



Photon-photon scattering in collisions of laser pulses

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- Vacuum polarisation introduction
Heisenberg-Euler Lagrangian



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- Vacuum birefringence and dichroism in interacting lasers beams



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- Elastic photon-photon scattering

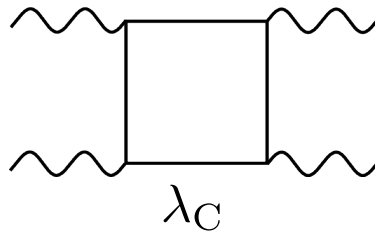


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- Elastic photon-photon scattering
- Vacuum four-wave mixing

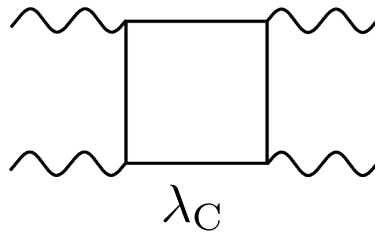


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- Elastic photon-photon scattering
- Vacuum four-wave mixing
- Summary



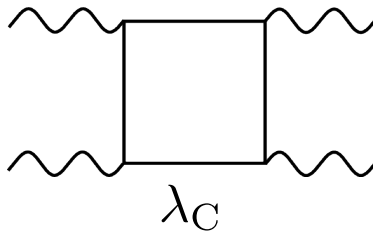


$$\Delta E \Delta t \sim \hbar$$



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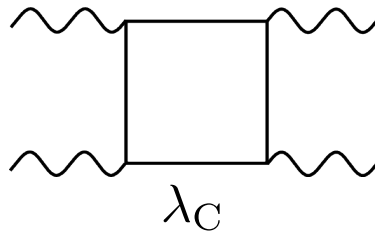
$$\sigma_{\gamma\gamma}[\text{cm}^2] = 7.3 \times 10^{-66} (\hbar\omega[\text{eV}])^6$$



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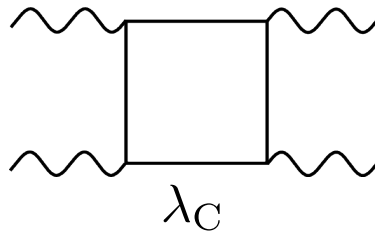
$$N_\gamma \approx 6 \times 10^{18} \mathcal{E}[\text{J}] \hbar\omega[\text{eV}]$$



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D. Bernard (LPNHE) et al.
EPJD **10**, 141-145 (2000)

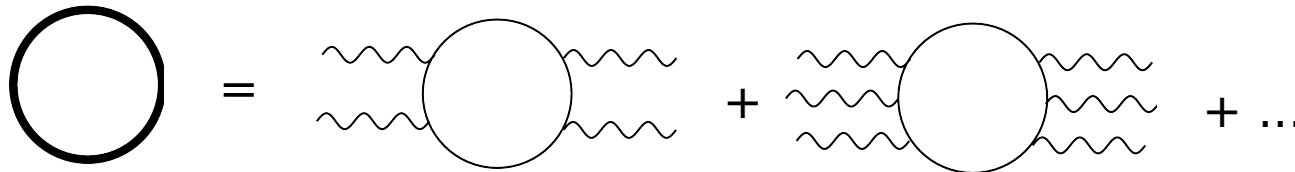


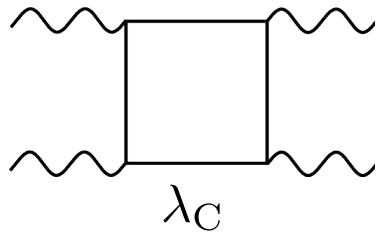
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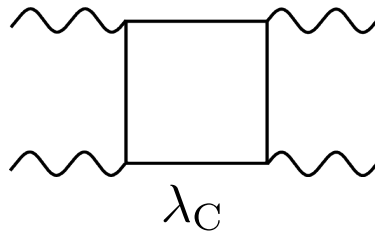
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Vacuum Polarisation



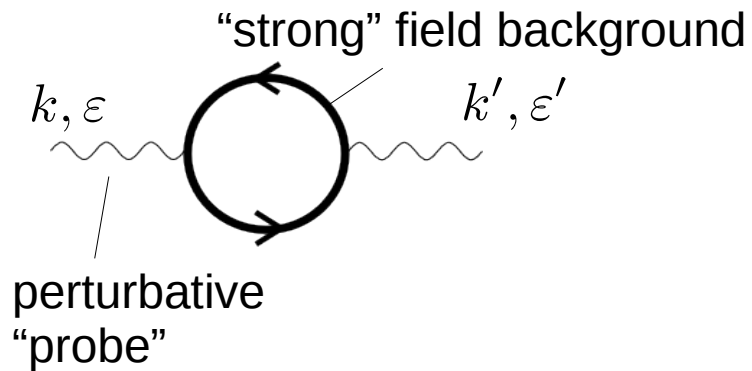
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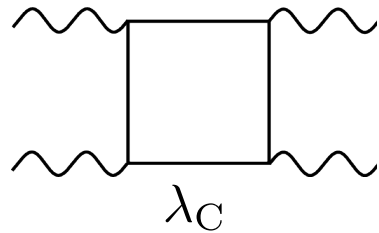
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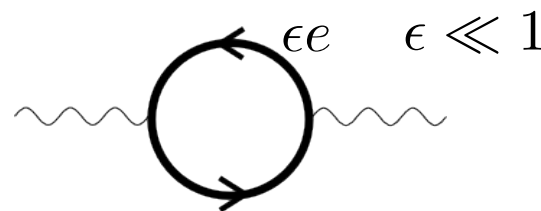
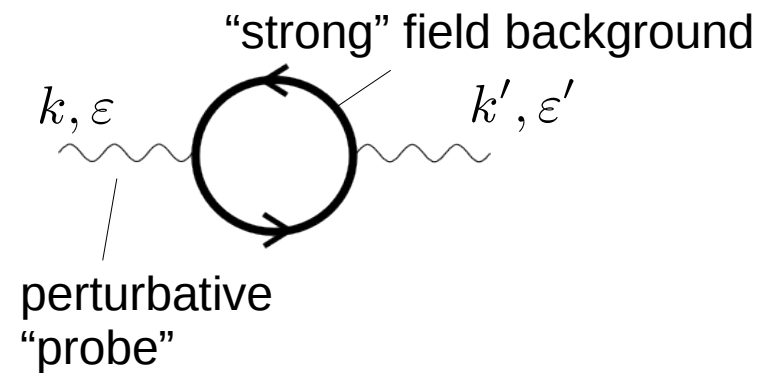
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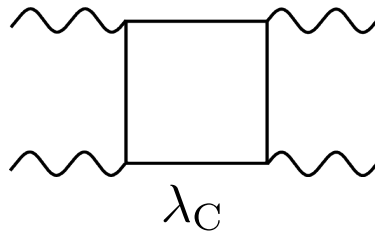
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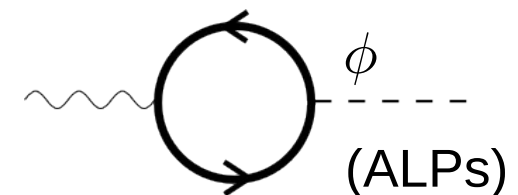
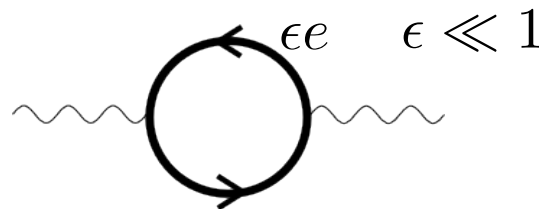
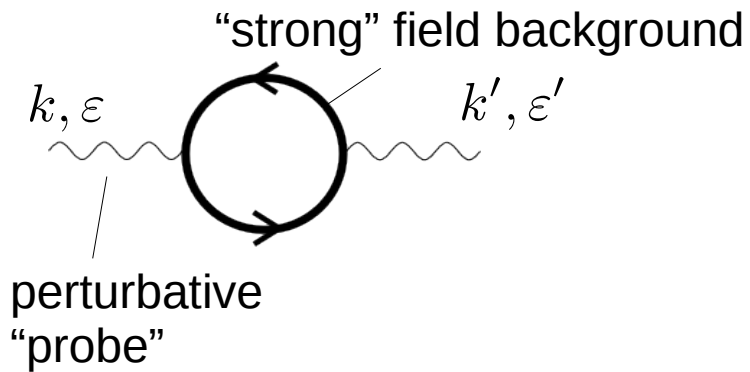
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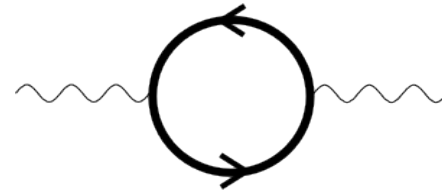
Photon-photon scattering





$$\hbar\omega \ll mc^2 \quad (\lambda \ll \lambda_c)$$

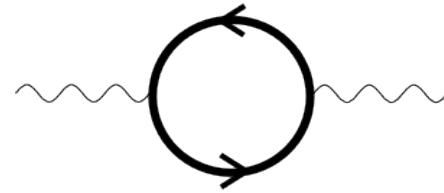
$$\hat{A} \rightarrow A$$





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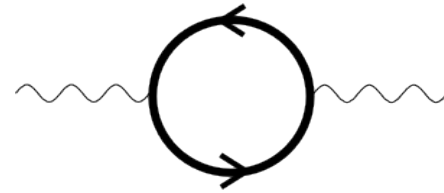


$$\mathcal{L}_{\text{eff.}} = -i \text{Tr} \ln \left(\frac{\gamma(p + eA) + m}{\gamma p + m} \right)$$



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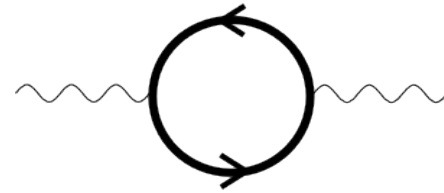
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$$\alpha(E/E_{\text{cr}})^2 \ll 1 \quad E/E_{\text{cr}} \lesssim 2 \times 10^{-4}$$



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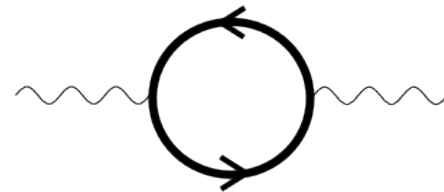
$$\mathcal{L} = \frac{1}{2}(E^2 - B^2) + c_1(E^2 - B^2)^2 + c_2(\mathbf{E} \cdot \mathbf{B})^2 \quad (c_{1,2} \sim \frac{\alpha}{45\pi} \frac{1}{E_{\text{cr}}^2})$$

Photon-photon scattering



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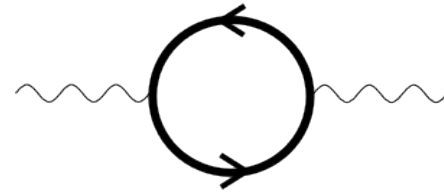
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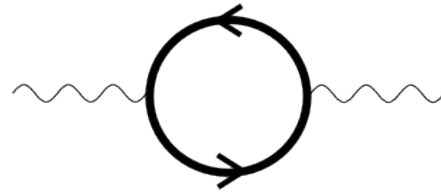
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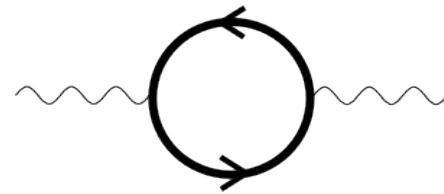
$$\mathbf{P}_{\text{vac}} = c'_1(E^2 - B^2)\mathbf{E} + c'_2(\mathbf{E} \cdot \mathbf{B})\mathbf{B}$$

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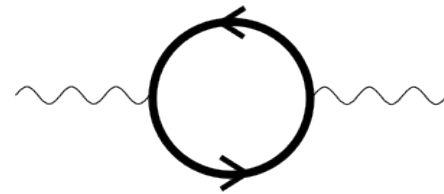
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$$\mathbf{E} \approx \mathbf{E}^{(0)} + \mathbf{E}_d$$

