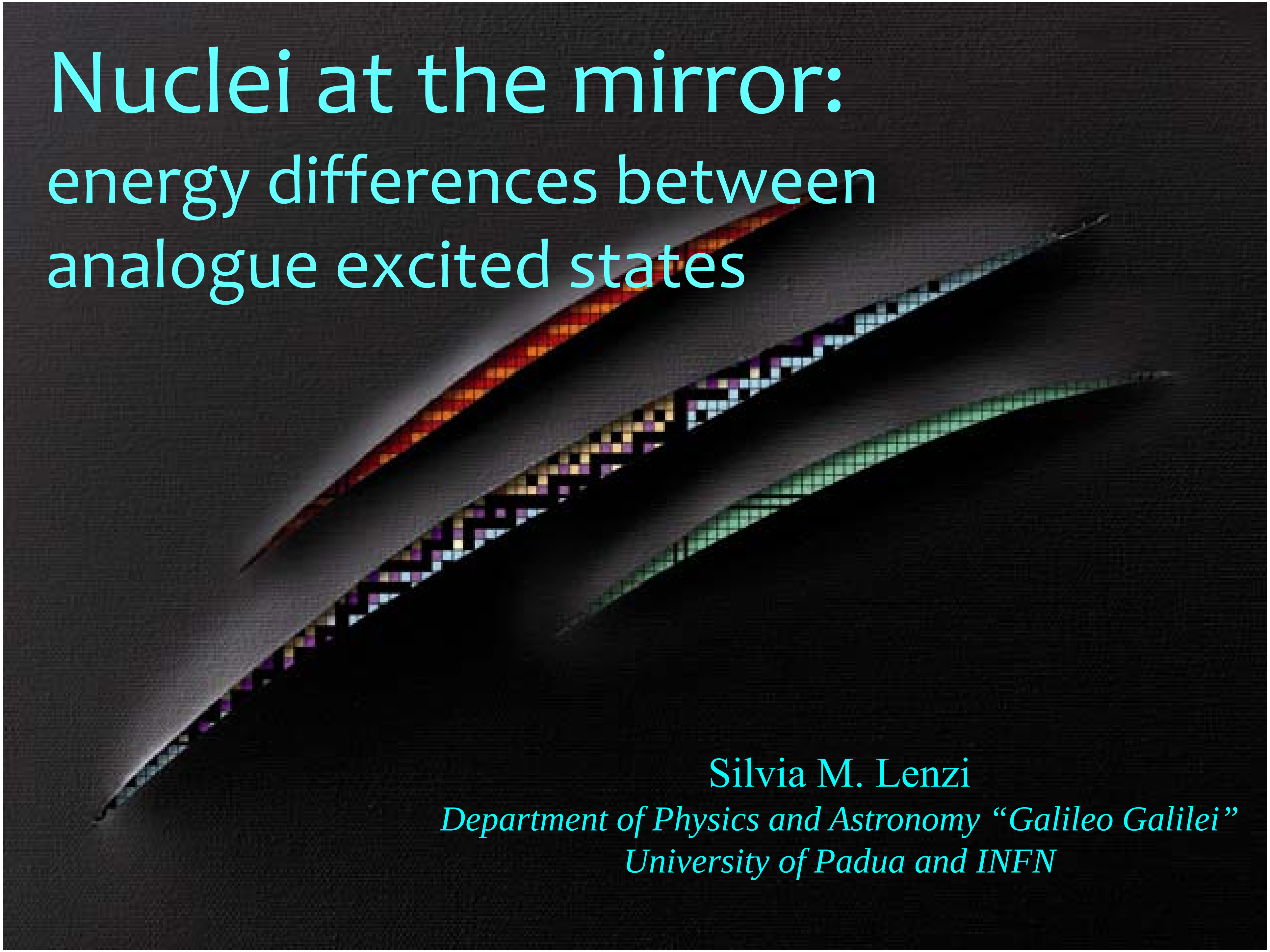
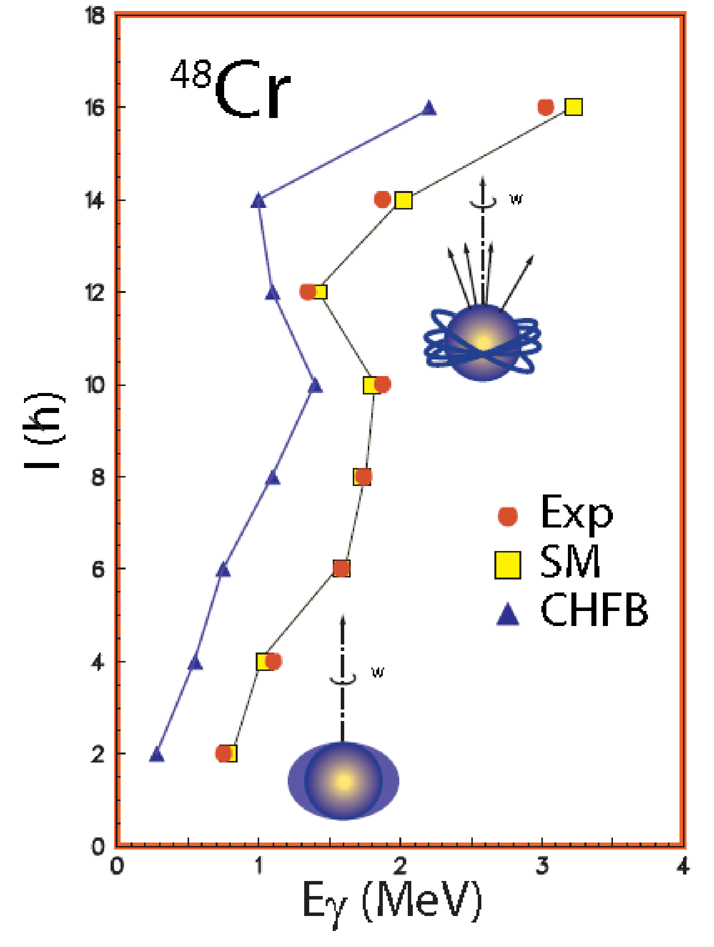


