Dipion Spectrum : e^+e^- annihilations and τ decays

Presented by

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LPNHE Paris 6/7

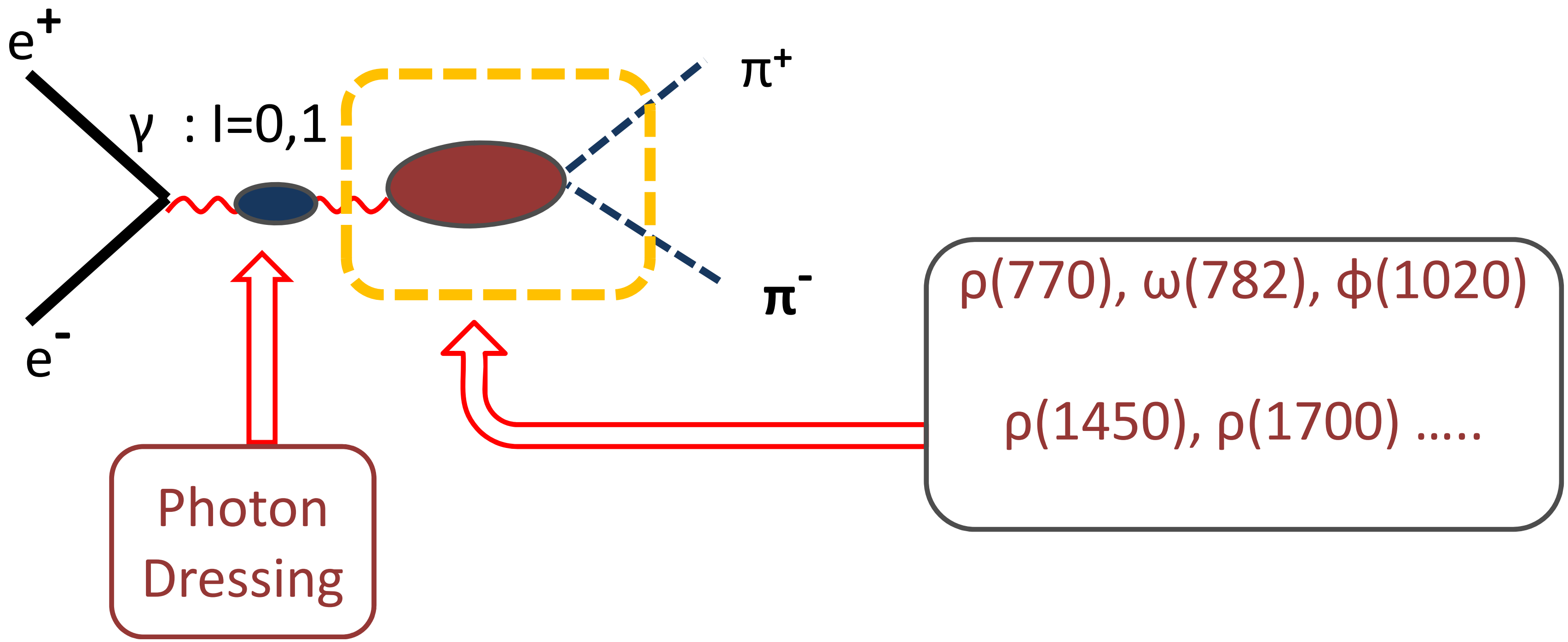
Work done together with

L. DelBuono, P. David, O. Leitner, H. O'Connell

Un Rapide Rappel

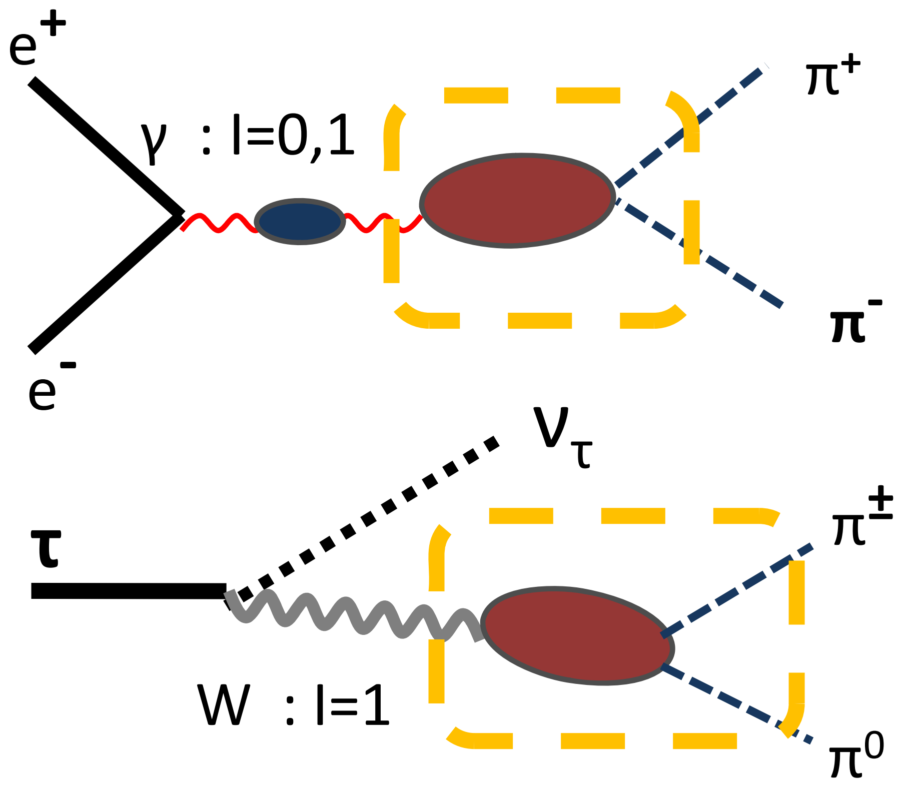
Qu'est ce que le Facteur de Forme du pion ?

The Pion Form Factor : Not A Pointlike Coupling



BUT SAME OBJECT IN τ DECAYS

The Pion Form Factor : CVC assumption



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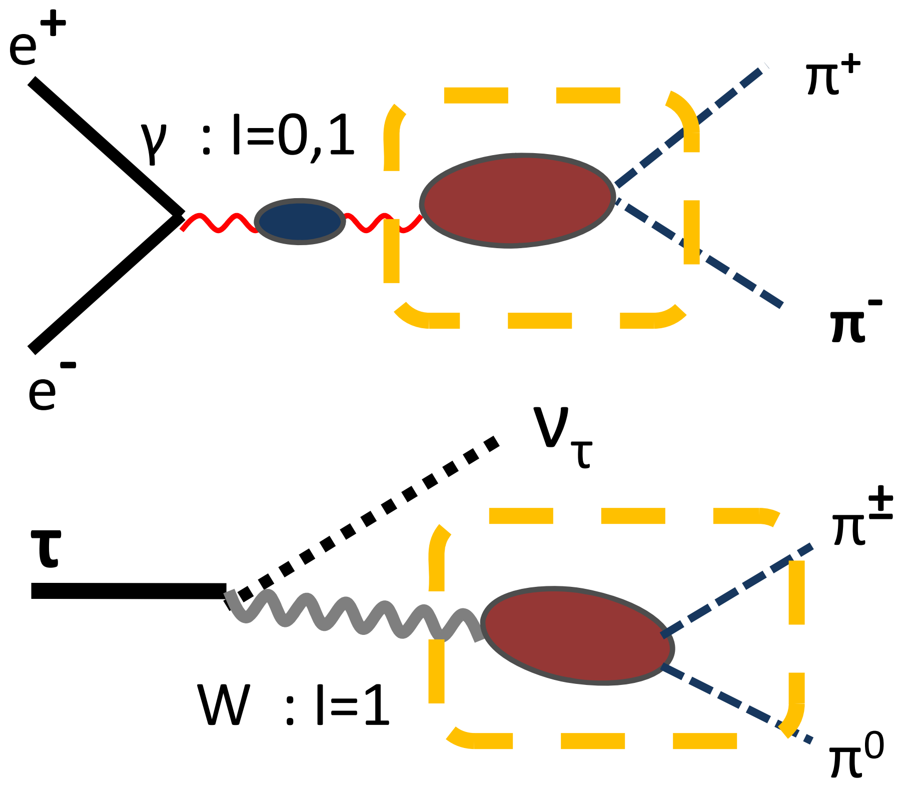


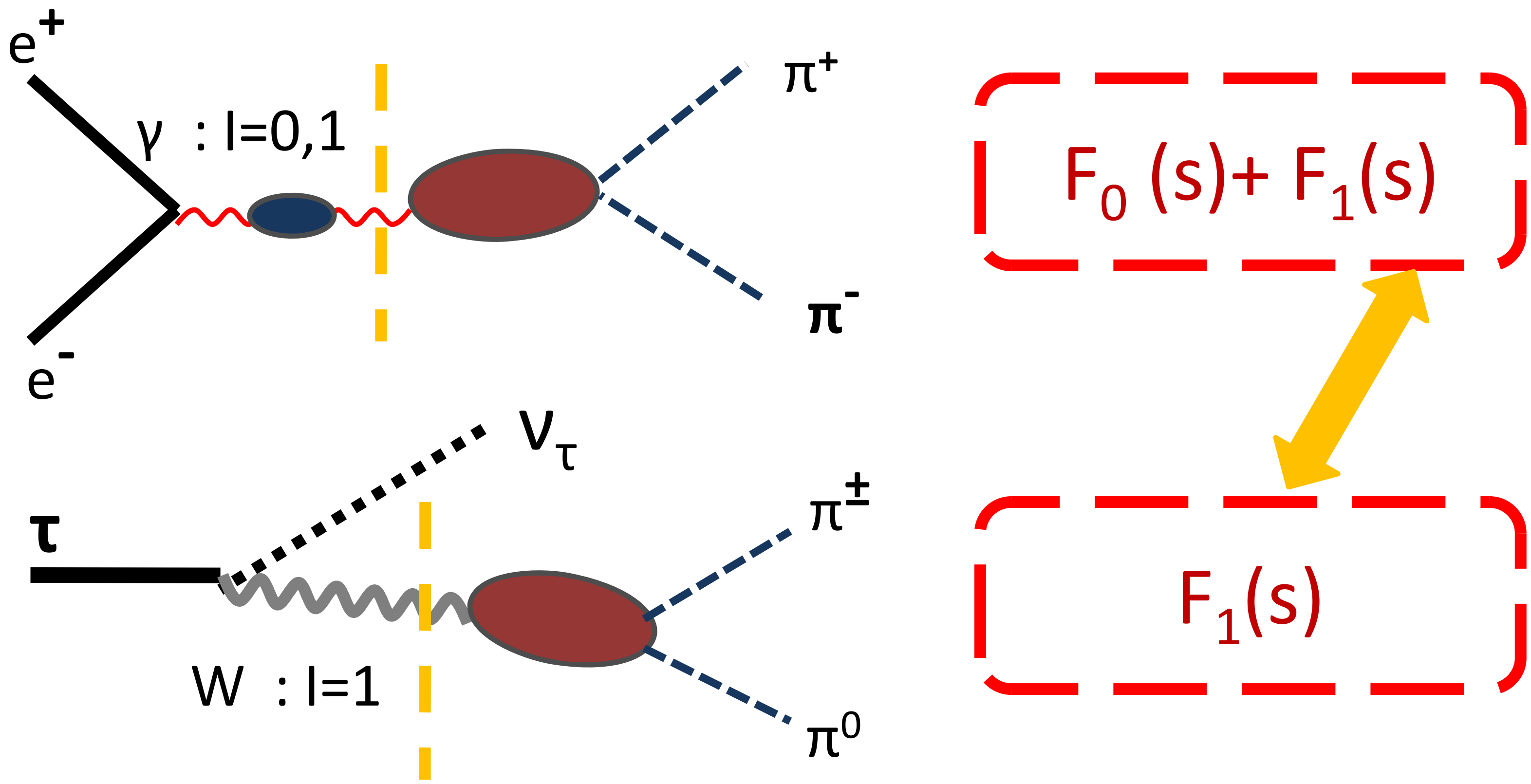
ρ(770), ω(782), ϕ(1020)

ρ(1450),…, ρ(1700) …..

ρ(770), ρ(1450), ρ(1700)

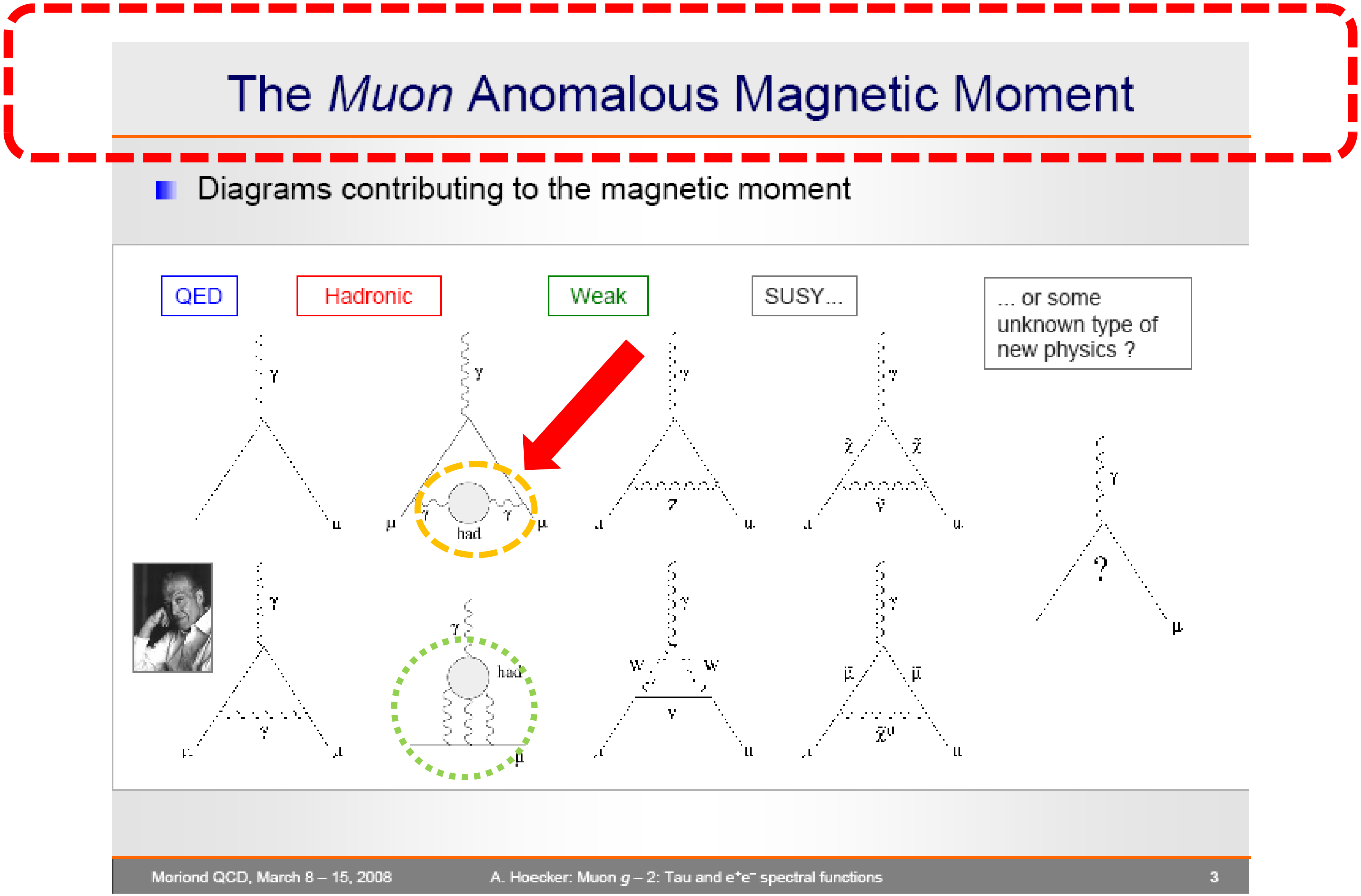
…..