Vacuum birefringence

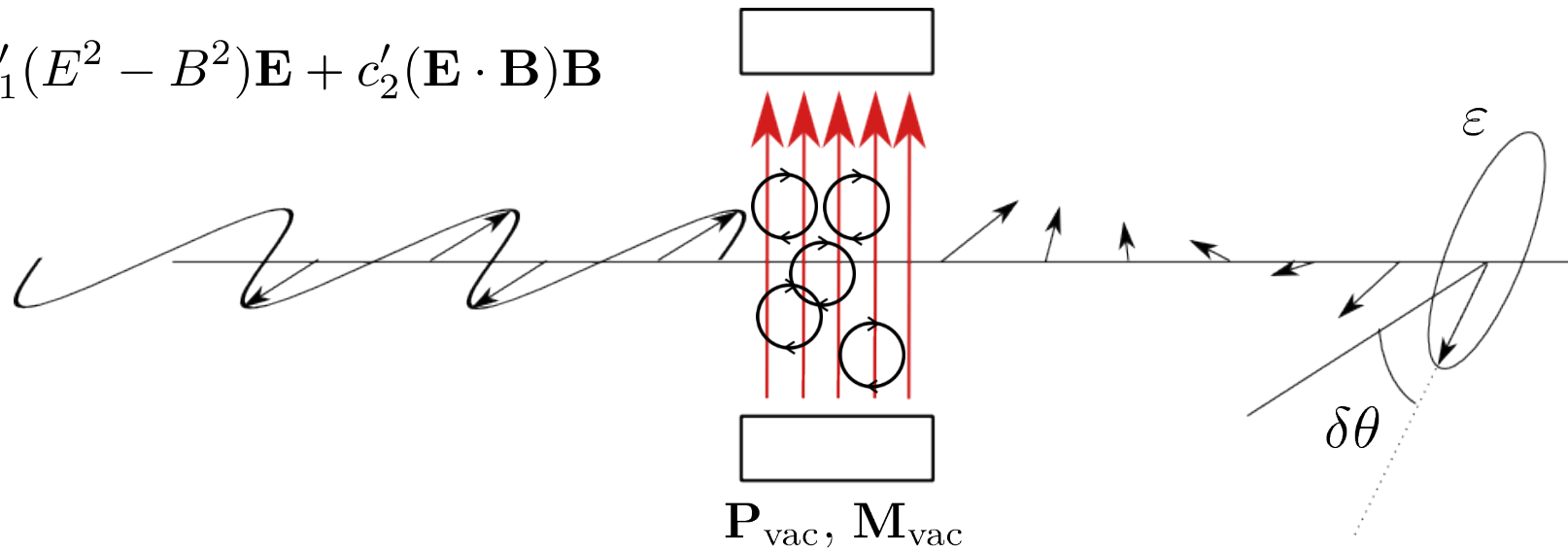




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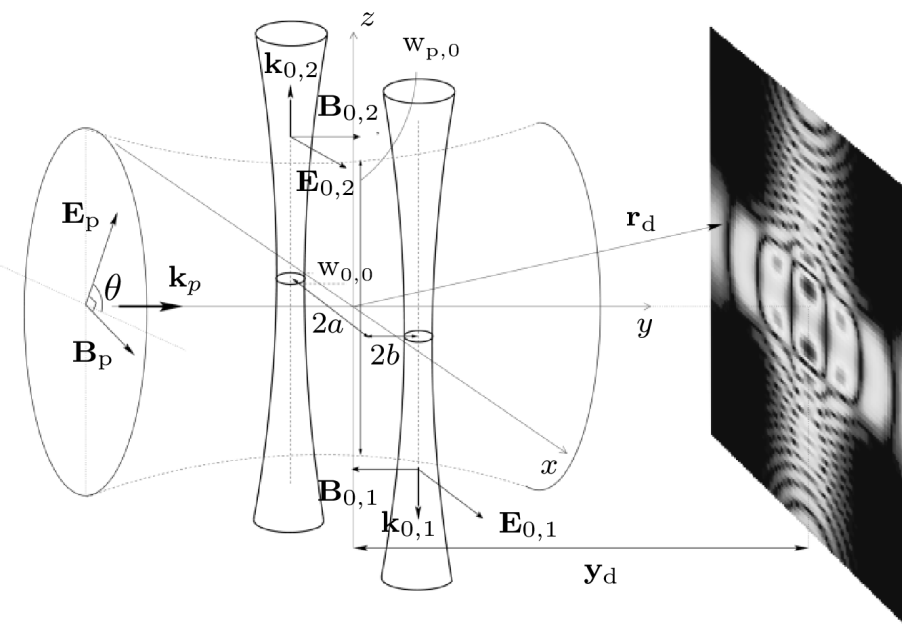
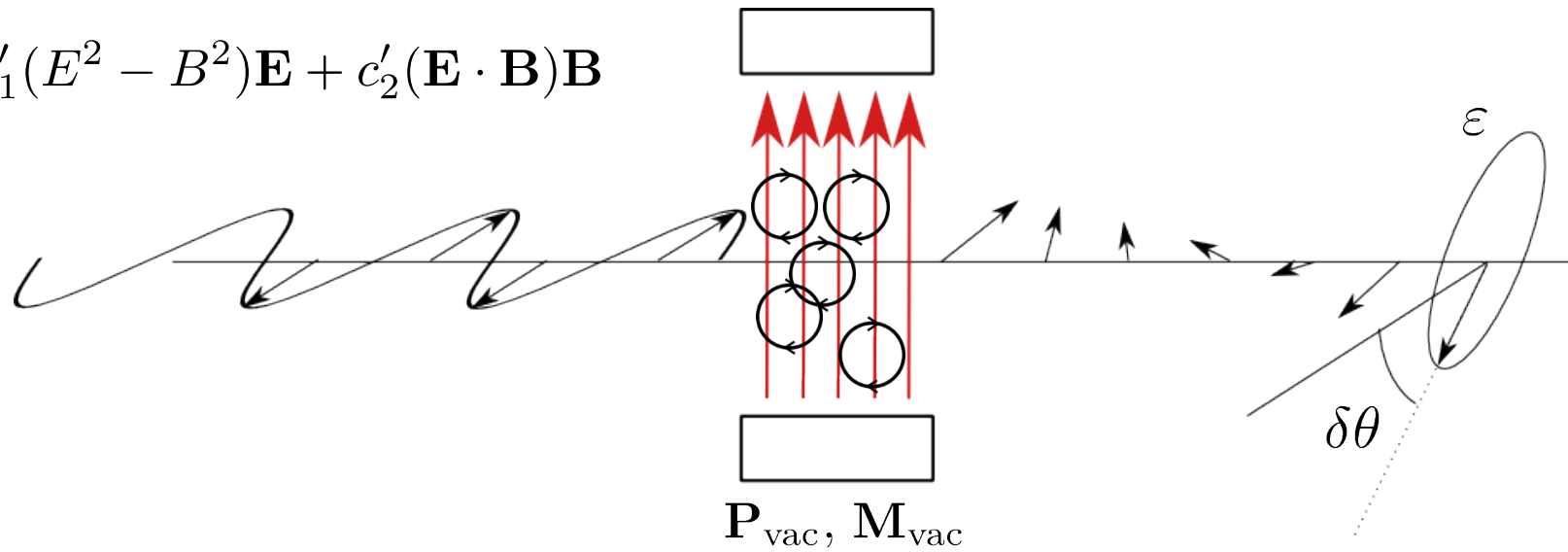
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$$I_s = 10^{24} \text{ Wcm}^{-2}$$

$$\omega_s = 1.5 \text{ eV}$$

$$\tau_s = 100 \text{ fs}$$

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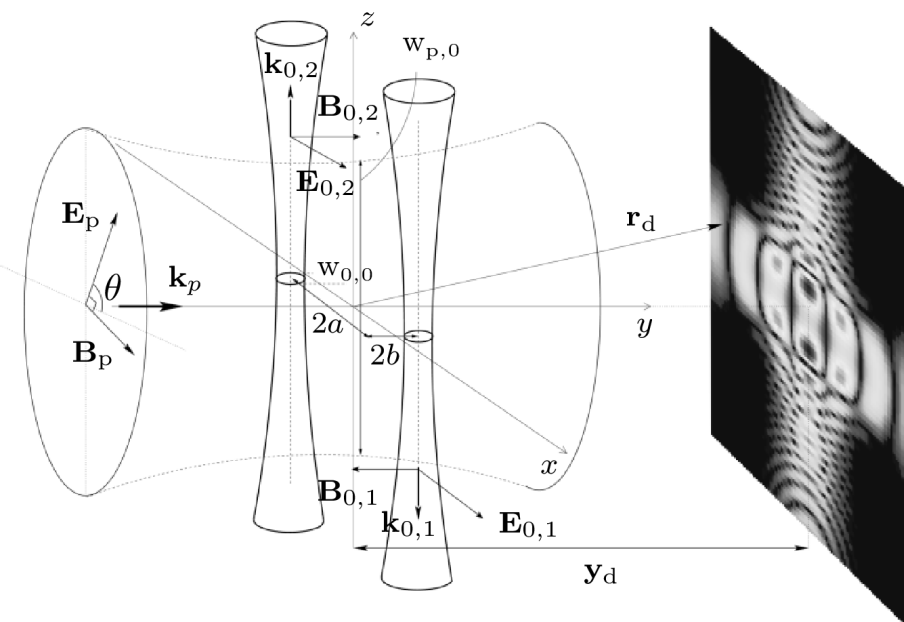
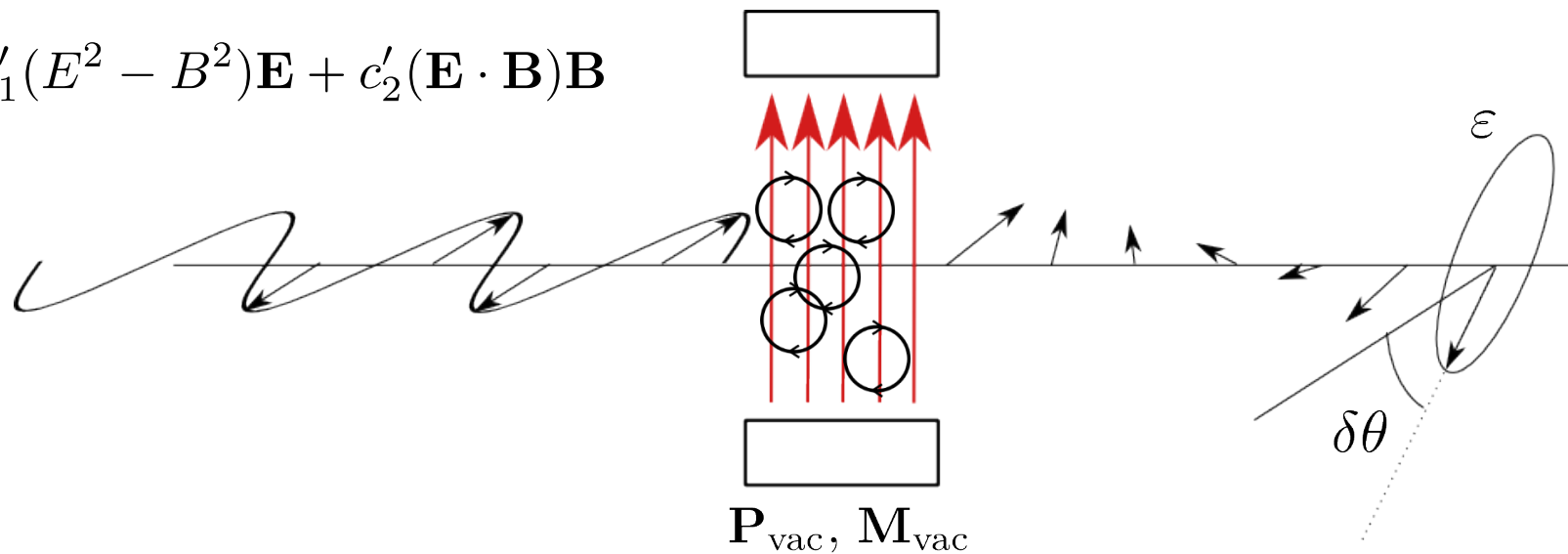
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Vacuum birefringence



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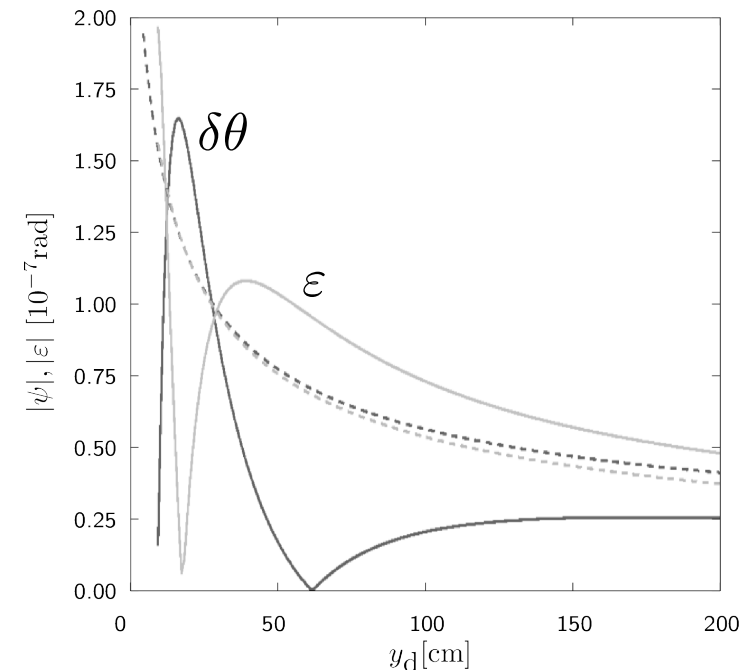
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BK, A. Di Piazza and C. H. Keitel, PRA **82**, 032114 (2010)

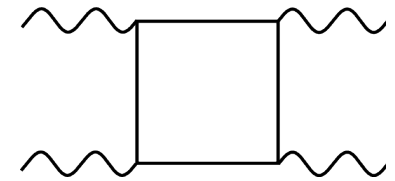
Photon-photon scattering



Photon-photon scattering



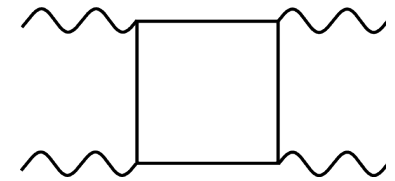
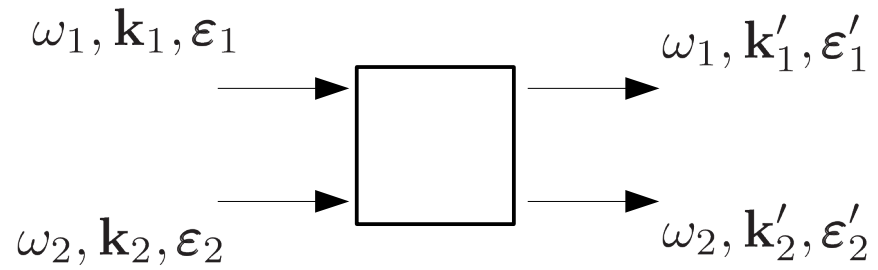
LMU



R. Karplus and M. Neuman
Phys. Rev. **80** 380-385 (1950)



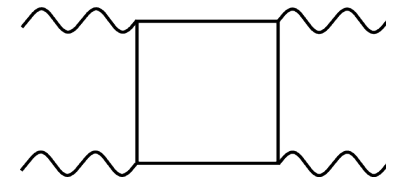
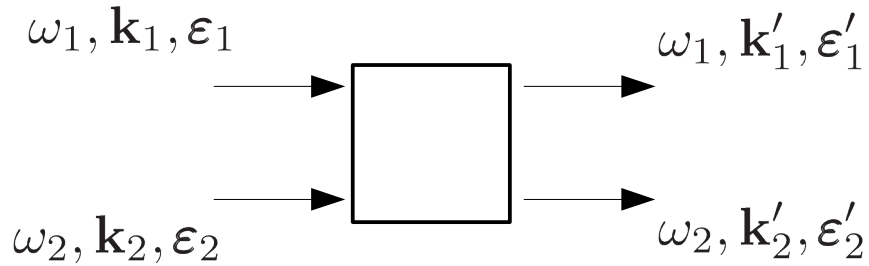
Elastic photon-photon scattering



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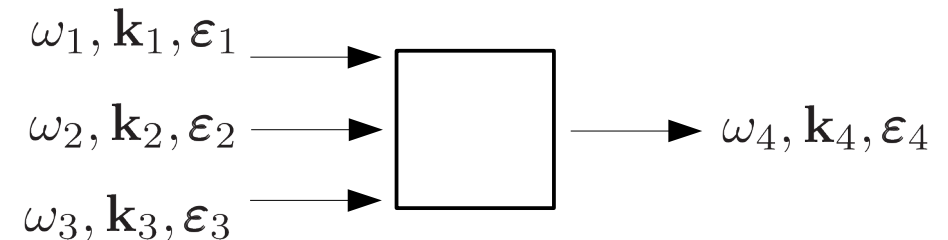
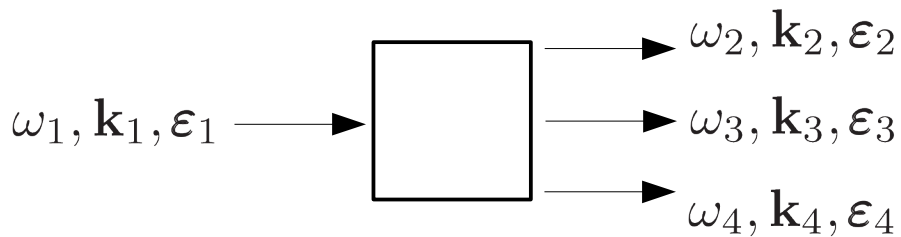


Elastic photon-photon scattering



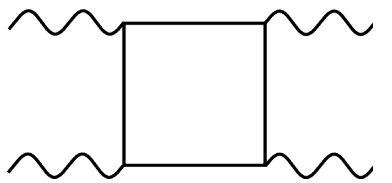
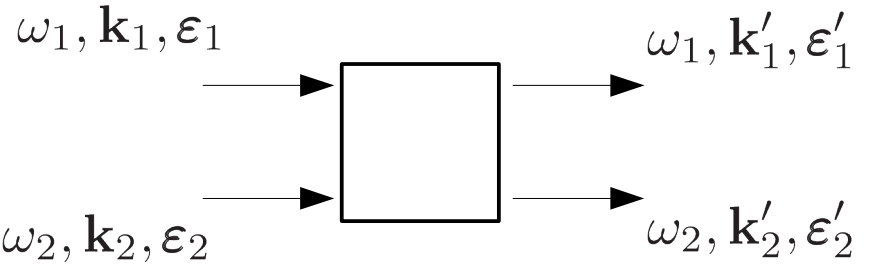
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Inelastic photon-photon scattering (four-wave mixing)



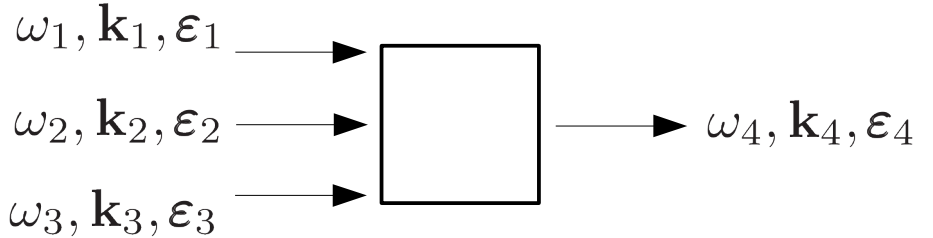
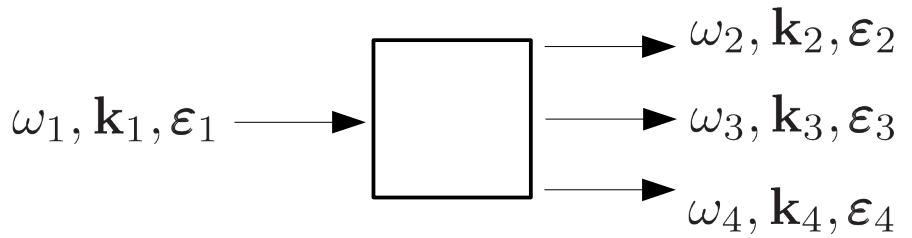


