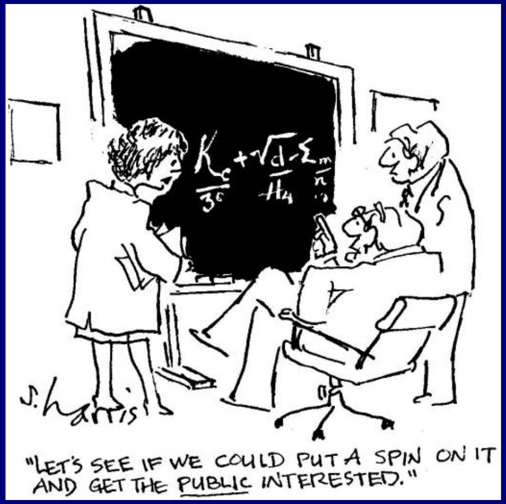
Physics at
A Fixed Target ExpeRiment (AFTER)
using the LHC beams

Spin theory:
A short taste



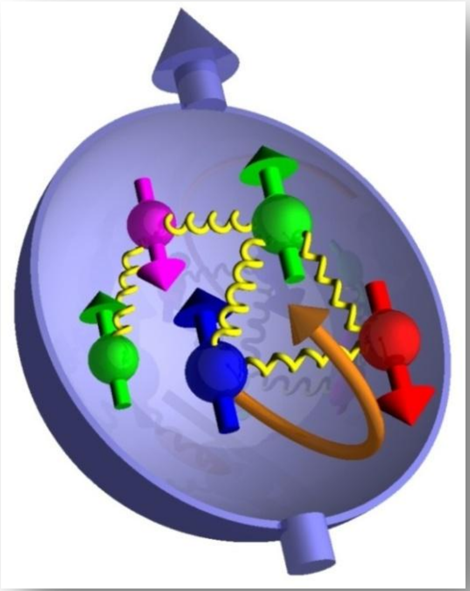
Cédric Lorcé

Institut de Physique Nucléaire d'Orsay
and
Laboratoire de Physique Théorique

February 11, 2013, ECT*, Trento, Italy

Introduction

Basic questions



Introduction

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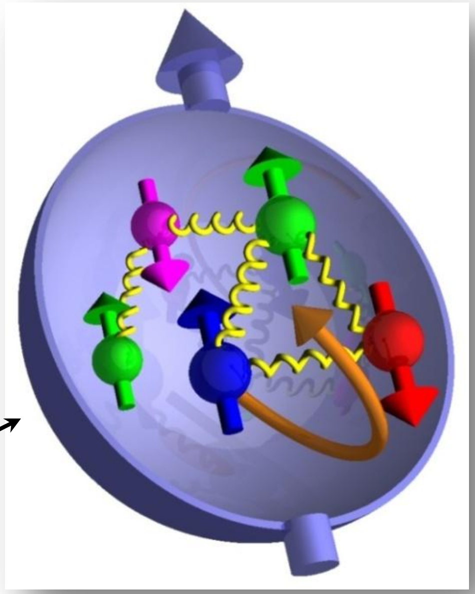
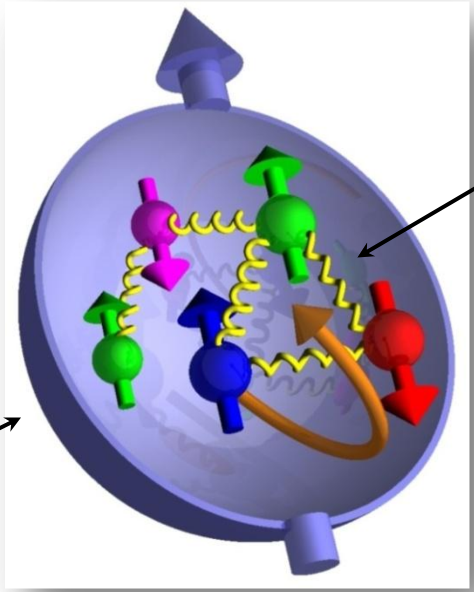


Parton distributions ?

Introduction

Basic questions

Quark-gluon interactions ?



Parton distributions ?

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Proton spin decomposition ?

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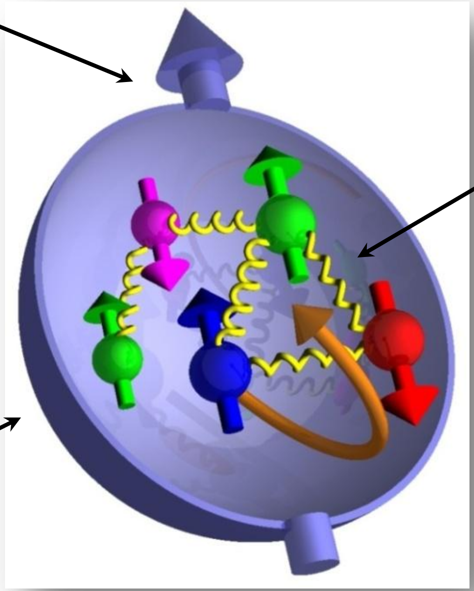

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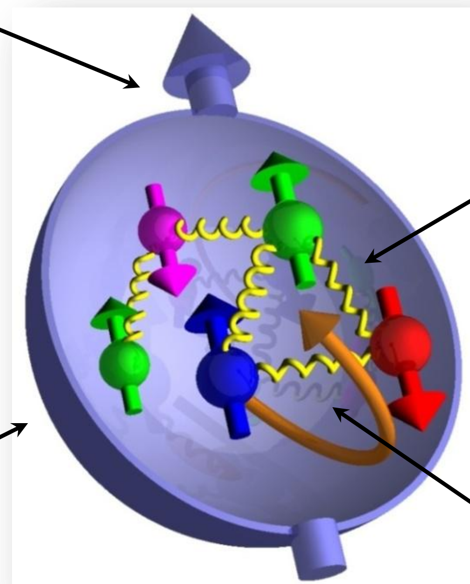

Parton distributions ?

Spin-orbit correlations ?

Angular momentum

Spin and OAM

Orbital angular momentum

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

Angular momentum

Spin and OAM

$$\boxed{[J^i, J^j] = i\epsilon^{ijk} J^k}$$

Angular momentum

Orbital angular momentum

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Angular momentum

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Angular momentum

Orbital angular momentum

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

Spin

$$\boxed{\vec{J}\,\big|_{\vec{p}=\vec{0}} = \vec{S}}$$

Angular momentum

Polarizations

Relativity couples spin and OAM

$$\boxed{[K^i, K^j] = -i\epsilon^{ijk} J^k}$$

Angular momentum

Polarizations

Relativity couples spin and OAM

$$\boxed{[K^i, K^j] = -i\epsilon^{ijk} J^k}$$

Rest frame

$$|\vec{0}, \uparrow\rangle$$

Angular momentum

Polarizations

Relativity couples spin and OAM

$$\boxed{[K^i, K^j] = -i\epsilon^{ijk} J^k}$$

Rest frame **Moving frame**

$$|\vec{0}, \uparrow\rangle \xrightarrow{\vec{K}} |\vec{p}, s = +\tfrac{1}{2}\rangle$$

Canonical spin

Angular momentum

Polarizations

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Rest frame

Moving frame

$$|\vec{0}, \uparrow\rangle \xrightarrow{\vec{K}} |\vec{p}, s = +\tfrac{1}{2}\rangle$$

Canonical spin

$$|\vec{0}, \uparrow\rangle \xrightarrow{R(\hat{e}_z \to \hat{p})\, K^z} |\vec{p}, h = +1\rangle$$

Helicity

Angular momentum

Polarizations

Relativity couples spin and OAM

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Rest frame | **Moving frame**

$$|\vec{0}, \uparrow\rangle \xrightarrow{\vec{K}} |\vec{p}, s = +\tfrac{1}{2}\rangle$$

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Helicity

$$|\vec{0}, \uparrow\rangle \xrightarrow{\vec{B}^{\perp}_{LF}\, B^z_{LF}} |\vec{p}, \lambda = +1\rangle$$

Light-front helicity

Angular momentum

Polarizations

Relativity couples spin and OAM

$$\boxed{[K^i, K^j] = -i\epsilon^{ijk} J^k}$$

Rest frame | **Moving frame**

$|\vec{0}, \uparrow\rangle$

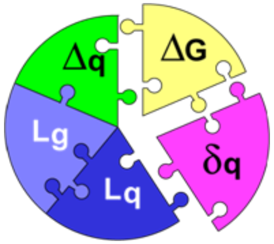
$\xrightarrow{\vec{K}}$ $|\vec{p}, s = +\tfrac{1}{2}\rangle$ **Canonical spin**

$\xrightarrow{R(\hat{e}_z \to \hat{p})\, K^z}$ $|\vec{p}, h = +1\rangle$ **Helicity**

