A Minimal Model of Neutrino Flavor

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous

Mode

NII O

UV Completic

Vacuum Alignmen

Conclusions

A Minimal Model of Neutrino Flavor

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Based on arXiv:1210.1197 in collaboration with Christoph Luhn (IPPP, Durham) & Akın Wingerter (LPSC Grenoble)

Neutrino Mixing: The PMNS Matrix

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

Mode

NLO

Completic

Vacuum Alignmen

Conclusions

Neutrino mixing established⇒ neutrino mass, neutrino mixing.

lacktriangle Consider charged lepton mass matrix M_ℓ and neutrino mass matrix $M_
u$

Mass matrices diagonalized by singular value decomposition:

$$\hat{M}_{\ell} = D_L M_{\ell} D_R^{\dagger}, \quad \hat{M}_{\nu} = U_L M_{\nu} U_R^{\dagger}$$

Neutrino Mixing: The PMNS Matrix

A Minimal Model of Neutrino Flavor

Introduction

Symmetri

Previous Work

Mode

NLO

Completion

Alignment

Conclusions

▶ If neutrinos have different masses, a basis where both the matrices are diagonal need not exist.

- ► Shift to the basis in which the charged lepton mass matrix is diagonal
 - →Neutrino mixing described by Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

B. Pontecorvo, "Mesonium and antimesonium," Sov. Phys. JETP 6 (1957) 429

Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \underbrace{D_L U_L^{\dagger}}_{U_{\rm PMNS}} \quad \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass eigen states

Standard Parametrisation

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

....

NLO

UV Completion

Vacuum Alignmen

Conclusions

$$\hat{M}_{\nu} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}; \quad \Delta m_{ji}^2 = m_j^2 - m_i^2$$

► For Dirac neutrinos: 3 angles $(\theta_{12}, \theta_{23}, \theta_{13})$; 1 Dirac phase δ $(c_{23} \equiv \cos \theta_{23} \text{ etc})$

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta}s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

► For Majorana neutrinos: 2 extra Majorana phases

$$U_{\mathrm{PMNS}} = \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{smallmatrix} \right) \left(\begin{smallmatrix} c_{13} & 0 & \mathrm{e}^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -\mathrm{e}^{i\delta} s_{13} & 0 & c_{13} \end{smallmatrix} \right) \left(\begin{smallmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{smallmatrix} \right) \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{i\alpha} & 0 \\ 0 & 0 & \mathrm{e}^{i\beta} \end{smallmatrix} \right)$$

Standard Parametrisation

A Minimal Model of Neutrino Flavor

Introduction

Family Symmeti

Previous Work

Model

UV Complet

Vacuum Alignmen

Conclusions

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Standard Parametrisation

A Minimal Model of Neutrino Flavor

Introduction

Family

Previous

.....

....

NLU

Completio

Vacuum Alignmen

Conclusions

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Tribimaximal Mixing Paradigm

A Minimal Model of Neutrino Flavor

Introduction

amily

Previous Work

Mode

...

UV Completi

Vacuum Alignmen

Conclusions

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167. hep-ph/0202074

► Conjecture:
$$U_{\text{PMNS}} = U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$ightharpoonup \sin^2 \theta_{12} = 1/3$$
, $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{13} = 0$

$$\theta_{12} \approx 35.26^{\circ}, \ \theta_{23} = 45^{\circ}, \ \theta_{13} = 0^{\circ}$$

Current Experimental Status

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetri

Previous Work

Mode

NLC

Completic

Vacuum Alignment

Conclusions

M. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, "Global fit to three neutrino mixing: critical look at present precision," 1209.3023

(http://www.nu-fit.org/)

Parameter	Best Fit	3σ range	TBM
θ ₁₂ [°]	33.3 ± 0.8	31 – 36	35.26
θ ₂₃ [°]	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$	36 — 55	45
$ heta_{13}[^{\circ}]$	$8.6^{+0.44}_{-0.46}$	7.2 - 9.5	0
$\frac{\Delta m_{21}^2}{10^{-5}eV^2}$	7.5 ± 0.185	7.00 - 8.09	
$\frac{\Delta m_{31}^2}{10^{-3} eV^2} (N)$	$2.47^{+0.069}_{-0.067}$	2.27 - 2.69	
$\frac{\Delta m_{32}^2}{10^{-3} eV^2} (I)$	$-2.43^{+0.042}_{-0.065}$	-2.65 - 2.24	

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

Model

UV Completi

Vacuum Alignmen

Conclusions

- Family symmetry is a symmetry relating the quark or lepton generations.
- Discrete groups have been a preferred choice
- ▶ TBM had been seen as roadsign for discrete symmetries. "Discrete non abelian groups naturally emerge as suitable flavor symmetries. In fact the TB mixing matrix immediately suggests rotations by fixed, discrete angles."-
 - G. Altarelli and F. Feruglio, "Discrete Flavor Symmetries and Models of Neutrino Mixing," Rev. Mod. Phys. 82 (2010) 2701–2729, 1002.0211

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previou

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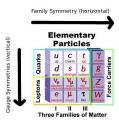
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Completion

vacuum Alignment

Conclusions

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A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

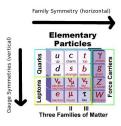
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UV

Vacuum

Conclusions

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A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previou Work

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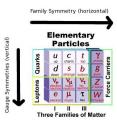
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Completio

Vacuum Alignment

Conclusions

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Previous Work

A Minimal Model of Neutrino Flavor

Introductio

Family Symmetries

Previous Work

Mode

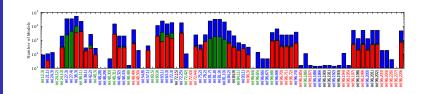
NLO

Completio

Alignment

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

- ➤ Scanned all discrete groups ≤ 100 with 3D irreps that can be scanned within a stipulated time
- ▶ Green bars number of models allowed at 3σ (2010)
- Red bars give number of TBM models
- \succ T_7 the smallest group that gave TBM
- ▶ An $A_4 \times \mathbb{Z}_3$ model with $\theta_{13} \approx 5^\circ$



Previous Work: Table of Small Groups

A Minimal Model of Neutrino Flavor

ntroductio

Family Symmetries

Previous Work

Model

NI O

Completion

Vacuum Alignment

Conclusions

The first few of the 1048 groups of order \leq 100 in the Table or	ıline
$\checkmark = U(n)$ and $\checkmark = SU(n)$ for $n = 2, 3$	

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A ₄
[1, 1]	1	X	X	Х	Х	Х
[2, 1]	C_2	X	Х	Х	X	Х
[3, 1]	C ₃	X	Х	Х	×	Х
[4, 1]	C ₄	X	Х	Х	X	Х
[4, 2]	$C_2 \times C_2$	X	Х	Х	×	Х
[5, 1]	C ₅	X	Х	Х	X	Х
[6, 1]	S_3	X	V	~	~	Х
[6, 2]	C ₆	X	Х	Х	X	Х
[7, 1]	C ₇	X	Х	Х	×	Х
[8, 1]	C ₈	X	X	Х	X	X

http://lpsc.in2p3.fr/wingerter/documents/listof100smallgroups.pdf

The Minimal Model of Neutrino Flavour

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetric

Previous Work

Model

Completio

Vacuum Alignment

Conclusions

C. Luhn, K. M. Parattu, A. Wingerter, arXiv:1210.1197

• Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \underbrace{T_7}_{\mathrm{Minimal Symmetry}} \times \mathrm{U}(1)_R$$

2 Particle content and charges (Majorana neutrinos assumed)

Field	$SU(2)_L \times U(1)_Y$	T ₇	$U(1)_R$
L	(2,-1)	3	1
e	(1, 2)	1	1
μ	(1, 2)	1′	1
τ	(1, 2)	1"	1
hu	(2, 1)	1	0
h _d	(2,-1)	1	0
φ	(1, 0)	3	0
\widetilde{arphi}	(1, 0)	3′	0

Two flavon fields φ and $\widetilde{\varphi}$.

