Exotic physics: Antigravity?

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Full title: “Exotic physics from standard Einstein theory: Topology and antigravity?”

References on the last slide.

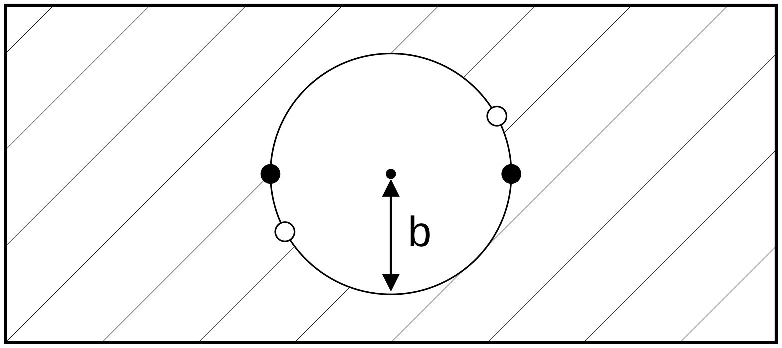
Manifold [1, 2]:

$$M_4 = \mathbb{R} \times M_3 \, . \tag{1}$$

The 3-space M_3 is a noncompact, orientable, nonsimply-connected manifold without boundary. In fact, there is the following homeomorphism:

$$M_3 \simeq \mathbb{R}P^3 - \{\text{point}\} \, . \tag{2}$$

Explicit construction by surgery from $\mathbb{R}^3$: interior of ball with radius b removed and antipodal points on boundary of ball identified.



Standard Cartesian or spherical coordinates over $\mathbb{R}^3$ are not good coordinates over M_3.

Better coordinates with three charts, each surrounding one of the Cartesian coordinate axes. Coordinates of one chart (around x^2 axis):

$$X = \begin{cases} \phi & \text{for} \quad 0 < \phi < \pi\,, \\ \phi - \pi & \text{for} \quad \pi < \phi < 2\pi\,, \end{cases} \tag{3a}$$

$$Y = \begin{cases} r - b & \text{for} \quad 0 < \phi < \pi\,, \\ b - r & \text{for} \quad \pi < \phi < 2\pi\,, \end{cases} \tag{3b}$$

$$Z = \begin{cases} \theta & \text{for} \quad 0 < \phi < \pi\,, \\ \pi - \theta & \text{for} \quad \pi < \phi < 2\pi\,, \end{cases} \tag{3c}$$

with ranges $X \in (0,\,\pi)$, $Y \in (-\infty,\,\infty)$, and $Z \in (0,\,\pi)$.

Spherically symmetric *Ansatz* [3] for the metric over M_4 :

$$ds^2 = -\widetilde{\mu}(W)^2\, dT^2 + \widetilde{\kappa}(W)^2\, dY^2 + W\left(dZ^2 + \sin^2 Z\, dX^2\right), \tag{4a}$$

$$W \equiv b^2 + Y^2, \tag{4b}$$

$$Y \in (-\infty,\, \infty), \tag{4c}$$

$$b > 0, \tag{4d}$$

with $c = 1$ and showing only the coordinates of one chart.

Einstein equations now have the following exact vacuum solution [3]:

$$\widetilde{\kappa}^2 = \frac{1 - b^2/(Y^2+b^2)}{1 - \widetilde{L}/\sqrt{Y^2+b^2}}\,, \tag{5a}$$

$$\widetilde{\mu}^2 = 1 - \widetilde{L}/\sqrt{Y^2+b^2}\,, \tag{5b}$$

with constant $\widetilde{L}$, 'radial' coordinate $Y \in (-\infty,\, +\infty)$, and defect parameter $b > 0$.

Defining $\widetilde{L} \equiv 2G_N\widetilde{M}$, the solution over M_4 gives asymptotically:

$$\widetilde{\kappa}^2 \sim \frac{1}{1 - 2G_N\widetilde{M}/Y}\,, \tag{6a}$$

$$\widetilde{\mu}^2 \sim (1 - 2G_N\widetilde{M}/Y)\,. \tag{6b}$$

This has the same form as the standard Schwarzschild metric upon the identification $Y \to r$. But there is a crucial difference: the scalar

$$S_2 \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 12 \, \frac{\widetilde{L}^2}{W^3} \tag{7}$$

is **finite** over the **whole** of M_4, because $W \geq b^2 > 0$, whereas, for the Schwarzschild metric, $S_2 \propto M^2/r^6$ diverges at $r = 0$.

Two general remarks:

1. For $\widetilde{M} = 0$, the metric differs from the standard Minkowski metric ($\to$ modified propagation of photons and neutrinos?).
2. For $\widetilde{M} < 0$, a point mass far away from the defect core will not be attracted towards it but repulsed.

The actual value of the free parameter $\widetilde{M}$ in the metric from (5) will have to be determined by adding matter fields (the same applies for the determination of M in the standard Schwarzschild solution).

Is it then possible to get $\widetilde{M} < 0$, corresponding to antigravity?