Nuclei at the mirror:

energy differences between analogue excited states

Silvia M. Lenzi
Department of Physics and Astronomy “Galileo Galilei”
University of Padua and INFN

The success of Shell Model

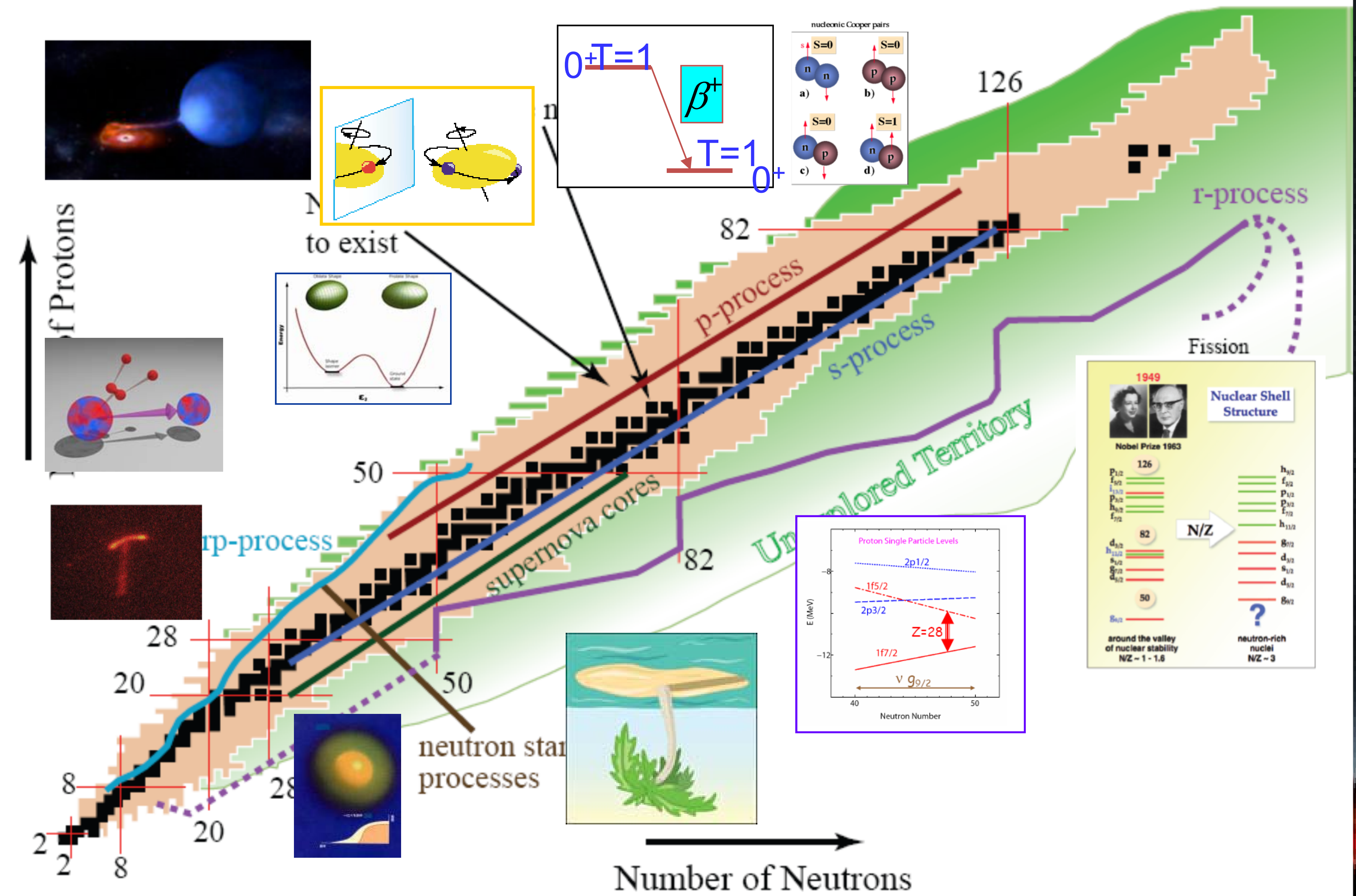


Rotational motion in ^{48}Cr

Theory: E. Caurier et al., Phys. Rev. Lett. 75 (1995) 225.

Experiment: SML et al., Z. Phys. A354 (1996) 117.