Breaking the family symmetry

$$\langle \varphi \rangle = (v_{\varphi}, v_{\varphi}, v_{\varphi}), \quad \langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$$

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

Model

1420

Completion

Vacuum Alignmen

Conclusions

Terms that can be contracted to an invariant, have 2 leptons and mass dimension ≤ 5 (charged leptons) or 6 (neutrinos). Λ-a common cut-off above the scale of family symmetry breaking.

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} Leh_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L\mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L\tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} LLh_u h_u + y_2 \frac{\varphi}{\Lambda^2} LLh_u h_u$$

- ▶ Blue charged lepton terms, black neutrino terms
- Contract family indices (need to know Clebsch-Gordan coefficients of T₇)
- ▶ Contract $SU(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:
- Mass matrices

$$\begin{aligned} & e & \mu & \tau \\ M_{\ell^+} &= -\frac{v_{\nu}v_{\varphi}}{\sqrt{6}\Lambda} \times \frac{L_1^{(2)}}{L_2^{(2)}} \begin{pmatrix} y_{e} & 0 & 0 \\ 0 & y_{\mu} & 0 \\ 0 & 0 & y_{\tau} \end{pmatrix}, & M_{\nu} &= \frac{v_{\nu}^2}{12N^2} \times \frac{L_1^{(1)}}{L_2^{(1)}} \begin{pmatrix} L_1^{(1)} & L_2^{(1)} & L_2^{(1)} \\ \sqrt{2}y_2v_{\varphi} + 2y_1v_{\varphi} & -\frac{1}{2}\sqrt{2}y_2v_{\varphi} & -\frac{1}{2}\sqrt{2}y_2v_{\varphi} \\ -\frac{1}{2}\sqrt{2}y_2v_{\varphi} & \sqrt{2}y_2v_{\varphi} + 2y_1v_{\varphi} \end{pmatrix} \\ & L_3^{(1)} & L_3^{(1)} & -\frac{1}{2}\sqrt{2}y_2v_{\varphi} & -\frac{1}{2}\sqrt{2}y_2v_{\varphi} + 2y_1v_{\varphi} \\ & -\frac{1}{2}\sqrt{2}y_2v_{\varphi} & -\frac{1}{2}\sqrt{2}y_2v_{\varphi} + 2y_1v_{\varphi} \end{pmatrix}$$

A Minimal Model of Neutrino Flavor

Introduction

Symmetric

Previous Work

Model

Completio

Vacuum Alignment

Conclusions

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A Minimal Model of Neutrino Flavor

Introduction

Symmetric

Previous Work

Model

1420

Completio

Vacuum Alignmen

Conclusions

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A Minimal Model of Neutrino Flavor

Introductioi

Family Symmetrie

Previous Work

Model

HV

Completio

Vacuum Alignment

Conclusions

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$$\begin{split} M_{L^{0}} &= \begin{pmatrix} e & \mu & \tau \\ L_{1}^{12} & y_{e} & 0 & 0 \\ L_{3}^{12} & 0 & y_{\mu} & 0 \\ L_{3}^{12} & 0 & 0 & y_{\tau} \end{pmatrix}, \qquad M_{\nu} &= \frac{v_{e}^{2}}{12\lambda^{2}} \times L_{2}^{(1)} \begin{pmatrix} L_{1}^{(1)} & L_{2}^{(1)} & L_{3}^{(1)} \\ \sqrt{2}A + 2B & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A \\ -\frac{1}{2}\sqrt{2}A & \sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \end{pmatrix} \end{split}$$

A Minimal Model of Neutrino Flavor

Introduction

Family

Previous Work

Model

UV Completion

Vacuum Alignment

Conclusions

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$$M_{\ell^+} = -\frac{v_{\ell}v_{\overline{\nu}}}{\sqrt{6}\Lambda} \times \frac{L_1^{(2)}}{L_3^{(2)}} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_{\mu} & 0 \\ 0 & 0 & y_{\tau} \end{pmatrix}, \qquad M_{\nu} = \frac{v_{\ell}^2}{12\Lambda^2} \times \frac{L_1^{(1)}}{L_2^{(1)}} \begin{pmatrix} L_2^{(1)} & L_2^{(1)} & L_3^{(1)} \\ \sqrt{2}A + 2B & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A \\ -\frac{1}{2}\sqrt{2}A & \sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \\ -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \end{pmatrix}$$

Our T₇ Model at Leading Order

A Minimal Model of Neutrino Flavor

ntroduction

Family Symmetr

Previous Work

Model

NLC

Completi

Vacuum Alignmen

Conclusions

➤ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^{\dagger}, \quad \hat{M}_{\nu} = U_L M_{\nu} U_R^{\dagger}$$

➤ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} \, U_{\mathrm{L}}^{\dagger} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \stackrel{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form-diagonalizable i.e Mixing is independent of the neutrino masses

Mixing angles: $\theta_{12}=35.26^\circ,~\theta_{23}=45^\circ,~\theta_{13}=0^\circ$ Tribimaximal

Our T₇ Model at Leading Order

A Minimal Model of Neutrino Flavor

troductio

Family Symmetri

Previous Work

Model

NLC

Completion

Vacuum Alignmen

Conclusions

➤ Singular value decomposition

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Our T₇ Model at Leading Order

A Minimal Model of Neutrino Flavor

troductio

Family Symmetri

Previous Work

Model

NLC

Completion

Vacuum Alignmen

Conclusions

➤ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^{\dagger}, \quad \hat{M}_{\nu} = U_L M_{\nu} U_R^{\dagger}$$

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Form-diagonalizable i.e Mixing is independent of the neutrino masses.

ightharpoonup Mixing angles: $\theta_{12}=35.26^{\circ},~\theta_{23}=45^{\circ},~\theta_{13}=0^{\circ}$ Tribimaximal

Constraining the Yukawas: Mass squared differences.

A Minimal Model of Neutrino Flavor

Introduction

Symmetrie

Previous Work

Model

NII O

Completion

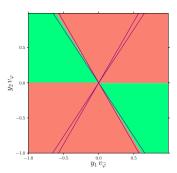
Vacuum Alignmen

Conclusions

Neutrino masses:

$$(m_1^{\nu}, m_2^{\nu}, m_3^{\nu}) = (2y_1v_{\widetilde{\varphi}} + \sqrt{6}y_2v_{\varphi}, 2y_1v_{\widetilde{\varphi}}, -2y_1v_{\widetilde{\varphi}} + \sqrt{6}y_2v_{\varphi})\frac{v_u^2}{6\Lambda^2}.$$

► Contour lines of $\Delta m_{31}^2/\Delta m_{21}^2 \equiv 30$ with normal hierarchy.



