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Extraction of TMDs with global fits

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Outline

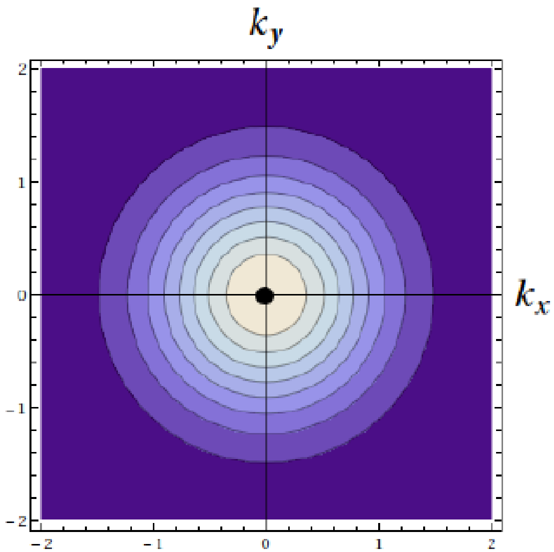
- The Sivers function in SIDIS & DY, TMD evolution
- Transversity and Collins functions
- Boer-Mulders & Cahn effect in SIDIS, BM in DY

Extraction of the Sivers function from $lp^{\uparrow} \to l'h + X$ (SIDIS) data

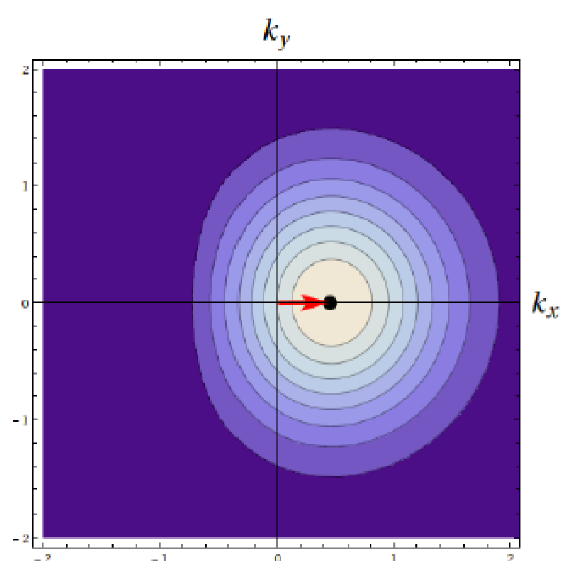
The Sivers function

- The Sivers function describes the distortion of the (unpolarized) quark distribution due to the fact that the proton is polarized

$$
\begin{aligned}
f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) - f_{q/p^\downarrow}(x, \boldsymbol{k}_\perp) &= \Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \boldsymbol{S}_T \cdot (\hat{\boldsymbol{P}} \times \hat{\boldsymbol{k}}_\perp) \\
&= \Delta^N f_{q/p^\uparrow}(x, k_\perp) \sin(\varphi - \phi_S)
\end{aligned}
$$



Unpolarized proton



Transversely polarized proton

Fig. Courtesy by Alexei Prokudin

The Sivers function from SIDIS data

- The Sivers contribution can selected weighting the cross section

$$A_{UT}^{\sin(\phi_h-\phi_S)} = 2\,\frac{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow - d\sigma^\downarrow]\, \sin(\phi_h-\phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow + d\sigma^\downarrow]}$$

- In details:

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\; \Delta^N f_{q/p^\uparrow}(x,k_\perp)\sin(\varphi-\phi_S)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\; D_q^h(z,p_\perp)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\; f_{q/p}(x,k_\perp)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\; D_q^h(z,p_\perp)}$$

Turin standard approach (DGLAP)

- Unpolarized TMDs are factorized in x and $k_\perp$. Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Turin standard approach (DGLAP)

- The Sivers function is factorized in x and $k_\perp$ and proportional to the unpolarized PDF.

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) \quad = \quad 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)$$

Proportional to the unpolarized TMD

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\,e^{-k_\perp^2/M_1^2}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x,k_\perp)$$

Turin standard approach (DGLAP)

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$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)$$
$$= 2\mathcal{N}_q(x)f_{q/p}(x;Q)\sqrt{2e}\frac{k_\perp}{M_1}\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}$$

Collinear PDF (DGLAP)

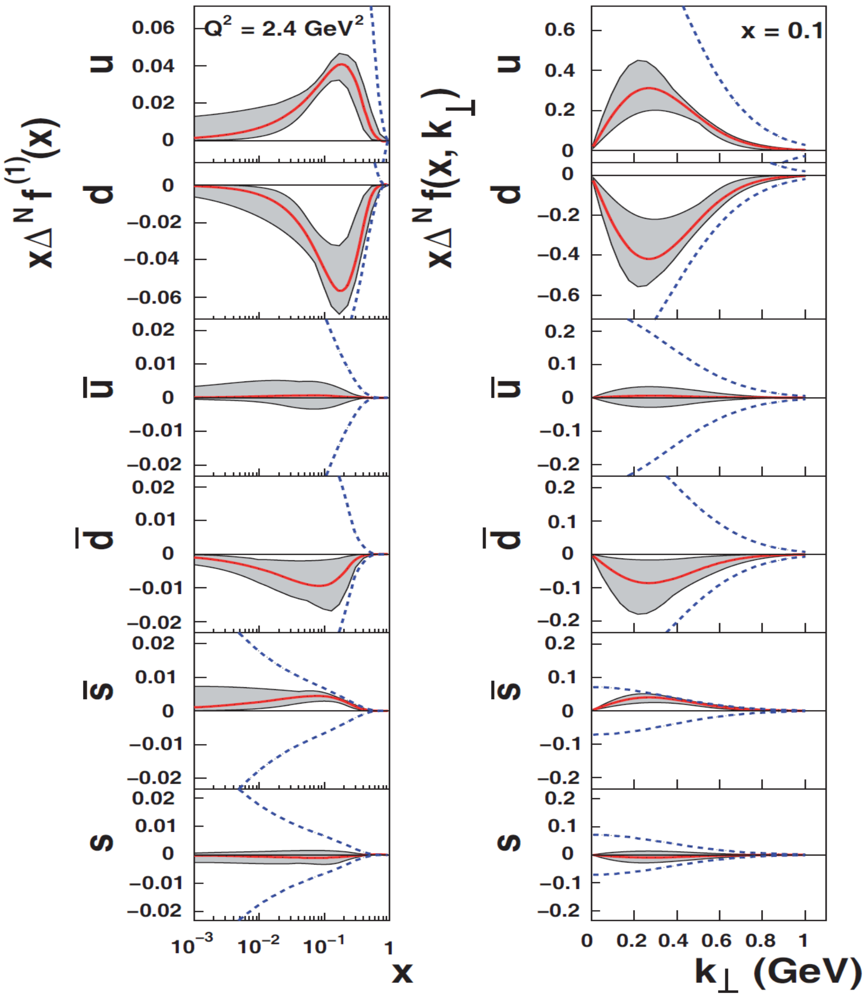
$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p}f_{1T}^\perp(x,k_\perp)$$

Sivers function in SIDIS

- In 2009 we performed a fit of HERMES (2002-2005) and COMPASS (Deuteron 2003-2004) data on π and K production



✔Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp u} < 0$
- $\Delta^N f_{d/p^\uparrow} < 0 \quad \Rightarrow f_{1T}^{\perp d} > 0$

✔Sea quarks

- $\Delta^N f_{\bar{s}/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp \bar{s}} < 0$

→ $\Delta^N f_q^{(1)}(x) \equiv \int d^2 \boldsymbol{k}_\perp \, \frac{k_\perp}{4m_p} \, \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$

Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

Sivers function in SIDIS

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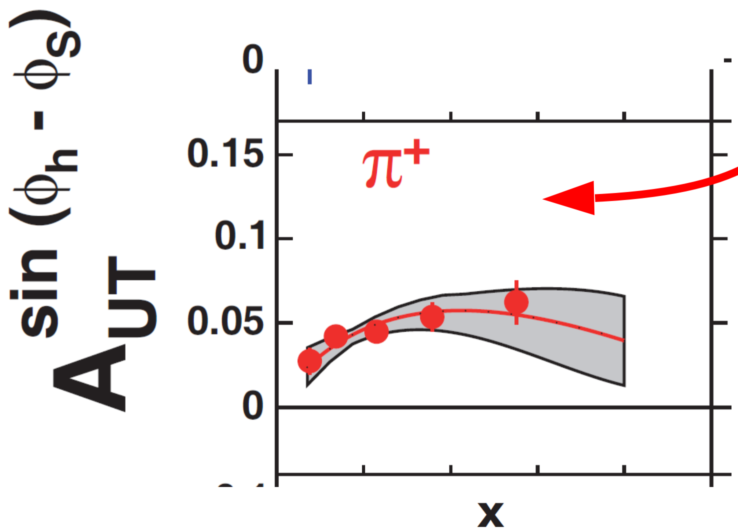
HERMES (Proton target)



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Sivers function in SIDIS

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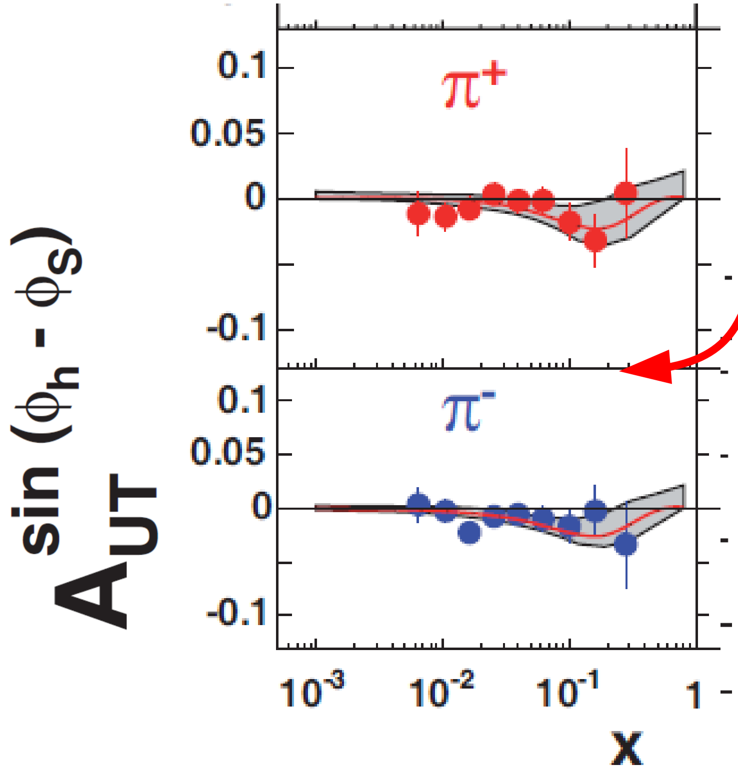
COMPASS (Deuteron target)



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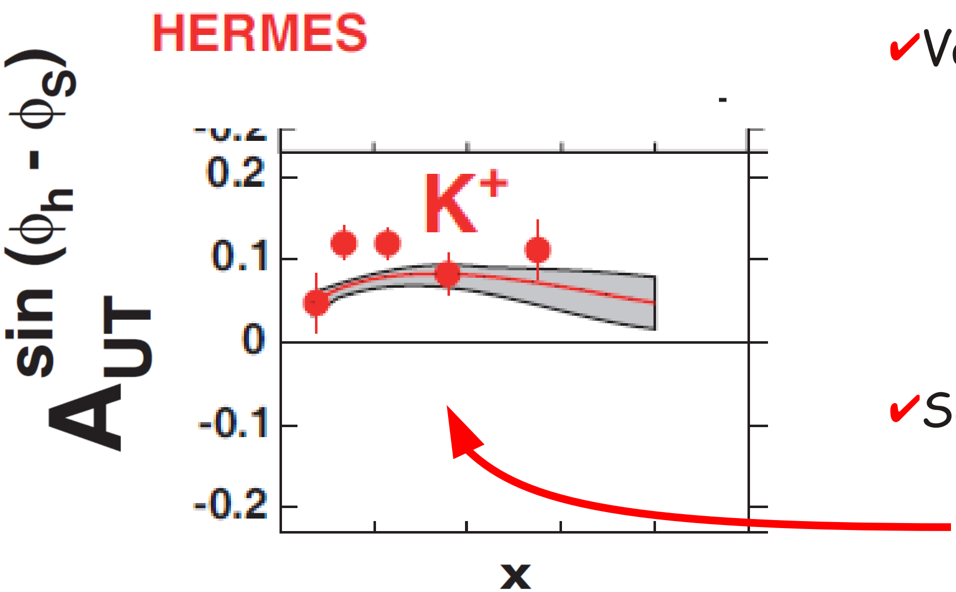

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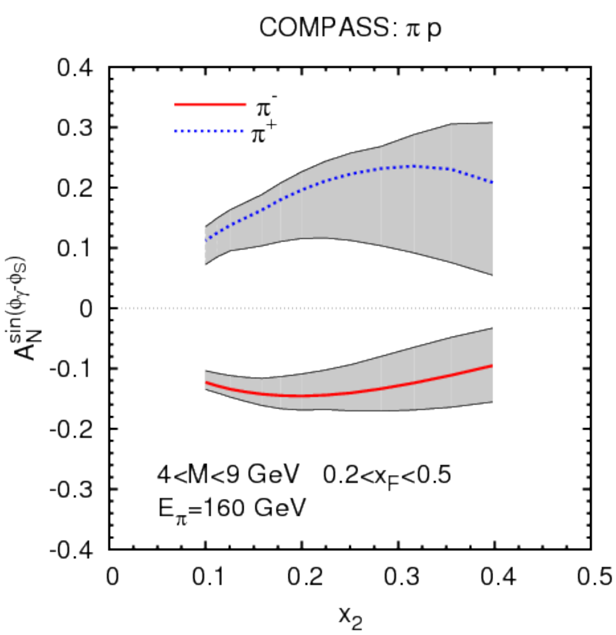
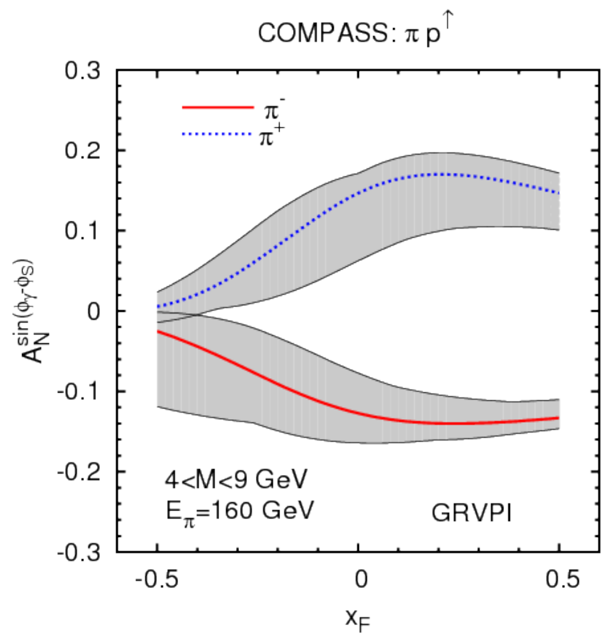
Sivers in DY processes (DGLAP)

- Polarized NH_3
- Pion beam
- Valence region for the Sivers function





• Anselmino et al. Phys. Rev. D79,054010

Sivers function in SIDIS

- New theoretical tool: TMD evolution equation!

What are the consquences from the phenomenological point of view??

TMD evolution formalism*

*
- J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.
- S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]
- S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]

TMD evolution formalism

- Let us denote with $\widetilde{F}$ either a PDF (or a FF) or the first derivative of the Sivers function in the impact parameter space:

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) \rightarrow \widetilde{f}_{q/p}(x,\boldsymbol{b}_T;Q)$$ Unpolarized PDF

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) \rightarrow \widetilde{D}_{h/q}(z,\boldsymbol{b}_T;Q)$$ Unpolarized FF

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) \rightarrow \widetilde{f}_{1T}^{\prime\perp f}(x,\boldsymbol{b}_T;Q)$$ First derivative of the Sivers function

TMD evolution formalism

➢ At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

Corresponding to Eq. 44 of Ref [*] with $\tilde{K}$=0 and : $\mu^2 = \zeta_F = \zeta_D = Q^2$

- *[*]S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

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Output function at the scale **Q**
in the impact parameter space

Input function at the scale **Q_0**
in the impact parameter space

Evolution kernel

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

TMD evolution formalism

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$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0) \widetilde{R}(Q, Q_0, b_T) \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

➤ **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{\mathrm{d}\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right) \right\}$$

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x,\boldsymbol{b}_T;Q)=\widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q,Q_0,b_T)\equiv\exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu')+\int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu)=\alpha_s(\mu)\,\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu;\frac{Q^2}{\mu^2})=\alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2}-\ln\frac{Q^2}{\mu^2}\right)$$

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

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Scale that separates the perturbative region
from the non perturbative one

TMD evolution formalism

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$$\mu_b = \frac{C_1}{b_*(b_T)} \qquad b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}} \qquad C_1 = 2e^{-\gamma_E}$$

One of the possible prescription to separate the perturbative region from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- **Non Perturbative** (scale independent) part of the evolution kernel that needs to be empirically modeled

$$g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2$$

Common choice used in the unpolarized DY data analyses in the CSS formalism

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

- Model/parametrization

Parametrization of the input functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Example: unpolarized pdf

$$\tilde{F}(x,b_T;Q_0) = \tilde{f}_{q/p}(x,b_T;Q_0) \xrightarrow{\text{Fourier transf.}} \widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Parametrization of the input functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\widetilde{f}_{q/p}(x,b_T;Q_0) = f_{q/p}(x,Q_0)\exp\left\{-\alpha^2\, b_T^2\right\}$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\alpha^2 = \langle k_\perp^2\rangle/4$$

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q_0) = \frac{1}{z^2} D_{h/q}(z, Q_0)\, \exp\{-\beta^2\, b_T^2\}$$

$$\widehat{D}_{h/q}(z, p_\perp; Q_0) = D_{h/q}(z, Q_0)\, \frac{1}{\pi \langle p_\perp^2 \rangle}\, e^{-p_\perp^2/\langle p_\perp^2 \rangle}$$

$$\beta^2 = \langle p_\perp^2 \rangle / 4z^2$$

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$\widetilde{f}'^{\perp}_{1T}(x, b_T; Q_0) = -2\,\gamma^2\, f^{\perp}_{1T}(x; Q_0)\, b_T\, e^{-\gamma^2\, b_T^2}$$

$$\widehat{f}^{\perp}_{1T}(x, k_\perp; Q_0) = f^{\perp}_{1T}(x; Q_0)\, \frac{1}{4\,\pi\,\gamma^2}\, e^{-k_\perp^2/4\gamma^2}$$

$$4\,\gamma^2 \equiv \langle k_\perp^2 \rangle_S = \frac{M_1^2\, \langle k_\perp^2 \rangle}{M_1^2 + \langle k_\perp^2 \rangle}$$

Parametrization of the input functions

- Then the evolution equations for unpolarized TMDs become simply:

$$\widetilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0)\ \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q) = \frac{1}{z^2} D_{h/q}(z, Q_0)\ \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\beta^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

- While for the Sivers function we have:

$$\widetilde{f}'^{\perp}_{1T}(x, b_T; Q) = -2\,\gamma^2\, f^{\perp}_{1T}(x; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, b_T\ \exp\left\{-b_T^2\left(\gamma^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

TMD evolution formalism

- One can get the TMD in the momentum space by Fourier trasforming:

$$\widehat{f}_{q/p}(x, k_\perp; Q) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}b_T \, b_T \, J_0(k_\perp b_T) \, \widetilde{f}_{q/p}(x, b_T; Q)$$

$$\widehat{D}_{h/q}(z, p_\perp; Q) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}b_T \, b_T \, J_0(\mathrm{k}_T b_T) \, \widetilde{D}_{h/q}(z, b_T; Q)$$

$$\widehat{f}_{1T}^{\perp f}(x, k_\perp; Q) = \frac{-1}{2\pi k_\perp} \int_0^\infty \mathrm{d}b_T \, b_T \, J_1(k_\perp b_T) \, \widetilde{f}'^{\perp q}_{1T}(x, b_T; Q)$$

$$\begin{aligned} f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp, \boldsymbol{S}; Q) &= f_{q/p}(x, k_\perp; Q) - f_{1T}^{\perp q}(x, k_\perp; Q) \frac{\epsilon_{ij} \, k_\perp^i \, S^j}{M_p} \\ &= f_{q/p}(x, k_\perp; Q) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp; Q) \frac{\epsilon_{ij} \, k_\perp^i \, S^j}{k_\perp} \end{aligned}$$

Fit of HERMES and COMPASS SIDIS data

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \Delta^N f_{q/p^\uparrow}(x,k_\perp,Q)\sin(\varphi-\phi_S)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x,k_\perp,Q)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q_0) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2}$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\, e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

11 free parameters

N_{u_v}	N_{d_v}	N_s
$N_{\bar u}$	$N_{\bar d}$	$N_{\bar s}$
α_{u_v}	α_{d_v}	α_{sea}
β	M_1 (GeV/c) .	

Fixed parameters

$$\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$$
[*] $g_2 = 0.68\ \mathrm{GeV}^2$
$$b_{max} = 0.5\ \mathrm{GeV}^{-1}$$

[*]Landry et al. Phys Rev D67, 073016

Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD evolution (exact)
$\chi^2_{\rm tot} = 255.8$ $\chi^2_{\rm d.o.f} = 1.02$

DGLAP evolution
$\chi^2_{\rm tot} = 315.6$ $\chi^2_{\rm d.o.f} = 1.26$

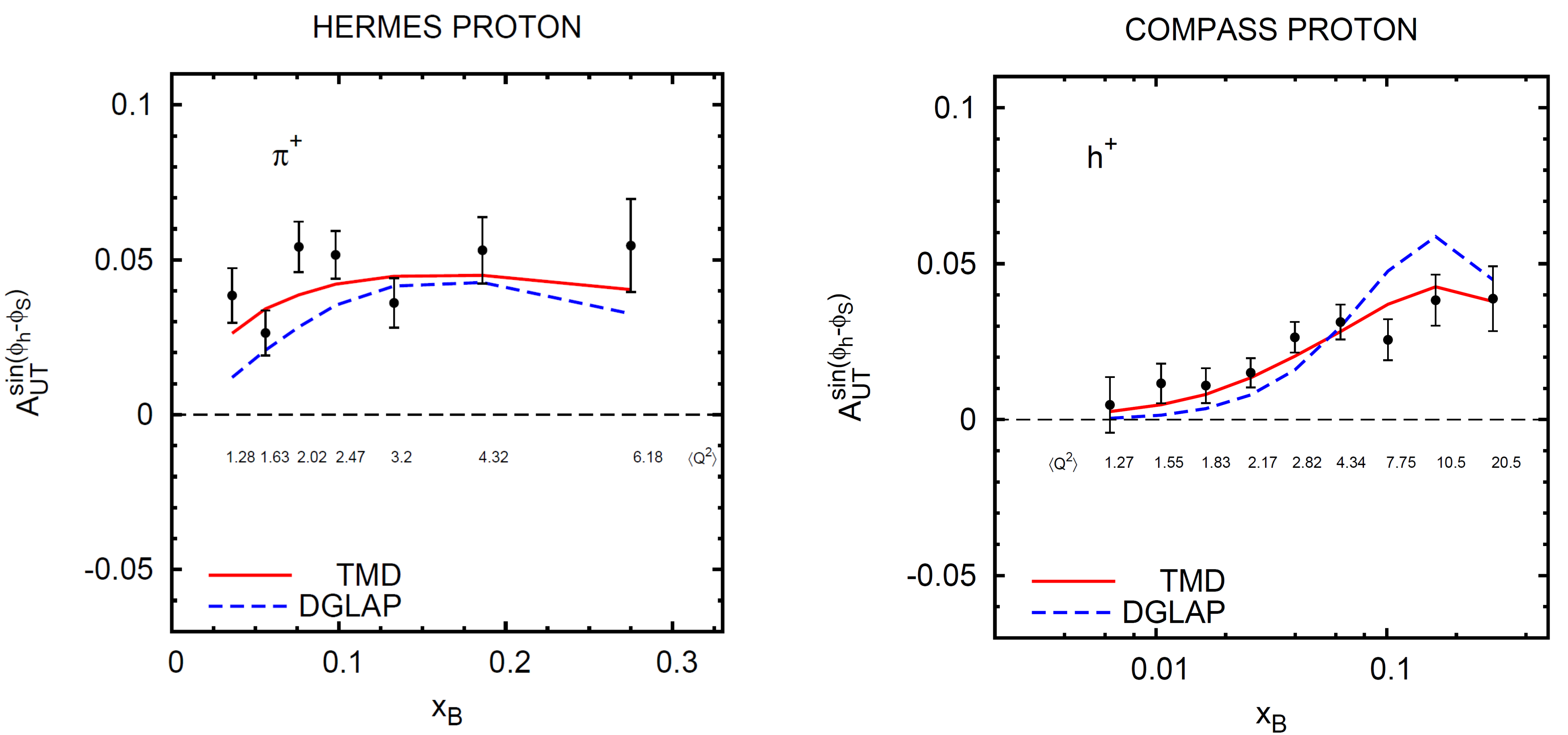
Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

	TMD Evolution (Exact)		DGLAP Evolution
	$\chi^2_{tot} = 255.8$		$\chi^2_{tot} = 315.6$
	$\chi^2_{d.o.f} = 1.02$		$\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\chi^2_x = 10.7$	7 points	$\chi^2_x = 27.5$
	$\chi^2_z = 4.3$		$\chi^2_z = 8.6$
	$\chi^2_{P_T} = 9.1$		$\chi^2_{P_T} = 22.5$
COMPASS h^+	$\chi^2_x = 6.7$	9 points	$\chi^2_x = 29.2$
	$\chi^2_z = 17.8$		$\chi^2_z = 16.6$
	$\chi^2_{P_T} = 12.4$		$\chi^2_{P_T} = 11.8$

Fit of HERMES and COMPASS SIDIS data



Anselmino, Boglione, Melis, PRD 86 (2012) 014028

Consequences on DY data and warnings

- A rigorous fit need a 'fresh restart' i.e. the analysis of the SIDIS and DY unpolarized data

Fixed parameters in the fit

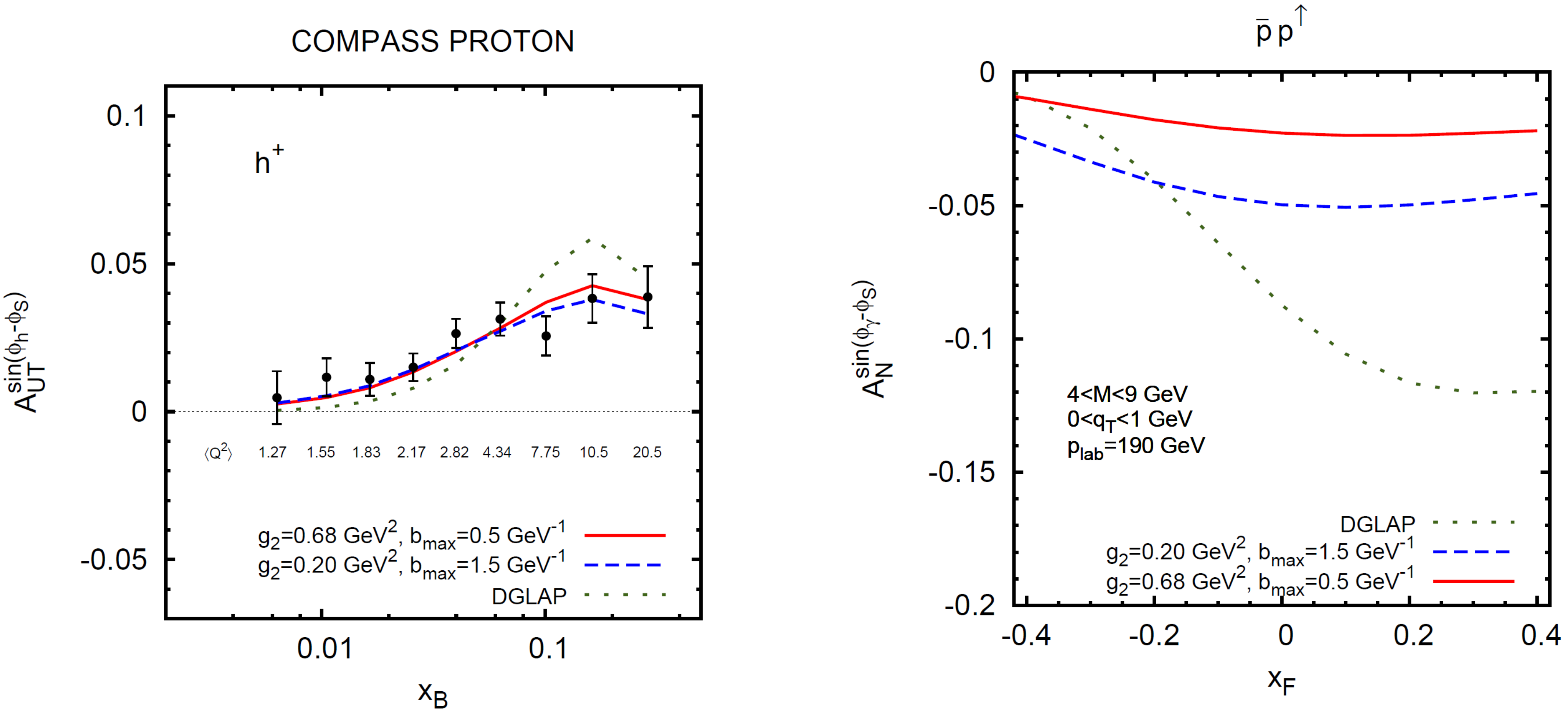
$$\langle k_\perp^2 \rangle = 0.25\ \mathrm{GeV}^2$$
$$\langle p_\perp^2 \rangle = 0.20\ \mathrm{GeV}^2$$
[*] $$g_2 = 0.68\ \mathrm{GeV}^2$$
$$b_{max} = 0.5\ \mathrm{GeV}^{-1}$$

- In SIDIS, the Sivers asymmetry is not so strongly sensitive to these values.

