Laboratoire Kastler Brossel
Physique quantique et applications

Determination of the fine structure constant using atom interferometry

Ph.D Students

R. Battesti (2003)
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M. Cadoret (2008)
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Postdoc

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Permanents

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F. Nez
F. Biraben

Overview

- Fundamental tests using high precision measurements
  - Metrology
  - Simple and calculable system
  - Precision at the level of 10^{-9} and below
- Atomic physics.
  - Laser cooling and trapping
  - Atom interferometry

Quantum electrodynamics by an atomic physicist

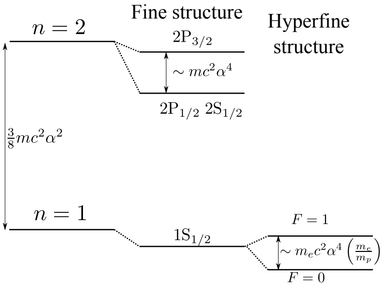
- 1916 : A. Sommerfeld tried to include special relativity in the Bohr model in order to explain the fine structure observed in hydrogen atoms.

$$\alpha = \frac{v_e}{c} = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$\alpha \simeq 1/137$$

  - e : elementary charge
  - $\hbar = h/2\pi$: reduced Planck constant
  - c : speed of ligh in vacuum
  - ϵ_0 : permittivity of free space

- 1928 : Dirac equation. Full explanation of the spin of the electron
  - Hydrogen fine structure



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  - Magnetic moment of the electron

    Electron in a magnetic field :
    - Cyclotron frequency : $\omega_{\mathrm{cyc}} = \frac{eB}{m}$
    - Larmor frequency: $\omega_{\mathrm{lar}} = \gamma B$

    Landé g-factor : $g_{\mathrm{e}} = \frac{\omega_{\mathrm{lar}}}{\omega_{\mathrm{cyc}}}$

    Dirac equation predicts that : $g_{\mathrm{e}} = 2$

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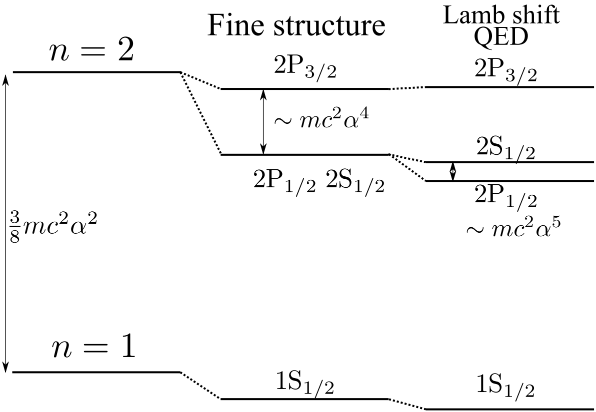
- 1947-1948
  - Lamb shift
  - Anomalous magnetic moment of the electron

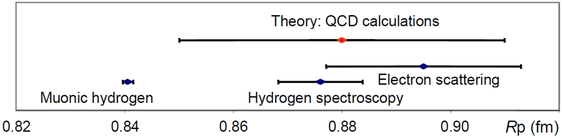
Formulation of the QED



Quantum electrodynamics: vacuum fluctuations

Measurement of the Lamb shift

- Optical frequency measurement in hydrogen
  - Max-Planck-Institut (Garching)
  - Laboratoire Kastler Brossel (Paris)
- The contribution of QED to the Lamb shift is limited by the knowledge of the proton charge radius.
- Independant measurement of the proton charge radius:
  - determination with e-p scattering
  - recent determination using muonic hydrogen (R. Pohl, A. Antognini, F. Nez *et al.*, Nature 466, 213 (2010))



Anomalous magnetic moment of the electron

Landé g-factor : $g_\mathrm{e} = \frac{\omega_\mathrm{lar}}{\omega_\mathrm{cyc}}$

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In 1948, Kusch and Foley performed a measurement of g_e. The value they obtain is not compatible with Dirac equation.

Prediction of QED (Schwinger, 1948):

$$\frac{g}{2} = 1 + \left(\frac{\alpha}{2\pi}\right) + \ldots \quad (1)$$

Renormalization

α is the coupling constant of QED
$\alpha \simeq 1/137$

General formula:

$$\frac{g_\mathrm{e}}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + \ldots \quad (2)$$

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Prediction of QED (Schwinger, 1948):

Renormalization

Kinoshita *et al.*

A life dedicated to the calculation of g_e
2003 : The C_4 term that requires 819 Feynman diagrams
2012 : Calculation of the C_5 term : 12672 diagrams

$$\frac{g_{\mathrm{e}}}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + \ldots \qquad (2)$$