A Minimal Model of Neutrino Flavor

Introduction

Family

Previous

NLO

UV Completic

Vacuum Alignment

Conclusions

Superpotential with Next-to-Leading-Order(NLO) terms (now mass dimension ≤6 (charged leptons) or 7(neutrinos))

```
\begin{array}{c} C_{e} \ Le \ h_{d} \ \widetilde{\varphi} \ + \ C_{\mu} \ L \ \mu \ h_{d} \ \widetilde{\varphi} \ + \ C_{\tau} \ L \ \tau \ h_{d} \ \widetilde{\varphi} \ + \\ C_{1}^{e} \ Le \ h_{d} \ \varphi \ \varphi \ + \ C_{2}^{e} \ Le \ h_{d} \ \varphi \ \widetilde{\varphi} \ + \ C_{3}^{e} \ Le \ h_{d} \ \widetilde{\varphi} \ \widetilde{\varphi} \ + \\ C_{1}^{\mu} \ L \ \mu \ h_{d} \ \varphi \ \varphi \ + \ C_{2}^{\mu} \ L \ \mu \ h_{d} \ \widetilde{\varphi} \ \widetilde{\varphi} \ + \\ C_{1}^{\tau} \ L \ \tau \ h_{d} \ \varphi \ \varphi \ + \ C_{2}^{\tau} \ L \ \tau \ h_{d} \ \widetilde{\varphi} \ \widetilde{\varphi} \ + \\ C_{1}^{\tau} \ L \ L \ h_{u} \ h_{u} \ \varphi \ \varphi \ + \ C_{3}^{\tau} \ L \ \tau \ h_{d} \ \widetilde{\varphi} \ \widetilde{\varphi} \ + \\ C_{1} \ L \ L \ h_{u} \ h_{u} \ \varphi \ \varphi \ + \ C_{2} \ L \ L \ h_{u} \ h_{u} \ \widetilde{\varphi} \ \varphi \ + \\ C_{1}^{\nu} \ (L \ L)_{3} \ h_{u} \ h_{u} \ \varphi \ \varphi \ + \ C_{2}^{\nu} \ (L \ L)_{3}, \ h_{u} \ h_{u} \ \widetilde{\varphi} \ \widetilde{\varphi} \ + \\ C_{5}^{\nu} \ (L \ L)_{3} \ h_{u} \ h_{u} \ \varphi \ \widetilde{\varphi} \ + \ C_{6}^{\nu} \ (L \ L)_{2}, \ h_{u} \ h_{u} \ \mu \ \widetilde{\varphi} \ \widetilde{\varphi} \ \end{array}
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Leading-order terms, NLO terms

A Minimal Model of Neutrino Flavor

Introductio

Family Symmetrie:

Previous Work

Mode

NLO

UV Completion

Vacuum Alignmen

Conclusions

The Charged Lepton Sector, putting back the cut-off scale
 Λ

$$\begin{split} \Delta \textit{M}_{\ell} &= \frac{\textit{v}_{\textit{d}}}{\Lambda^2} \, \frac{1}{3} \bigg[\textit{C}_{1}^{e} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \textit{C}_{2}^{e} \left(\begin{smallmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{smallmatrix} \right) \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{e} \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\widetilde{\varphi}}^2 \\ & + \textit{C}_{1}^{\mu} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \textit{C}_{2}^{\mu} \left(\begin{smallmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{\mu} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) \textit{v}_{\widetilde{\varphi}}^2 \\ & + \textit{C}_{1}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \textit{C}_{2}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\widetilde{\varphi}}^2 \\ & + \textit{C}_{1}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \textit{C}_{2}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\widetilde{\varphi}}^2 \\ & + \textit{C}_{1}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \textit{C}_{2}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\widetilde{\varphi}}^2 \\ & + \textit{C}_{1}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \textit{C}_{2}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}}^2 + \textit{C}_{3}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 \\ & + \textit{C}_{2}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}}^2 + \textit{C}_{3}^{\tau} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 \right)$$

The Neutrino Sector

$$\begin{split} \Delta \textit{M}_{\nu} &= \frac{\textit{v}_{\textit{u}}^2}{\textit{\Lambda}^3} \frac{1}{9\sqrt{2}} \bigg[\textit{C}_{1}^{\nu} \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \sqrt{3} \textit{C}_{2}^{\nu} \left(\begin{smallmatrix} 2 & -\omega^2 & -\omega \\ -\omega^2 & 2\omega & -1 \\ -\omega & -1 & 2\omega^2 \end{smallmatrix} \right) \textit{v}_{\varphi}^2 + \sqrt{6} \textit{C}_{3}^{\nu} \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{smallmatrix} \right) \textit{v}_{\overline{\varphi}}^2 \\ &+ \sqrt{6} \textit{C}_{4}^{\nu} \left(\begin{smallmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{smallmatrix} \right) \textit{v}_{\overline{\varphi}}^2 + \sqrt{2} \textit{C}_{5}^{\nu} \left(\begin{smallmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} + \textit{C}_{6}^{\nu} \left(\begin{smallmatrix} 2 & -\omega & -\omega^2 \\ -\omega & 2\omega^2 & -1 \\ -\omega^2 & -1 & 2\omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} \end{split}$$

Color Code

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous Work

Mode

NLO

UV Completion

Vacuum Alignment

Conclusions

The Charged Lepton Sector, putting back the cut-off scale
 Λ

$$\begin{split} \Delta M_{\ell} &= \frac{v_d}{\Lambda^2} \, \frac{1}{3} \bigg[C_1^e \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \, v_{\varphi}^2 + C_2^e \left(\begin{smallmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{smallmatrix} \right) \, \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \\ &+ C_1^\mu \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \, v_{\varphi}^2 + C_2^\mu \left(\begin{smallmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \\ &+ C_1^\tau \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\varphi}^2 + C_2^\tau \left(\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \end{split}$$

The Neutrino Sector

$$\begin{split} \Delta \textit{M}_{\nu} &= \frac{\textit{v}_{\textit{u}}^{2}}{\textit{\Lambda}^{3}} \frac{1}{\textit{9}\sqrt{2}} \bigg[\textit{C}_{\textit{1}}^{\nu} \left(\begin{smallmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{3} \textit{C}_{\textit{2}}^{\nu} \left(\begin{smallmatrix} 2 & -\omega^{2} & -\omega \\ -\omega & 2\omega & -1 \\ -\omega & -1 & 2\omega^{2} \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{6} \textit{C}_{\textit{3}}^{\nu} \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{smallmatrix} \right) \textit{v}_{\overline{\varphi}}^{2} \\ &+ \sqrt{6} \textit{C}_{\textit{4}}^{\nu} \left(\begin{smallmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{2} \textit{C}_{\textit{5}}^{\nu} \left(\begin{smallmatrix} 1 & \omega & \omega^{2} \\ \omega & \omega^{2} & 1 & \omega \\ \omega^{2} & 1 & \omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} + \textit{C}_{\textit{6}}^{\nu} \left(\begin{smallmatrix} 2 & -\omega & -\omega^{2} \\ -\omega & 2\omega^{2} & -1 \\ -\omega^{2} & -1 & 2\omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} \end{split} \right]$$