New phenomena far from stability



The proton-rich side

- Which components of the NN interaction break the isospin symmetry?
- How does isospin symmetry work with shape coexistence?
- Will new decay modes be observed far from stability?
- Do we understand the interplay of T=0 and T=1 pairing?
- Do proton skins / halos exist?
- Connection with other fields: Astrophysics, Standard Model

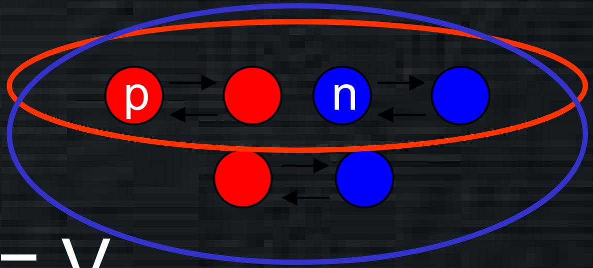
Neutron-proton exchange symmetry

Charge symmetry : $V_{pp} = V_{nn}$

Nuclear force slightly asymmetric

e.g. scattering lengths

Machleidt and Muther, PRC 58(2001)1393



Charge independence: $(V_{pp} + V_{nn})/2 = V_{np}$

some charge dependence exists

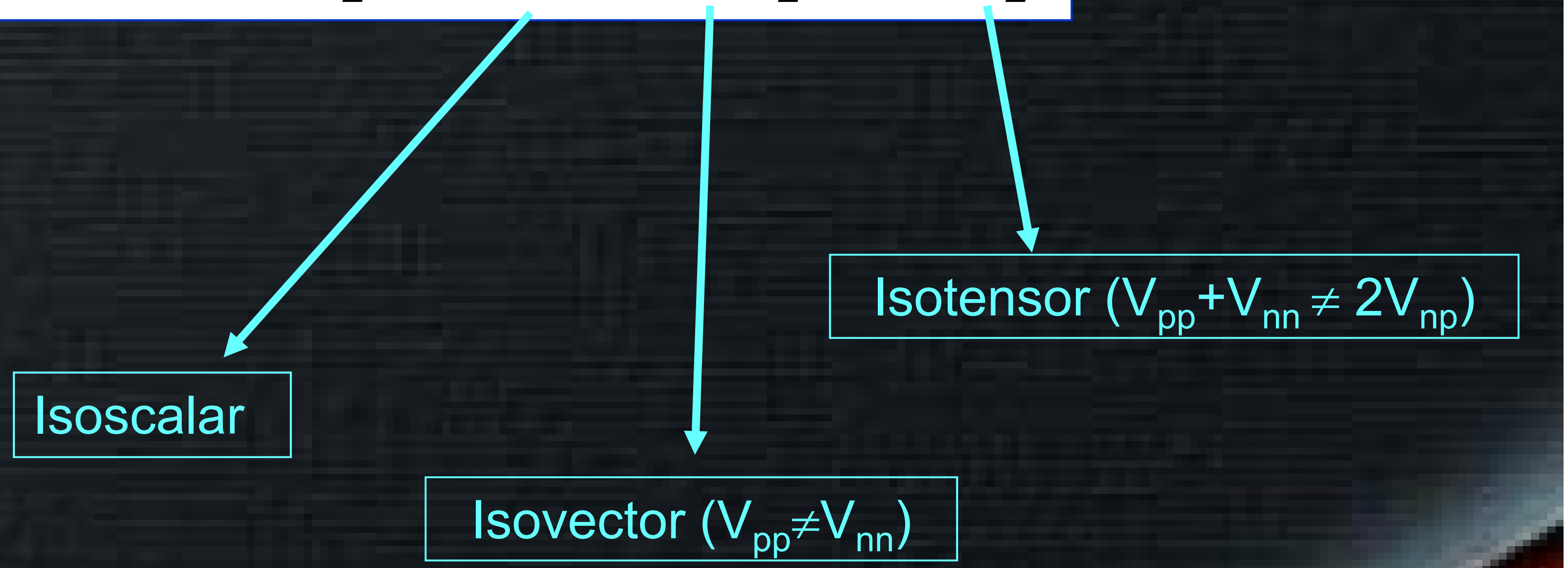
González Trotter et al., PRL 83 (1999)3788

Deviations are small

Isobaric Multiplet Mass Equation

A simple relationship between mass and T_z for a set of analogue states (E.P.Wigner 1957)

