Shell-model description of isospin-symmetry breaking in nuclei

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Shell Model as Unified View of Nuclear Structure
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Physics motivation

Isospin symmetry is broken in nuclear physics

- Coulomb force
- Charge-dependent nuclear forces $v_{pp} \neq v_{nn} \neq v_{pn}^{T=1}$

Experimental evidence on isospin-symmetry breaking

- Splittings of isobaric multiplets
- Isospin-forbidden processes (isospin-forbidden proton emission, Fermi β-decay to non-analogue states, $E1$-transitions in $N = Z$ nuclei, etc)

Importance of precise theoretical description of the isospin-symmetry breaking

Tests of fundamental symmetries underlying the Standard Model, such as in superallowed $0^+ \to 0^+$ decay and other weak interaction processes in nuclei

Aim of the present study is to construct a shell-model isospin non-conserving Hamiltonian

Shell-model description of isospin-symmetry breaking

- We start with an isospin-symmetry invariant shell-model Hamiltonian $[\hat{H}, \hat{T}] = 0$

$$\hat{H}\Psi_{TT_z} \equiv (\hat{H}_0 + \hat{V})\Psi_{TT_z} = E_T\Psi_{TT_z}, \quad \Psi_{TT_z} = \sum_k a_{T\,k}\Phi_{TT_z\,k}$$

W.E. Ormand, B.A. Brown NPA491 (1989)

Y.H. Lam, Ph.D. thesis, CENBG (2011); Y.H. Lam, N. Smirnova, E. Caurier, submitted (2012).

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- We consider an isospin-symmetry non-conserving term

$$\hat{V}_{INC} = \lambda_C \hat{V}_C + \lambda_\pi \hat{V}_\pi + \lambda_\rho \hat{V}_\rho + \lambda_0 \hat{V}^{T=1} + \hat{H}_0^{IV}$$

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- Within perturbation theory:

$$\langle \Psi_{TT_z} | \hat{V}_{INC} | \Psi_{TT_z} \rangle = E^{(0)}(\alpha, T) + E^{(1)}(\alpha, T) T_z + E^{(2)}(\alpha, T) \left[3T_z^2 - T(T+1) \right]$$

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- The strength parameters are obtained in a fit to experimental coefficients of the Isobaric Mass Multiplet Equation (IMME):

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T) T_z + c(\alpha, T) T_z^2 \,,$$

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- Finally, we solve the eigenproblem for an isospin non-conserving Hamiltonian $[\hat{H}_{INC}, \hat{T}] \neq 0$:

$$\hat{H}_{INC} \Psi(\alpha_p, \alpha_n) \equiv (\hat{H}_0 + \hat{V} + \hat{V}_{INC}) \Psi(\alpha_p, \alpha_n) = E \Psi(\alpha_p, \alpha_n)$$

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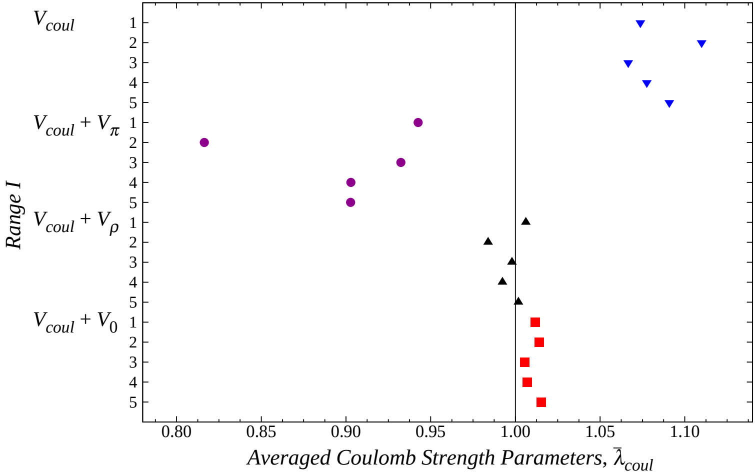
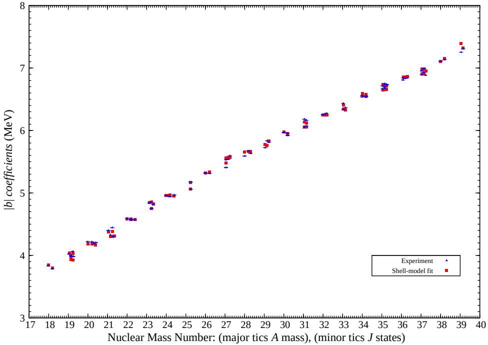
Coulomb strength parameters