- … however in DY they are crucial, in particular g_2

[*]Landry et al. Phys Rev D67, 073016

Consequences on DY data and warnings



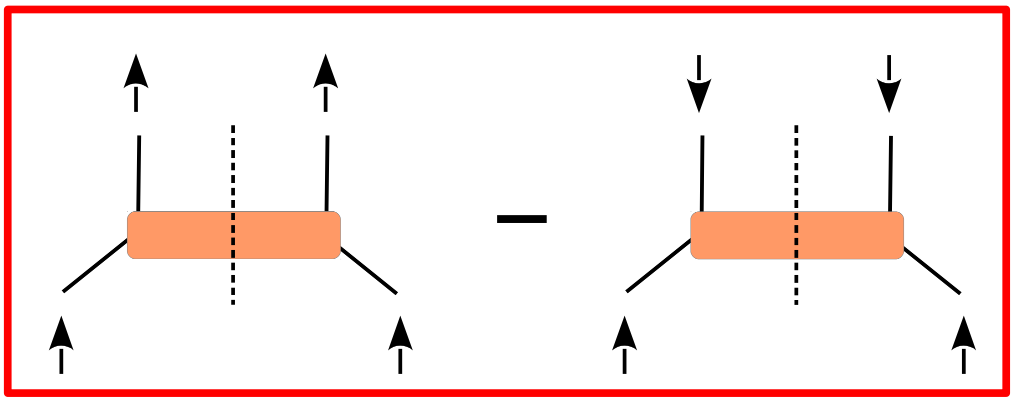
Conclusions I

- Sivers functions are definitively different from zero!
- There are indications supporting TMD evolution in SIDIS
- Asymmetry in DY are more sensitive to TMD evolution

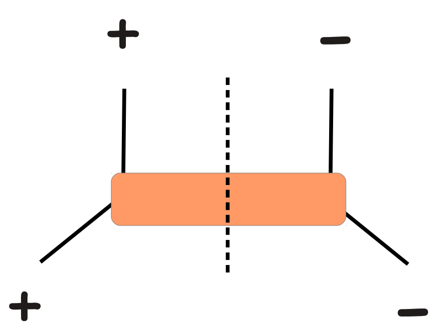
Transversity&Collins functions

The Transversity Function

- The transversity is a twist two, collinear, distribution of transversely polarized quarks inside a transversely polarized hadron



- Or in the helicity basis: $|\uparrow,\downarrow\rangle = |+\rangle \pm i|-\rangle$

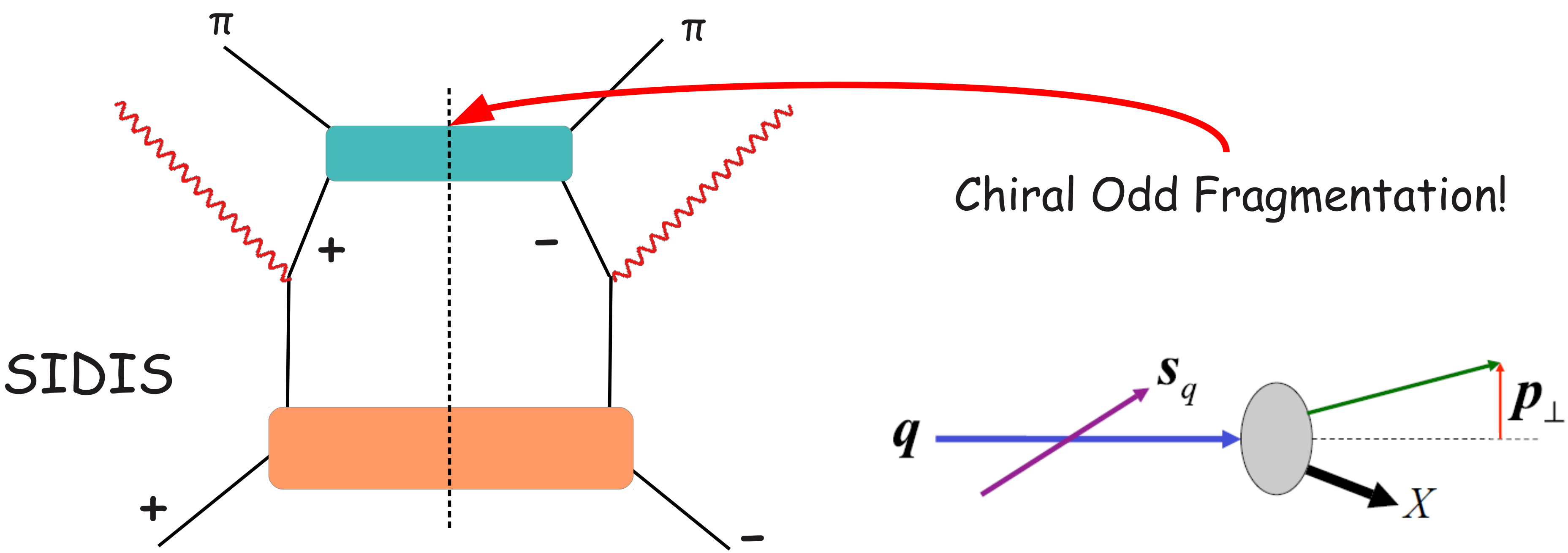


- Off diagonal in helicity basis: Chiral Odd!

$$F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} \longrightarrow F^{+-}_{+-}$$

Accessing the transversity

- Let us consider the SIDIS instead of the DIS process



- Collins fragmentation function:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp) = D_{\pi/q^\uparrow}(z,p_\perp) - D_{\pi/q^\downarrow}(z,p_\perp)$$

Extraction of the transversity & Collins functions

- Azimuthal asymmetry in polarized SIDIS

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_q h_{1q}(x, \boldsymbol{k}_\perp) \otimes \mathrm{d}\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

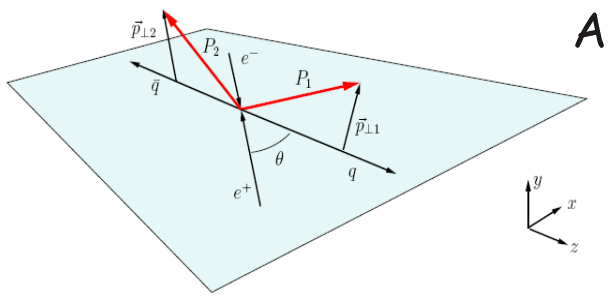
Transversity

Collins function

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2 \frac{\int \mathrm{d}\phi \, \mathrm{d}\phi_S \, [\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}] \, \sin(\phi + \phi_S)}{\int \mathrm{d}\phi \, \mathrm{d}\phi_S \, [\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}$$

Extraction of transversity & Collins functions

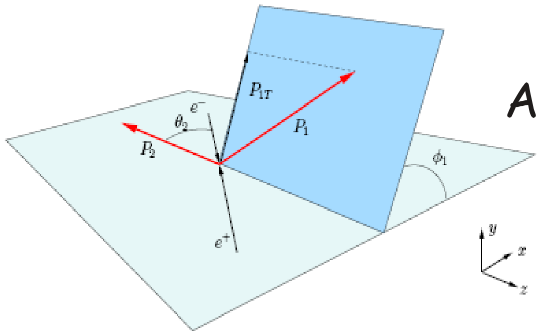
- $e^+e^- \to h_1\, h_2\, X$ BELLE Data



A_{12} asymmetry → Thrust axis method

$$A(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1+\cos^2\theta} \cos(\varphi_1+\varphi_2) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$



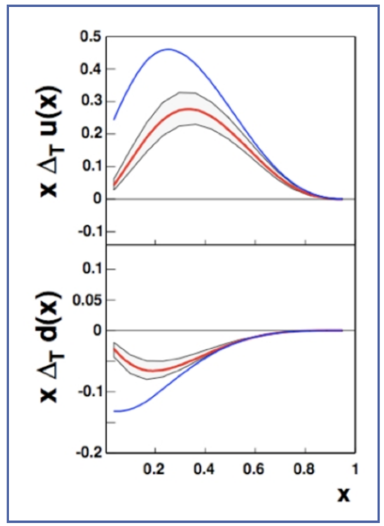
A_0 asymmetry → Hadronic plane method

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1\, z_2}{z_1^2 + z_2^2} \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \cos(2\,\phi_1) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

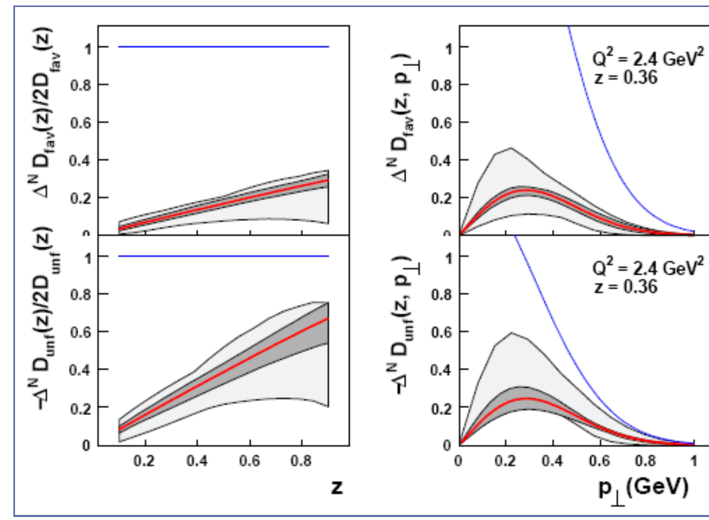
Extraction of transversity & Collins functions

- Simultaneous fit of HERMES, COMPASS and BELLE data

$\chi^2_{dof}=1.3$



Transversity



Collins functions

$N_u^T = 0.64 \pm 0.34$	$N_d^T = -1.00 \pm 0.02$
$\alpha = 0.73 \pm 0.51$	$\beta = 0.84 \pm 2.30$
$N_{fav}^C = 0.44 \pm 0.07$	$N_{unf}^C = -1.00 \pm 0.06$
$\gamma = 0.96 \pm 0.08$	$\delta = 0.01 \pm 0.05$
$M_h^2 = 0.91 \pm 0.52$ GeV^2	

- Anselmino et. al arXiv: 0812.4366v1

News on the Collins function

- New data from COMPASS (proton target, 2010-11)
- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905

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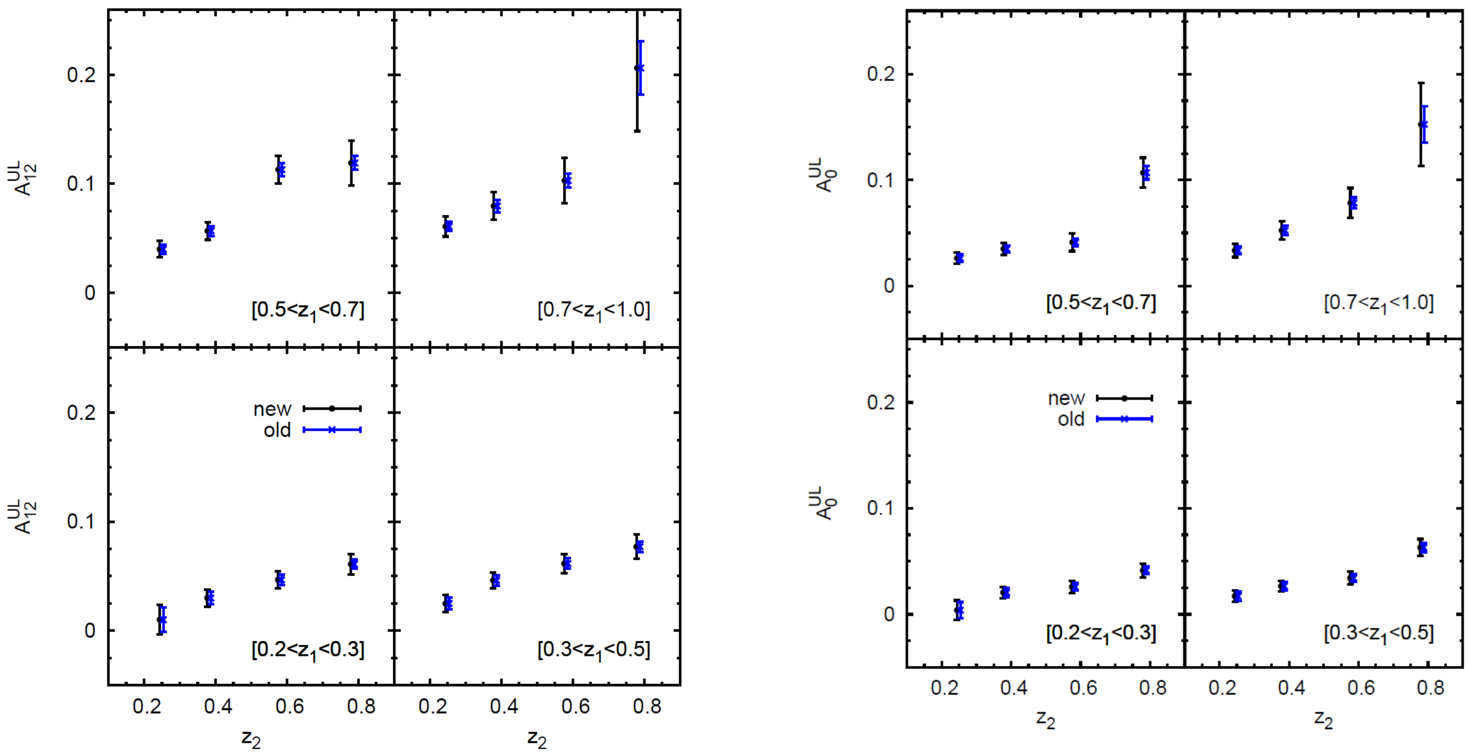
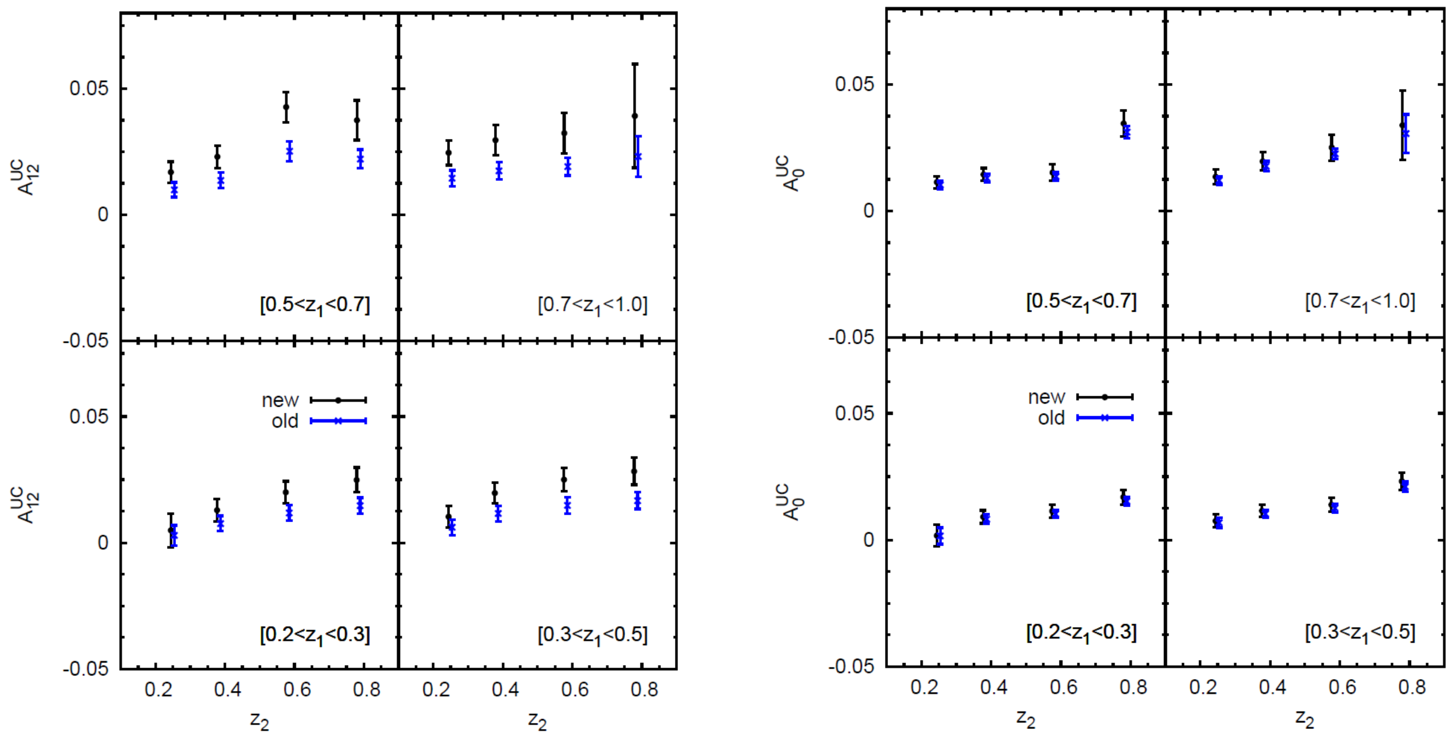


A^{UL}: **<u>Same central values</u>**, larger errors

News on the Collins function

- New data from COMPASS (proton target, 2010-11)
- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905

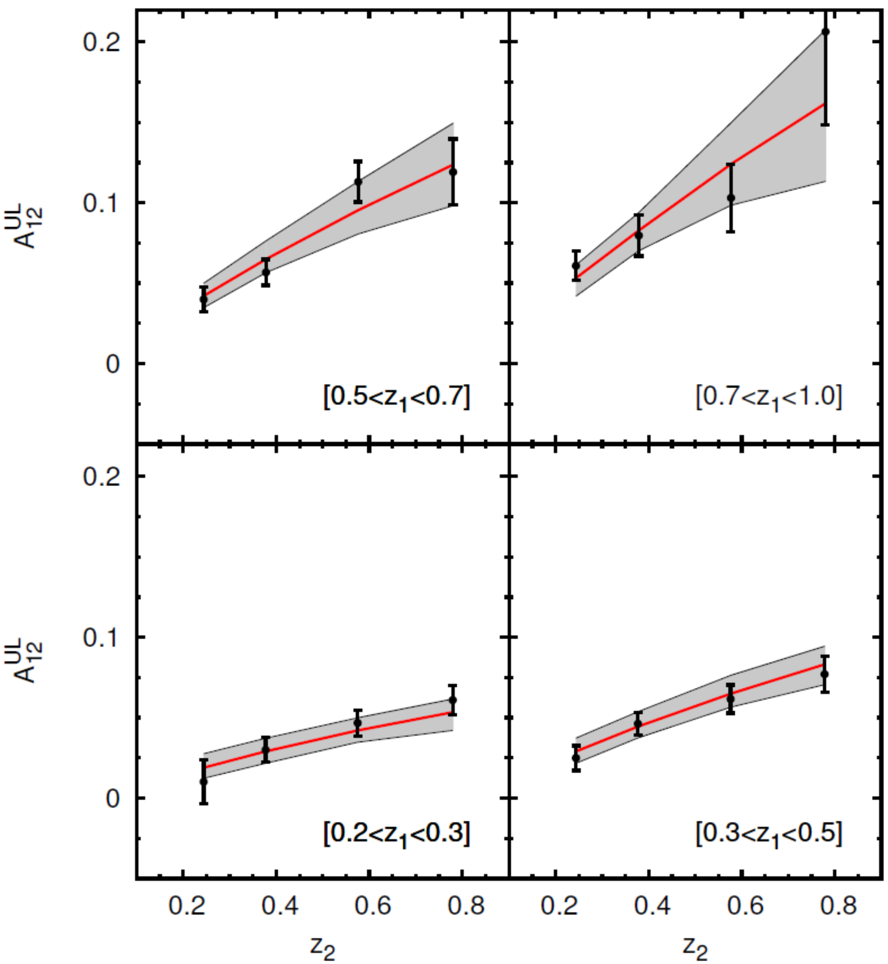


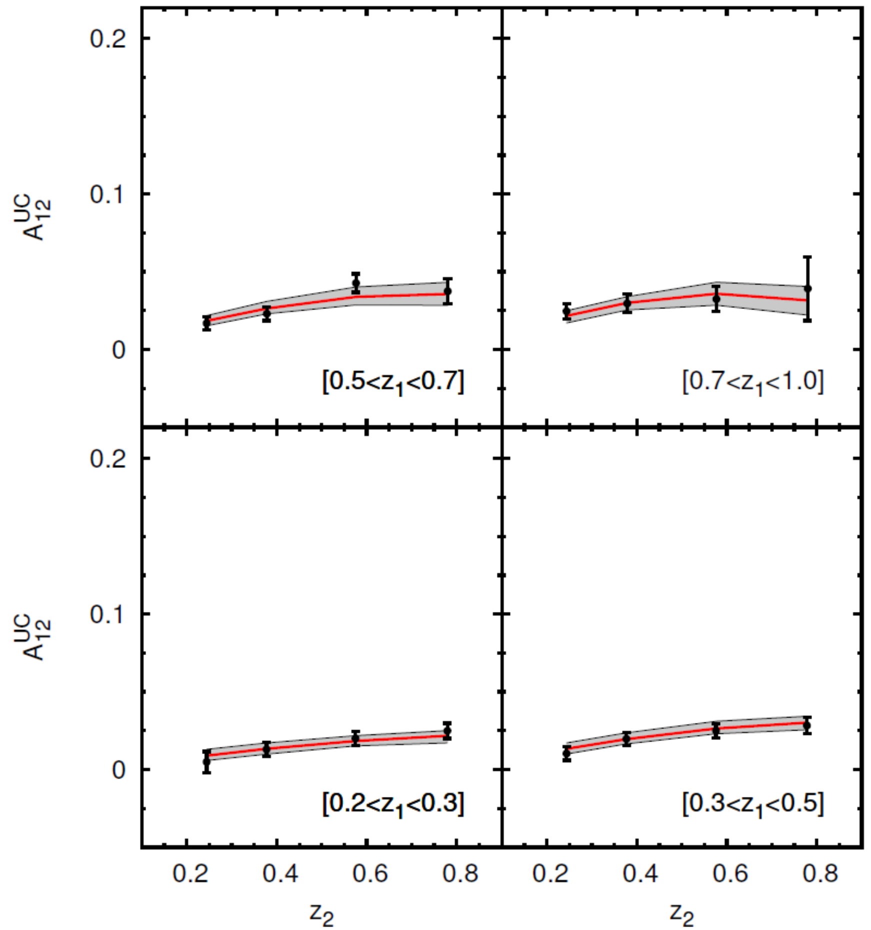
A^{UC}: Different normalization, larger errors
Good news! Previously partial incompatibility between UL & UC

News on the Collins function

- New analysis:
  - HERMES (2009) π+ π-
  - COMPASS Deuteron (2004) π+ π-
  - COMPASS Proton (2011) h+ h-
  - BELLE A_{12}