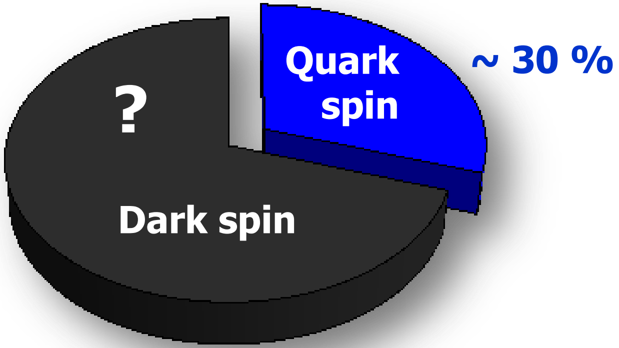
$\xrightarrow{\vec{B}^{\perp}_{LF}\, B^z_{LF}}$ $|\vec{p}, \lambda = +1\rangle$ **Light-front helicity**

Light-front dominance $\quad \dfrac{1}{Q} \sim \dfrac{1}{P^+} \ll 1$

Proton spin puzzle

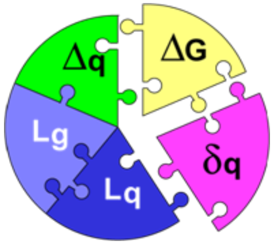


Sum rule



$$\boxed{\frac{1}{2} = S_q + L_q + S_g + L_g}$$

Proton spin puzzle



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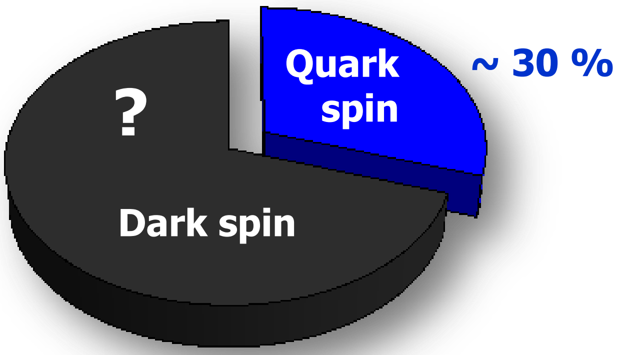
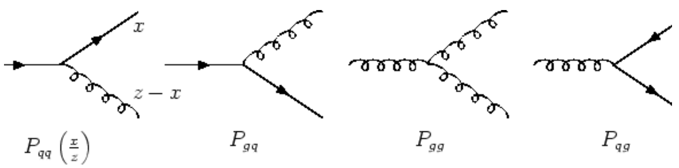
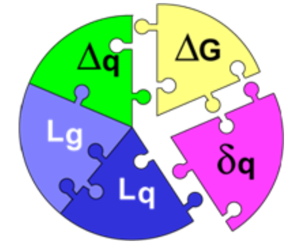


$$\boxed{\frac{1}{2} = {\color{blue}S_q(\mu)} + L_q(\mu) + S_g(\mu) + L_g(\mu)}$$

$$\partial_\mu M^{\mu\nu\rho} = \underbrace{\partial_\mu M_q^{\mu\nu\rho}}_{\neq 0} + \underbrace{\partial_\mu M_g^{\mu\nu\rho}}_{\neq 0} = 0$$

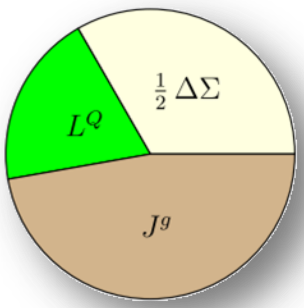


Proton spin puzzle



Ji

[Ji (1997)]



$$\vec{J}_{\rm QCD} = \frac{1}{2}\int \mathrm{d}^3r\,\psi^\dagger\vec{\Sigma}\psi + \int \mathrm{d}^3r\,\psi^\dagger\vec{r}\times(-i\vec{D})\psi + \int \mathrm{d}^3r\,\vec{r}\times(\vec{E}^a\times\vec{B}^a)$$

Kinetic

Pros:
- **Gauge-invariant decomposition**
- **Accessible in DIS and DVCS**

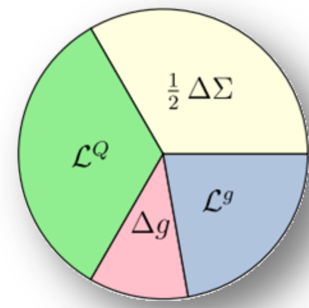
Cons:
- **Does not satisfy canonical relations**
- **Incomplete decomposition**

News:
- **Complete decomposition**

[Wakamatsu (2009,2010)]

Jaffe-Manohar

[Jaffe, Manohar (1990)]



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Canonical

Pros:
- **Satisfies canonical relations**
- **Complete decomposition**

Cons:
- **Gauge-variant decomposition**
- **Missing observables for the OAM**

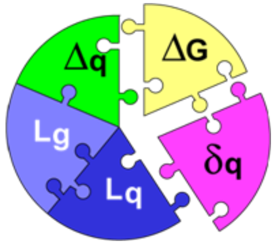
News:
- **Gauge-invariant extension**

[Chen *et al.* (2008)]

- **OAM accessible *via* Wigner distributions**

[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (2011)]
[Hatta (2011)]

Proton spin puzzle



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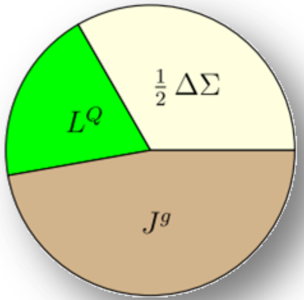

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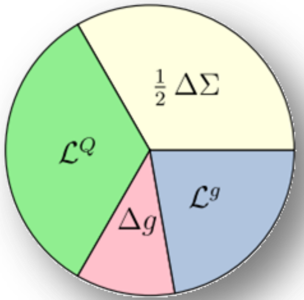

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onical

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- **observables for the OAM**

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- **OAM accessible *via* Wigner distributions**

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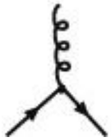
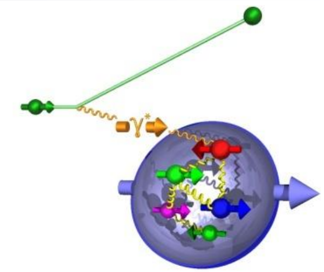
[Hatta (2011)]

$$L_q \leftrightarrow \mathcal{L}_q$$

Quark-gluon interaction
Twist-3 effect

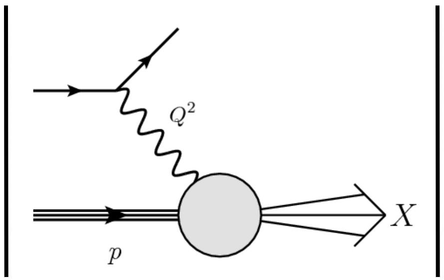


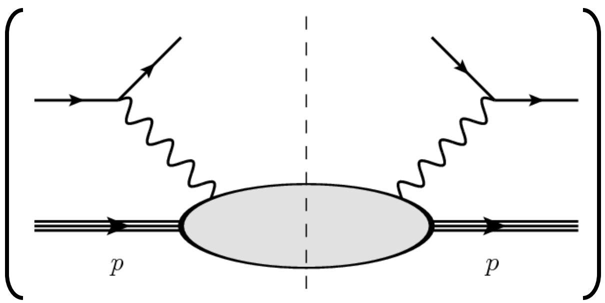
Probing the proton structure



Inelastic scattering

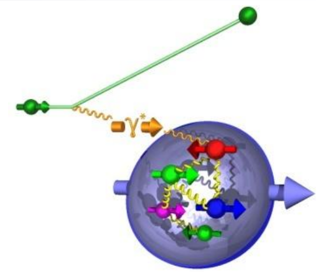
Coherent scattering

$\sum_X$  ^{2}

$\sim$ Im 

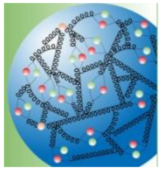
Extremely hard to describe with QCD !

