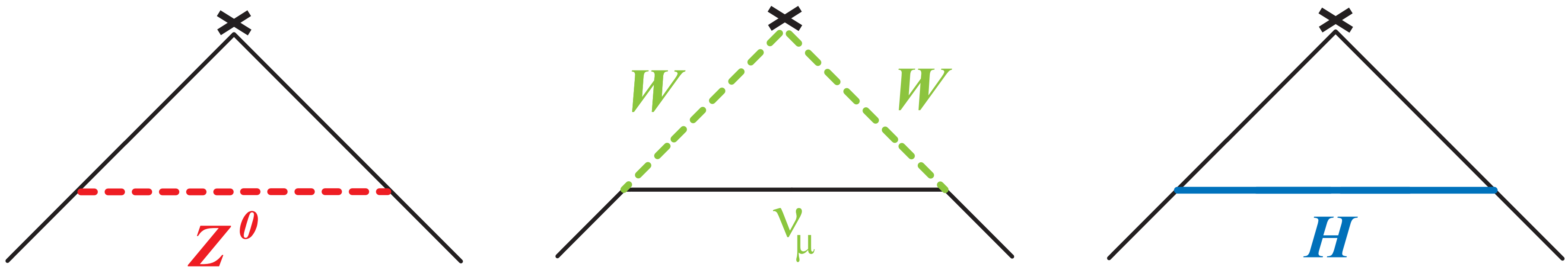
more conservative estimate

$$a_e^{\text{HLxL}} = (0.035 \pm 0.010) \cdot 10^{-12}$$ [J. Prades, E. de Rafael, A. Vainshtein, in *Lepton Dipole Mom*

$\longrightarrow$ *talks by A. Vainshtein, V. Pauk, I. Prado Longhi*

Contributions from weak interactions

- Weak contributions : W, Z,... loops



$$
\begin{aligned}
a_\mu^{\text{weak(1)}} &= \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right] \\
&= 19.48\times 10^{-10}
\end{aligned}
$$

[W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)]

[G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)]

[R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)]

[I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)]

[M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)]

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_\mu^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi}\right) \cdot (-79.3)$$

[A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)]

Two-loop fermionic contributions

[A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)]

[M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)]

$$a_\mu^{\text{weak}} = (15.4 \pm 0.1) \cdot 10^{-10}$$

$$a_e^{\text{weak}} = (0.0297 \pm 0.0005) \cdot 10^{-12}$$

Summary and conclusions

SM prediction ?

$\longrightarrow$ requires an input for the fine structure constant α that matches the experimental accuracy on a_e

$$\frac{\Delta a_e}{a_e} = 0.24\text{ppb} \;\rightarrow\; \frac{\Delta\alpha}{\alpha} \sim 0.24\text{ppb} \;\rightarrow\; \Delta\alpha \lesssim 2\cdot 10^{-12}$$

- quantum Hall effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \qquad [19.7\text{ppb}]$$

[P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)]