News on the Collins function

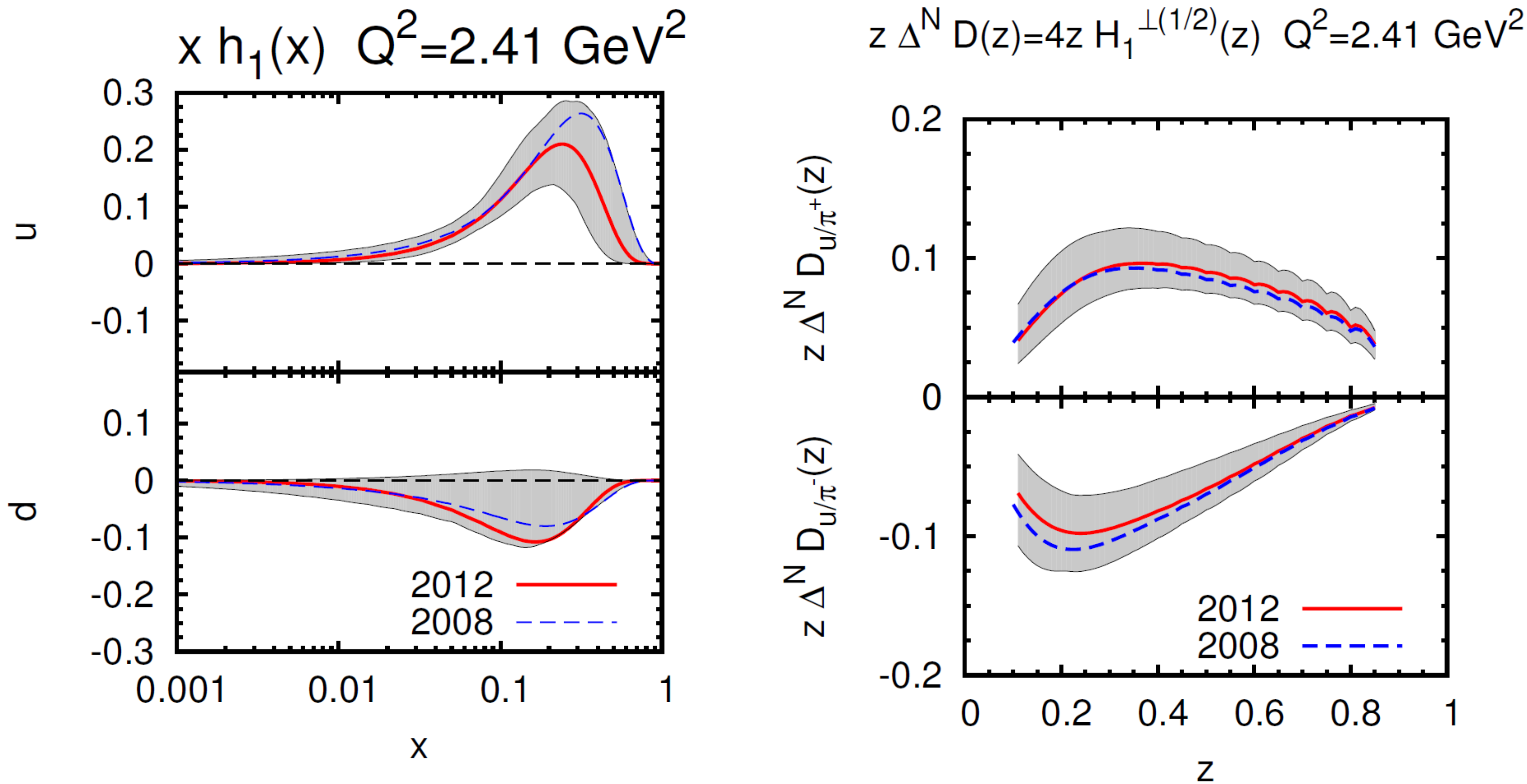




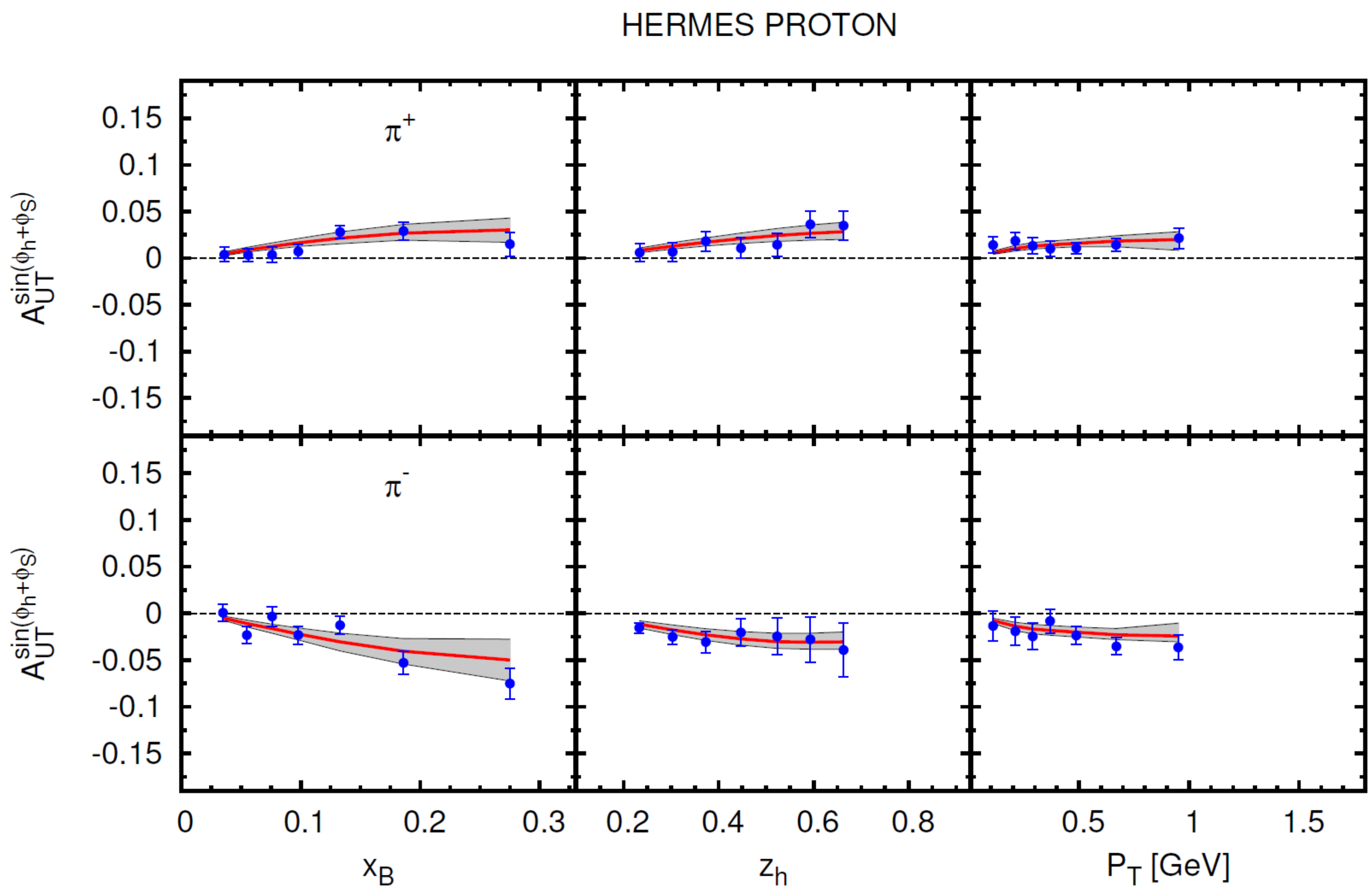
Full compatibility between UL e UC

News on the Collins function

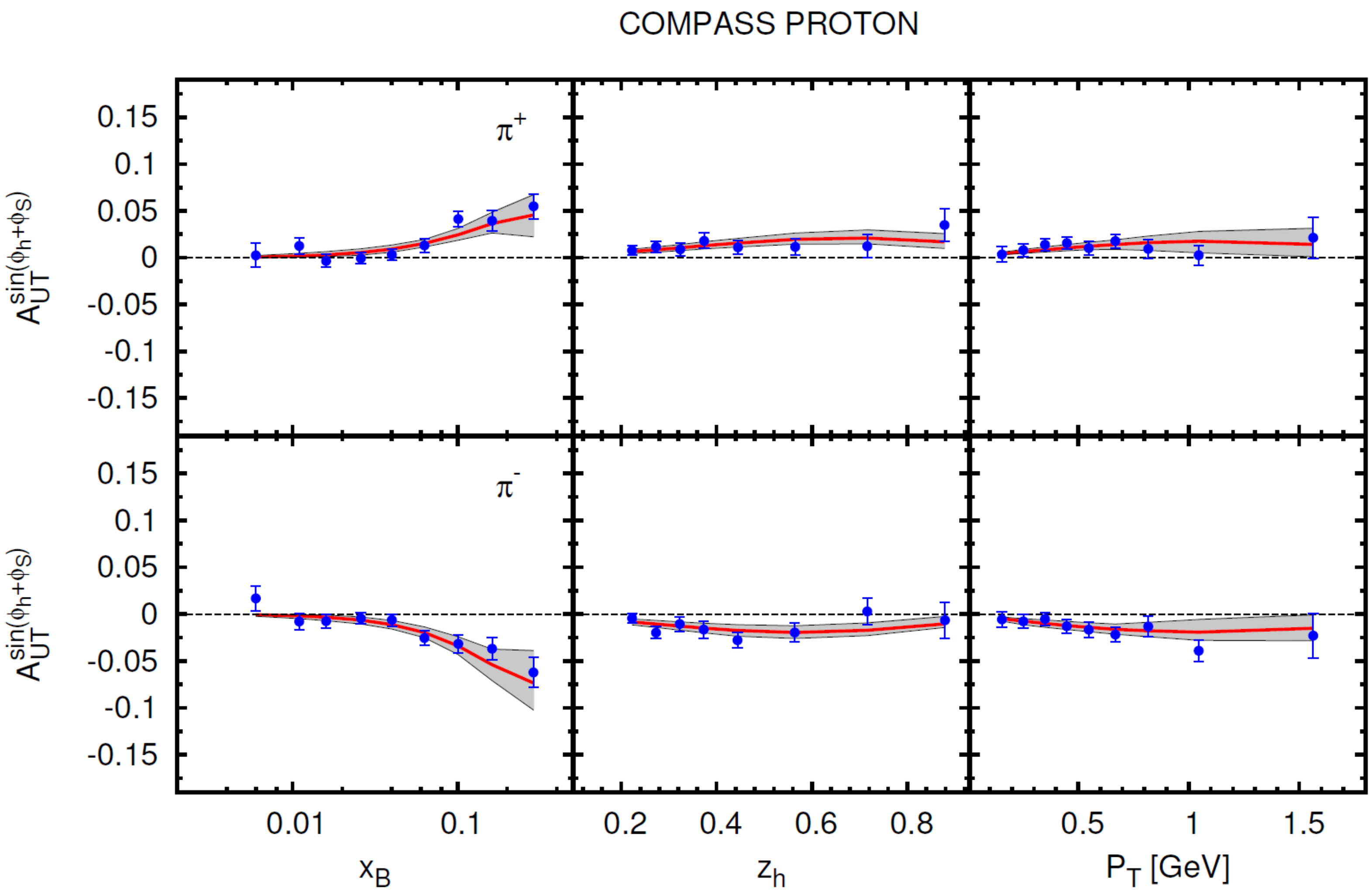
$$\chi^2_{d.o.f} = 0.8 \qquad \chi^2_{tot} = 135 \qquad \#\text{points} = 146\ (SIDIS) + 32\ (e^+e^-)$$



Extraction of transversity & Collins functions

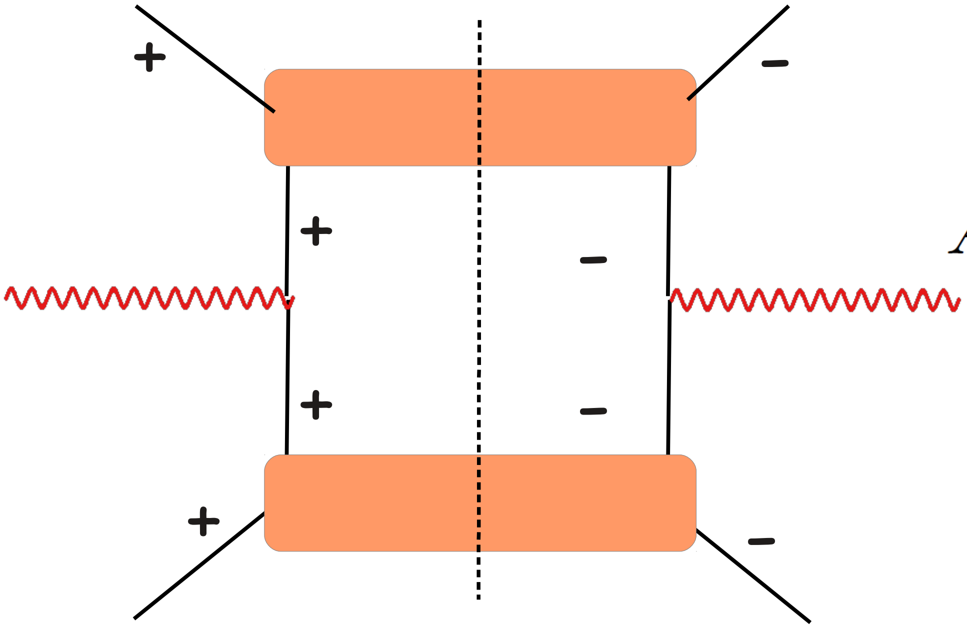


Extraction of transversity & Collins functions



Transversity in Drell-Yan processes

- Golden channel: Double transversely polarized DY



$$A_{TT} \propto \frac{h_1(x_1)\bar{h}_1(x_2)}{f_1(x_1)\bar{f}_1(x_2)} \cos(2\phi)$$

Not experimentally performed yet:
Very small in pp@RHIC: 1-2% (upper bound)
Feasible in $p\bar{p}$ @PAX ...

Transversity in Drell-Yan processes

- TMD way: Single transversely polarized DY: the transversity couples to another TMD, namely, the Boer-Mulders function

$$F_{UT}^{\sin(2\phi-\phi_b)} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} h_1^{\perp} \bar{h}_1\right]$$

Boer-Mulders function

Transversity

- The Boer-Mulders function can be interpreted as the probability to find a transversely polarized quark in an unpolarized proton
- (Chiral odd and T-odd)

Conclusions II

- Transversity functions are definitively different from zero!
- BELLE Erratum: Good News, better description of data

Boer-Mulders function and Cahn effect in unpolarized SIDIS

Boer-Mulders functions in unpolarized SIDIS

- The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A =\propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $B \propto \frac{1}{Q}(f_1 \otimes D_1 + h_1^{\perp} \otimes H_1^{\perp})$ subleading Cahn+Boer-Mulders effect
- $C \propto h_1^{\perp} \otimes H_1^{\perp} + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

Boer-Mulders functions in unpolarized SIDIS

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- $C \propto h_1^{\perp} \otimes H_1^{\perp} + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect+???

Extraction of the Boer-Mulders function

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$$A^{\cos 2\phi} = 2\frac{\int d\sigma \cos 2\phi}{\int d\sigma} = \frac{C}{A}$$

Extraction of the Boer-Mulders functions

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Unpolarized PDF&FF gaussian as in Anselmino et al. [1]

[1] Anselmino et. Phys. ReV D71, 074006 (2005)

Extraction of the Boer-Mulders functions

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Collins function as in Anselmino et. al arXiv: 0812.4366v1

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BM that we want to extract from the fit of $A^{\cos 2\phi}$ data

Extraction of the Boer-Mulders functions

- Simple parametrization of the Boer-Mulders functions:

  - $h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$ for valence quarks
  - $h_1^{\perp q}(x, k_\perp) = -|f_{1T}^{\perp q}(x, k_\perp)|$ for sea quarks

- Inspired by models:

$$h_1^{\perp q}(x, k_\perp) = \frac{\mathcal{K}_T^q}{\mathcal{K}^q} f_{1T}^{\perp q}(x, k_\perp)$$

Tensor magnetic moment

Anomalous magnetic moment

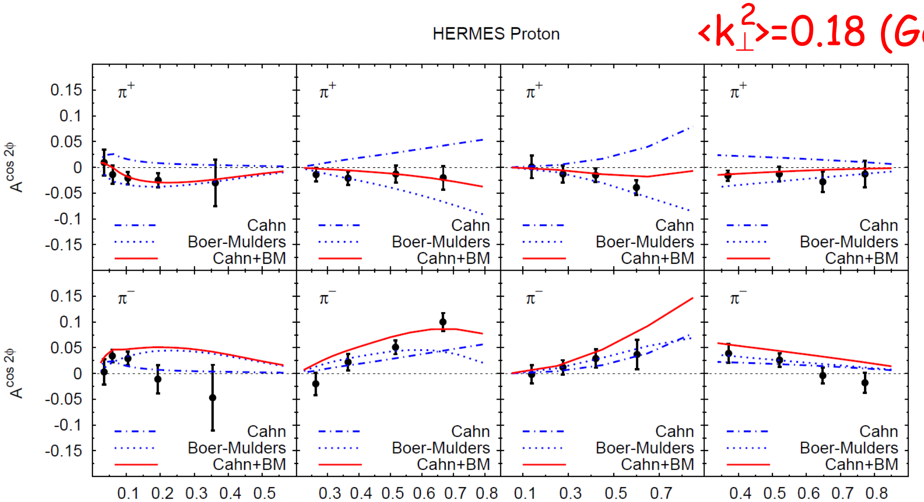
Burkardt, Phys. Rev. D72, 094020 (2005)

Gockeler, Phys.Rev.Lett.98:222001,2007.

Extraction of the Boer-Mulders function

arXiv:0901.2438

$\langle k_\perp^2 \rangle = 0.18\ (GeV/c)^2$



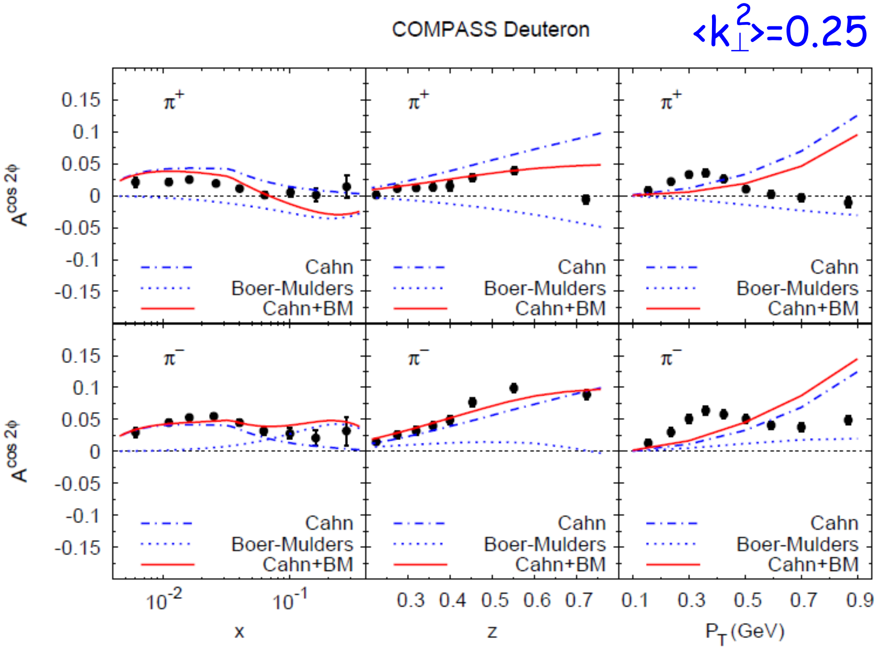
$$h_1^{\perp q}(x, k_\perp) = \lambda_q\, f_{1T}^{\perp q}(x, k_\perp)$$

$\Rightarrow h_1^{\perp d}$ and $h_1^{\perp u}$ both negative

Compatible with models predictions

arXiv 0808.0114

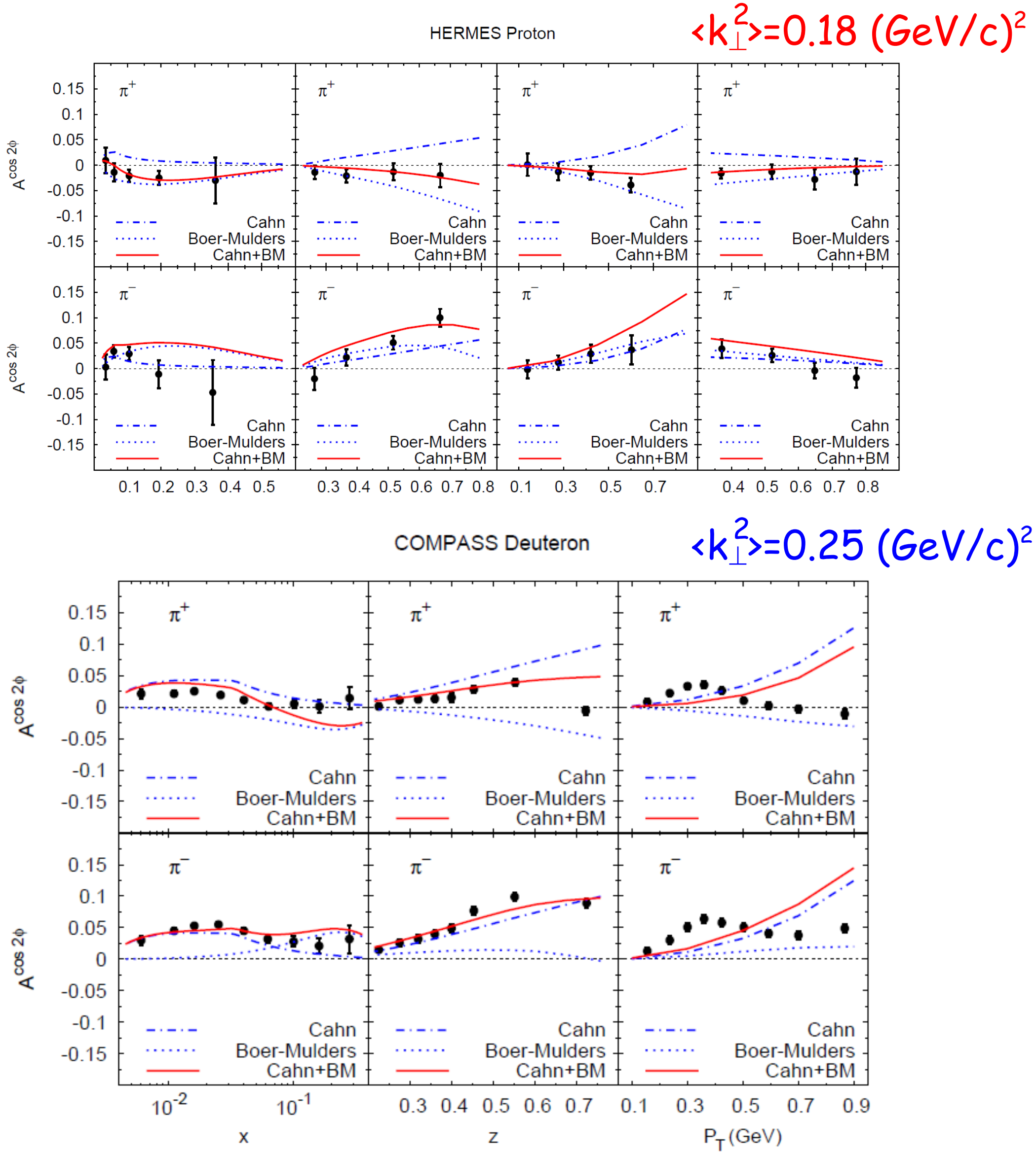
$\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$



- $\diamond\ \chi^2/d.o.f. = 2.41$
- $\bullet\ \lambda_u = 2.1 \pm 0.1$
- $\bullet\ \lambda_d = -1.11^{+0.00}_{-0.02}$

V. Barone, S. Melis and A. Prokudin Phys. Rev. D81, 114026 (2010)

Extraction of the Boer-Mulders function



- ✔Cahn effect (Twist-4) comparable to BM effect
- ✔Same sign of Cahn contribution for positive and negative pion
- ✔Different average transverse momenta are preferred
- ✔BM contribution opposite in sign for positive and negative pions

V. Barone, S. Melis and A. Prokudin Phys. Rev. D81, 114026 (2010)

Boer-Mulders function in DY?