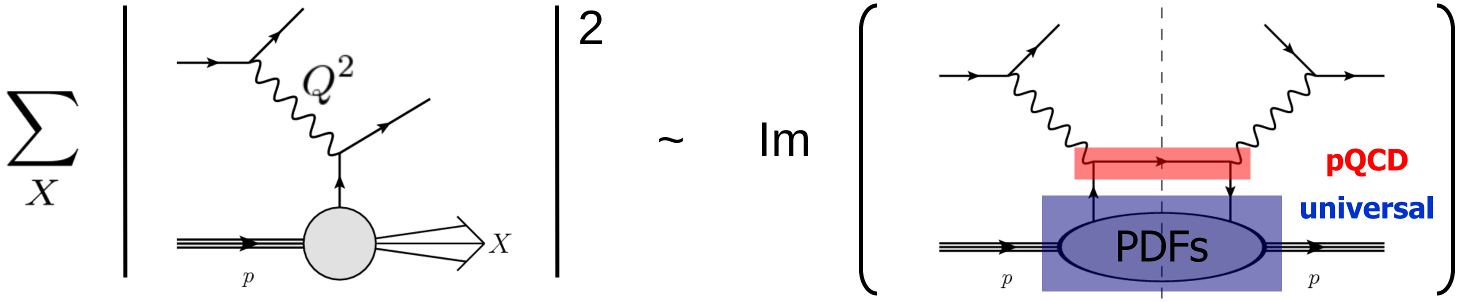
Probing the proton structure



Deep inelastic scattering

Incoherent scattering



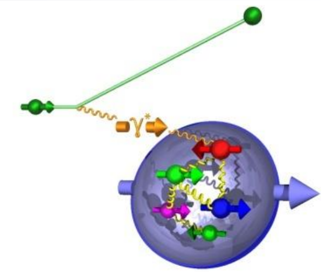


Factorization

$$\boxed{\mathrm{d}\sigma^{lp\to lX} \sim \sum_i \int_x^1 \frac{\mathrm{d}y}{y}\, C_i\left(\frac{x}{y}, \frac{Q^2}{\mu_F^2}; \alpha_S(\mu_R^2)\right) f_i(y, \mu_F^2)}$$

$i = q_f, \bar{q}_f, g$

Probing the proton structure



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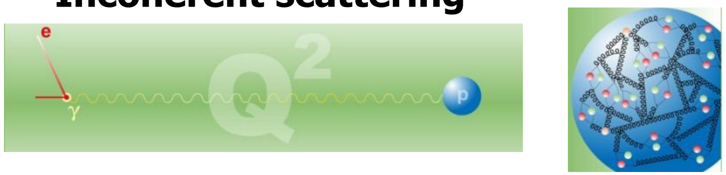

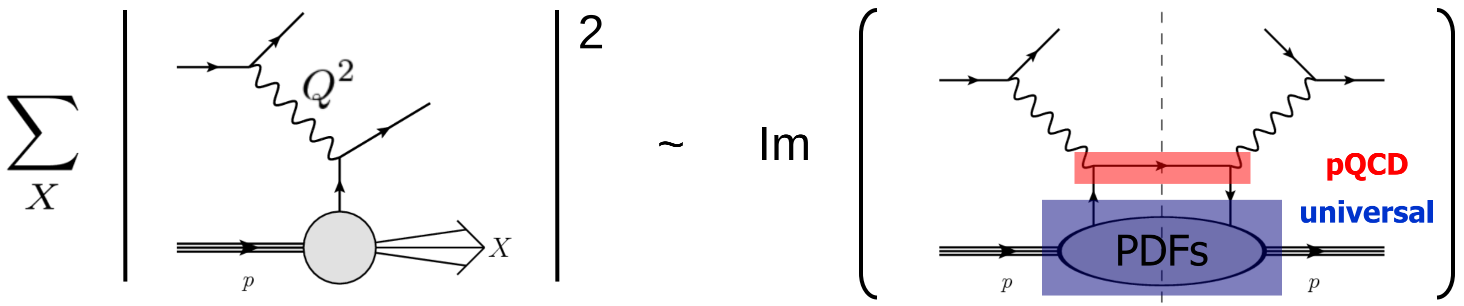

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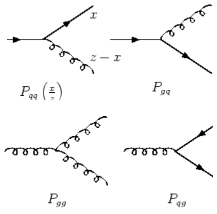
$i = q_f, \bar{q}_f, g$

DGLAP

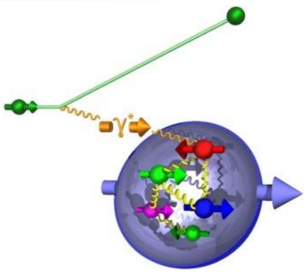
$$\mu_F \frac{\mathrm{d}}{\mathrm{d}\mu_F} C_i\left(x, \frac{Q^2}{\mu_F^2}; \alpha_S(\mu_R^2)\right) = -\sum_j \int_x^1 \frac{\mathrm{d}y}{y}\, C_j\left(y, \frac{Q^2}{\mu_F^2}; \alpha_S(\mu_R^2)\right) P_{ji}\left(\frac{x}{y}; \alpha_S(\mu_R^2)\right)$$

$$\mu_F \frac{\mathrm{d}}{\mathrm{d}\mu_F} f_i(y,\mu_F^2) = \sum_j \int_y^1 \frac{\mathrm{d}z}{z}\, P_{ij}\left(\frac{y}{z}; \alpha_S(\mu_R^2)\right) f_j(z,\mu_F^2)$$



In practice $\mu_F^2 = \mu_R^2 = Q^2$

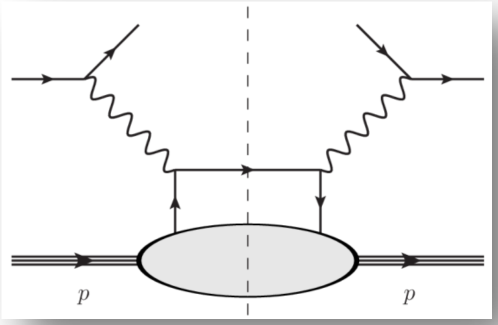
Probing the proton structure

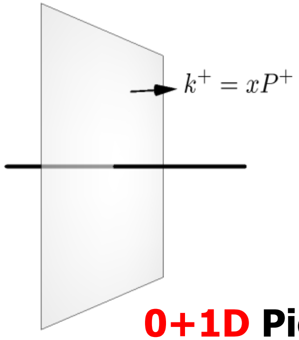


Parton Distribution Functions

$$\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ik^+z^-}\,\langle p^+,\vec{0}_\perp,\Lambda'|\overline{\psi}(-\tfrac{z^-}{2})\Gamma\mathcal{W}\psi(\tfrac{z^-}{2})|p^+,\vec{0}_\perp,\Lambda\rangle$$

DIS





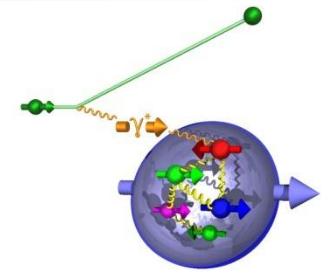
0+1D Picture

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f			
T_x		h		
T_y			h	
L				g

Probing the proton structure



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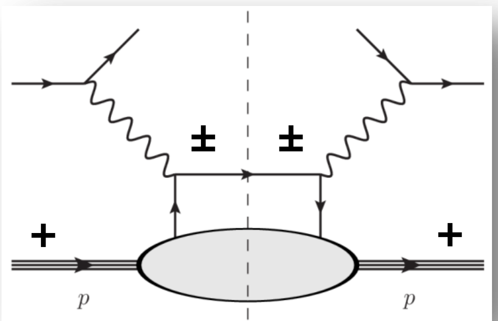

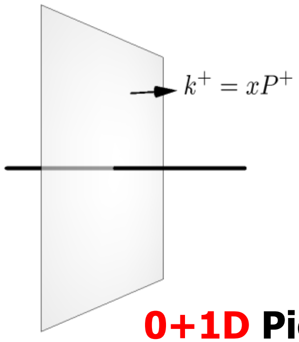

0+1D Picture

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Nucleon polarization

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U	f			
T_x		h		
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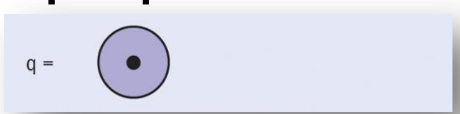
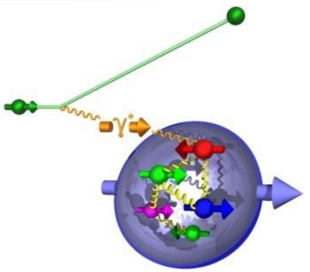
Spin-spin correlations

Vector



Probing the proton structure



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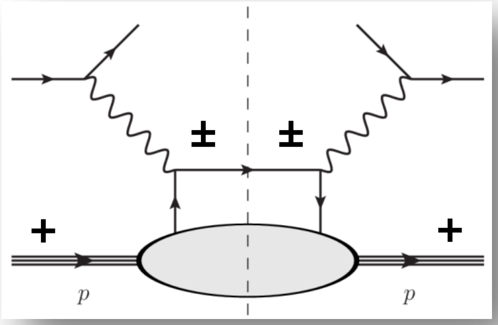

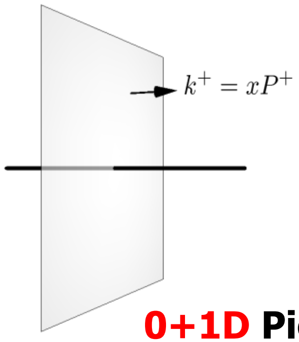

0+1D Picture

Quark polarization

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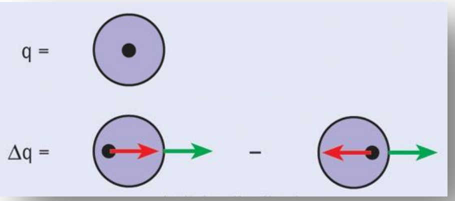
	U	T_x	T_y	L
U	f			
T_x		h		
T_y			h	
L				g

