Unified dispersive approach to real and virtual photon-photon scattering into two pions

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Introduction

- Goal: representations for amplitudes

  $\gamma\gamma^*(q^2) \to \pi\pi$ or $\gamma^*(q^2) \to \gamma\pi\pi$ $\qquad (|q^2| \lesssim 1\ \text{GeV}^2)$

  Method: combine nonperturbative QCD tools [Unitarity, analyticity, Chiral symmetry, soft photon theorems]

- Relevance for muon $g-2$
  - → HVP contribution from $\gamma\pi\pi$ channels [reduced model dependence]
  - → Future: 2π contribution in light-by-light amplitude $\gamma\gamma^*(q_2^2) \to \pi\pi \to \gamma^*(q_3^2)\gamma^*(q_4^2)$ ([Hoferichter et al.])

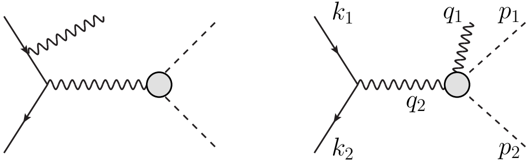
- Account for $\pi\pi$ re-scattering (exp. progress recently: [NA48/2],[DIRAC]...)

Experimental aspects

- $q^2 > 4m_\pi^2$:

  $e^+e^- \to \gamma\pi^0\pi^0$ direct relation to $\gamma^* \to \gamma\pi^0\pi^0$.
  Measurements performed ($q < 1$ GeV):
  SND: [*Phys. Lett. B* **537** *(2002) 201*]
  CMD-2: [*Phys. Lett. B* **580** *(2004) 119*]
  also KLOE [*EPJ C49 (2007) 473*] but $q = m_\phi$

  $e^+e^- \to \gamma\pi^+\pi^-$ ISR and FSR amplitudes interfere.

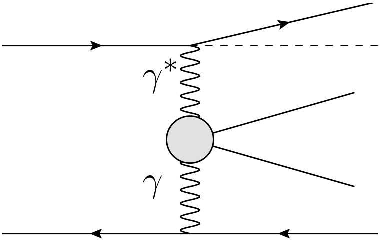


(A) (B)

must measure angular distributions

- $q^2 < 0$:

  From (in principle) $e^+e^- \to e^+e^-\pi^0\pi^0, \to e^+e^-\pi^+\pi^-$ with single tagging



Theory:

- Unitarity key ingredient to FSI
  $\pi\pi$ scattering elastic: $s \lesssim 1$ GeV^2

- Fermi-Watson theorem. Valid for $\gamma\gamma^*(q^2) \to \pi\pi$?
  Depends on q^2
  - → $q^2 \leq 4m_\pi^2$:

$$\mathrm{Im}\,(\gamma\gamma^* \to \pi\pi)_J = (\gamma\gamma^* \to \pi\pi)_J^*\,(\pi\pi \to \pi\pi)_J$$

  - → $q^2 > 4m_\pi^2$:

$$\mathrm{Im}\,(\gamma\gamma^* \to \pi\pi)_J = (\gamma\gamma^* \to \pi\pi)_J^*\,(\pi\pi \to \pi\pi)_J + (\gamma^* \to \pi\pi)(\gamma\pi\pi \to \pi\pi)_J$$

[Creutz,Einhorn PR D1 (1970)2537.]

Chiral low energy expansion:

- Valid when $|q^2|, \ |s| << 1 \text{ GeV}^2$

  $\pi^0\pi^0$ probes **loops**, NLO calculations

  [*Bijnens, Cornet NP B296 (1988) 557*] $(q^2 = 0)$

  [*Donoghue,Holstein, PR D48 (1993) 137*] $(q^2 \neq 0)$

$$
\begin{aligned}
H^n_{++}\Big|_{NLO} &= \frac{2(s-m_\pi^2)}{F_\pi^2}\,\bar{\mathcal{G}}(s,q^2) \\
H^c_{++}\Big|_{NLO} &= \frac{s}{F_\pi^2}\,\bar{\mathcal{G}}(s,q^2) + (\bar{l}_6 - \bar{l}_5)\,\frac{s-q^2}{48\pi^2 F_\pi^2} + H^{Born}_{++}
\end{aligned}
$$

with

$$
\bar{\mathcal{G}}(s,q^2) = \frac{s\bar{G}_\pi(s) - q^2\bar{G}_\pi(q^2)}{s-q^2} - q^2\,\frac{\bar{J}_\pi(s) - \bar{J}_\pi(q^2)}{s-q^2}
$$

→ $\mathrm{Im}\,\bar{G}_\pi(z), \bar{J}_\pi(z) \neq 0$ when $z > 4m_\pi^2$

Analyticity in QCD

- Combine unitarity with analyticity [*Omnès, NC 8 (1958) 316*]. PW amplitudes analytic s with two cuts
  - → right-hand cut: $[4m_\pi^2, \infty]$
  - → left-hand cut: $[-\infty, 0]$ (usually !)

