A Minimal Model of Neutrino Flavor

Krishna Mohan Parattu

The Inter-University Centre for Astronomy and Astrophysics, Pune, India

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Based on arXiv:1210.1197 in collaboration with
Christoph Luhn (IPPP, Durham) & Akın Wingerter (LPSC Grenoble)

Neutrino Mixing: The PMNS Matrix

- Neutrino mixing established
  $\Rightarrow$ neutrino mass, neutrino mixing.
- Consider charged lepton mass matrix M_ℓ and neutrino mass matrix M_ν
- Mass matrices diagonalized by singular value decomposition:

$$\hat{M}_\ell = D_L M_\ell D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

Neutrino Mixing: The PMNS Matrix

- If neutrinos have different masses, a basis where both the matrices are diagonal need not exist.
- Shift to the basis in which the charged lepton mass matrix is diagonal
  $\rightarrow$Neutrino mixing described by
  Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

B. Pontecorvo, "Mesonium and antimesonium," *Sov. Phys. JETP* **6** (1957) 429

Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \underbrace{D_L U_L^\dagger}_{U_{\mathrm{PMNS}}} \underbrace{\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}}_{\text{Mass eigen states}}$$

Standard Parametrisation

- $\hat{M}_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}; \quad \Delta m^2_{ji} = m^2_j - m^2_i$
- For Dirac neutrinos: 3 angles (θ_{12}, θ_{23}, θ_{13}); 1 Dirac phase δ ($c_{23} \equiv \cos\theta_{23}$ etc)

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & \text{-}s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta}s_{13} \\ 0 & 1 & 0 \\ \text{-}e^{i\delta}s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ \text{-}s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ \text{-}c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & \text{-}s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- For Majorana neutrinos: 2 extra Majorana phases

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Tribimaximal Mixing Paradigm

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167, `hep-ph/0202074`

- Conjecture: $U_{\mathrm{PMNS}} = U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$
- $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{13} = 0$
- $\theta_{12} \approx 35.26^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$

Current Experimental Status

M. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, "Global fit to three neutrino mixing: critical look at present precision," 1209.3023

(http://www.nu-fit.org/)

Parameter	Best Fit	3σ range	TBM
$\theta_{12}[^\circ]$	33.3 ± 0.8	$31 - 36$	35.26
$\theta_{23}[^\circ]$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$	$36 - 55$	45
$\theta_{13}[^\circ]$	$8.6^{+0.44}_{-0.46}$	$7.2 - 9.5$	0
$\frac{\Delta m^2_{21}}{10^{-5} eV^2}$	7.5 ± 0.185	$7.00 - 8.09$	
$\frac{\Delta m^2_{31}}{10^{-3} eV^2}$ (N)	$2.47^{+0.069}_{-0.067}$	$2.27 - 2.69$	
$\frac{\Delta m^2_{32}}{10^{-3} eV^2}$ (I)	$-2.43^{+0.042}_{-0.065}$	$-2.65 - 2.24$	

Family Symmetries

- Family symmetry is a symmetry relating the quark or lepton generations.
- Discrete groups have been a preferred choice
- TBM had been seen as roadsign for discrete symmetries.

"Discrete non abelian groups naturally emerge as suitable flavor symmetries. In fact the TB mixing matrix immediately suggests rotations by fixed, discrete angles."-

G. Altarelli and F. Feruglio, "Discrete Flavor Symmetries and Models of Neutrino Mixing," *Rev. Mod. Phys.* **82** (2010) 2701–2729, 1002.0211

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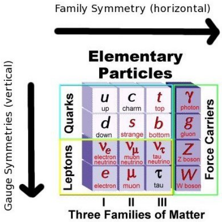

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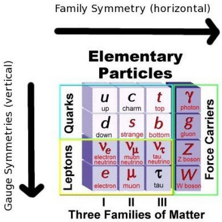

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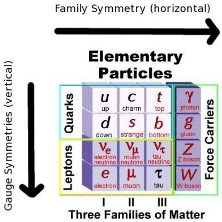

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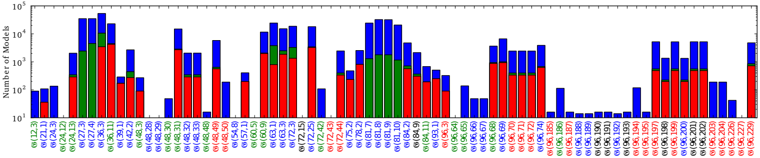
Previous Work

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, `1012.2842`

- Scanned all discrete groups$\leq$ 100 with 3D irreps that can be scanned within a stipulated time
- Green bars number of models allowed at 3σ (2010)
- Red bars give number of TBM models
- T_7 the smallest group that gave TBM
- An $A_4 \times \mathbb{Z}_3$ model with $\theta_{13} \approx 5^\circ$



Previous Work: Table of Small Groups

The first few of the 1048 groups of order ≤ 100 in the Table online
✔ $= \mathrm{U}(n)$ and ✔ $= \mathrm{SU}(n)$ for $n = 2, 3$

GAP ID	Group	**3**	$\mathrm{U}(3)$	$\mathrm{U}(2)$	$\mathrm{U}(2)\times\mathrm{U}(1)$	A_4
$[1, 1]$	1	✗	✗	✗	✗	✗
$[2, 1]$	C_2	✗	✗	✗	✗	✗
$[3, 1]$	C_3	✗	✗	✗	✗	✗
$[4, 1]$	C_4	✗	✗	✗	✗	✗
$[4, 2]$	$C_2 \times C_2$	✗	✗	✗	✗	✗
$[5, 1]$	C_5	✗	✗	✗	✗	✗
$[6, 1]$	S_3	✗	✔	✔	✔	✗
$[6, 2]$	C_6	✗	✗	✗	✗	✗
$[7, 1]$	C_7	✗	✗	✗	✗	✗
$[8, 1]$	C_8	✗	✗	✗	✗	✗

http://lpsc.in2p3.fr/wingerter/documents/listof100smallgroups.pdf

The Minimal Model of Neutrino Flavour

C. Luhn, K. M. Parattu, A. Wingerter, arXiv:1210.1197

❶ Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \underbrace{T_7}_{\text{Minimal Symmetry}} \times \mathrm{U}(1)_R$$

❷ Particle content and charges (Majorana neutrinos assumed)

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	T_7	$\mathrm{U}(1)_R$
L	$(\mathbf{2},-1)$	$\mathbf{3}$	1
e	$(\mathbf{1},\ 2)$	$\mathbf{1}$	1
μ	$(\mathbf{1},\ 2)$	$\mathbf{1}'$	1
τ	$(\mathbf{1},\ 2)$	$\mathbf{1}''$	1
h_u	$(\mathbf{2},\ 1)$	$\mathbf{1}$	0
h_d	$(\mathbf{2},-1)$	$\mathbf{1}$	0
φ	$(\mathbf{1},\ 0)$	$\mathbf{3}$	0
$\widetilde{\varphi}$	$(\mathbf{1},\ 0)$	$\mathbf{3}'$	0

Two flavon fields φ and $\widetilde{\varphi}$.