Spin-spin correlations

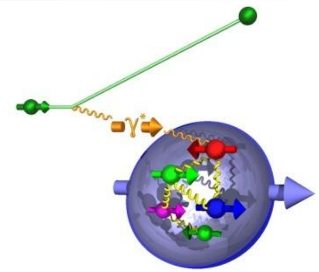
Vector

Axial



S_q, S_g

Probing the proton structure



Parton Distribution Functions

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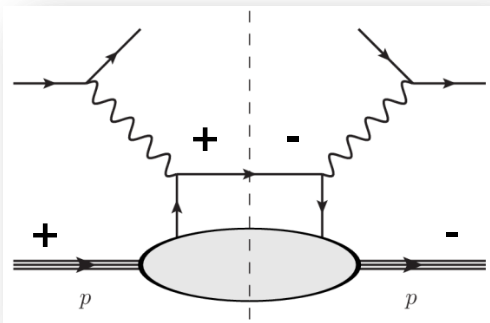

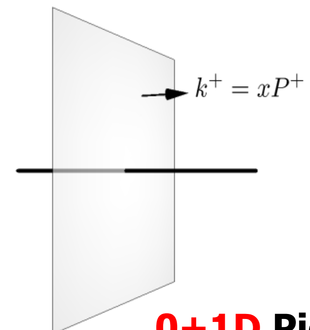

0+1D Picture

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Nucleon polarization

	U	T_x	T_y	L
U	f			
T_x		h		
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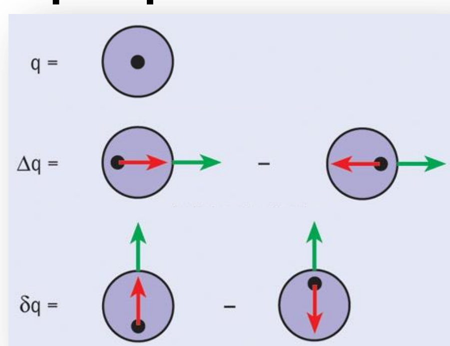
Spin-spin correlations

Vector

Axial

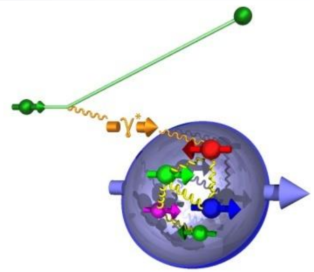
Tensor



S_q, S_g

Not accessible in DIS !

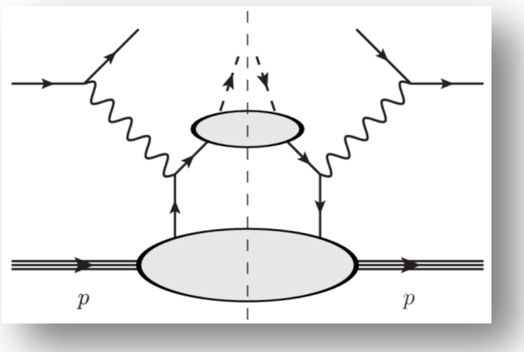
Probing the proton structure



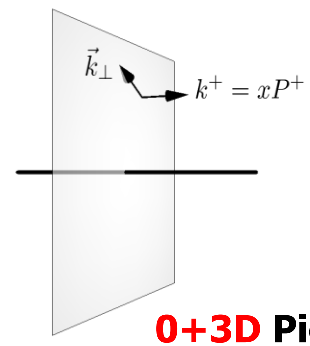
Transverse-Momentum dependent PDFs

$$\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p^+,\vec{0}_\perp,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2})|p^+,\vec{0}_\perp,\Lambda\rangle\Big|_{z^+=0}$$

SIDIS



[Kotzinian (1995)]
[Boer, Mulders (1998)]
[Bacchetta *et al.* (2007)]
[Anselmino *et al.* (2011)]



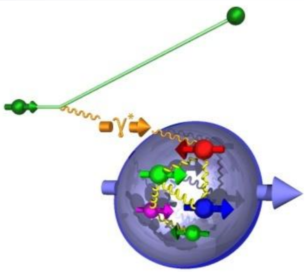
0+3D Picture

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\,h_1^\perp$	$i\frac{k_x}{M}\,h_1^\perp$	
T_x	$-i\frac{k_y}{M}\,f_{1T}$	$h_1+\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$\frac{k_x}{M}\,g_{1T}$
T_y	$i\frac{k_x}{M}\,f_{1T}$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$h_1-\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_y}{M}\,g_{1T}$
L		$\frac{k_x}{M}\,h_{1L}^\perp$	$\frac{k_y}{M}\,h_{1L}^\perp$	g_{1L}

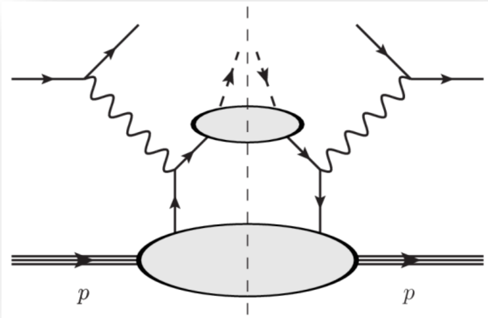
Probing the proton structure



Transverse-Momentum dependent PDFs

$$\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p^+,\vec{0}_\perp,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2})|p^+,\vec{0}_\perp,\Lambda\rangle\Big|_{z^+=0}$$

SIDIS



[Kotzinian (1995)]
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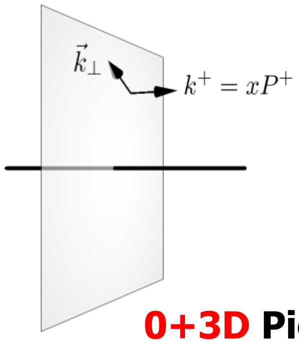


0+3D Picture

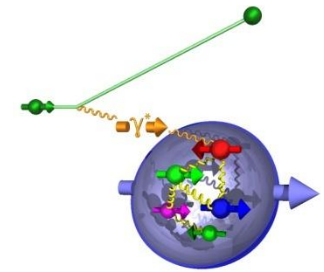
Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\,h_1^\perp$	$i\frac{k_x}{M}\,h_1^\perp$	
T_x	$-i\frac{k_y}{M}\,f_{1T}$	$h_1+\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$\frac{k_x}{M}\,g_{1T}$
T_y	$i\frac{k_x}{M}\,f_{1T}$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$h_1-\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_y}{M}\,g_{1T}$
L		$\frac{k_x}{M}\,h_{1L}^\perp$	$\frac{k_y}{M}\,h_{1L}^\perp$	g_{1L}

Monopole

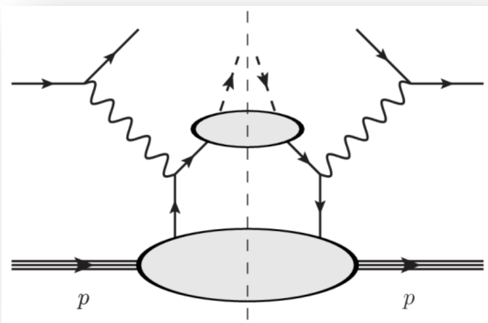
Probing the proton structure



Transverse-Momentum dependent PDFs

$$\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p^+,\vec{0}_\perp,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2})|p^+,\vec{0}_\perp,\Lambda\rangle\Big|_{z^+=0}$$

SIDIS



[Kotzinian (1995)]
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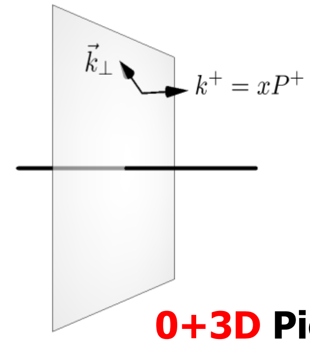
[Anselmino *et al.* (2011)]



0+3D Picture

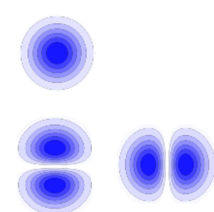
Quark polarization

Nucleon polarization

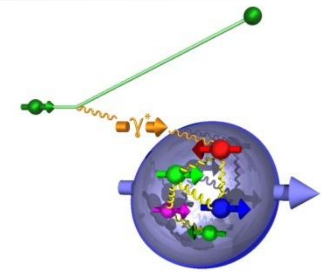
	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\,h_1^\perp$	$i\frac{k_x}{M}\,h_1^\perp$	
T_x	$-i\frac{k_y}{M}\,f_{1T}$	$h_1+\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$\frac{k_x}{M}\,g_{1T}$
T_y	$i\frac{k_x}{M}\,f_{1T}$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$h_1-\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_y}{M}\,g_{1T}$
L		$\frac{k_x}{M}\,h_{1L}^\perp$	$\frac{k_y}{M}\,h_{1L}^\perp$	g_{1L}