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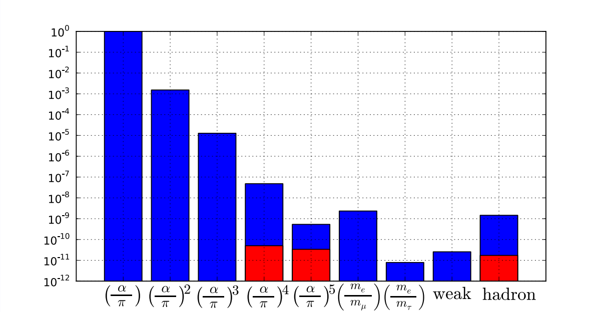
Contributions to g_e



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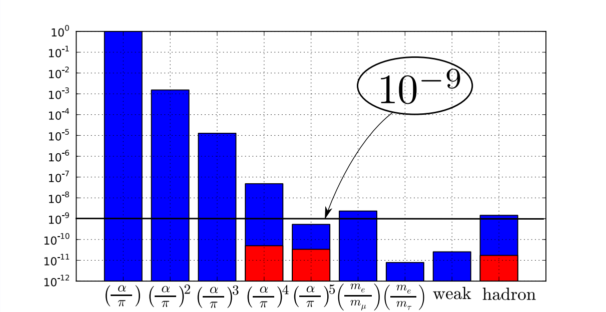

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Experiment of Dehmelt and Gabrielse

1984 and 2008

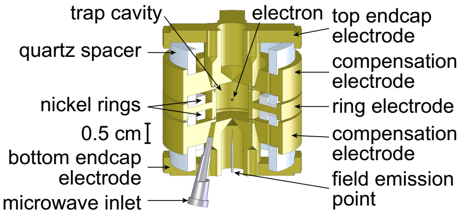
Electron in a Penning trap.

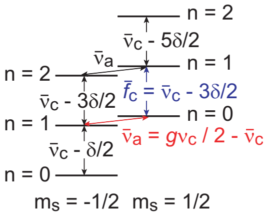
Measurement of the :

- Larmor frequency
- Cyclotron frequency

$$g/2 = 1.001\,159\,652\,180\,73(28)$$
$$[0.28 \times 10^{-12}]$$

One Electron Quantum Cyclotron





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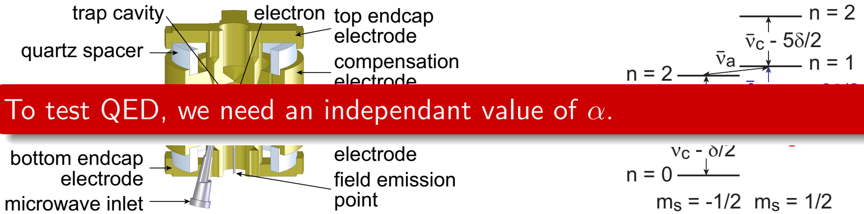
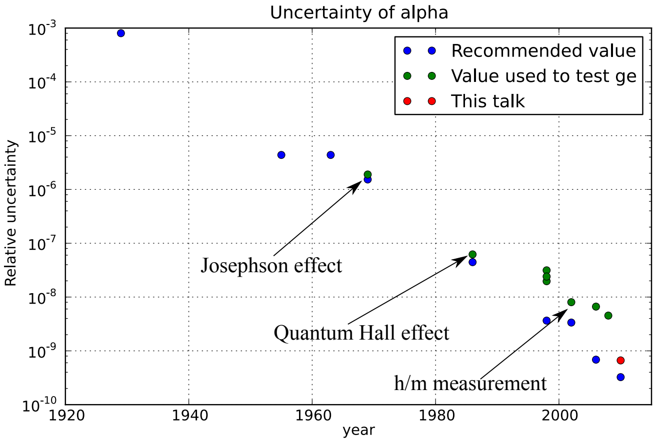


To test QED, we need an independant value of α.

History of the determinations of α

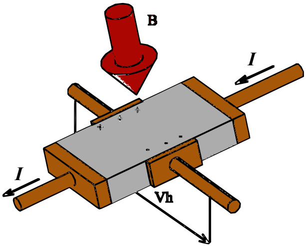


The Quantum Hall effect

Hall effect (1879)

- E field (Coulomb force)
- B field (Lorentz force)

$$V_h = \frac{BI}{en_\mathrm{e}}$$



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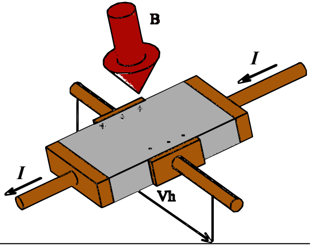
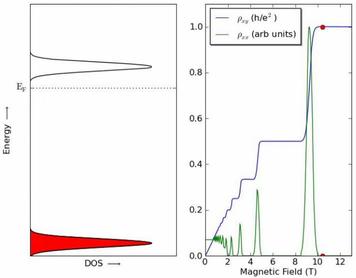