Results of the fit to *b* coefficients in *sd*-shell

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T)T_z + c(\alpha, T)T_z^2$$

USD (*Brown, Wildenthal, 1988*) or USDA/USDB (*Brown, Richter, 2006*) plus the INC term ($\hat{V}_C$, $\hat{V}_\rho$ or $\hat{V}^{T=1}$, $\hat{H}_0^{IV}$)

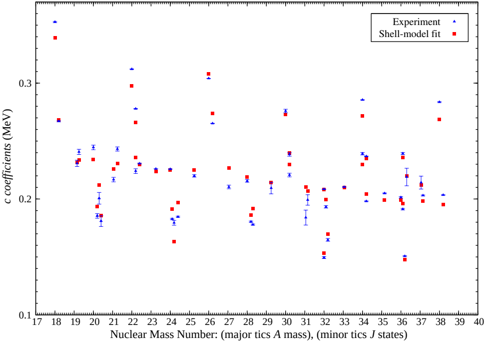


- b coefficients $(v_{pp} - v_{nn})$; 81 data points ($T = 1/2, 1, 3/2, 2$);
- rms $\approx$ 32 keV

Results of the fit to c coefficients in sd-shell

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T)T_z + c(\alpha, T)T_z^2$$

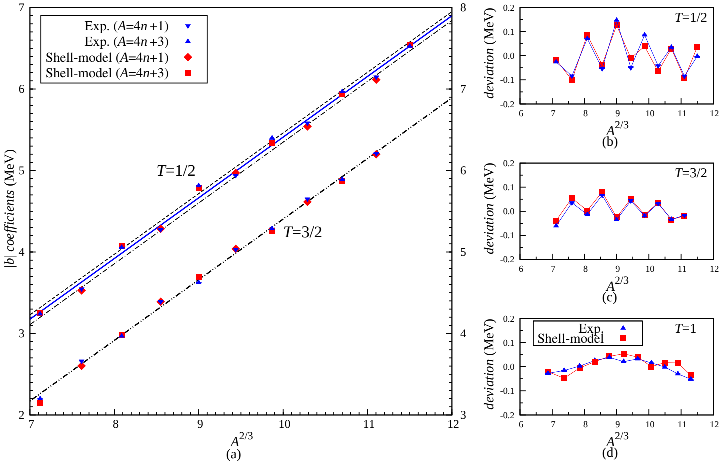
USD (*Brown, Wildenthal, 1988*) or USDA/USDB (*Brown, Richter, 2006*) plus the INC term ($\hat{V}_C$, $\hat{V}_\rho$ or $\hat{V}^{T=1}$, $\hat{H}_0^{IV}$)



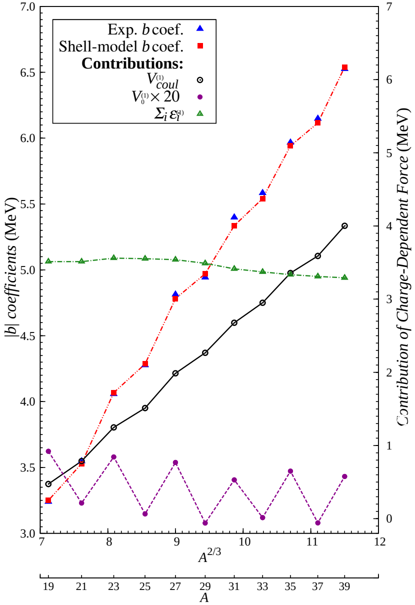
- c coefficients ($v_{pp} + v_{nn} - 2v_{pn}$); 51 data points ($T = 1, 3/2, 2$);
- rms $\approx$ 10 keV