Monopole

Dipole



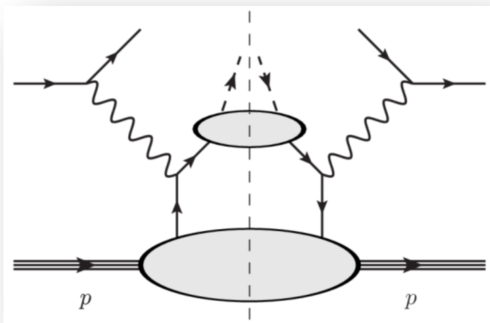
Probing the proton structure



Transverse-Momentum dependent PDFs

$$\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p^+,\vec{0}_\perp,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2})|p^+,\vec{0}_\perp,\Lambda\rangle\Big|_{z^+=0}$$

SIDIS



[Kotzinian (1995)]
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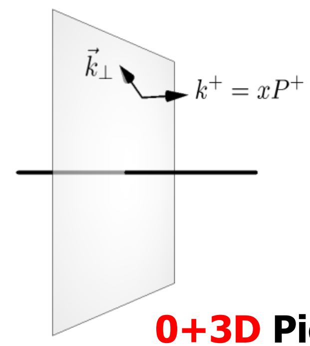
[Anselmino *et al.* (2011)]



0+3D Picture

Quark polarization

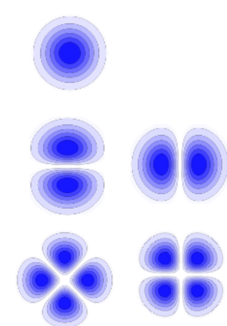
Nucleon polarization

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\,h_1^\perp$	$i\frac{k_x}{M}\,h_1^\perp$	
T_x	$-i\frac{k_y}{M}\,f_{1T}$	$h_1+\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$\frac{k_x}{M}\,g_{1T}$
T_y	$i\frac{k_x}{M}\,f_{1T}$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$h_1-\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_y}{M}\,g_{1T}$
L		$\frac{k_x}{M}\,h_{1L}^\perp$	$\frac{k_y}{M}\,h_{1L}^\perp$	g_{1L}

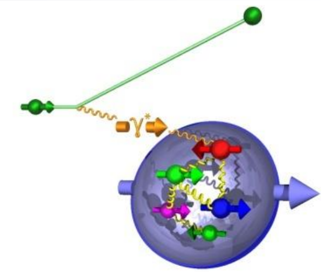
Monopole

Dipole

Quadrupole



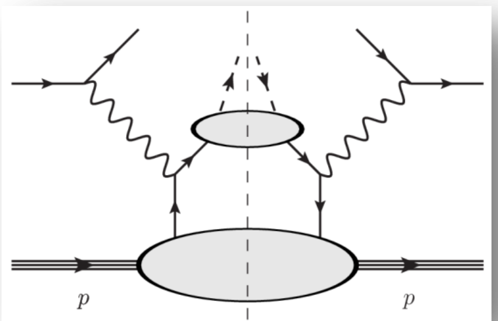
Probing the proton structure



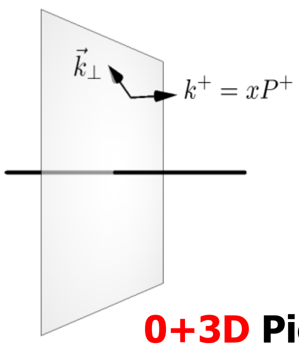
Transverse-Momentum dependent PDFs

$$\frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p^+,\vec{0}_\perp,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2})|p^+,\vec{0}_\perp,\Lambda\rangle\Big|_{z^+=0}$$

SIDIS



[Kotzinian (1995)]
[Boer, Mulders (1998)]
[Bacchetta *et al.* (2007)]
[Anselmino *et al.* (2011)]



0+3D Picture

Quark polarization

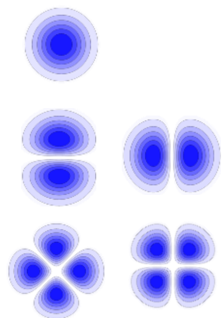
Nucleon polarization

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\,h_1^\perp$	$i\frac{k_x}{M}\,h_1^\perp$	
T_x	$-i\frac{k_y}{M}\,f_{1T}$	$h_1+\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$\frac{k_x}{M}\,g_{1T}$
T_y	$i\frac{k_x}{M}\,f_{1T}$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$h_1-\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_y}{M}\,g_{1T}$
L		$\frac{k_x}{M}\,h_{1L}^\perp$	$\frac{k_y}{M}\,h_{1L}^\perp$	g_{1L}

Monopole

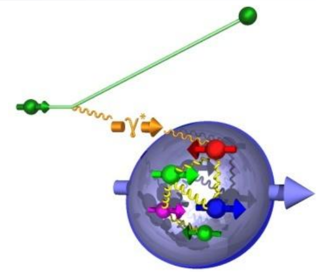
Dipole

Quadrupole



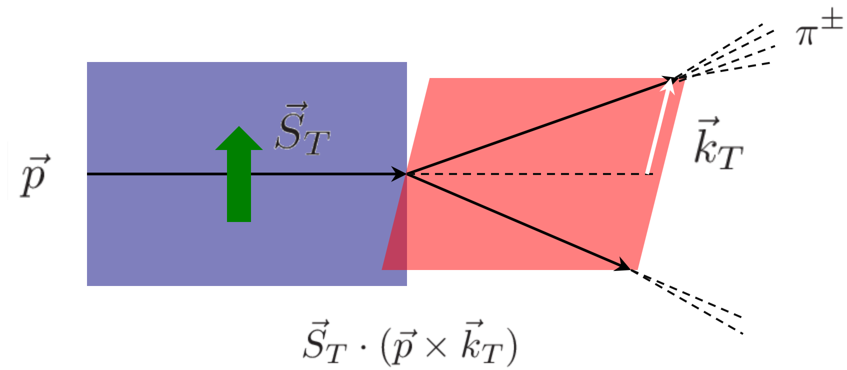
$\left.\right\} L_q, L_g$ **?**

Probing the proton structure



Transverse-Momentum dependent PDFs

Transverse single spin asymmetry $\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$



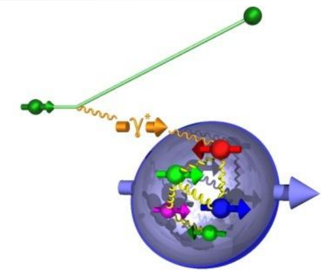
Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M} h_1^{\perp}$	$i\frac{k_x}{M} h_1^{\perp}$	
T_x	$-i\frac{k_y}{M} f_{1T}$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^{\perp}$	$\frac{k_x k_y}{M^2} h_{1T}^{\perp}$	$\frac{k_x}{M} g_{1T}$
T_y	$i\frac{k_x}{M} f_{1T}$	$\frac{k_x k_y}{M^2} h_{1T}^{\perp}$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^{\perp}$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^{\perp}$	$\frac{k_y}{M} h_{1L}^{\perp}$	g_{1L}

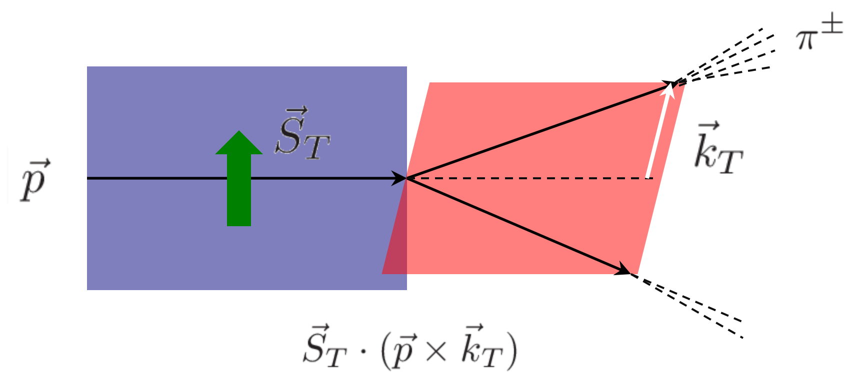
Naive T-odd

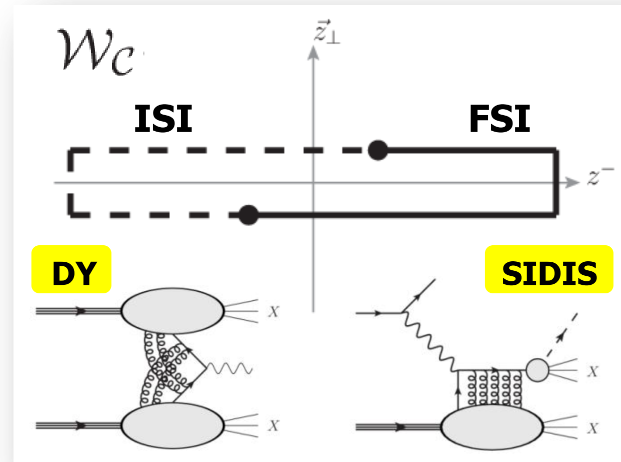
Probing the proton structure



Transverse-Momentum dependent PDFs

Transverse single spin asymmetry $\frac{\mathrm{d}\sigma^{\uparrow}-\mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow}+\mathrm{d}\sigma^{\downarrow}}$