Quantum Hall Effect : Quantification of the density of moving electron in a semi-conductor layer at low temperature in a high magnetic field:

$$n_{\mathrm{e}} = \nu \frac{eB}{h}, \quad \nu = 1, 2, 3, \ldots \qquad (3)$$



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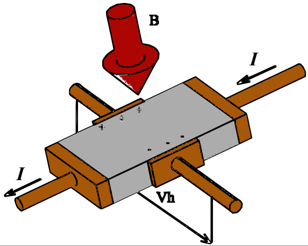

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$$R_K = \frac{h}{e^2}$$

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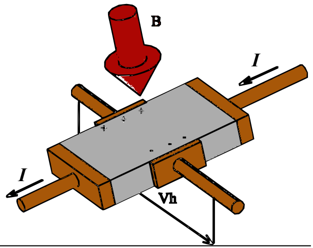

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$$R_K = \frac{h}{e^2} \rightarrow R_K = \frac{Z_0}{2\alpha}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$
$$Z_0 = \frac{1}{\epsilon_0 c} = 377\ \Omega$$

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance, K. Klitzing *et al.*, *Phys. Rev. Lett.* **45** (1980)

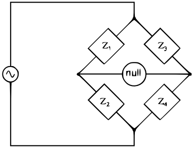
Measurement of the quantum Hall effect

Universality of the Quantum Hall effect :

GaAs heterostructure (< 1 ppb),
Si-MOSFET (< 1 ppb),
Epitaxial graphene (3 ppb).

Measurement in SI :

Quantum Hall Device
(500 μm)



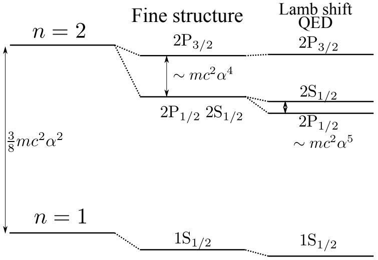
AC bridge, ..

Calculable capacitor
Lampard
(50 cm)

$$R_K = 25\,812.808\,16(47)\ \Omega \quad [1.8 \times 10^{-8}]$$

Measurement of alpha with h/m

Hydrogen atom



$$\boxed{hR_\infty c = \frac{1}{2}m_e c^2\alpha^2}$$

R_∞ known at the level of 10^{-12}. Conversion between an energy expressed in terms of frequency and mass.

Measuring the ratio h/m_e :

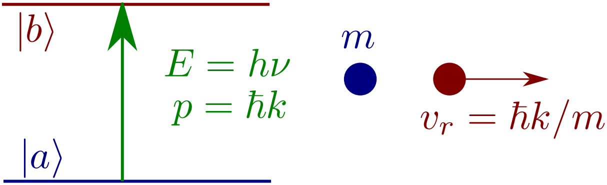
- Mass ratio are well known
- Neutron
- Alkali atoms

$$m_e/m_p : 4.2 \times 10^{-10}$$
$$m_{Rb}/m_p : 1.4 \times 10^{-10}$$

$$\alpha^2 = \frac{2}{c}R_\infty \frac{m_p}{m_e}\frac{m_{Rb}}{m_p}\frac{h}{m_{Rb}}$$

The recoil velocity

Absorption of a photon by a atom



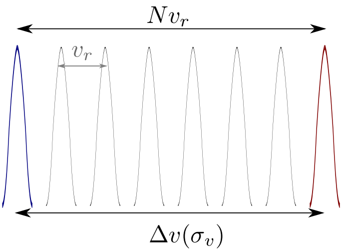
Rubidium atoms

- $v_r = 6$ mm/s
- $E_r = 3$ kHz (10^{-11} eV)

Principle of the experiment

Principle

- Transfer to atoms as many recoils as possible
  Bloch oscillations
- Measure the change of velocity
  Doppler shift, atom interferometry



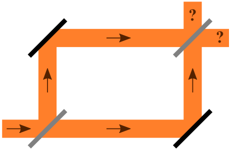
Uncertainty :

$$\sigma_{v_r} = \frac{\sigma_v}{N} \qquad (4)$$

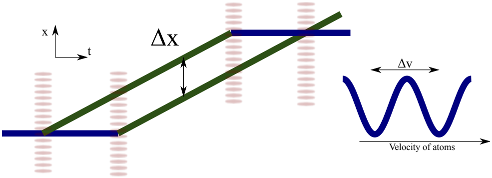
Atom interferometry

Atom interferometers are similar to optical interferometers

- **Beamsplitter** (split and recombine a wave)
- Measure the **phase difference** between the two paths