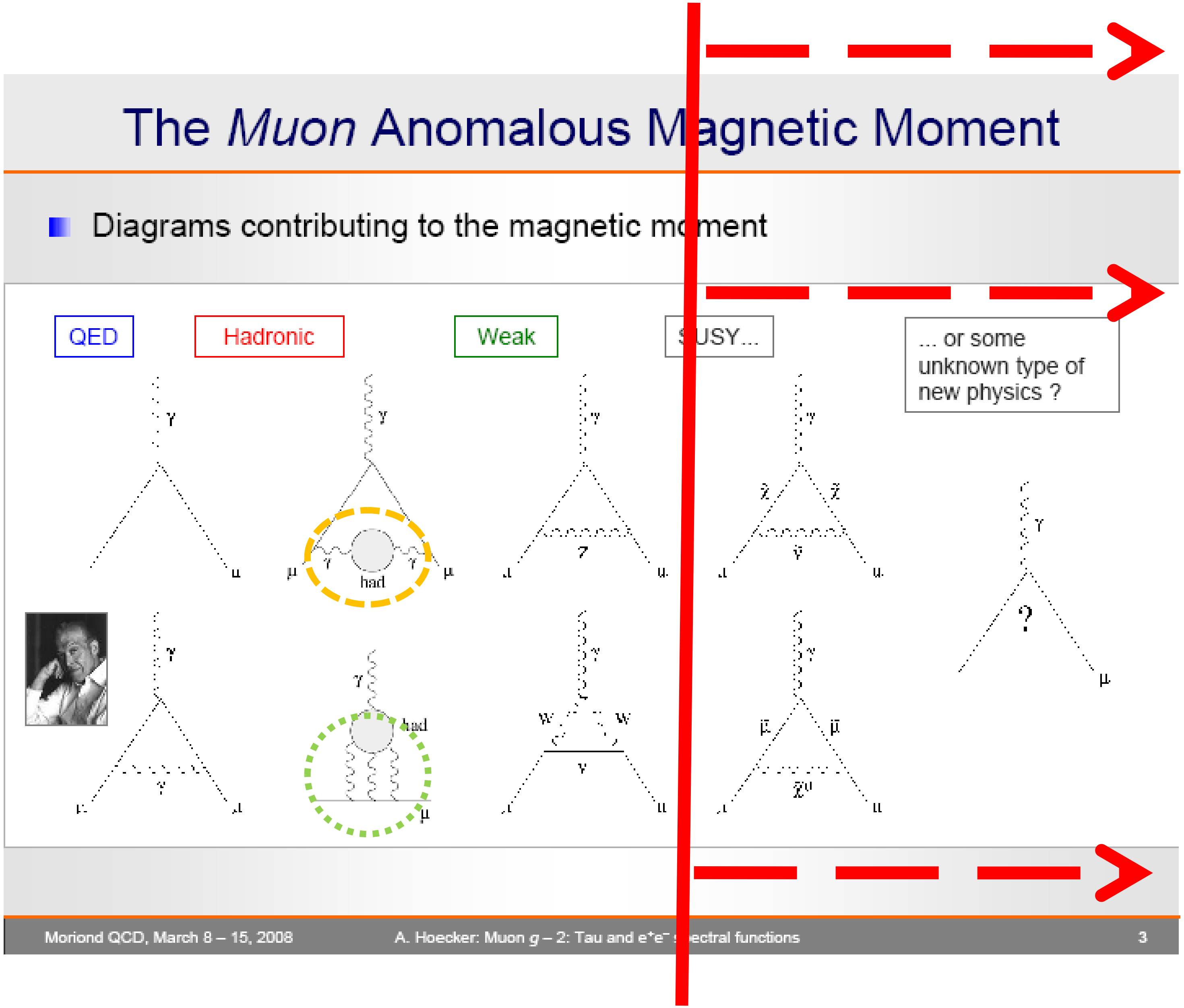
The Pion Form Factor : CVC assumption



Un Autre Rapide Rappel

A quoi sert le Facteur de Forme du pion ?


The Muon Anomalous Magnetic Moment
Diagrams contributing to the magnetic moment
QED
Hadronic
Weak
SUSY...
... or some unknown type of new physics ?
had
?
Moriond QCD, March 8 – 15, 2008
A. Hoecker: Muon g – 2: Tau and e+e− spectral functions
3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Hadronic Contributions to g-2

Dominated By Non Perturbative Contributions, Estimated by Dispersion Integrals

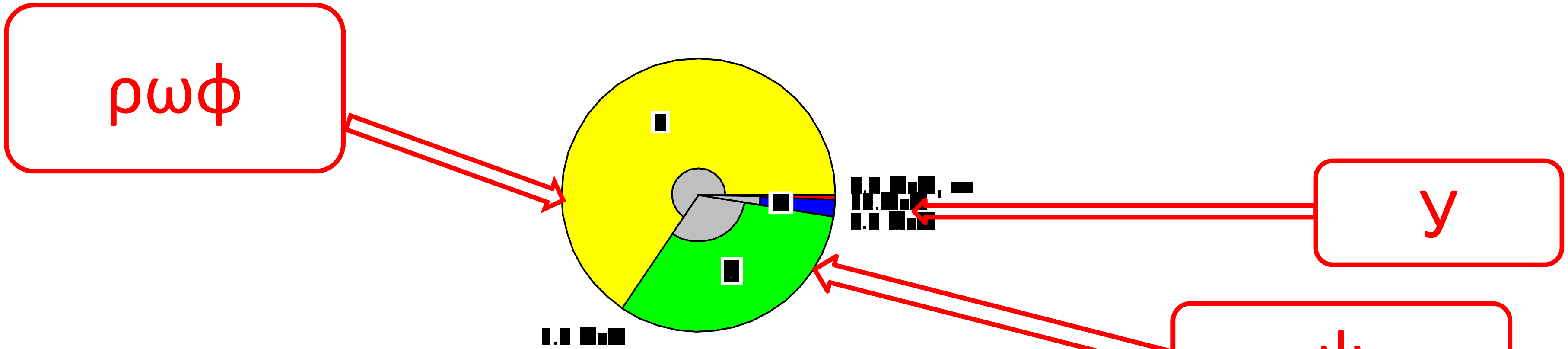
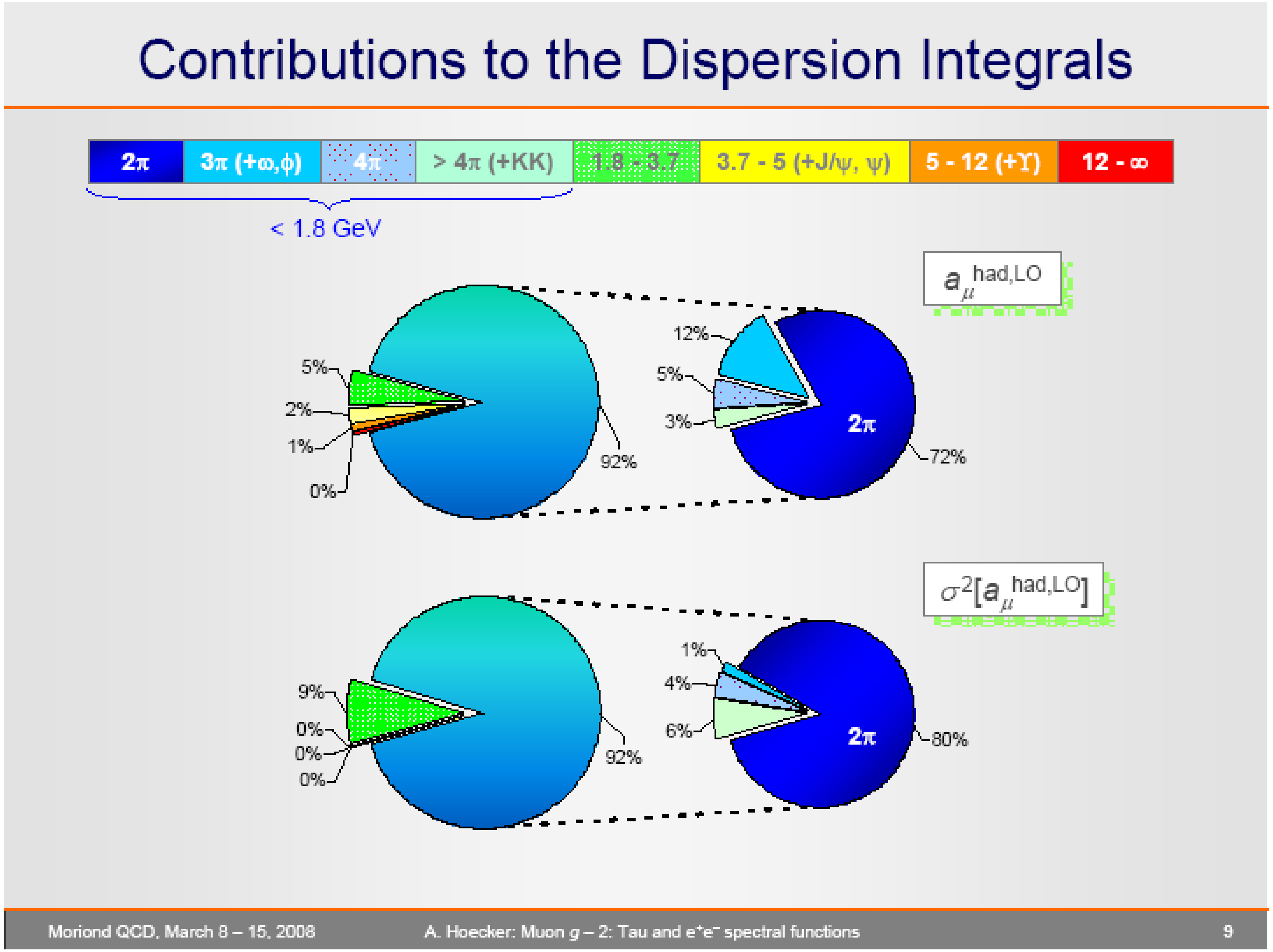


Figure 2. The distribution of contributions an errors (shaded areas scaled up by 10) for a_{μ}^{had} .


Contributions to the Dispersion Integrals
2π
3π (+ω,φ)
4π
> 4π (+KK)
1.8 - 3.7
3.7 - 5 (+J/ψ, ψ)
5 - 12 (+ϒ)
12 - ∞
< 1.8 GeV
a_μ had,LO
5%
2%
1%
0%
92%
12%
5%
3%
2π
72%
σ²[a_μ had,LO]
9%
0%
0%
0%
92%
1%
4%
6%
2π
80%
Moriond QCD, March 8 – 15, 2008
A. Hoecker: Muon g – 2: Tau and e⁺e⁻ spectral functions
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The Most Accurate Measurement In Quantum World

Experimental Progress: from CERN to BNL

Miller-de Rafael-Roberts, Rept.Prog.Phys.70:795,2007 [hep-ex/0602035]

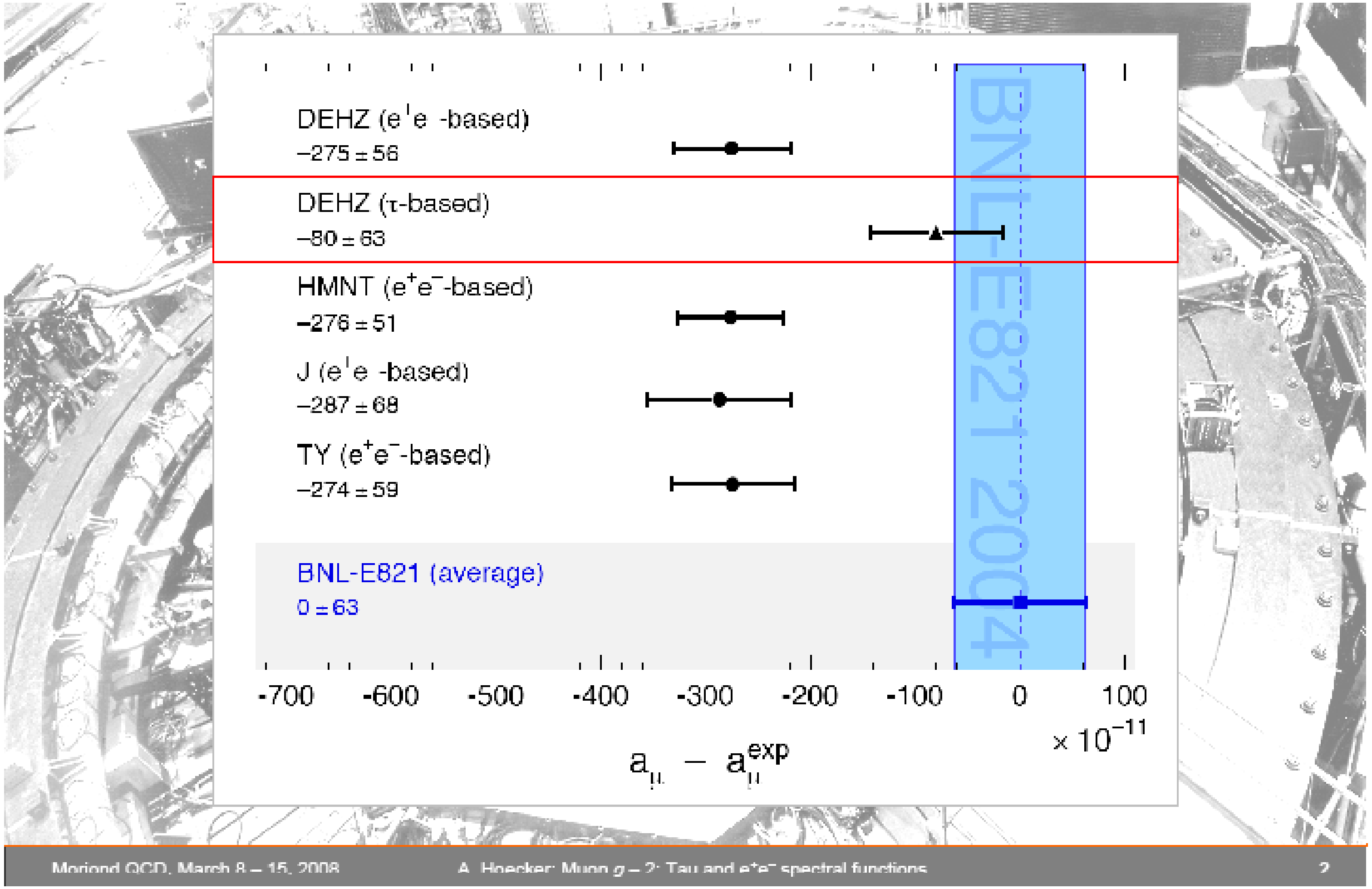
Experiment	Beam	Measurement	$\delta a_\mu / a_\mu$	Required th. terms
Columbia-Nevis ('57)	μ^+	g = 2.00(0.10)		$g = 2$
Columbia-Nevis ('59)	μ^+	0.001 13(+16)(–12)	12.4%	α/π
CERN 1 ('61)	μ^+	0.001 145(22)	1.9%	α/π
CERN 1 ('62)	μ^+	0.001 162(5)	0.43%	$(\alpha/\pi)^2$
CERN 2 ('68)	μ^+	0.001 166 16(31)	265 ppm	$(\alpha/\pi)^3$
CERN 3 ('75)	$\mu^\pm$	0.001 165 895(27)	23 ppm	$(\alpha/\pi)^3$ + had
CERN 3 ('79)	$\mu^\pm$	0.001 165 911(11)	7.3 ppm	$(\alpha/\pi)^3$ + had
BNL E821 ('00)	μ^+	0.001 165 919 1(59)	5 ppm	$(\alpha/\pi)^3$ + had
BNL E821 ('01)	μ^+	0.001 165 920 2(16)	1.3 ppm	$(\alpha/\pi)^4$ + had + weak
BNL E821 ('02)	μ^+	0.001 165 920 3(8)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?
BNL E821 ('04)	μ^-	0.001 165 921 4(8)(3)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?