Antiquark BM

Boer-Mulders function in DY from fits

- General expression for the dilepton angular distributions in the dilepton rest frame:

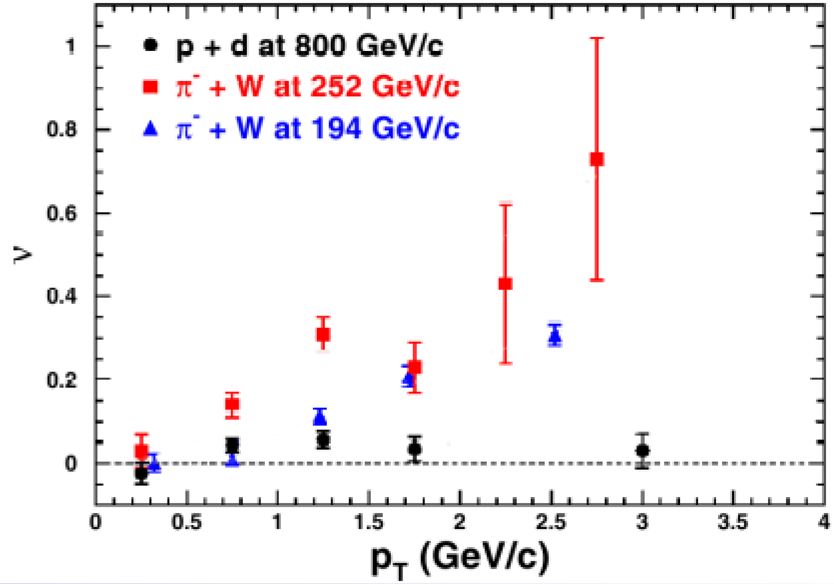
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi(\lambda+3)}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+(\nu/2)\ \sin^2\theta\cos 2\phi\right]$$

- TMDs approach

Boer-Mulders functions

$$\nu \propto \frac{h_1^{\perp a}\otimes h_1^{\perp b}}{f_1^a \otimes f_1^b}$$

Unpolarized PDFs



Boer-Mulders function in DY from fits

➤ In 2010 we performed an analysis of E866 data on pp and pD Drell-Yan

✎ $\bar{u}$ and $\bar{d}$ Boer-Mulders extraction from DY data:

$$h_1^{\perp \bar{q}}(x, k_\perp) = \lambda_{\bar{q}} f_{1T}^{\perp \bar{q}}(x, k_\perp)$$ [*]

✎ u and d Boer-Mulders functions as extracted from SIDIS

✎ Gaussian smearing for PDFs

$$f_{q/p}(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}$$

[**] $\langle k_\perp^2 \rangle = 0.25 \ (\mathrm{GeV}/c)^2$

[*] Sivers functions : Anselmino et al. Eur. Phys. J. A39,89

[**] Anselmino et. Phys. ReV D71, 074006 (2005)

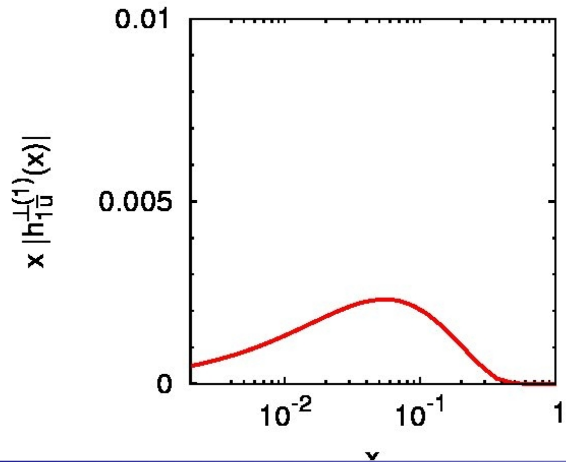
Boer-Mulders function in DY from fits

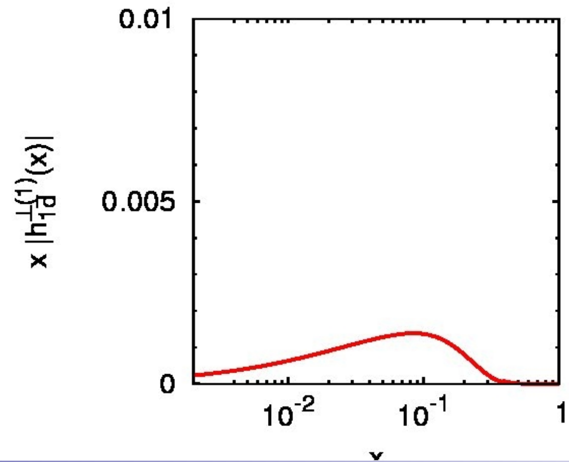
- Results of the analysis of E866 data on pp and pD Drell-Yan

$$h_1^{\perp \bar{q}}(x, k_\perp) = \lambda_{\bar{q}} f_{1T}^{\perp \bar{q}}(x, k_\perp)$$

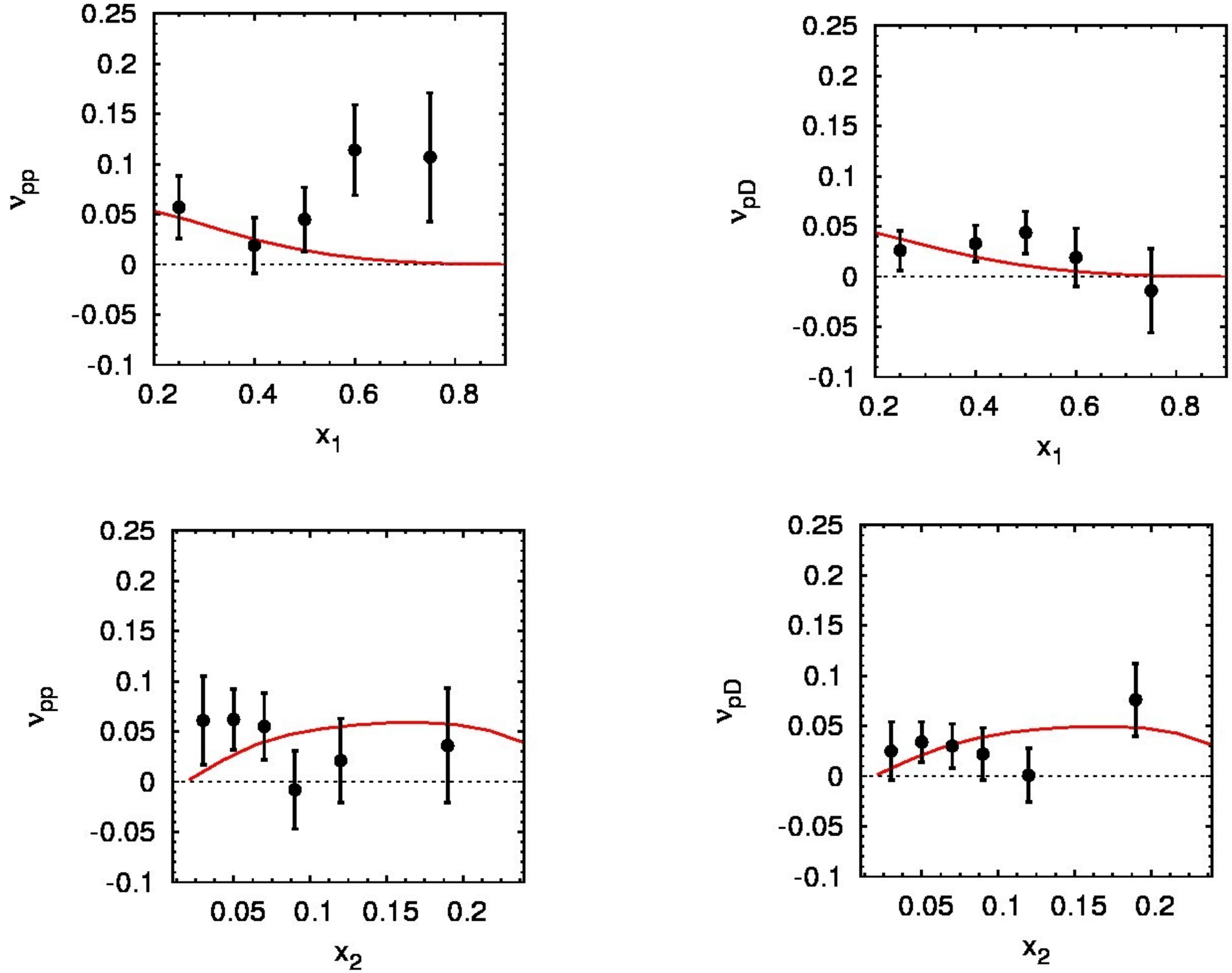
$$\lambda_{\bar{u}} = 3.25 \pm 0.75 \qquad \chi^2_{d.o.f} = 1.24$$
$$\lambda_{\bar{d}} = -0.15 \pm 0.13$$

FIT I

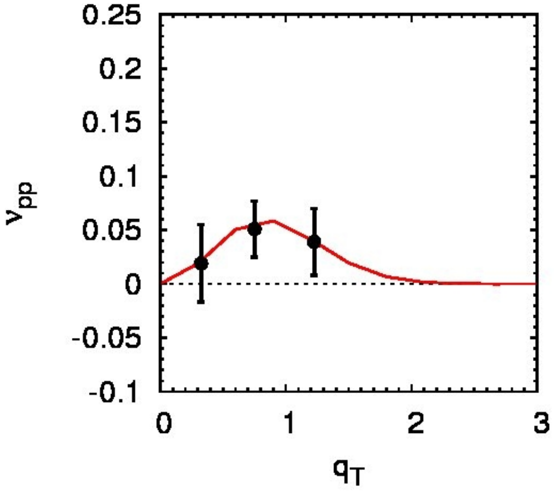


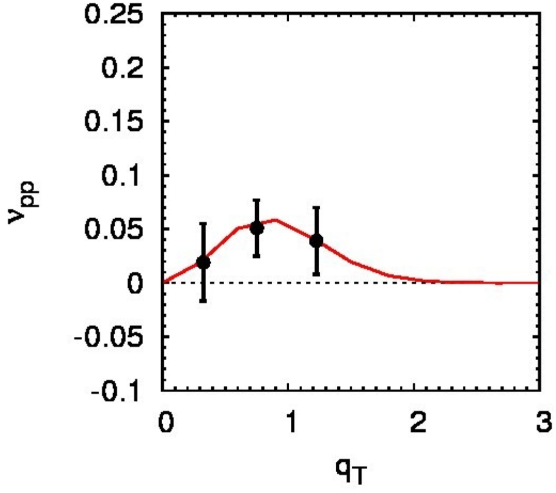


Boer-Mulders function in DY from fits



Boer-Mulders function in DY from fits





Boer-Mulders function in DY from fits

- Can we safely assume that the average transverse momentum is the same in SIDIS and in DY?

✎ Gaussian smearing for unpolarized PDFs

- $f_{q/p}(x, k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$

From SIDIS: $\langle k_\perp^2\rangle = 0.25\ (\mathrm{GeV}/c)^2$

Typical DY : $\langle k_\perp^2\rangle \simeq 0.5 - 1\ (\mathrm{GeV}/c)^2$

➔ Let us try to change this value

Boer-Mulders function in DY from fits

- Notice taht BM functions are proportional to the unpolarized pdf

✎ $$h_1^{\perp q}(x, k_T^2) = \lambda_q \, f_{1T}^{\perp q}(x, k_T^2) = \lambda_q \, \rho_q(x) \, \eta(k_T) \, f_1^q(x, \mathbf{k}_T^2)$$

Unpolarized PDF

Boer-Mulders function in DY from fits

- As an exercise let us assume different average transverse momentum in the unpolarized PDF.

FIT II as Fit I but with $\langle k_\perp^2 \rangle \simeq 0.64\ (\mathrm{GeV}/c)^2$ [*]

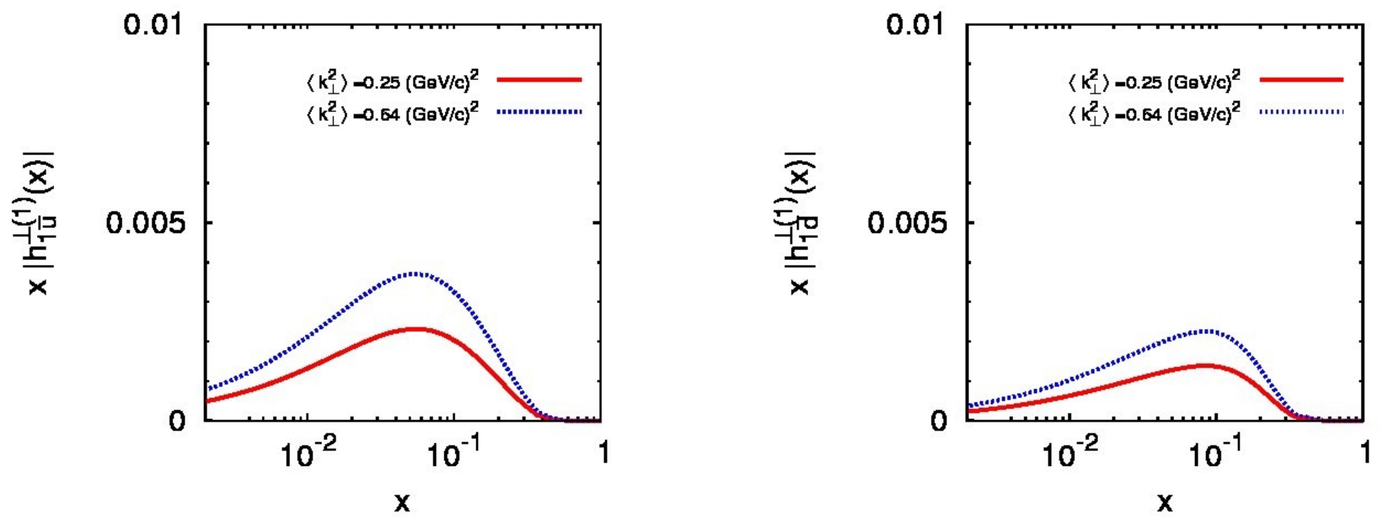
[*] U. D'Alesio and F. Murgia, Phys. Rev. D67,

Boer-Mulders function in DY from fits

$$\lambda_{\bar{u}} = 5.5 \pm 1.5$$
$$\lambda_{\bar{d}} = -0.25 \pm 0.20$$
$$\chi^2_{d.o.f} = 1.24$$

FIT II

Same description of the data!



Conclusions III

- From <cos 2φ> analysis BM compatible with models
- Large Cahn effect
- Different average transverse momenta for different experiments.
- Antiquark BM are not vanishing
- Different transverse momenta for different processes &/or Q^2?

Extraction of the Boer-Mulders functions

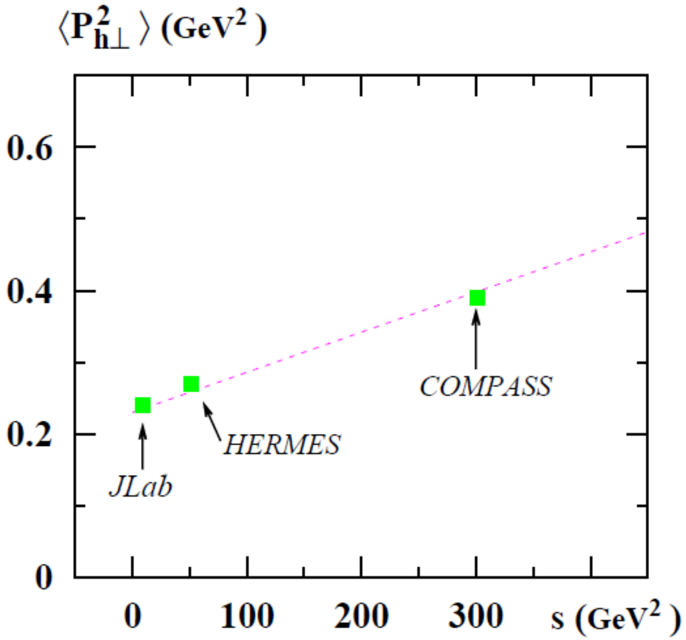
- Simple parametrization of the Boer-Mulders functions:
  - $h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$ for valence quarks
  - $h_1^{\perp q}(x, k_\perp) = -|f_{1T}^{\perp q}(x, k_\perp)|$ for sea quarks

V. Barone, S. Melis and A. Prokudin Phys. Rev. D81, 114026 (2010)

Extraction of the Boer-Mulders function

✔Different average transverse momenta are preferred

Schweitzer, Teckentrup, Metz (2010)

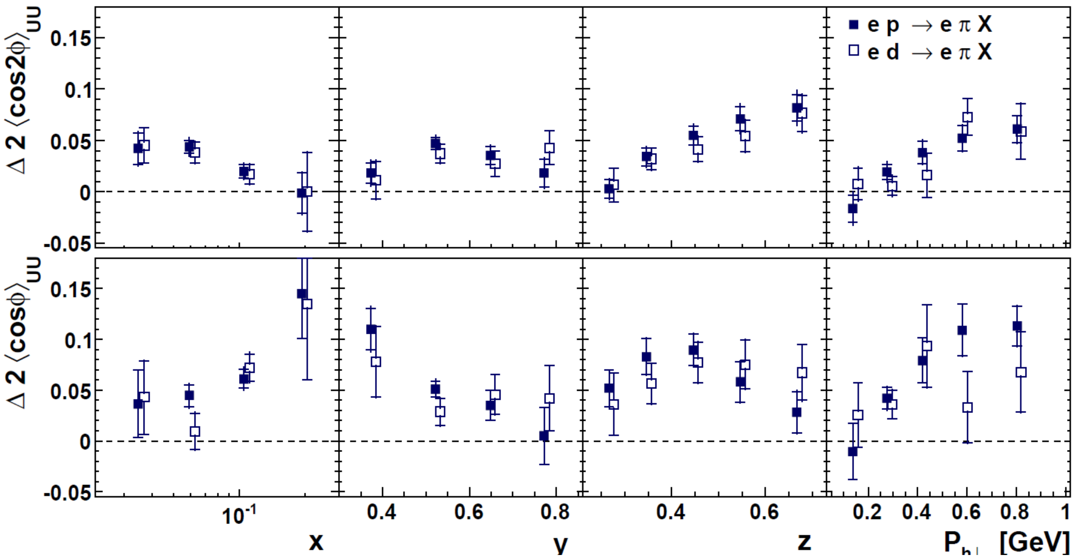


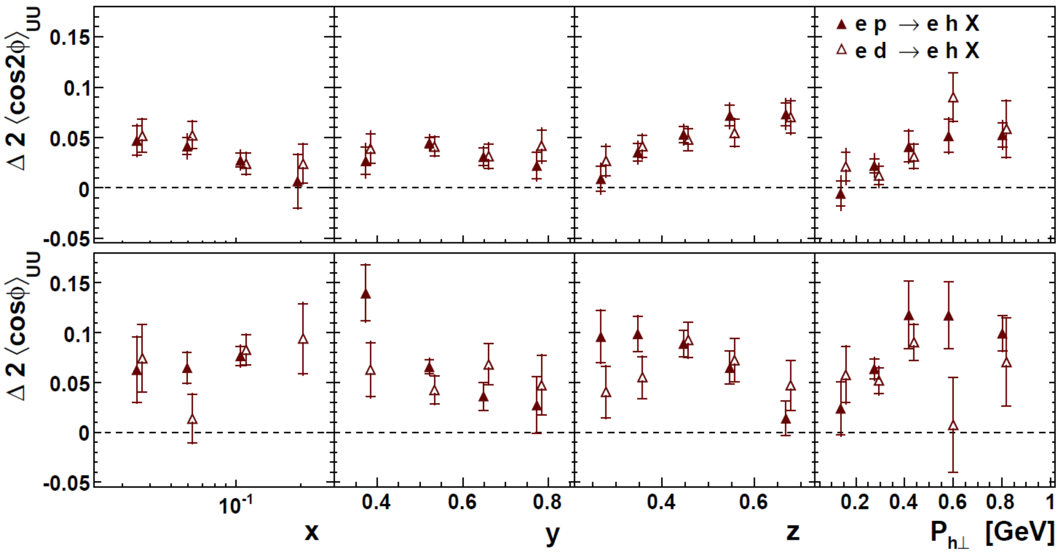
Extraction of the Boer-Mulders function

- ✔Same sign of Cahn contribution for positive and negative pion
- ✔BM contribution opposite in sign for positive and negative pions

$$\langle \cos 2\phi \rangle \propto h_1^{\perp} H_1^{\perp} + \text{Cahn}$$

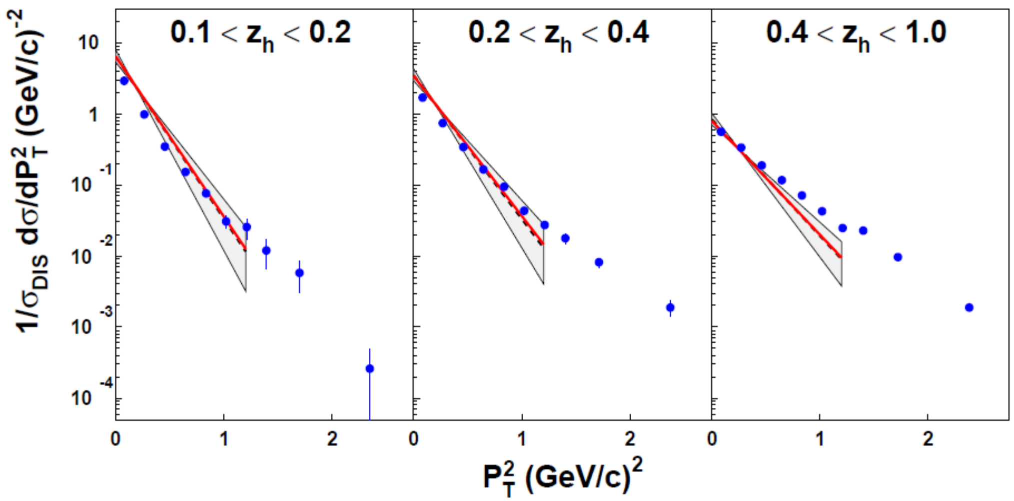
$$\langle \cos \phi \rangle \propto -h_1^{\perp} H_1^{\perp} - \text{Cahn}$$

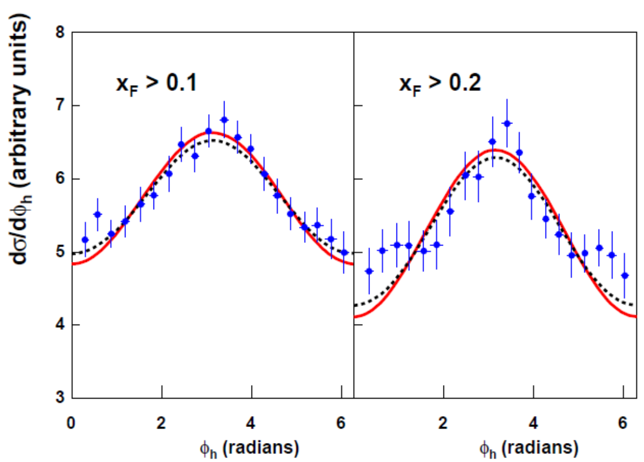




Extraction of the Boer-Mulders function

✔.. large cahn effect!



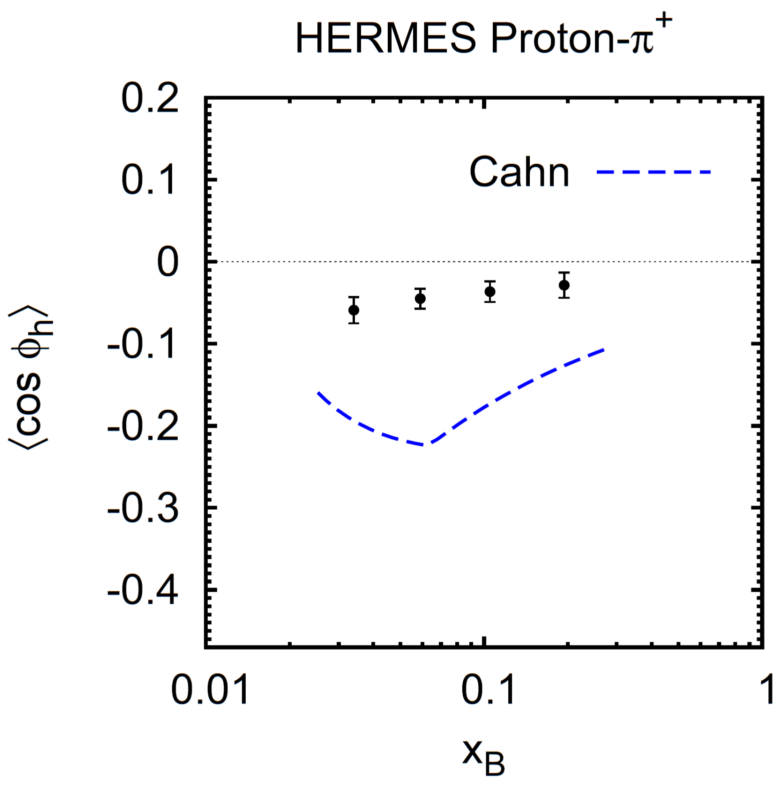


Fit of EMC data: Anselmino et al (2005)

...but...

Extraction of the Boer-Mulders function

✔... large cahn effect!



Why such a large Cahn effect?

- The Cahn effect is suppressed by powers of Q:

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A = \propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $B \propto \frac{1}{Q}\left(f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp\right)$ subleading Cahn+Boer-Mulders effect
- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

$$\frac{k_\perp}{Q} \ll 1 \text{ ??}$$

Why such a large Cahn effect?