► Color Code: Null

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous Work

Mode

NLO

UV Completion

> Vacuum Alignment

Conclusions

The Charged Lepton Sector, putting back the cut-off scale
 Λ

$$\begin{split} \Delta M_{\ell} &= \frac{v_d}{\Lambda^2} \, \frac{1}{3} \Bigg[C_1^e \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\varphi}^2 + C_2^e \left(\begin{smallmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \\ &+ C_1^\mu \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\varphi}^2 + C_2^\mu \left(\begin{smallmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \\ &+ C_1^\tau \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\varphi}^2 + C_2^\tau \left(\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \end{bmatrix} \end{split}$$

▶ The Neutrino Sector

$$\begin{split} \Delta \textit{M}_{\nu} &= \frac{\textit{v}_{\textit{u}}^{2}}{\textit{\Lambda}^{3}} \frac{1}{9\sqrt{2}} \bigg[\textit{C}_{1}^{\nu} \left(\begin{smallmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{3} \textit{C}_{2}^{\nu} \left(\begin{smallmatrix} 2 & -\omega^{2} & -\omega \\ -\omega^{2} & 2\omega & -1 \\ -\omega & -1 & 2\omega^{2} \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{6} \textit{C}_{3}^{\nu} \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{smallmatrix} \right) \textit{v}_{\overline{\varphi}}^{2} \\ &+ \sqrt{6} \textit{C}_{4}^{\nu} \left(\begin{smallmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{smallmatrix} \right) \textit{v}_{\overline{\varphi}}^{2} + \sqrt{2} \textit{C}_{5}^{\nu} \left(\begin{smallmatrix} 1 & \omega & \omega^{2} \\ \omega & \omega^{2} & 1 \\ \omega^{2} & 1 & \omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} + \textit{C}_{6}^{\nu} \left(\begin{smallmatrix} 2 & -\omega & -\omega^{2} \\ -\omega & 2\omega^{2} & -1 \\ -\omega^{2} & -1 & 2\omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} \end{split}$$

► Color Code: Null, LO structure

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous Work

Mode

NLO

UV Completion

> Vacuum Alignment

Conclusions

The Charged Lepton Sector, putting back the cut-off scale
 Λ

$$\begin{split} \Delta M_{\ell} &= \frac{v_d}{\Lambda^2} \, \frac{1}{3} \Bigg[C_1^e \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\varphi}^2 + C_2^e \left(\begin{smallmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \\ &+ C_1^\mu \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\varphi}^2 + C_2^\mu \left(\begin{smallmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \\ &+ C_1^\tau \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\varphi}^2 + C_2^\tau \left(\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \frac{v_{\varphi}}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) v_{\widetilde{\varphi}}^2 \end{bmatrix} \end{split}$$

▶ The Neutrino Sector

$$\begin{split} \Delta \textit{M}_{\nu} &= \frac{\textit{v}_{\textit{u}}^{2}}{\textit{\Lambda}^{3}} \frac{1}{9\sqrt{2}} \bigg[\textit{C}_{1}^{\nu} \left(\begin{smallmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{3} \textit{C}_{2}^{\nu} \left(\begin{smallmatrix} 2 & -\omega^{2} & -\omega \\ -\omega^{2} & 2\omega & -1 \\ -\omega & -1 & 2\omega^{2} \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{6} \textit{C}_{3}^{\nu} \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{smallmatrix} \right) \textit{v}_{\overline{\varphi}}^{2} \\ &+ \sqrt{6} \textit{C}_{4}^{\nu} \left(\begin{smallmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{smallmatrix} \right) \textit{v}_{\overline{\varphi}}^{2} + \sqrt{2} \textit{C}_{5}^{\nu} \left(\begin{smallmatrix} 1 & \omega & \omega^{2} \\ \omega & \omega^{2} & 1 \\ \omega^{2} & 1 & \omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} + \textit{C}_{6}^{\nu} \left(\begin{smallmatrix} 2 & -\omega & -\omega^{2} \\ -\omega & 2\omega^{2} & -1 \\ -\omega^{2} & -1 & 2\omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\overline{\varphi}} \end{split}$$

► Color Code: Null, LO structure

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous Work

Mode

NI O

UV Completio

Vacuum Alignment

Conclusions

The Charged Lepton Sector, putting back the cut-off scale
 Λ

$$\begin{split} \Delta \textit{M}_{\ell} &= \frac{\textit{v}_{d}}{\textit{\Lambda}^{2}} \frac{1}{3} \bigg[\textit{C}_{1}^{e} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} \textit{v}_{\varphi}^{2} + \textit{C}_{2}^{e} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ \end{pmatrix} \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{e} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} \textit{v}_{\varphi}^{2} \\ &+ \textit{C}_{1}^{\mu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} \textit{v}_{\varphi}^{2} + \textit{C}_{2}^{\mu} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ \end{pmatrix} \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{\mu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} \textit{v}_{\widetilde{\varphi}}^{2} \\ &+ \textit{C}_{1}^{\tau} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} \textit{v}_{\varphi}^{2} + \textit{C}_{2}^{\tau} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ \end{pmatrix} \frac{\textit{v}_{\varphi}}{\sqrt{3}} \textit{v}_{\widetilde{\varphi}} + \textit{C}_{3}^{\tau} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ \end{pmatrix} \textit{v}_{\widetilde{\varphi}}^{2} \end{split}$$

The Neutrino Sector

$$\begin{split} \Delta \textit{M}_{\nu} &= \frac{\textit{v}_{u}^{2}}{\text{Λ^{3}}} \frac{1}{9\sqrt{2}} \bigg[\textit{C}_{1}^{\nu} \left(\begin{smallmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{3} \textit{C}_{2}^{\nu} \left(\begin{smallmatrix} 2 & -\omega^{2} & -\omega \\ -\omega^{2} & 2\omega & -1 \\ -\omega & -1 & 2\omega^{2} \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{6} \textit{C}_{3}^{\nu} \left(\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} \\ &+ \sqrt{6} \textit{C}_{4}^{\nu} \left(\begin{smallmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{smallmatrix} \right) \textit{v}_{\varphi}^{2} + \sqrt{2} \textit{C}_{5}^{\nu} \left(\begin{smallmatrix} 1 & \omega & \omega^{2} \\ \omega^{2} & 1 & \omega \\ \omega^{2} & 1 & \omega \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\varphi}^{2} + \textit{C}_{6}^{\nu} \left(\begin{smallmatrix} 2 & -\omega & -\omega^{2} \\ -\omega & 2\omega^{2} & -1 \\ -\omega^{2} & 2\omega^{2} & -1 \end{smallmatrix} \right) \textit{v}_{\varphi} \textit{v}_{\varphi}^{2} \end{split}$$

