

# Three-nucleon forces at neutron-rich extremes

Johannes Simonis

Institut für Kernphysik, TU Darmstadt

with Jason D. Holt, Javier Menéndez,  
Achim Schwenk (EMMI/TU Darmstadt)

Shell Model as a Unified View of Nuclear Structure  
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# Motivation

Why perform calculation of **medium-mass nuclei** with nuclear forces from **Chiral Effective Field Theory**?

- **No phenomenological fits**
- Understand phenomenological adjustments necessary for realistic interactions, e.g.  $G$ -matrix
- Hope for more controlled extrapolations for neutron-rich extremes

# Microscopic calculation of medium-mass nuclei

## Microscopic calculation of medium-mass nuclei including 3N forces

- Use **Chiral Effective Field Theory** (chiral EFT) interactions, includes naturally NN and 3N forces
- Perform a renormalization group evolution to  $V_{low k}$  **interaction** to enhance convergence of the MBPT calculation
- Apply **Many-Body Perturbation Theory** (MBPT) to obtain interactions to be used in **Shell Model** (SM) calculations

⇒ **3N forces** are naturally included

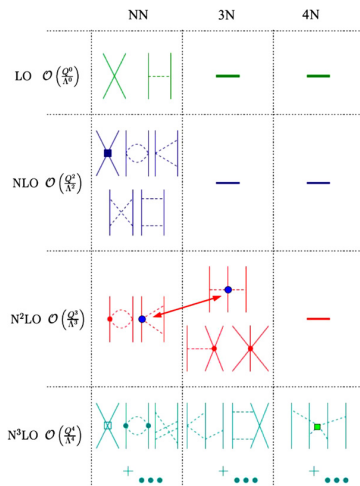
Shown necessary to reproduce properties of light nuclei

⇒ All the parameters that appear in the SM hamiltonian calculated from the input of the **microscopic interaction** (no fits!)

Test nuclear forces for stable and exotic nuclei

# NN+3N forces in Chiral EFT

## Systematic expansion for nuclear forces



- NN force couplings fitted to NN,  $\pi$ N data
- 3N force couplings:  $c_i$ 's consistent with NN;  $c_D, c_E$  fitted to few-body data:  ${}^3\text{H}$  binding energy,  ${}^4\text{He}$  charge radius
- Chiral EFT potentials for NN at N<sup>3</sup>LO and 3N at N<sup>2</sup>LO
- But: N<sup>3</sup>LO 3N  $\sim 1/3$  of N<sup>2</sup>LO 3N

Tews et al., arXiv:1206.0025

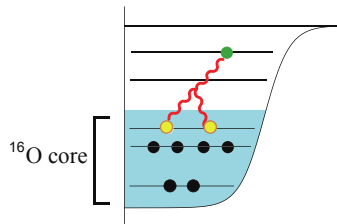
# Shell Model interactions

- Results with NN+3N forces included to 3rd order in MBPT
- In addition to standard sd and pf shell
  - O isotopes:  $sdf_{7/2}p_{3/2}$  valence space
  - Ca isotopes:  $pf_{9/2}$  valence space
- Full diagonalizations using ANTOINE  
Caurier et al., *RMP* **77** 427 (2005).
- MBPT with enlarged valence space to include as many non-perturbative orbits as possible
- Valence space beyond one major shell: center-of-mass contamination?
  - Singular value decomposition Hagen et al., *PRC* **82** 034330 (2010).
  - Lee-Suzuki trafo to major shell,
  - In-medium SRG Tsukiyama et al., *PRC* **85** 061304 (2012).