- HERMES and COMPASS: $\langle Q^2 \rangle \simeq 2 \ \mathrm{GeV}^2$

$$Q^2 > 1 \ \mathrm{GeV}^2$$

- Analytical integration of the transverse momenta

$$f_{q/p}(x, k_\perp) = f(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$\langle k_\perp^2 \rangle \simeq 0.25 \ (\mathrm{GeV}/c)^2$$

$$\int d^2\boldsymbol{k}_\perp \Rightarrow \int_0^{2\pi} d\varphi \int_0^\infty dk_\perp \, k_\perp$$

Bounds on the intrinic transverse momenta

- ✔The integration from 0 to infinity can be a crude assumption
- ✔The parton model provides kinematical limits on the transverse momentum size

➢By requiring the energy of the parton to be smaller than the energy of its parent hadron, we have

$$k_\perp^2 \leq (2 - x_B)(1 - x_B)Q^2 \quad , \quad 0 < x_B < 1$$

➢By requiring the parton not to move backward with respect to its parent hadron, we find

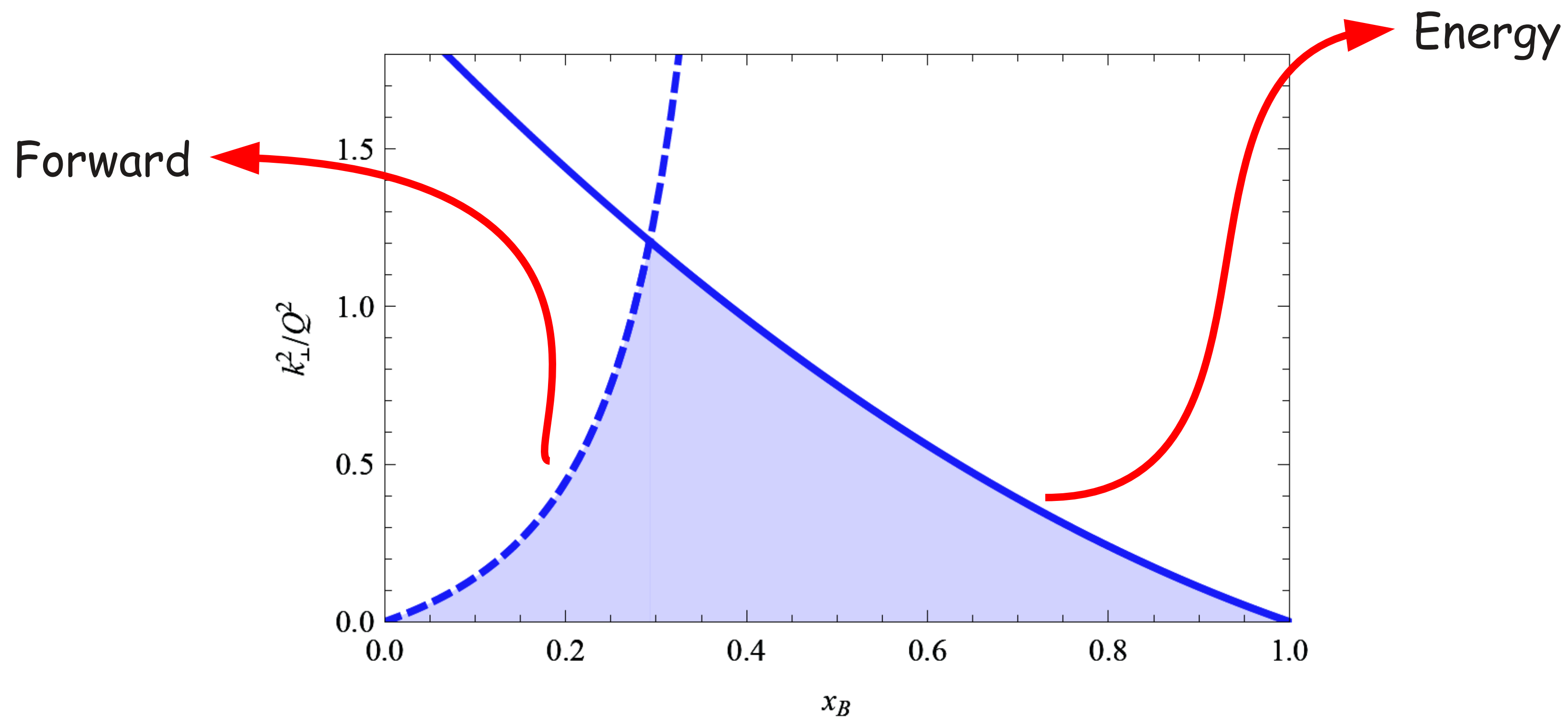
$$k_\perp^2 \leq \frac{x_B(1 - x_B)}{(1 - 2x_B)^2} Q^2 \ , \quad x_B < 0.5$$

Boglione, Melis, Prokudin, Phys. Rev. D 84, 034033 (2011)

Bounds on the intrinic transverse momenta

✔The integration from 0 to infinity can be a crude assumption

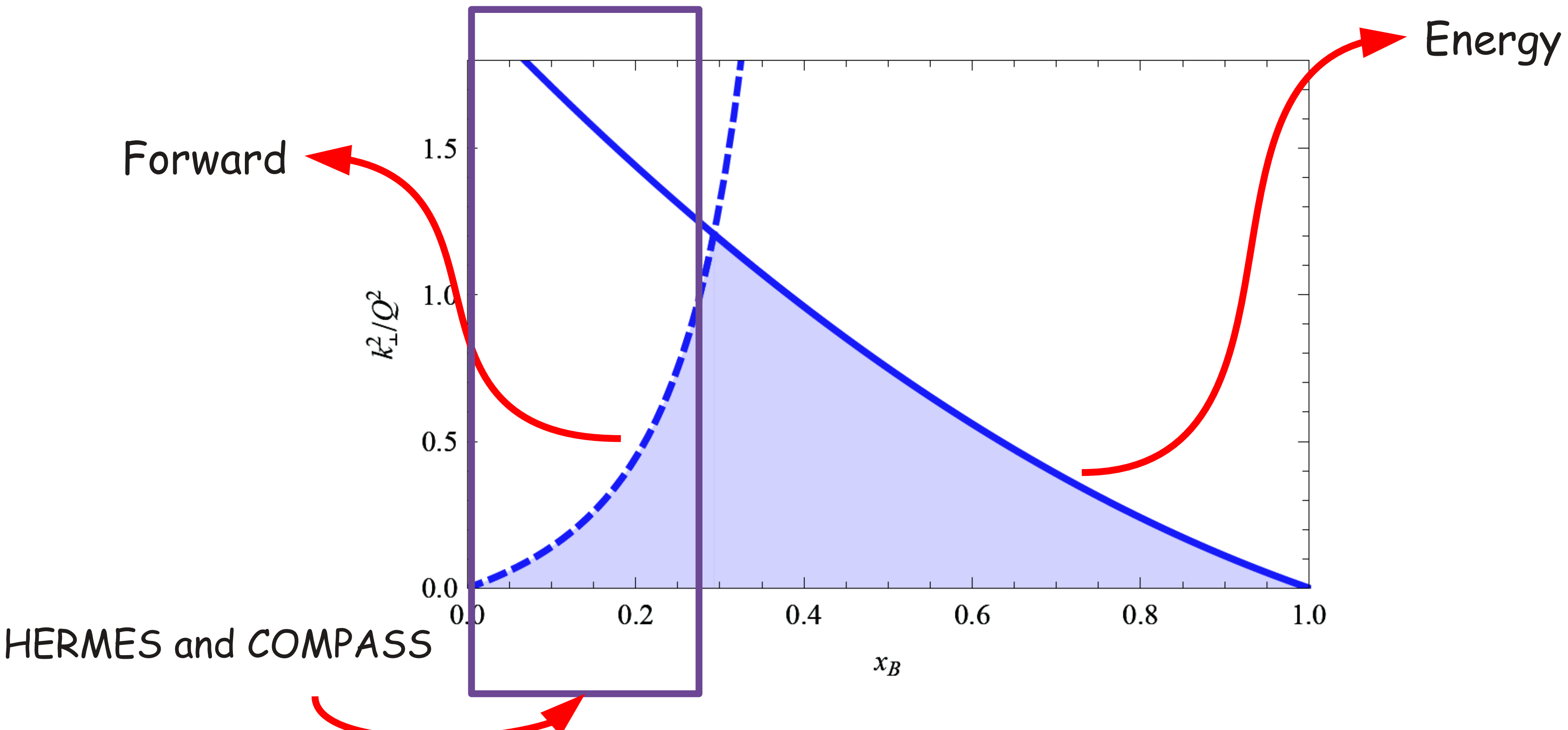
✔The parton model provides kinematical limits on the transverse momentum size



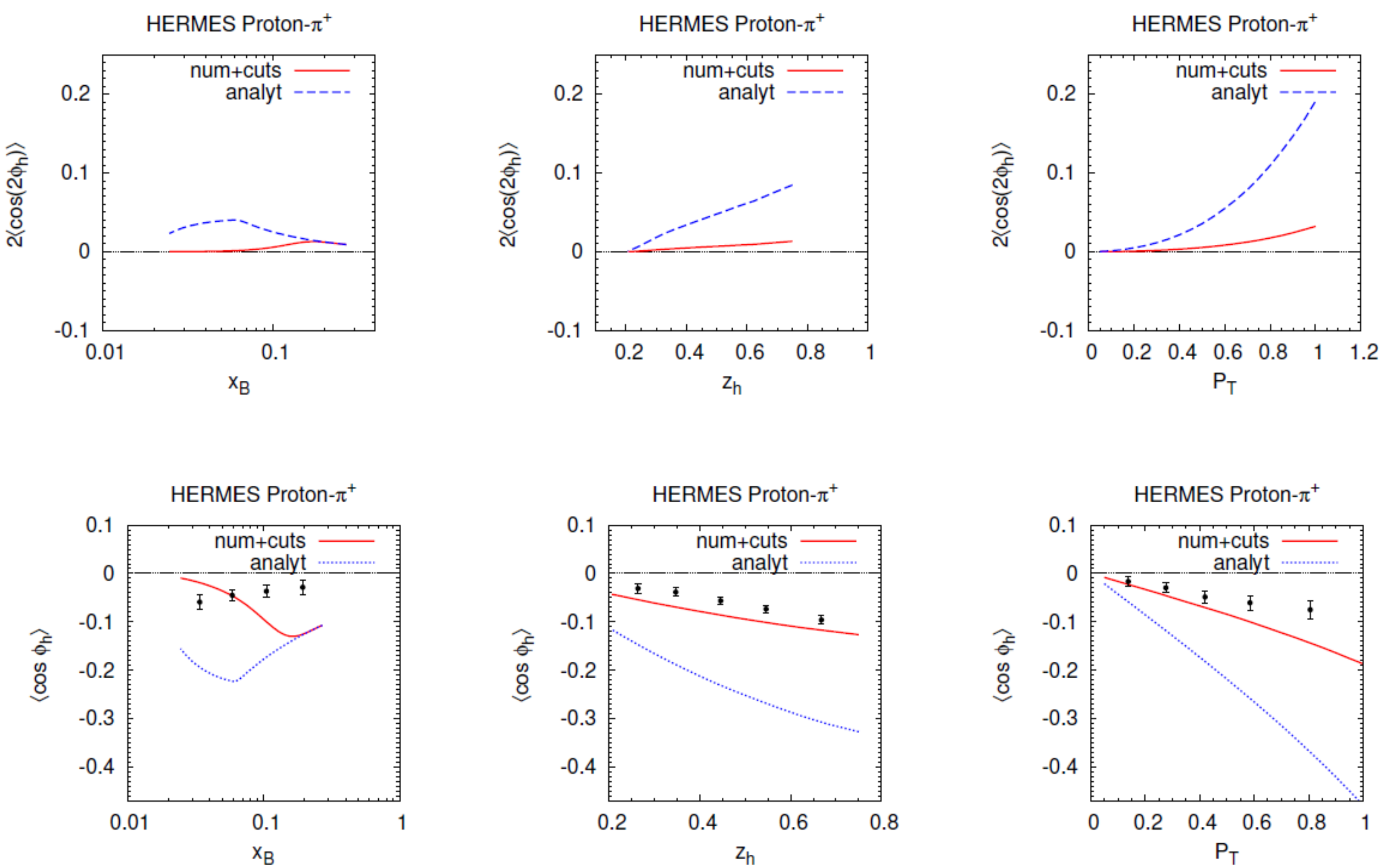
Bounds on the intrinic transverse momenta

✔The integration from 0 to infinity can be a crude assumption

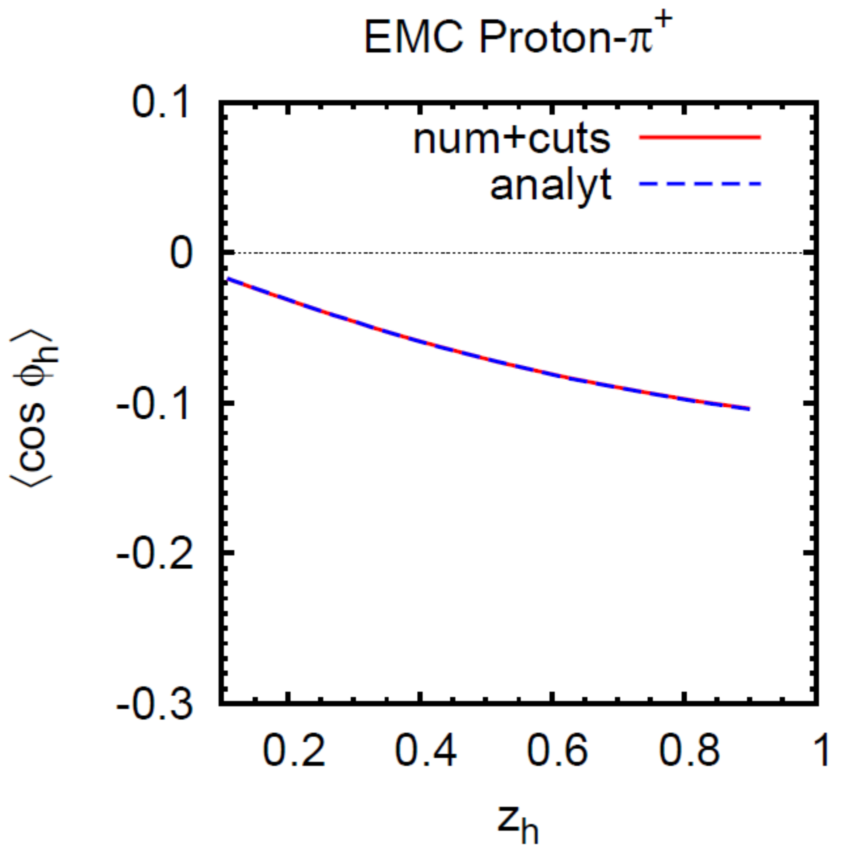
✔The parton model provides kinematical limits on the transverse momentum size



Smaller Cahn effect...



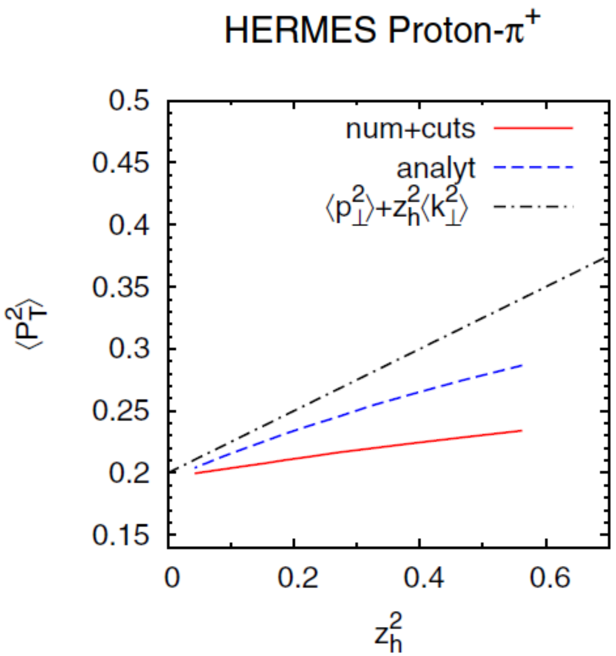
No effects in “true” DIS regime...



EMC like kinematics:

$$Q^2 \geq 5 \text{ GeV}^2$$

$\langle P_T^2 \rangle$



Very often the relation

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

is used in phenomenological analysis
But is wrong unless you integrate from
0 to infinity P_T which is never
the case experimentally

$$f_1(x,\mathbf{k}_\perp^2) = N\, f_1(x)\, \mathrm{e}^{-\mathbf{k}_\perp^2/\overline{\mathbf{k}_\perp^2}} \qquad D_1(z,\mathbf{p}_\perp^2) = N\, D_1(z)\, \mathrm{e}^{-\mathbf{p}_\perp^2/\overline{\mathbf{p}_\perp^2}}$$

$$\langle \mathbf{k}_\perp^2 \rangle \equiv \int \mathrm{d}^2\mathbf{k}_\perp\, \mathbf{k}_\perp^2\, f_1(x,\mathbf{k}_\perp^2) \qquad \langle \mathbf{p}_\perp^2 \rangle \equiv \int \mathrm{d}^2\mathbf{p}_\perp\, \mathbf{p}_\perp^2\, D_1(z,\mathbf{p}_\perp^2)$$

If you integrate from 0 to infinity! $\langle \mathbf{k}_\perp^2 \rangle = \overline{\mathbf{k}_\perp^2}$ $\langle \mathbf{p}_\perp^2 \rangle = \overline{\mathbf{p}_\perp^2}$

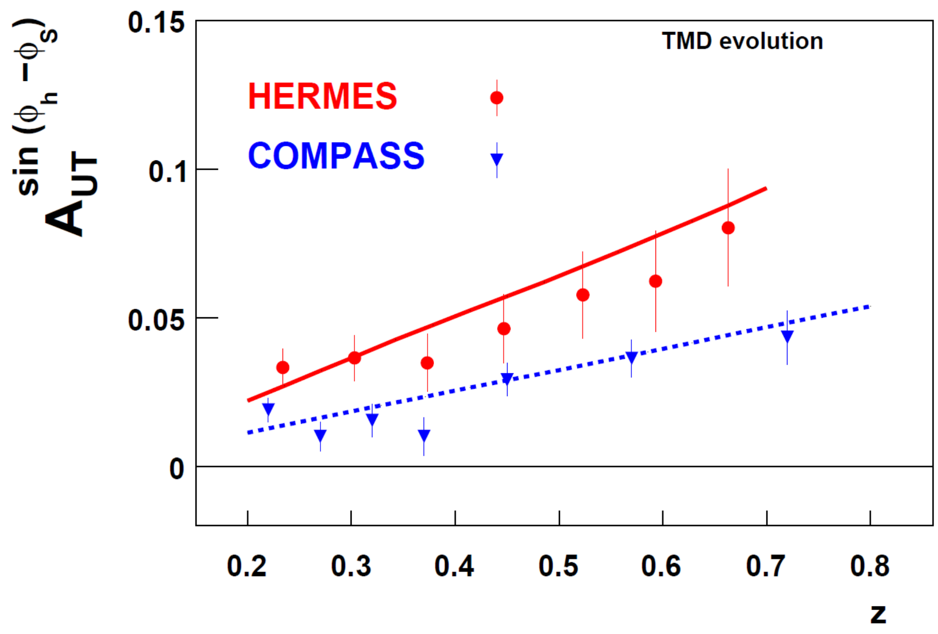
$$F_{UU} = \sum_a e_a^2 \int \mathrm{d}^2\mathbf{k}_\perp \int \mathrm{d}^2\mathbf{p}_\perp\, \delta^2(\mathbf{p}_\perp + z_h\mathbf{k}_\perp - \mathbf{P}_{h\perp})\, f_1^a(x_B,\mathbf{k}_\perp^2)\, D_1^a(z_h,\mathbf{p}_\perp^2)$$

$$F_{UU} = \sum_a e_a^2\, f_1^a(x_B)\, D_1^a(z_h)\, \frac{\mathrm{e}^{-\mathbf{P}_{h\perp}^2/\overline{\mathbf{P}_{h\perp}^2}}}{\pi\,\overline{\mathbf{P}_{h\perp}^2}}$$

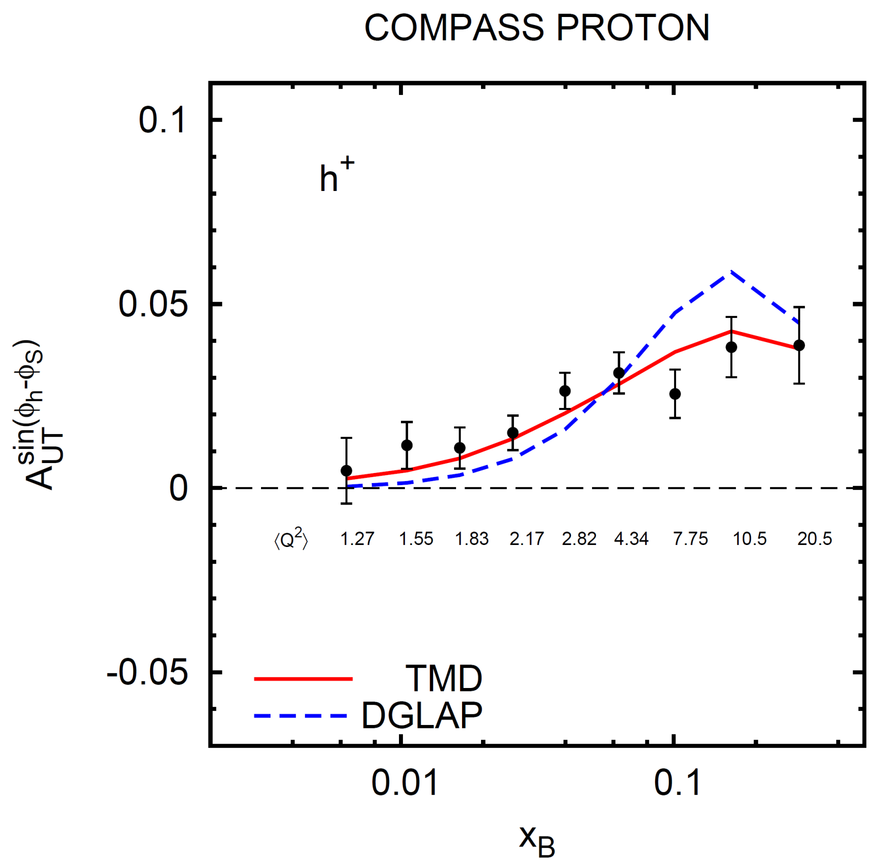
$$\overline{\mathbf{P}_{h\perp}^2} = \overline{\mathbf{p}_\perp^2} + z_h^2\, \overline{\mathbf{k}_\perp^2}$$

$\langle \mathbf{P}_{h\perp}^2 \rangle = \overline{\mathbf{P}_{h\perp}^2}$ <u>Only if you integrate from 0 to infinity!</u>

Sivers function in SIDIS



Aybat, Prokudin, Rogers, PRL 108 (2012) 242003



Anselmino, Boglione, Melis, PRD 86 (2012) 014028

Sivers function in SIDIS: Pavia analysis

- The Sivers function can shed light on the partonic angular momentum. Naively, the distortion in the transverse momentum space corresponds to an orbitating quark in the position space.

Lensing function

GPD

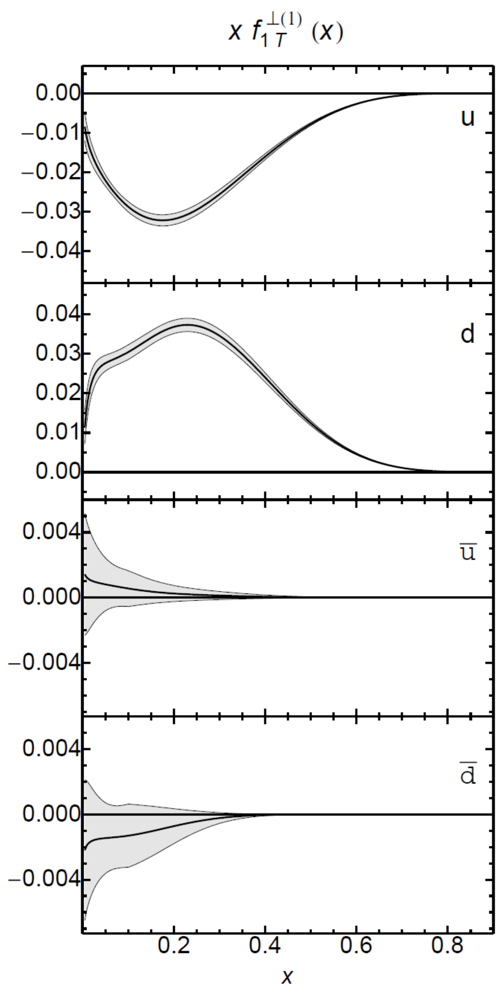
$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2)$$

(Model dependent)

Burkardt, PRD 66 (2002) 114005

$$J^a(Q^2) = \frac{1}{2}\int_0^1 dx\, x\left(H^a(x,0,0;Q^2) + E^a(x,0,0;Q^2)\right)$$

Sivers function in SIDIS: Pavia analysis



Bacchetta and Radici, PRL 107 (2011) 212001

Simple model

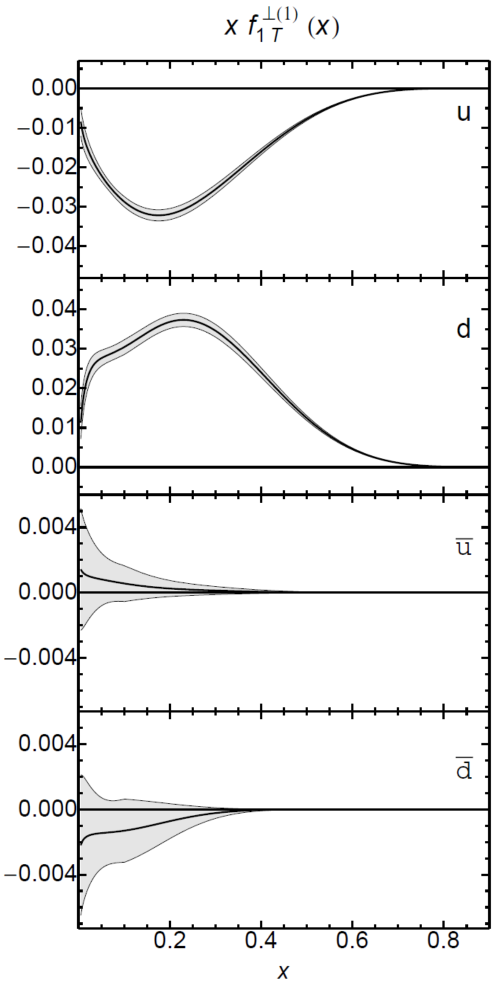
$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2)$$

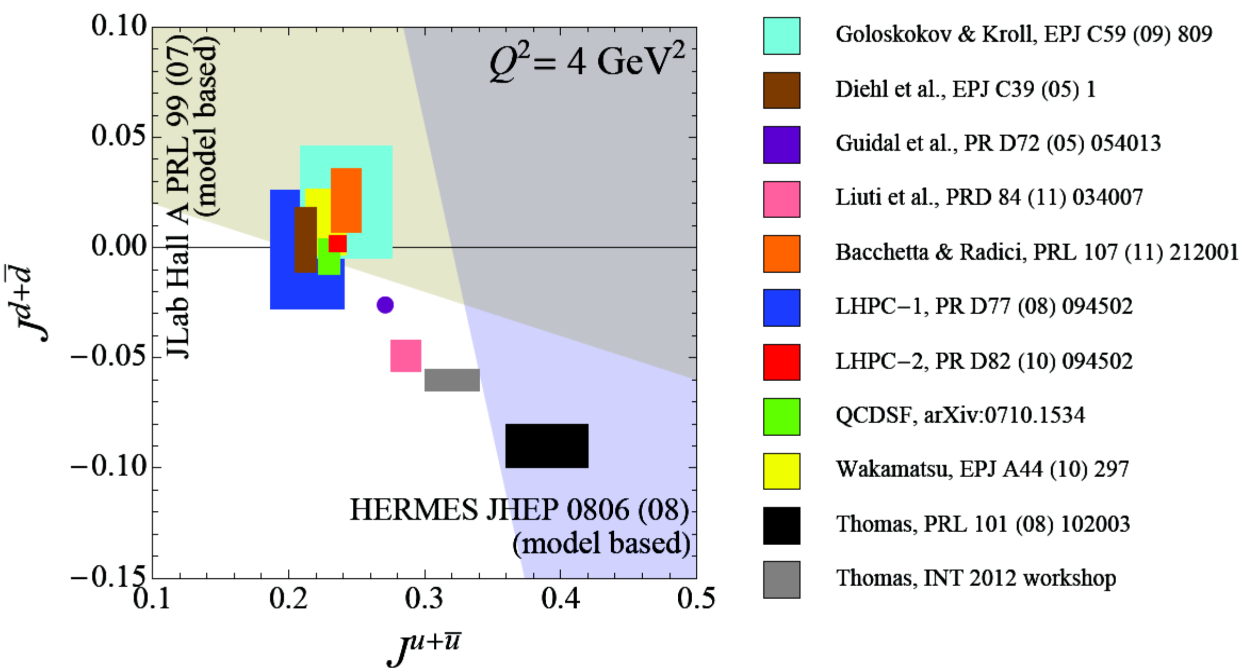
SIDIS data (HERMES&COMPASS)

First moment constrained by the anomalous magnetic moments

Sivers function in SIDIS

- The sivers function can shed light on the partonic angular momentum





Bacchetta and Radici, PRL 107 (2011) 212001

Gluon Sivers function

- Almost no information...

- Burkardt Sum Rule: The net transverse Sivers momentum from all quark flavors plus the gluons vanishes. (Like net force of of a classic multi-particle system with only internal forces)

$$\sum_a \int dx\, d^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp\, f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \sum_a \langle \boldsymbol{k}_\perp^a \rangle = 0$$

$$\langle \boldsymbol{k}_\perp^a \rangle = \left[\frac{\pi}{2} \int_0^1 dx \int_0^\infty dk_\perp\, k_\perp^2\, \Delta^N f_{a/p^\uparrow}(x, k_\perp) \right] (\boldsymbol{S} \times \hat{\boldsymbol{P}})$$

$$= m_p \int_0^1 dx\, \Delta^N f_{q/p^\uparrow}^{(1)}(x)\, (\boldsymbol{S} \times \hat{\boldsymbol{P}}) \equiv \langle k_\perp^a \rangle\, (\boldsymbol{S} \times \hat{\boldsymbol{P}})$$

Gluon Sivers function

- Almost no information...