Staggering of *b*-coefficients of *sd*-shell nuclei

J. Jänecke (1966,1969); K.T. Hecht (1968); Y.H. Lam, N. Smirnova, E. Caurier (2012)



Contributions of various INC terms to *b* coefficients



Staggering of *c*-coefficients of *sd*-shell nuclei

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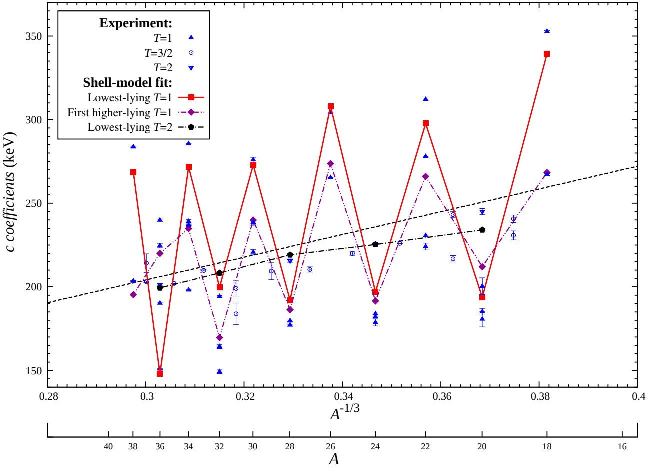
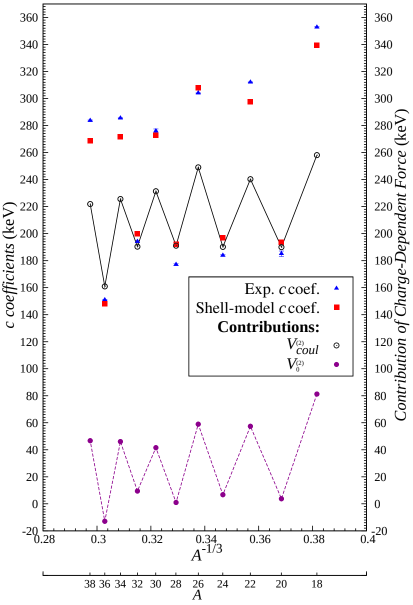


Contributions of various INC terms to c coefficients



1. IMME beyond a quadratic form

Quadratic IMME has been deduced at first order in perturbation theory. Higher-order terms in T_z may be present due to

- isospin-symmetry breaking three-body (or four-body) interactions;
- Coulomb effects at second order in perturbation theory (isospin mixing)

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T)T_z + c(\alpha, T)T_z^2 + d(\alpha, T)T_z^3 + e(\alpha, T)T_z^4$$

Theoretical estimations of $d \sim 1$ keV.
E.M. Henley, C.E. Lacy (1969); J. Jänecke (1969); G.F. Bertsch, S. Kahana (1970)

$J^\pi = 0^+$ quintet ($T = 2$) in $A = 32$ isobars

Nuclide	Mass excess (keV)	References
^{32}Ar	-2200.4 (18)	K. Blaum et al (2003)
^{32}Cl	-8288.8 (10)	C. Wrede et al (2010); A. Kankainen et al (2010)
^{32}S	-13967.57 (28)	W. Shi et al (2005); S. Triambak et al (2006)
^{32}P	-19232.46 (15)	Redshow et al (2008)
	-19232.78 (20)	AME03, P.M. Endt (1998)
^{32}Si	-24080.86 (77)	A. Paul et al (2001)
	-24080.92 (5)	M. Redshow et al (2008)
	-24077.69 (30)	A.A. Kwiatkowski et al (2009)

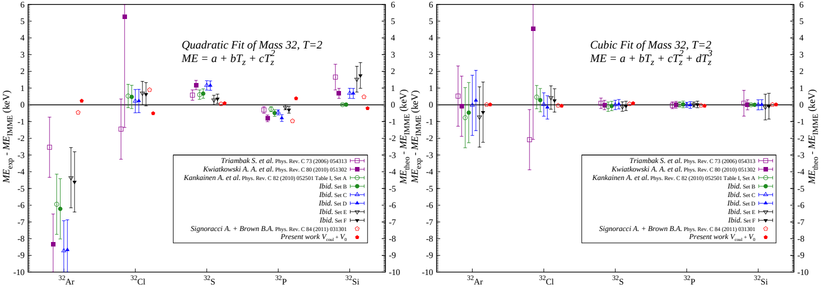
Experimental and theoretical d coefficients for the 0^+ quintet in $A = 32$

