How to define the entropy of dense QCD states

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R.P, arXiv:1211.6911 and PRD (2013)
R.P. (2013) to appear

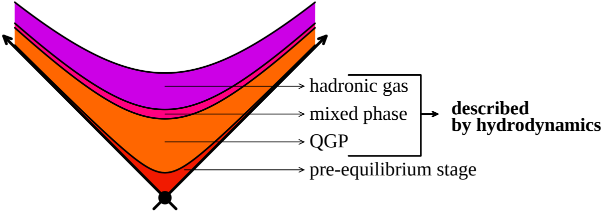
Outline

1. **Why**
2. **How**
3. **How much**

Why

QCD processes in heavy-ion collisions

Aim: Glueing together two "languages"



(from F.Gélis)

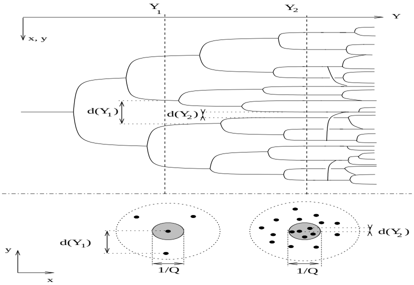
- Initial Stage: Scattering, Partons, Feynman diagrams, Weak coupling
- Second stage: Flow, Temperature, Hydrodynamics, Strong coupling
- Final stage: Particles/Hadrons

Key concept: ENTROPY

The Saturation Mechanism

Theory: Rapidity evolution at weak coupling

(from S.Munier & R.P. 2003)



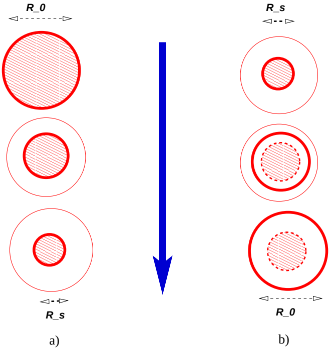
- I: First gluon branching $Y_0 \to Y_1,\ d(Y) \gg 1/Q_s$
- II: Exponential growth from Branching $Y_1 \to Y_2,\ d(Y) \gtrsim 1/Q_s$
- III: Branching + Recombination = Saturation $Y > Y_2,\ d(Y) \ll 1/Q_s$

Branching equilibrated by Recombination

How

Variation of the QCD color correlation length

Hint: Out-of-equilibrium Thermodynamics



- Left: Compression due to Saturation
- Right: Expansion $\longrightarrow$ Non-Reversibility

Generalized Entropy for Non-equilibrium processes

Theory: Non-equilibrium work identities

Jarzynski (1997), Crooks(1998), Hatano-Sasa(2001)

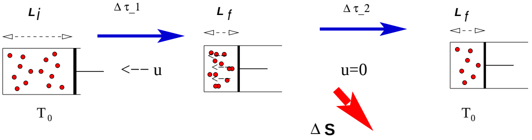
- Dissipative Work Distribution: a puzzling property

$$\boxed{\left\langle e^{-\mathcal{W}_{diss}/T} \right\rangle = 1}$$

- $\langle f(X) \rangle \geq f(\langle X \rangle)$, If $f(X)$ convex — Jensen(1906)

$$\boxed{\Delta S \equiv \frac{1}{T} \langle \mathcal{W}_{diss} \rangle \geq 0}$$

- Explicit Example: "Piston model" — Lua & Grosberg (2008)



Generalized Entropy for CGC States

- **QCD Input:** Unintegrated Gluon Distribution $\phi(k, Y) \sim \phi(k/Q_s(Y))$
- Probability Distribution $\mathcal{P}(k; Y)$

$$\langle\ [\cdots]\ \rangle_{\mathcal{P}(k;Y)} \equiv \int d^2k\ [\cdots]\ \left\{ \frac{\phi(k,Y)}{\int \phi(k,Y)\ d^2k} \right\}$$

- Jarzynski-Crooks-Sasa identity for CGC

$$\left\langle e^{-\left\{\log \frac{\mathcal{P}(k;Y_2)}{\mathcal{P}(k;Y_1)}\right\}} \right\rangle_{\mathcal{P}(k;Y_2)} = \int d^2k\ \mathcal{P}(k;Y_1) \equiv\ 1\ ,$$

- Generalized Entropy

$$\Delta\Sigma^{Y_1 \to Y_2} = \left\langle \log \frac{\mathcal{P}(k;Y_2)}{\mathcal{P}(k;Y_1)} \right\rangle_{\mathcal{P}(k;Y_2)} \equiv \int d^2k\ \mathcal{P}(k;Y_2) \left\{ \log \frac{\mathcal{P}(k;Y_2)}{\mathcal{P}(k;Y_1)} \right\} \geq\ 0\ ,$$

Application to Gaussian CGC models

- Probability Distribution $\phi(k;Y) = (k^2/Q_s^2)^{\kappa-1} e^{-k^2/Q_s^2}$

$$\mathcal{P}_\kappa(k;Y)\, d^2k \equiv \mathcal{P}(u = k^2/Q_s^2)\, du = \frac{1}{\Gamma(\kappa)}\, u^{\kappa-1}\, e^{-u}\, du\ ,$$