  Discontinuity on RHC:

$$\text{disc}(\gamma\gamma^* \to \pi\pi) = (\gamma\gamma^* \to \pi\pi)_{s-i\epsilon}(\pi\pi \to \pi\pi)_{s+i\epsilon}$$

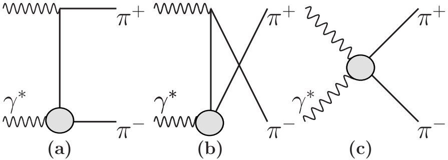
  also when $q^2 > 4m_\pi^2$. FSI problem solved

- Amplitude from LHC from Muskhelishvili equation

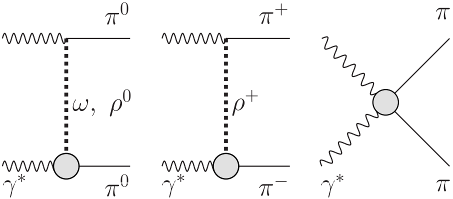
- Appl. to $\gamma\gamma \to \pi\pi$[*Gourdin, Martin NC 17 (1960) 224*]
  Matching w. ChPT [*Morgan, Pennington PL B272 (1991) 134, Donoghue, Holstein PR D48 (1993) 137*]

Phenomenology of LHC

- Pion pole (Born Amplitude)



- Resonance “poles”



- $q^2 \neq 0$: form-factors

- Born helicity amplitudes:

$$H^{Born}_{\lambda\lambda'}(s,q^2,\theta) = F^v_\pi(q^2)\,\bar{H}^{Born}_{\lambda\lambda'}(s,q^2,\theta)$$

$$\bar{H}^{Born}_{++}(s,q^2,\theta) = \frac{2(4m_\pi^2 - q^2(1-\sigma_\pi^2\cos^2\theta))}{(s-q^2)(1-\sigma_\pi^2\cos^2\theta)}$$

$$\bar{H}^{Born}_{+-}(s,q^2,\theta) = \frac{2(s-4m_\pi^2)\sin^2\theta}{(s-q^2)(1-\sigma_\pi^2\cos^2\theta)}$$

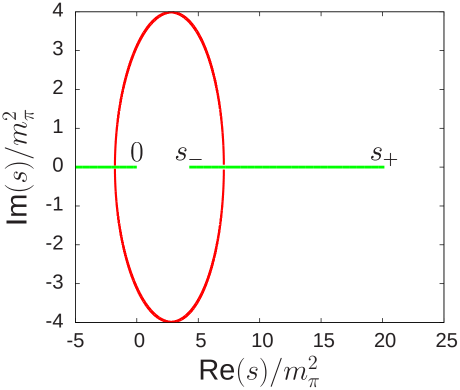
$$\bar{H}^{Born}_{+0}(s,q^2,\theta) = \frac{2\sqrt{2\,q^2}\,(s-4m_\pi^2)\sin\theta\cos\theta}{\sqrt{s}(s-q^2)(1-\sigma_\pi^2\cos^2\theta)}$$

- Partial-wave $J=0$

$$\bar{h}^{Born}_{0,++}(s,q^2) = \frac{1}{s-q^2}\left[\frac{4m_\pi^2}{\sigma_\pi(s)}\log\frac{1+\sigma_\pi(s)}{1-\sigma_\pi(s)} - 2q^2\right],$$

Singularities: LHC $[-\infty, 0]$, pole $s=q^2$ (soft photon)

- Resonance exchange partial-waves: LHC in case $q^2 > 4m_\pi^2$

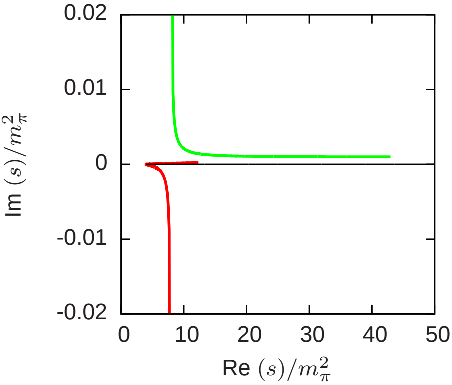


- Omnès applicable ?