$$BE(\alpha T T_z) = a + bT_z + cT_z^2$$

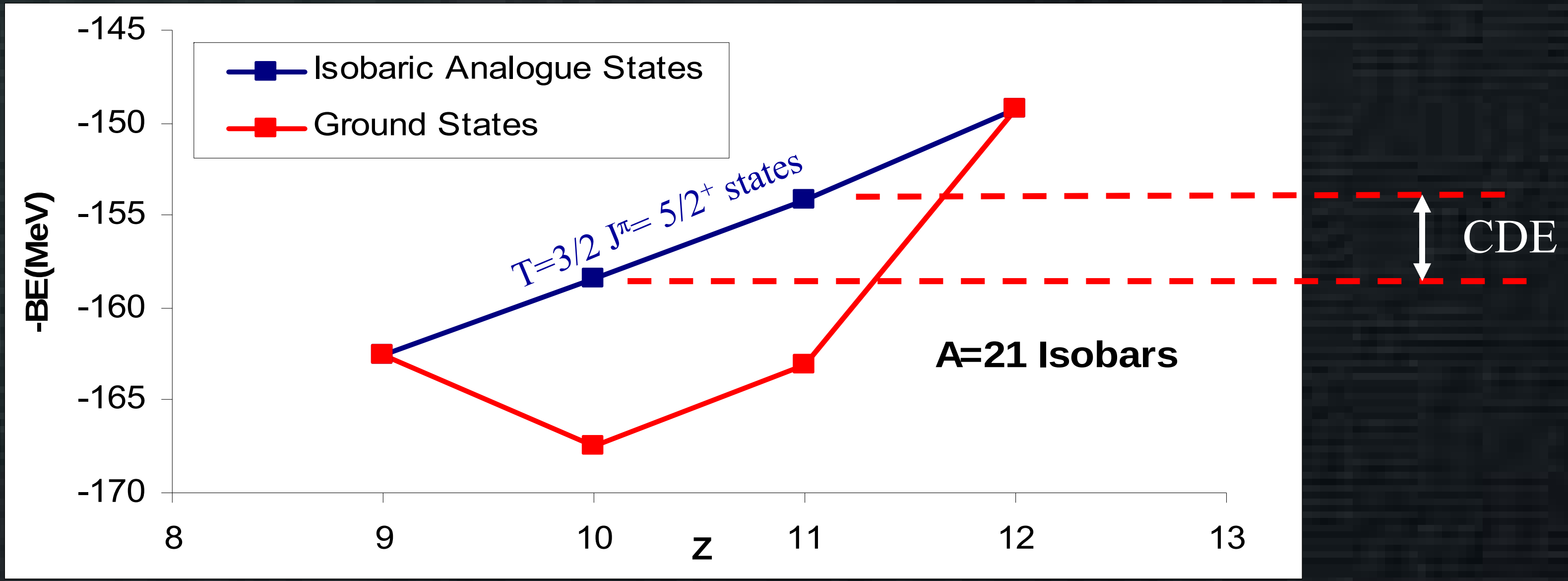


Isoscalar

Isovector ($V_{pp} \neq V_{nn}$)

Isotensor ($V_{pp} + V_{nn} \neq 2V_{np}$)

Coulomb displacement energies



CDE : For any two members of a multiplet of isospin T, transformed through exchange of k protons for neutrons is given by

$$(CDE)_{\alpha,T,T_z} = M_{\alpha,T,T_z+k} - M_{\alpha,T,T_z} + k\Delta M_{np}$$

(α = Isobaric analogue state, defined by A, T, J)

The Nolen-Schiffer Anomaly

IMME works well, but…reproducing magnitude of coefficients → historical problem

For two adjacent nuclei,

$$\begin{aligned}\mathrm{CDE}_{\alpha,T,T_z} &= M_{\alpha,T,T_z} - M_{\alpha,T,T_z+1} + \Delta M_{np} \\ &= -b - c(2T_z+1) + \Delta M_{np}\end{aligned}$$

Nolen & Schiffer (1969) calculated CDE for wide range of isobaric multiplets…

Used independent-particle models (with exchange term) → Corrected for other phenomena (e.g. electromagnetic spin-orbit, magnetic effects, core-polarisation, isospin impurity, etc.).

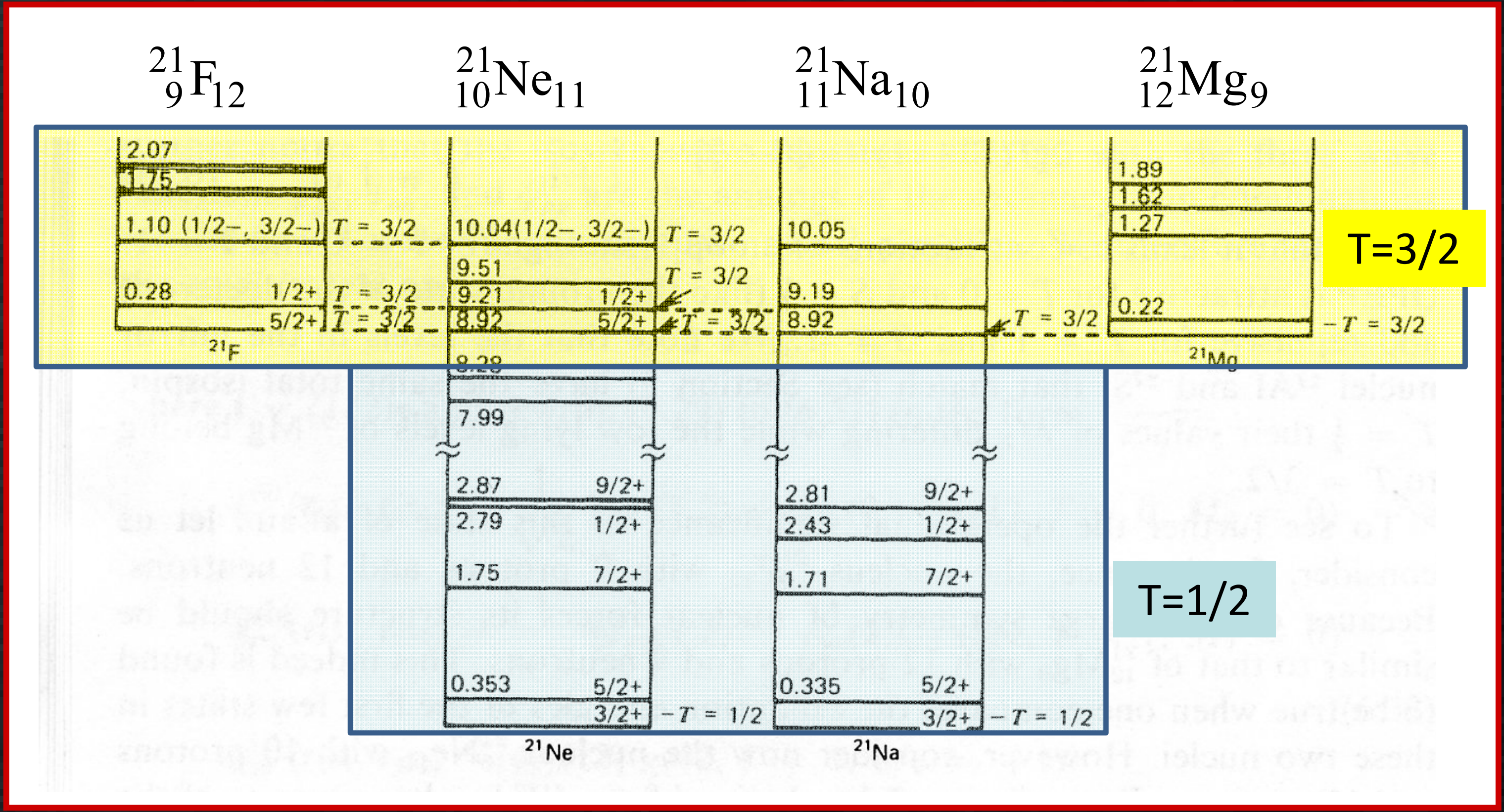
But…magnitude of predicted CDE always ~5-8% lower than experimental values. Underestimated by ~500 keV!