Golec-Biernat,Wüsthoff: $\kappa = 2$

- Generalized Entropy: “compression” $Q_1 \to Q_2$ due to saturation

$$\Sigma^{Q_1 \to Q_2} = \kappa \left\{ \left(Q_2^2/Q_1^2 - 1\right) - \log\left(Q_2^2/Q_1^2\right) \right\} \geq 0\ .$$

- Reversed process: “expansion” $Q_2 \to Q_1$ due *e.g.* to cooling

$$\Sigma^{Q_2 \to Q_1} = \kappa \left\{ \log\left(Q_2^2/Q_1^2\right) - \left(1 - Q_1^2/Q_2^2\right) \right\} \geq 0\ .$$

Strong Irreversibility

How Much

"Microscopic" *vs.* "macroscopic" Entropy

- "Macroscopic Entropy" $\sim$ gluon multiplicity K.Kutak (2011)

$$\boxed{\left\{ dE = Td\Sigma \ ; \ T = \frac{Q_s}{2\pi} \ ; \ E = N_g \times Q_s, N_g \propto Q_s^2 \right\} \Rightarrow \Sigma \propto Q_s^2}$$

{ Equilibrium ; Unruh-Karzeev-Tuchin ; Dense-Dilute gluon production }

- Comparison with a 1-dimensional gas model

$$\Sigma^{L_1 \to L_2} \sim \frac{1}{T} \left\{ \langle \mathcal{W} \rangle - \Delta F^{L_1 \to L_2} \right\} \propto \frac{L_i}{L_f} - 1$$

$$\Sigma^{L_2 \leftarrow L_1} \sim -\Delta F^{L_2 \to L_1} \propto \log \left(L_1 / L_2 \right)$$

- CGC $\sim$ Effective 2-dimensional Thermal System at $T \sim Q_s$

The Initial State Problem

- Entropy of the Glasma with correlation scale $Q(Y)$

$$\mathcal{P}_{glasma}\ d^2k = \frac{\phi_{glasma}(p_\perp;Q)\ d^2k}{\int \phi_{glasma}(p_\perp;Q)\ d^2k} \Rightarrow \Sigma_{glasma}^{Q_1\to Q_2} \equiv \left\langle \log \frac{\mathcal{P}_{glasma}(p_\perp;Q_1)}{\mathcal{P}_{glasma}(p_\perp;Q_2)} \right\rangle_{Q_2} \geq\ 0$$

cf. Y-Evolution from T.Lappi (2011)

- Weak/Strong coupling transition

$$\epsilon^{glasma} = \frac{N_c^2-1}{4\pi^2 N_c}\cdot Q^4_{glasma} \quad ; \quad \Sigma^{glasma} = \kappa_{gl}\ \frac{N_c^2-1}{4\pi^2 N_c}\cdot Q^2_{glasma}$$

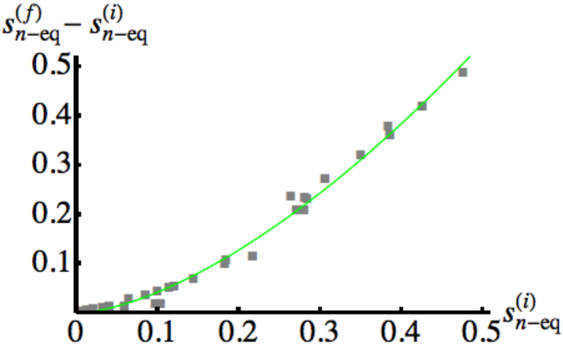
$$\epsilon_i^{AdS/CFT} = N_c^2\cdot \tfrac{3}{8}\pi^2\cdot T_i^4 \quad ; \quad \Sigma_i^{AdS/CFT} = N_c^2\cdot \tfrac{1}{2}\pi^2\cdot T_i^2\cdot s_i$$

s_i = initial entropy

- Weak/Strong Matching

$$\boxed{T_i \propto \sqrt{s_i}\ Q_{glasma}}$$

Final *vs.* Initial Entropy



Heller,Janik,Witaszczyk (2012)

- Exemple: (Unruh/Kharzeev-Tuchin) and GBW apply

$$T_i = \frac{Q_{glasma}}{2\pi} \Rightarrow s_i \sim .2 \; s_f \sim .35$$

Conclusions and Outlook

- **Hints from non-equilibrium statistical physics**

  New Tools: Non-Equilibrium Work Identities

- **Generalized Entropy of CGC states**

  Definition of an entropy increase with saturation: $\Delta\Sigma^{Q_1 \to Q_2}$

- **Application to Gaussian (GBW) models**

  $\Delta\Sigma^{Q_1 \to Q_2} \propto Q_2^2/Q_1^2$ "micro" $\equiv$ "macro" entropy *cf*. Kutak (2011)

- **Initial Heavy-Ion Entropy and Glasma-AdS/CFT matching**

$$\boxed{s_i \propto Q^2_{glasma}/T^2_{AdS/CFT}}$$

- **Outlook: Gauge Theory/Statistical Physics: a new duality?:**

  Hard-Soft/fluctuation-dissipation

  Collisions parton-CGC/Particle-thermal interactions

  other...