Quark polarization

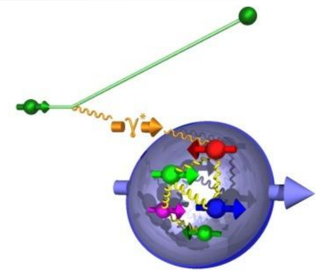
Nucleon polarization

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}h_1^{\perp}$	$i\frac{k_x}{M}h_1^{\perp}$	
T_x	$-i\frac{k_y}{M}f_{1T}$	$h_1+\frac{k_x^2-k_y^2}{2M^2}h_{1T}^{\perp}$	$\frac{k_x k_y}{M^2}h_{1T}^{\perp}$	$\frac{k_x}{M}g_{1T}$
T_y	$i\frac{k_x}{M}f_{1T}$	$\frac{k_x k_y}{M^2}h_{1T}^{\perp}$	$h_1-\frac{k_x^2-k_y^2}{2M^2}h_{1T}^{\perp}$	$\frac{k_y}{M}g_{1T}$
L		$\frac{k_x}{M}h_{1L}^{\perp}$	$\frac{k_y}{M}h_{1L}^{\perp}$	g_{1L}

Naive T-odd

$$\boxed{\begin{aligned} f_{1T}^{\perp,DY}(x,k_T) &= -f_{1T}^{\perp,SIDIS}(x,k_T) \\ h_1^{\perp,DY}(x,k_T) &= -h_1^{\perp,SIDIS}(x,k_T) \end{aligned}}$$

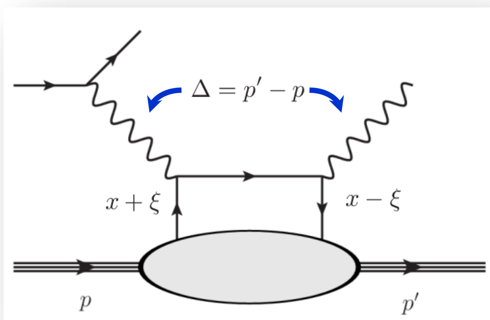
Probing the proton structure

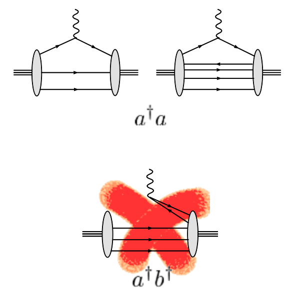


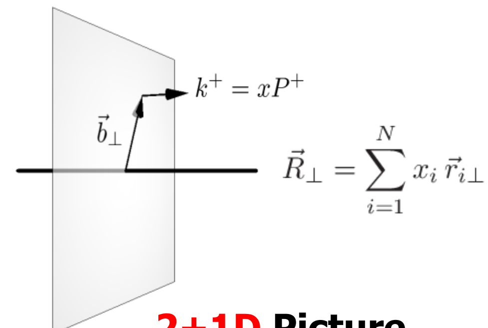
Generalized PDFs

$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ik^+z^-}\langle p'^+,-\tfrac{\vec{\Delta}_\perp}{2},\Lambda'|\overline{\psi}(-\tfrac{z^-}{2})\Gamma\mathcal{W}\psi(\tfrac{z^-}{2})|p^+,\tfrac{\vec{\Delta}_\perp}{2},\Lambda\rangle}$$

DVCS







2+1D Picture

[Burkardt (2000,2003)]

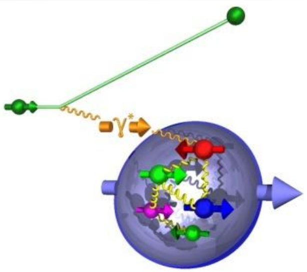
$$\boxed{\rho(x,\vec{b}_\perp) = \int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,GPD(x,0,-\vec{\Delta}_\perp^2)}$$

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}_T$	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}_T$	
T_x	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}$
T_y	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

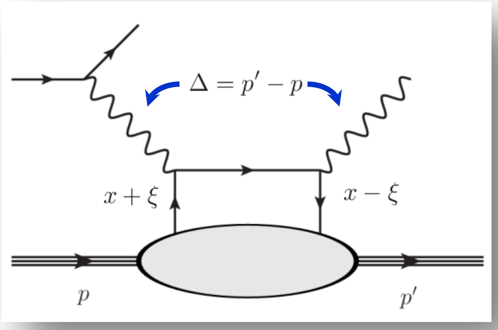
Probing the proton structure

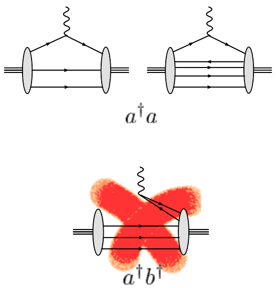


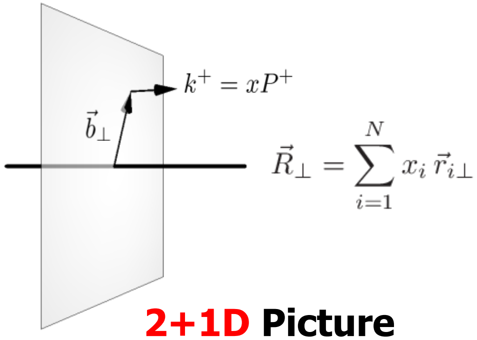
Generalized PDFs

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ik^+z^-}\langle p'^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda'|\overline{\psi}(-\tfrac{z^-}{2})\Gamma\mathcal{W}\psi(\tfrac{z^-}{2})|p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda\rangle$$

DVCS







2+1D Picture

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}_T$	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}_T$	
T_x	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}$
T_y	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

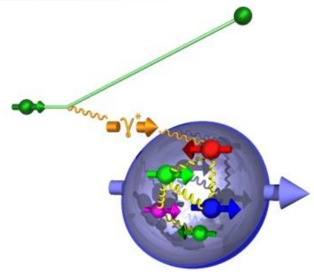
[Burkardt (2000,2003)]

$$\rho(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, GPD(x,0,-\vec{\Delta}_\perp^2)$$

Ji's sum rule **[Ji (1997)]**

$$J^{q,g} = \frac{1}{2}\int dx\, x\,[H^{q,g}(x,0,0) + E^{q,g}(x,0,0)]$$

Probing the proton structure



Generalized TMDs

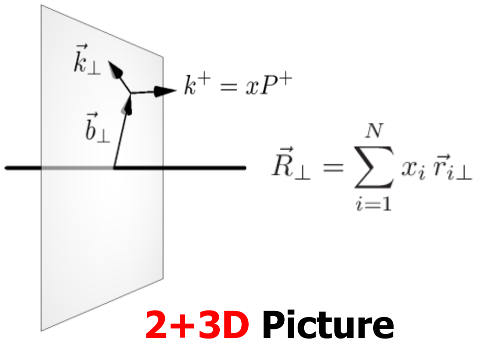
$$\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p^+,\tfrac{\vec{\Delta}_\perp}{2},\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2})|p^+,-\tfrac{\vec{\Delta}_\perp}{2},\Lambda\rangle\Big|_{z^+=0}$$

???

???

Quasi-probabilistic interpretation

[Wigner (1932)]
[Belitsky , Ji, Yuan (2004)]
[C.L., Pasquini (2011)]



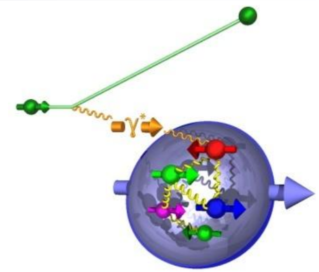
2+3D Picture

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	$\mathcal{F}_{11}$	$\frac{i}{2M}\left(k_y\mathcal{H}_{11}+\Delta_y\mathcal{H}_{12}\right)$	$-\frac{i}{2M}\left(k_x\mathcal{H}_{11}+\Delta_x\mathcal{H}_{12}\right)$	$i\mathcal{G}^q_{11}$
T_x	$\frac{i}{2M}\left(k_y\mathcal{F}_{12}+\Delta_y\mathcal{F}_{13}+\xi\Delta_x\mathcal{F}_{14}\right)$	$\cdots$	$\cdots$	$\frac{1}{2M}\left(k_x\mathcal{G}_{12}+\Delta_x\mathcal{G}_{13}+\Delta_y\mathcal{G}_{11}\right)$
T_y	$-\frac{i}{2M}\left(k_x\mathcal{F}_{12}+\Delta_x\mathcal{F}_{13}-\xi\Delta_y\mathcal{F}_{14}\right)$	$\cdots$	$\cdots$	$\frac{1}{2M}\left(k_y\mathcal{G}_{12}+\Delta_y\mathcal{G}_{13}-\Delta_x\mathcal{G}_{11}\right)$
L	$-i\mathcal{F}_{14}$	$\frac{1}{2M}\left(k_x\mathcal{H}_{17}+\Delta_x\mathcal{H}_{18}\right)$	$\frac{1}{2M}\left(k_y\mathcal{H}_{17}+\Delta_y\mathcal{H}_{18}\right)$	$\mathcal{G}^q_{14}$