- From the 2009 analysis: the B.S.R. is almost saturated by u and d quarks alone at Q^2=2.4 GeV^2

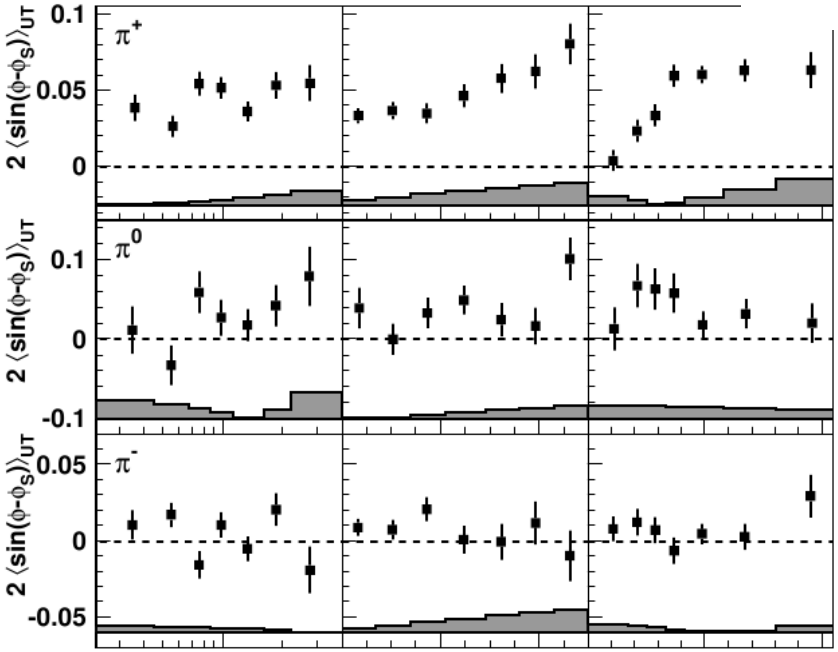
$$\langle k_\perp^u \rangle + \langle k_\perp^d \rangle = -17^{+37}_{-55} \ (\mathrm{MeV}/c)$$

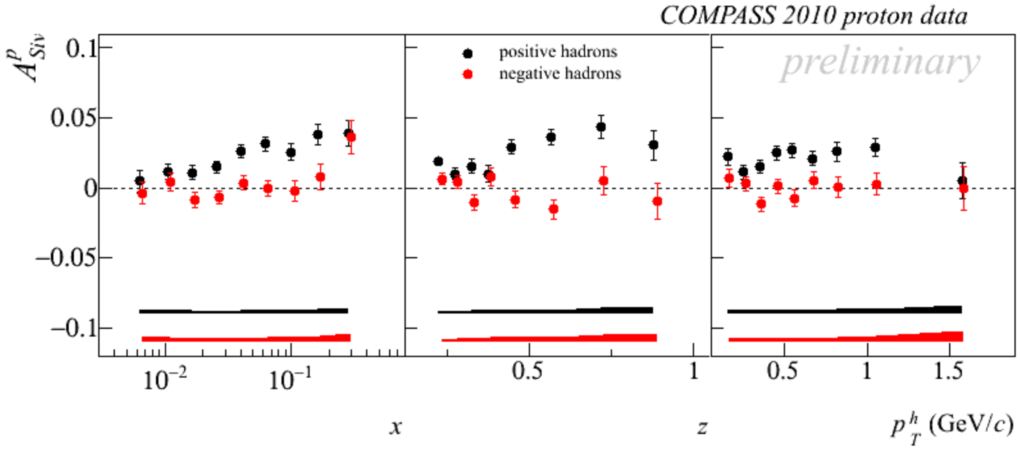
$$\langle k_\perp^{\bar{u}} \rangle + \langle k_\perp^{\bar{d}} \rangle + \langle k_\perp^s \rangle + \langle k_\perp^{\bar{s}} \rangle = -14^{+43}_{-66} \ (\mathrm{MeV}/c)$$

- ...thus leaving little room for a gluon Sivers function. However data are only in a limited x region (almost a valence region)

Sivers function in SIDIS

- New SIDIS data from HERMES and COMPASS



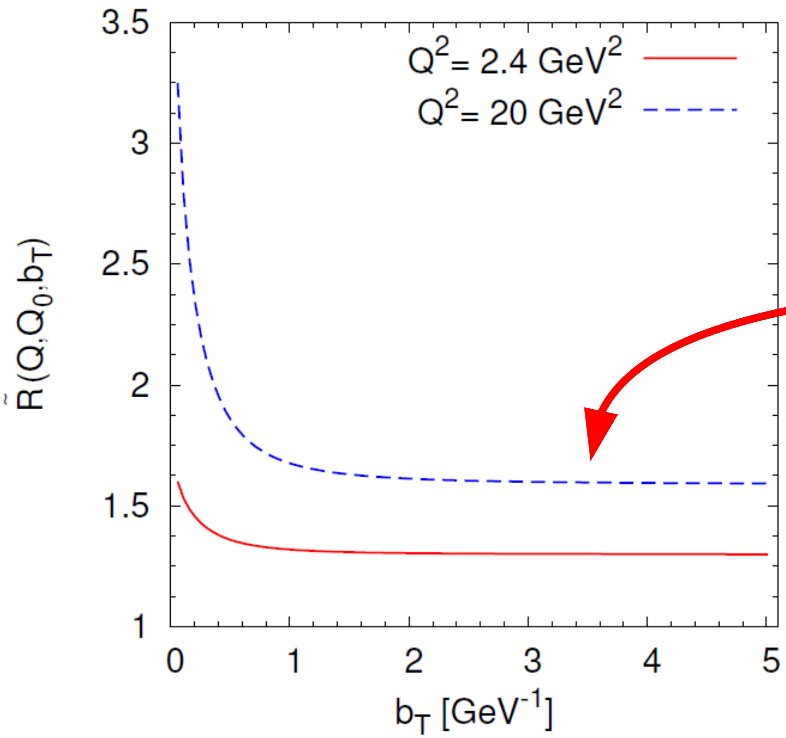


Phys.Rev.Lett.103:152002,2009

Bradamante, Transversity 2011

Analytical (approximated) solution of the TMD evolution equation

- $\tilde{R}(Q,Q0,b_T)$ exhibits a non trivial dependence on b_T that prevents any analytical integration



$\tilde{R}(Q,Q0,b_T)$ becomes **constant** for $b_T > 1$ GeV $^{-1}$

We can therefore neglect the $\tilde{R}$ dependence on b_T and define:

$$R(Q,Q_0) \equiv \tilde{R}(Q,Q_0,b_T \to \infty)$$

Good approximation for large b_T i.e. small $k_\perp$

Analytical (approximated) solution of the TMD evolution equation

- For instance, replacing $\tilde{R}$ with R in the unpolarized, we get:

$$\widetilde{f}_{q/p}(x, \boldsymbol{b}_T; Q) = f_{q/p}(x, Q_0)\; R(Q, Q_0)\; \exp\left\{ -b_T^2 \left(\alpha^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right) \right\}$$

Which is Gaussian in b_T, and will then Fourier-transform into a Gaussian in $k_\perp$

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x, Q_0)\; R(Q, Q_0)\; \frac{e^{-k_\perp^2/w^2}}{\pi w^2}$$

$$w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}$$

Analytical (approximated) solution of the TMD evolution equation

- Similarly, for the unpolarized TMD fragmentation function, we have

$$\widehat{D}_{h/q}(z, p_\perp; Q) = D_{h/q}(z, Q_0)\, R(Q, Q_0)\, \frac{e^{-p_\perp^2/w_F^2}}{\pi w_F^2}$$

$$w_F^2 \equiv w_F^2(Q, Q_0) = \langle p_\perp^2 \rangle + 2z^2 g_2 \ln \frac{Q}{Q_0}$$

Analytical (approximated) solution of the TMD evolution equation

- For the Sivers distribution function, we find:

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = \frac{k_\perp}{M_1}\sqrt{2e}\,\frac{\langle k_\perp^2\rangle_S^2}{\langle k_\perp^2\rangle}\,\Delta^N f_{q/p^\uparrow}(x,Q_0)\,R(Q,Q_0)\,\frac{e^{-k_\perp^2/w_S^2}}{\pi w_S^4}$$

$$w_S^2(Q,Q_0) = \langle k_\perp^2\rangle_S + 2g_2 \ln\frac{Q}{Q_0}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x,k_\perp)$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2 + \langle k_\perp^2\rangle}$$

Consequences on DY data and warnings

- Numerator of the asymmetry in analytical approximation for a SIDIS process

$$N_{SIDIS} \propto \Delta^N f(x,Q_0) D(z,Q_0) \sqrt{2e} \frac{P_T}{M_1} \frac{z \langle k_\perp^2 \rangle_{Siv}^2}{\langle k_\perp^2 \rangle \langle P_T^2 \rangle_{Siv}^2} e^{-P_T^2/\langle P_T^2 \rangle_{Siv}}$$

$$\langle P_T^2 \rangle_{Siv}^{SIDIS} = z^2 \omega_{Siv}^2 + \omega_{FF}^2$$

$$w_S^2(Q,Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}$$

$$w_F^2 \equiv w_F^2(Q,Q_0) = \langle p_\perp^2 \rangle + 2 z^2 g_2 \ln \frac{Q}{Q_0}$$

- Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

- $0.2 < z < 0.8$

Consequences on DY data and warnings

➢Numerator of the asymmetry in analytical approximation for a DY process

$$N_{DY} \propto \Delta^N f(x_1, Q_0) f(x_2, Q_0) \sqrt{2e} \frac{P_T}{M_1} \frac{\langle k_\perp^2 \rangle_{Siv}^2}{\langle k_{\perp 1}^2 \rangle \langle P_T^2 \rangle_{Siv}^2} e^{-P_T^2/\langle P_T^2 \rangle_{Siv}}$$

$$\langle P_T^2 \rangle_{Siv}^{DY} = \omega_{Siv}^2 + \omega_2^2$$

$$w_S^2(Q, Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}$$

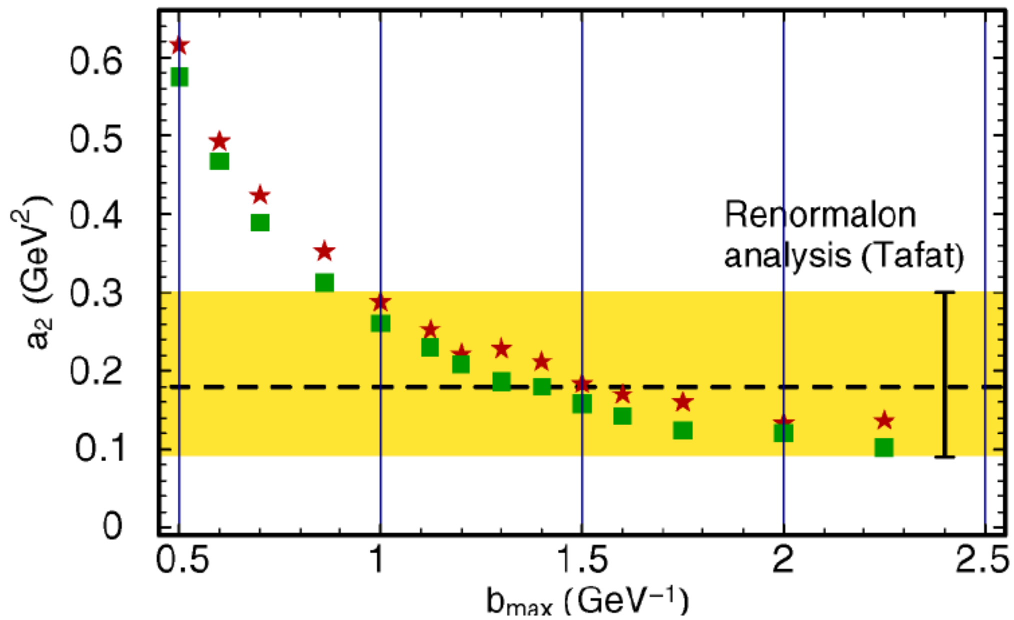
$$w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}$$

➢Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

➢g_2 is more crucial for DY processes than for the present SIDIS data (because of a wider kinematical range in Q^2)

Consequences on DY data and warnings

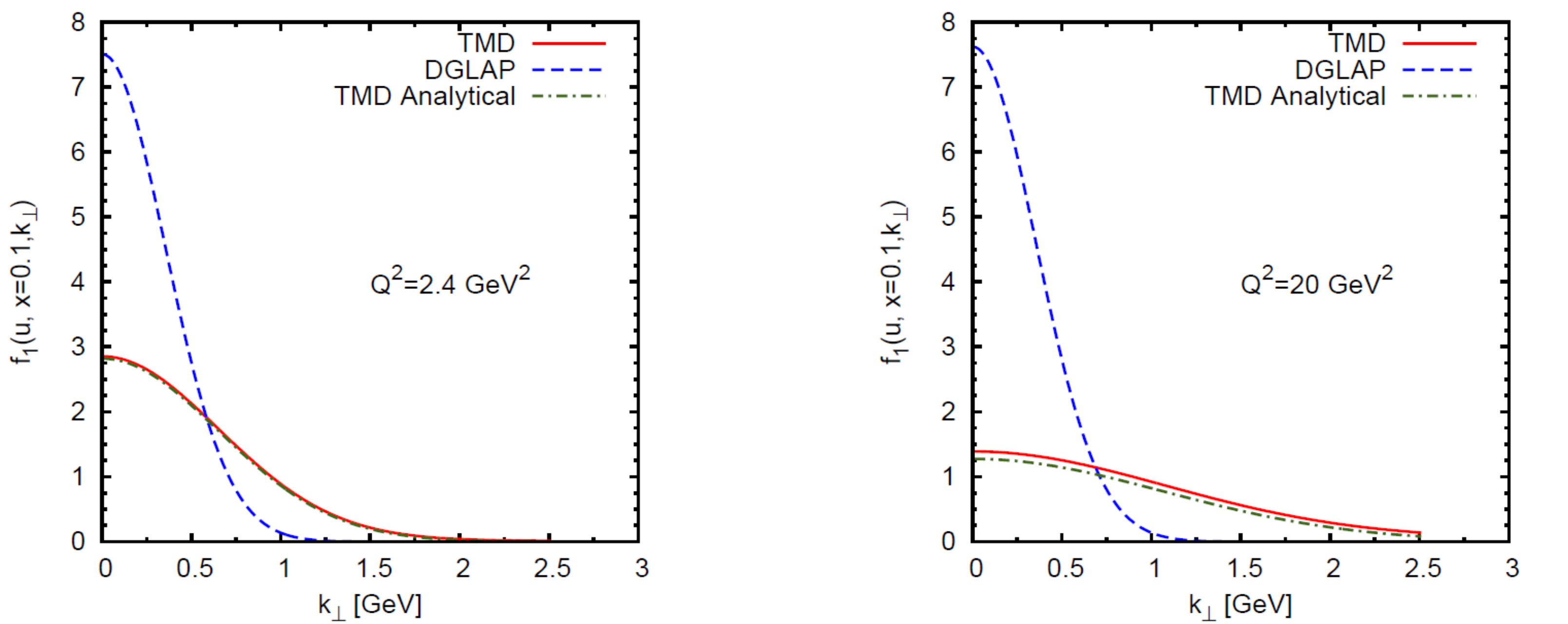
➢ g_2 depends on the prescription for the separation of the perturbative region from the non -perturbative one. Depends also on the "order" at which you stop in the perturbative expansion.



$a_2=g_2$, stars correspond to the choice $C1=2\exp(-\gamma_e)$, squares to $C1=4\exp(-\gamma_e)$

Konychev and Nadolsky Phys. Lett. B633 (2006)

Comparative analysis of TMD evoultion equations

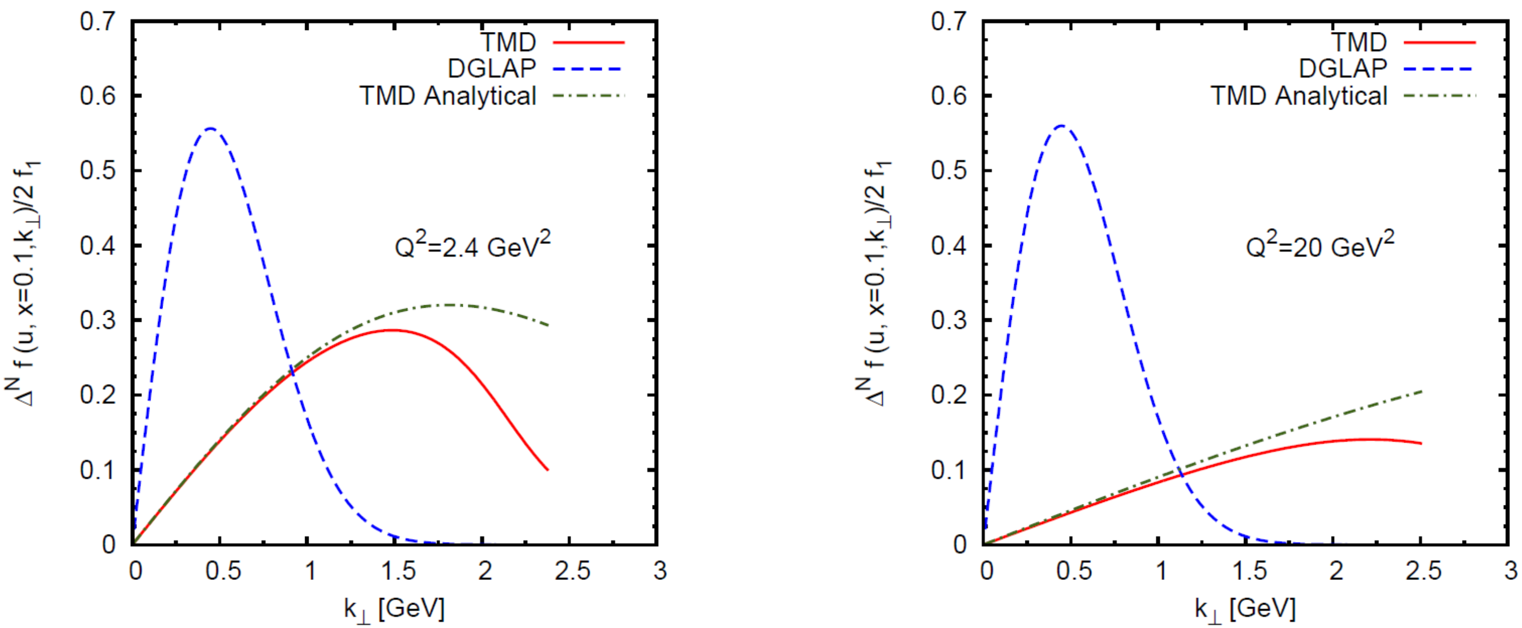


Starting scale Q_0=1 GeV
Same function at Q_0

DGLAP evolution is slow at moderate x and in this range of Q^2

For the unpolarized PDF, the analytical approximation holds up to large $k_\perp$

Comparative analysis of TMD evoultion equations



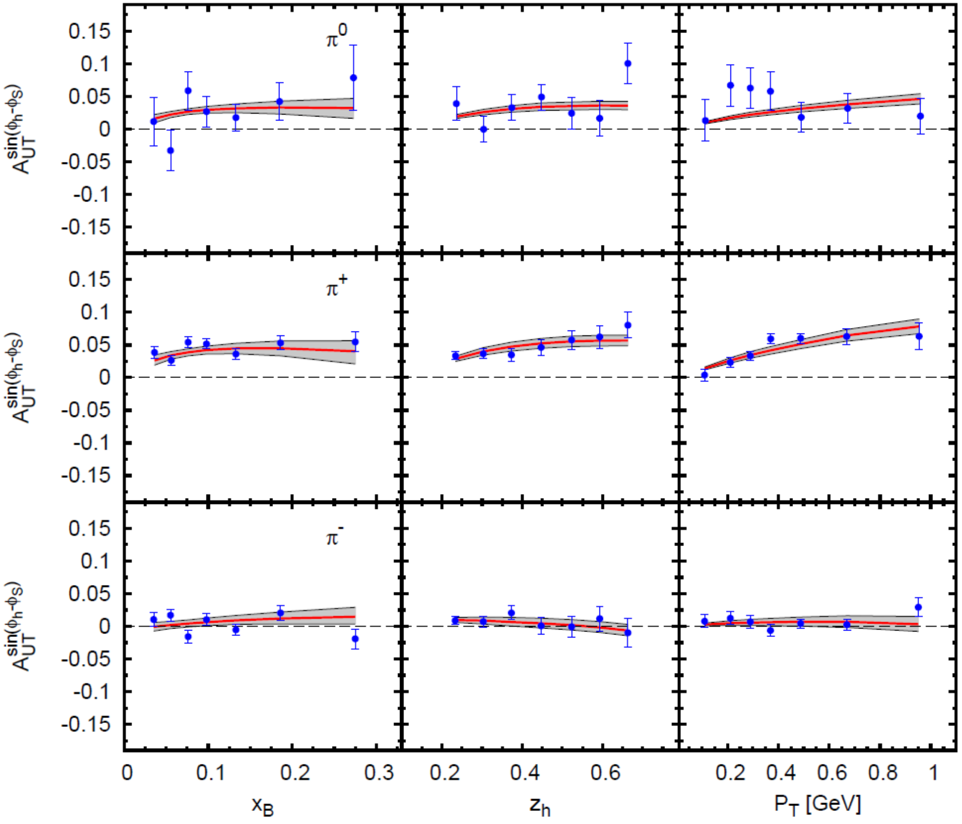
Starting scale Q_0=1 GeV
Same function at Q_0

For the Sivers function, the analytical approximation breaks down at large $k_\perp$ values

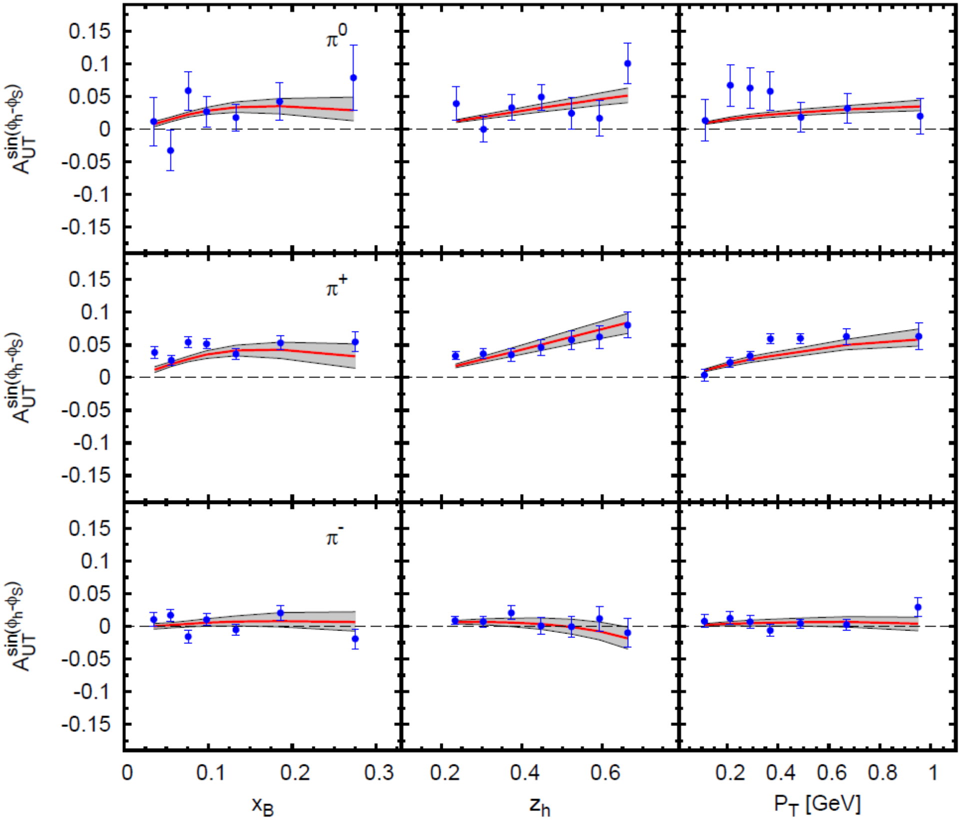
Fit of HERMES and COMPASS SIDIS data

A. Airapetian et al., Phys. Rev. Lett. 103, 152002 (2009), arXiv:0906.3918 [hep-ex]

HERMES PROTON - TMD

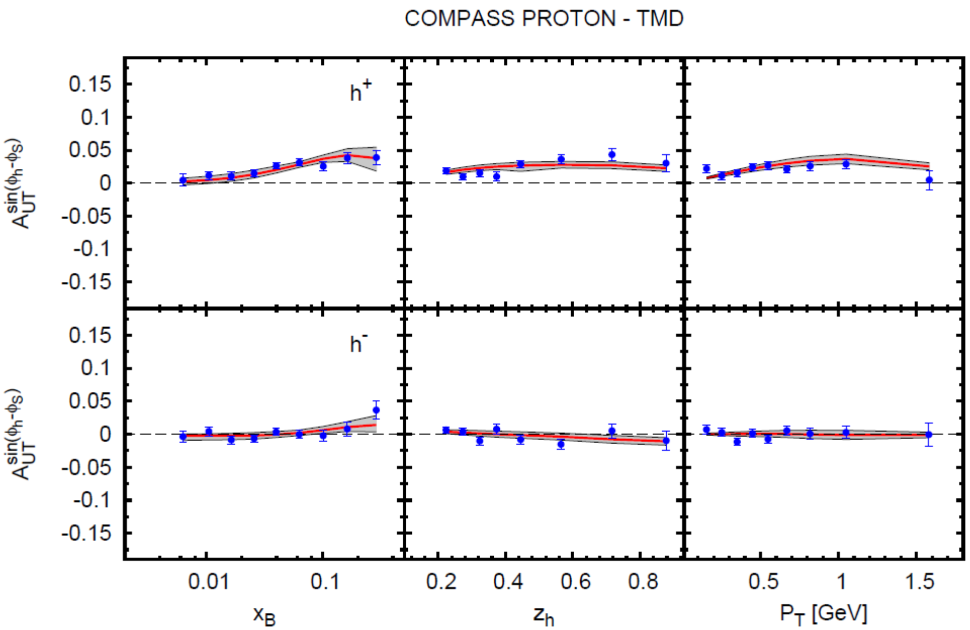


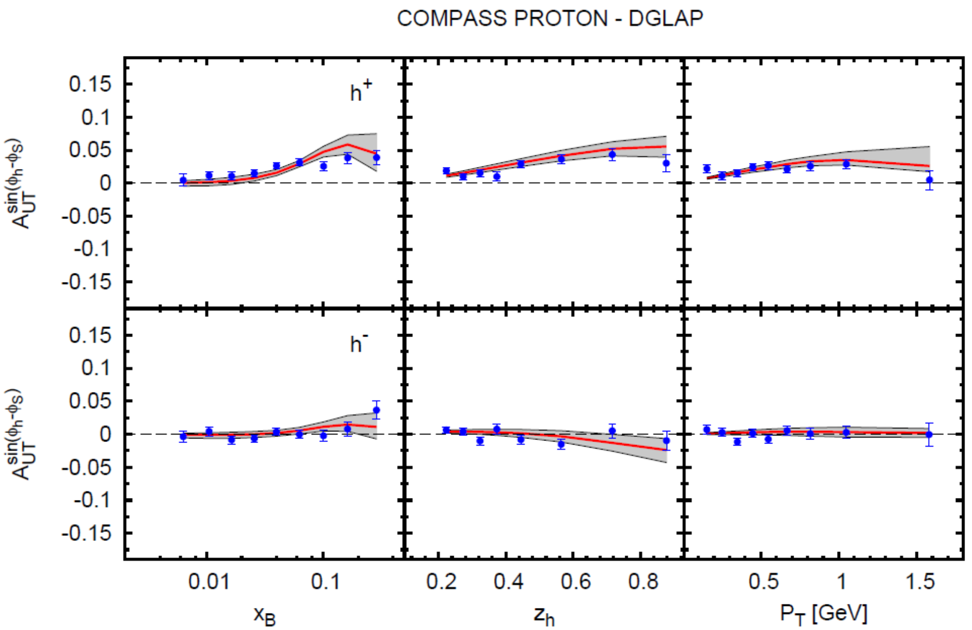
HERMES PROTON - DGLAP



Fit of HERMES and COMPASS SIDIS data

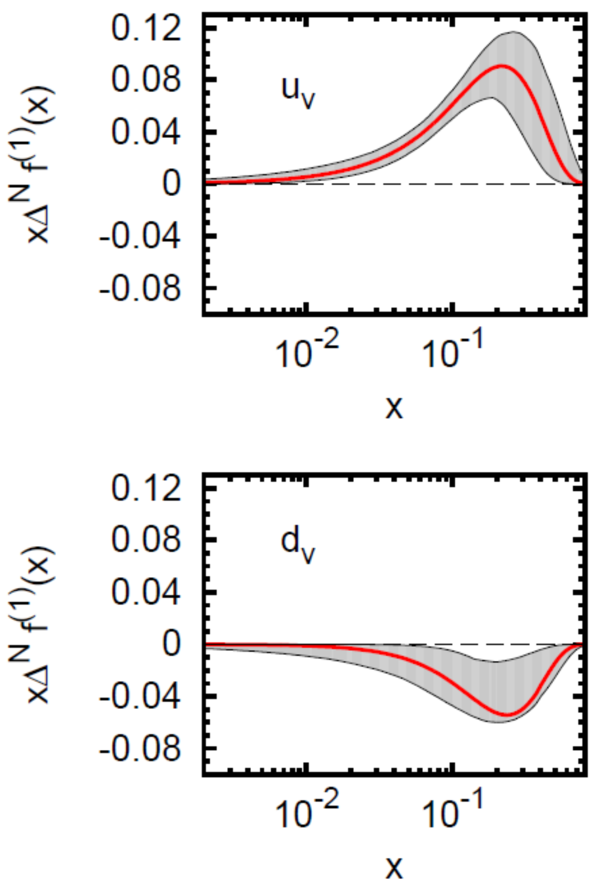
F. Bradamante, arXiv:1111.0869 [hep-ex]