- atomic recoil velocity through photon absorption

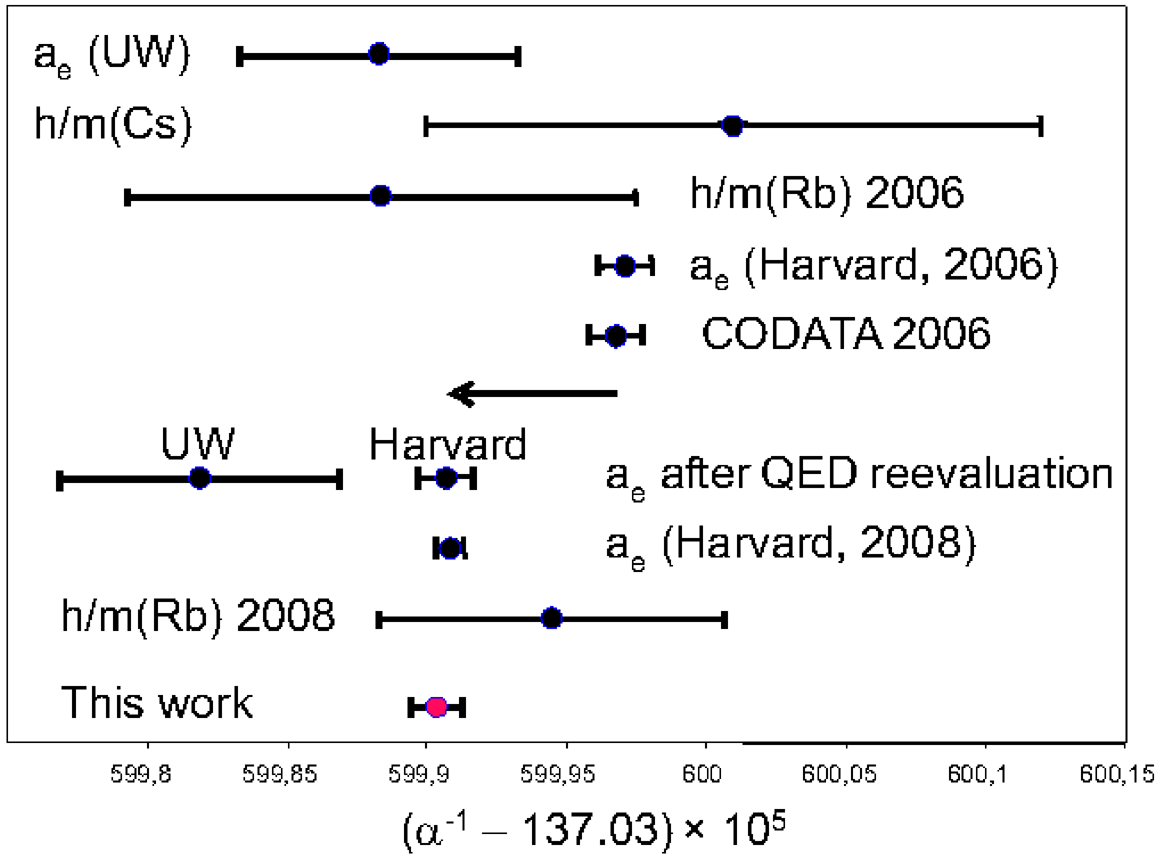
$$\alpha^2 = \frac{2R_\infty}{c}\cdot\frac{M_{\text{atom}}}{m_e}\cdot\frac{h}{M_{\text{atom}}} \qquad \Delta R_\infty = 7\cdot 10^{-12} \qquad \Delta\left(\frac{M_{\text{Rb}}}{m_e}\right) = 4.4\cdot 10^{-10}$$

$$\alpha^{-1}[Rb\,11] = 137.035\,999\,037(91) \qquad [0.66\text{ppb}]$$

[R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)]