❸ Breaking the family symmetry

$$\langle\varphi\rangle = (v_\varphi, v_\varphi, v_\varphi), \quad \langle\widetilde{\varphi}\rangle = (v_{\widetilde{\varphi}}, 0, 0)$$

The T_7 Model at Leading Order

- Terms that can be contracted to an invariant, have 2 leptons and mass dimension ≤ 5 (charged leptons) or 6 (neutrinos). Λ-a common cut-off above the scale of family symmetry breaking.

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

- Blue charged lepton terms, black neutrino terms
- Contract family indices (need to know Clebsch-Gordan coefficients of T_7)
- Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:
- Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\widetilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \!\!\begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \!\!\begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \sqrt{2} y_2 v_\varphi + 2 y_1 v_{\widetilde{\varphi}} & -\frac{1}{2}\sqrt{2} y_2 v_\varphi & -\frac{1}{2}\sqrt{2} y_2 v_\varphi \\ -\frac{1}{2}\sqrt{2} y_2 v_\varphi & \sqrt{2} y_2 v_\varphi & -\frac{1}{2}\sqrt{2} y_2 v_\varphi + 2 y_1 v_{\widetilde{\varphi}} \\ -\frac{1}{2}\sqrt{2} y_2 v_\varphi & -\frac{1}{2}\sqrt{2} y_2 v_\varphi + 2 y_1 v_{\widetilde{\varphi}} & \sqrt{2} y_2 v_\varphi \end{pmatrix} \end{array}$$

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Our T_7 Model at Leading Order

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form-diagonalizable i.e Mixing is independent of the neutrino masses.

➢ Mixing angles: $\theta_{12} = 35.26°$, $\theta_{23} = 45°$, $\theta_{13} = 0°$ Tribimaximal ✔

Our T_7 Model at Leading Order

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

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Our T_7 Model at Leading Order

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\rm PMNS} = D_{\rm L} U_{\rm L}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form-diagonalizable i.e Mixing is independent of the neutrino masses.

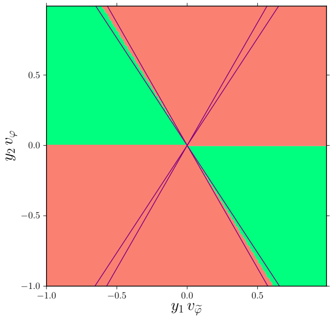
➢ Mixing angles: $\theta_{12} = 35.26^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ Tribimaximal ✔

Constraining the Yukawas: Mass squared differences.

- Neutrino masses:

$$(m_1^\nu, m_2^\nu, m_3^\nu) \;=\; (2y_1 v_{\widetilde{\varphi}} + \sqrt{6} y_2 v_\varphi \,,\; 2y_1 v_{\widetilde{\varphi}} \,,\; -2y_1 v_{\widetilde{\varphi}} + \sqrt{6} y_2 v_\varphi) \frac{v_u^2}{6\Lambda^2} \;.$$

- Contour lines of $\Delta m^2_{31}/\Delta m^2_{21} \equiv 30$ with normal hierarchy.



Effective NLO Corrections

- Superpotential with Next-to-Leading-Order(NLO) terms (now mass dimension $\leq$6 (charged leptons) or 7(neutrinos))

$$
\begin{aligned}
C_e\ L\,e\,h_d\,\widetilde{\varphi} + C_\mu\ L\,\mu\,h_d\,\widetilde{\varphi} + C_\tau\ L\,\tau\,h_d\,\widetilde{\varphi} + \\
C_1^e\,L\,e\,h_d\,\varphi\,\varphi + C_2^e\,L\,e\,h_d\,\varphi\,\widetilde{\varphi} + C_3^e\,L\,e\,h_d\,\widetilde{\varphi}\,\widetilde{\varphi} + \\
C_1^\mu\,L\,\mu\,h_d\,\varphi\,\varphi + C_2^\mu\,L\,\mu\,h_d\,\varphi\,\widetilde{\varphi} + C_3^\mu\,L\,\mu\,h_d\,\widetilde{\varphi}\,\widetilde{\varphi} + \\
C_1^\tau\,L\,\tau\,h_d\,\varphi\,\varphi + C_2^\tau\,L\,\tau\,h_d\,\varphi\,\widetilde{\varphi} + C_3^\tau\,L\,\tau\,h_d\,\widetilde{\varphi}\,\widetilde{\varphi} + \\
C_1\ L\,L\,h_u\,h_u\,\varphi + C_2\ L\,L\,h_u\,h_u\,\widetilde{\varphi} + \\
C_1^\nu\ (L\,L)_{\mathbf{3}}\,h_u\,h_u\,\varphi\,\varphi + C_2^\nu\ (L\,L)_{\mathbf{3'}}\,h_u\,h_u\,\varphi\,\varphi + \\
C_3^\nu\ (L\,L)_{\mathbf{3}}\,h_u\,h_u\,\widetilde{\varphi}\,\widetilde{\varphi} + C_4^\nu\ (L\,L)_{\mathbf{3'}}\,h_u\,h_u\,\widetilde{\varphi}\,\widetilde{\varphi} + \\
C_5^\nu\ (L\,L)_{\mathbf{3}}\,h_u\,h_u\,\varphi\,\widetilde{\varphi} + C_6^\nu\ (L\,L)_{\mathbf{3'}}\,h_u\,h_u\,\varphi\,\widetilde{\varphi}
\end{aligned}
$$