Light splits the atomic wave packet in two paths

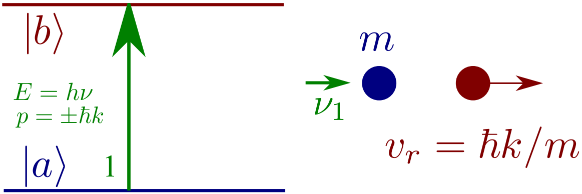


An atom interferometer measures the **kinetic energy**

Sensitivity in velocity

Heisenberg principle : $\Delta v \simeq \frac{h}{m\Delta x}$
Example : $\Delta x = 1\ \mathrm{mm} \Rightarrow \Delta v = 5\ \mu\mathrm{m/s}$ for rubidium atoms.

Two photon transition



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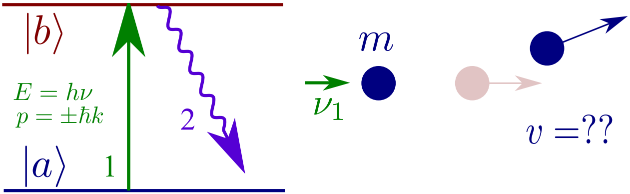
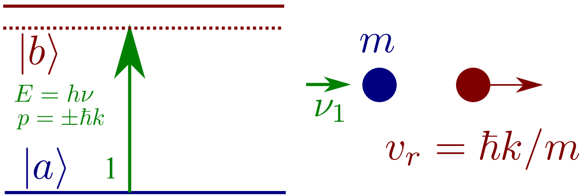


Spontaneous emission

Two photon transition



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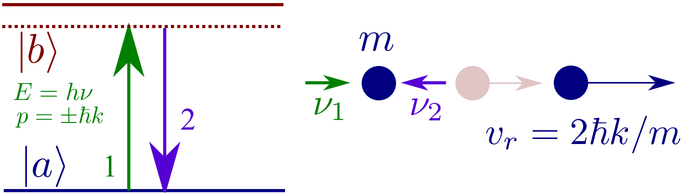
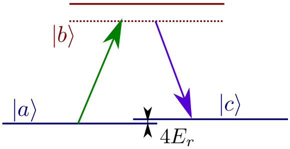
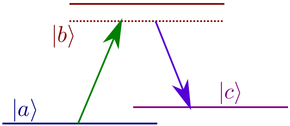


Two photon transition to supress spontaneous emission.



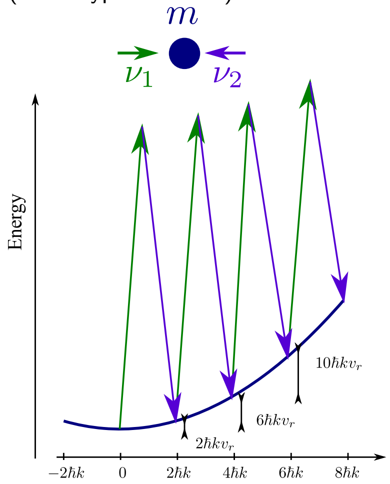
Same internal state



Two different internal states

Coherent acceleration of atoms

Succession of stimulated Raman transitions (same hyperfine level)



Bloch oscillations

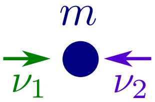
$$\delta = \nu_1 - \nu_2 \propto t$$

Adiabatic passage : acceleration of the atoms

The atoms are placed in an accelerated standing wave : in its frame, they are submitted to an inertial force
$\rightarrow$ Bloch oscillations in a periodic potential

Ben Dahan, *et al.*, PRL 76, (1996) (group of C. Salomon, LKB, Paris)

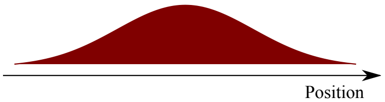
Atom in an accelerated lattice



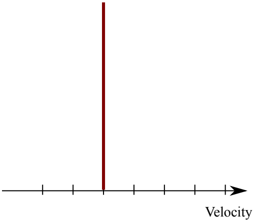
$v = (\nu_1 - \nu_2)/2k$

Light shifts : periodic potential
$U(x,t) = \frac{U_0}{2}\cos(2k(x - vt))$

Position



Velocity



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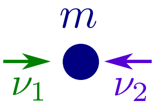

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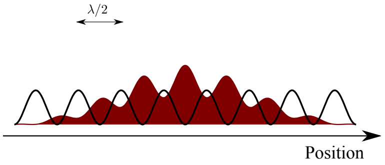