Elastic photon-photon scattering



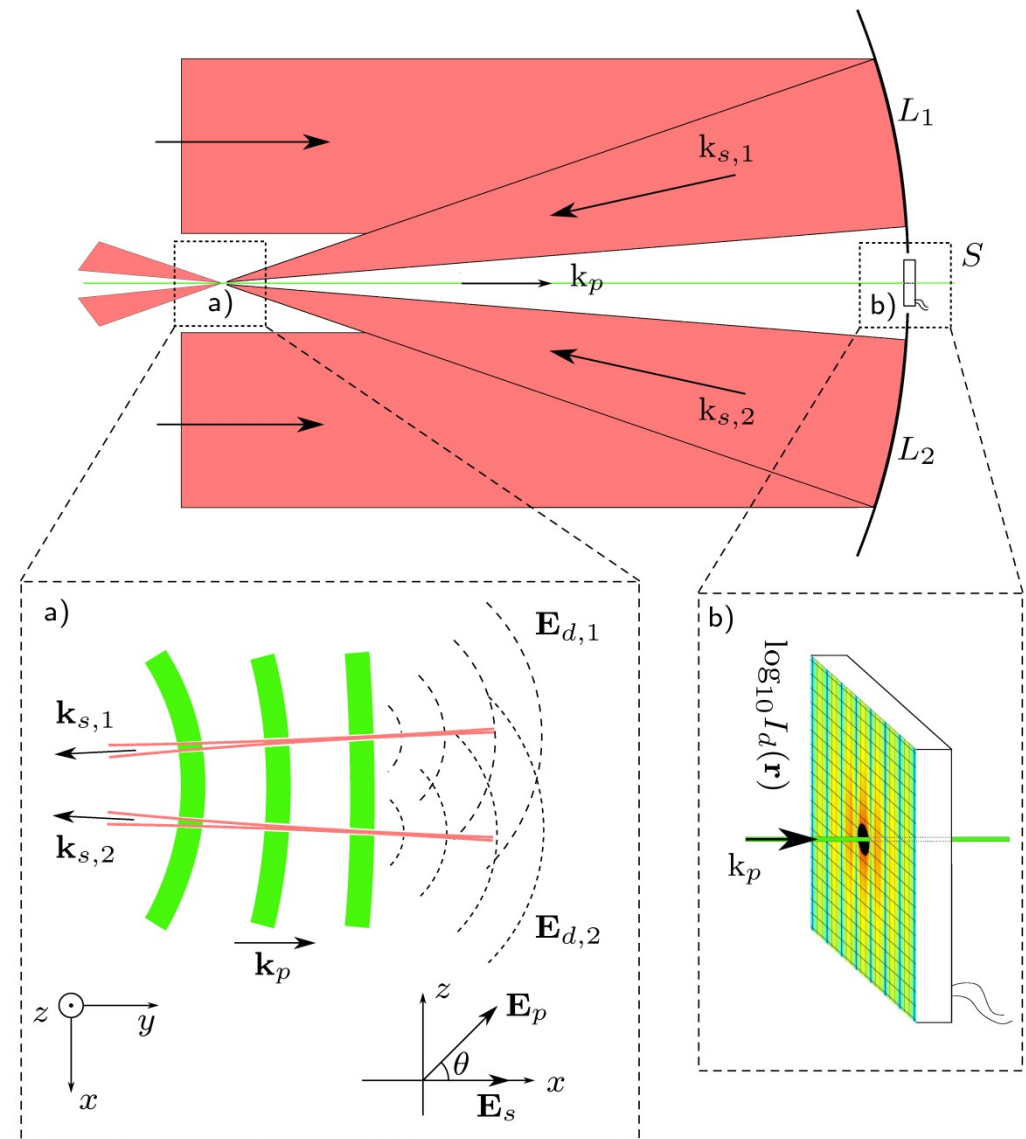
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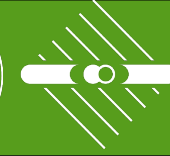
Inelastic photon-photon scattering (four-wave mixing)



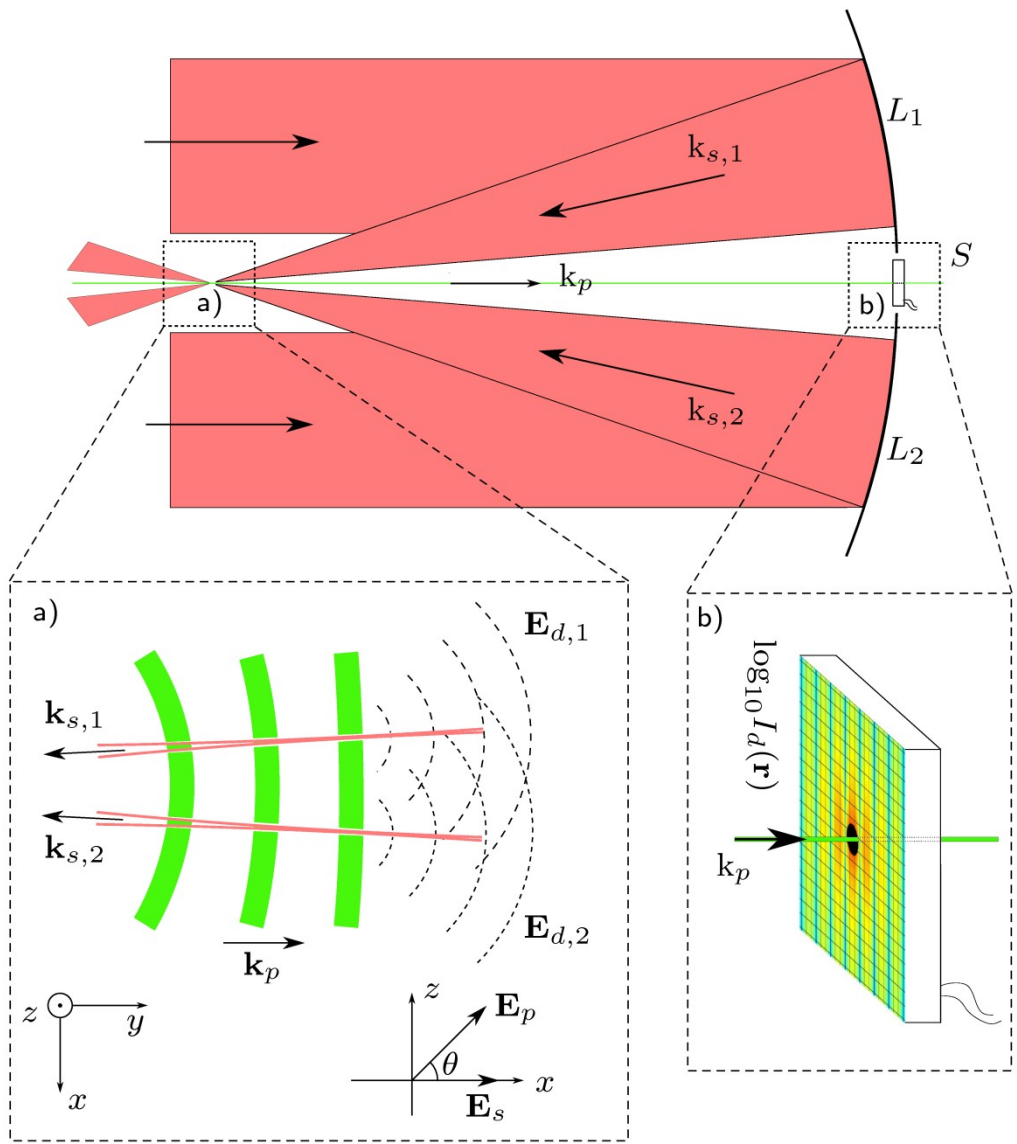
$$k_1^\mu + k_2^\mu + k_3^\mu + k_4^\mu = 0$$



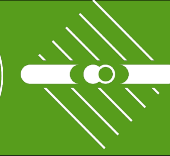




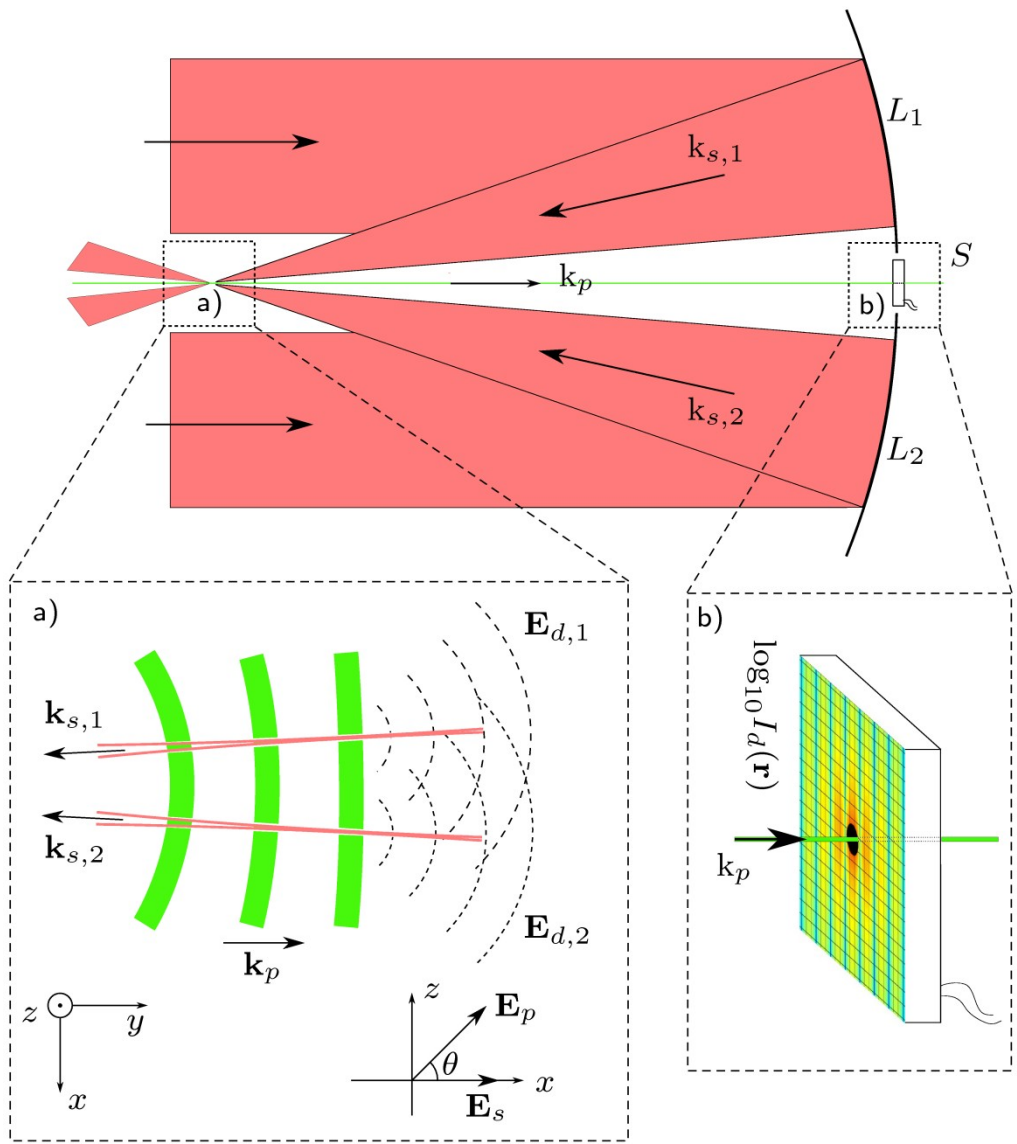
- Strong laser: 150 PW, 800 nm, 30 fs
diffraction limited



Elastic photon-photon scattering

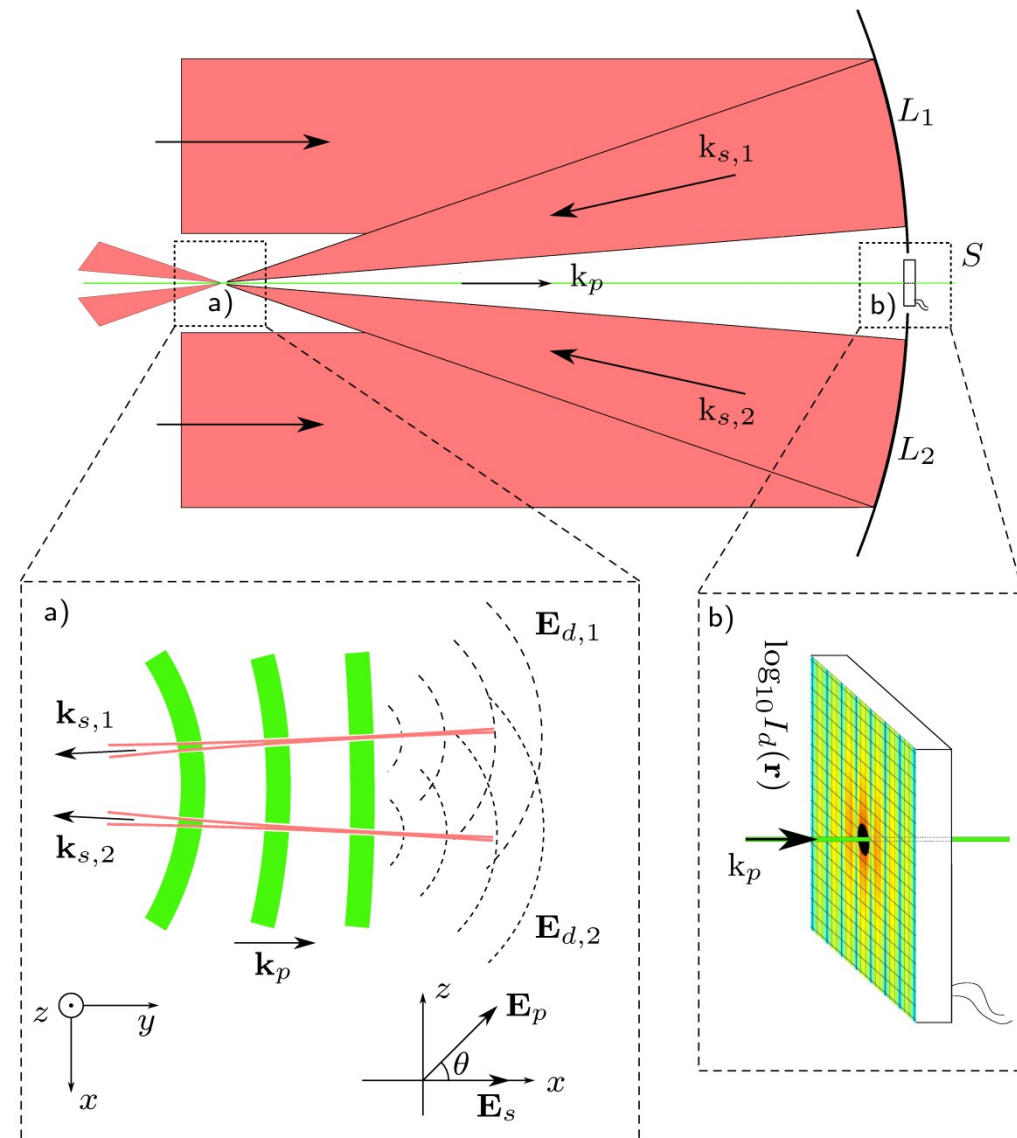


- Strong laser: 150 PW, 800 nm, 30 fs
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- Probe laser: 200 TW, 527 nm, 100 fs
focused to 300 μm