Auerbach (1983) improves the theoretical description but the anomaly remained

Duflo and Zuker (2002) reduce the difference to ~100-200 keV

Energy differences between excited states

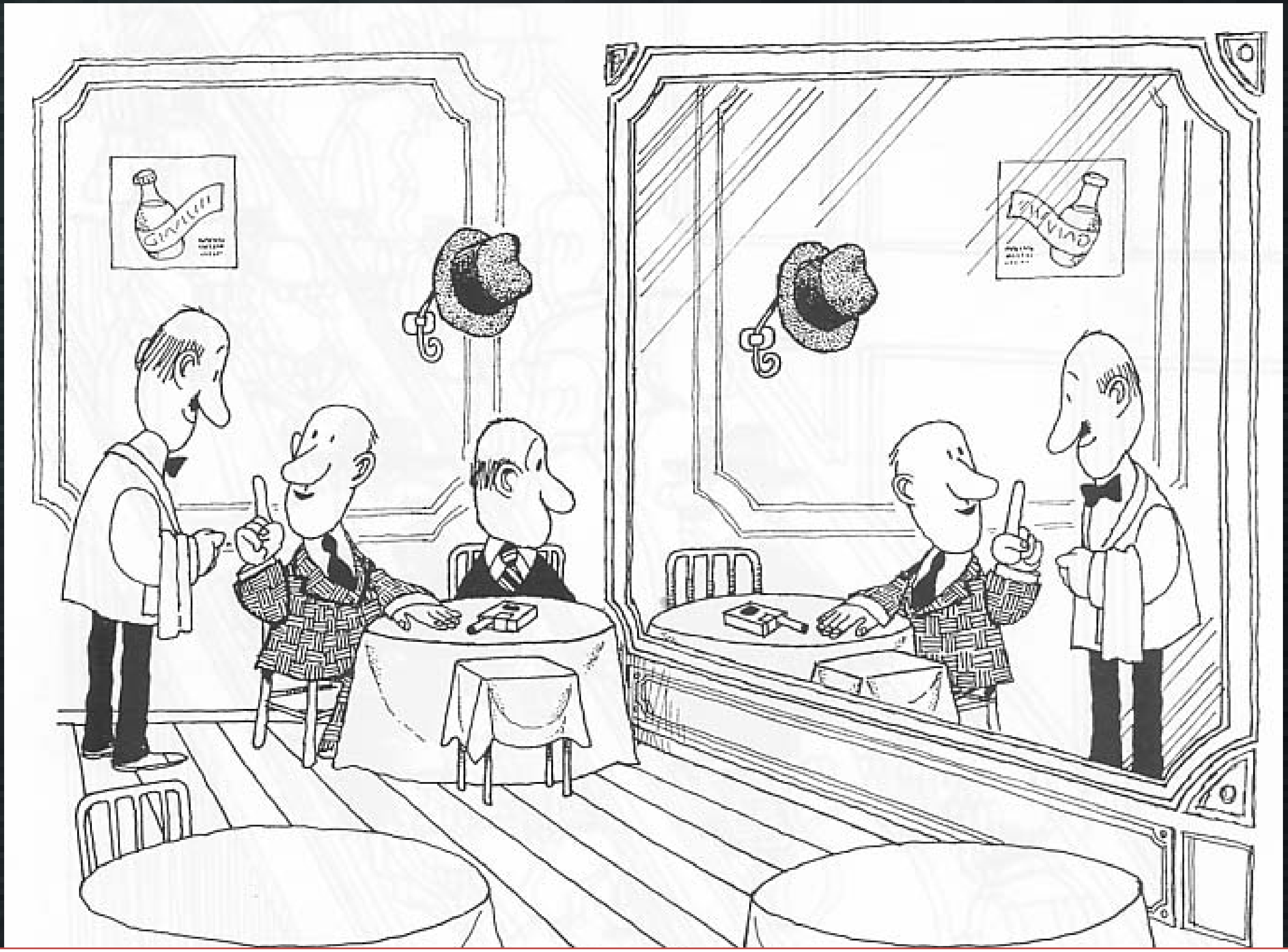
Analogue excited states



LARGE differences in mass/binding energy mainly due to Coulomb effects

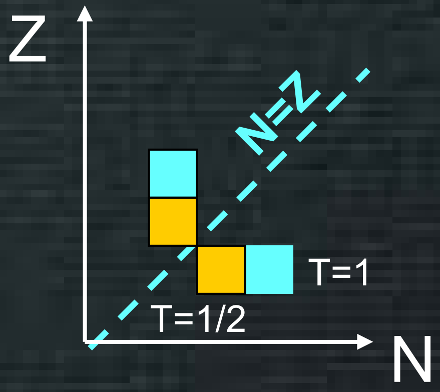
What about the SMALL differences in excitation energy?