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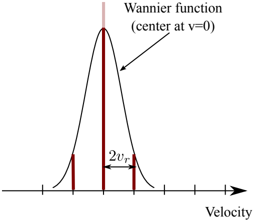

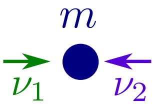

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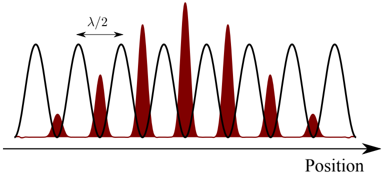

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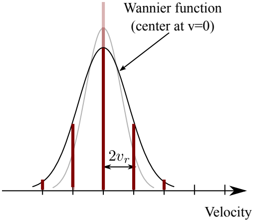

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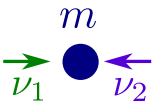

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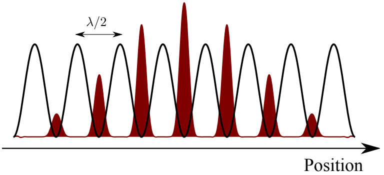

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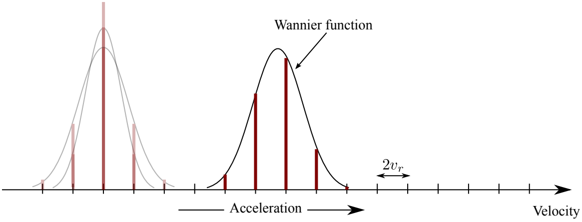

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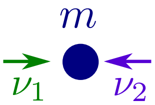

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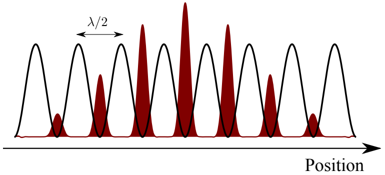

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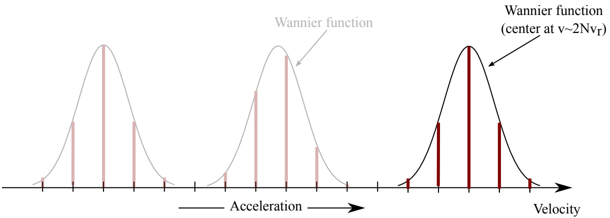

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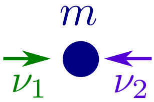

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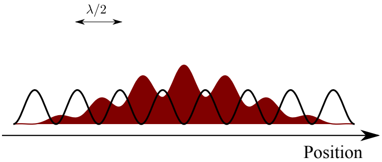
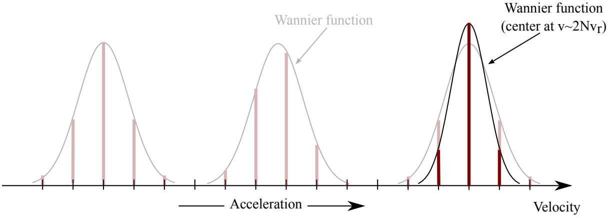
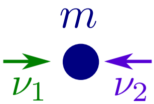


