Field-theoretical formulations of MOND-like gravity

(MOND = MOdified Newtonian Dynamics)

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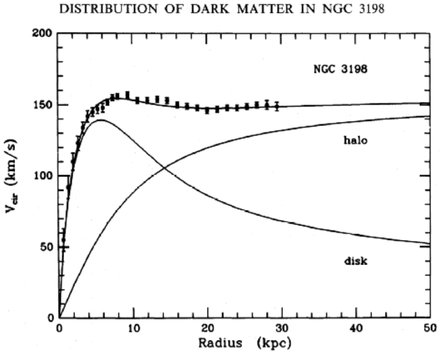
Discussions with L. Blanchet, T. Damour, C. Deffayet, B. Fort, G. Mamon, Y. Mellier, M. Milgrom, J. Moffat, R. Sanders, J.-P. Uzan, R. Woodard, *etc.*

April 30th, 2008

Dark matter and galaxy rotation curves

- $\Omega_\Lambda \approx 0.7$ (SNIa) and $\Omega_\Lambda + \Omega_m \approx 1$ (CMB) $\Rightarrow \Omega_m \approx 0.3$, at least $10\times$ greater than estimates of baryonic matter.
- Rotation curves of galaxies and clusters: almost rigid bodies



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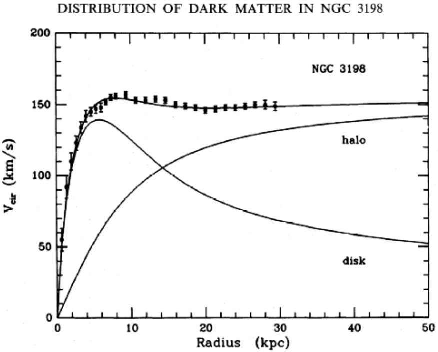

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Milgrom's MOND proposal [1983]

MOdified Newtonian Dynamics
for small accelerations (i.e., at large distances)

$$a = a_N = \frac{GM}{r^2} \quad \text{if } a > a_0 \approx 1.2 \times 10^{-10}\,\text{m.s}^{-2}$$

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- Automatically recovers the Tully-Fisher law [1977]

$$v_\infty^4 \propto M_{\text{baryonic}}$$

- Superbly accounts for galaxy rotation curves
  (but clusters still require some dark matter)

  [Sanders & McGaugh, Ann. Rev. Astron. Astrophys. **40** (2002) 263]

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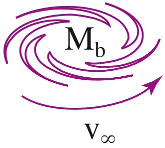
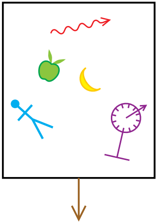


Modified gravity or modified inertia?

General Relativity

$$S = \underbrace{\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\,R}_{\text{Einstein-Hilbert}} + \underbrace{S_{\text{matter}}[\text{matter}, g_{\mu\nu}]}_{\text{Metric coupling}}$$

freely falling elevator



Einstein-Hilbert $\Rightarrow$ pure spin 2
Metric coupling $\Rightarrow$ equivalence principle
Example: $S_{\text{point particle}} = -\int mc\sqrt{-g_{\mu\nu}(x)\,v^\mu v^\nu}\,dt$

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  Galileo invariance $\Rightarrow$ nonlocal! ($\Rightarrow$ causality?)

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- Other possibilities: Both modifications, or none?

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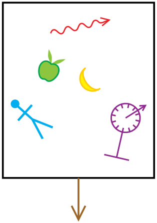

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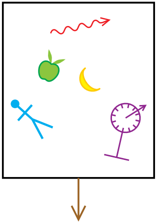

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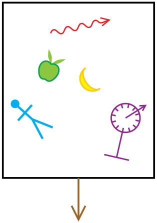

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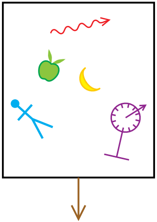

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  Constant coefficient $2/b$ instead of $\sqrt{M}$.

  Some papers write actions which depend on the galaxy mass M
  $\Rightarrow$ They are actually using a different theory for each galaxy!