- Leading-order terms, NLO terms

Effective NLO Corrections

- The Charged Lepton Sector, putting back the cut-off scale Λ

$$\Delta M_\ell = \frac{v_d}{\Lambda^2}\frac{1}{3}\Bigg[C_1^e \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + C_2^e \begin{pmatrix} 1&0&0\\1&0&0\\1&0&0\end{pmatrix}\frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \begin{pmatrix} 1&0&0\\0&0&0\\0&0&0\end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\mu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + C_2^\mu \begin{pmatrix} 0&1&0\\0&1&0\\0&1&0\end{pmatrix}\frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \begin{pmatrix} 0&0&0\\0&1&0\\0&0&0\end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + C_2^\tau \begin{pmatrix} 0&0&1\\0&0&1\\0&0&1\end{pmatrix}\frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&1\end{pmatrix} v_{\widetilde{\varphi}}^2\Bigg]$$

- The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{\Lambda^3}\frac{1}{9\sqrt{2}}\Bigg[C_1^\nu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + \sqrt{3}C_2^\nu \begin{pmatrix} 2&-\omega^2&-\omega\\-\omega^2&2\omega&-1\\-\omega&-1&2\omega^2\end{pmatrix} v_\varphi^2 + \sqrt{6}C_3^\nu \begin{pmatrix} 1&0&0\\0&0&1\\0&1&0\end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ \sqrt{6}C_4^\nu \begin{pmatrix} 2&0&0\\0&0&-1\\0&-1&0\end{pmatrix} v_{\widetilde{\varphi}}^2 + \sqrt{2}C_5^\nu \begin{pmatrix} 1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega\end{pmatrix} v_\varphi v_{\widetilde{\varphi}} + C_6^\nu \begin{pmatrix} 2&-\omega&-\omega^2\\-\omega&2\omega^2&-1\\-\omega^2&-1&2\omega\end{pmatrix} v_\varphi v_{\widetilde{\varphi}}\Bigg]$$

- Color Code:

Effective NLO Corrections

- The Charged Lepton Sector, putting back the cut-off scale Λ

$$\Delta M_\ell = \frac{v_d}{\Lambda^2}\frac{1}{3}\Bigg[C_1^e \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^e \begin{pmatrix} 1&0&0\\1&0&0\\1&0&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \begin{pmatrix} 1&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\mu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\mu \begin{pmatrix} 0&1&0\\0&1&0\\0&1&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\tau \begin{pmatrix} 0&0&1\\0&0&1\\0&0&1 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&1 \end{pmatrix} v_{\widetilde{\varphi}}^2 \Bigg]$$

- The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{\Lambda^3}\frac{1}{9\sqrt{2}}\Bigg[C_1^\nu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + \sqrt{3} C_2^\nu \begin{pmatrix} 2&-\omega^2&-\omega\\-\omega^2&2\omega&-1\\-\omega&-1&2\omega^2 \end{pmatrix} v_\varphi^2 + \sqrt{6} C_3^\nu \begin{pmatrix} 1&0&0\\0&0&1\\0&1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ \sqrt{6} C_4^\nu \begin{pmatrix} 2&0&0\\0&0&-1\\0&-1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2 + \sqrt{2} C_5^\nu \begin{pmatrix} 1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}} + C_6^\nu \begin{pmatrix} 2&-\omega&-\omega^2\\-\omega&2\omega^2&-1\\-\omega^2&-1&2\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}} \Bigg]$$

- Color Code: Null

Effective NLO Corrections

- The Charged Lepton Sector, putting back the cut-off scale Λ

$$\Delta M_\ell = \frac{v_d}{\Lambda^2}\frac{1}{3}\Bigg[C_1^e \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^e \begin{pmatrix} 1&0&0\\1&0&0\\1&0&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \begin{pmatrix} 1&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+C_1^\mu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\mu \begin{pmatrix} 0&1&0\\0&1&0\\0&1&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+C_1^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\tau \begin{pmatrix} 0&0&1\\0&0&1\\0&0&1 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&1 \end{pmatrix} v_{\widetilde{\varphi}}^2\Bigg]$$

- The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{\Lambda^3}\frac{1}{9\sqrt{2}}\Bigg[C_1^\nu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + \sqrt{3}C_2^\nu \begin{pmatrix} 2&-\omega^2&-\omega\\-\omega^2&2\omega&-1\\-\omega&-1&2\omega^2 \end{pmatrix} v_\varphi^2 + \sqrt{6}C_3^\nu \begin{pmatrix} 1&0&0\\0&0&1\\0&1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+\sqrt{6}C_4^\nu \begin{pmatrix} 2&0&0\\0&0&-1\\0&-1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2 + \sqrt{2}C_5^\nu \begin{pmatrix} 1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}} + C_6^\nu \begin{pmatrix} 2&-\omega&-\omega^2\\-\omega&2\omega^2&-1\\-\omega^2&-1&2\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}}\Bigg]$$

- Color Code: Null, LO structure

Effective NLO Corrections

- The Charged Lepton Sector, putting back the cut-off scale Λ

$$\Delta M_\ell = \frac{v_d}{\Lambda^2}\frac{1}{3}\Big[C_1^e \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^e \begin{pmatrix} 1&0&0\\1&0&0\\1&0&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \begin{pmatrix} 1&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\mu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\mu \begin{pmatrix} 0&1&0\\0&1&0\\0&1&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\tau \begin{pmatrix} 0&0&1\\0&0&1\\0&0&1 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&1 \end{pmatrix} v_{\widetilde{\varphi}}^2 \Big]$$

- The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{\Lambda^3}\frac{1}{9\sqrt{2}}\Big[C_1^\nu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + \sqrt{3} C_2^\nu \begin{pmatrix} 2&-\omega^2&-\omega\\-\omega^2&2\omega&-1\\-\omega&-1&2\omega^2 \end{pmatrix} v_\varphi^2 + \sqrt{6} C_3^\nu \begin{pmatrix} 1&0&0\\0&0&1\\0&1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ \sqrt{6} C_4^\nu \begin{pmatrix} 2&0&0\\0&0&-1\\0&-1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2 + \sqrt{2} C_5^\nu \begin{pmatrix} 1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}} + C_6^\nu \begin{pmatrix} 2&-\omega&-\omega^2\\-\omega&2\omega^2&-1\\-\omega^2&-1&2\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}} \Big]$$