► Color Code: Null, LO structure, Major change only in $\theta_{13} © \rightarrow \mathsf{Keep}$

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous Work

Mode

NI O

UV Completion

Vacuum Alignment

Conclusions

The Charged Lepton Sector, putting back the cut-off scale
 Λ

The Neutrino Sector

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► Color Code: Null, LO structure, Major change only in $\theta_{13} \odot \rightarrow \text{Keep}$ Bad Bad Terms! KILL!

Scan over the C's for Mixing Angles

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

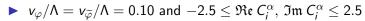
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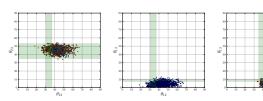
NLO

Completio

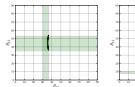
Vacuum

Conclusions

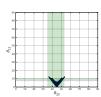












Scan over the C's for Mixing Angles

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

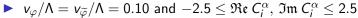
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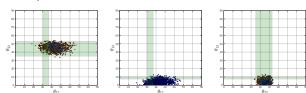
NLO

UV Completio

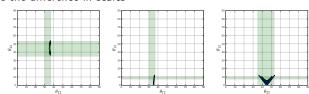
Vacuum

Conclusions





ho $v_{\varphi}/\Lambda=0.25, \ v_{\widetilde{\varphi}}/\Lambda=0.05, \ -2\leq \mathfrak{Re}\ C_5^{\nu},\ \mathfrak{Im}\ C_5^{\nu}\leq 2$ Note the difference in scales



 θ_{13} can be driven up without spoiling the other 2 angles.

A Renormalizable Realization

A Minimal

IIV

Completion

 \triangleright An SU(2), triplet Higgs \triangle . Three $SU(2)_I$ singlet fermion messenger pairs $\Theta, \Theta^c, \Sigma, \Sigma^c \text{ and } \Omega, \Omega^c$

Field	Δ	Θ	Θ^c	Σ	Σ	Ω	Ω¢
T ₇	1	3	3	3	3	3	3
U(1) _Y	2	-2	2	-2	2	1	-1
$U(1)_R$	0	1	1	2	0	1	1

Renormalizable superpotential

$$\begin{split} W_{\ell}^{\text{ren}} &\sim L h_d \Theta^c + \Theta \widetilde{\varphi} \overline{e} + \Theta \widetilde{\varphi} \mu^c + \Theta \widetilde{\varphi} \tau^c + \Theta \Theta^c (M_{\Theta} + \varphi + \widetilde{\varphi}) \\ W_{\nu}^{\text{ren}} &\sim L L \Sigma^c + \Sigma \widetilde{\varphi} \Delta + \Sigma \Sigma^c (M_{\Sigma} + \varphi + \widetilde{\varphi}) + L \varphi \Omega + L \widetilde{\varphi} \Omega \\ &+ \Omega^c \Delta L + \Omega \Omega^c (M_{\Omega} + \varphi + \widetilde{\varphi}) + L \Sigma^c \Omega^c + \Sigma^c \Omega^c \Omega^c \end{split}$$

- \triangleright For a small parameter ϵ , assume the hierarchies:
- \triangleright And we suppress all contributions other than from the $C_{\rm E}^{\nu}$

A Renormalizable Realization

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetr

Previous Work

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UV Completion

Vacuum

__ Conclusions An $SU(2)_L$ triplet Higgs Δ. Three $SU(2)_L$ singlet fermion messenger pairs Θ , Θ^c , Σ , Σ^c and Ω , Ω^c

Field	Δ	Θ	Θ^c	Σ	Σ	Ω	Ω¢
T ₇	1	3	3	3	3	3	3
U(1) _Y	2	-2	2	-2	2	1	-1
$U(1)_R$	0	1	1	2	0	1	1

Renormalizable superpotential

$$\begin{split} W_{\ell}^{\mathrm{ren}} &\sim L h_{d} \Theta^{c} + \Theta \widetilde{\varphi} \overline{e} + \Theta \widetilde{\varphi} \mu^{c} + \Theta \widetilde{\varphi} \tau^{c} + \Theta \Theta^{c} (M_{\Theta} + \varphi + \widetilde{\varphi}) \\ W_{\nu}^{\mathrm{ren}} &\sim L L \Sigma^{c} + \Sigma \widetilde{\varphi} \Delta + \Sigma \Sigma^{c} (M_{\Sigma} + \varphi + \widetilde{\varphi}) + L \varphi \Omega + L \widetilde{\varphi} \Omega \\ &+ \Omega^{c} \Delta L + \Omega \Omega^{c} (M_{\Omega} + \varphi + \widetilde{\varphi}) + L \Sigma^{c} \Omega^{c} + \Sigma^{c} \Omega^{c} \Omega^{c} \end{split}$$

- For a small parameter ϵ , assume the hierarchies: $v_{\tilde{\alpha}} \sim \epsilon^{2k+1} M$, $v_{\omega} \sim \epsilon^{k+1} M$, $M_{\Sigma} \sim \epsilon^{k} M$, $M_{\Omega} \sim M$
- And we suppress all contributions other than from the C_5^{ν}

A Minimal Model of Neutrino Flavor

Introduction

Symmeti

Previous Work

IIV

Completion

Vacuum Alignment

onclusions

An $SU(2)_L$ triplet Higgs Δ. Three $SU(2)_L$ singlet fermion messenger pairs Θ , Θ^c , Σ , Σ^c and Ω , Ω^c

Field	Δ	Θ	Θ^c	Σ	Σ	Ω	Ω^c
T ₇	1	3	3	3	3	3	3
U(1) _Y	2	-2	2	-2	2	1	-1
$U(1)_R$	0	1	1	2	0	1	1

► Renormalizable superpotential

$$\begin{split} W_{\ell}^{\mathrm{ren}} &\sim L h_d \Theta^c + \Theta \widetilde{\varphi} \overline{e} + \Theta \widetilde{\varphi} \mu^c + \Theta \widetilde{\varphi} \tau^c + \Theta \Theta^c (M_{\Theta} + \varphi + \widetilde{\varphi}) \\ W_{\nu}^{\mathrm{ren}} &\sim L L \Sigma^c + \Sigma \widetilde{\varphi} \Delta + \Sigma \Sigma^c (M_{\Sigma} + \varphi + \widetilde{\varphi}) + L \varphi \Omega + L \widetilde{\varphi} \Omega \\ &+ \Omega^c \Delta L + \Omega \Omega^c (M_{\Omega} + \varphi + \widetilde{\varphi}) + L \Sigma^c \Omega^c + \Sigma^c \Omega^c \Omega^c \end{split}$$

- For a small parameter ϵ , assume the hierarchies: $\mathbf{v}_{\tilde{o}} \sim \epsilon^{2k+1} M$, $\mathbf{v}_{\omega} \sim \epsilon^{k+1} M$, $\mathbf{M}_{\Sigma} \sim \epsilon^{k} M$, $\mathbf{M}_{O} \sim M$
- And we suppress all contributions other than from the C₅^{\(\nu\)} term!

A Minimal Model of Neutrino Flavor

Introductio

Family Symmetries

Previous Work

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UV Completi

Vacuum Alignment

Conclusions

 F-term aligment mechanism- Assume that SUSY is unbroken at the scale of family symmetry breaking.

- Introduce driving fields D_r in irrep r of T₇, uncharged under standard model symmetries.
- Flavon superpotential $W_{flav}(D_r, \varphi, \widetilde{\varphi})$

$$\Longrightarrow \text{F-term conditions } \frac{\partial W_{flav}}{\partial D_r} = 0.$$

- Not possible in our model to obtain the required flavon alignments without setting some coupling constants to zero.
- Then, what are the options?
 - Localize φ and $\bar{\varphi}$ on different branes in an extra-dimensional model. Two different driving fields is the 3 can give us the desired alignments.
 - Introduce a hidden symmetry that doesn't act on the leptons, higgses or flavons.

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous Work

Model

NLO

Completic

Vacuum Alignment

Conclusions

 F-term aligment mechanism- Assume that SUSY is unbroken at the scale of family symmetry breaking.