- Moffat [2004] proposes a consistent field theory (nonsymmetric $g_{\mu\nu}$)
  but predicts $a = kM^2/r$ instead of $\sqrt{M}/r$,
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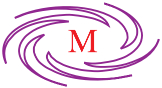

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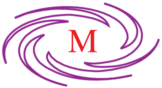

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  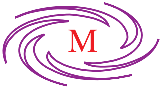

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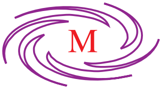

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Consistent field theories

- Field theories
  All predictions deriving from a single action
  ∃ proposed models in which 2 field equations are inconsistent with each other
  ⇒ violation of conservation laws

- Stability
  Full Hamiltonian should be bounded by below:
  no tachyon ($m^2 \geq 0$), no ghost ($E_{\text{kinetic}} \geq 0$)

- Well-posed Cauchy problem
  Hyperbolic field equations

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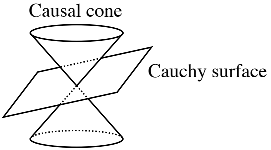
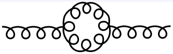


Quadratic gravity

- ’t Hooft & Veltman [1974]: Divergence of  needs

Counterterm

$$\Delta\mathcal{L} = \frac{\sqrt{-g}}{8\pi^2(d-4)}\left[\frac{53}{90}R^2_{\mu\nu\rho\sigma} - \frac{361}{180}R^2_{\mu\nu} + \frac{43}{72}R^2\right]$$

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$C_{\mu\nu\rho\sigma}$: Weyl tensor (fully traceless)
$\mathrm{GB} \equiv R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$: Gauss-Bonnet topological invariant

- Stelle’s thesis [1977]: If $\alpha \neq 0$ and $\beta \neq 0$,

Quadratic gravity is renormalizable

$$S_{\text{gravity}} = \int d^4x\sqrt{-g}\left[R + \alpha C^2_{\mu\nu\rho\sigma} + \beta R^2 + \gamma\mathrm{GB}\right]$$

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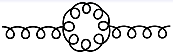
- 't Hooft & Veltman [1974]: Divergence of  needs

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f(curvature) and ghosts

But quadratic gravity is unstable!

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- Intuitive argument:

  Propagator $\quad \dfrac{1}{p^2 + \alpha p^4} = \dfrac{1}{p^2} \underset{\text{ghost!}}{-} \dfrac{1}{p^2 + \frac{1}{\alpha}}$

  N.B.: $\frac{1}{\alpha} = m^2$ of extra d° of freedom ⇒ negative α gives a tachyon, but anyway a ghost

- Full calculation

  [Stelle 1977; Hindawi, Ovrut, Waldram 1996; Tomboulis 1996]:

  - $R + f(R_{\mu\nu}, R_{\mu\nu\rho\sigma})$ ⇒ extra massive spin-2 ghost ⇒ unstable vacuum
  - $R + f(R)$ ⇒ extra massive spin-0 scalar with $E_{\text{kin}} > 0$

N.B.: Strings predict $C^2_{\mu\nu\rho\sigma}$, but also any higher derivative (nonlocal)

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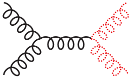

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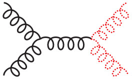

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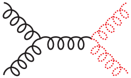

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$f(R)$ as scalar-tensor theories

- Introduction of a Lagrange multiplier Φ [Teyssandier & Tourrenc 1983]

$$
\begin{aligned}
S_{\text{gravity}} &= \int d^4x\sqrt{-g}\, f(R) \\
&\Leftrightarrow \int d^4x\sqrt{-g}\,\{f(\Phi) + (R-\Phi)f'(\Phi)\} \\
&= \int d^4x\sqrt{-g}\Big\{f'(\Phi)R - \underbrace{0\,(\partial_\mu\Phi)^2}_{\omega_{\text{BD}}=0} - \underbrace{[\Phi f'(\Phi) - f(\Phi)]}_{\text{potential}}\Big\}
\end{aligned}
$$

N.B.: If potential $= 0$, solar system $\Rightarrow \omega_{\text{BD}} > 40000$

- Similarly $f(R, \Box R, \ldots, \Box^n R) \Rightarrow$ Einstein plus $n+1$ scalar fields [Gottlöber, Schmidt, Starobinsky 1990; Wands 1994]