➔ Current world average: a_μ^{exp}=11 659 208.0 ± 5.4 ± 3.3 × 10^{-10}

Dominated by by BNL-E821: [PRD73(06)072003, hep-ex/0602035]

Moriond QCD, March 8 – 15, 2008 A. Hoecker: Muon $g-2$: Tau and e^+e^- spectral functions 4

The “Problem”



DEHZ (e⁺e⁻-based)
−275 ± 56
DEHZ (τ-based)
−80 ± 63
HMNT (e⁺e⁻-based)
−276 ± 51
J (e⁺e⁻-based)
−287 ± 68
TY (e⁺e⁻-based)
−274 ± 59
BNL-E821 (average)
0 ± 63
BNL-E821 2004
-700 -600 -500 -400 -300 -200 -100 0 100
× 10⁻¹¹
$a_\mu - a_\mu^{exp}$
Moriond QCD, March 8 – 15, 2008
A. Hoecker: Muon g – 2: Tau and e⁺e⁻ spectral functions
2

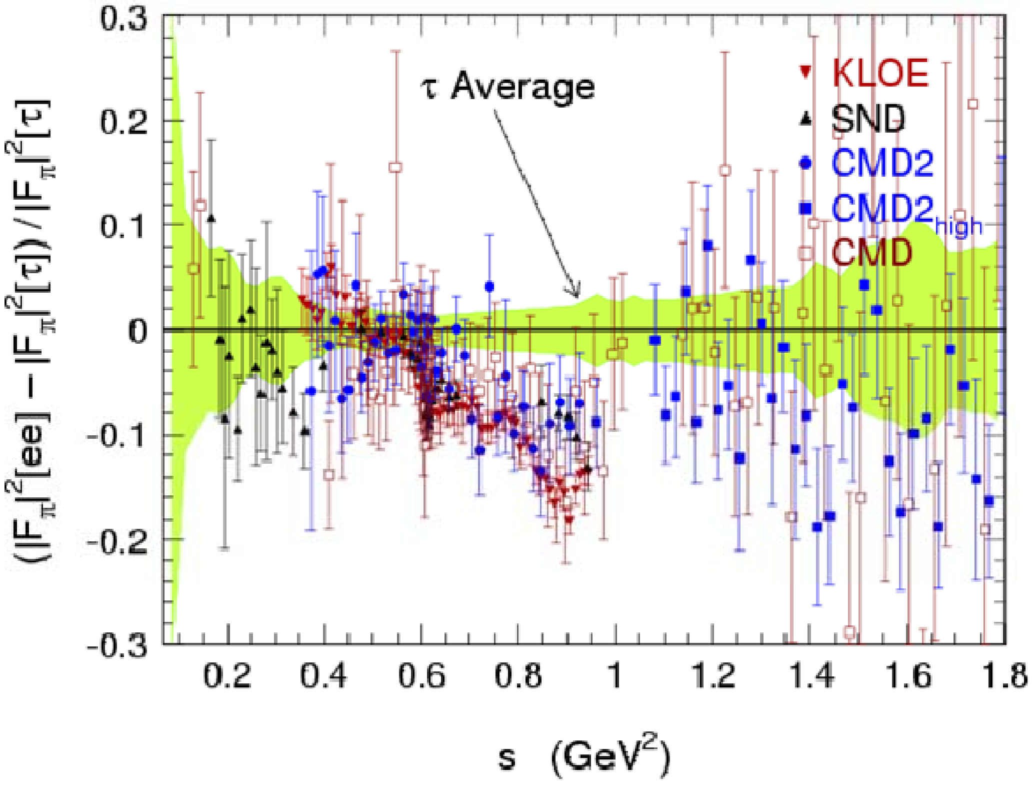
The Pion Form factor in e^+e^- and τ Data

- Since the advent of τ data, disagreement with e^+e^- data
- Large activity in identifying isospin symmetry breaking in both e^+e^- annihilation and τ decay
- Disagreement survived accounting for identified isospin breaking corrections!
- Is there a missing piece, a systematic effect (in e^+e^- or τ data) or new physics?

The Latest Account

- M. Davier NP Proc. Supp. 169 (2007) 288

- Inv. Mass dependent missing effect



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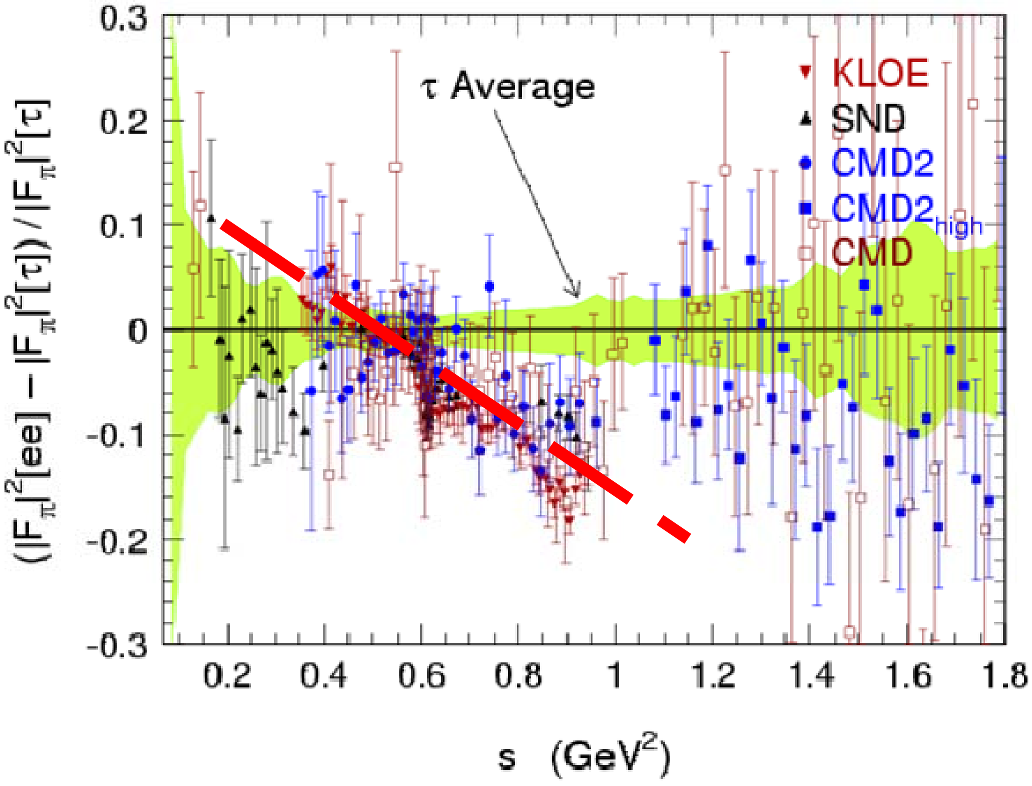
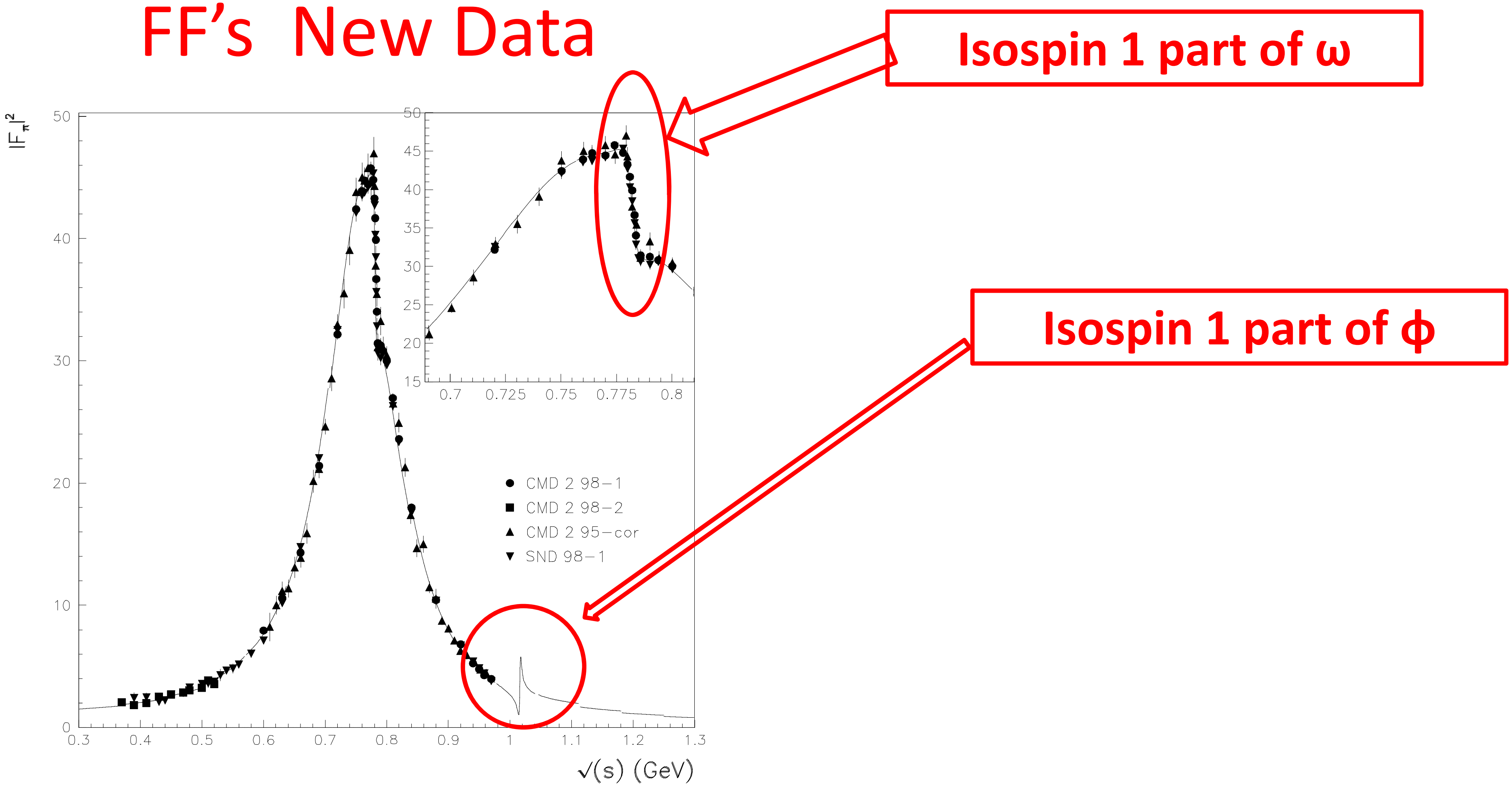
- **Inv. Mass dependent missing effect !**



Is it isospin breaking?