# 3N forces for valence-shell theories

- Contribution to valence neutron interactions

Effective one body



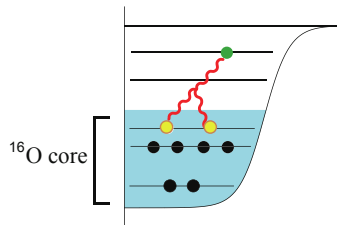
$$\langle a | V_{3N,eff} | a' \rangle = \frac{1}{2} \sum_{\alpha, \beta \in \text{core}} \langle \alpha \beta a | V_{3N} | \alpha \beta a' \rangle$$

⇒ (Effective) single-particle energies  
(SPE)

## 3N forces for valence-shell theories

- Contribution to valence neutron interactions

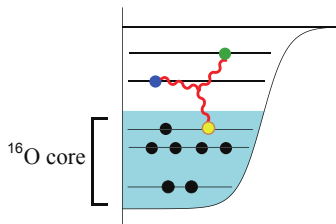
Effective one body



$$\langle a | V_{3N,eff} | a' \rangle = \frac{1}{2} \sum_{\alpha, \beta \in core} \langle \alpha \beta a | V_{3N} | \alpha \beta a' \rangle$$

⇒ (Effective) single-particle energies  
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Effective two body

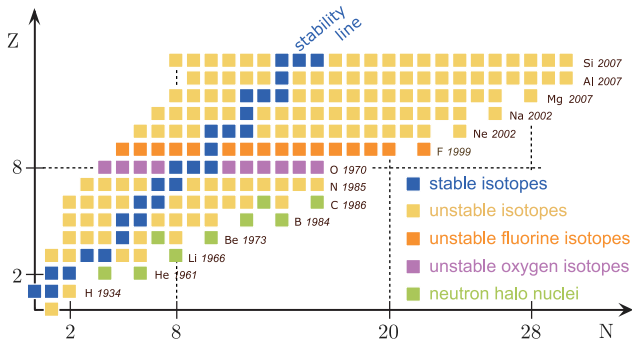


$$\langle ab | V_{3N,eff} | a' b' \rangle = \sum_{\alpha \in core} \langle \alpha ab | V_{3N} | \alpha a' b' \rangle$$

⇒ (Effective) two-body matrix  
elements (TBME)

# The oxygen anomaly

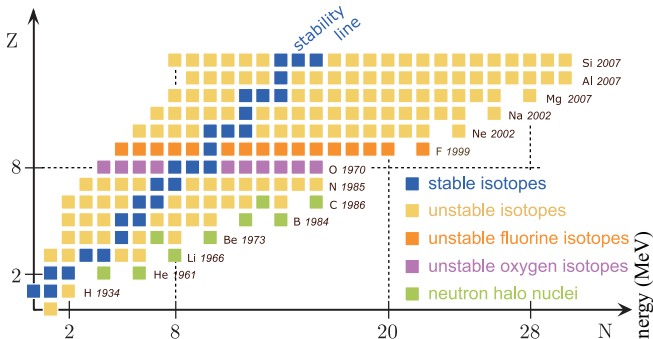
Where is the neutron dripline?



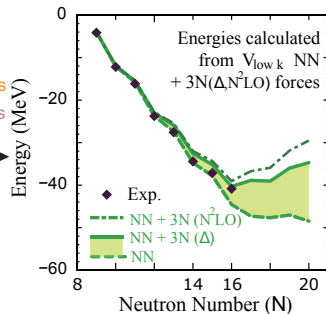


# The oxygen anomaly

Where is the neutron dripline?



- **NN forces too attractive**
- Inclusion of **3N force** leads to **repulsion**  
 $\Rightarrow$  Correct prediction of dripline



Otsuka et al., *PRL* **105** 032501 (2010).

# 3N forces for valence-shell theories

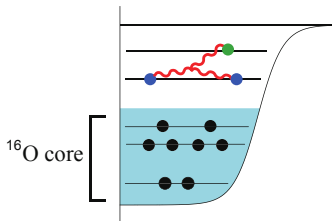
When going to neutron-rich extremes must include

- Residual three body

Friman, Schwenk, arXiv:1101.4858.