Velocity



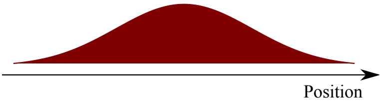
Atom in an accelerated lattice



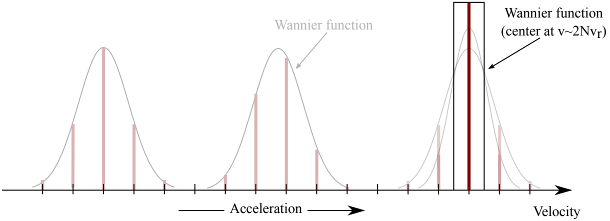
$v = (\nu_1 - \nu_2)/2k$

Light shifts : periodic potential
$U(x,t) = \frac{U_0}{2}\cos(2k(x - vt))$

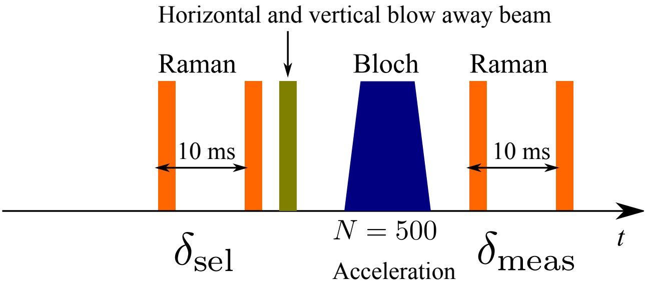
Position



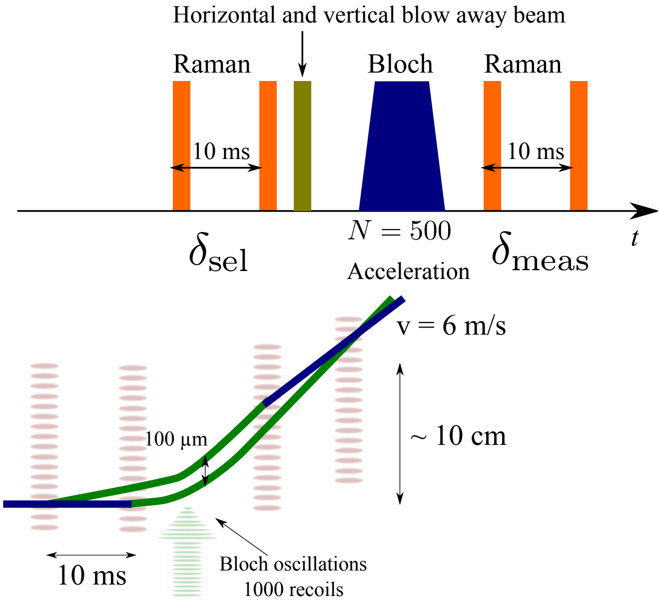
Velocity



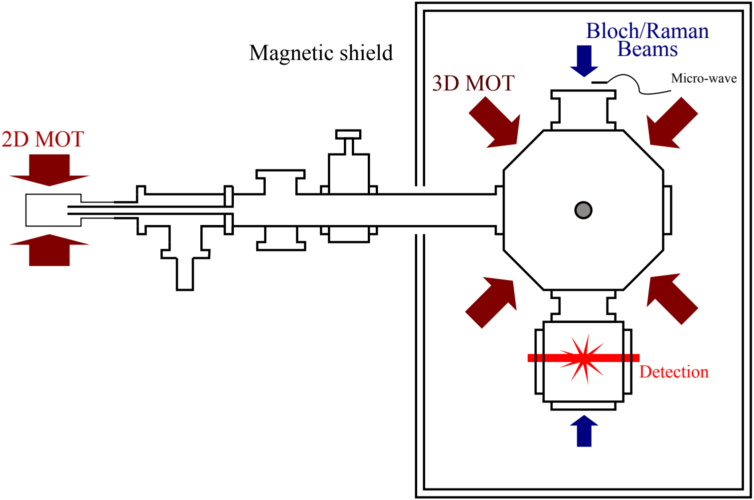
Temporal sequence



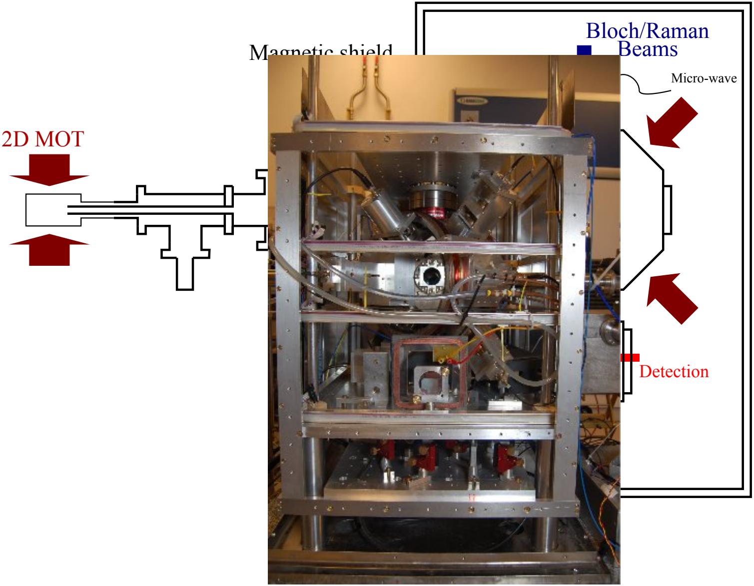
Temporal sequence



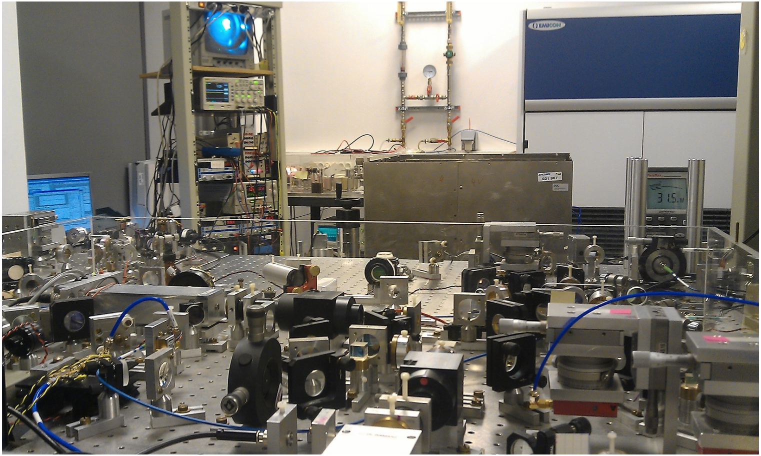
Experimental setup



Experimental setup

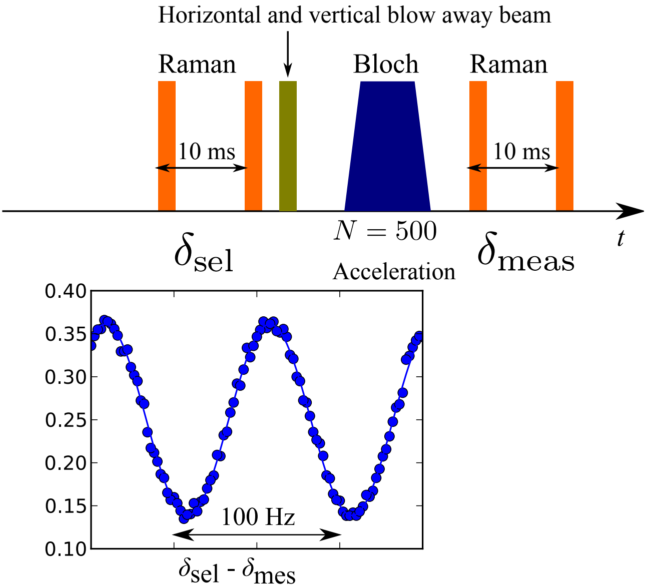


Experimental room

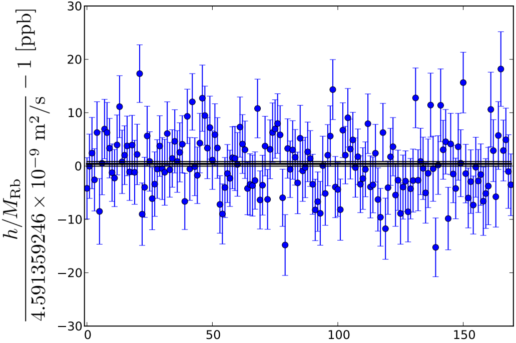


LKB

Temporal sequence



Results



170 measurements (14 hours)

Each measurement : 6×10^{-9} (h/m) and 3×10^{-9} (α)

Relative uncertainty on h/m : 4.4×10^{-10} and 2.2×10^{-10} on α.

Error Budget

Source	Correction	Uncertainty 10^{-10}
Laser frequencies		1.3
Beams alignment	-3.3	3.3
Wavefront curvature and Gouy phase	-25.1	3.0
2nd order Zeeman effect	4.0	3.0
Gravity gradient	-2.0	0.2
Light shift (one photon transition)		0.1
Light shift (two photon transition)		0.01
Light shift (Bloch oscillations)		0.5