Possible Missing Effect : (ρ-ω-ϕ) Mixing

FF's New Data



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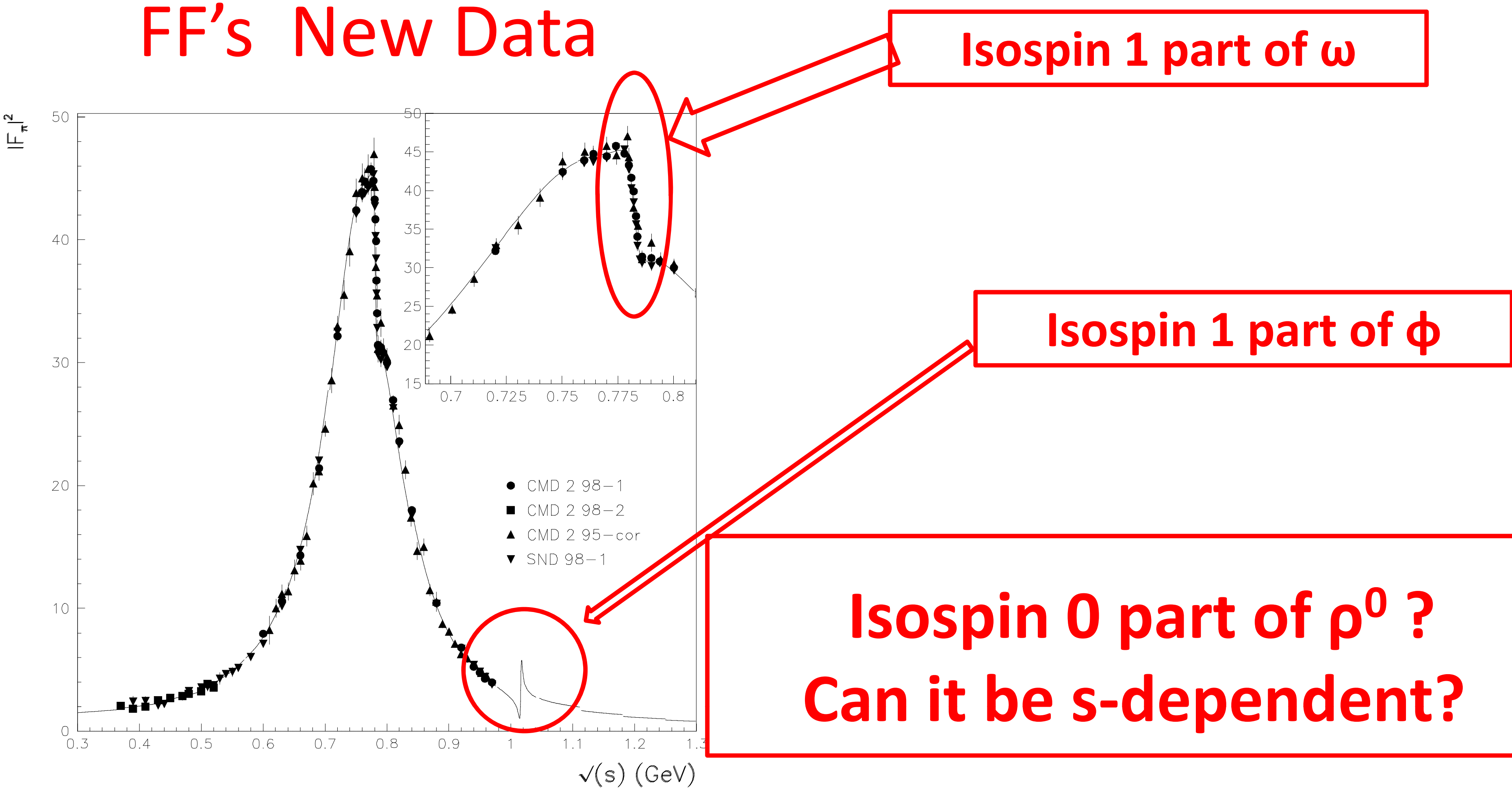

Isospin 0 part of ρ^0 ?
Can it be s-dependent?

OUTLINE

- A VMD-like Model : HLS Model (briefly)
- Breaking of U(3)/SU(3) Symmetries (briefly)
- The Anomalous Sector (briefly)
- The Pion Form Factor in e^+e^- Annihilation and τ Decay (Isospin Breaking)
- Loop Transition Effects in e^+e^- : Physical ρ^0, ω, ϕ
- Extended Data Sample submitted to fit, Why?
- Fit results & Plots
- Conclusions

The Hidden Local Symmetry Model

- Vector Mesons ≡ gauge bosons of a HL symmetry
-

M.Bando, T. Kugo & K. Yamawaki Phys. Rep. 164 (1988) 217
M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

- Define $\xi_{L/R} = e^{[\mp i P / f_\pi]}$
- Define covariant derivatives $D_\mu \xi_L \,,\, D_\mu \xi_R$
- Then $L/R = D_\mu \xi_{L/R} \xi^\dagger_{L/R}$ and $\mathcal{L}_{A/V} = -\frac{f_\pi^2}{4} Tr[L \mp R]^2$
- The HLS Lagrangian $\mathcal{L}_{HLS} = \mathcal{L}_A + a\, \mathcal{L}_V$

Expanded form: M.Benayoun & H.O'Connell PR D 58 (1998) 074006

- VMD : a=2 , Phenomenology a ~ 2.4

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The Covariant Derivatives

- Covariant derivatives ≠ for left- right-ξ fields :

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} - igV_\mu \xi_{L/R} + i\xi_{L/R} G_{L/R}$$

- With :

$$G_R = eQA_\mu \quad , \quad G_L = eQA_\mu + \frac{g_2}{\sqrt{2}}\left(W_\mu^+ T_+ + W_\mu^- T_-\right)$$

$T_\pm$ is CKM matrix reduced to V_{us} and V_{ud} terms

Breaking of SU(3) Flavor Symmetry

Several possible schemes

$$L_{A/V} = -\frac{f_\pi^2}{4} Tr\left[L \mp R\right]^2 \Rightarrow$$

Benayoun & O'Connell op. cit.

$$L_{A/V} = -\frac{f_\pi^2}{4} Tr\left[(L \mp R) X_{A/V}\right]^2$$

With Breaking matrices $X_{A/V} = Diag(1, 1, z_{A/V})$

z_V related to vector meson masses, $z_A = \left[\frac{f_K}{f_\pi}\right]^2$

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z_V related to vector meson masses, $z_A = \left[\frac{f_K}{f_\pi}\right]^2 \approx 1.5$

Bando Kugo Yamawaki op. cit.

Nonet Symmetry Breaking in HLS Model

Nonet Symmetry Breaking accounted for by adding determinant terms to HLS Lagrangian

M. Benayoun L. DelBuono H. O'Connell EPJ C 17 (2000) 593

Effective way :

$$P_8^{'} + xP_0^{'} = X_A^{1/2} \left(P_8 + P_0 \right) X_A^{1/2}$$

P.J. O'Donnell RMP 53 (1981) 673

Radiative decays of light mesons vs glue :

no glue $\Rightarrow$ $x \sim 0.9$ but $x = 1 \Rightarrow$ *glue in* (η, η')

M. Benayoun et al. PR D 59 (1999) 114027

Anomalous Sector of the HLS Model

- HLS Model has an anomalous sector for VPγ and Pγγ couplings ; can be derived from :

$$L = C\varepsilon^{\mu\nu\rho\sigma} Tr\left[X_T \partial_\mu \left(eQA_\nu + gV_\nu \right) X_T^{-2} \partial_\rho \left(eQA_\sigma + gV_\sigma \right) X_T P \right]$$

M. Benayoun et al. PR D 59 (1999) 114027

A. Bramon, A. Grau & G. Pancheri PL B 345 (1995) 263

- X_T allows for correct account of K* rad. decays

  [C = - 3 /(4 π² f_π)]

G. Morpurgo PR D 42 (1990) 1497

- (η, η') mixing angle related with (x, z_A) vanishes when no SU(3) breaking

MB, LD & HO EPJ C 17 (2000) 593

The Pion form Factor in e$^+$e$^-$ and τ Physics

- Without Symmetry Breaking :

$$F_\pi^{e/\tau}(s) = \left[\left(1 - \frac{a}{2}\right) - \frac{F_\rho^{\gamma/W}(s) g_{\rho\pi\pi}}{D_\rho(s)} \right]$$

- Isospin symmetry breaking: mass splittings +

$$\tau \Rightarrow \times S_{EW} G_{EM}(s) \quad , e^+e^- \Rightarrow - \left[\frac{F_\omega^{\gamma/W}(s) g_{\omega\pi\pi}}{D_\omega(s)} + \frac{F_\varphi^{\gamma/W}(s) g_{\varphi\pi\pi}}{D_\varphi(s)} \right]$$

W. Marciano & A. Sirlin PRL 71 (1993) 3629

V. Cirigliano G. Ecker & H. Neufeld PL B 513 (2001) 361

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HLS

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$$F_\pi^{e/\tau}(s) = \left[\left(1-\frac{a}{2}\right) - \frac{F_\rho^{\gamma/W}(s)\, g_{\rho\pi\pi}}{D_\rho(s)}\right]$$

?

?

δm^2

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$$\tau \Rightarrow \times S_{EW} G_{EM}(s) \qquad e^+e^- \Rightarrow -\left[\frac{F_\omega^\gamma(s) g_{\omega\pi\pi}}{D_\omega(s)} + \frac{F_\varphi^\gamma(s) g_{\varphi\pi\pi}}{D_\varphi(s)}\right]$$

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γ/W- ρ Transitions

- No isospin breaking :

$$F_{\rho}^{\gamma/W}(s) = agf_{\pi}^{2} - \Pi_{(\gamma/W)\rho}(s)$$

- Isospin symmetry breaking :

ρ$^{\pm}$ component

$$F_{\rho}^{W}(s) = agf_{\pi}^{2} - \Pi_{W\rho}(s)$$

$$F_{\rho}^{\gamma}(s) = agf_{\pi}^{2} + \ldots - \Pi_{\gamma\rho}(s)$$

ρ_I component

ϕ_I and ω_I components

Transitions among vector fields at one loop

- At tree level **ideal fields ≡ mass *eigenstates***
- At one loop, the HLS Lagrangian piece

$$\left(\rho_I + \omega_I - \sqrt{2}\, z_V\, \varphi_I\right) K^- \overleftrightarrow{\partial} K^+ + \left(\rho_I - \omega_I + \sqrt{2}\, z_V\, \varphi_I\right) K^0 \overleftrightarrow{\partial} \overline{K}^0$$

induces transitions among ideal fields

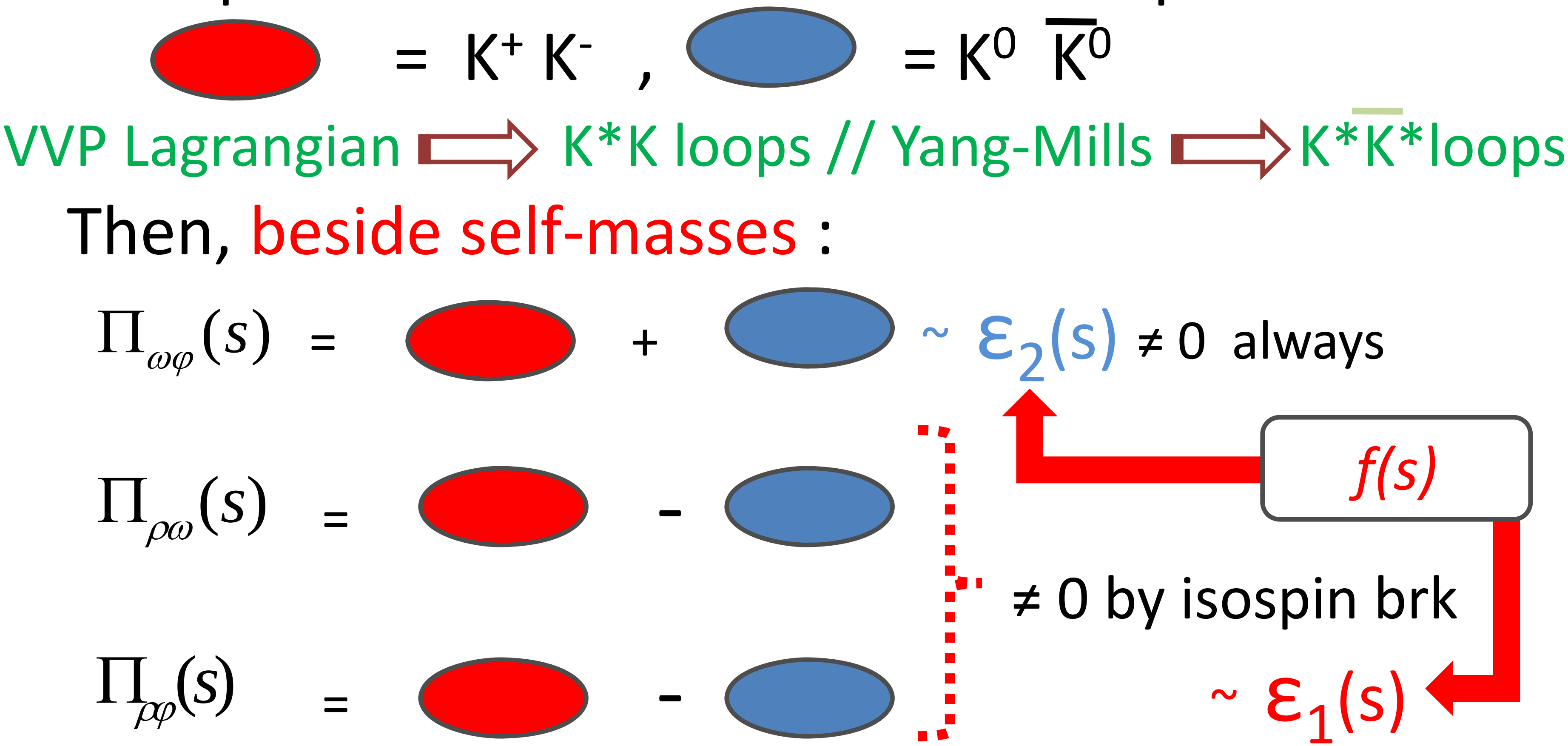
ideal fields ≢ mass *eigenstates*

isospin symmetry breaking :

$$m_{K^\pm} \neq m_{K^0}$$

Transitions among vector fields at one loop

- Dispersion Relations define the loops :



The Modified Vector Mass Matrix

$$M^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) + \varepsilon_2(s) & \varepsilon_1(s) & -\mu\varepsilon_1(s) \\ \varepsilon_1(s) & m^2 + \varepsilon_2(s) & -\mu\varepsilon_2(s) \\ -\mu\varepsilon_1(s) & -\mu\varepsilon_2(s) & z_V m^2 + \mu^2\varepsilon_2(s) \end{pmatrix}$$

(columns: ρ_I, ω_I, ϕ_I)

With $m^2 = a\, g^2 f_\pi^2$ and $\mu = z_V \sqrt{2}$

No SU(3) Brk :: $z_V = 1$, *No SU(2) Brk*:: $\varepsilon_1(s) = 0$

Blue : Loop effects magenta : SU(3) breaking

red : isospin breaking

Loop Corrections : (ω , ϕ) Mixing

$$\rho_I \qquad \omega_I \qquad \phi_I$$

$$M^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) + \varepsilon_2(s) & \varepsilon_1(s) & -\mu\varepsilon_1(s) \\ \varepsilon_1(s) & m^2 + \varepsilon_2(s) & \boxed{-\mu\varepsilon_2(s)} \\ -\mu\varepsilon_1(s) & \boxed{-\mu\varepsilon_2(s)} & z_V m^2 + \mu^2\varepsilon_2(s) \end{pmatrix}$$

[Blue : Loop effects magenta : SU(3) breaking
red : isospin breaking]

Isospin Breaking : (ρ, ω , ϕ) Mixing

$$\begin{matrix} & \rho_I & \omega_I & \phi_I \end{matrix}$$

$$M^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) + \varepsilon_2(s) & \varepsilon_1(s) & -\mu\varepsilon_1(s) \\ \varepsilon_1(s) & m^2 + \varepsilon_2(s) & -\mu\varepsilon_2(s) \\ -\mu\varepsilon_1(s) & -\mu\varepsilon_2(s) & z_V m^2 + \mu^2\varepsilon_2(s) \end{pmatrix}$$

[Blue : Loop effects magenta : SU(3) breaking
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The Mass Matrix Eigen System