- Introduce driving fields D_r in irrep ${\bf r}$ of T_7 , uncharged under standard model symmetries.
- Flavon superpotential $W_{flav}(D_r, \varphi, \widetilde{\varphi})$

$$\Longrightarrow \text{F-term conditions } \frac{\partial W_{flav}}{\partial D_r} = 0.$$

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 - Introduce a hidden symmetry that doesn't act on the leptons, higgses or flavons.

A Minimal Model of Neutrino Flavor

Introductio

Family Symmetrie:

Previous Work

Mode

Completic

Alignment

 F-term alignment mechanism- Assume that SUSY is unbroken at the scale of family symmetry breaking.

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- Not possible in our model to obtain the required flavon alignments without setting some coupling constants to zero.
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 - Localize φ and $\widetilde{\varphi}$ on different branes in an extra-dimensional model. Two different driving fields in the ${\bf 3}$ can give us the desired alignments.
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A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie:

Previous Work

Mode

υv

Vacuum

Alignment

 F-term aligment mechanism- Assume that SUSY is unbroken at the scale of family symmetry breaking.

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- Not possible in our model to obtain the required flavon alignments without setting some coupling constants to zero.
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 - Localize φ and φ on different branes in an extra-dimensional model. Two different driving fields in the 3 can give us the desired alignments.
 - Introduce a hidden symmetry that doesn't act on the leptons, higgses or flavons.

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

Mode

.._0

Completio

Vacuum Alignment

Constant

- F-term aligment mechanism- Assume that SUSY is unbroken at the scale of family symmetry breaking.
 - Introduce driving fields D_r in irrep ${\bf r}$ of T_7 , uncharged under standard model symmetries.
 - Flavon superpotential $W_{\mathit{flav}}(D_r, \varphi, \widetilde{\varphi})$

$$\Longrightarrow \text{F-term conditions } \frac{\partial W_{flav}}{\partial D_r} = 0.$$

- Not possible in our model to obtain the required flavon alignments without setting some coupling constants to zero.
- ▶ Then, what are the options?
 - Localize φ and $\widetilde{\varphi}$ on different branes in an extra-dimensional model. Two different driving fields in the **3** can give us the desired alignments.
 - Introduce a hidden symmetry that doesn't act on the leptons, higgses or flavons.

Vacuum Alignment through a hidden sector

A Minimal Model of Neutrino Flavor

Introduction

Symmetrie

Previous Work

VVork

NLO

UV

Vacuum Alignment

Conclusions

Field	χ	ξ'	ψ	$\widetilde{\zeta}$	\widetilde{D}_{χ}	D_{ψ}	$O_{\chi\widetilde{\zeta}}$	$O_{\psi\widetilde{\zeta}}$
<i>T</i> ₇	3	1′	3	3	3	3	1	1
$\mathbb{Z}_{\it N}^{ m hid}$	х	x	у	z	-2x	-2 <i>y</i>	-x-z	-y-z
$\mathrm{U}(1)_R$	0	0	0	0	2	2	2	2

- ▶ Set of hidden fields χ , ξ' , ψ , $\widetilde{\zeta}$ and the corresponding driving fields.
- $ightharpoonup \mathbb{Z}_N^{\mathrm{hid}}$ is a symmetry acting only on this sector.
- ► F-term conditions of the driving fields solved to get the vevs of the hidden fields.

Vacuum Alignment through a hidden sector

A Minimal Model of Neutrino Flavor

Introduction

Symmetrie

Previous

Mode

.....

UV Completi

Vacuum Alignment

Conclusions

Field	$O_{\chi\widetilde{arphi}}$	$O_{\chi\widetilde{arphi}}'$	$\mathcal{O}_{\widetilde{\zeta}arphi}$	$O'_{\widetilde{\zeta}arphi}$
T ₇	1	1′	1	1′
$\mathbb{Z}_{N}^{\mathrm{hid}}$	-x	-x	-z	-z
$\mathrm{U}(1)_R$	2	2	2	2

- Set of driving fields coupling the hidden sector to flavons φ and $\widetilde{\varphi}$.
- ► F-term conditions of these driving fields, alongwith the vevs of the hidden fields, give the required alignments.
- ▶ Minimal \mathbb{Z}_N : N = 6 with (x, y, z) = (2, 1, 5)

Conclusions

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

Mode

NLO

Completion

Vacuum Alignment

Conclusions

A minimal model of neutrino flavour

- Pros
 - Used \mathcal{T}_7 , the second smallest group with 3-dim irreps, with no extra $\mathrm{U}(1)$ or \mathbb{Z}_N
 - Only two flavon fields
 - TBM at lowest order; UV completion possible that will bring the angles in tune with the experiment at NLO.
- Cons
 - UV completion has been constructed for our purpose rather than being built up from an underlying priniciple.
 - Hierarchies of vevs and masses need to be tuned for controlling higher order corrections
 - Vev alignment mechanism is rather complicated

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous

Model

UV Completic

Vacuum Alignmen

Conclusions

THANK YOU!

Backup

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous

Work

ivioue

UV Completic

Vacuum Alignment

Conclusions

Backup Slides

A Minimal Model of Neutrino Flavor

Introduction

Symmet

Previous Work

ivioae

NLO

UV Completion

Vacuum Alignment

Conclusions

- One can argue that there is no pattern and no symmetry!

 A. de Gouvea and H. Murayama, "Neutrino Mixing Anarchy: Alive and Kicking," 1204.1249
- One can create models that give mixings close to the experimental values at lowest order.

R. d. A. Toorop, F. Feruglio, and C. Hagedorn, "Discrete Flavour Symmetries in Light of T2K," Phys.Lett. B703 (2011) 447–451, 1107.3486

Produce TBM at lowest order. Use NLO corrections to drive θ_{13} up, taking care not to disturb the other angles too much.

A Minimal Model of Neutrino Flavor

Introduction

Symmet

Previous Work

VVOIR

iviouc

UV

Completio

Vacuum Alignment

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A Minimal Model of Neutrino Flavor

Introduction

Symmet

Previous Work

Mode

NLO

JV Completio

Vacuum Alignment

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A Minimal Model of Neutrino Flavor

Introduction

Symmet

Previous Work

Mode

MI O

U**V** Completic

Vacuum Alignment

Conclusions

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A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous

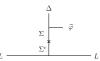
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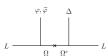
UV

Vacuum

Conclusions