$\longrightarrow$ *Colloquium by P. Cladé*

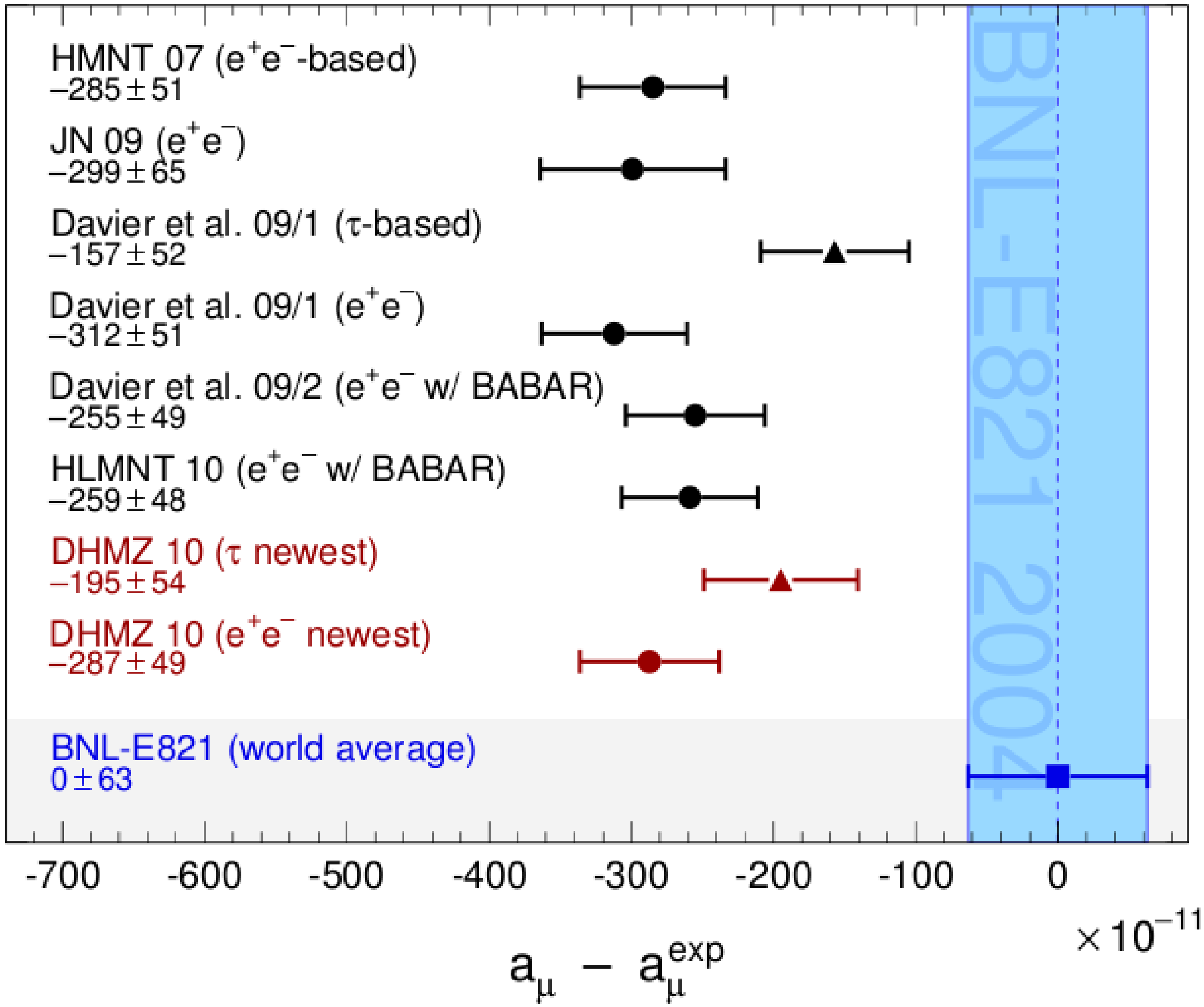
$$a_e(HV08) - a_e(\text{theory}) = -1.05(0.82) \cdot 10^{-12}$$



[R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)]

$$\alpha[a_e(HV\,08)] = 137.035\,999\,172\,7(68)_{\alpha^4}(46)_{\alpha^5}(19)_{\text{had+weak}}(331)_{\text{exp}} \quad [0.25\text{ppb}]$$