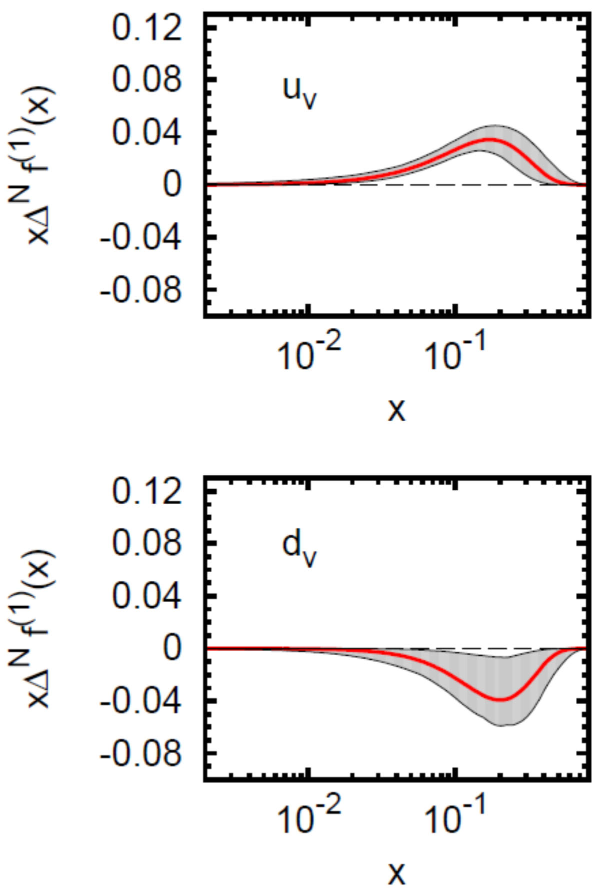
Fit of HERMES and COMPASS SIDIS data

TMD Evolution

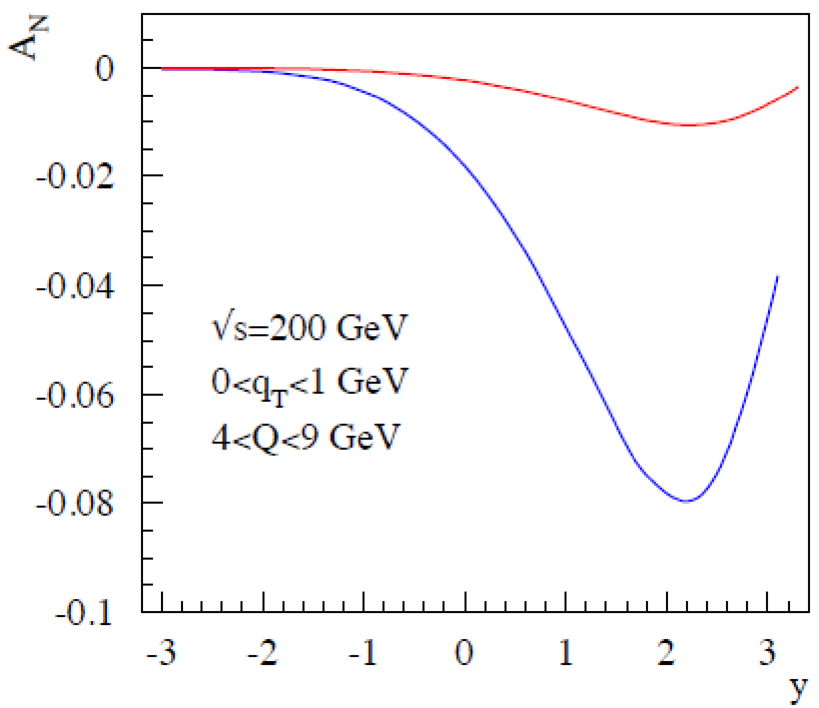


Q_0=1 GeV

DGLAP Evolution



Consequences on DY data and warnings

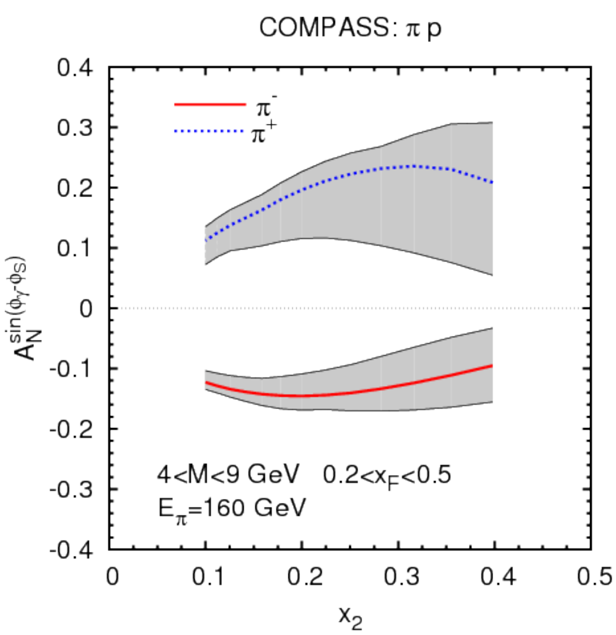


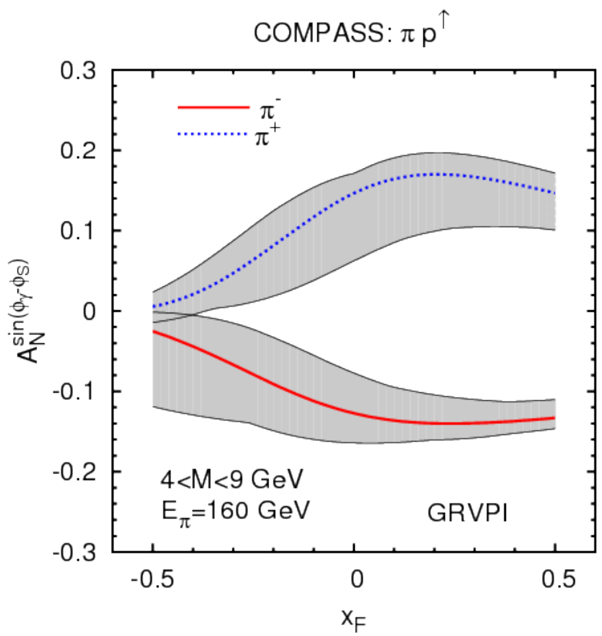
- Blue curve: bare parton model (using Torino TMD with Gaussian ansatz from SIDIS)
- Red curve: resummed formalism (using Torino TMD to calculate $T_F(x, x)$ as the initial input function, then evolve)

Presented by Zhongbo Kang, QCD Evolution 2012, JLAB

Predictions for COMPASS DY(DGLAP)

- Polarized NH_3
- Pion beam
- Valence region for the Sivers function





- Anselmino et al. Phys. Rev. D79,054010

Extraction of the transversity and the Collins function

- Parametrization of Transversity function:

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)[f_{q/p}(x) + \Delta q(x)] \frac{1}{\pi\langle k_\perp^2\rangle} e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Unpolarized PDF

Helicity PDF

$$\mathcal{N}_q^T(x) = N_q^T\, x^{\alpha_q}(1-x)^{\beta_q}\, \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

N_q^T, α, β free parameters

Extraction of the transversity and the Collins function

➢ Parametrization of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z,p_\perp)$$

- $\mathcal{N}_q^C(z) = N_q^C\, z^\gamma (1-z)^\delta \dfrac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$
- $h(p_\perp) = \sqrt{2e}\,\dfrac{p_\perp}{M_h}\, e^{-p_\perp^2/M_h^2}$

Unpolarized FF

N_q^C, γ, δ, M_h free parameters

✔Bound:

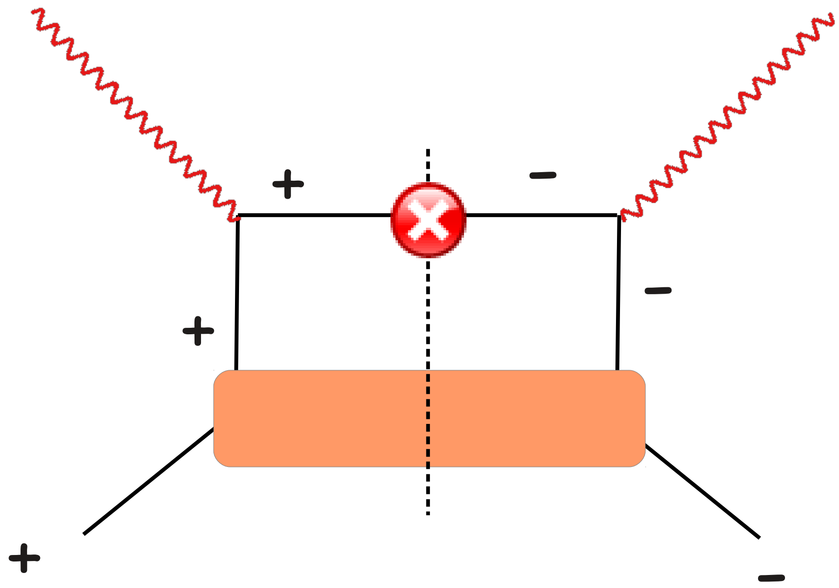
$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp)$$

✔Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp)$$

Chiral Odd nature of Transversity

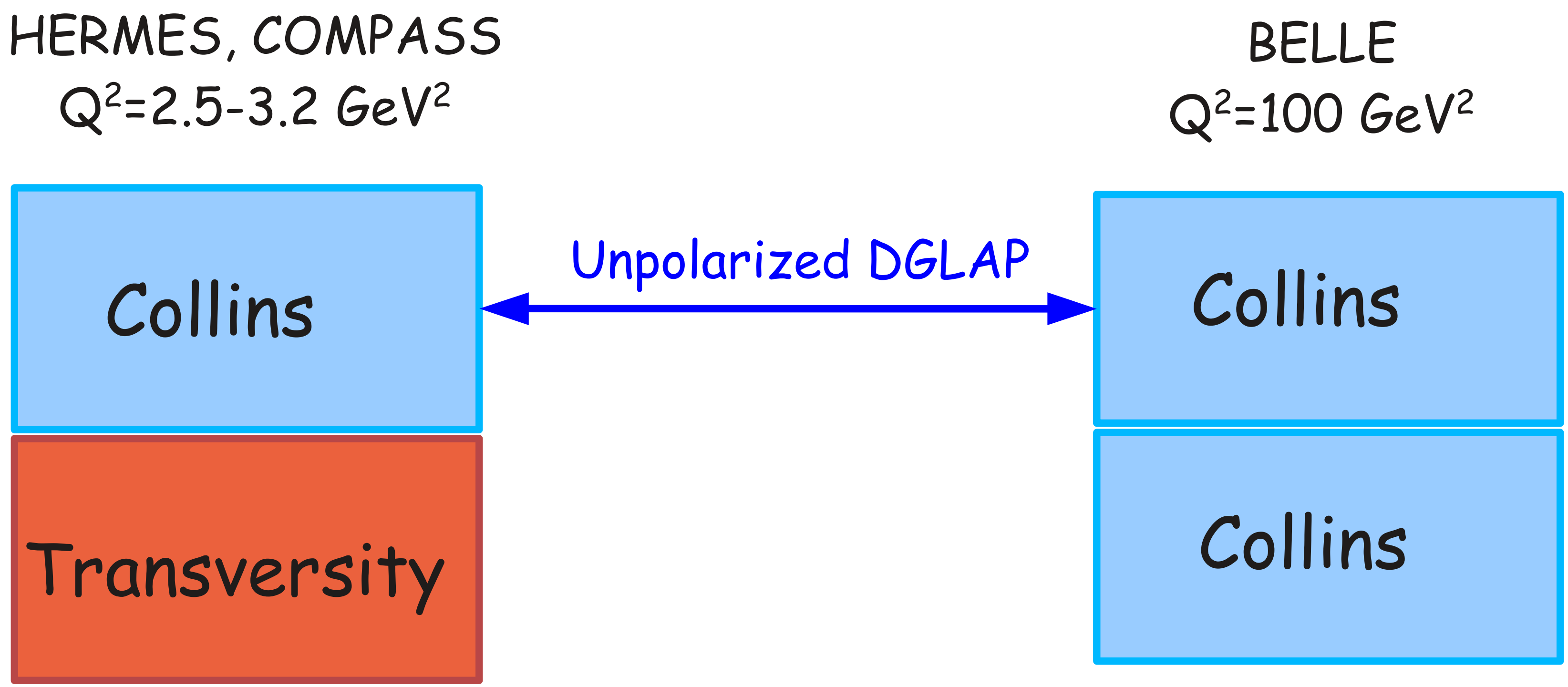
- Chiral Odd: It cannot be measured in DIS processes!



- It needs to be coupled with another chiral odd quantity....

TMD evolution

- TMD evolution for the Collins function is still unknown.
- TMD evolution can suppress the Collins function at large Q^2 (Boer, 2001)



TMD evolution

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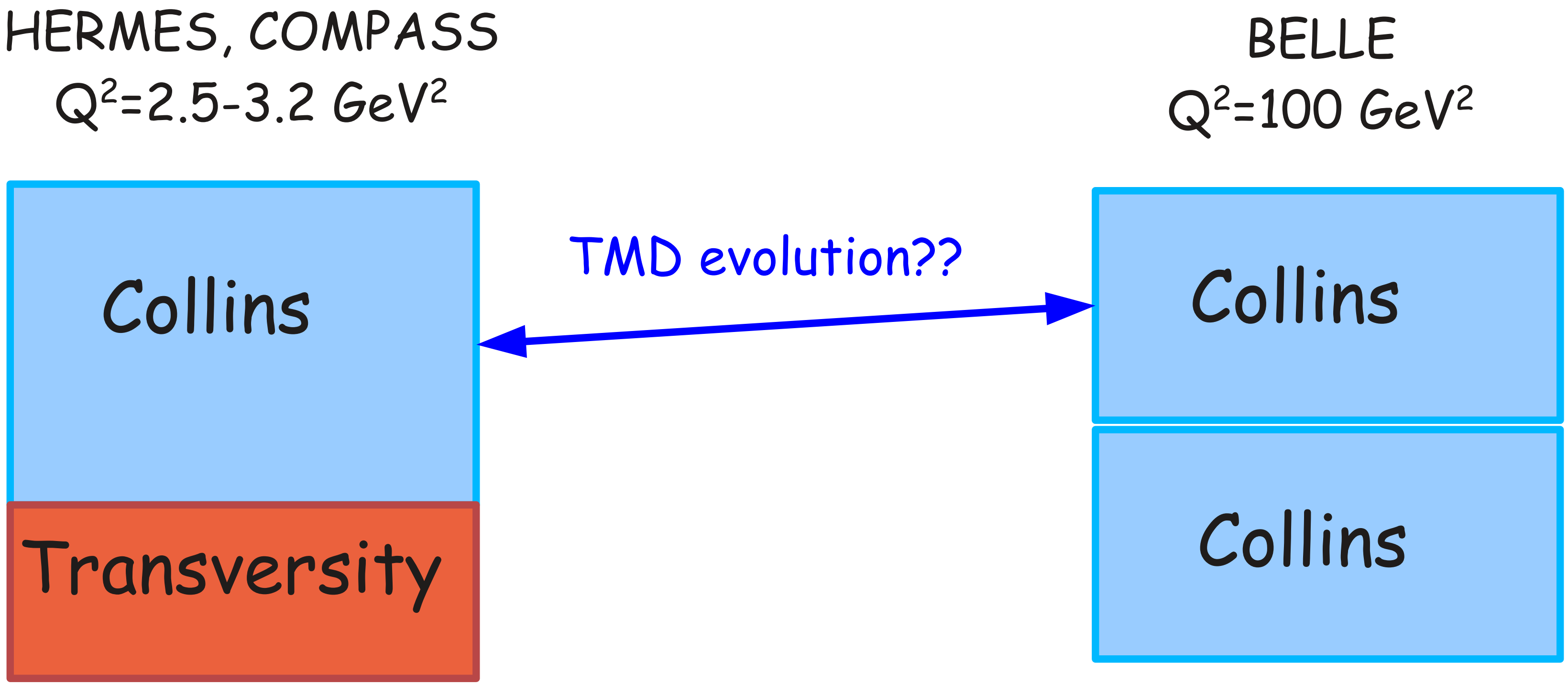
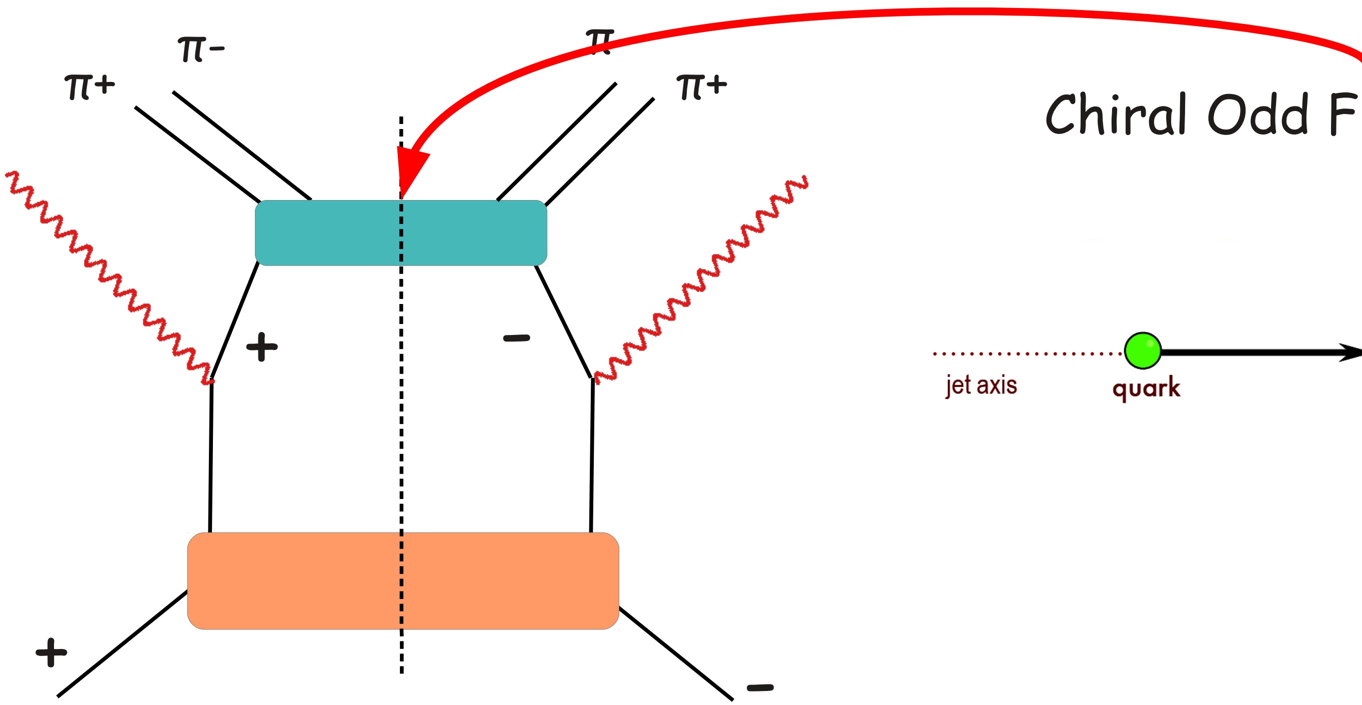
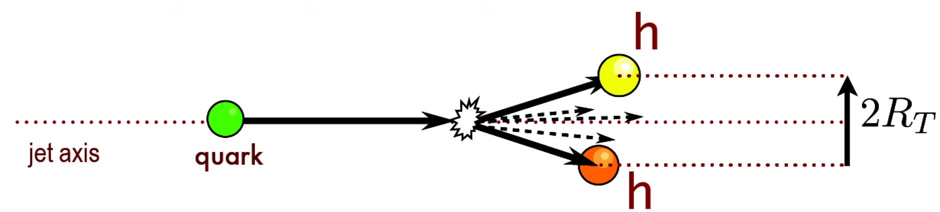
  [D. Boer, Nucl. Phys. B603 (2001); Nucl. Phys. B806 (2009)]



The dihadron way



Chiral Odd Fragmentation!