- **Expect :** $\left(m^2, \Pi_{\pi\pi}(s)\right) \gg \varepsilon_2(s) \gg \varepsilon_1(s)$
- Then solve for the eigensystem **perturbatively** :

$$M_0^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) + \varepsilon_2(s) & 0 & 0 \\ 0 & m^2 + \varepsilon_2(s) & 0 \\ 0 & 0 & z_V m^2 + \mu^2 \varepsilon_2(s) \end{pmatrix}$$

and :

$$\delta M^2(s) = \begin{pmatrix} 0 & \varepsilon_1(s) & -\mu\varepsilon_1(s) \\ \varepsilon_1(s) & 0 & -\mu\varepsilon_2(s) \\ -\mu\varepsilon_1(s) & -\mu\varepsilon_2(s) & 0 \end{pmatrix}$$

From Ideal To Physical Fields I

$$\begin{pmatrix} \rho^0 \\ \omega \\ \varphi \end{pmatrix} = R(s) \begin{pmatrix} \rho_I^0 \\ \omega_I \\ \varphi_I \end{pmatrix}$$

$R(s)$: Real analytic matrix function

fulfills Unitarity Condition

$$R(s+i\varepsilon)\tilde{R}(s+i\varepsilon) = 1$$

$$R(s) = \begin{pmatrix} 1 & \dfrac{\varepsilon_1}{\Pi_{\pi\pi} - \varepsilon_2} & \dfrac{-\mu\varepsilon_1}{(1-z_V)m^2 + \Pi_{\pi\pi} - \mu^2\varepsilon_2} \\ \dfrac{-\varepsilon_1}{\Pi_{\pi\pi} - \varepsilon_2} & 1 & \dfrac{-\mu\varepsilon_2}{(1-z_V)m^2 + (1-\mu^2)\varepsilon_2} \\ \dfrac{\mu\varepsilon_1}{(1-z_V)m^2 + \Pi_{\pi\pi} - \mu^2\varepsilon_2} & \dfrac{\mu\varepsilon_2}{(1-z_V)m^2 + (1-\mu^2)\varepsilon_2} & 1 \end{pmatrix}$$

$$+O(\varepsilon_i^2)$$

From Ideal To Physical Fields II

Mass term derived from the eigenvalues of $M^2(s)$

:

$$\frac{m^2}{2}\left[\rho_I^2+\omega_I^2+z_V\varphi_I^2\right] \Rightarrow \frac{1}{2}\left[m^2+\Pi_{\pi\pi}(s)+\varepsilon_2(s)\right]\rho^2$$

$$+\frac{1}{2}\left[m^2+\varepsilon_2(s)\right]\omega^2+\frac{1}{2}\left[z_V m^2+\mu^2\varepsilon_2(s)\right]\varphi^2$$

(Self masses include subtraction polynomials)

Leading order propagators become $\sim\left[s-\lambda_V(s)\right]^{-1}$

V π π Couplings

$$\frac{iag}{2}\rho_I \bullet \pi^- \overleftrightarrow{\partial} \pi^+ \Rightarrow$$

$$\frac{iag}{2}\left[\rho^0 - \frac{\varepsilon_1}{\Pi_{\pi\pi} - \varepsilon_2}\omega + \frac{\mu\varepsilon_1}{(1-z_V)m^2 + \Pi_{\pi\pi} - \mu^2\varepsilon_2}\varphi\right] \bullet \pi^- \overleftrightarrow{\partial} \pi^+$$

*At leading order : **ρ term unchanged**

***s-dependent ω and φ couplings** generated

V π π Couplings

$$\frac{iag}{2}\rho_I^0 \bullet \pi^- \overleftrightarrow{\partial} \pi^+ \Rightarrow$$

Orsay Phase ≈90° At peak

$$\frac{iag}{2}\left[\rho^0 - \frac{\varepsilon_1}{\Pi_{\pi\pi} - \varepsilon_2}\omega + \frac{\mu\varepsilon_1}{(1-z_V)m^2 + \Pi_{\pi\pi} - \mu^2\varepsilon_2}\varphi\right] \bullet \pi^- \overleftrightarrow{\partial} \pi^+$$

*At leading order : **ρ term unchanged** $+O(\varepsilon_i^2)$

$$g_{\rho\pi\pi} \Rightarrow g_{\rho\pi\pi} + O(\varepsilon_i^2)$$

***s-dependent ω and ϕ couplings** generated

γ – V Couplings

- In terms of Ideal Fields

$$-e\, agf_\pi^2 \left[\rho_I^0 + \frac{1}{3}\omega_I - \frac{\sqrt{2}}{3} z_V \phi_I \right]_\mu \bullet A^\mu \Longrightarrow$$

- Becomes

$$\Longrightarrow -e \left[f_\rho^\gamma \rho^0 + f_\omega^\gamma \frac{1}{3}\omega - f_\phi^\gamma \frac{\sqrt{2}}{3} z_V \phi \right]_\mu \bullet A^\mu$$

With $agf_\pi^2 \Longrightarrow f_V^\gamma \equiv f_V^\gamma(s) = agf_\pi^2 \left[1 + O(\varepsilon_1(s))\right]$!!!

Vector Meson Couplings to γ/W

- (γ/W) V transitions : constant + (PP,VP…) loops

- loop term : disp. relation (subtractions) $\Pi^{\gamma/W}(s)$
- tree terms :

$$f_\rho^\gamma = a\, g\, f_\pi^2 \left[1 + \frac{1}{3} \frac{\varepsilon_1}{\Pi_{\pi\pi} - \varepsilon_2} + \frac{1}{3} \frac{\mu^2 \varepsilon_1}{(1 - z_V) m^2 + \Pi_{\pi\pi} - \mu^2 \varepsilon_2} \right]$$

$$f_\rho^W = a g f_\pi^2$$

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- (γ/W) V transitions : constant + (PP,VP…) loops

- loop term : disp. relation (subtractions) $\Pi^{\gamma/W}(s)$
- tree terms :

$$f_\rho^\gamma = a\,g\,f_\pi^2 \left[1 + \frac{1}{3}\frac{\varepsilon_1}{\Pi_{\pi\pi} - \varepsilon_2} + \frac{1}{3}\frac{\mu^2 \varepsilon_1}{(1 - z_V)\,m^2 + \Pi_{\pi\pi} - \mu^2 \varepsilon_2} \right]$$

$$f_\rho^W = a g f_\pi^2$$

ω_I and ϕ_I components

Parameter Freedom

- If only using the pion form factor (e^+e^-,τ) the parameter freedom is too large:
- a (HLS), g , z_A , δm^2 ~4 par.
- Subtraction polynomials in :

$$\Pi_{\pi\pi}^{\rho}(s), \Pi_{\pi\pi}^{\gamma/W}(s), \varepsilon_1(s), \varepsilon_2(s) \quad \text{~8 par.}$$

⇨ **Too many parameters, too few structures**

- Only way out : **extend the fitted data sample**

⇨ **more information, less correlations**

The Extended Data Sample

- Add anomalous decay modes VPγ, Pγγ :

$$\rho^0/\omega/\varphi \to \pi^0\gamma/\eta\gamma \;\|\; \varphi \to \eta'\gamma \;\|\; \eta' \to (\rho^0/\omega)\gamma$$

$$K^* \to K\gamma \;\|\; \eta/\eta' \to \gamma\gamma \;\|\; \rho^\pm \to \pi^\pm\gamma$$

- Price : x, z_T,z_V,(z_A) for 14 modes

$(\rho^0)\ \omega/\varphi \to e^+e^-$ for free

Modulus and phase $(\varphi \to \pi^+\pi^-)$ for free

+ 4 measured data :: **Total 18 add. data**

M.B.,L.D. ,S.E, V.I. & H.OC, PR D 59 (1999) 114027

M.B, H.OC EPJ C22 (2001) 503

The Main Guess

- Main Guess ≈ Proof of Principle

18 Decay Modes + Pion FF in e^+e^- annihilation

- Fully reconstruct

The Pion FF in τ decay

(improvement of parameter fit values)

The X^2 contributions to fits

Data Set (#data points)	Full Data Fit	No τ data	No Spacelike Data
Decays (18+1)	11.13	11.52	11.48
New Timelike (127+1)	128.1	122.0	125.8
Old Timelike (82+1)	59.1	54.7	55.2
Spacelike (59+2)	65.7	55.2	**89.8/(59)**
τ ALEPH (33)	23.9	**42.3/(33)**	20.8
τ CLEO (25+1)	26.1	**26.2/(25)**	29.7
X^2/dof Probability	313.8/331 74%	257.7/274 75%	238.8/272 93%

The X^2 contributions to fits

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The X^2 contributions to fits

τ DATA **OUTSIDE FIT** : X^2 distance to ***prediction***

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Decays (18+1)	11.13	11.52	11.48
New Timelike (127+1)	128.1	122.0	125.8
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!!!!!!!

The X^2 contributions to fits

Spacelike data **OUTSIDE FIT**

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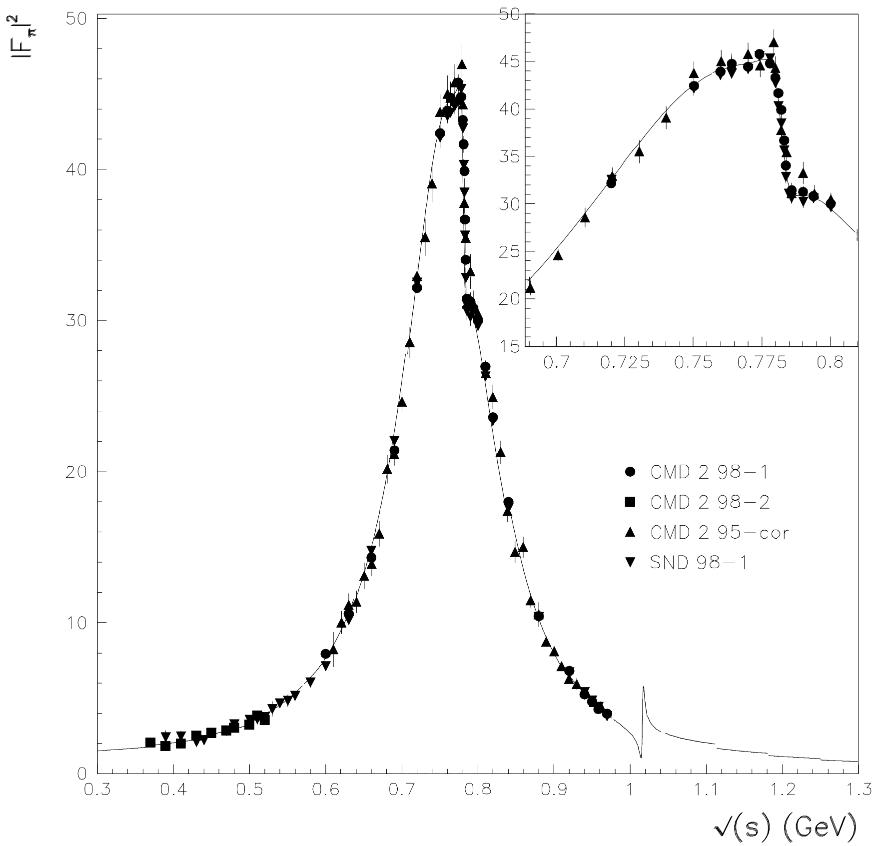
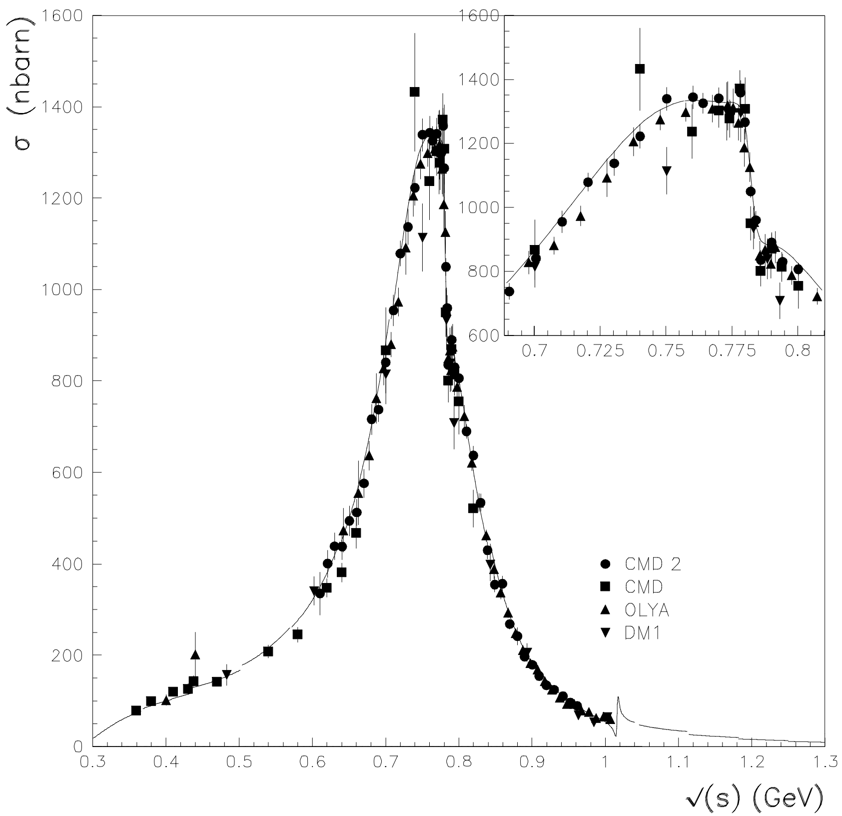
Global Fit to e^+e^- Data