- Color Code: Null, LO structure

Effective NLO Corrections

- The Charged Lepton Sector, putting back the cut-off scale Λ

$$\Delta M_\ell = \frac{v_d}{\Lambda^2}\frac{1}{3}\Big[C_1^e \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^e \begin{pmatrix} 1&0&0\\1&0&0\\1&0&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^e \begin{pmatrix} 1&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\mu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\mu \begin{pmatrix} 0&1&0\\0&1&0\\0&1&0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\mu \begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ C_1^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + C_2^\tau \begin{pmatrix} 0&0&1\\0&0&1\\0&0&1 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\widetilde{\varphi}} + C_3^\tau \begin{pmatrix} 0&0&0\\0&0&0\\0&0&1 \end{pmatrix} v_{\widetilde{\varphi}}^2 \Big]$$

- The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{\Lambda^3}\frac{1}{9\sqrt{2}}\Big[C_1^\nu \begin{pmatrix} 0&0&0\\0&0&0\\0&0&0 \end{pmatrix} v_\varphi^2 + \sqrt{3} C_2^\nu \begin{pmatrix} 2&-\omega^2&-\omega\\-\omega^2&2\omega&-1\\-\omega&-1&2\omega^2 \end{pmatrix} v_\varphi^2 + \sqrt{6} C_3^\nu \begin{pmatrix} 1&0&0\\0&0&1\\0&1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2$$
$$+ \sqrt{6} C_4^\nu \begin{pmatrix} 2&0&0\\0&0&-1\\0&-1&0 \end{pmatrix} v_{\widetilde{\varphi}}^2 + \sqrt{2} C_5^\nu \begin{pmatrix} 1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}} + C_6^\nu \begin{pmatrix} 2&-\omega&-\omega^2\\-\omega&2\omega^2&-1\\-\omega^2&-1&2\omega \end{pmatrix} v_\varphi v_{\widetilde{\varphi}} \Big]$$

- Color Code: Null, LO structure , Major change only in θ_{13} ☺→ Keep

Effective NLO Corrections

- The Charged Lepton Sector, putting back the cut-off scale Λ

$$\Delta M_\ell = \frac{v_d}{\Lambda^2}\frac{1}{3}\left[C_1^e \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + C_2^e \begin{pmatrix}1&0&0\\1&0&0\\1&0&0\end{pmatrix}\frac{v_\varphi}{\sqrt{3}} v_{\tilde{\varphi}} + C_3^e \begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix} v_{\tilde{\varphi}}^2 \right.$$
$$+ C_1^\mu \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + C_2^\mu \begin{pmatrix}0&1&0\\0&1&0\\0&1&0\end{pmatrix}\frac{v_\varphi}{\sqrt{3}} v_{\tilde{\varphi}} + C_3^\mu \begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix} v_{\tilde{\varphi}}^2$$
$$\left. + C_1^\tau \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + C_2^\tau \begin{pmatrix}0&0&1\\0&0&1\\0&0&1\end{pmatrix}\frac{v_\varphi}{\sqrt{3}} v_{\tilde{\varphi}} + C_3^\tau \begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} v_{\tilde{\varphi}}^2 \right]$$

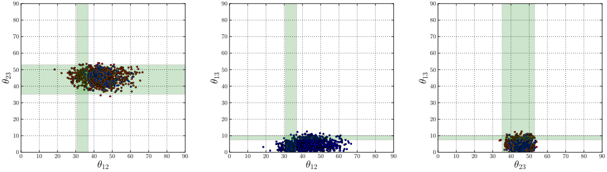
- The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{\Lambda^3}\frac{1}{9\sqrt{2}}\left[C_1^\nu \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} v_\varphi^2 + \sqrt{3}C_2^\nu \begin{pmatrix}2&-\omega^2&-\omega\\-\omega^2&2\omega&-1\\-\omega&-1&2\omega^2\end{pmatrix} v_\varphi^2 + \sqrt{6}C_3^\nu \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} v_{\tilde{\varphi}}^2 \right.$$
$$\left. + \sqrt{6}C_4^\nu \begin{pmatrix}2&0&0\\0&0&-1\\0&-1&0\end{pmatrix} v_{\tilde{\varphi}}^2 + \sqrt{2}C_5^\nu \begin{pmatrix}1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega\end{pmatrix} v_\varphi v_{\tilde{\varphi}} + C_6^\nu \begin{pmatrix}2&-\omega&-\omega^2\\-\omega&2\omega^2&-1\\-\omega^2&-1&2\omega\end{pmatrix} v_\varphi v_{\tilde{\varphi}}\right]$$

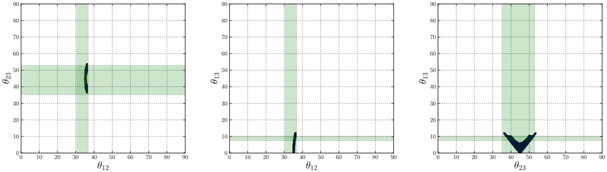
- Color Code: Null, LO structure ,
  Major change only in θ_{13} ☺ $\rightarrow$ Keep
  Bad Bad Terms! **KILL!**

Scan over the C's for Mixing Angles

- $v_\varphi/\Lambda = v_{\widetilde{\varphi}}/\Lambda = 0.10$ and $-2.5 \leq \mathfrak{Re}\, C_i^\alpha,\ \mathfrak{Im}\, C_i^\alpha \leq 2.5$

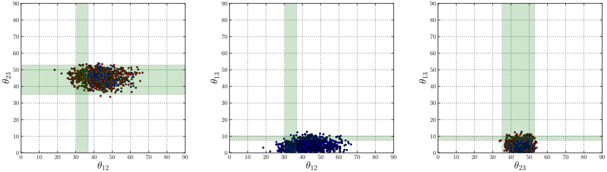


- $v_\varphi/\Lambda = 0.25,\ \ v_{\widetilde{\varphi}}/\Lambda = 0.05,\ \ -2 \leq \mathfrak{Re}\, C_5^\nu,\ \mathfrak{Im}\, C_5^\nu \leq 2$
  Note the difference in scales

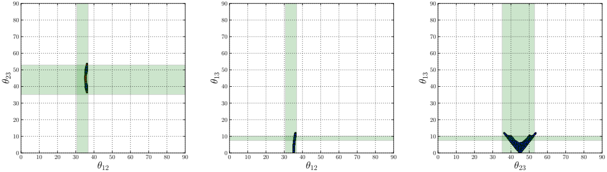


Scan over the C's for Mixing Angles

- $v_\varphi/\Lambda = v_{\widetilde{\varphi}}/\Lambda = 0.10$ and $-2.5 \le \mathfrak{Re}\, C_i^\alpha,\ \mathfrak{Im}\, C_i^\alpha \le 2.5$



- $v_\varphi/\Lambda = 0.25, \quad v_{\widetilde{\varphi}}/\Lambda = 0.05, \quad -2 \le \mathfrak{Re}\, C_5^\nu,\ \mathfrak{Im}\, C_5^\nu \le 2$
  Note the difference in scales



θ_{13} can be driven up without spoiling the other 2 angles.