- Yes
  a) Use Källen-Lehmann representation for propagators

$$\widetilde{BW}_V(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt' \frac{\sigma(t', m_V, \gamma_V)}{(t'-t)}$$

  b) Limiting prescriptions: $q^2 = \lim_{\epsilon\to 0} q^2 + i\epsilon$
- (generalized) LHC does not intersect RHC



- Representation based on 2-subtracted DR

$$H^I_{++}(s,q^2,z) = F^v_\pi(q^2)\,\bar{H}^{I,Born}_{++}(s,q^2,z) + \sum_{V=\rho,\omega} F_{V\pi}(q^2)\,\bar{H}^{I,V}_{++}(s,q^2,z)$$
$$+\Omega^I_0(s)\Big\{(s-q^2)\,b^I(q^2) + s\,F^v_\pi(q^2)\Big[\frac{s\,(J^{I,\pi}(s,q^2)-J^{I,\pi}(q^2,q^2))}{s-q^2} - q^2\hat{J}^{I,\pi}(q^2)\Big]$$
$$+s\sum_{V=\rho,\omega} F_{V\pi}(q^2)\Big[s J^{I,V}(s,q^2) - q^2 J^{I,V}(q^2,q^2)\Big]\Big\}\,.$$

Remarks:

- → H^I_{++} isospin amplitude, H^n_{++}, H^c_{++} linear combination
- → First line: tree diagrams
- → Omnès function

$$\Omega^I_0(s) = \exp\left(\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'(s'-s)}\,\delta^I_0(s')\right)$$

- (Continued)

→ $J^{I,\pi}(s,q^2)$,$J^{I,V}(s,q^2)$: integrals of $h_0^{I,Born}$,$h_0^{I,V}$, phase-shifts

$$J^{I,\pi}(s,q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^2(s'-s)} \frac{\sin\delta_0^I(s')}{|\Omega_0^I(s')|} \bar{h}_{0,++}^{I,\pi}(s',q^2)$$
$$J^{I,V}(s,q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^2(s'-s)} \frac{\sin\delta_0^I(s')}{|\Omega_0^I(s')|} \bar{h}_{0,++}^{I,V}(s',q^2)$$

(integrals well defined)

→ also

$$\hat{J}^{I,\pi}(q^2) = \left.\frac{\partial J^{I,\pi}(s,q^2)}{\partial s}\right|_{s=q^2}$$

→ Satisfies soft-photon theorem

→ Extra polynomial in s: paramatrize higher energy portions of cuts. Two unknown functions: $b^I(q^2)$

Chiral symmetry constraints

- $\pi^0\pi^0$ amplitude at $s=0$

$$H^n_{++}(0,q^2,z) = \sum_V H^{n,V}_{++}(0,q^2,z) - q^2 b^n(q^2)$$

→ Adler zero ⇒ $\boxed{b^n(q^2) = O(m_\pi^2)}$

Except if $q^2 = 0$! NLO ChPT:

$$\left. H^n_{++}(0,q^2,z)\right|_{NLO} = \frac{-2m_\pi^2}{F_\pi^2}\left(\bar{G}_\pi(q^2) - \bar{J}_\pi(q^2)\right) = \frac{q^2}{96\pi^2 F_\pi^2}\left(1 + \frac{q^2}{15 m_\pi^2} + \cdots\right)$$

- Parametrization of subtraction functions:

$$b^n(q^2) = b^n(0)\,\bar{F}(q^2) \;+\beta_\rho(GS_\rho(q^2)-1)+\beta_\omega(BW_\omega(q^2)-1)$$
$$b^c(q^2) = b^c(0) \;+\beta_\rho(GS_\rho(q^2)-1)+\beta_\omega(BW_\omega(q^2)-1)$$

→ $\bar{F}(q^2)$ from chiral NLO

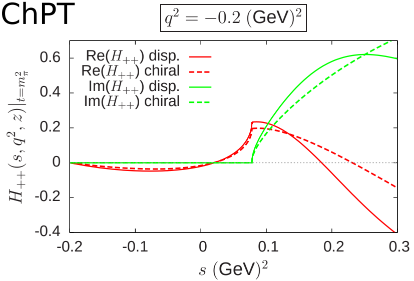
→ $b^n(0)$, $b^c(0)$: comp. with chiral amplitude
Note: relation to pion polarizabilities

$$\alpha_{\pi^0}-\beta_{\pi^0} = \frac{2\alpha}{m_\pi}\left[\lim_{s=0}\frac{1}{s}H^{n,V}_{++}(s,0,\theta)+b^n(0)\right]$$
$$\alpha_{\pi^+}-\beta_{\pi^+} = \frac{2\alpha}{m_\pi}\left[\lim_{s=0}\frac{1}{s}H^{c,V}_{++}(s,0,\theta)+b^c(0)\right]$$