- Estimated to be suppressed  $N_v/N_c$

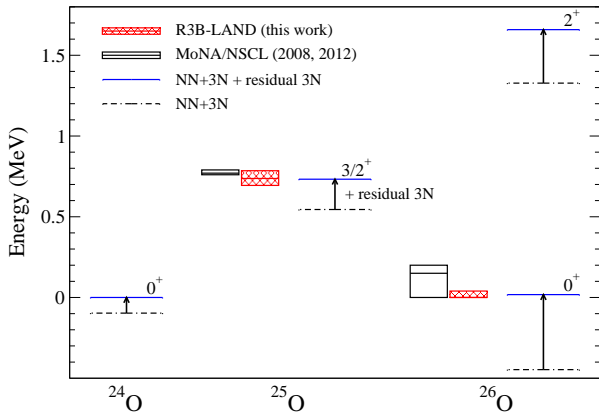
$$\Delta E \sim \frac{N_v}{N_c} E_{2\text{-body}}^{\text{normal-ordered}}$$



- Becomes important for most neutron-rich isotopes
- Included perturbatively using wave function from ANTOINE

$$\Delta E = \langle \Psi | V_{3N} | \Psi \rangle$$

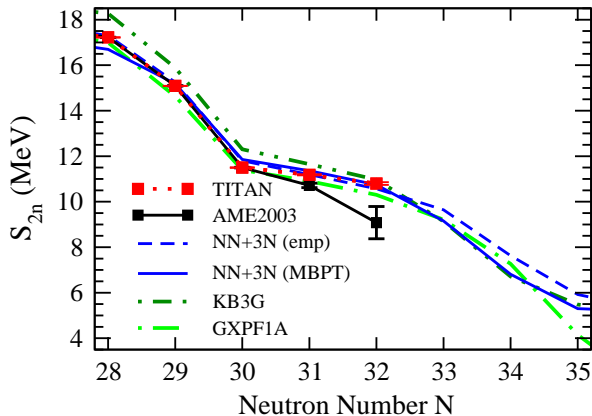
# Theory vs. experiment - arXiv:1209.0156.



- Residual three-body contribution is repulsive, small compared to normal-ordered 3N
- Increases with neutron number as expected
- Very good agreement with resonances in  $^{25,26}\text{O}$

Hoffman et al., *PRL* **100** 152502 (2008).; Lunderberg et al., *PRL* **108** 142503 (2012).

# Two-neutron separation energies of Ca isotopes



- New mass measurement at **TITAN** for  $^{51,52}\text{Ca}$
- 1.74 MeV deviation for  $^{52}\text{Ca}$  from AME2003
- KB3G/GXPF1A in good agreement
- Effect of residual three body? smaller than in O as expected from  $N_v/N_c$

Gallant et al., *PRL* **109** 032506 (2012).

# Summary and Outlook

Microscopic calculation based on chiral EFT (NN+3N forces) with inclusion of residual three-body contribution **for the first time**

Contribution of residual three body **small**, but **increases with neutron number** in the shell model up to 0.4 MeV in  $^{26}\text{O}$

Very good agreement with experiment for neutron-rich oxygen isotopes

## Outlook:

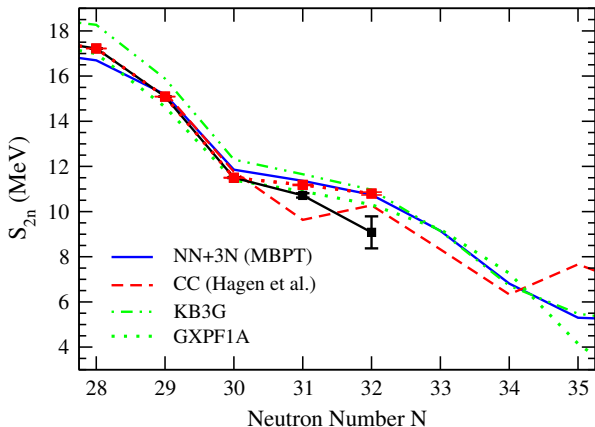
Explore effect of residual three body in Ca and other chains

Error estimate by cutoff- and  $c_i$ -variation

Inclusion of  $\text{N}^3\text{LO}$  3N forces

# Thank you for your attention!

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