A Renormalizable Realization

- An $\mathrm{SU}(2)_L$ triplet Higgs Δ.
  Three $\mathrm{SU}(2)_L$ singlet fermion messenger pairs
  Θ, Θ^c, Σ, Σ^c and Ω, Ω^c

Field	Δ	Θ	Θ^c	Σ	Σ^c	Ω	Ω^c
T_7	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{U}(1)_Y$	2	-2	2	-2	2	1	-1
$\mathrm{U}(1)_R$	0	1	1	2	0	1	1

- Renormalizable superpotential

$$W_\ell^{\mathrm{ren}} \sim L h_d \Theta^c + \Theta\widetilde{\varphi}\bar{e} + \Theta\widetilde{\varphi}\mu^c + \Theta\widetilde{\varphi}\tau^c + \Theta\Theta^c(M_\Theta + \varphi + \widetilde{\varphi})$$

$$W_\nu^{\mathrm{ren}} \sim LL\Sigma^c + \Sigma\widetilde{\varphi}\Delta + \Sigma\Sigma^c(M_\Sigma + \varphi + \widetilde{\varphi}) + L\varphi\Omega + L\widetilde{\varphi}\Omega$$
$$+ \Omega^c\Delta L + \Omega\Omega^c(M_\Omega + \varphi + \widetilde{\varphi}) + L\Sigma^c\Omega^c + \Sigma^c\Omega^c\Omega^c$$

- For a small parameter ϵ, assume the hierarchies:
  $v_{\widetilde{\varphi}} \sim \epsilon^{2k+1}M, \quad v_\varphi \sim \epsilon^{k+1}M, \quad M_\Sigma \sim \epsilon^k M, \quad M_\Omega \sim M$
- And we suppress all contributions other than from the C_5^ν term!

A Renormalizable Realization

- An $\mathrm{SU}(2)_L$ triplet Higgs Δ.
  Three $\mathrm{SU}(2)_L$ singlet fermion messenger pairs Θ, Θ^c, Σ, Σ^c and Ω, Ω^c

Field	Δ	Θ	Θ^c	Σ	Σ^c	Ω	Ω^c
T_7	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{U}(1)_Y$	2	-2	2	-2	2	1	-1
$\mathrm{U}(1)_R$	0	1	1	2	0	1	1

- Renormalizable superpotential

$$W_\ell^{\mathrm{ren}} \sim L h_d \Theta^c + \Theta \widetilde{\varphi} \bar{e} + \Theta \widetilde{\varphi} \mu^c + \Theta \widetilde{\varphi} \tau^c + \Theta \Theta^c (M_\Theta + \varphi + \widetilde{\varphi})$$

$$W_\nu^{\mathrm{ren}} \sim L L \Sigma^c + \Sigma \widetilde{\varphi} \Delta + \Sigma \Sigma^c (M_\Sigma + \varphi + \widetilde{\varphi}) + L \varphi \Omega + L \widetilde{\varphi} \Omega$$
$$+ \Omega^c \Delta L + \Omega \Omega^c (M_\Omega + \varphi + \widetilde{\varphi}) + L \Sigma^c \Omega^c + \Sigma^c \Omega^c \Omega^c$$

- For a small parameter ϵ, assume the hierarchies:
  $v_{\widetilde{\varphi}} \sim \epsilon^{2k+1} M, \quad v_\varphi \sim \epsilon^{k+1} M, \quad M_\Sigma \sim \epsilon^k M, \quad M_\Omega \sim M$
- And we suppress all contributions other than from the C_5^ν term!

A Renormalizable Realization

- An $\mathrm{SU}(2)_L$ triplet Higgs Δ.
  Three $\mathrm{SU}(2)_L$ singlet fermion messenger pairs
  Θ, Θ^c, Σ, Σ^c and Ω, Ω^c

Field	Δ	Θ	Θ^c	Σ	Σ^c	Ω	Ω^c
T_7	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{U}(1)_Y$	2	-2	2	-2	2	1	-1
$\mathrm{U}(1)_R$	0	1	1	2	0	1	1

- Renormalizable superpotential

$$W_\ell^{\mathrm{ren}} \sim L h_d \Theta^c + \Theta\widetilde{\varphi}\bar{e} + \Theta\widetilde{\varphi}\mu^c + \Theta\widetilde{\varphi}\tau^c + \Theta\Theta^c(M_\Theta + \varphi + \widetilde{\varphi})$$

$$\begin{aligned} W_\nu^{\mathrm{ren}} \sim\ & LL\Sigma^c + \Sigma\widetilde{\varphi}\Delta + \Sigma\Sigma^c(M_\Sigma + \varphi + \widetilde{\varphi}) + L\varphi\Omega + L\widetilde{\varphi}\Omega \\ & + \Omega^c\Delta L + \Omega\Omega^c(M_\Omega + \varphi + \widetilde{\varphi}) + L\Sigma^c\Omega^c + \Sigma^c\Omega^c\Omega^c \end{aligned}$$

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Vacuum Alignment

- F-term aligment mechanism- Assume that SUSY is unbroken at the scale of family symmetry breaking.
  - Introduce driving fields D_r in irrep **r** of T_7, uncharged under standard model symmetries.
  - Flavon superpotential $W_{flav}(D_r, \varphi, \widetilde{\varphi})$

$$\Longrightarrow \text{F-term conditions } \frac{\partial W_{flav}}{\partial D_r} = 0.$$

- Not possible in our model to obtain the required flavon alignments without setting some coupling constants to zero.
- Then, what are the options?
  - Localize φ and $\widetilde{\varphi}$ on different branes in an extra-dimensional model. Two different driving fields in the 3 can give us the desired alignments.
  - Introduce a hidden symmetry that doesn't act on the leptons, higgses or flavons.