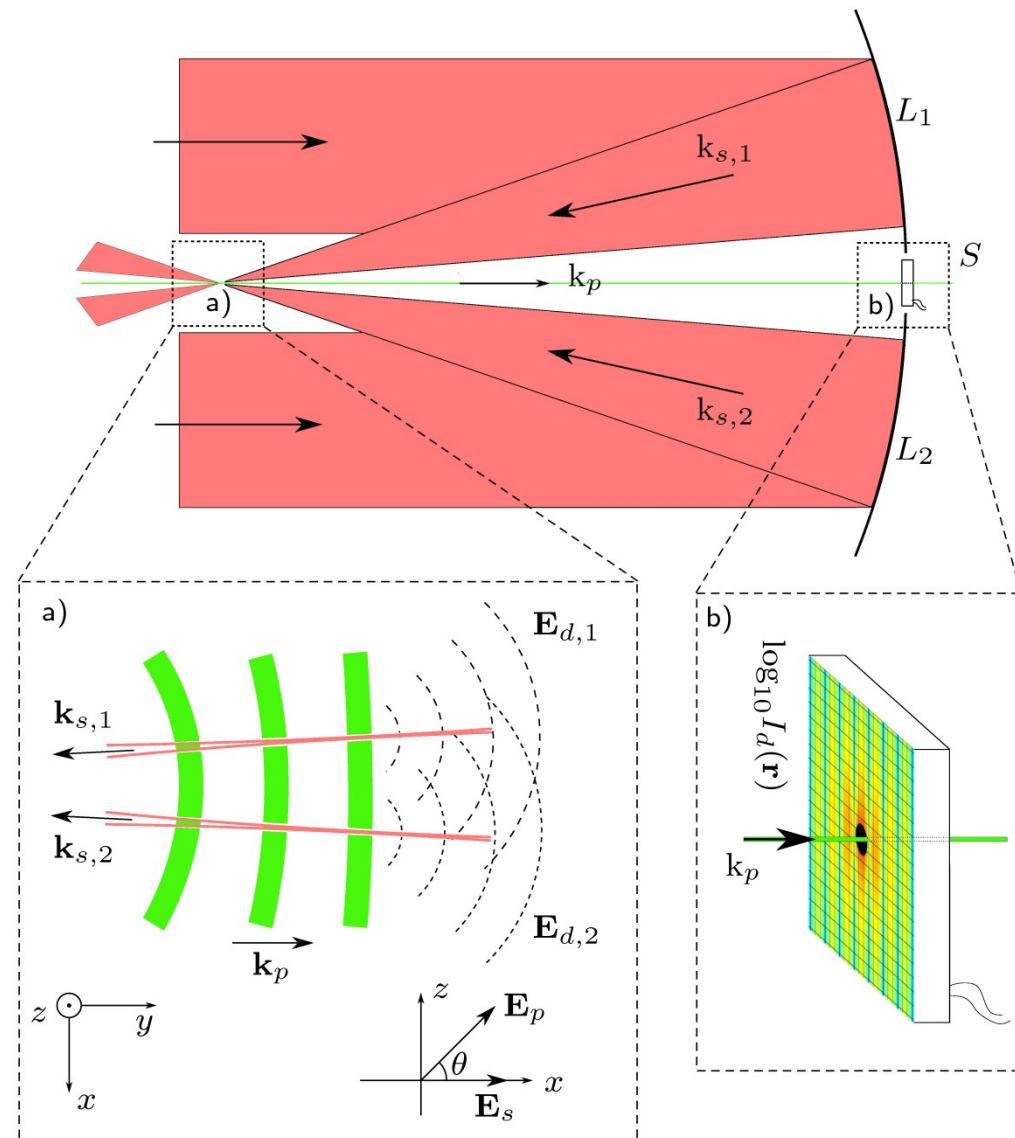


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monochromatic



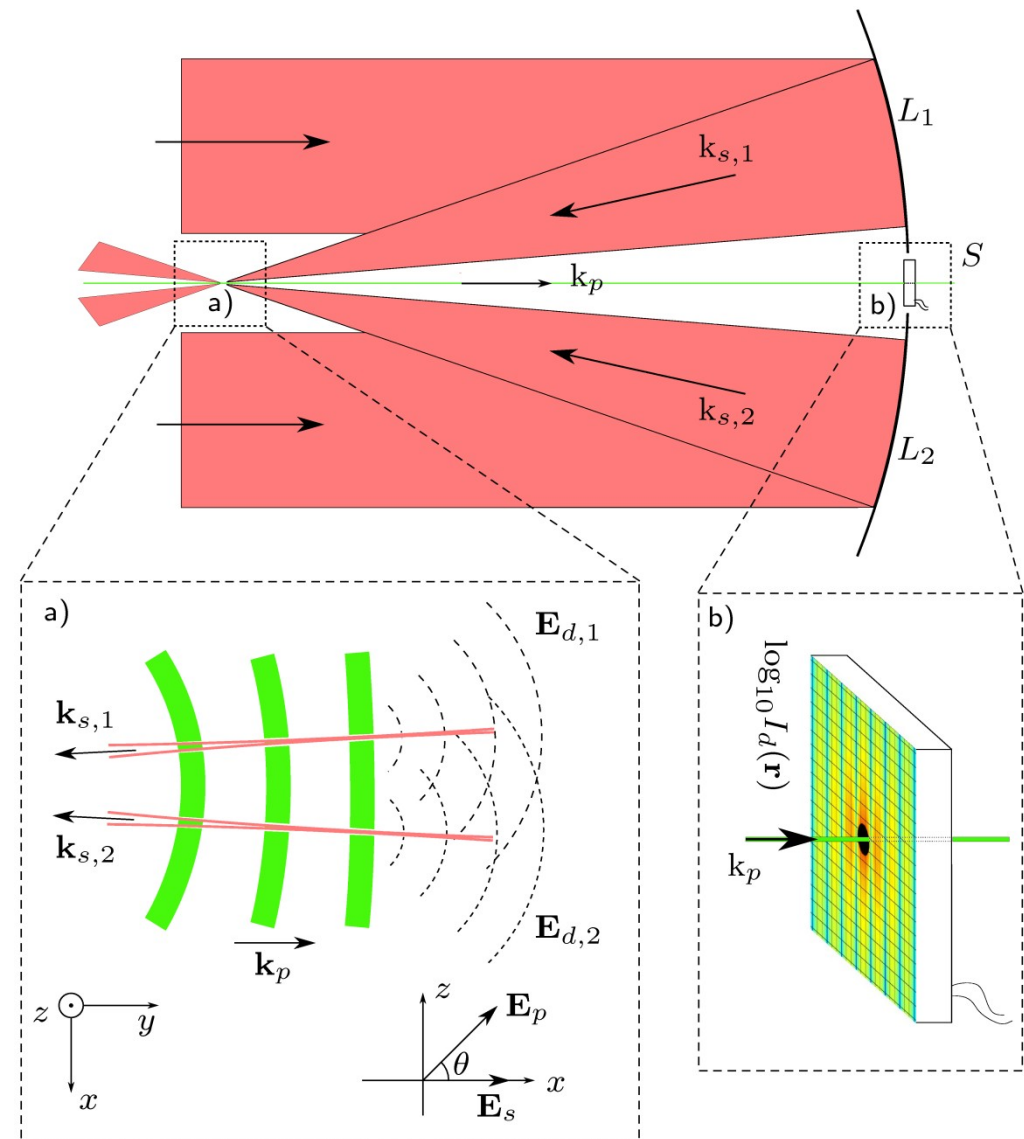


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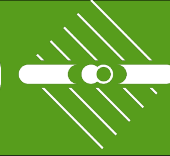
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- Required vacuum pressure: $P \leq 10^{-6}$ torr



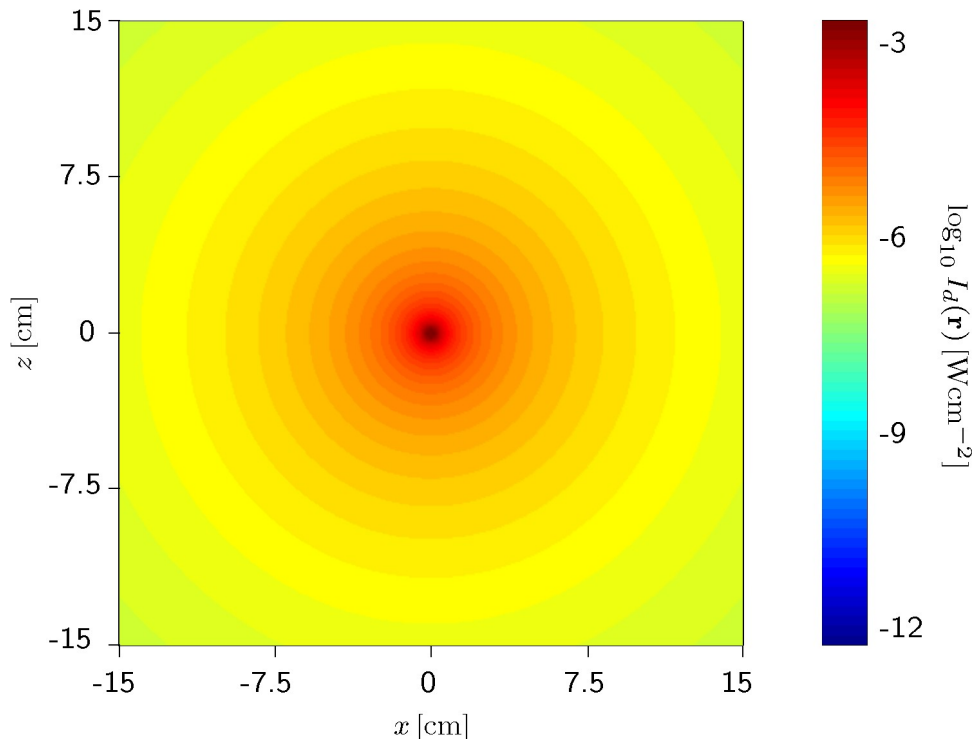




$$\frac{1}{2} \langle |\mathbf{E}_p + \mathbf{E}_d|^2 \rangle = I_p + I_{pd} + I_d$$

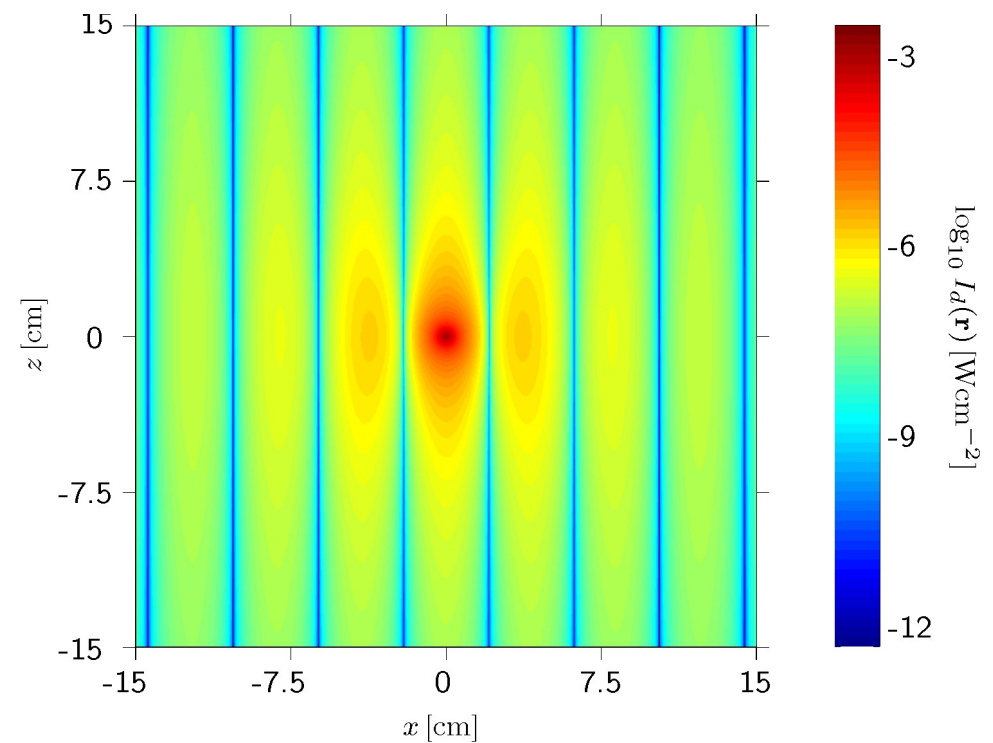
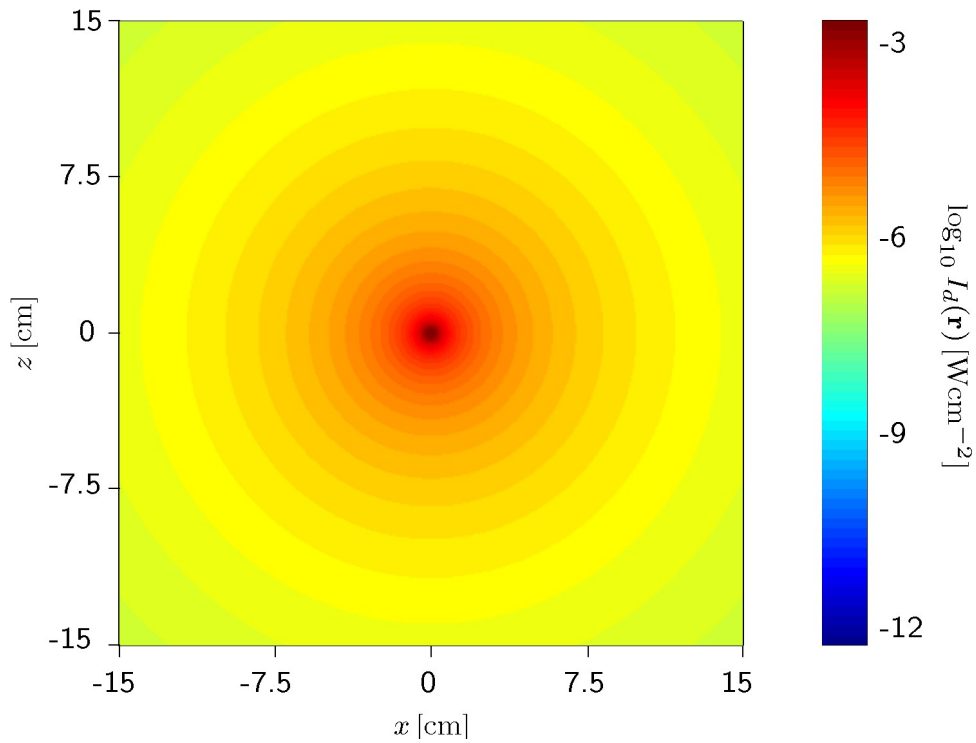


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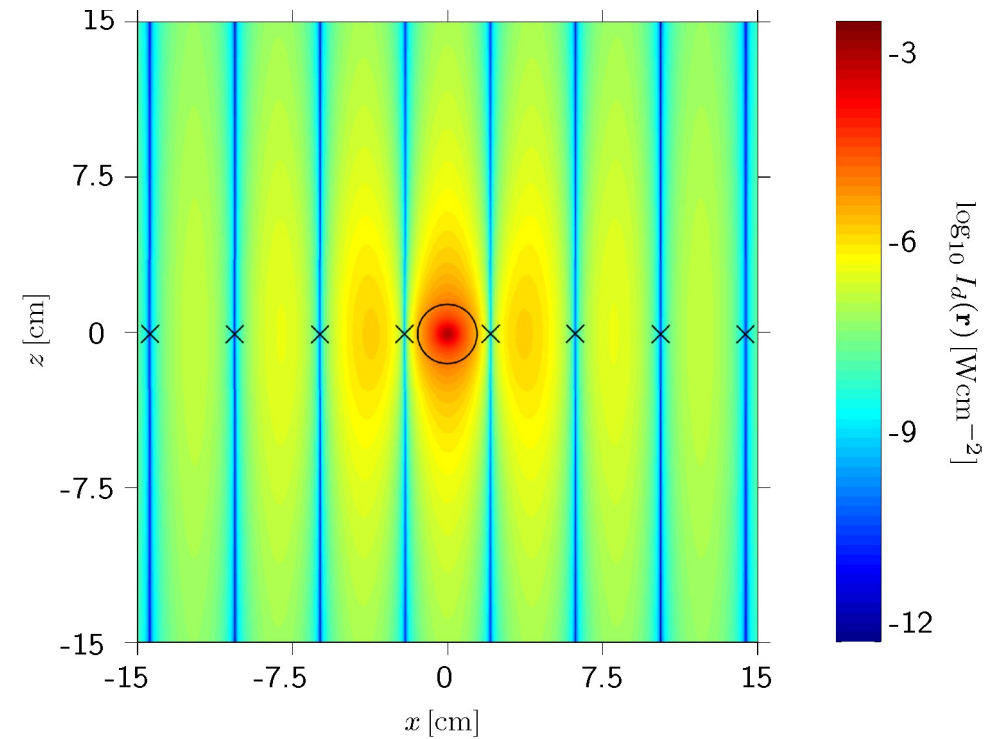
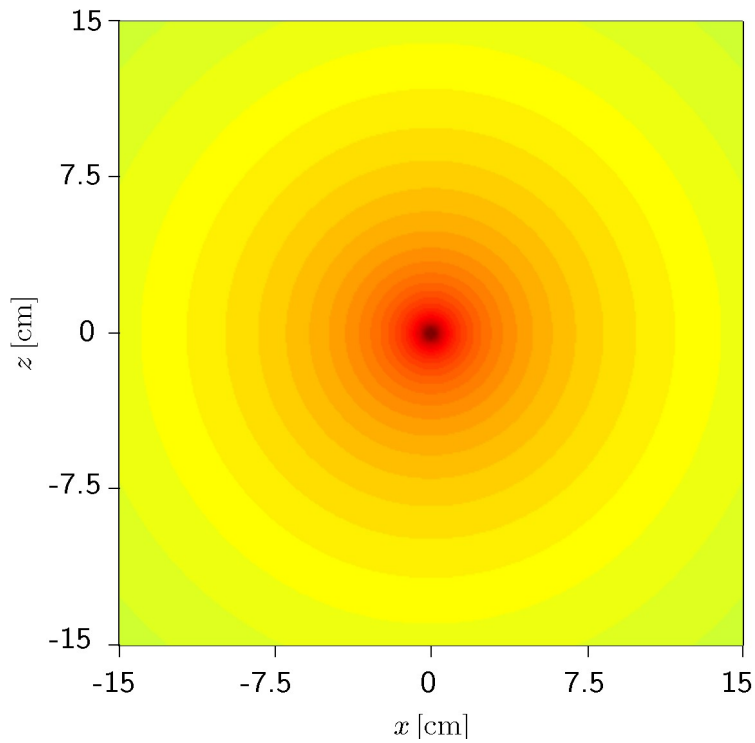