- Such scalar fields give generically Yukawa potentials $\propto \dfrac{e^{-mr}}{r}$
$\Rightarrow$ not MOND (potential $\propto \ln r$)

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N.B.: If potential $=0$, solar system $\Rightarrow \omega_{\text{BD}} > 40000$

- Similarly $f(R, \Box R, \ldots, \Box^n R) \Rightarrow$ Einstein plus $n+1$ scalar fields [Gottlöber, Schmidt, Starobinsky 1990; Wands 1994]

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graviton

scalar

Relativistic aquadratic Lagrangians

"RAQUAL" models

$$S = \int d^4x\sqrt{-g^*}\Big\{R^* - \underbrace{f\left(g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right)}_{\text{"k-essence"}} - V(\varphi)\Big\}$$

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The nonlinearity of $f(\partial_\mu\varphi\partial^\mu\varphi)$ now allows us to reproduce a MOND-like potential $\sim \sqrt{GMa_0}\ln r$ [Bekenstein & Sanders]:

$$\frac{1}{r^2}\partial_r\left(r^2 f'\left[(\partial_r\varphi)^2\right]\partial_r\varphi\right) \propto T \text{ (matter source)}$$

- $f'(x) \to$ constant for large x : Newtonian limit
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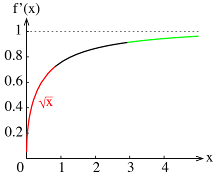

Consistency conditions on $f(\partial_\mu\varphi\partial^\mu\varphi)$

Hyperbolicity of the field equations + Hamiltonian bounded by below

- $\forall x, \quad f'(x) > 0$
- $\forall x, \quad 2\,x f''(x) + f'(x) > 0$

N.B.: If $f''(x) > 0$, the scalar field propagates faster than gravitons, but still causally
$\Rightarrow$ no need to impose $f''(x) \leq 0$

These conditions become much more complicated *within matter*

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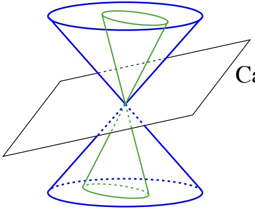

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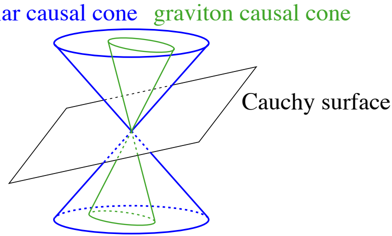

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Light deflection

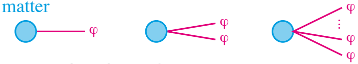
RAQUAL models

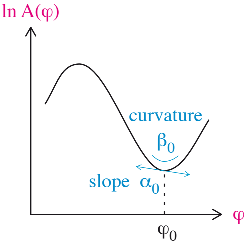
$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\left\{R^* - f\left(g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) - V(\varphi)\right\}$$

$$+ S_{\rm matter}[\text{matter}, \underbrace{g_{\mu\nu}}_{\text{physical metric}} \equiv \underbrace{A^2(\varphi)}_{\text{matter-scalar coupling}} \times \underbrace{g^*_{\mu\nu}}_{\text{spin-2 metric}}]$$

- Matter-scalar coupling function

$$\ln A(\varphi) = \alpha_0\,(\varphi-\varphi_0) + \frac{1}{2}\beta_0\,(\varphi-\varphi_0)^2 + \ldots$$





- Effective gravitational constant

Scalar field ⇒ extra attractive force

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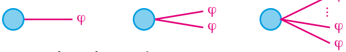
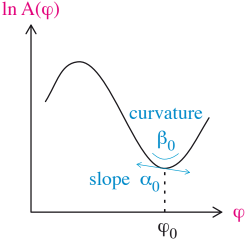
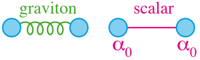