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Vacuum Alignment through a hidden sector

Field	χ	ξ'	ψ	$\widetilde{\zeta}$	$\widetilde{D}_\chi$	D_ψ	$O_{\chi\widetilde{\zeta}}$	$O_{\psi\widetilde{\zeta}}$
T_7	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbb{Z}_N^{\mathrm{hid}}$	x	x	y	z	$-2x$	$-2y$	$-x-z$	$-y-z$
$\mathrm{U}(1)_R$	0	0	0	0	2	2	2	2

- Set of hidden fields χ, ξ', ψ, $\widetilde{\zeta}$ and the corresponding driving fields.
- $\mathbb{Z}_N^{\mathrm{hid}}$ is a symmetry acting only on this sector.
- F-term conditions of the driving fields solved to get the vevs of the hidden fields.

Vacuum Alignment through a hidden sector

Field	$O_{\chi\widetilde{\varphi}}$	$O'_{\chi\widetilde{\varphi}}$	$O_{\widetilde{\zeta}\varphi}$	$O'_{\widetilde{\zeta}\varphi}$
T_7	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$
$\mathbb{Z}_N^{\rm hid}$	$-x$	$-x$	$-z$	$-z$
$\mathrm{U}(1)_R$	2	2	2	2

- Set of driving fields coupling the hidden sector to flavons φ and $\widetilde{\varphi}$.
- F-term conditions of these driving fields, alongwith the vevs of the hidden fields, give the required alignments.
- Minimal $\mathbb{Z}_N$: $N = 6$ with $(x, y, z) = (2, 1, 5)$

Conclusions

A minimal model of neutrino flavour

- Pros
  - Used T_7, the second smallest group with 3-dim irreps, with no extra $\mathrm{U}(1)$ or $\mathbb{Z}_N$
  - Only two flavon fields
  - TBM at lowest order; UV completion possible that will bring the angles in tune with the experiment at NLO.
- Cons
  - UV completion has been constructed for our purpose rather than being built up from an underlying priniciple.
  - Hierarchies of vevs and masses need to be tuned for controlling higher order corrections
  - Vev alignment mechanism is rather complicated

THANK YOU!

Backup

Backup

Slides

Discrete Flavour Symmetries and a non-zero θ_{13}

- One can argue that there is no pattern and no symmetry!
  A. de Gouvea and H. Murayama, "Neutrino Mixing Anarchy: Alive and Kicking," 1204.1249

- One can create models that give mixings close to the experimental values at lowest order.
  R. d. A. Toorop, F. Feruglio, and C. Hagedorn, "Discrete Flavour Symmetries in Light of T2K," *Phys.Lett.* **B703** (2011) 447–451, 1107.3486

- Produce TBM at lowest order. Use NLO corrections to drive θ_{13} up, taking care not to disturb the other angles too much.

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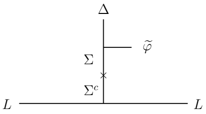
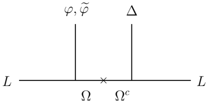
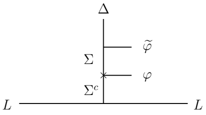
A Renormalizable Realization

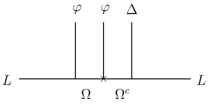
- Diagrams contributing to the leading order superpotential.





- Two diagrams contributing to the NLO superpotential.





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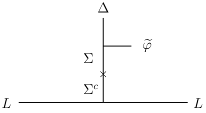

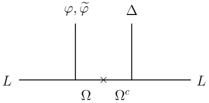

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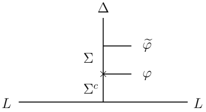

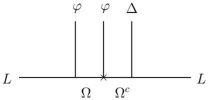

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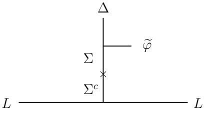

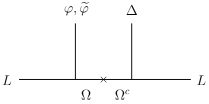

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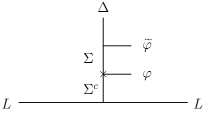

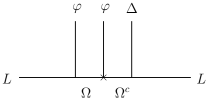

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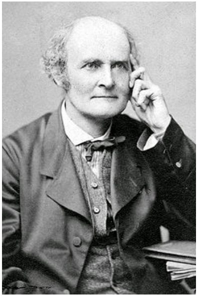
Which Flavor Symmetry?

Arthur Cayley (1821-1895) is the first to systematically construct groups; in 1854, he determined all groups of order 4 and 6 . . .



The Small Groups library
All groups (423,164,062) of order $\leq$ 2000 except 1024
Hans Ulrich Besche, Bettina Eick and Eamonn O'Brien

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
$[1,1]$	1	✗	---	---	---	✗
$[2,1]$	$\mathbb{Z}_2$	✗	---	---	---	✗
$[3,1]$	$\mathbb{Z}_3$	✗	---	---	---	✗
$[4,1]$	$\mathbb{Z}_4$	✗	---	---	---	✗
$[4,2]$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
$[5,1]$	$\mathbb{Z}_5$	✗	---	---	---	✗
$[6,1]$	S_3	✗	✔	✔	✔	✗
$[6,2]$	$\mathbb{Z}_6$	✗	---	---	---	✗
$[7,1]$	$\mathbb{Z}_7$	✗	---	---	---	✗
$[8,1]$	$\mathbb{Z}_8$	✗	---	---	---	✗

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[8, 2]	$\mathbb{Z}_4 \times \mathbb{Z}_2$	✗	---	---	---	✗
[8, 3]	D_4	✗	✔	✔	✔	✗
[8, 4]	Q_8	✗	✔	✔	✔	✗
[8, 5]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[9, 1]	$\mathbb{Z}_9$	✗	---	---	---	✗
[9, 2]	$\mathbb{Z}_3 \times \mathbb{Z}_3$	✗	---	---	---	✗
[10, 1]	D_5	✗	✔	✔	✔	✗
[10, 2]	$\mathbb{Z}_{10}$	✗	---	---	---	✗
[11, 1]	$\mathbb{Z}_{11}$	✗	---	---	---	✗
[12, 1]	$\mathbb{Z}_3 \rtimes \mathbb{Z}_4$	✗	✔	✔	✔	✗