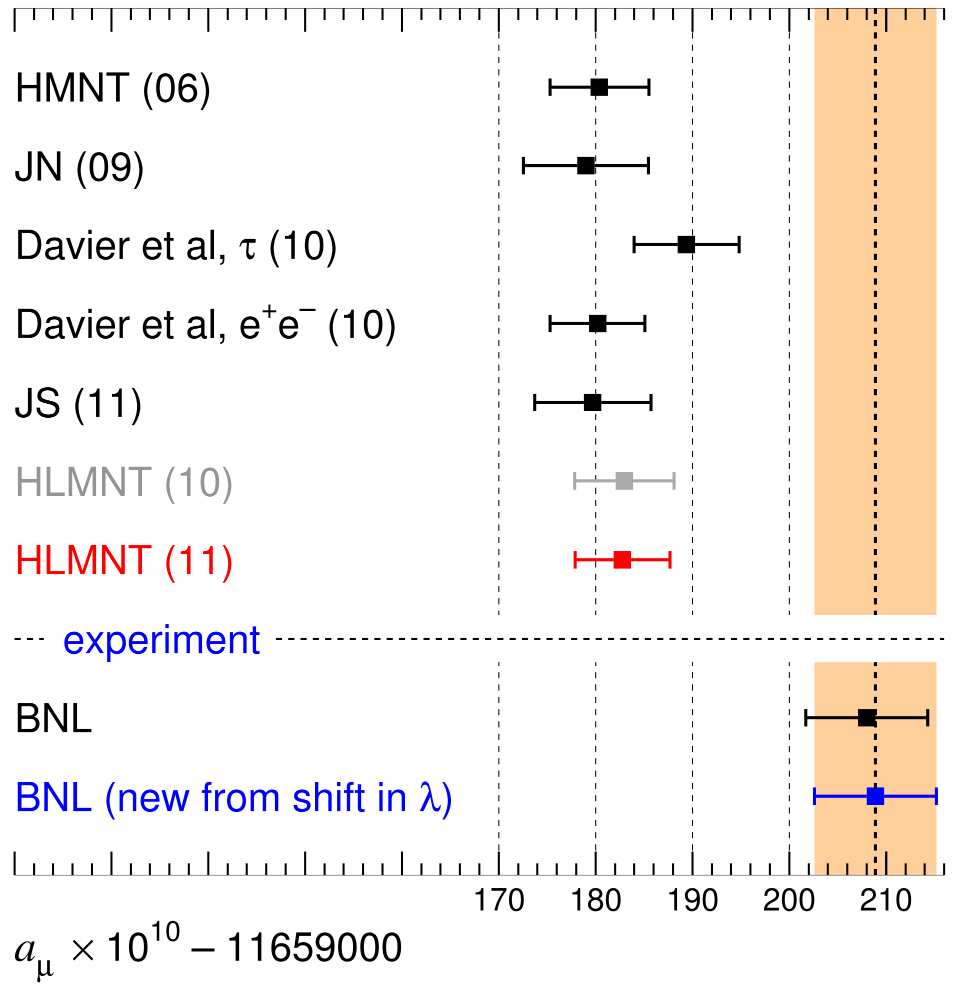
[T. Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]



[Davier, Hoecker, Malaescu, Zhang, Eur. Phys. J. C 71, 1515 (2011)]

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (28.7 \pm 8.0) \cdot 10^{-10} \quad [3.6\sigma] \quad \text{for } a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$$

$$(a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.6 \pm 8.6) \cdot 10^{-10} \quad [3.2\sigma] \quad \text{for } a_\mu^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10})$$



[K. Hagiwara et al., J. Phys. G 38, 085003 (2011)]

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \cdot 10^{-10} \;\; [3.3\sigma] \quad \text{for } a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$$

$$(a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.0 \pm 8.6) \cdot 10^{-10} \;\; [2.9\sigma] \quad \text{for } a_\mu^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10})$$

- a_e and a_μ have been measured very precisely (to 0.24ppb and to 0.54ppm, respectively), improving previous measurements by several factors

- a_e essentially provides a test for QED, although recent measurements of the fine-structure constant in atomic physics allow to reach the level of precision required in order to see also contributions from the strong interactions

- a_e still provides the most precise determination of the fine-structure constant