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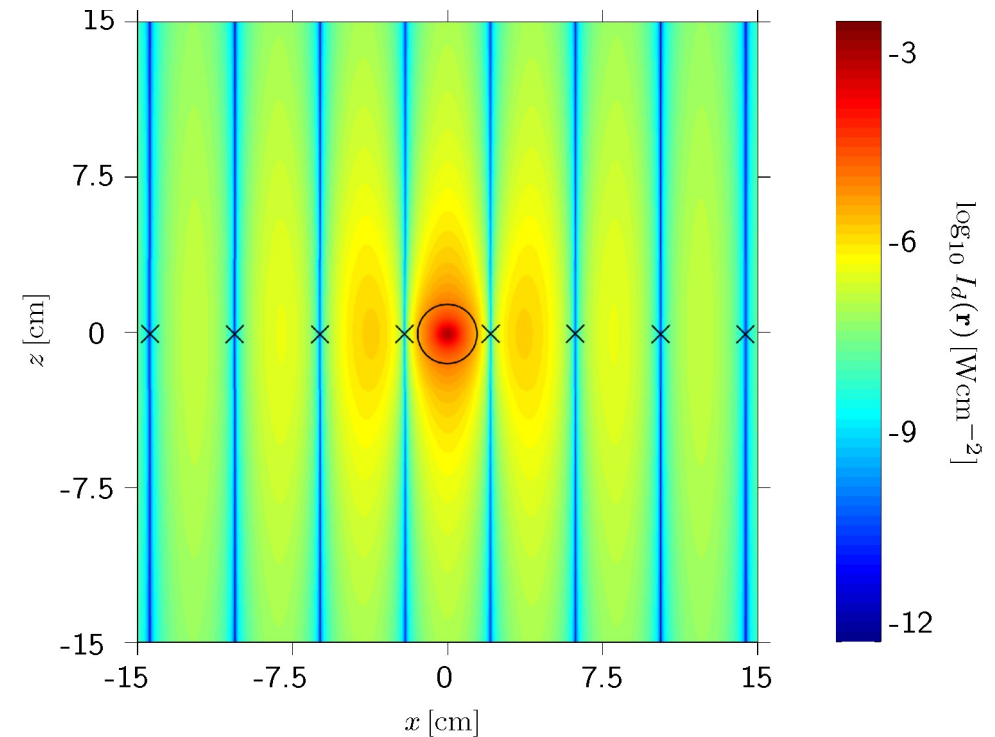
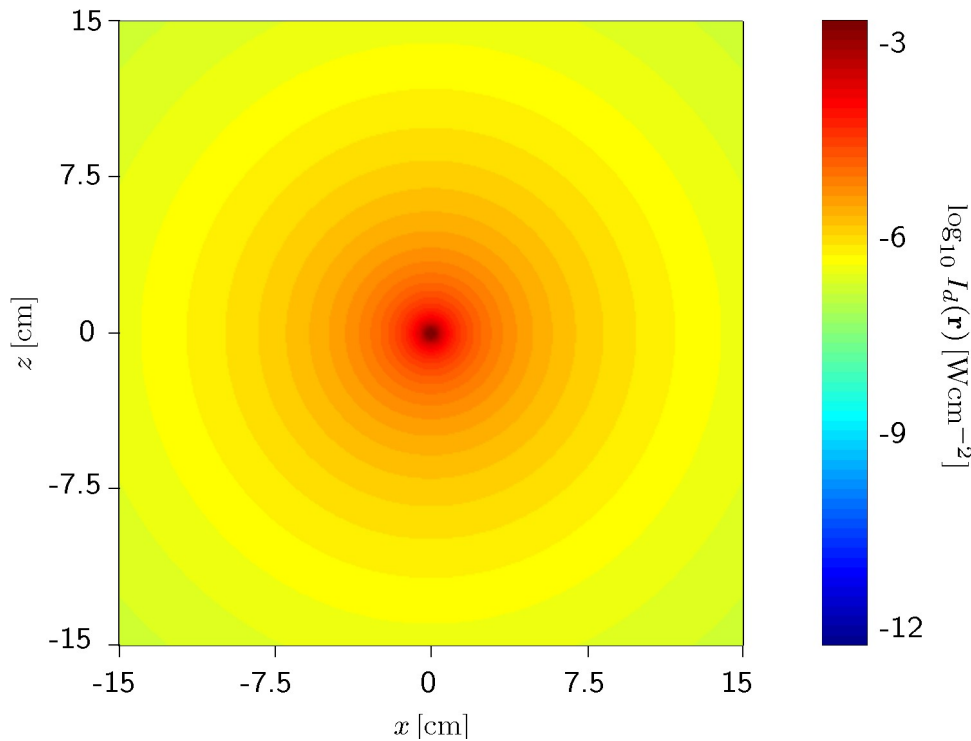


▪ Double-slit minima:

$$(m + 1/2)\lambda_p = D \sin \theta$$



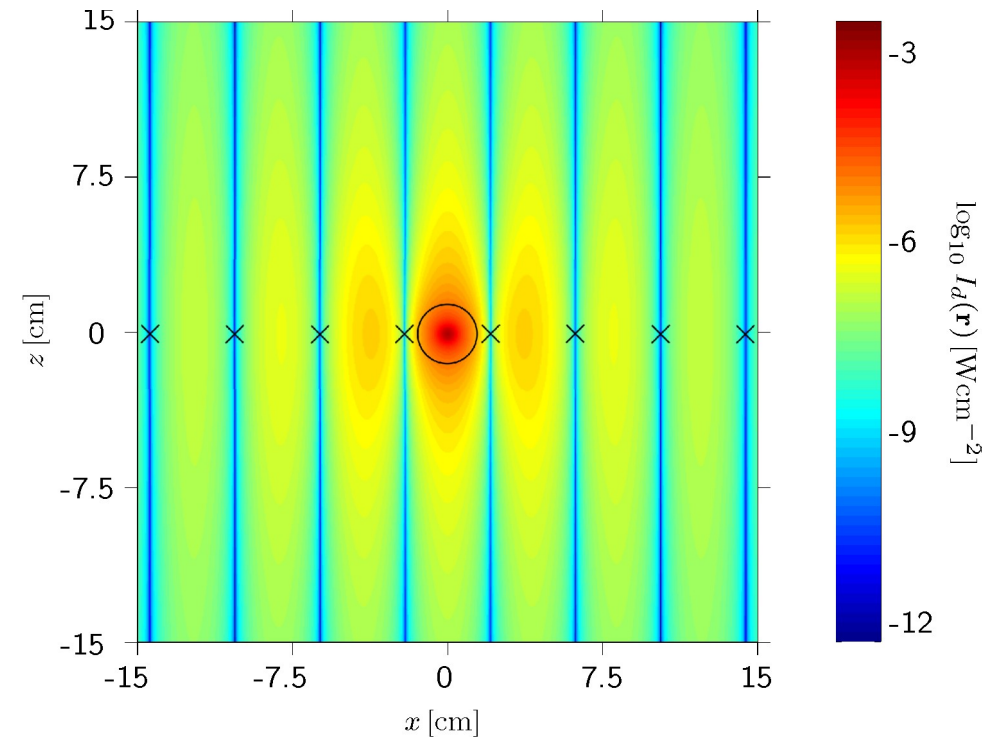
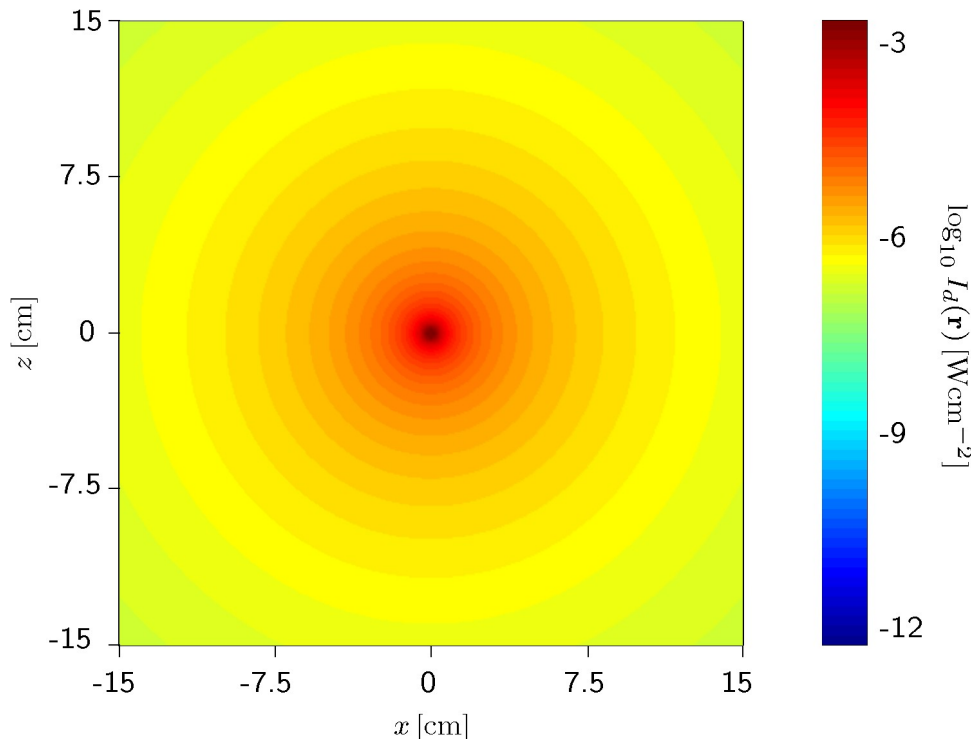
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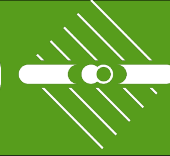
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- “Detectable” vacuum signal: $I_d > 100(I_p + I_{pd})$

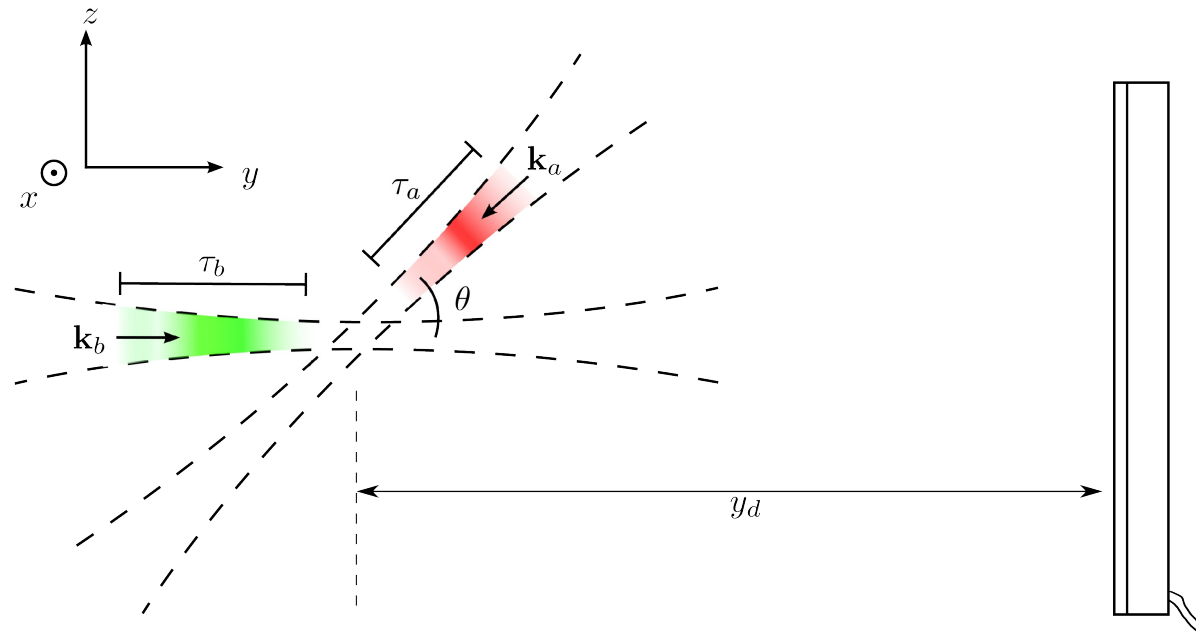


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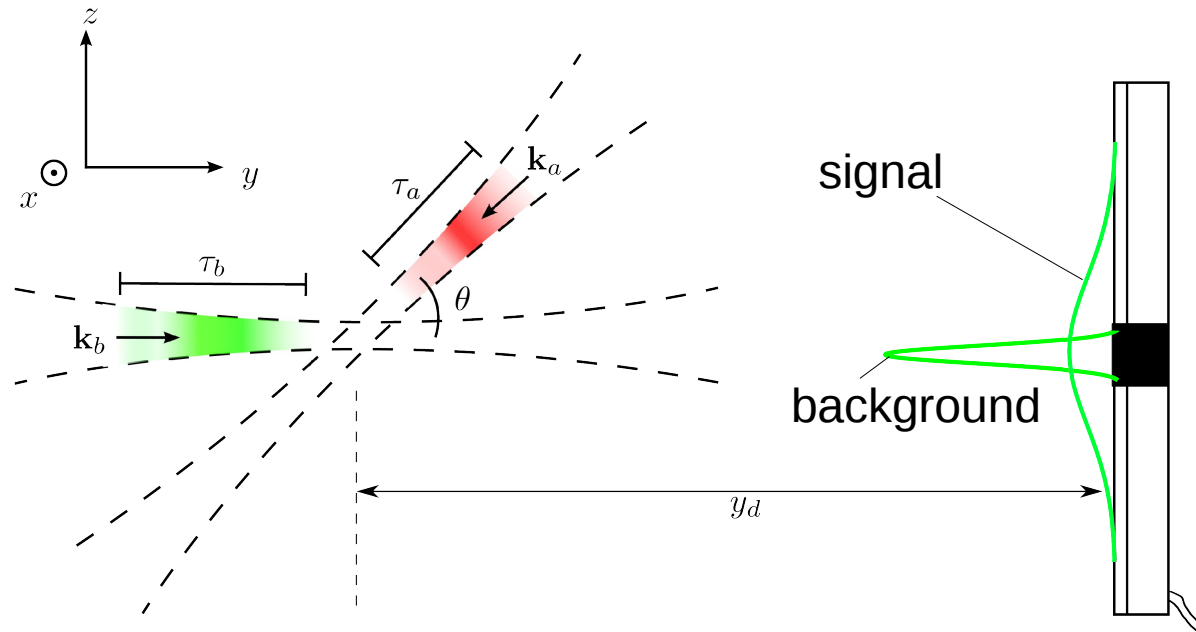