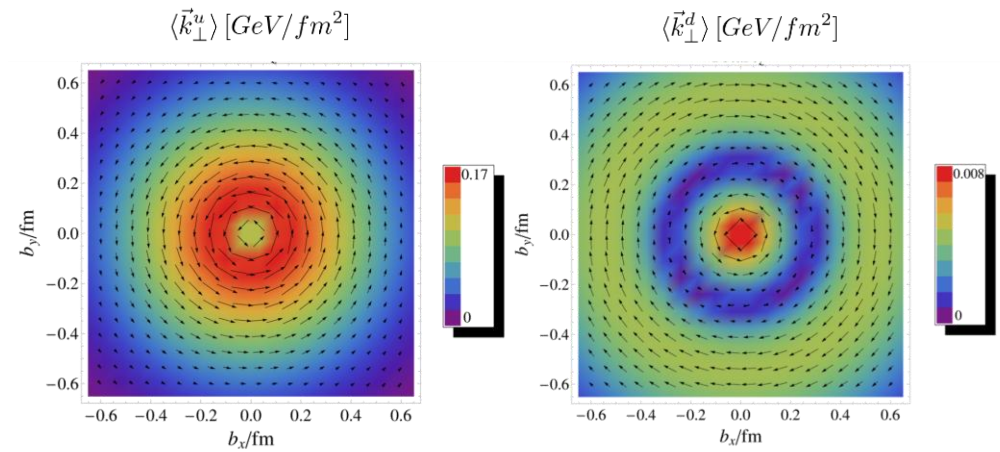
Probing the proton structure



Generalized TMDs

OAM **[C.L., Pasquini (2011)]**

$$\ell_z = \int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \mathrm{d}^2b_\perp \left(\vec{k}_\perp \times \vec{b}_\perp\right)_z \rho(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= -\int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \frac{\vec{k}_\perp^2}{M^2}\, F_{14}(x,\vec{k}_\perp,\vec{0}_\perp)$$



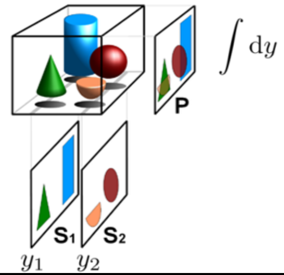
[C.L. *et al.* (2012)]

Quark polarization

Nucleon polarization

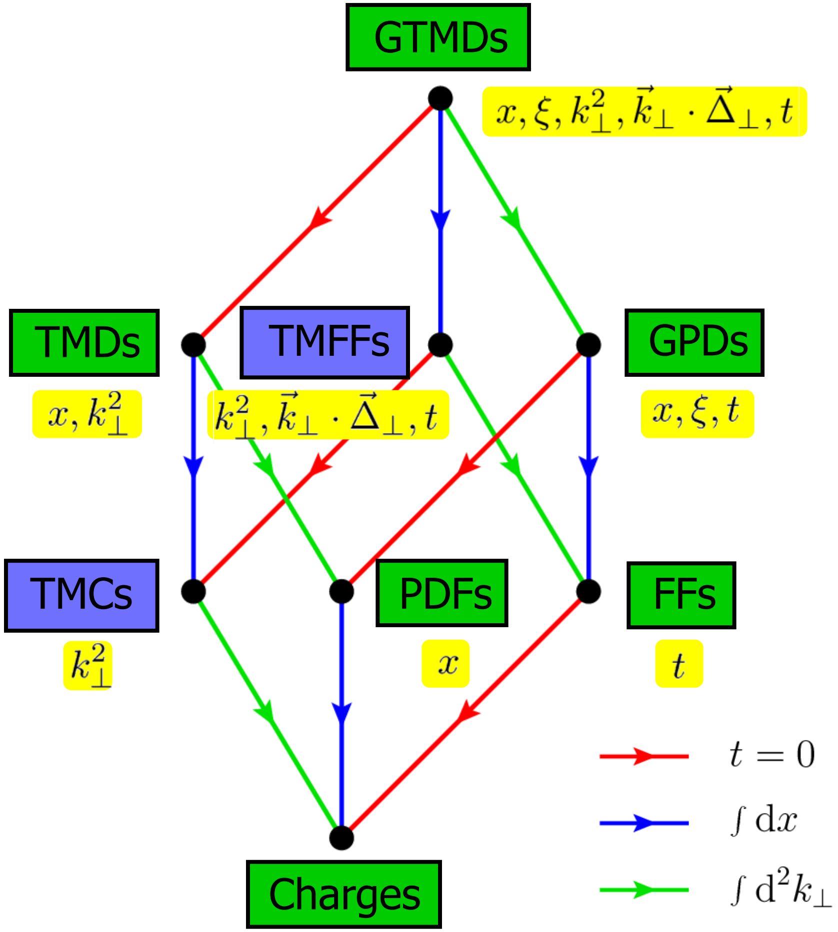
	U	T_x	T_y	L
U	$\mathcal{F}_{11}$	$\frac{i}{2M}(k_y\mathcal{H}_{11}+\Delta_y\mathcal{H}_{12})$	$-\frac{i}{2M}(k_x\mathcal{H}_{11}+\Delta_x\mathcal{H}_{12})$	$i\mathcal{G}_{11}^q$
T_x	$\frac{i}{2M}(k_y\mathcal{F}_{12}+\Delta_y\mathcal{F}_{13}+\xi\Delta_x\mathcal{F}_{14})$	$\cdots$	$\cdots$	$\frac{1}{2M}(k_x\mathcal{G}_{12}+\Delta_x\mathcal{G}_{13}+\Delta_y\mathcal{G}_{11})$
T_y	$-\frac{i}{2M}(k_x\mathcal{F}_{12}+\Delta_x\mathcal{F}_{13}-\xi\Delta_y\mathcal{F}_{14})$	$\cdots$	$\cdots$	$\frac{1}{2M}(k_y\mathcal{G}_{12}+\Delta_y\mathcal{G}_{13}-\Delta_x\mathcal{G}_{11})$
L	$-i\mathcal{F}_{14}$	$\frac{1}{2M}(k_x\mathcal{H}_{17}+\Delta_x\mathcal{H}_{18})$	$\frac{1}{2M}(k_y\mathcal{H}_{17}+\Delta_y\mathcal{H}_{18})$	$\mathcal{G}_{14}^q$

Parton distribution zoo



Complete set

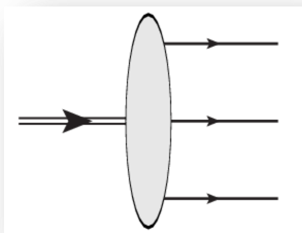
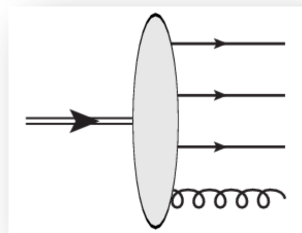
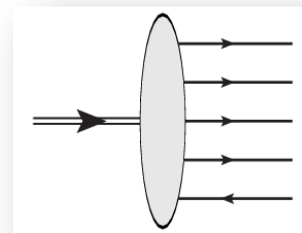
[C.L., Pasquini, Vanderhaeghen (2011)]



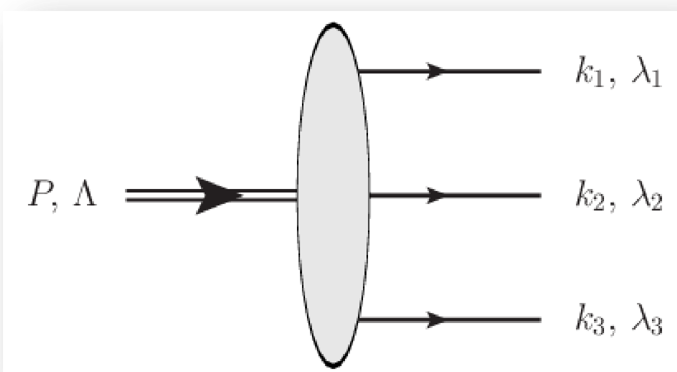
Overlap representation

Fock expansion of the proton state

$$|p\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqqq\bar{q}}\,|qqqq\bar{q}\rangle + \cdots$$

Fock states



Simultaneous eigenstates of

$$\left.\begin{aligned} P^+ &= \sum_{i=1}^{N} k_i^+ \\ \vec{0}_\perp = \vec{P}_\perp &= \sum_{i=1}^{N} \vec{k}_{i\perp} \end{aligned}\right\} \quad \textbf{Momentum}$$