- Effective gravitational constant

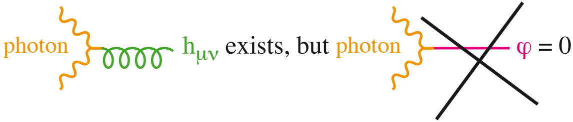
$$G_{\text{eff}} = G\,(\,1 + \alpha_0^{\,2}\,)$$



Scalar field ⇒ extra attractive force

Light deflection (continued)

- Conformally related metrics $g_{\mu\nu} \equiv A^2(\varphi) g^*_{\mu\nu}$
  $\Rightarrow$ Light rays do not feel the scalar field:
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = 0 \Leftrightarrow g^*_{\mu\nu} dx^\mu dx^\nu = 0$



- Light deflection angle

$$\Delta\theta = \frac{4GM}{bc^2} \quad \text{same as G.R.}$$

$$= \underbrace{\frac{4G_{\text{eff}}M}{bc^2(1+\alpha_0^2)} < \frac{4G_{\text{eff}}M}{bc^2}}$$

interpreted as smaller than G.R. because $G_{\text{eff}} > G_{\text{bare}}$

[N.B.: $\exists$ an erroneous theorem (overstatement) by Bekenstein & Sanders about this]

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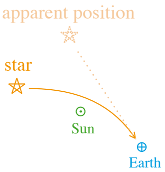

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"Disformal" coupling

Stratified theories
[Ni, Sanders, Bekenstein (TeVeS)]

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\left\{R^* - 2f(g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi)\right\}$$
$$+S_{\text{matter}}\left[\text{matter}\ ;\ g_{\mu\nu} \equiv A^2(\varphi, U)g^*_{\mu\nu} + B(\varphi, U)U_\mu U_\nu\right]$$

- U_μ is either a new vector field, or $\partial_\mu\varphi$ itself
- $A^2 > 0$ and $A^2 + Bg_*^{\mu\nu}U_\mu U_\mu > 0$ necessary for hyperbolicity (in addition to the previous conditions on f).

Stratified theories

Trick to increase light deflection (as dark matter does)

- Since Schwarzschild is such that $-g_{00} = g_{rr}^{-1} = \left(1 - \frac{2GM}{rc^2}\right)$, let us couple φ inversely to g_{00}^* and g_{ij}^*, say

$$g_{00} \equiv e^{2\varphi} g_{00}^* \quad \text{and} \quad g_{ij} \equiv e^{-2\varphi} g_{ij}^*$$

- Covariant rewriting:
  Let a vector $U_\mu = (1, 0, 0, 0)$ in this preferred frame.
  Then $-U_\mu U_\nu = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ behaves as g_{00}^*, and $\left(g_{\mu\nu}^* + U_\mu U_\nu\right)$ as g_{ij}^*
  $\Rightarrow$ Define the physical metric (minimally coupled to matter) as

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- N.B.1: Other factors depending on φ would give a different light deflection
  $\Rightarrow$ this is *ad hoc*!
- N.B.2: Preferred-frame effects strongly constrained in solar system

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- No MOND regime in dense Universe at high redshifts
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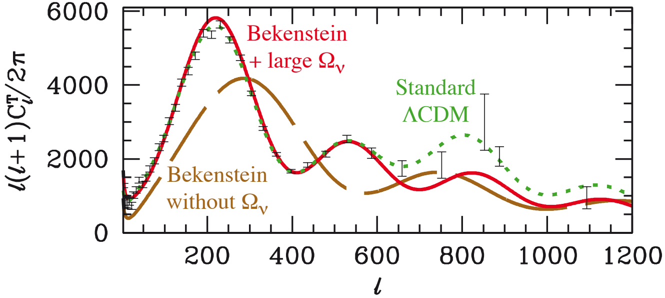