	$\chi^2/n_{\text{quadr.}}$	χ^2/n_{cubic}	d (keV)
S. Triambak et al (2006)	6.5	0.77	0.54 (16)
A. Kwiatkowski et al (2009)	30.6	0.48	1.00 (9)
Set A from A. Kankainen et al (2010)	9.9	0.86	0.52 (12)
Set B *Ibid.*	12.3	0.31	0.60 (13)
Set C *Ibid.*	28.3	0.002	0.90 (12)
Set D *Ibid.*	30.8	0.09	1.00 (13)
Set E *Ibid.*	6.5	0.74	0.51 (15)
Set F *Ibid.*	8.3	0.28	0.62 (16)
A. Signoracci, B.A. Brown (2011)	1.09	0.005	0.39
Present work (2012)	0.26	0.02	-0.19

Comparison of b, c, d, and e coefficients for the 0^+ quintet in $A = 32$

		b, c (keV)	b, c, d (keV)	b, c, e (keV)	b, c, d, e (keV)
Experiment	b	−5471.85 (27)	−5472.83 (29)	−5470.45 (29)	−5472.64 (68)
	c	208.55 (14)	207.12 (23)	204.92 (23)	206.89 (75)
	d	—	0.89 (11)	—	0.83 (22)
	e	—	—	0.69 (11)	0.06 (19)
	χ^2/n	32.15	0.10	13.80	
Signoracchi, Brown (2011)	b	−5417.7	−5419.0	−5417.7	−5419.0
	c	209.1	209.1	209.0	209.0
	d	—	0.39	—	0.39
	e	—	—	0.03	0.03
	χ^2/n	1.09	0.006	2.17	—
Present work	b	−5464.4	−5463.8	−5464.4	−5463.8
	c	207.6	207.6	207.4	207.4
	d	—	−0.19	—	−0.19
	e	—	—	0.045	0.045
	χ^2/n	0.26	0.017	0.50	—

IMME beyond the quadratic form in the $A = 32$ quintet



Isospin-mixing leads to breaking of the quadratic IMME.

A. Signoracchi, B.A.Brown, PRC (2011).

Y.H. Lam, N. Smirnova, E. Caurier, submitted to PRC (2012)

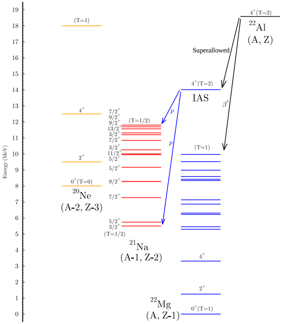
Analysis of 0^+ quintets in $A = 24$ and $A = 28$

		b, c (keV)	b, c, d (keV)	b, c, e (keV)	b, c, d, e (keV)
$A = 24$ Experiment	b	-4178.8 (9)	-4177.7 (11)	-4176.0 (11)	-4175.6 (32)
	c	225.9 (4)	223.4 (16)	221.9 (16)	221.7 (30)
	d	—	0.91 (58)	—	-0.24 (176)
	e	—	—	0.61 (58)	0.75 (107)
	χ^2/n	1.48	0.49	0.02	
$A = 24$ Theory	b	-4179.0	-4178.95	-4179.0	-4178.95
	c	224.3	224.3	219.7	219.7
	d	—	-0.02	—	-0.02
	e	—	—	**1.04**	1.04
	χ^2/n	8.9	0.24	**0.0008**	
$A = 28$ Experiment	b	-4808 (2)	-4803 (4)	-4800 (4)	-4810 (16)
	c	216 (1)	208 (7)	207 (7)	212 (12)
	d	—	2.77 (229)	—	9.35 (1354)
	e	—	—	1.32 (229)	-3.49 (707)
	χ^2/n	0.85	0.24	0.48	
$A = 28$ Theory	b	-4869.7	-4869.6	-4869.7	-4869.6
	c	216.8	216.8	204.5	204.5
	d	—	-0.05	—	-0.05
	e	—	—	**2.78**	2.78
	χ^2/n	63.6	31.8	**0.007**	

Y.H. Lam, N. Smirnova, E. Caurier (in preparation).