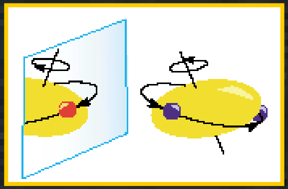
Mirror symmetry is (slightly) broken



Isospin symmetry breakdown, mainly due to the Coulomb field, manifests when comparing mirror nuclei. This constitutes an efficient observatory for a direct insight into nuclear structure properties.

Mirror energy differences

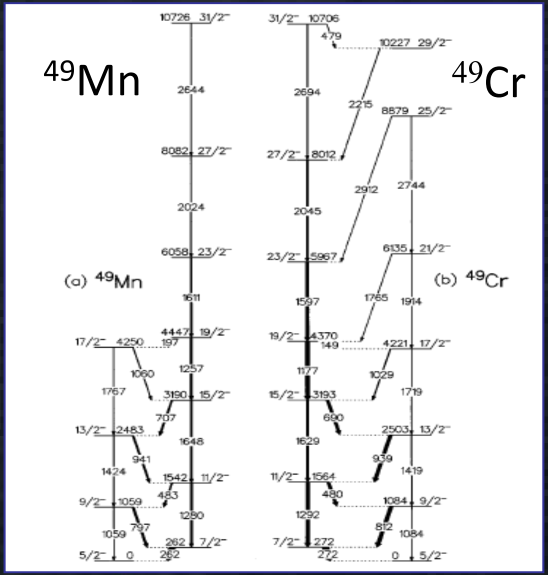




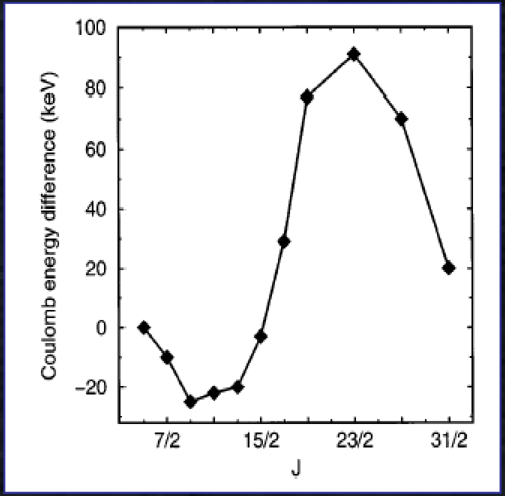
difference in excitation energies

$$\mathrm{MED}_J = \mathrm{E}^*{}_{J,T_z=-T} - \mathrm{E}^*{}_{J,T_z=T}$$

Test the charge symmetry of the interaction



MED



Measuring the Isospin Symmetry Breaking

Can we reproduce such small energy differences?
What can we learn from them?

Interestingly they contain a richness of information about spin-dependent structural phenomena

We measure **nuclear** structure features:

- How the nucleus generates its angular momentum
- Evolution of radii (deformation) along a rotational band
- Learn about the configuration of the states
- Isospin non-conserving terms in the nuclear interaction

What do we need to study MED?

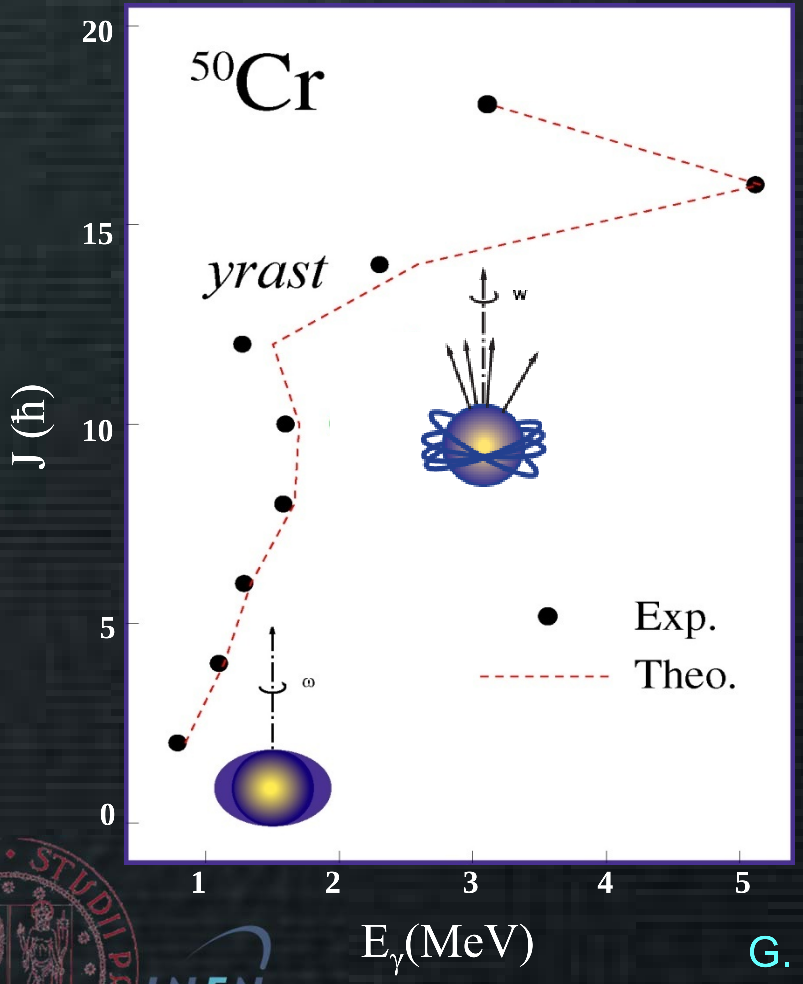
✓A good description of the nuclear structure