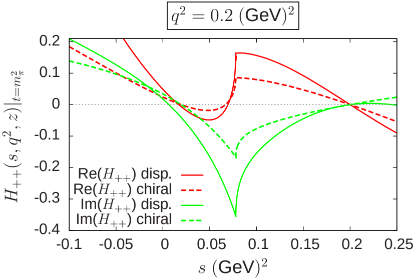
→ β_ρ, β_ω: experimental inputs

Comparison with NLO ChPT

$q^2 = -0.2\ \text{GeV}^2$

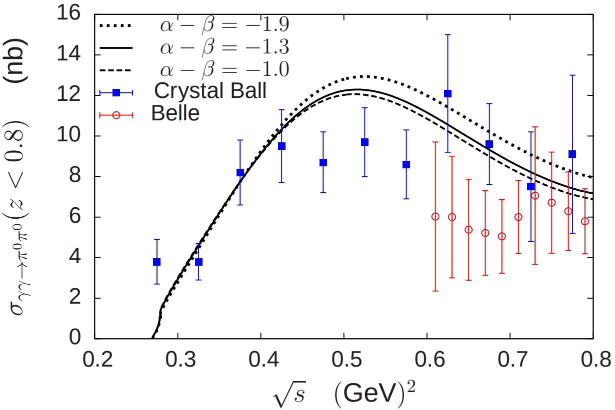


$q^2 = 0.2\ \text{GeV}^2$



→ Form factor =1 at NLO

■ $q^2 = 0$: comparison w. data [$\gamma\gamma \to \pi^0\pi^0$]

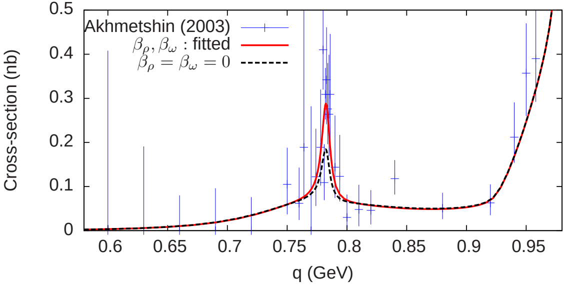


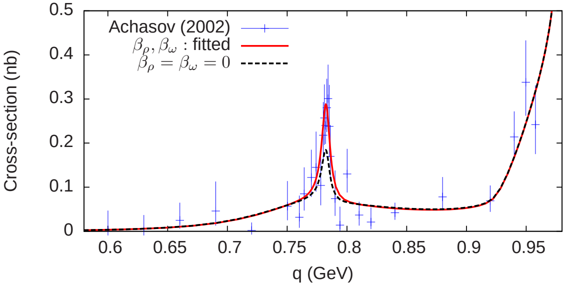
Crystal Ball: [*PR D41 (1990) 3324*]
Belle : [*PR D78 (2008) 052004*]
KLOE-2 [*expected*]

- Fit of $\sigma_{e^+e^-\to\gamma\pi^0\pi^0}(q^2)$ data

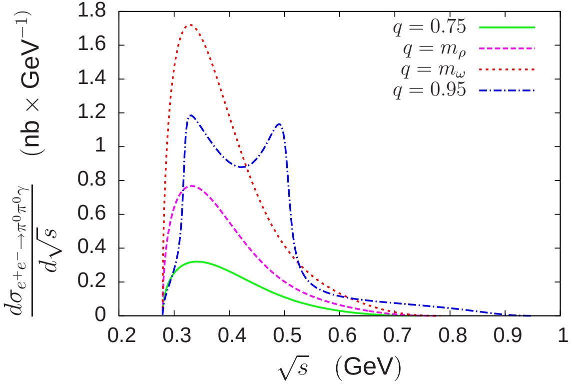
β_ρ	β_ω	χ^2/N_{dof}	ref.
0.14 ± 0.12	$(-0.39\pm0.12)\,10^{-1}$	20.2/27	SND (2002)
-0.13 ± 0.15	$(-0.31\pm0.15)\,10^{-1}$	15.0/21	CMD-2 (2003)
0.05 ± 0.09	$(-0.37\pm0.09)\,10^{-1}$	38.1/50	Combined

Fit of $\sigma_{e^+e^-\to\gamma\pi^0\pi^0}(q^2)$ data (cont.)