Index of refraction atomic cloud and atom interactions		2.0
Global systematic effects	-26.4	5.9
Statistical uncertainty		2.0
Rydberg constant and mass ratio		2.2
Total uncertainty		6.6

Error Budget

Source	Correction	Uncertainty 10^{-10}
Laser frequencies		1.3
Beams alignment	-3.3	3.3
Wavefront curvature and Gouy phase	-25.1	3.0

Wavefront curvature and Gouy phase shift

What is the momentum of a photon ?

$p = \hbar\frac{\partial\phi}{\partial z}$ where ϕ is the phase of the laser beam.

$p = \hbar k$ holds only for perfect plane-wave.

For a Gaussian beam :

$$\frac{\partial\phi}{\partial z} = \frac{1}{2k}\left(\frac{4}{w^2} - \frac{4r^2}{w^4} + \frac{r^2k^2}{R^2}\right) \quad (5)$$

where w is the waist of the beam, R the wavefromnt curvature and r the distance from the propagation axes of the beam.

Error Budget

Source	Correction	Uncertainty 10^{-10}
Laser frequencies		1.3
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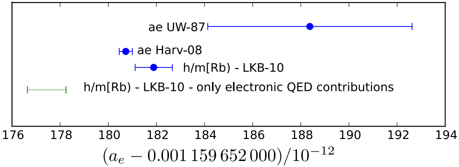
Measurements of α

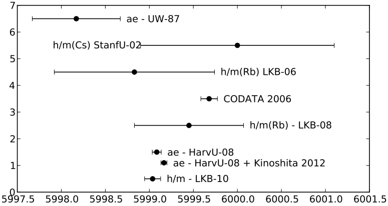
LKB-10 (Paris) : $\alpha^{-1} = 137.035\,999\,037\,(91) \quad [6.6 \times 10^{-10}]$