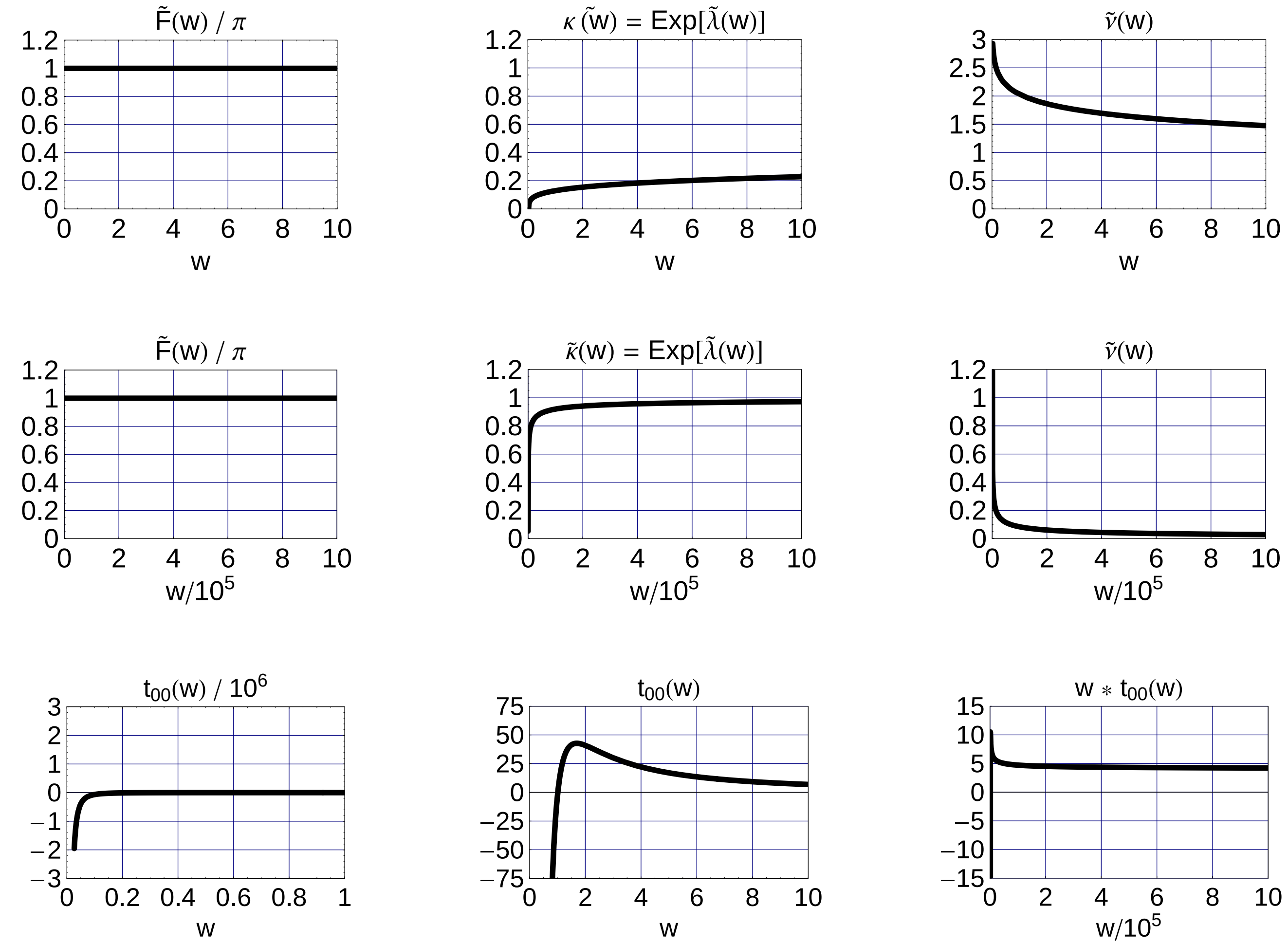
The answer appears to be affirmative.

We consider standard Einstein gravity coupled to an $SO(3) \times SO(3)$ chiral model of scalars (Skyrme model [4]), with an additional self-interaction term (coupling constant $\gamma > 0$) allowing for negative energy-density contributions.

Dimensionless variables, for example, $w = y^2 + (y_0)^2$. Also define

$$\widetilde{\kappa}(w) \equiv \exp\left[\widetilde{\lambda}(w)\right], \quad \widetilde{\mu}(w) \equiv \exp\left[\widetilde{\nu}(w)\right]. \tag{8}$$

Numerical solution [3]:



From these numerical results, we appear to have at least one solution with a negative dimensionless parameter,

$$\widetilde{l} \approx -28\,, \tag{9}$$

which gives the following dimensional quantity:

$$\widetilde{M} \approx -14 \times \frac{1}{e\,f\,G_N}\,, \tag{10}$$

where f is the energy scale of the Skyrme model and e the dimensionless coupling constant of the Skyrme self-interaction term.

Referring to the asymptotic form (6), the conclusion is that this particular defect leads to **antigravity**, meaning that the gravitational force on a distant point mass is **repulsive**.

$\widetilde{M}$ can be interpreted as the Schwarzschild mass. Alternatively, consider the Komar mass at spatial infinity. That quantity can be written as

$$\widehat{M}_{\text{Komar}} = M_{\text{mat}} + M_{\text{grav}} + M_{\text{def}} \,, \tag{11}$$

with <u>bulk</u> contributions M_{mat} and M_{grav} which are <u>nonnegative</u> and a <u>surface</u> contribution M_{def} (integral over sphere $W = b^2$) which is <u>negative</u>.

Precisely this antigravity property and the presence of negative energy density at the defect core <u>invalidate</u> the Singularity Theorem 2.1 of Gannon [5], which applies to spacetimes with asymptotically-flat nonsimply-connected spacelike slices.

As mentioned before, a random distribution of such defects affects the propagation of photons and neutrinos, even with $\widetilde{M} = 0$?

There exist stringent bounds on LV effects from static defects in Minkowski spacetime [1], but analysis must be revisited for the new metric.

Possible connection of small-scale structure of spacetime (with length scale $\hbar\, c/E_{\text{Planck}}$) to Higgs-boson and top-quark masses [6]?

Back to the defect solution over the nonsimply-connected manifold M_4:

- Consequences of exact vacuum solution need to be explored.
- Numerical analysis needs to be verified and extended.

5. References

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