Difficulties of RAQUAL models

- Action/reaction, light deflection & CMB: $\exists$ solutions
- Complicated Lagrangians (unnatural)
- Fine tuning ($\approx$ fit rather than predictive models):
  Possible to predict different lensing and rotation curves
- Discontinuities: can be cured
- In TeVeS [Bekenstein], gravitons & scalar are slower than photons
  $\Rightarrow$ gravi-Cerenkov radiation suppresses high-energy cosmic rays
  [Moore *et al.* 2001–05]
  Solution: Accept slower photons than gravitons
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- Solar-system tests $\Rightarrow$ matter *a priori* weakly coupled to φ
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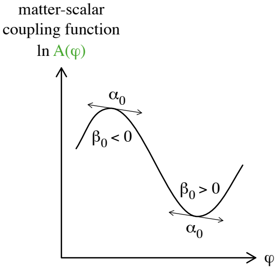
- Binary-pulsar tests $\Rightarrow$ matter must be weakly coupled to φ



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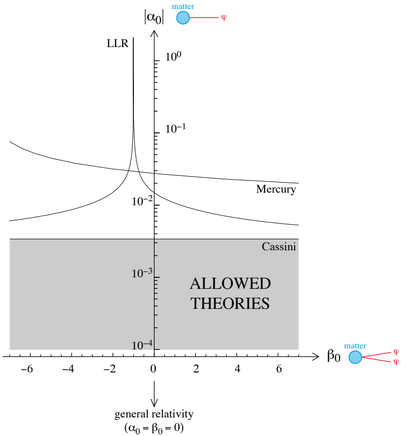

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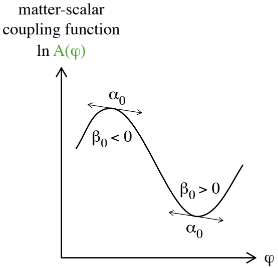

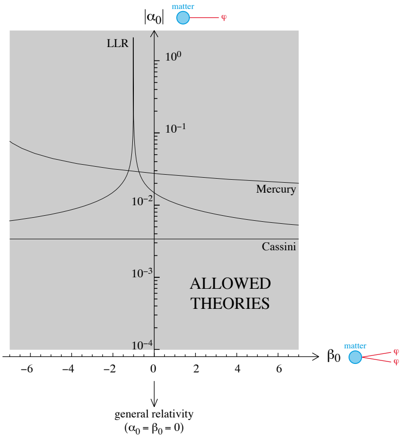

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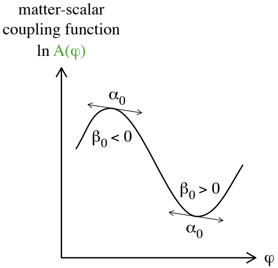

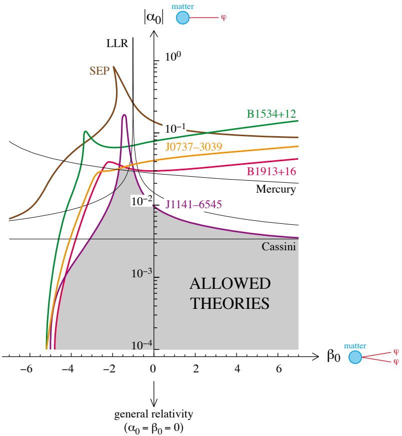

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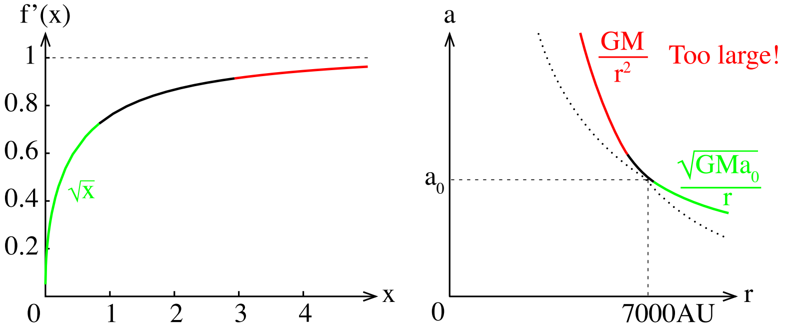

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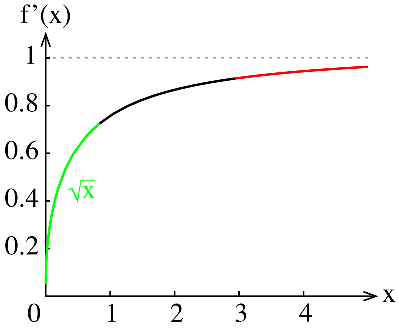