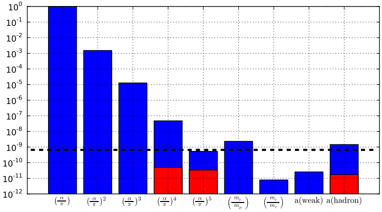
$$\Rightarrow a_{\mathrm{e}} = 0.001\,159\,652\,181\,88\,(78)$$

HarvU-08 : $a_{\mathrm{e}} = 0.001\,159\,652\,180\,73\,(28)$

$$\Rightarrow \alpha^{-1} = 137.035\,999\,173\,(35) \quad [2.6 \times 10^{-10}]$$







First verification of the muonic and hadronic contributions to a_{e}

Conclusion and perspective

Conclusion

- We have measured the fine structure constant with an unertainty of 6.6×10^{-10}.
- Most precise input value of α used to test the QED
- Most stringent test of QED
- Set a limitation on exotic theory

Perspectives

- Improve the uncertainty to few parts in 10^{-10}
- Redefinition of the SI

Perspective : 10^{-10}

$$\alpha^2 = \frac{2}{c} R_\infty \frac{m_\mathrm{p}}{m_\mathrm{e}} \frac{m}{m_\mathrm{p}} \frac{h}{m_\mathrm{Rb}}$$

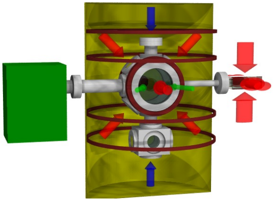
- Calculation of a_e below 10^{-10}
- Measurement of a_e : 2.6×10^{-10}. Group of G. Gabrielse
- Measurement of $\frac{m_\mathrm{p}}{m_\mathrm{e}}$: 4×10^{-10}: hydrogen like atoms. Other methods using spectroscopy of HD^+ and antiprotonic helium.
- Measurement of $\frac{m}{m_\mathrm{p}}$. Below 10^{-10}
- Measurement of h/m : Group of Paris (1.3×10^{-9}) and Berkeley (4×10^{-9} in 2013).

How to improve the determination of h/m

- Increasing the sensitivity
  Building a new generation of beamsplitters that will increase the distance between the arms of the interferometer
- Improving the accuracy
  Experimental and theoretical study of systematic effects at the level of 10^{-10}

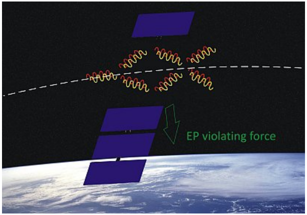
New experimental setup

- Ultracold atomic source (BEC)
- Large Momentum Beamsplitter using Bloch oscillations

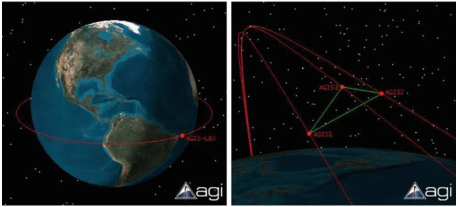


Atom interferometer in large instruments

- STE-QUEST (ESA) : equivalence principle



- AGIS-LEO : gravitational wave detection



Redefinition of the SI

- **mass** : the kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
- **electric current** : The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
- **amount of substance** : The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is “mol.”.

Two problems :

- The kilogram is defined with an artefact.
- Electrical units are independant from the SI : V_{90} and Ω_{90} derived from conventional value of K_J and R_K.

Redefinition of the SI

Proposal for a new redefinitioon of the base units :

- h has a fixed value $\rightarrow$ kilogram (watt balance)
- e has a fixed value $\rightarrow$ ampere
- N_A has a fixed value $\rightarrow$ mole (silicium sphere)

The measurement of h/M and the fine structure constants plays important roles. **Before** the redefinition:

- Test of the validity of the Josephson effect and the quantum Hall effect used for the pratical realisation of the base units
- Link between the Planck constant, the Avogadro constant and the molar mass:

$$\frac{h}{m_{^{87}\mathrm{Rb}}} \times \frac{m_{^{87}\mathrm{Rb}}}{m_{^{12}\mathrm{C}}} = \frac{hN_A}{M\left(^{12}\mathrm{C}\right)}$$

Redefinition of the SI

Proposal for a new redefinitioon of the base units :

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The measurement of h/M and the fine structure constants plays important roles. **After** the redefinition:

- Measurement of h/m will be a realisation of the atomic unit of mass in SI
- Some fundamental constant will be directely linked to α.

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \tag{6}$$

For example μ_0 will no longer be $4\pi \times 10^{-7}$.

The team