FF's New Data

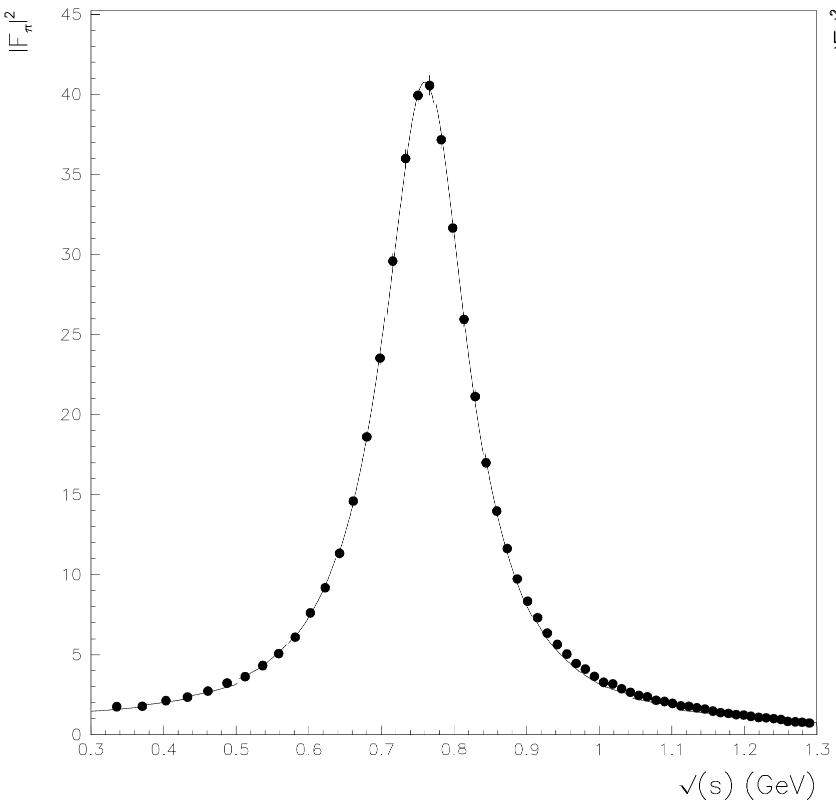


Cross Sections Old Data

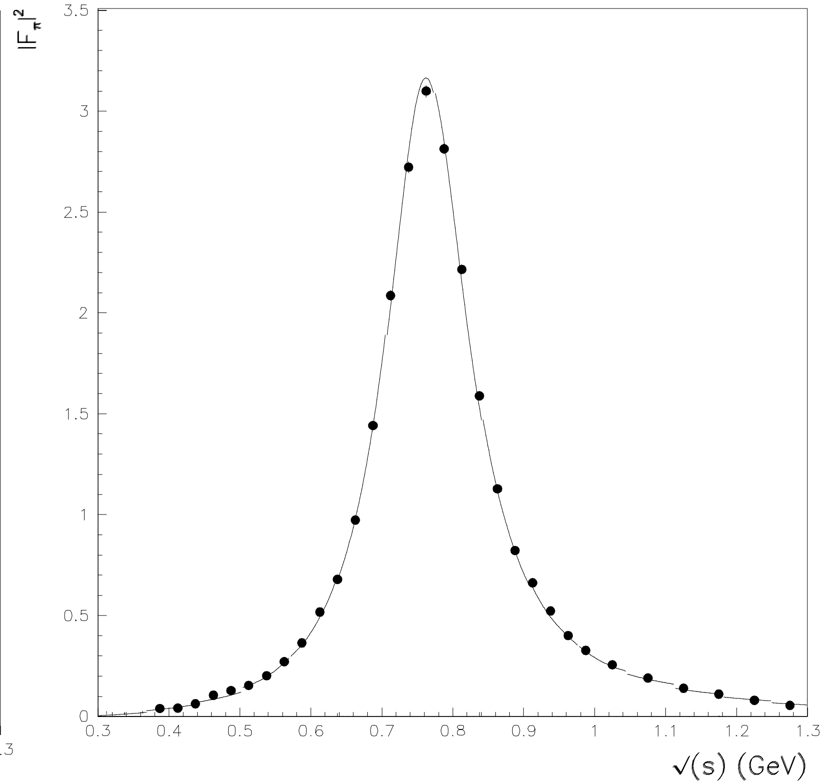


Fits to τ Data

ALEPH Data

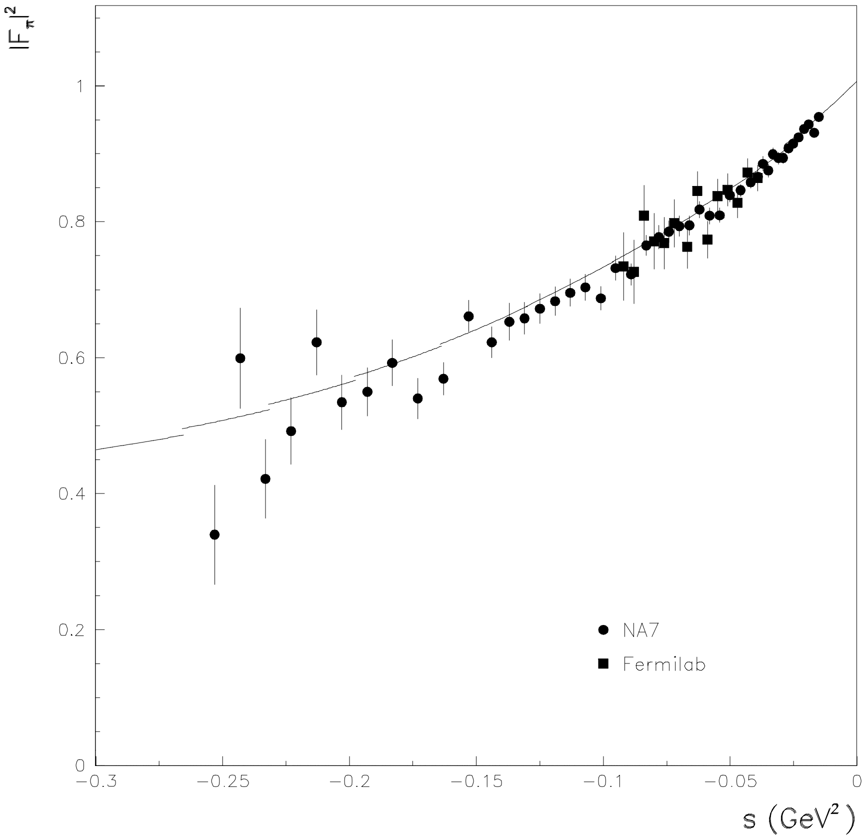


CLEO Data



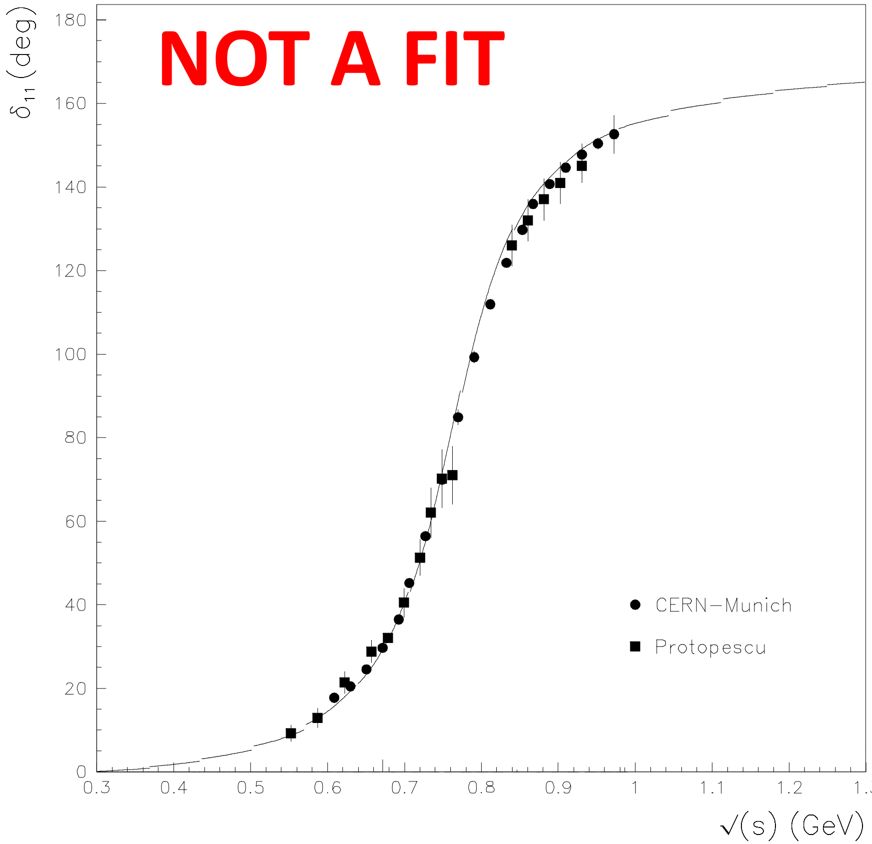
Spacelike Data & ππ Phase Shift

SpaceLike Data

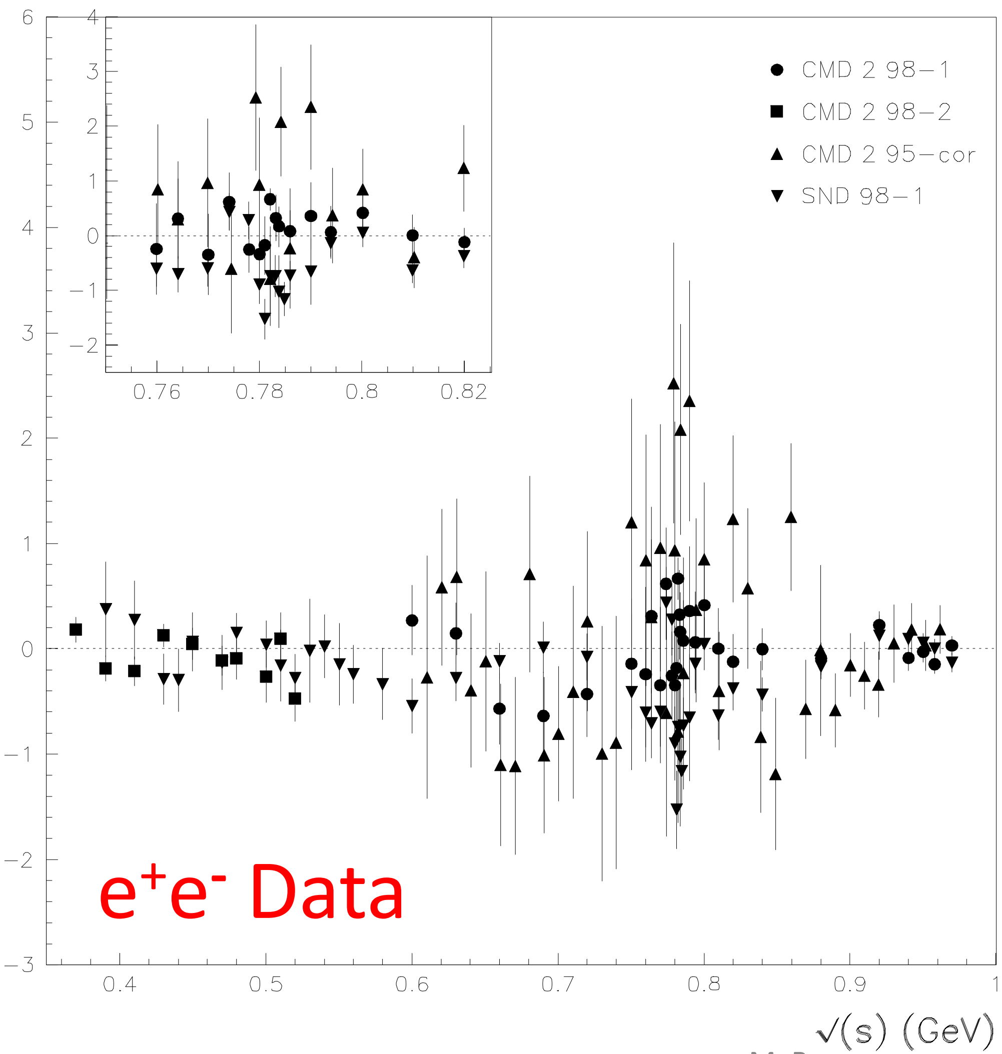


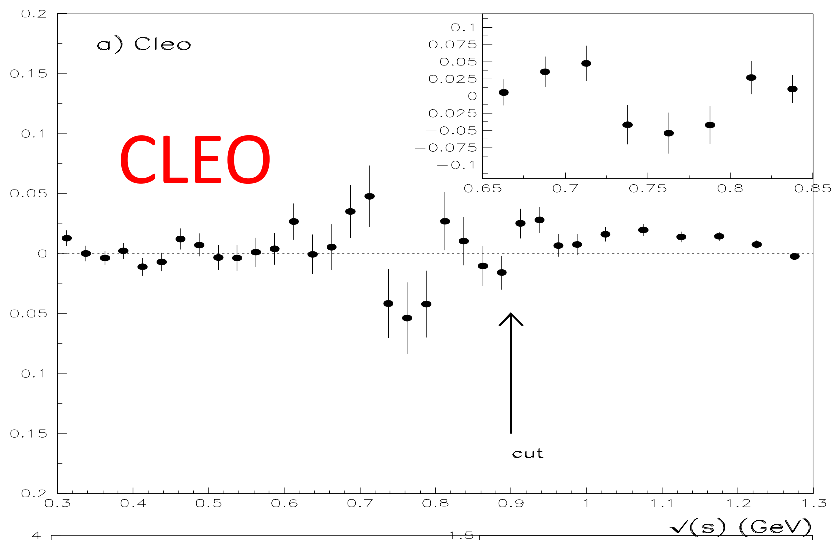
I=1 ππ Phase shift

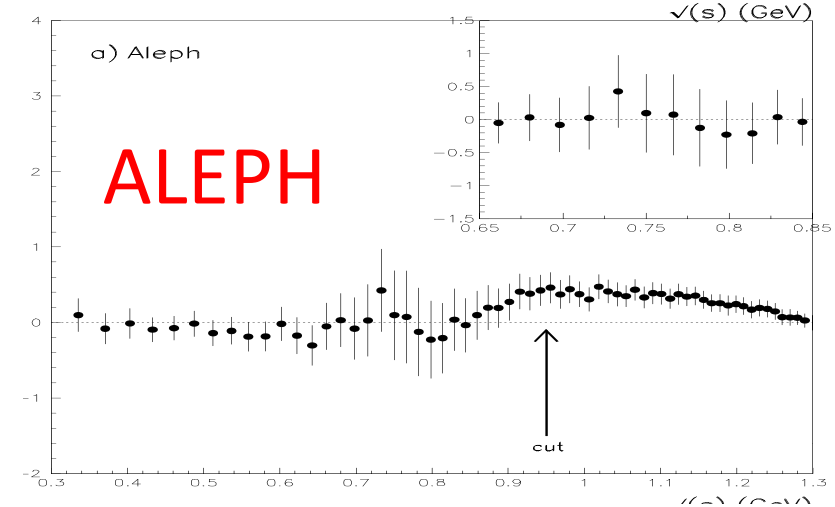
NOT A FIT



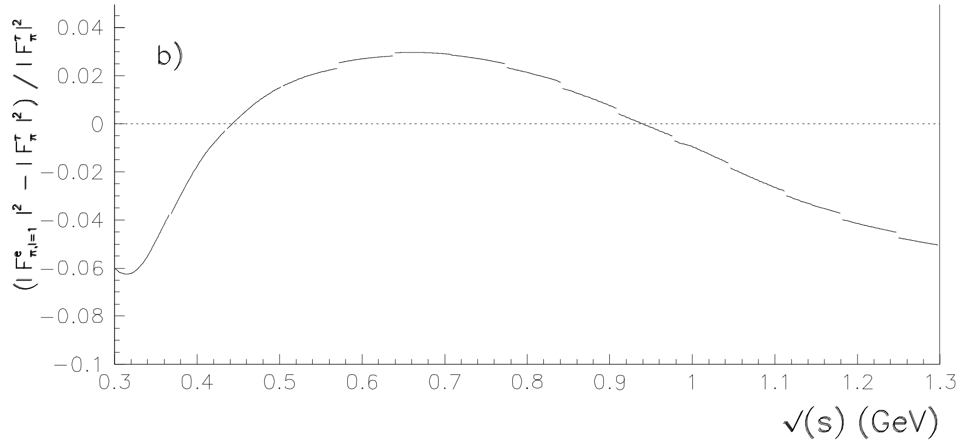
Fit Residuals : No Structure







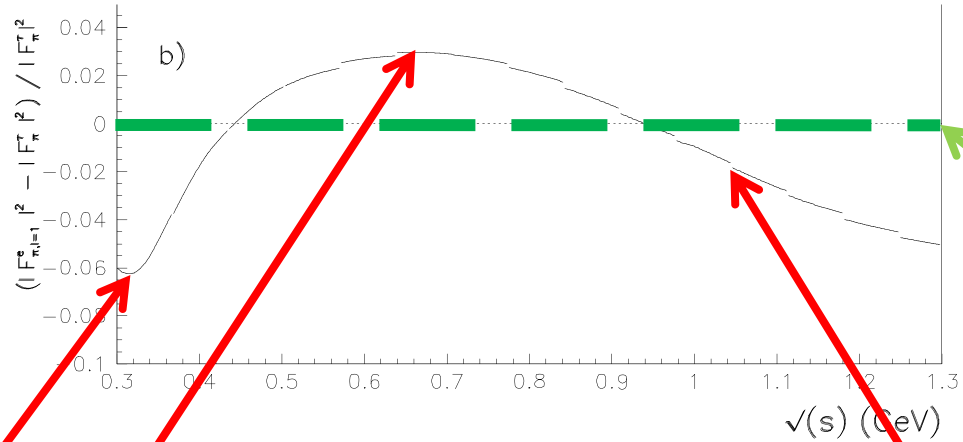
Isospin Symmetry Breaking : ρ^0 VS $\rho^{\pm}$