Diagrams contributing to the leading order superpotential.





▶ Two diagrams contributing to the NLO superpotential.





▶ For a small parameter ϵ , assume the hierarchies:

$$v_{\widetilde{\varphi}} \sim \epsilon^{2k+1} M \,, \quad v_{\varphi} \sim \epsilon^{k+1} M \,, \quad M_{\Sigma} \sim \epsilon^k M \,, \quad M_{\Omega} \sim M \,,$$

And we suppress all contributions other than from the C₅^{\(\nu\)} term!

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous

VVOFK

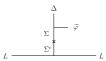
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UV Completic

Vacuum Alignment

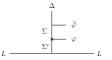
Conclusions

Diagrams contributing to the leading order superpotential.





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$$v_{\widetilde{\varphi}} \sim \epsilon^{2k+1} M$$
, $v_{\varphi} \sim \epsilon^{k+1} M$, $M_{\Sigma} \sim \epsilon^{k} M$, $M_{\Omega} \sim M$,

And we suppress all contributions other than from the C₅^t term!

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetrie

Previous

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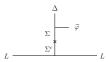
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UV Completion

Vacuum Alignmen

Conclusions

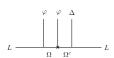
Diagrams contributing to the leading order superpotential.





▶ Two diagrams contributing to the NLO superpotential.





 \blacktriangleright For a small parameter ϵ , assume the hierarchies:

$$v_{\widetilde{\varphi}} \sim \epsilon^{2k+1} M$$
, $v_{\varphi} \sim \epsilon^{k+1} M$, $M_{\Sigma} \sim \epsilon^{k} M$, $M_{\Omega} \sim M$,

And we suppress all contributions other than from the C₅^{\(\nu\)} term!

A Minimal Model of Neutrino Flavor

Introduction

Family

Previou

Work

Mode

NLO

Completio

Vacuum Alignment

Conclusions

Arthur Cayley (1821-1895) is the first to systematically construct groups; in 1854, he determined all groups of order 4 and $6\ldots$



The Small Groups library All groups (423,164,062) of order \leq 2000 except 1024 Hans Ulrich Besche, Bettina Eick and Eamonn O'Brien

A Minimal Model of Neutrino Flavor

1012.2842

ntroduction

Family Symmetries

Previous Work

Model

NLO

Completio

Vacuum Alignment

Conclusions

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4
[1,1]	1	X				X
[2, 1]	\mathbb{Z}_2	X				X
[3, 1]	\mathbb{Z}_3	X				X
[4, 1]	\mathbb{Z}_4	Х				X
[4, 2]	$\mathbb{Z}_2 imes \mathbb{Z}_2$					X
[5, 1]	\mathbb{Z}_5	Х				X
[6, 1]	S_3		V	~	~	X
[6, 2]	\mathbb{Z}_6	Х				X
[7, 1]	\mathbb{Z}_7	X				X
[8, 1]	\mathbb{Z}_{8}	Х				Х

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011,

A Minimal Neutrino Flavor

1012.2842

Conclusions

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4
[8, 2]	$\mathbb{Z}_4 imes \mathbb{Z}_2$	X				X
[8, 3]	D_4	X	V	~	~	X
[8, 4]	Q_8	X	~	~	✓	X
[8, 5]	$\mathbb{Z}_2 imes \mathbb{Z}_2 imes \mathbb{Z}_2$	X				X
[9, 1]	\mathbb{Z}_9	X				X
[9, 2]	$\mathbb{Z}_3 imes \mathbb{Z}_3$	X				X
[10, 1]	D_5	X	~	~	✓	X
[10, 2]	\mathbb{Z}_{10}	X				X
[11, 1]	\mathbb{Z}_{11}	X				X
[12, 1]	7/3 × 10 7/4	X	V	V	V	X

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011,

A Minimal Model of Neutrino Flavor

1012.2842

Introduction

Family Symmetries

Previous Work

Model

NLO

Completion

Vacuum Alignment

Conclusions

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GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4
[12, 2]	\mathbb{Z}_{12}	X				X
[12, 3]	A_4	~	V	Х		~
[12, 4]	D_6	X	~	~	~	X
[12, 5]	$\mathbb{Z}_6 imes \mathbb{Z}_2$	X				X
[13, 1]	\mathbb{Z}_{13}	X				X
[14, 1]	D ₇	X	~	~	~	X
[14, 2]	\mathbb{Z}_{14}	X				X
[15, 1]	\mathbb{Z}_{15}	X				X
[16,1]	\mathbb{Z}_{16}	X				X
[16, 2]	$\mathbb{Z}_{A} \times \mathbb{Z}_{A}$	X				X

A Minimal Model of Neutrino Flavor

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011,

A Minimal Model of Neutrino Flavor

1012.2842

ntroduction

Family Symmetries

Previous Work

Model

NLC

Completio

Vacuum Alignment

Conclusions

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A ₄
	Стоир		(3)	0(2)	○(2)×○(1)	714
[16, 3]	$(\mathbb{Z}_4 imes \mathbb{Z}_2) times_{arphi} \mathbb{Z}_2$	X	~	X	~	X
[16, 4]	$\mathbb{Z}_4 \rtimes_{\varphi} \mathbb{Z}_4$	X	~	X	✓	X
[16,5]	$\mathbb{Z}_8 imes \mathbb{Z}_2$	X				X
[16, 6]	$\mathbb{Z}_8 times_{arphi} \mathbb{Z}_2$	X	~	~	~	X
[16, 7]	D_8	X	~	~	~	X
[16, 8]	QD ₈	X	V	~	~	X
[16, 9]	Q_{16}	X	V	~	✓	X
[16,10]	$\mathbb{Z}_4 imes \mathbb{Z}_2 imes \mathbb{Z}_2$	X				X
[16,11]	$\mathbb{Z}_2 imes D_4$	X	~	X	✓	X
[16, 12]	$\mathbb{Z}_2 imes Q_8$	Х	V	Х	/	Х

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011,

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

Model

NLC

Completion

Vacuum Alignment

Conclusions

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011, 1012.2842

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4
[16, 13]	$(\mathbb{Z}_4 imes \mathbb{Z}_2) times_{arphi} \mathbb{Z}_2$	X	~	~	V	X
[16, 14]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	X				X
[17, 1]	\mathbb{Z}_{17}	X				X
[18, 1]	D_9	X	V	~	V	X
[18, 2]	\mathbb{Z}_{18}	X				X
[18, 3]	$\mathbb{Z}_3 imes \mathcal{S}_3$	X	V	~	~	X
[18, 4]	$(\mathbb{Z}_3 imes \mathbb{Z}_3) times_{arphi} \mathbb{Z}_2$	X	X	X	X	X
[18, 5]	$\mathbb{Z}_6 imes \mathbb{Z}_3$	X				X
[19, 1]	\mathbb{Z}_{19}	X				X
[20, 1]	715 X 10 711	X	V	V	V	X

A Minimal Model of Neutrino Flavor

1012.2842

ntroduction

Family Symmetries

Previous Work

Model

NLO

Completio

Vacuum Alignment

Conclusions

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4
[20, 2]	\mathbb{Z}_{20}	X				Х
[20, 3]	$\mathbb{Z}_5 \rtimes_{\varphi} \mathbb{Z}_4$	X	X	Х	X	X
[20, 4]	D_{10}	X	~	~	✓	X
[20, 5]	$\mathbb{Z}_{10} imes \mathbb{Z}_2$	X				X
[21, 1]	$\mathbb{Z}_7 \rtimes_{\varphi} \mathbb{Z}_3$	~	V	Х	X	
[21, 2]	\mathbb{Z}_{21}	X				X
[22, 1]	D_{11}	X	~	~	✓	X
[22, 2]	\mathbb{Z}_{22}	X				X
[23, 1]	\mathbb{Z}_{23}	X				Х
[24, 1]	Z3 X10 Z8	X	~	~	✓	X

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011,

A Minimal Model of Neutrino Flavor

ntroduction

Family Symmetries

Previous Work

Model

NI O

Completio

Vacuum Alignment

Conclusions

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys.Rev. D84 (2011) 013011, 1012.2842

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4	
[24, 2]	\mathbb{Z}_{24}	Х				X	
[24, 3]	SL(2,3)	~	/	/	~	Х	