✓ Good data up to high spin on N~Z nuclei

to allow a systematic analysis

Istituto Nazionale
di Fisica Nucleare

Understanding structure features



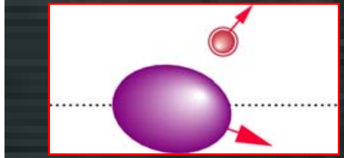
Most of the structure features of nuclei in the $f_{7/2}$ shell are very well described by shell model calculations in the full fp valence space

What happens at the backbending?

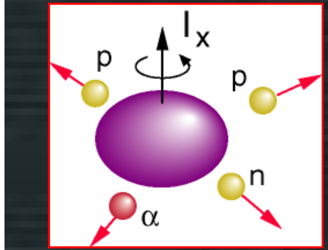
- band-crossing?
- alignment?
- which nucleons are aligning?

G. Martinez-Pinedo et al., PRC54, R2150 (1996)
S.M. Lenzi et al., PRC **56**, 1313 (1997)

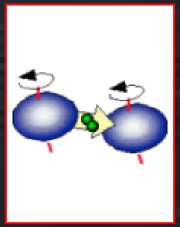
Techniques for proton-rich spectroscopy



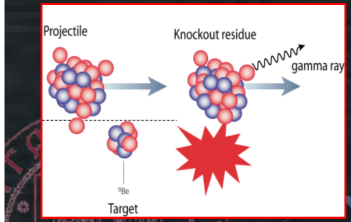
Fusion-evaporation 1: Gamma-ray array + recoil mass spectrometer + focal plane detectors - identify A,Z of recoiling nucleus → tag emitted gamma-rays



Fusion-evaporation 2: Gamma-ray array + clean identification of all emitted particles from reaction - needs a high efficiency & high granularity charged-particle detector (p, α) + neutron detector array



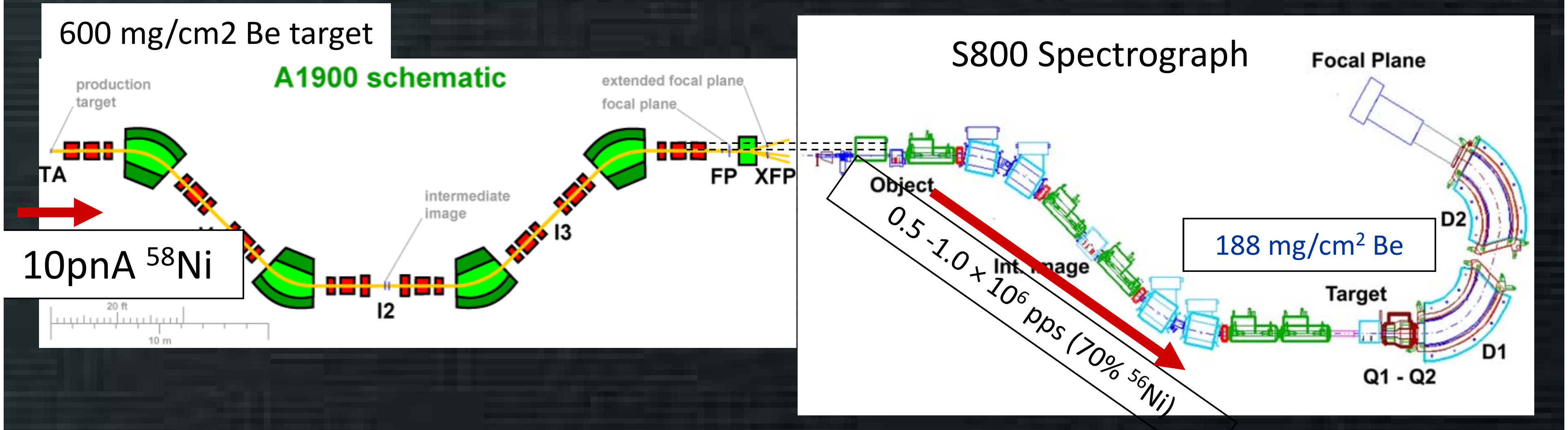
Multinucleon transfer: Gamma-ray array + identify cleanly the recoiling nuclei - needs identify A,Z of recoiling nucleus → tag emitted gamma-rays



Fragmentation and knockout: high efficiency gamma-ray array (limited to relatively low spins)

Mirrored fragmentation of N=Z nuclei

MSU experiment


600 mg/cm2 Be target

10pnA ^{58}Ni

0.5 -1.0 × 10^6 pps (70% ^{56}Ni)