Experimental study
of βp emission:

- *M.D. Cable et al (1982)*
  $4^+(IAS) \rightarrow 3/2^+_{gs}, 5/2^+_1$
- *B. Blank et al (1997)*
  $4^+(IAS) \rightarrow 7/2^+_1$
- *N.L. Achouri et al (2006)*
  $4^+(IAS) \rightarrow 7/2^+_1, 9/2^+_1$



Beta-delayed proton emission from ^{22}Al

Present results: from USD interaction and INC term.
θ_l^2 are spectroscopic factors, Γ_p are proton widths, $\Gamma_p = 2\sum_l \theta_l^2 \gamma^2 P_l(Q_p)$

^{21}Na	E_{exc}^{th}	Present work				Brown (1990)			
	(MeV)	$10^4\theta_l^2$ $l=0$	$10^4\theta_l^2$ $l=2$	Γ_p (keV)	BR (%)	$10^4\theta_l^2$ $l=0$	$10^4\theta_l^2$ $l=2$	Γ_p (keV)	BR (%)
$3/2^+$	0.0		0.090(2)	0.04(0)	1.6(0)		0.13	0.055	5.3
$5/2^+$	0.236		0.267(4)	0.11(0)	4.3(1)		0.18	0.071	6.9
$7/2^+$	1.779	0.009(4)	4.73(21)	1.21(6)	45.7(22)	0.04	1.71	0.412	40.0
$9/2^+$	2.780	0.006(0)	2.54(23)	0.38(4)	15.4(14)	0.09	0.56	0.144	14.0
$5/2^+$	3.694		0.28(7)	0.02(1)	1.0(2)		0.24	0.017	1.6
$11/2^+$	4.428		1.91(30)	0.09(1)	3.7(6)		1.22	0.041	4.0
$5/2^+$	4.556		0.37(5)	0.02(0)	0.6(1)		1.34	0.043	4.1
$3/2^+$	4.785		0.25(2)	0.01(0)	0.3(0)		0.49	0.012	1.1
$7/2^+$	5.328	0.184(4)	0.72(17)	0.08(1)	3.3(2)	0.00	0.11	0.001	0.1
$3/2^+$	5.784		0.10(0)	0.00(0)	0.0(0)		0.07	0.000	0.0
$9/2^+$	6.078	0.90(5)	0.17(2)	0.18(1)	7.1(4)	0.10	0.07	0.008	0.8
$13/2^+$	6.141		0.54(3)	0.00(0)	0.1(0)		0.78	0.004	0.4
$9/2^+$	6.192	1.05(3)	6.9(2)	0.21(1)	8.5(2)	2.29	5.69	0.169	16.5
$7/2^+$	6.274	1.02(7)	14.2(4)	0.21(1)	8.4(5)	0.40	11.90	0.043	4.1

Remark: USDA interaction produces strong mixing for the 4^+ ($T = 2$ IAS) in ^{22}Mg.

Summary and Outlook

- We constructed a very precise **INC shell-model Hamiltonian** in *sd* shell with a large potential of applications.
- The INC Hamiltonian accurately describes the **staggering** of the IMME b and c coefficients with mass numbers.
- The INC Hamiltonian allows to study the **validity of the quadratic IMME** in quartets and quintets.
- **Isospin-forbidden nucleon emission** branching ratios (like in ^{22}Al) are very sensitive to the detailed structure of the INC potential.
- Isospin-mixing correction to superallowed β decay (see next talk by Y.H. Lam).
- Extension to *sdpf* and *pf* shell nuclei is in progress.
- Experimental information on proton-rich nuclei with $N \sim Z$ (displacement of energy levels in mirror nuclei, Q-values, lifetimes, branching ratios, spectroscopic factors and partial widths isospin-forbidden particle emission, isospin-forbidden Fermi decay, . . .) is important to constrain the model.