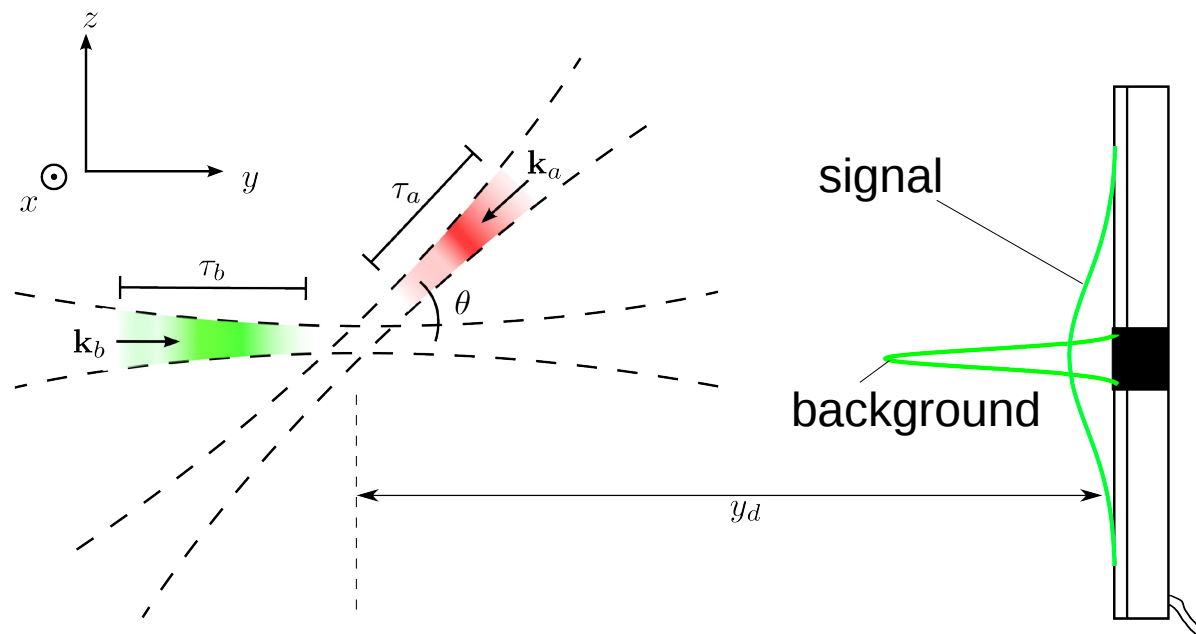
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- $\sim 40 / 4$ diffracted photons per shot





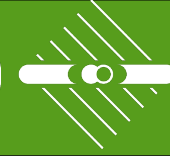
Photon-photon scattering scenario





$$\mathbf{E}_b = \epsilon_b E_b f(x, y, z, t, w_b, \omega_b) g(t - y)$$

$$\mathbf{E}_a = \epsilon_a E_a f(\tilde{x} - \tilde{x}_0, \tilde{y}, \tilde{z} - \tilde{z}_0, t - \Delta t, w_a, \omega_a) g(t - \Delta t + \tilde{y})$$





Vulcan laser parameters:

$$\lambda_a = \lambda_b = 0.91 \mu\text{m}$$

$$\tau_a = \tau_b = 30 \text{ fs}$$

$$P_a = P_b = 5 \text{ PW}$$

$$w_a = 0.91 \mu\text{m}$$

$$w_b = 100 \mu\text{m}$$

Elastic scattering



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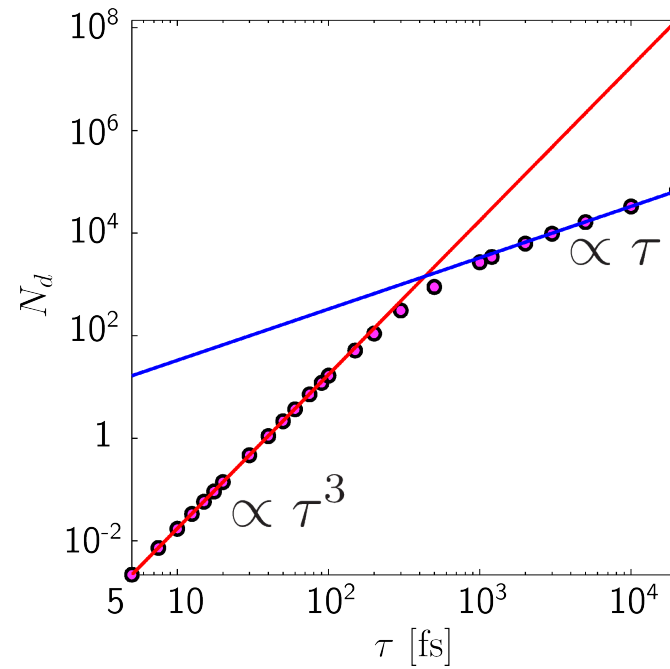
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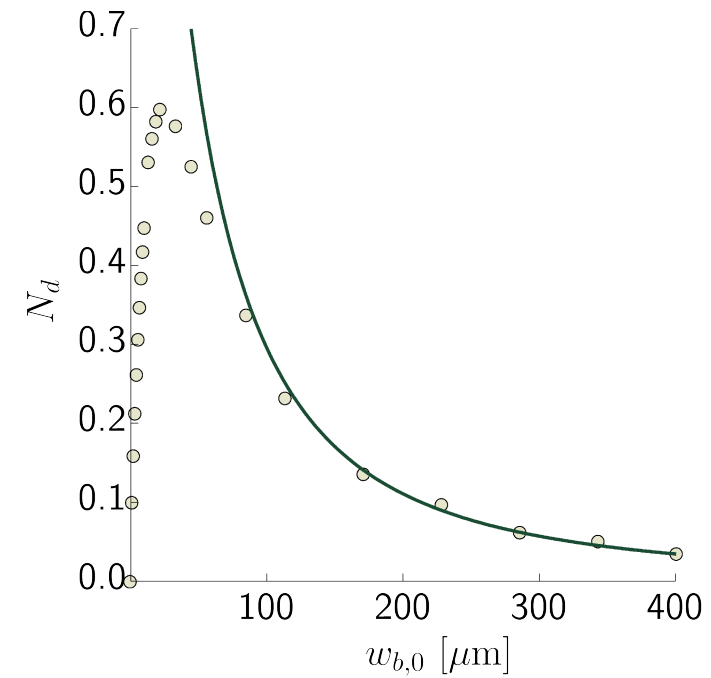
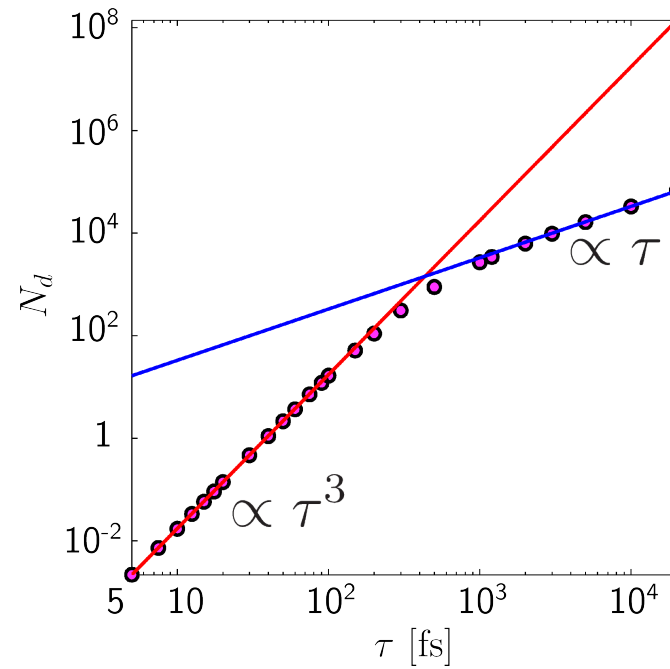
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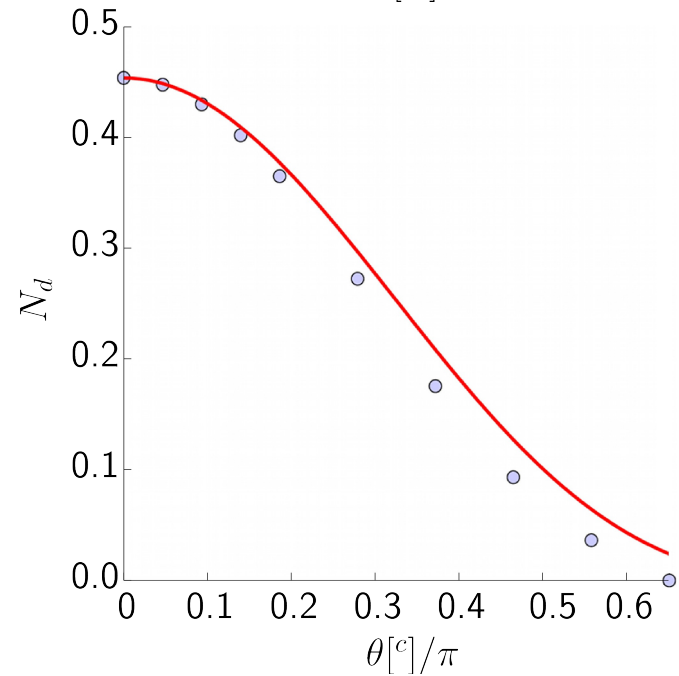
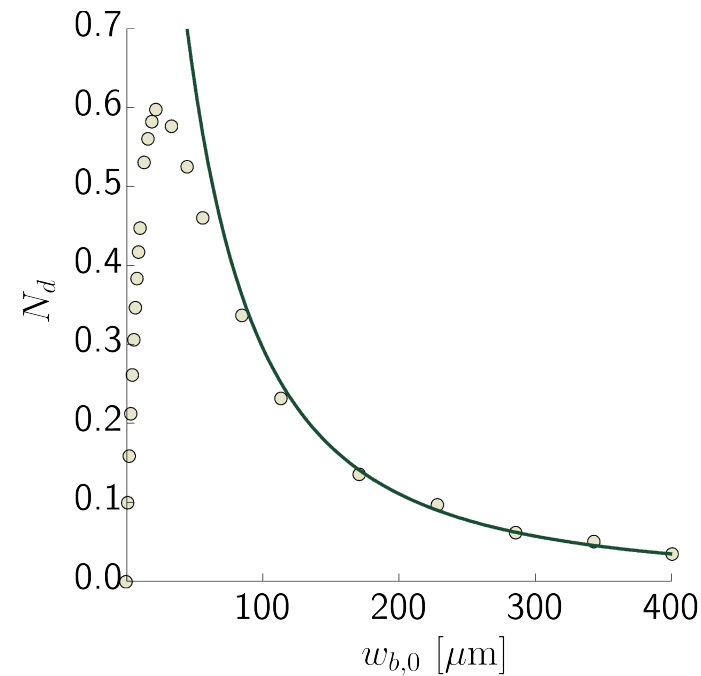
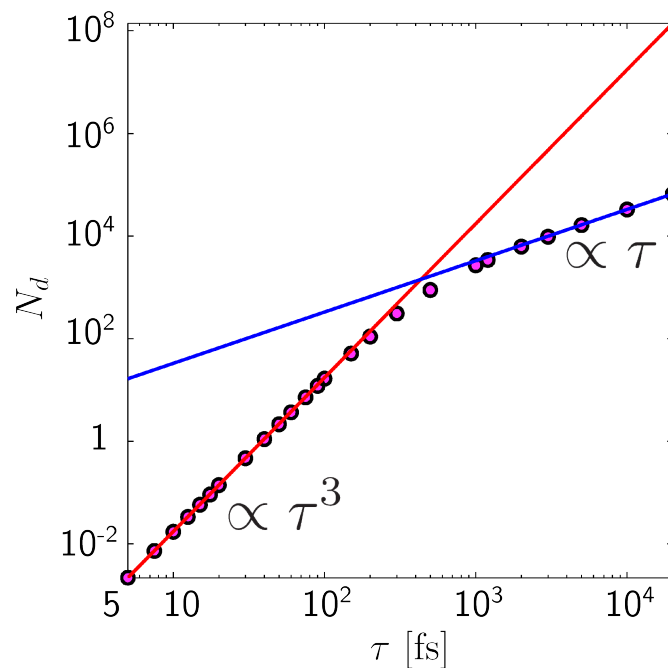
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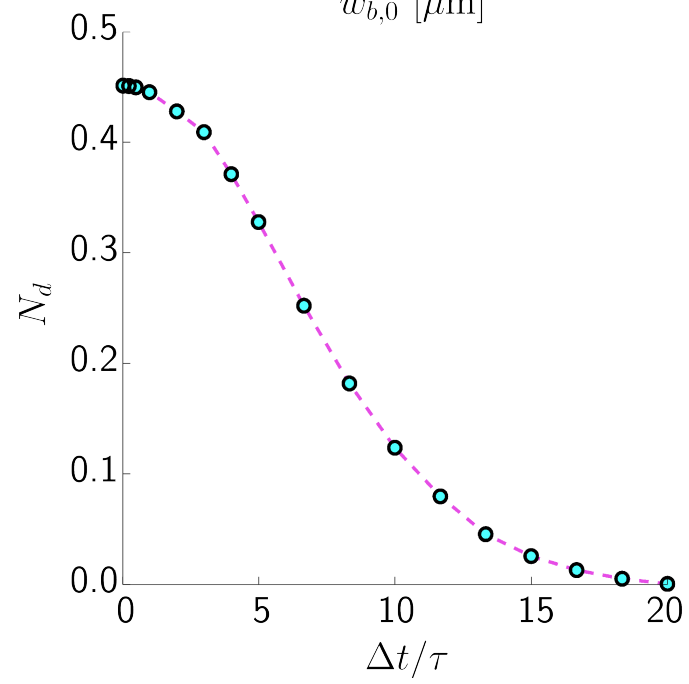
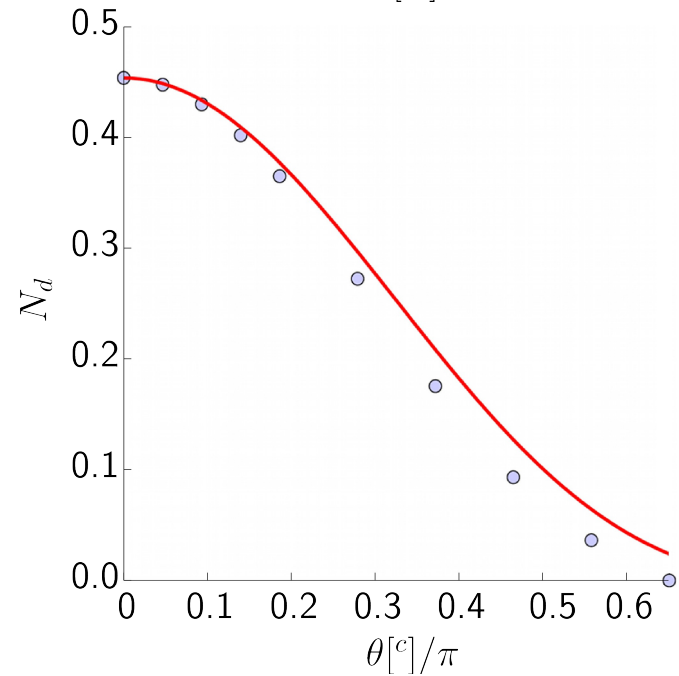
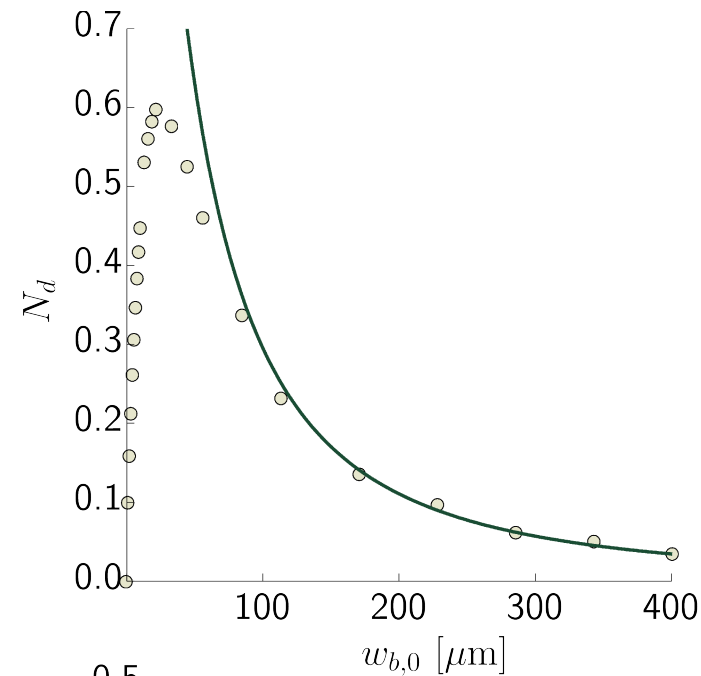
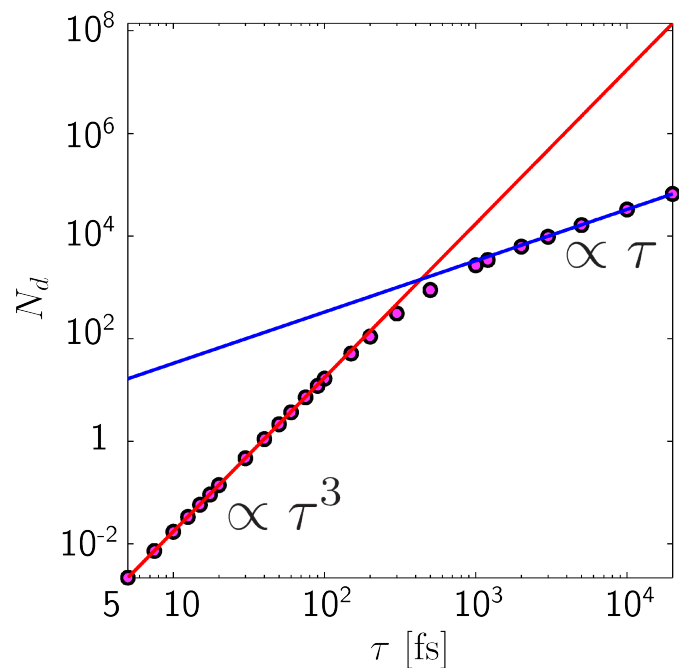
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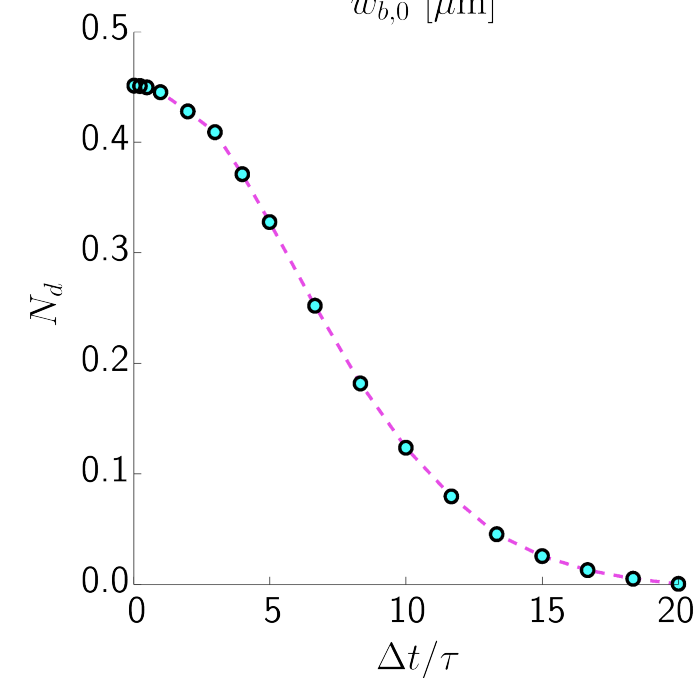
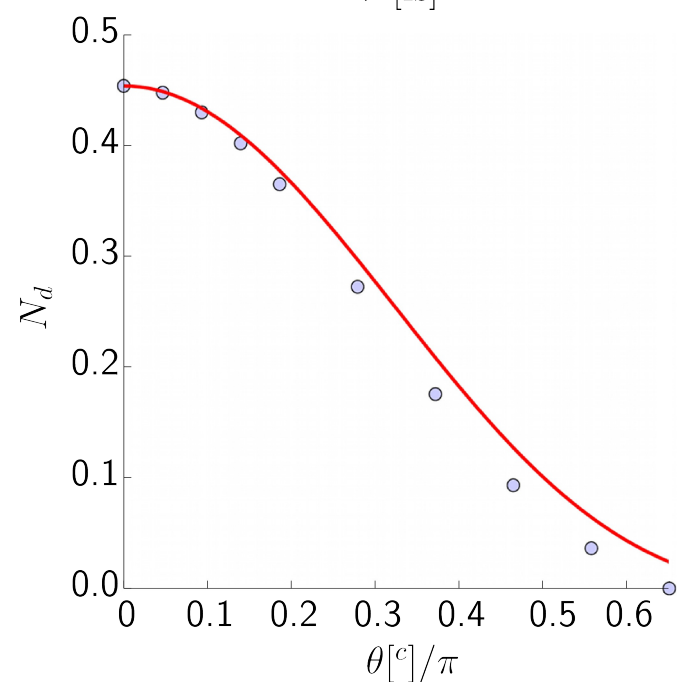
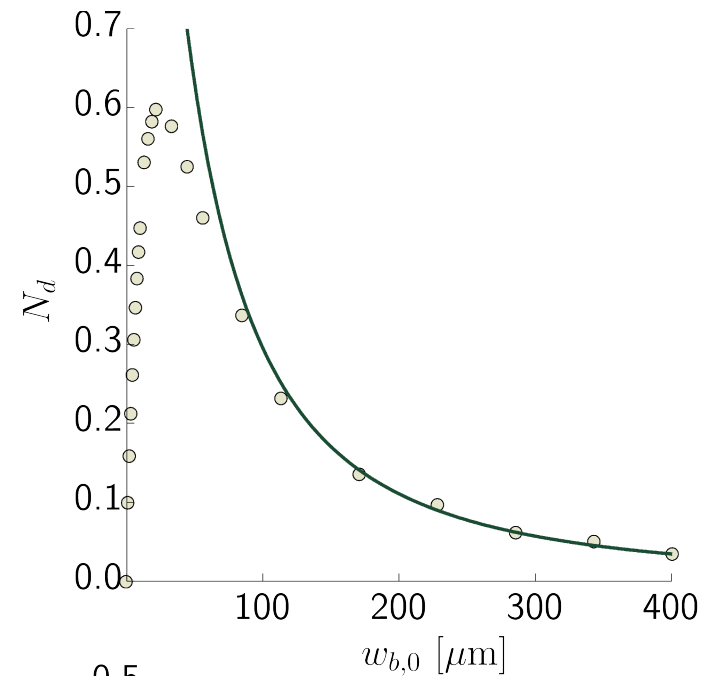
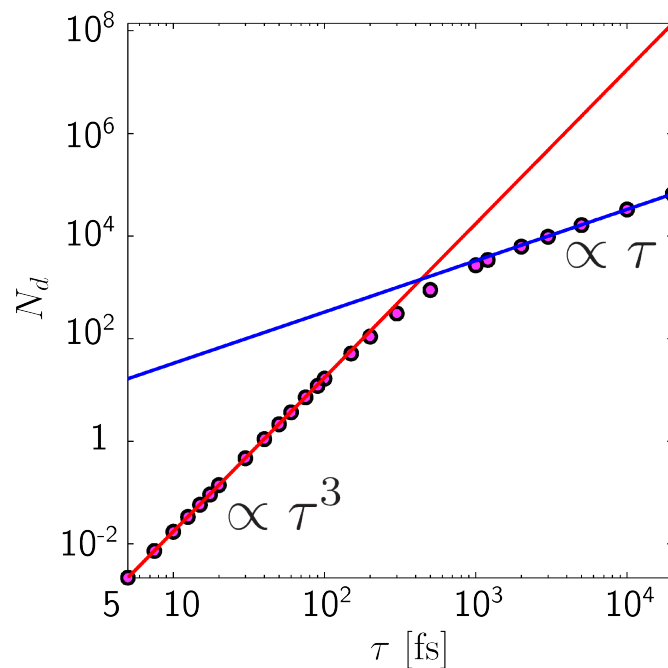
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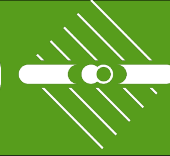
$$\theta = 0.1$$