$$\frac{\left(|F^{e}_{\pi\ "I=1"}(s)|^2 - |F^{\tau}_{\pi}(s)|^2\right)}{|F^{\tau}_{\pi}(s)|^2}$$

$$I = 1 \equiv \rho^0 \ part$$

Isospin Symmetry Breaking : ρ^0 VS $\rho^{\pm}$



0 :: NO IS Brk

Threshold : -6%

$$\frac{\left(|F_{\pi\ "I=1"}^{e}(s)|^2 - |F_{\pi}^{\tau}(s)|^2\right)}{|F_{\pi}^{\tau}(s)|^2}$$

ρ Peak : + 3%

Φ Mass : -2%

The ρ^0 - $\rho^\pm$ Mass Difference

$$M_{\rho^0} - M_{\rho^\pm} \simeq 1.35 \pm 0.15_{def.} \pm 0.53_{stat./syst.}\ MeV$$

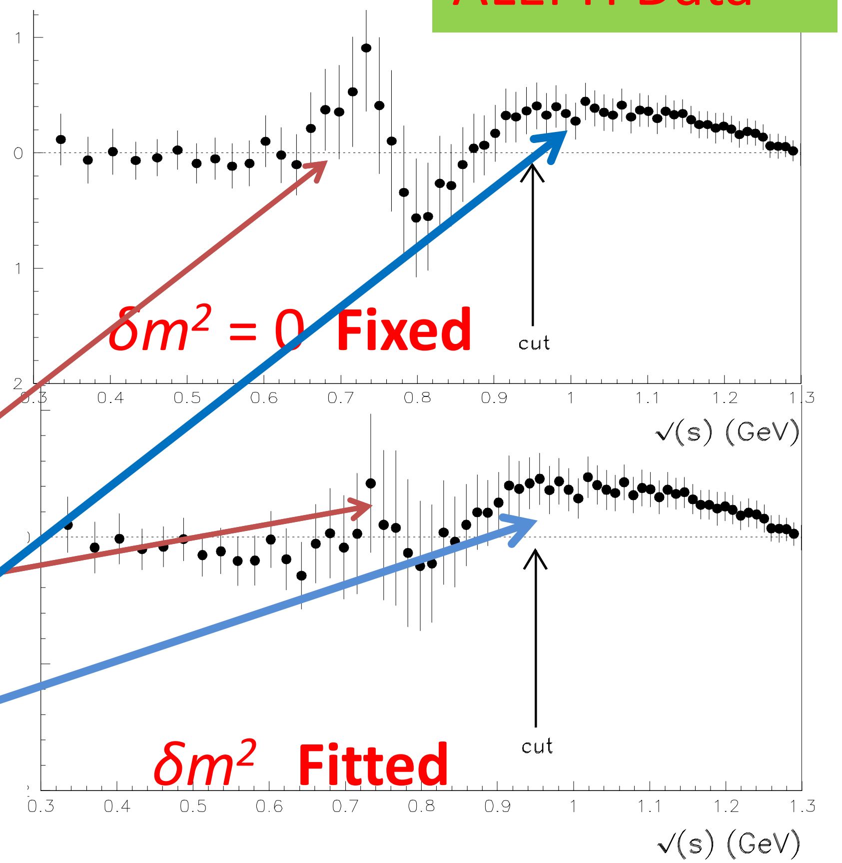
ALEPH Data

The ρ^0-$\rho^\pm$ Mass Difference :

1/ Only visible in ALEPH data

2/ ~1 .2 σ from ChPT

J. Bijnens & P. Gosdzinsky PL B 388 (1996) 203



ρ Peak Region

High mass Vector mesons

Fit Decay Modes Vs PDG

Decay Mode	FIT/PDG	Remark
$\rho^0 \to \pi^0\gamma$	0.86 ±0.15	
$\rho^\pm \to \pi^\pm\gamma$	1.12±0.11	
$\rho^0 \to \eta\gamma$	1.04±0.11	
$K^{*\pm} \to K^\pm\gamma$	1.00±0.14	
$K^{*0} \to K^0\gamma$	0.98±0.09	
$\omega \to \pi^0\gamma$	0.93±0.03	***
$\omega \to \eta\gamma$	1.35±0.11	***
$\phi \to \pi^0\gamma$	0.99±0.08	
$\phi \to \eta\gamma$	0.99±0.03	

Decay Mode	FIT/PDG	Remark
$\eta' \to \rho^0\gamma$	1.13±0.04	
$\eta' \to \omega\gamma$	1.04±0.11	
$\phi \to \eta'\gamma$	0.97±0.12	
$\eta \to \gamma\gamma$	0.90±0.02	!!!!!
$\eta' \to \gamma\gamma$	0.99±0.07	
$\omega \to e^+e^-$	1.00±0.02	
$\phi \to e^+e^-$	1.00±0.02	
$\phi \to \pi^+\pi^-$	0.98±0.29	
Phase [$\phi \to \pi^+\pi^-$]	0.79±0.15	

Two New Results

Process	FIT	PDG
$\rho^0 \to e^+e^-$ [x10^5]	5.56±0.06	4.70±0.08
$\omega \to \pi^+\pi^-$ (%)	1.13±0.08	1.70±0.27
$\rho^0 \to \pi^+\pi^-$ [MeV]	144.5±0.6	149.4±1.0

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~ 15 σ apart starting from the same data!

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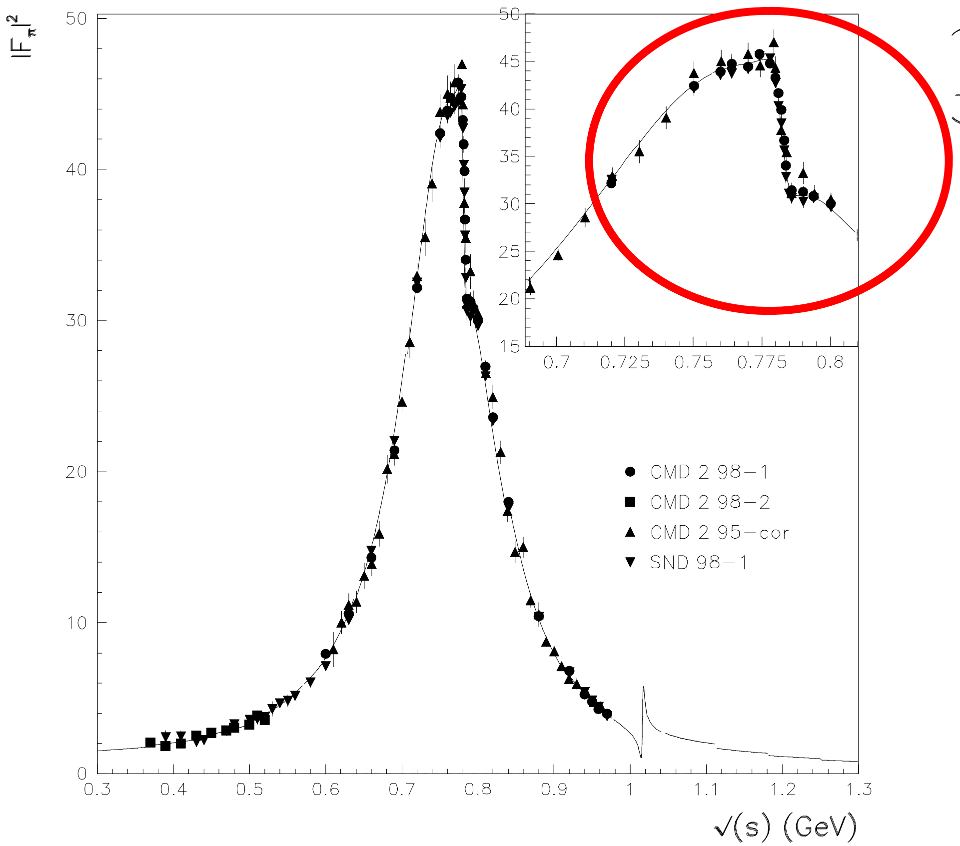
$\rho^0 \to \pi^+\pi^-$ [MeV]	144.5±0.6	149.4±1.0

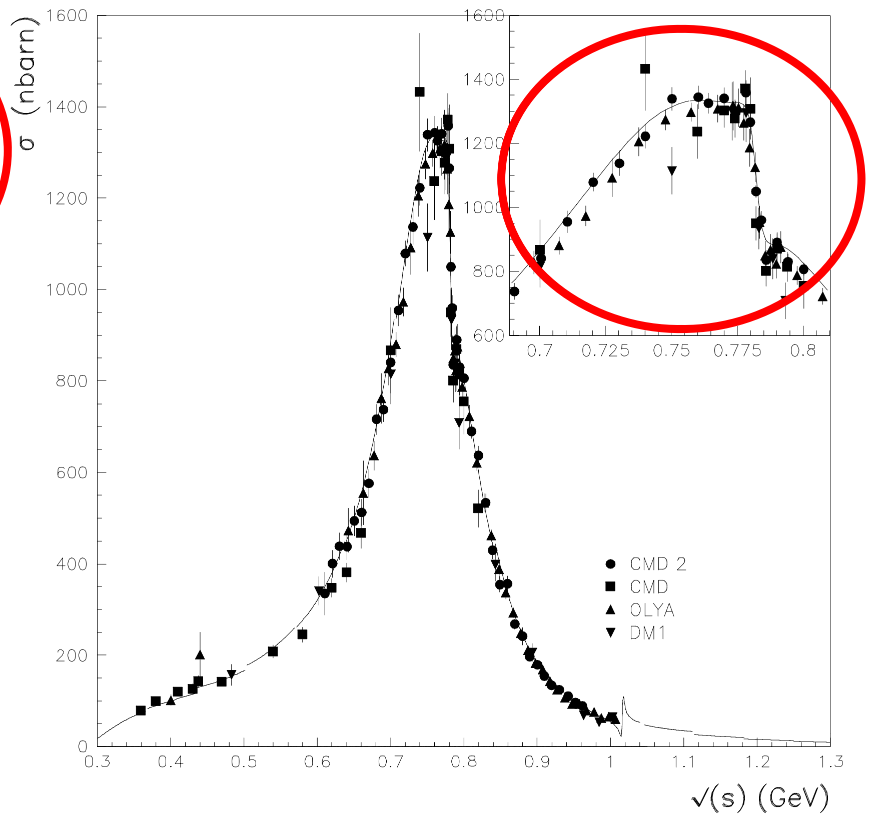
May change the global fit to ω branching ratios

Fit Of The ω Mass Region : Perfect

FF's New Data



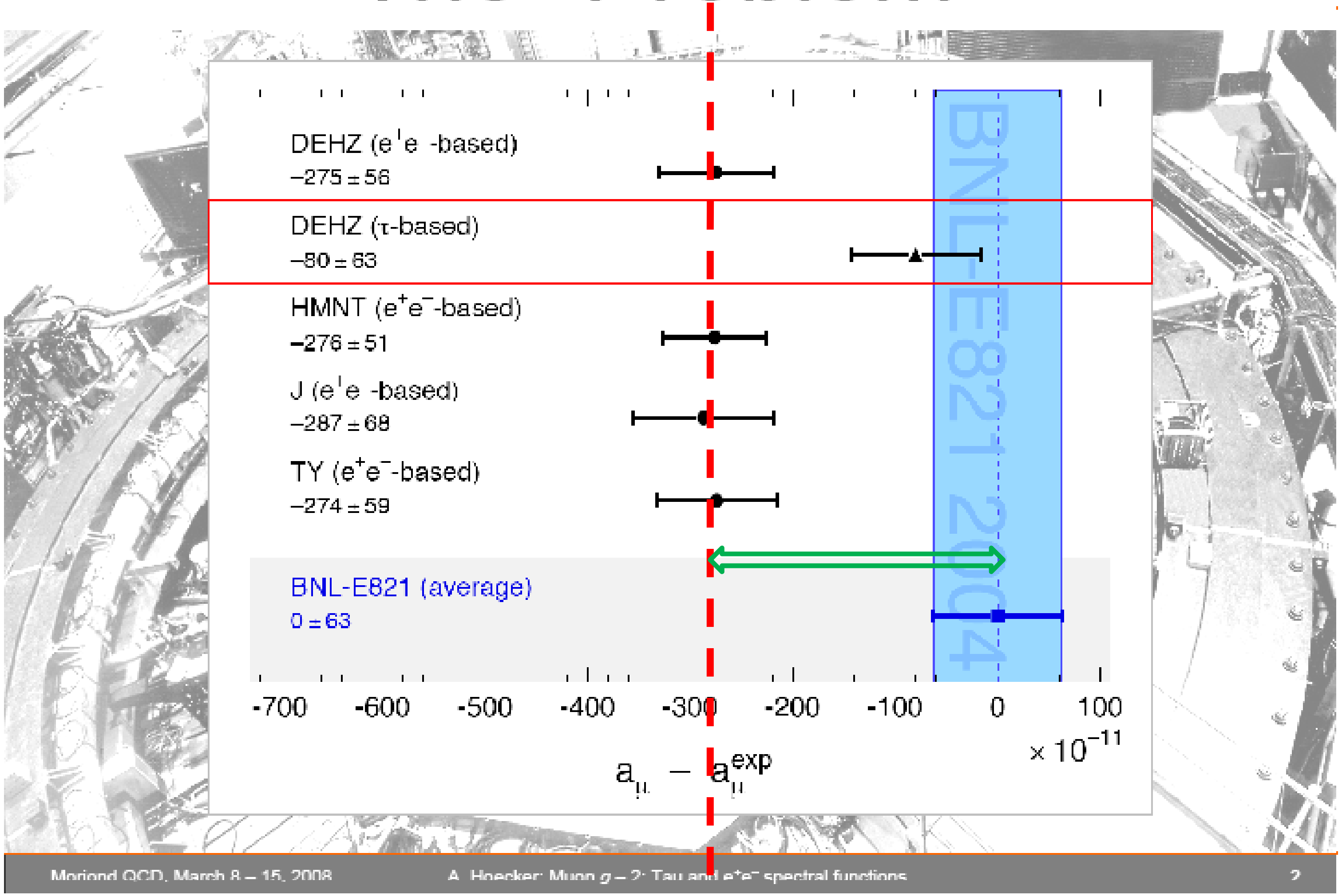
Cross Sections Old Data



Conclusions

- **No mismatch between e^+e^- and τ data (CVC is OK)**
- Radiative decays & pion form factor in e^+e^- annihilation ⟹ **predict the observed pion form factor in τ decay**
- Previously unaccounted for effects : I=0 (ω_I, ϕ_I) components inside the ρ^0 meson.
- The 3.3 σ discrepancy between prediction (using e^+e^- data) and the BNL measurement for the muon anomalous moment is confirmed