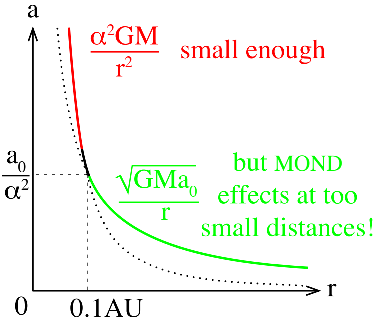

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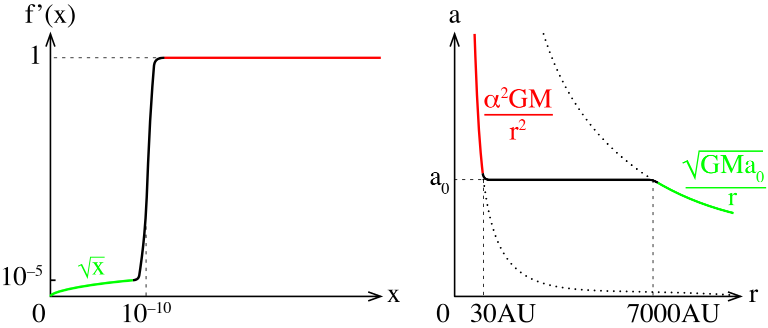

Quite unnatural! (and not far from being experimentally ruled out)

Non-minimal metric coupling

Neither "modified gravity" nor "modified inertia"

$$S = \underbrace{\frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\,R^*}_{\text{Einstein-Hilbert}} + \underbrace{S_{\text{matter}}[\text{matter}, g_{\mu\nu} \equiv g^*_{\mu\nu} + f(R^*_{\mu\nu\rho\sigma}, \ldots)]}_{\text{Metric coupling}}$$

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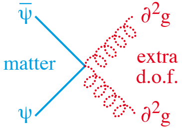

Non-minimal metric coupling: problems

But exhibits all generic problems

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- Near a spherical body, $R^*_{\mu\nu\rho\sigma}$ and its covariant derivatives give access to M and r independently
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Generic instability of higher-derivative theories

- Consider a "non-degenerate" $\mathcal{L}(q, \dot{q}, \ddot{q})$:

$$p_2 \equiv \frac{\partial \mathcal{L}}{\partial \ddot{q}} \text{ invertible} \Rightarrow \ddot{q} = f(q, \dot{q}, p_2)$$

- Ostrogradski [1850] defines

$$q_1 \equiv q \qquad p_1 \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \ddot{q}}\right)$$

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is such that $\dot{q}_i = \partial \mathcal{H}/\partial p_i$ and $\dot{p}_i = -\partial \mathcal{H}/\partial q_i$ reproduce the Euler-Lagrange equations derived from $\mathcal{L}$.

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Nonminimal metric coupling (unstable within matter)

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\,R^* \qquad \text{pure G.R. in vacuum}$$
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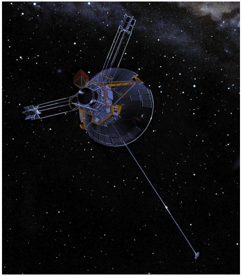
- Extra acceleration $\sim 8.5 \times 10^{-10}\,\mathrm{m.s}^{-2}$ towards the Sun between 30 and 70 AU
- Simpler problem than galaxy rotation curves ($M_{\mathrm{dark}} \propto \sqrt{M_{\mathrm{baryon}}}$), because we do not know how this acceleration is related to $M_\odot$
- $\Rightarrow$ *several* stable & well-posed solutions



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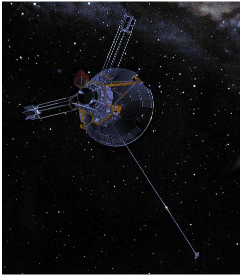

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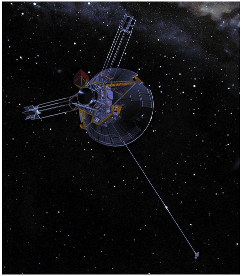

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Conclusions

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