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GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[12, 2]	$\mathbb{Z}_{12}$	✗	---	---	---	✗
[12, 3]	A_4	✔	✔	✗	✗	✔
[12, 4]	D_6	✗	✔	✔	✔	✗
[12, 5]	$\mathbb{Z}_6 \times \mathbb{Z}_2$	✗	---	---	---	✗
[13, 1]	$\mathbb{Z}_{13}$	✗	---	---	---	✗
[14, 1]	D_7	✗	✔	✔	✔	✗
[14, 2]	$\mathbb{Z}_{14}$	✗	---	---	---	✗
[15, 1]	$\mathbb{Z}_{15}$	✗	---	---	---	✗
[16, 1]	$\mathbb{Z}_{16}$	✗	---	---	---	✗
[16, 2]	$\mathbb{Z}_4 \times \mathbb{Z}_4$	✗	---	---	---	✗

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GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[16, 3]	$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✗	✔	✗
[16, 4]	$\mathbb{Z}_4 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✗	✔	✗
[16, 5]	$\mathbb{Z}_8 \times \mathbb{Z}_2$	✗	---	---	---	✗
[16, 6]	$\mathbb{Z}_8 \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[16, 7]	D_8	✗	✔	✔	✔	✗
[16, 8]	QD_8	✗	✔	✔	✔	✗
[16, 9]	Q_{16}	✗	✔	✔	✔	✗
[16, 10]	$\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[16, 11]	$\mathbb{Z}_2 \times D_4$	✗	✔	✗	✔	✗
[16, 12]	$\mathbb{Z}_2 \times Q_8$	✗	✔	✗	✔	✗

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[16, 13]	$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[16, 14]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[17, 1]	$\mathbb{Z}_{17}$	✗	---	---	---	✗
[18, 1]	D_9	✗	✔	✔	✔	✗
[18, 2]	$\mathbb{Z}_{18}$	✗	---	---	---	✗
[18, 3]	$\mathbb{Z}_3 \times S_3$	✗	✔	✔	✔	✗
[18, 4]	$(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes_\varphi \mathbb{Z}_2$	✗	✗	✗	✗	✗
[18, 5]	$\mathbb{Z}_6 \times \mathbb{Z}_3$	✗	---	---	---	✗
[19, 1]	$\mathbb{Z}_{19}$	✗	---	---	---	✗
[20, 1]	$\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

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$[20, 2]$	$\mathbb{Z}_{20}$	✗	---	---	---	✗
$[20, 3]$	$\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$	✗	✗	✗	✗	✗
$[20, 4]$	D_{10}	✗	✔	✔	✔	✗
$[20, 5]$	$\mathbb{Z}_{10} \times \mathbb{Z}_2$	✗	---	---	---	✗
$[21, 1]$	$\mathbb{Z}_7 \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
$[21, 2]$	$\mathbb{Z}_{21}$	✗	---	---	---	✗
$[22, 1]$	D_{11}	✗	✔	✔	✔	✗
$[22, 2]$	$\mathbb{Z}_{22}$	✗	---	---	---	✗
$[23, 1]$	$\mathbb{Z}_{23}$	✗	---	---	---	✗
$[24, 1]$	$\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_8$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

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GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[24, 2]	$\mathbb{Z}_{24}$	✗	---	---	---	✗
[24, 3]	SL(2, 3)	✔	✔	✔	✔	✗
[24, 4]	$\mathbb{Z}_3 \rtimes_\varphi Q_8$	✗	✔	✔	✔	✗
[24, 5]	$\mathbb{Z}_4 \times S_3$	✗	✔	✔	✔	✗
[24, 6]	D_{12}	✗	✔	✔	✔	✗
[24, 7]	$\mathbb{Z}_2 \times (\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_4)$	✗	✔	✗	✔	✗
[24, 8]	$(\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[24, 9]	$\mathbb{Z}_{12} \times \mathbb{Z}_2$	✗	---	---	---	✗
[24, 10]	$\mathbb{Z}_3 \times D_4$	✗	✔	✔	✔	✗
[24, 11]	$\mathbb{Z}_3 \times Q_8$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
$[24, 12]$	S_4	✔	✔	✗	✗	✔
$[24, 13]$	$\mathbb{Z}_2 \times A_4$	✔	✔	✗	✗	✔
$[24, 14]$	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times S_3$	✗	✔	✗	✔	✗
$[24, 15]$	$\mathbb{Z}_6 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
$[25, 1]$	$\mathbb{Z}_{25}$	✗	---	---	---	✗
$[25, 2]$	$\mathbb{Z}_5 \times \mathbb{Z}_5$	✗	---	---	---	✗
$[26, 1]$	D_{13}	✗	✔	✔	✔	✗
$[26, 2]$	$\mathbb{Z}_{26}$	✗	---	---	---	✗
$[27, 1]$	$\mathbb{Z}_{27}$	✗	---	---	---	✗
$[27, 2]$	$\mathbb{Z}_9 \times \mathbb{Z}_3$	✗	---	---	---	✗

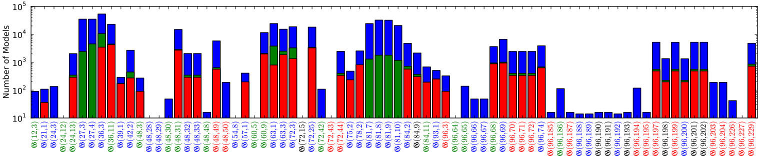
Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, `1012.2842`

`GAP ID`	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
$[27,3]$	$(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
$[27,4]$	$\mathbb{Z}_9 \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
$[27,5]$	$\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3$	✗	---	---	---	✗
$[28,1]$	$\mathbb{Z}_7 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗
$[28,2]$	$\mathbb{Z}_{28}$	✗	---	---	---	✗
$[28,3]$	D_{14}	✗	✔	✔	✔	✗
$[28,4]$	$\mathbb{Z}_{14} \times \mathbb{Z}_2$	✗	---	---	---	✗
$[29,1]$	$\mathbb{Z}_{29}$	✗	---	---	---	✗
$[30,1]$	$\mathbb{Z}_5 \times S_3$	✗	✔	✔	✔	✗
$[30,2]$	$\mathbb{Z}_3 \times D_5$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

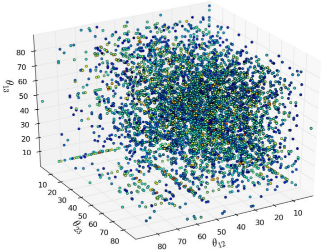


- 50% of groups have tribimaximal models
- Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$
- A_4 has nice geometric interpretation, but what does that mean? That humans like to think in terms of geometry?
- There is probably no special connection between A_4 and TBM!