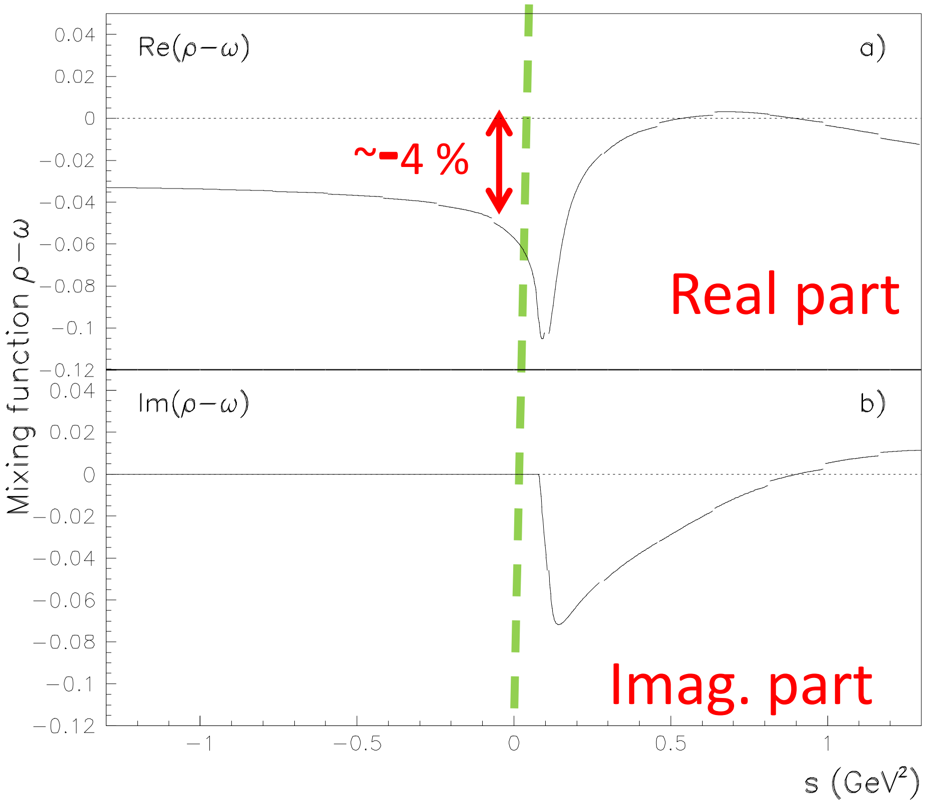
The “Problem”


DEHZ (e'e -based)
−275 ± 56
DEHZ (τ-based)
−80 ± 63
HMNT (e+e−-based)
−276 ± 51
J (e'e -based)
−287 ± 68
TY (e+e−-based)
−274 ± 59
BNL-E821 (average)
0 ± 63
BNL-E821 2004
-700 -600 -500 -400 -300 -200 -100 0 100
× 10^-11
a_μ − a_μ^exp
Moriond QCD, March 8 – 15, 2008
A. Hoecker: Muon g – 2: Tau and e+e− spectral functions
2

Additional Information

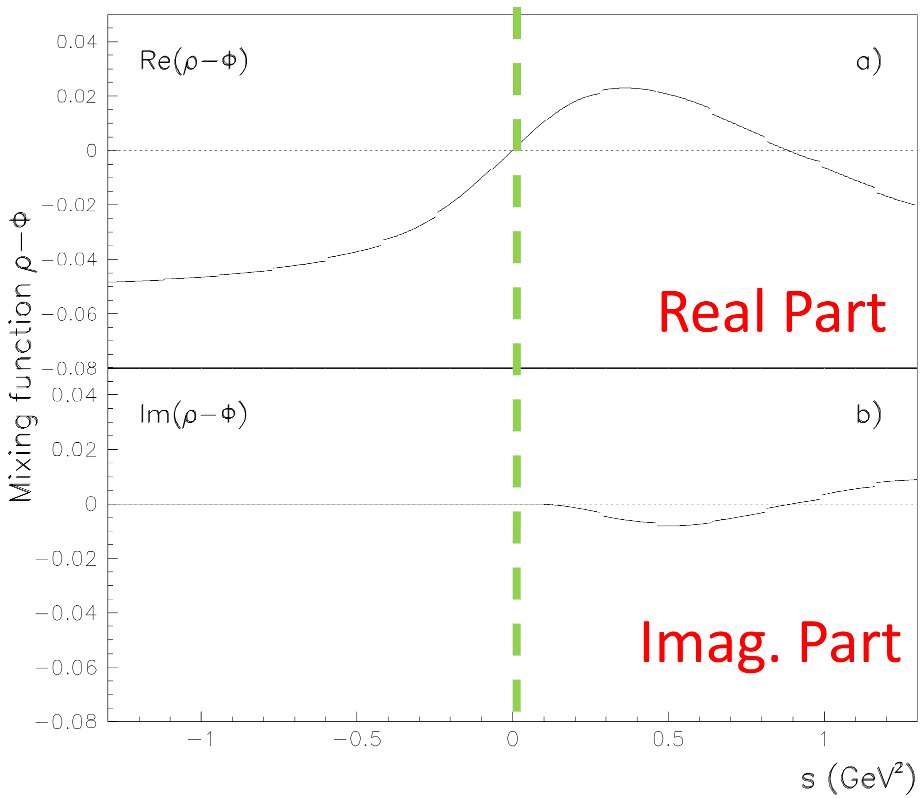
- THE MIXING « ANGLES »
- The (ρ,ω) and (ρ, ϕ) mixing «angles» are small (wrt 1) complex numbers
- The (ω, ϕ) mixing «angle» is real and small (wrt 1)

The (ρ, ω) Mixing «Angle»



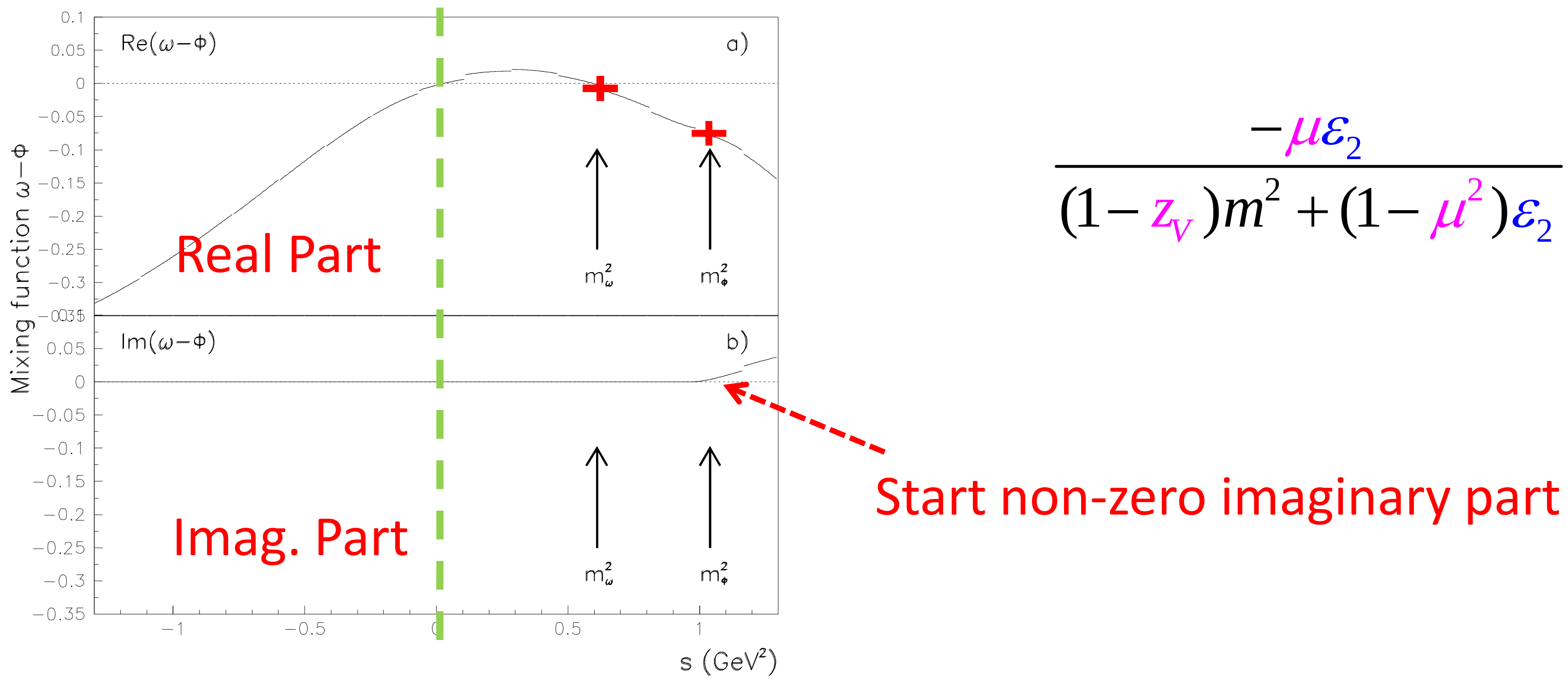
$$\frac{\varepsilon_1(s)}{\Pi_{\pi\pi}(s)-\varepsilon_2(s)}$$

The (ρ, ϕ) Mixing «Angle»



$$\frac{-\mu\varepsilon_1}{(1-z_V)m^2+\Pi_{\pi\pi}-\mu^2\varepsilon_2}$$

The (ω, ϕ) mixing «angle»

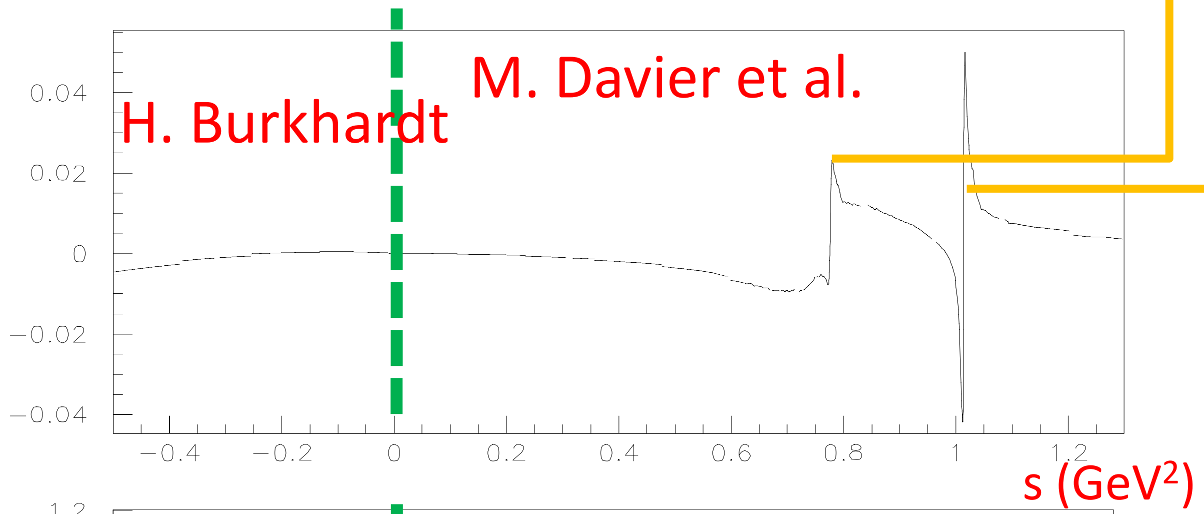


$$\frac{-\mu\varepsilon_2}{(1-z_V)m^2+(1-\mu^2)\varepsilon_2}$$

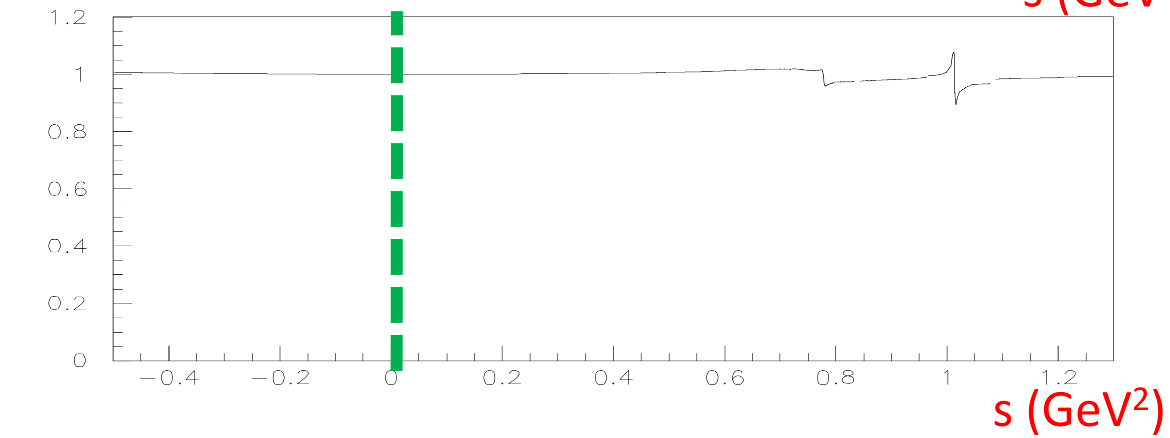
Hadronic Photon Vacuum Polarization

ω

ϕ



Photon Hadronic VP



Full Correction Factor
(Hadr. & Lept.)