[24, 4]	$\mathbb{Z}_3 times_{arphi} Q_8$	Х	>	/	✓	X	
[24, 5]	$\mathbb{Z}_4 imes S_3$	X	/	~	~	X	
[24, 6]	D_{12}	Х	/	/	✓	X	
[24, 7]	$\mathbb{Z}_2 imes (\mathbb{Z}_3 times_{arphi}\mathbb{Z}_4)$	Х	~	X	✓	X	
[24, 8]	$(\mathbb{Z}_6 imes\mathbb{Z}_2) times_arphi\mathbb{Z}_2$	X	/	✓	~	X	
[24, 9]	$\mathbb{Z}_{12} imes \mathbb{Z}_2$	X				X	
[24, 10]	$\mathbb{Z}_3 imes D_4$	X	/	✓	~	X	
[24, 11]	$\mathbb{Z}_2 imes Q_2$	X	V	V	V	X	

A Minimal Model of Neutrino Flavor

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troductior

Family Symmetries

Previous Work

Model

NLC

Completion

Vacuum Alignment

Conclusions

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4
[24, 12]	S ₄	~	/	X	X	~
[24, 13]	$\mathbb{Z}_2 \times A_4$	~	~	Х	X	~
[24, 14]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathcal{S}_3$	X	~	Х	~	X
[24, 15]	$\mathbb{Z}_6 imes \mathbb{Z}_2 imes \mathbb{Z}_2$	X				X
[25, 1]	\mathbb{Z}_{25}	X				X
[25, 2]	$\mathbb{Z}_5 imes \mathbb{Z}_5$	X				X
[26, 1]	D_{13}	X	~	~	✓	X
[26, 2]	\mathbb{Z}_{26}	X				X
[27, 1]	\mathbb{Z}_{27}	X				X
[27, 2]	$\mathbb{Z}_0 \times \mathbb{Z}_3$	Х				Х

A Minimal Model of Neutrino Flavor

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011,

A Minimal Model of Neutrino Flavor

troduction

Family Symmetries

Previous Work

Model

NLC

Completio

Vacuum Alignment

Conclusions

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys. Rev. D84 (2011) 013011, 1012.2842

GAP ID	Group	3	U(3)	U(2)	U(2)×U(1)	A_4	
[27, 3]	$(\mathbb{Z}_3 imes \mathbb{Z}_3) times_{arphi} \mathbb{Z}_3$	~	~	Х	X	Х	
[27, 4]	$\mathbb{Z}_9 \rtimes_{\varphi} \mathbb{Z}_3$	~	~	X	X	X	
[27, 5]	$\mathbb{Z}_3 imes \mathbb{Z}_3 imes \mathbb{Z}_3$	X				X	
[28, 1]	$\mathbb{Z}_7 times_{arphi} \mathbb{Z}_4$	X	~	~	✓	X	
[28, 2]	\mathbb{Z}_{28}	X				X	
[28, 3]	D_{14}	X	V	~	~	X	
[28, 4]	$\mathbb{Z}_{14} imes \mathbb{Z}_2$	X				X	
[29, 1]	\mathbb{Z}_{29}	X				X	
[30, 1]	$\mathbb{Z}_5 imes S_3$	X	~	~	~	X	
[30, 2]	$\mathbb{Z}_3 imes D_5$	X	V	1	V	Х	

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

Mode

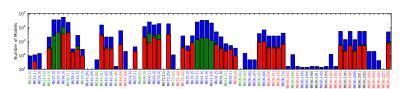
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Completio

Vacuum Alignment

Conclusions

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," Phys.Rev. D84 (2011) 013011, 1012.2842



- ▶ 50% of groups have tribimaximal models
- ▶ Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- ▶ Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$
- ► A₄ has nice geometric interpretation, but what does that mean? That humans like to think in terms of geometry?
- ▶ There is probably no special connection between A_4 and TBM!

A Minimal Model of Neutrino Flavor

Introduction

Family

Previous

Work

Mode

NLO

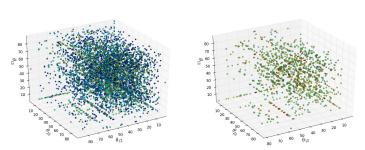
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Vacuum Alignmen

Conclusions

Group is $A_4 \times \mathbb{Z}_3$

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842



(a) The 5528 bins that are $\geq 1.$ (b) The 1287 bins that are $\geq 1000.$

A Minimal Model of Neutrino Flavor

ntroduction

Family

Previous

Work

ivioue

NLO

Completio

Vacuum Alignmen

Conclusions

• Fix the family symmetry (we considered 76 groups)

$$SU(2)_L \times U(1)_Y \times A_4 \times \mathbb{Z}_3 \times U(1)_R$$

2 Constant particle content; scan over all representations!

Field	$SU(2)_L \times U(1)_Y$	$A_4 imes \mathbb{Z}_3$	$U(1)_R$
1		3′	1
	(2,-1)		1
е	(1, 2)	1′	1
μ	(1, 2)	1 ⁽⁸⁾	1
τ	(1, 2)	1(5)	1
hu	(2, 1)	1	0
h _d	(2,-1)	1	0
φτ	(1, 0)	3	0
φ_{S}	(1, 0)	3′	0
ξ	(1, 0)	3′	0

8 Partial scan over vevs

$$\langle \varphi_T \rangle = (0/1, 0/1, 0/1), \quad \langle \varphi_S \rangle = (0/1, 0/1, 0/1), \quad \langle \xi \rangle = (0/1, 0/1, 0/1)$$

A Minimal Model of Neutrino Flavor

Introductio

Family Symmetri

Previous Work

Mode

NLU

Completio

Vacuum Alignment

Conclusions

- > We consider 2 models equivalent, if their Lagrangians are the same after contracting the family indices, but before the vevs are substituted
- ➤ In this sense, we have 39,900 inequivalent models/Lagrangians
- > 22,932 models have non-singular charged lepton and neutrino mass matrices:

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \qquad \hat{M}_\nu = U_L M_\nu U_R^\dagger, \qquad U_{\rm PMNS} \equiv D_{\rm L} U_{\rm L}^\dagger$$

- > 4,481 consistent w/experiment at 3σ level (19.5%) (obsolete!)
- > 4,233 are tribimaximal (18.5%)
- > Probably largest set of viable neutrino models ever constructed!

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

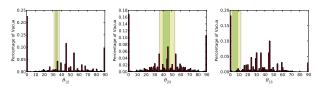
Mode

IVLU

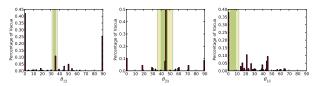
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Vacuum Alignmen

Conclusions



(c) Number of models that give θ_{ij} with no constraints on the other 2 angles. Each histogram has 15992118 entries.



(d) Number of models that give θ_{ij} with the other 2 angles restricted to their 3σ interval. The histograms have 838289, 148886 and 225844 entries, respectively.

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous Work

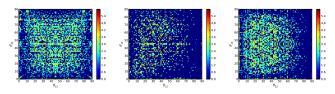
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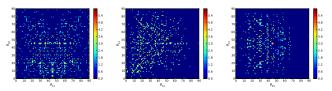
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Vacuum Alignmen

Conclusions



(e) Number of models that give θ_{ij} and θ_{mn} with no constraint on the remaining angle. Each histogram has 15992118 entries.



(f) Number of models that give θ_{ij} and θ_{mn} with the remaining angle restricted to its 3σ interval. The histograms have 2941000, 3675600 and 1057170 entries, respectively.

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous

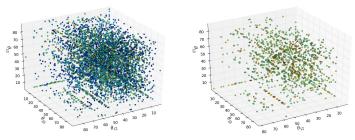
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Vacuum

Conclusions



(g) The 5528 bins that are ≥ 1 . (h) The 1287 bins that are ≥ 1000 .

A Minimal Model of Neutrino Flavor

Introduction

Family Symmetries

Previous

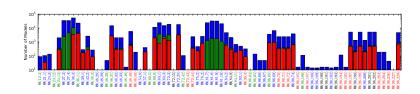
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Vacuum Alignment

Conclusions



- ► Histogram bars: All models, 3σ , TBM
- ▶ 38 groups (50%) have tribimaximal models
- ▶ Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- ▶ Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$. Special?
- ▶ Group names: $\mathfrak{g} \subset U(3)$, $\mathfrak{g} \supset A_4$, $A_4 \subset \mathfrak{g} \subset U(3)$