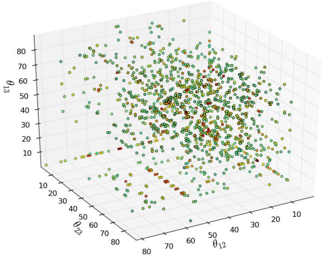
How did we generate this set of models?

Group is $A_4 \times \mathbb{Z}_3$

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842



(a) The 5528 bins that are ≥ 1.



(b) The 1287 bins that are ≥ 1000.

How did we generate this set of models?

❶ Fix the family symmetry (we considered 76 groups)

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times A_4 \times \mathbb{Z}_3 \times \mathrm{U}(1)_R$$

❷ Constant particle content; scan over all representations!

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	$A_4 \times \mathbb{Z}_3$	$\mathrm{U}(1)_R$
L	$(\mathbf{2},\text{-}1)$	$\mathbf{3'}$	1
e	$(\mathbf{1},\ 2)$	$\mathbf{1'}$	1
μ	$(\mathbf{1},\ 2)$	$\mathbf{1^{(8)}}$	1
τ	$(\mathbf{1},\ 2)$	$\mathbf{1^{(5)}}$	1
h_u	$(\mathbf{2},\ 1)$	$\mathbf{1}$	0
h_d	$(\mathbf{2},\text{-}1)$	$\mathbf{1}$	0
φ_T	$(\mathbf{1},\ 0)$	$\mathbf{3}$	0
φ_S	$(\mathbf{1},\ 0)$	$\mathbf{3'}$	0
ξ	$(\mathbf{1},\ 0)$	$\mathbf{3'}$	0

❸ Partial scan over vevs

$$\langle\varphi_T\rangle = (0/1, 0/1, 0/1), \quad \langle\varphi_S\rangle = (0/1, 0/1, 0/1), \quad \langle\xi\rangle = (0/1, 0/1, 0/1$$

How did we generate this set of models?

➢ We consider 2 models equivalent, if their Lagrangians are the same after contracting the family indices, but before the vevs are substituted

➢ In this sense, we have 39,900 inequivalent models/Lagrangians

➢ 22,932 models have non-singular charged lepton and neutrino mass matrices:

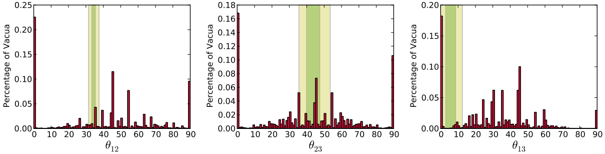
$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \qquad \hat{M}_\nu = U_L M_\nu U_R^\dagger, \qquad U_{\rm PMNS} \equiv D_{\rm L} U_{\rm L}^\dagger$$

➢ 4,481 consistent w/experiment at 3σ level (19.5%) (obsolete!)

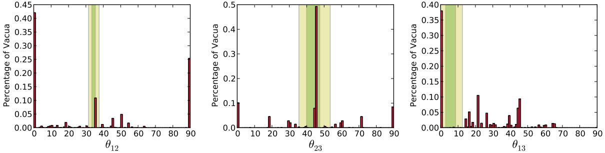
➢ 4,233 are tribimaximal (18.5%)

➢ Probably largest set of viable neutrino models ever constructed!

How did we generate this set of models?

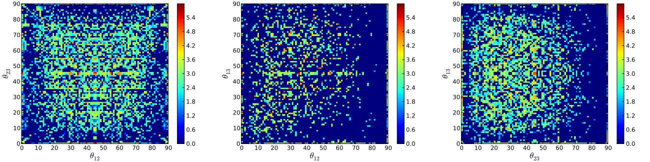


(c) Number of models that give θ_{ij} with no constraints on the other 2 angles. Each histogram has 15992118 entries.

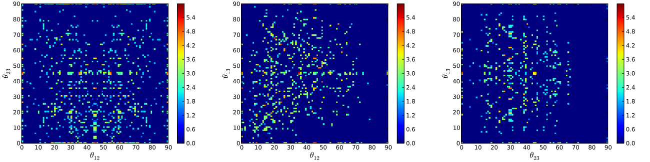


(d) Number of models that give θ_{ij} with the other 2 angles restricted to their 3σ interval. The histograms have 838289, 148886 and 225844 entries, respectively.

How did we generate this set of models?

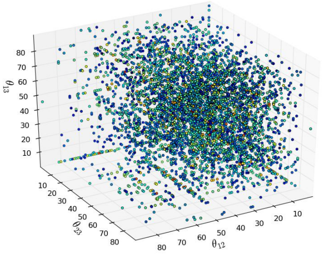


(e) Number of models that give θ_{ij} and θ_{mn} with no constraint on the remaining angle. Each histogram has 15992118 entries.

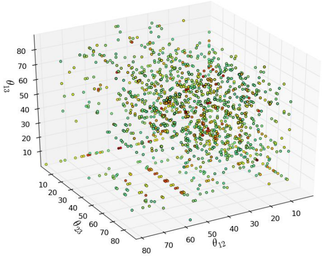


(f) Number of models that give θ_{ij} and θ_{mn} with the remaining angle restricted to its 3σ interval. The histograms have 2941000, 3675600 and 1057170 entries, respectively.

How did we generate this set of models?

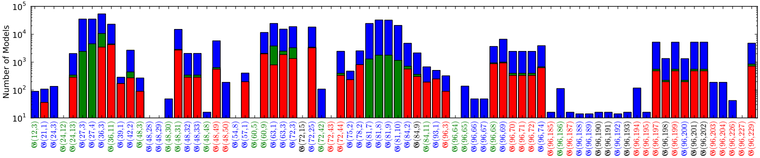


(g) The 5528 bins that are ≥ 1.



(h) The 1287 bins that are ≥ 1000.

How did we generate this set of models?



- Histogram bars: All models, 3σ, TBM
- 38 groups (50%) have tribimaximal models
- Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$. Special?
- Group names: $\mathfrak{g} \subset \mathrm{U}(3)$, $\mathfrak{g} \supset A_4$, $A_4 \subset \mathfrak{g} \subset \mathrm{U}(3)$