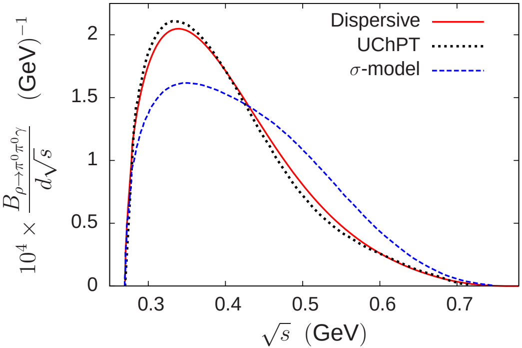
- Differential cross-sections $\frac{d\sigma}{ds}(s, q^2)$



→ shape changes when $q > (m_\rho + m_\pi)$

→ no σ meson “bump”

- Illustrative comp. w. other approaches
  - → Consider $\rho^0 \to \pi^0\pi^0\gamma$



– Chiral Lagr.+V + unitarized pion loop *[Palomar,Hirenzaki,Oset NP A707 (2002)161]*

– Chiral Lagr.+V +pion loop +sigma meson *[Bramon et al. PL B517 (2001) 345]*

Muon $(g-2)/2$:

- $\gamma\pi\pi$ contribution in HVP

$$a_\mu^{[\gamma\pi\pi]} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{q^2_{max}} dq^2\, K_\mu\left(q^2\right)\, \sigma_{e^+e^-\to\gamma^*\to\gamma\pi\pi}(q^2)$$

cross-sections: σ^c, σ^n

$$\sigma(q^2) = \frac{\alpha^3}{12(q^2)^3}\int_{4m_\pi^2}^{q^2} ds(q^2-s)\sigma_\pi(s)\int_{-1}^{1} dz \sum |H_{\lambda\lambda'}(s,q^2,\theta)|^2$$

- σ^c:
  Separate Born $|H^c_{\lambda\lambda'}|^2 = |H^{Born}_{\lambda\lambda'} + \hat{H}^c_{\lambda\lambda'}|^2$

  → $\sigma^c(q^2) = \sigma^{Born}(q^2) + \hat{\sigma}^{Born}(q^2) + \hat{\sigma}^c(q^2)$

  → Define σ^{Born} (e.g. add rad. corr. part $\sigma(e^+e^- \to \pi^+\pi^-)$)

- σ^{Born} corresponds to sQED

$$\sigma^{Born}(q^2) = \frac{\pi\alpha^2}{3\,q^2}\,\sigma_\pi^3(q^2)\,|F_\pi^v(q^2)|^2 \times \frac{\alpha}{\pi}\eta(q^2)$$

- Numerical results:

channel	cross-section	a_μ
$\gamma\pi^+\pi^-$	σ^{Born}	41.9×10^{-11}
$\gamma\pi^+\pi^-$	$\hat{\sigma}^{Born}$	$(1.31 \pm 0.30) \times 10^{-11}$
$\gamma\pi^+\pi^-$	$\hat{\sigma}^{c}$	$(0.16 \pm 0.05) \times 10^{-11}$
$\gamma\pi^0\pi^0$	σ^{n}	$(0.33 \pm 0.05) \times 10^{-11}$

Remarks

→ a_μ^{Born} comparable to $\Delta a_\mu^{SM} = \pm 49 \times 10^{-11}$

→ $a_\mu[\hat{\sigma}^{Born}] > 0$ unlike [*Dubinsky et al. EPJ C40 (2005)41*]

→ σ-meson approx: $a_\mu^{[\gamma\sigma]} = 1.2 \times 10^{-11}$ [*Narison (2003)*], $= 1.5 \times 10^{-11}$ [*Ahmadov,Kuraev,Volkov (2010)*]

Conclusions

- Analyticity based treatment of FSI in $\gamma\gamma \to \pi\pi$ extended to $\gamma\gamma^*(q^2)$
- Main issue: left-hand cut [pion, resonances] becomes generalized one, but properly defined
- Good description of experimental data $e^+e^- \to \gamma\pi^0\pi^0$ (2 parameters)
- a_μ: contributions from $\gamma\pi^0\pi^0$, $\gamma\pi^+\pi^-$ [$q < 0.95$ GeV]
- Other applications: pion generalized polarizabilities, sigma meson (pole)-γ form factor
- Extensions possible [$q \gtrsim 1$ GeV] (coupled-channel MO). Double virtual scattering ?