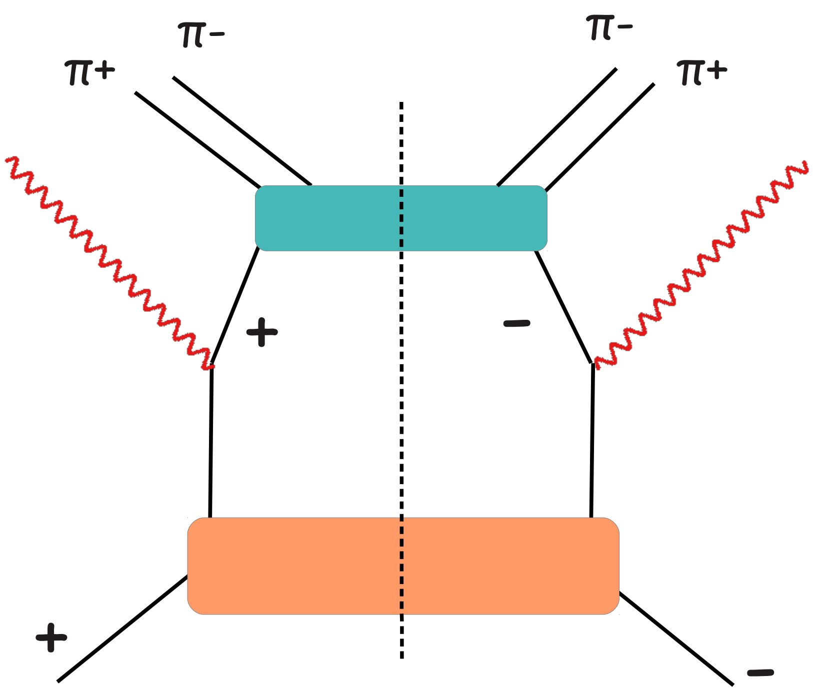
$$D_1^{q \to h_1 h_2}(z, M_h^2)$$

- Unpolarized DiFF

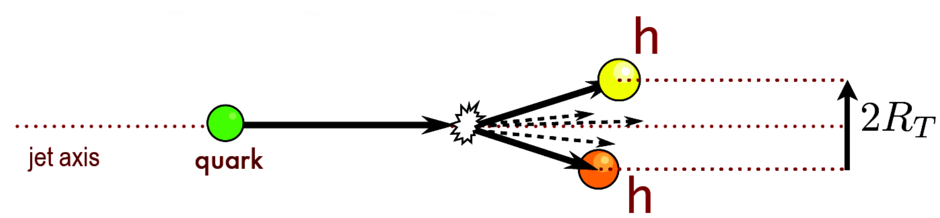
$$H_1^{\sphericalangle q}(z, M_h^2)$$

- Chiral-Odd DiFF

The dihadron way



Collinear evolution!



$$D_1^{q\rightarrow h_1 h_2}(z, M_h^2)$$

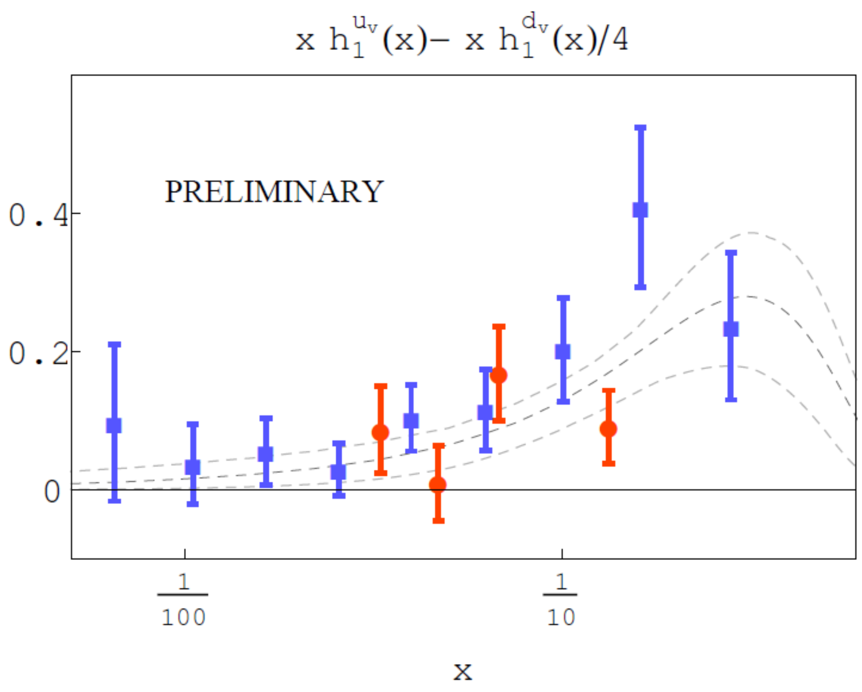
- Unpolarized DiFF

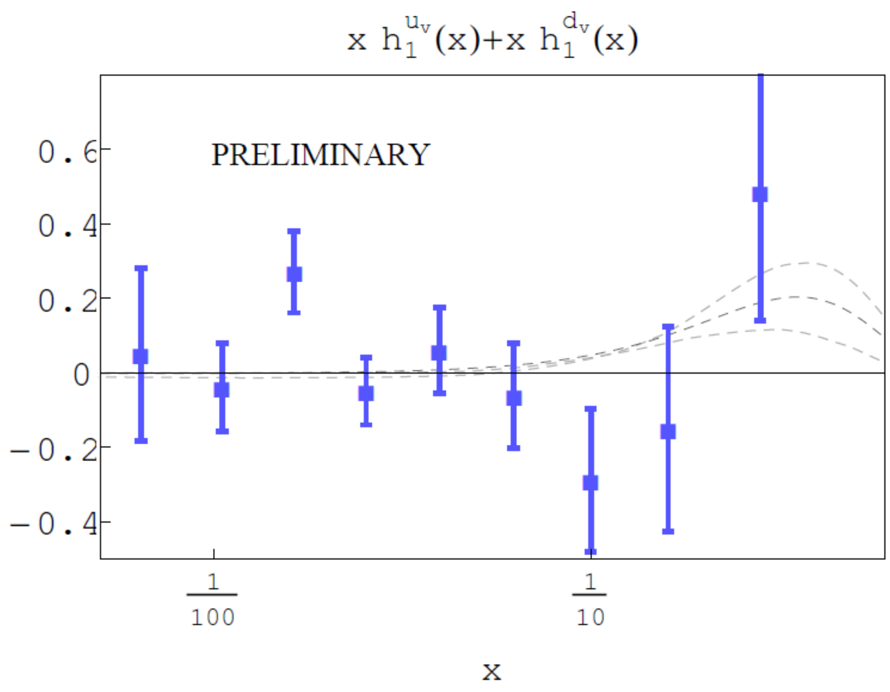
$$H_1^{\sphericalangle q}(z, M_h^2)$$

- Chiral-Odd DiFF

The dihadron way: Pavia group extraction

- Comparison Pavia-Torino

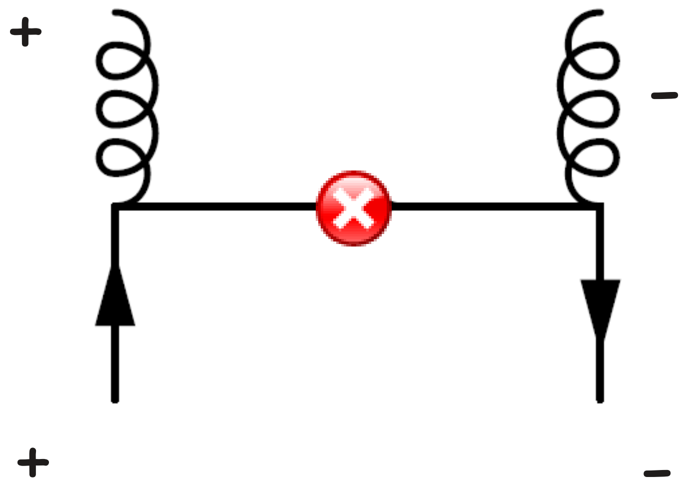




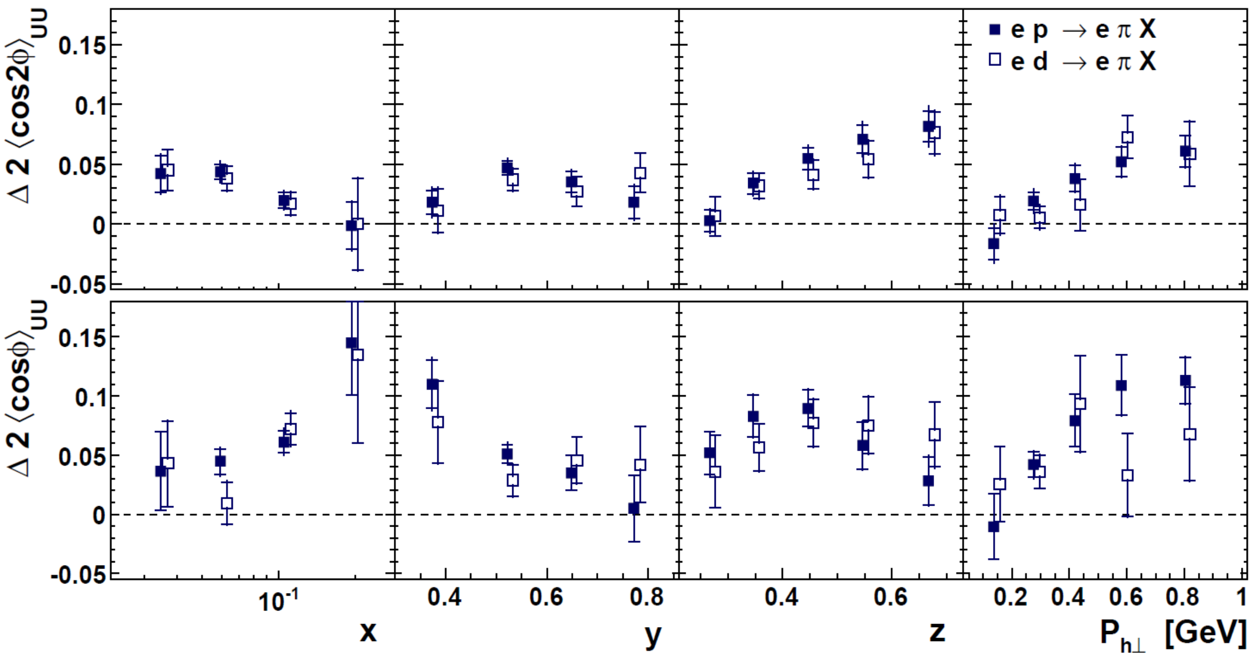
Bacchetta, Courtoy, Radici.,arXiv:1206.1836

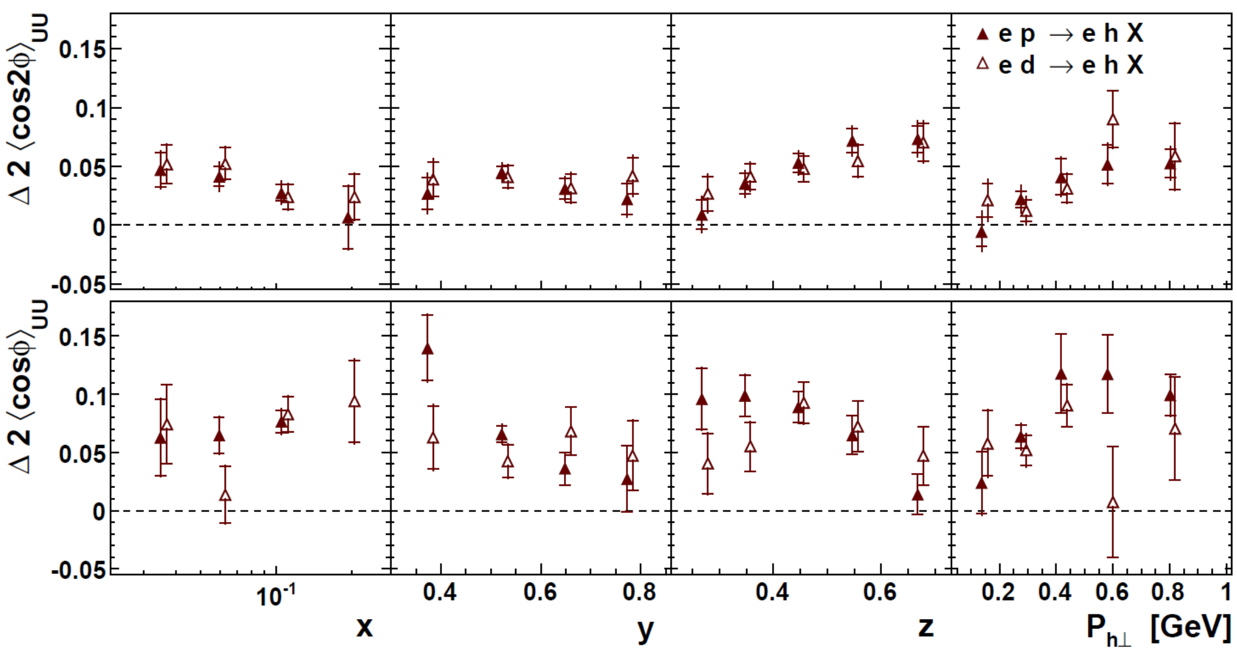
Transversity at low x?

- No gluon contributions in the evolution



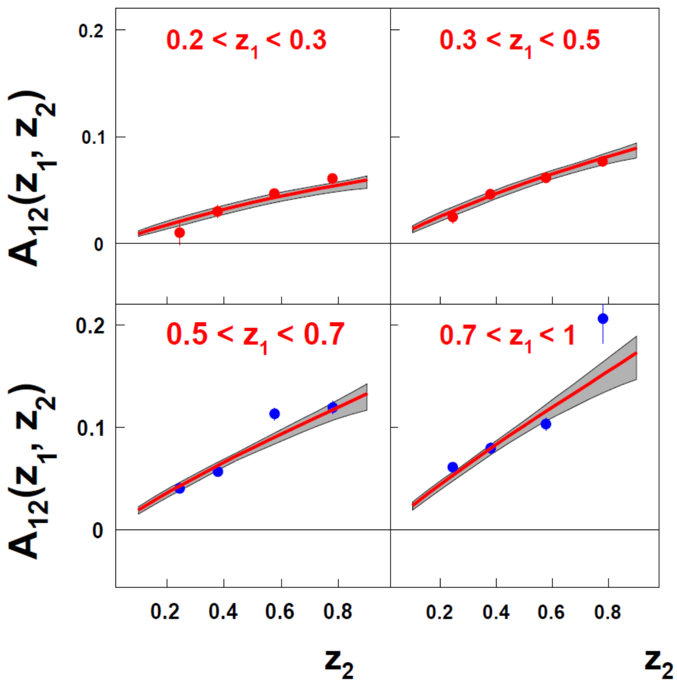
- Non singlet evolution of the transversity, h_1 suppressed at low x


Δ 2 ⟨cos2φ⟩UU
Δ 2 ⟨cosφ⟩UU
e p → e π X
e d → e π X
x
y
z
Ph⊥ [GeV]


Δ 2 ⟨cos2φ⟩UU
Δ 2 ⟨cosφ⟩UU
e p → e h X
e d → e h X
x
y
z
Ph⊥ [GeV]

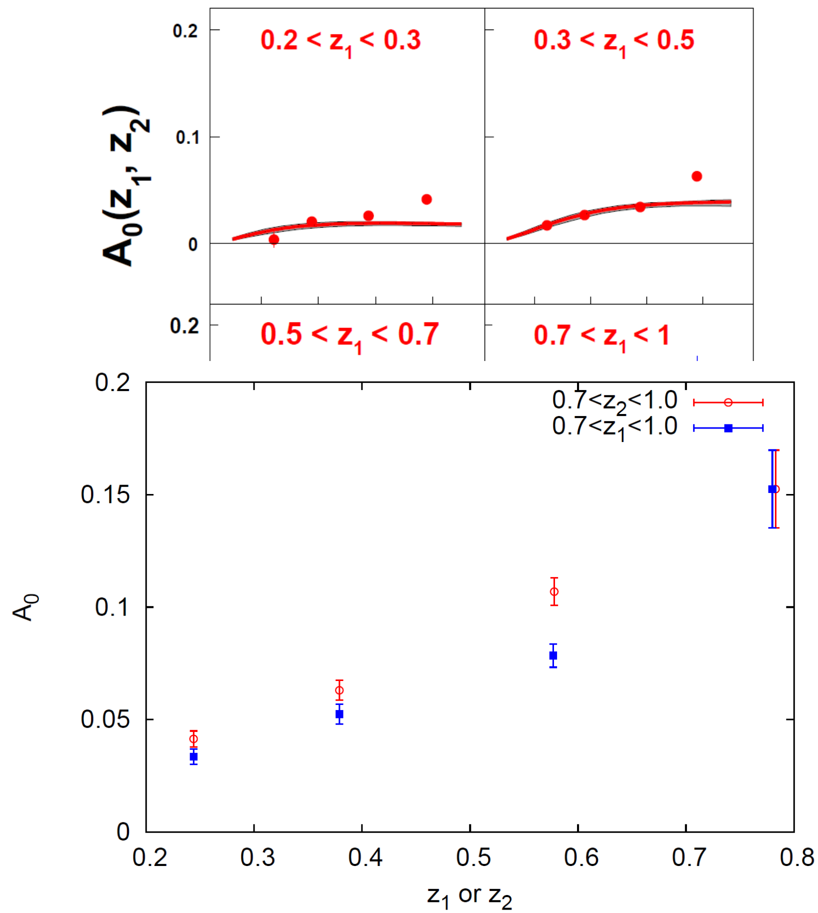
Extraction of the transversity and the Collins function

BELLE A_{12} (FIT)



◇ R. Seidl et al., Phys. Rev. D78

BELLE A_0 (Predicted)



• Anselmino et. al arXiv: 0812.4366v1

Extraction of the Boer-Mulders functions

- Simple parametrization of the Boer-Mulders functions:

  - $h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$ for valence quarks
  - $h_1^{\perp q}(x, k_\perp) = -|f_{1T}^{\perp q}(x, k_\perp)|$ for sea quarks

- Inspired by models:

$$h_1^{\perp q}(x, k_\perp) = \frac{\mathcal{K}_T^q}{\mathcal{K}^q} f_{1T}^{\perp q}(x, k_\perp)$$

Tensor magnetic moment

Anomalous magnetic moment

Burkardt, Phys. Rev. D72, 094020 (2005)

Gockeler, Phys.Rev.Lett.98:222001,2007.

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- Models inspired:

$$h_1^{\perp q}(x, k_\perp) = \frac{\mathcal{K}_T^q}{\mathcal{K}^q} f_{1T}^{\perp q}(x, k_\perp)$$

  - $h_1^{\perp u}(x, k_\perp) \simeq 1.80 f_{1T}^{\perp u}(x, k_\perp) < 0$
  - $h_1^{\perp d}(x, k_\perp) \simeq -0.94 f_{1T}^{\perp d}(x, k_\perp) < 0$

Extraction of the Boer-Mulders functions

FIT I

- HERMES proton and deuteron target (x,z,P_T) charged hadrons

HERMES, Giordano:arXiv:0901.2438

- COMPASS deuteron target (x,z) charged hadrons

COMPASS, Kafer: arXiv 0808.0114

- 2 free parameters:

$\lambda_u \quad \lambda_d$

✔GRV98 PDF

✔DSS FF

✔Gaussians: $\langle k_\perp^2 \rangle = 0.25\ (\mathrm{GeV}/c)^2$
$\langle p_\perp^2 \rangle = 0.20\ (\mathrm{GeV}/c)^2$
(from Cahn effect)

✔ $h_1^{\perp q}(x, k_\perp) = \lambda_q\, f_{1T}^{\perp q}(x, k_\perp)$

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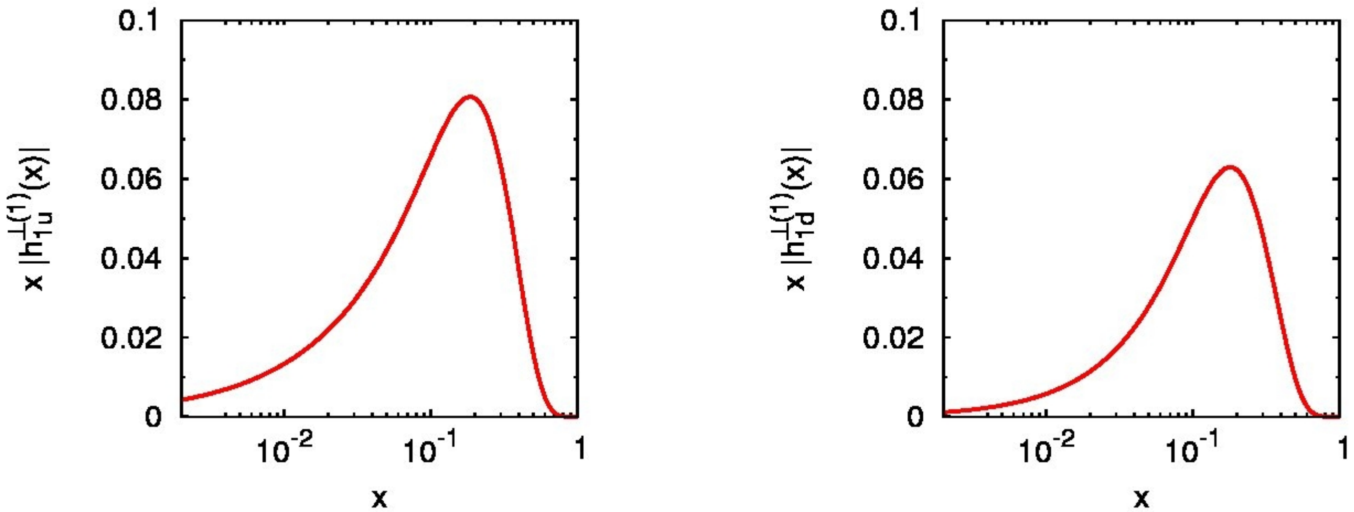
Sivers functions from
Anselmino et al. Eur. Phys. J. A39,89

Extraction of the Boer-Mulders functions



$\diamond \chi^2/d.o.f. = 3.73$

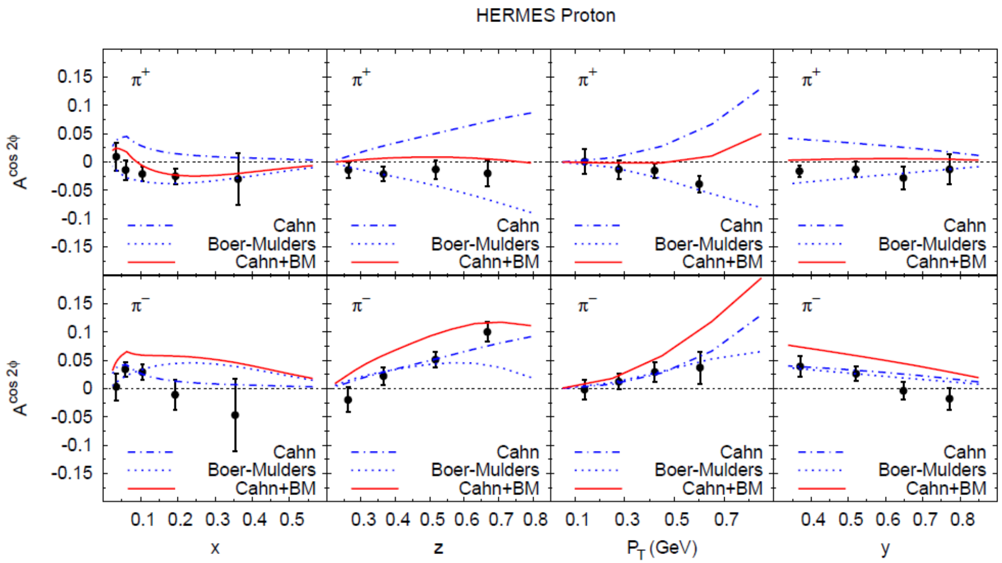
$\bullet \lambda_u = 2.0 \pm 0.1$

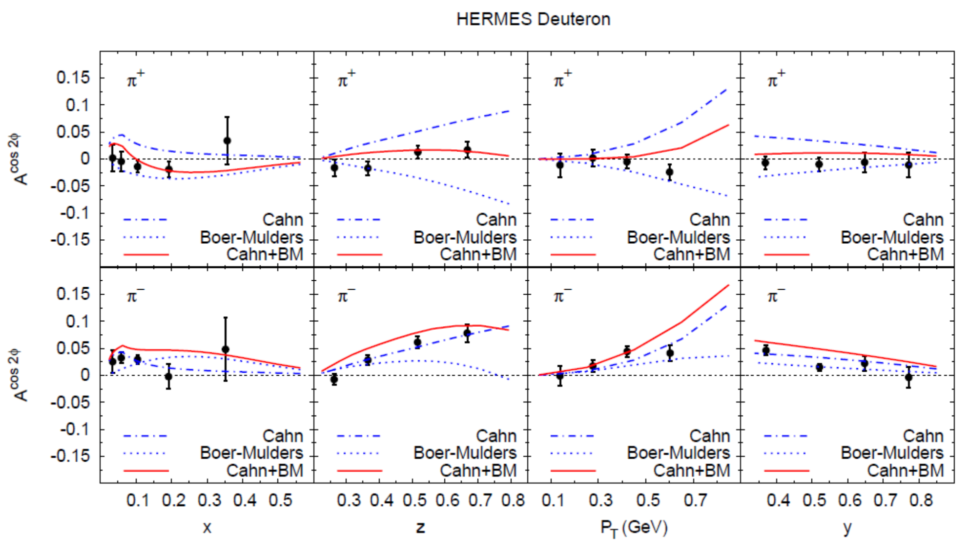
$\bullet \lambda_d = -1.11^{+0.00}_{-0.02}$

$\Rightarrow h_1^{\perp d}$ and $h_1^{\perp u}$ both negative

Compatible with models predictions

Extraction of the Boer-Mulders functions

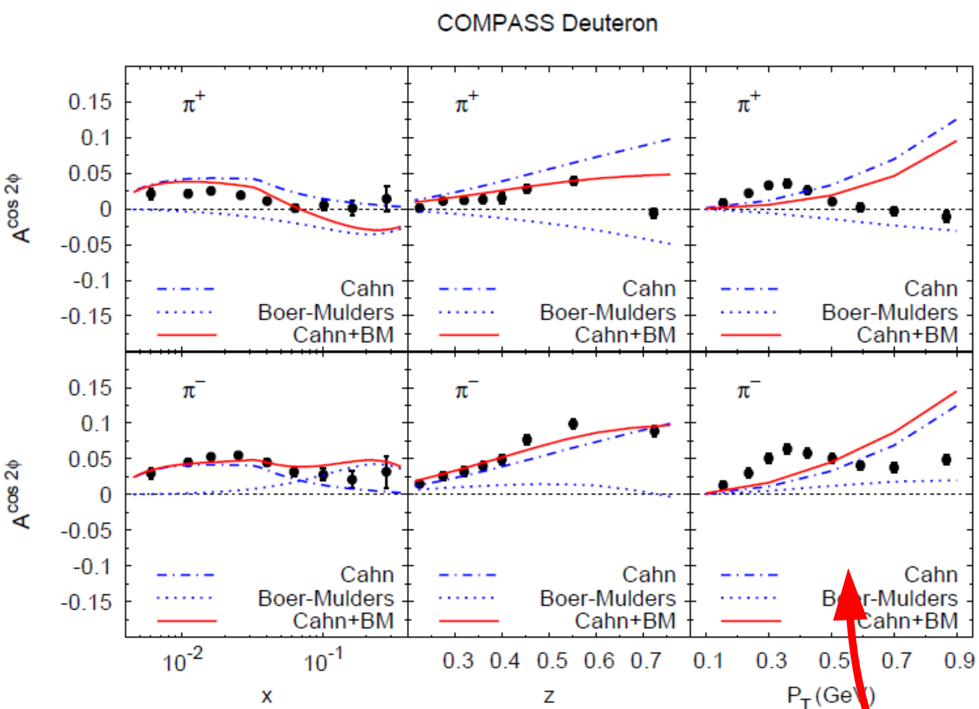




- ✔Cahn effect (Twist-4)comparable to BM effect
- ✔Same sign of Cahn contribution for positive and negative pions
- ✔BM contribution opposite in sign for positive and negative pions

HERMES, Giordano:arXiv:0901.2438

Extraction of the Boer-Mulders functions



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Data in p_T not included in the fit

COMPASS, Kafer: arXiv 0808.0114

Extraction of the Boer-Mulders Function

- The Cahn effect is a crucial ingredient

✔Gaussians: $\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$
$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$

} From Ref.[*]: analysis of Cahn $\cos\phi$ effect from EMC data

COMPASS	HERMES
$\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$	$\langle k_\perp^2 \rangle = 0.18\ (GeV/c)^2$
$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$	$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$
~EMC	~HERMES MC

[*] Anselmino et al. Phys. Rev. D71 074006 (2005)

Extraction of the Boer-Mulders Function

- FIT II

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~EMC

HERMES

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~HERMES MC

FIT II

$\diamond \chi^2/d.o.f. = 2.41$

$\bullet \lambda_u = 2.1 \pm 0.1$

$\bullet \lambda_d = -1.11^{+0.00}_{-0.02}$

Better description of HERMES but the BM is unchanged

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