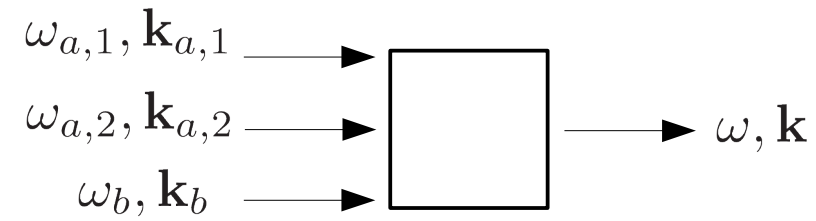
$$P_a = 2P_{\text{tot}}/3$$

$$N_d(\lambda_b = 0.46 \mu\text{m}) \approx 4$$

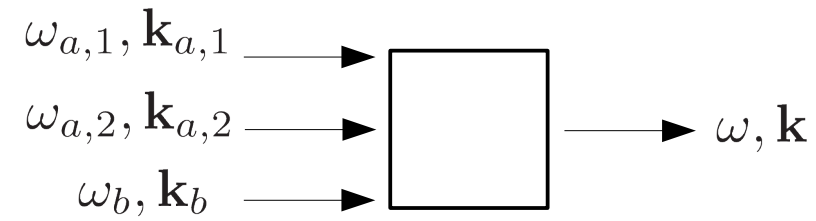
$$N_d(\lambda_b = 0.23 \mu\text{m}) \approx 16$$







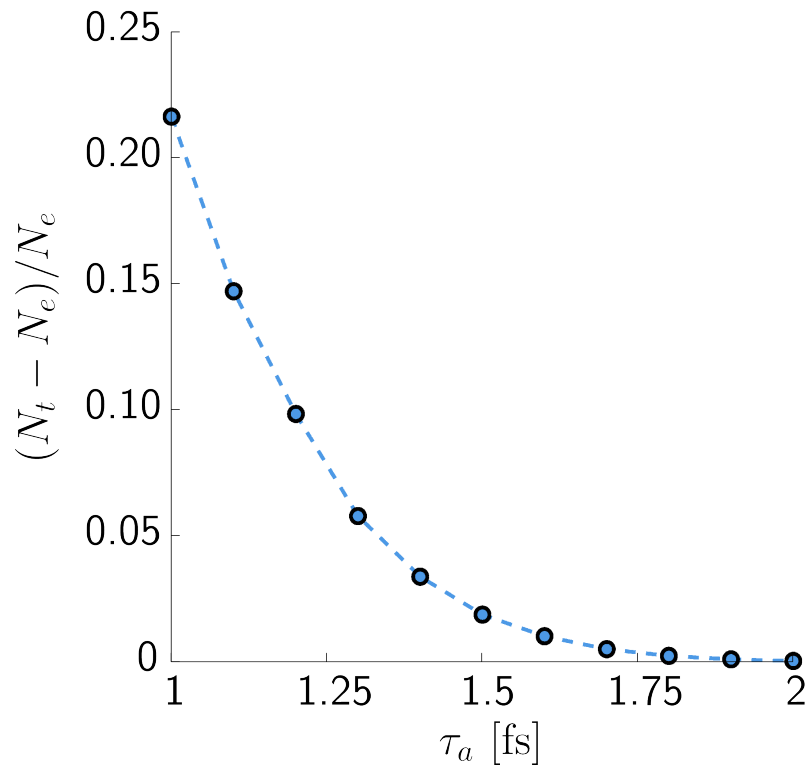
$$\begin{aligned}\omega &= \omega_{a,1} + \omega_{a,2} + \omega_b \\ \omega \frac{y}{r} &= \omega_{a,1} \cos \theta_{a,1} + \omega_{a,2} \cos \theta_{a,2} + \omega_b \cos \theta_b \\ \omega \frac{\rho}{r} &= \omega_{a,1} \sin \theta_{a,1} + \omega_{a,2} \sin \theta_{a,2} + \omega_b \sin \theta_b\end{aligned}$$

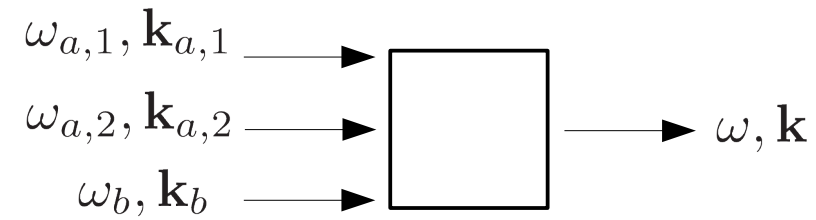


$$\omega = \omega_{a,1} + \omega_{a,2} + \omega_b$$

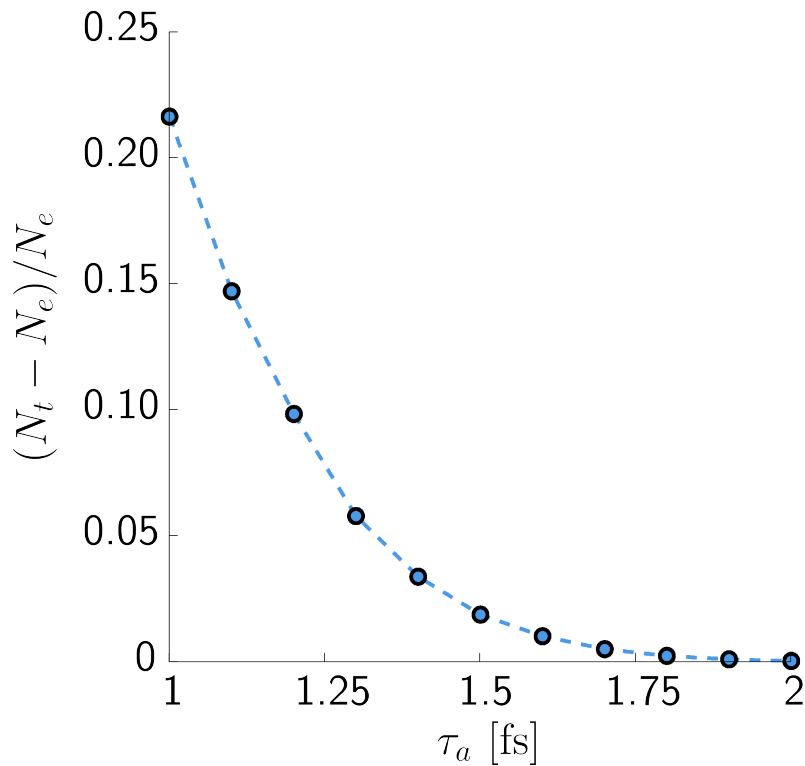
$$\omega \frac{y}{r} = \omega_{a,1} \cos \theta_{a,1} + \omega_{a,2} \cos \theta_{a,2} + \omega_b \cos \theta_b$$

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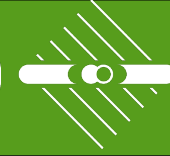
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$$\tau_a = 1 \text{ fs} \quad \tau_b = 30 \text{ fs}$$

$$N_d(\lambda_b = 0.46 \mu\text{m}) \approx 1$$

$$N_d(\lambda_b = 0.23 \mu\text{m}) \approx 4$$





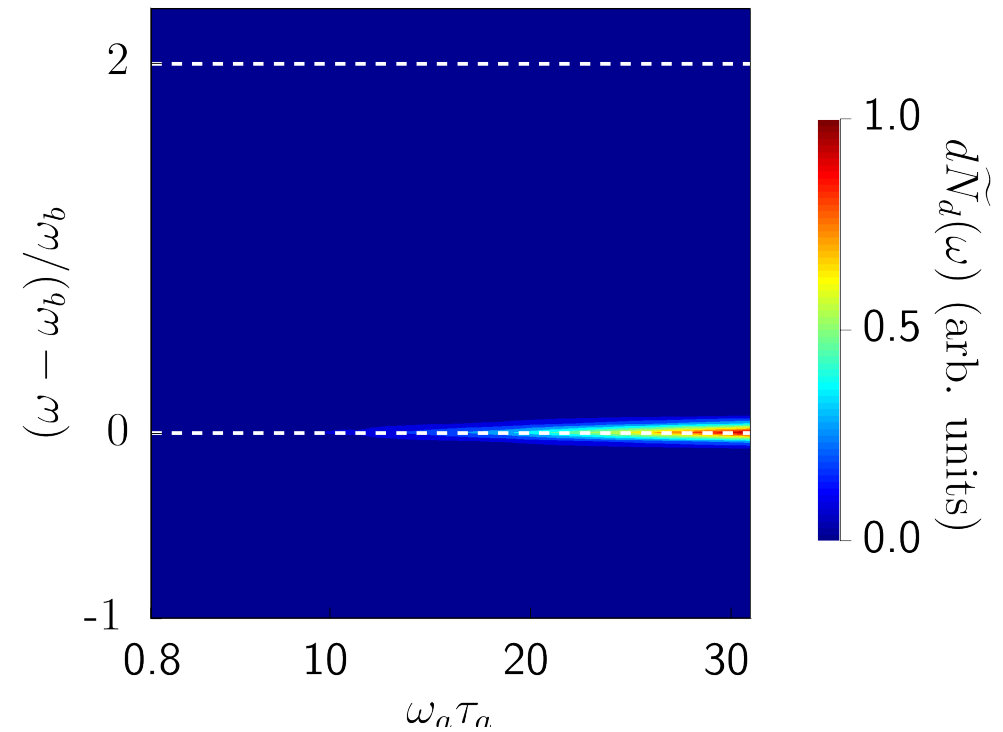
$$\mathbf{E}_a(y + t) = \varepsilon_a E_a \cos[\omega_a(y + t)] \operatorname{sech}[(y + t)/\tau_a]$$