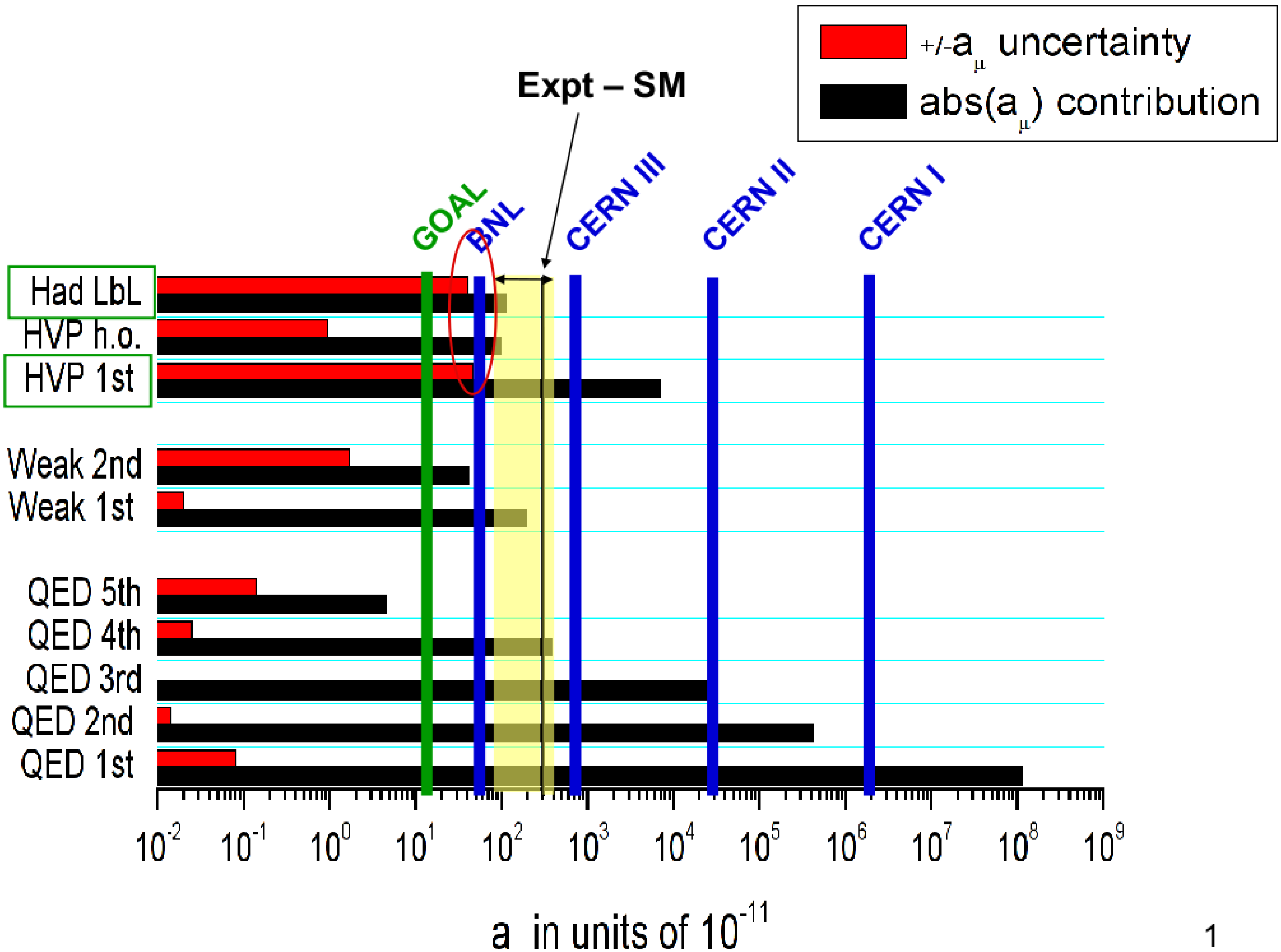
- it ist certainly of interest and worthwhile to pursue the efforts to measure the fine-structure constant with the accuracy required to test the prediction for a_e at the level where this quantity has been measured

- a_μ probes all the interactions of the standard model, and perhaps even beyond...

- there is a persistent discrepancy between the measured value and the SM prediction at the level of 3 to 3.6 σ

- whether this discrepancy is real or not will be probed soon by two forthcoming experiments, at FNAL and at J-PARC, which aim at a precision of 0.14ppm... $\longrightarrow$ talk by M. Lancaster

- the interpretation of future experiments (FNAL-E989, J-PARC) requires further theoretical improvement on the evaluation of the HLxL contributions



D. Hertzog, LPNHE Workshop, Paris Feb. 2010