λ_i **Light-front helicity**

Overlap representation

Light-front wave functions

Eigenstates of parton light-front helicity

$$\widehat{S}_{iz}\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = \lambda_i\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}$$

Eigenstates of total OAM

$$\widehat{L}_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}$$

$$\boxed{\Lambda = \sum_{i=1}^{N}\lambda_i + l_z}$$

$A^+ = 0$ **gauge**

Proton state

Probability associated with the *N,β* Fock state

$$\rho^{\Lambda}_{N,\beta} = \int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N\,\left|\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}\right|^2$$

Normalization

$$\sum_{N,\beta}\rho^{\Lambda}_{N,\beta} = 1$$

$$\boxed{\Lambda = s_z + \ell_z}$$

$$s_z = \langle\widehat{S}_z\rangle = \sum_{N,\beta}\sum_{i=1}^{N}\lambda_i\,\rho^{\Lambda}_{N,\beta}$$

$$\ell_z = \langle\widehat{L}_z\rangle = \sum_{N,\beta} l_z\,\rho^{\Lambda}_{N,\beta}$$

Comparison of different OAM

Overlap representation

[Hägler, Mukherjee, Schäfer (2004)]
[C.L., Pasquini, Xiong, Yuan (2011)]
[C.L., Pasquini (2011)]

Flavor contribution

$$\mathcal{L}_z^q = -\frac{i}{2}\sum_{N,\beta}\sum_{i=1}^{N}\delta_{qq_i}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N\left[\Psi^{*\uparrow}_{N,\beta}\left(\vec{k}_{i\perp}\times\overleftrightarrow{\nabla}_{k_{i\perp}}\right)\Psi^{\uparrow}_{N,\beta}\right]$$ **TMDs**

$$\ell_z^q = -\frac{i}{2}\sum_{N,\beta}\sum_{i=1}^{N}\delta_{qq_i}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N\sum_{n=1}^{N}(\delta_{ni}-x_n)\left[\Psi^{*\uparrow}_{N,\beta}\left(\vec{k}_{i\perp}\times\overleftrightarrow{\nabla}_{k_{n\perp}}\right)\Psi^{\uparrow}_{N,\beta}\right]$$ **GTMDs**

$$L_z^q = \frac{1}{2}\sum_{N,\beta}\sum_{i=1}^{N}\delta_{qq_i}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N\left\{(x_i-2\lambda_i)\left|\Psi^{\uparrow}_{N,\beta}\right|^2+Mx_i\sum_{n=1}^{N}(\delta_{ni}-x_n)\left[\Psi^{*\uparrow}_{N,\beta}\frac{\overleftrightarrow{\partial}}{\partial k_n^x}\Psi^{\downarrow}_{N,\beta}\right]\right\}$$ **GPDs**

Pure quark system

[C.L., Pasquini (2011)]

Conservation of transverse momentum

$$\sum_{i=1}^{N}\vec{k}_{i\perp}(\delta_{ni}-x_n)=\vec{k}_{n\perp}-x_n\sum_{i=1}^{N}\vec{k}_{i\perp}=\vec{k}_{n\perp}$$

Conservation of longitudinal momentum

$$\sum_{i=1}^{N}x_i(\delta_{ni}-x_n)=x_n\left(1-\sum_{i=1}^{N}x_i\right)=0$$

$$\boxed{\ell_z=\sum_q \ell_z^q=\sum_q \mathcal{L}_z^q=\sum_q L_z^q}$$

NB: also valid for N,β Fock states

$$\boxed{B(0)=0}$$

Anomalous gravitomagnetic sum rule!

[Brodsky, Hwang, Ma, Schmidt (2001)]

Comparison of different OAM

Light-front 3Q models

[C.L., Pasquini (2011)]

Model	LCCQM			LCχQSM			
q	u	d	Total	u	d	Total	
ℓ_z^q	0.131	-0.005	0.126	0.073	-0.004	0.069	GTMDs
L_z^q	0.071	0.055	0.126	-0.008	0.077	0.069	GPDs
$\mathcal{L}_z^q$	0.169	-0.042	0.126	0.093	-0.023	0.069	TMDs

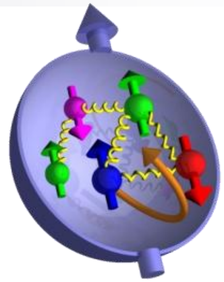
Models are not QCD

Truncation of Fock space can spoil Lorentz covariance

[Carbonell, Desplanques, Karmanov, Mathiot (1998)]

In model calculations, one should expect $\ell_z = L_z$ **but** $\ell_z^q \neq L_z^q$

Emerging picture

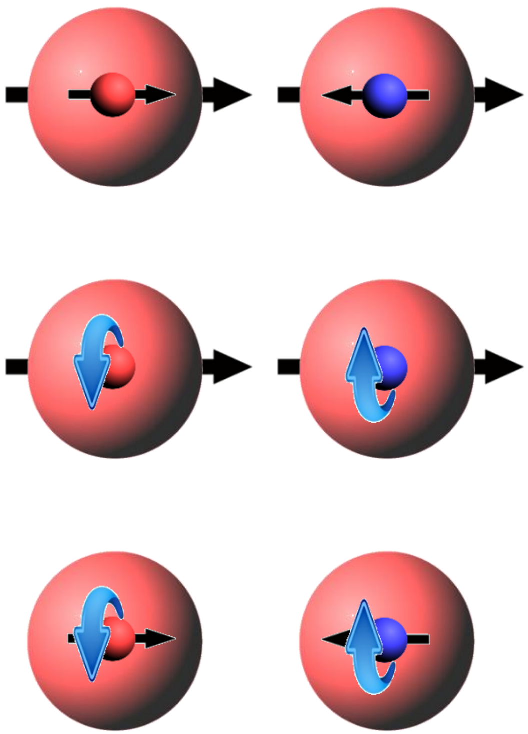


Longitudinal

$g^q_{1L} \leftrightarrow \tilde{\mathcal{H}}^q$

$\ell^q_z \leftrightarrow \tilde{\mathcal{F}}^q_{14}$

$C^q_z \leftrightarrow \tilde{\mathcal{G}}^q_{11}$



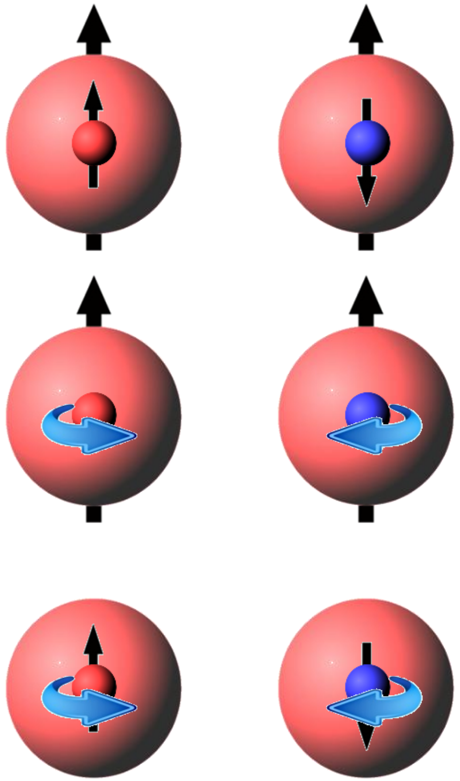
[C.L., Pasquini (2011)]

Transverse

$h^q_1 \leftrightarrow \mathcal{H}^q_T$

$f^{\perp q}_{1T} \leftrightarrow \mathcal{E}^q$

$h^{\perp q}_1 \leftrightarrow \mathcal{E}^q_T$

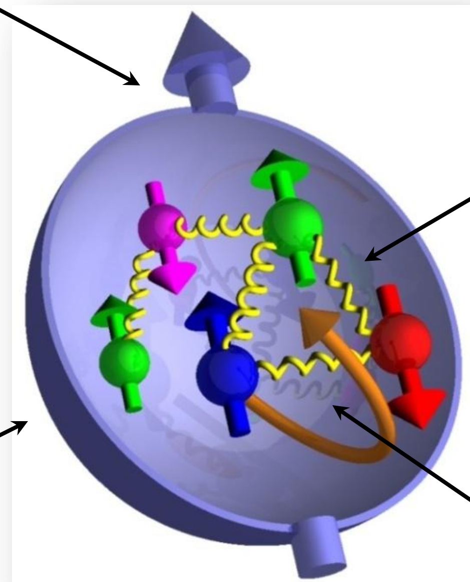


[Burkardt (2005)]
[Barone *et al.* (2008)]

Summary

Proton spin decomposition ?
- Canonical and kinetic AM
- Accessible in (semi-)inclusive and exclusive processes

Quark-gluon interactions ?
- Scale dependence
- Twist-3 effects



Parton distributions ?
- Factorization theorem
- Baryon tomography

Spin-orbit correlations ?
- Different types of polarization
- Multipolar structure

Thank you !