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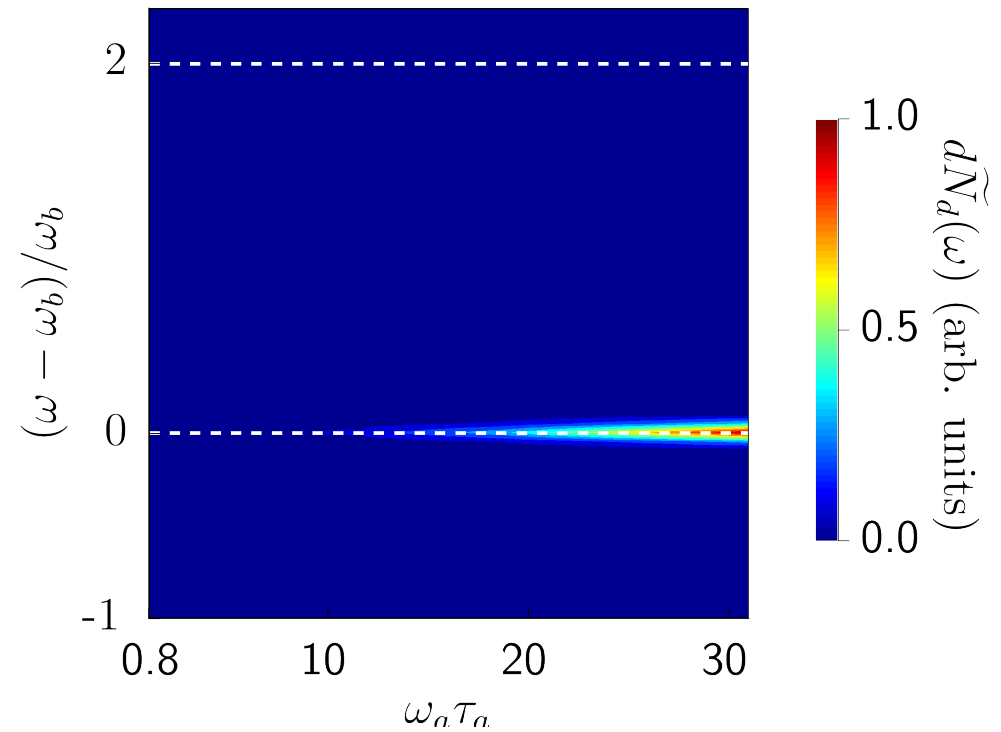
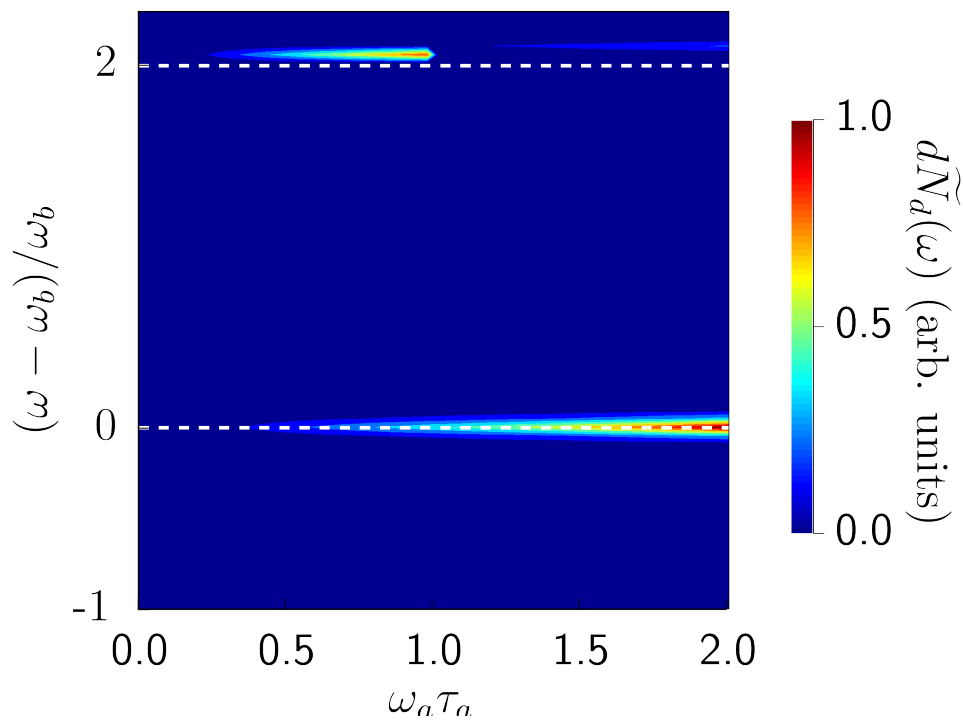
$$\mathbf{E}_b(y - t) = \varepsilon_b E_b \cos[\omega_b(y - t)] \operatorname{sech}[(y - t)/\tau_b]$$





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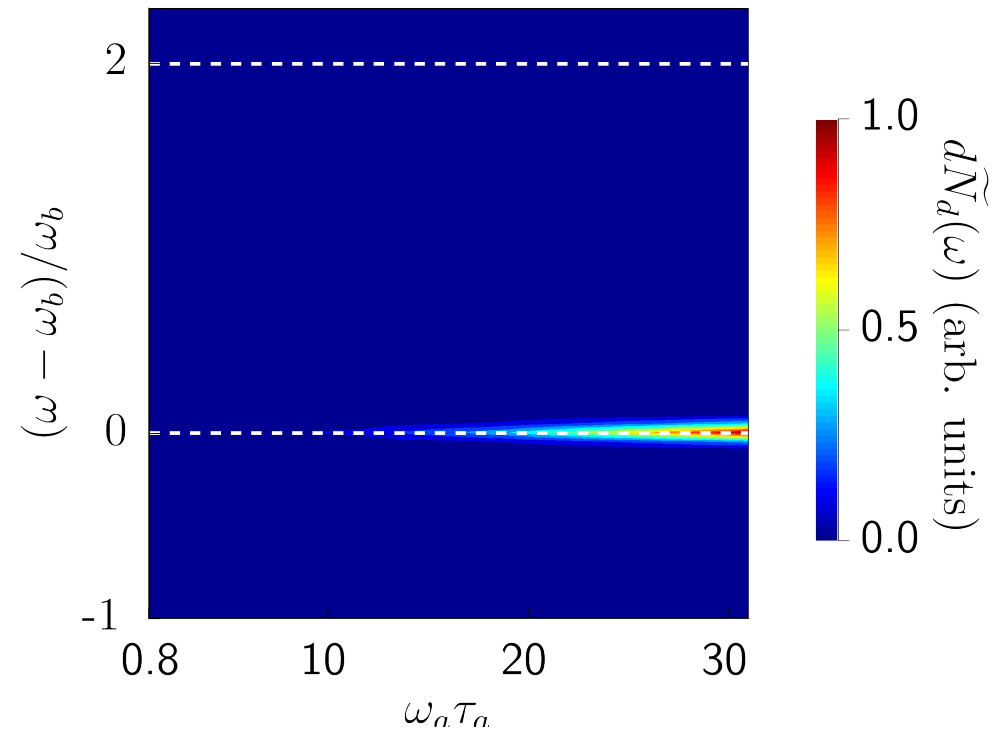
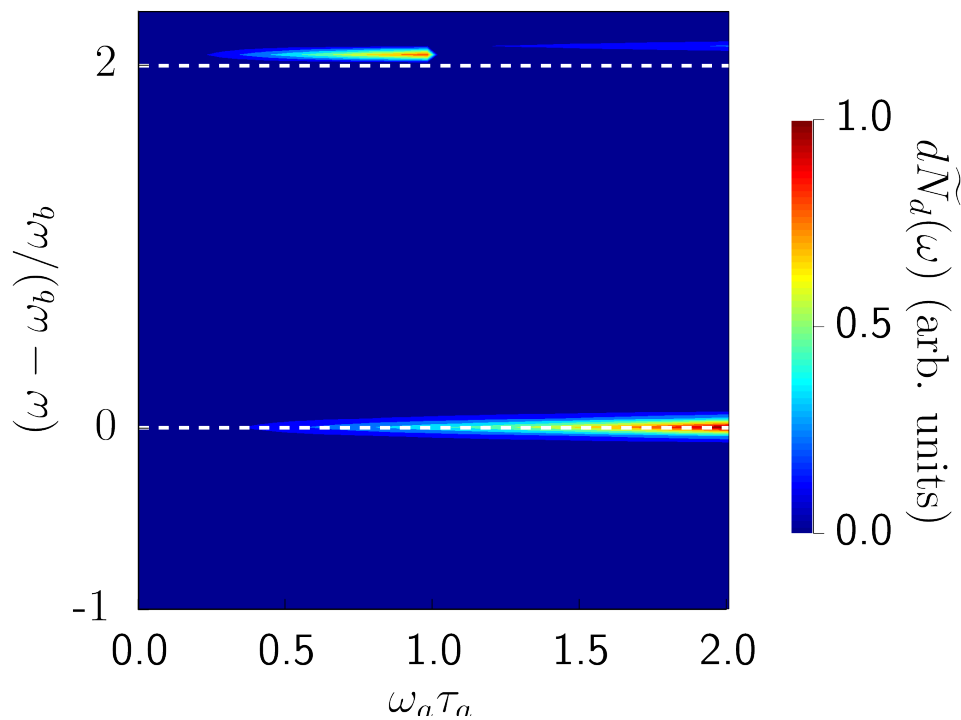
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$$\mathbf{E}_a^2 |\mathbf{E}_b| \sim \cos^2(\varphi_a) \cos(\varphi_b) = \frac{1}{2} [\cos(\varphi_b) + \cos(2\varphi_a) \cos(\varphi_b)]$$





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- Single 10 PW beam split into two pulses sufficient for measuring elastic photon-photon scattering

$$N_d(\lambda_p = 0.23 \mu\text{m}) \approx 16$$

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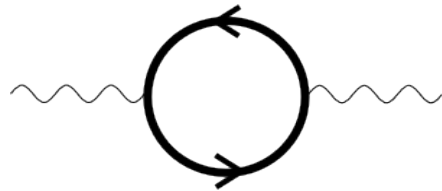
BK and C. H. Keitel, NJP **14** 103002 (2012) (see also NJP Highlights 2012)

BK, A. Di Piazza and C. H. Keitel, Nature Photon. **4**, 92 (2010)

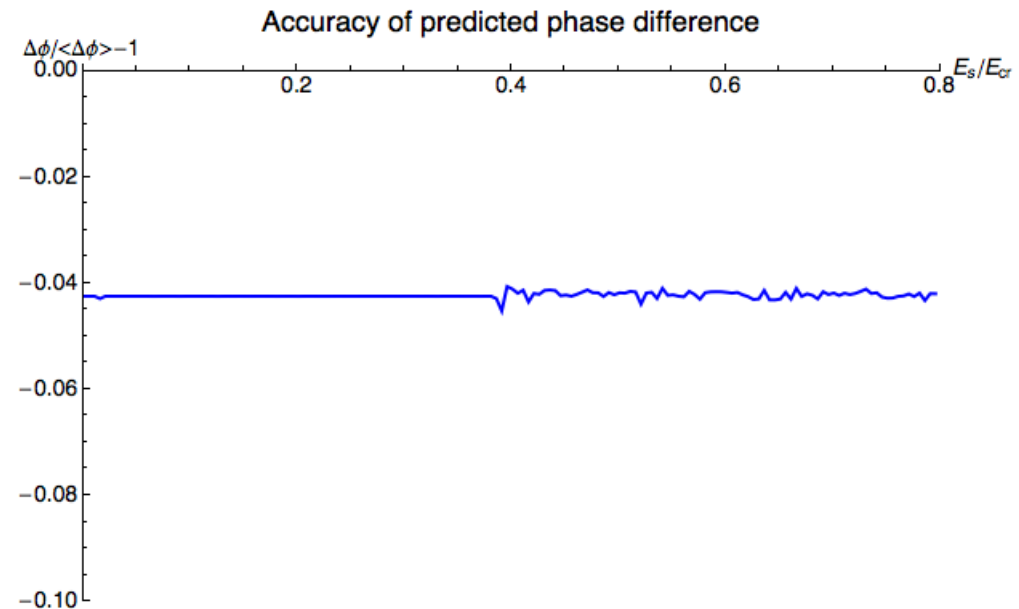
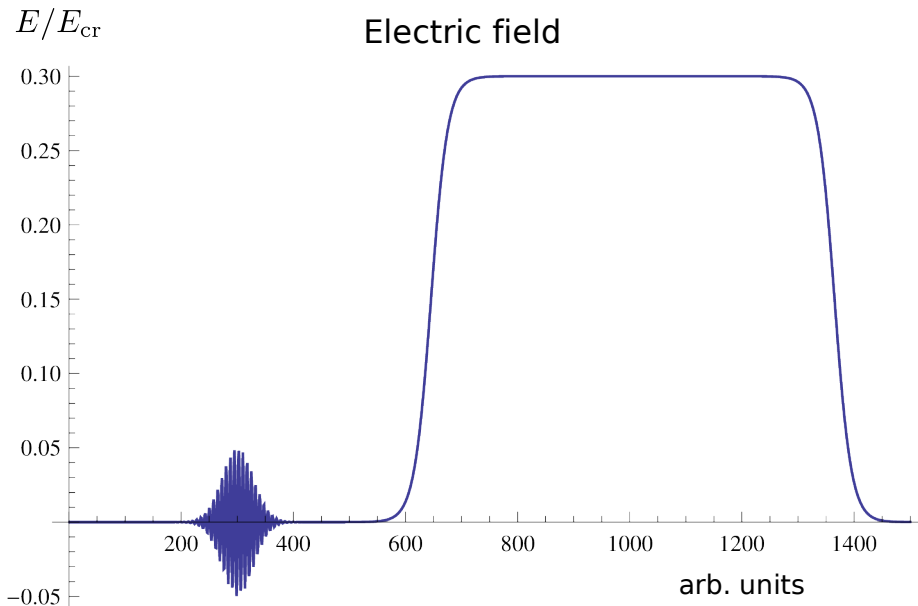
BK, A. Di Piazza and C. H. Keitel, PRA **82**, 032114 (2010)



$$\nabla^2 \mathbf{E} - \partial_t^2 \mathbf{E} = \mathbf{J}_{\text{vac}}(\mathbf{E})$$



$$\Delta\phi = \frac{11 \pm 3}{45\pi} \frac{\alpha I_s}{I_{\text{cr}}} \omega_p T$$



P. Boehl, BK and H. Ruhl (in preparation)



$$-\frac{\alpha}{(2\pi)^4} \int d^4x \text{Tr} \langle x | \frac{1}{\gamma(p + eA) - m} e^{-ikx} \gamma^\mu \frac{1}{\gamma(p + eA) - m} e^{ik'x} \gamma^\nu | x \rangle$$

$$a_0 = \frac{eE\lambda_C}{\hbar\omega},$$