188 mg/cm^2 Be

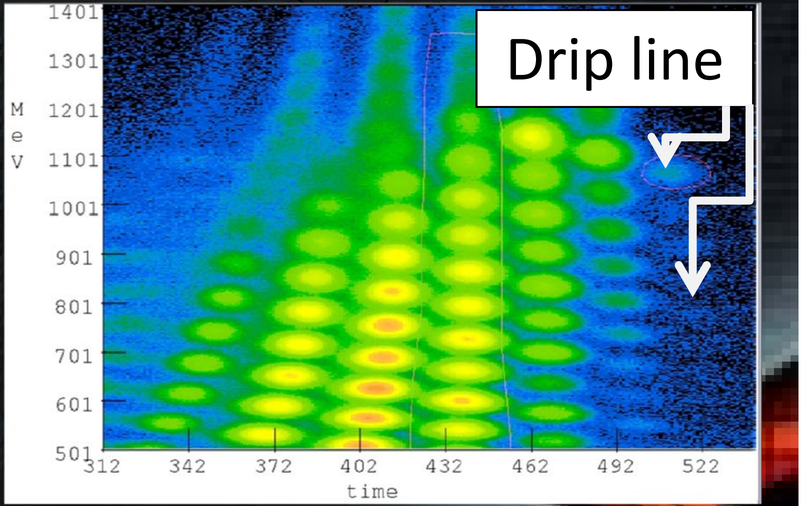
Primary → N=Z second. → “mirrored frag.”

^{58}Ni

^{56}Ni

^{53}Mn

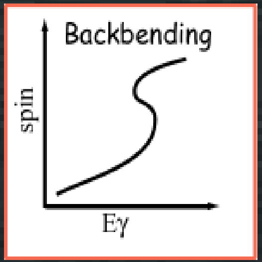
^{53}Ni

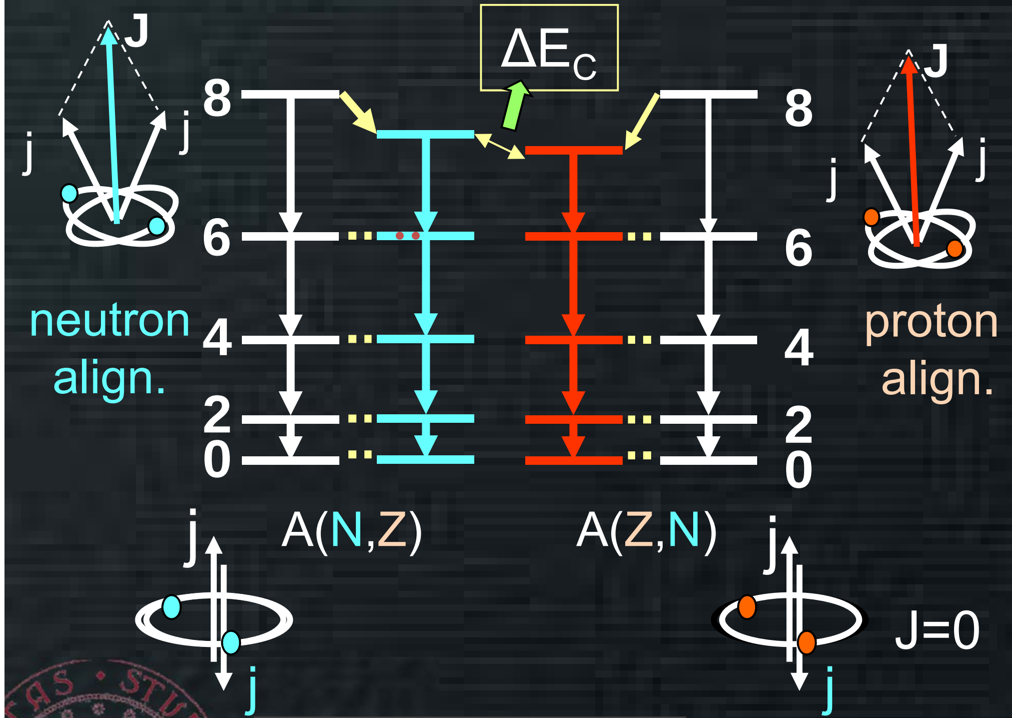

Drip line

J.R. Brown et al.,
Phys. Rev. C 80, 011306(R) (2009)

What do we learn from the study MED?

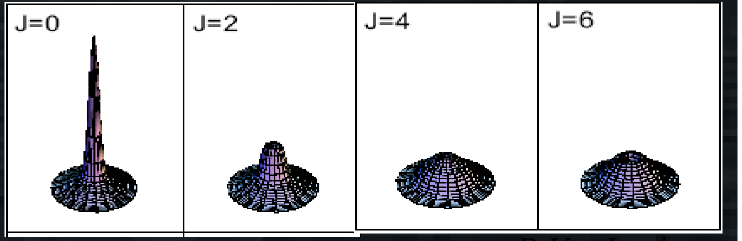
MED and nucleon spatial correlations



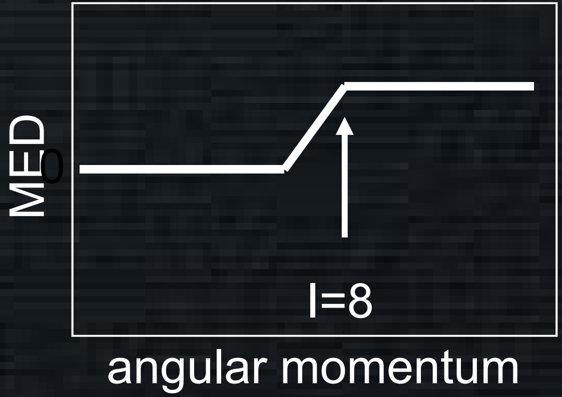


J.A. Cameron *et al.*,
PLB **235**, 239 (1990)

probability distribution for the relative distance of two like particles in the $f_{7/2}$ shell



courtesy P. Van Isacker



Shifts between the excitation energies of the mirror pair at the backbending indicate the type of nucleons that are aligning

Alignment and shell model

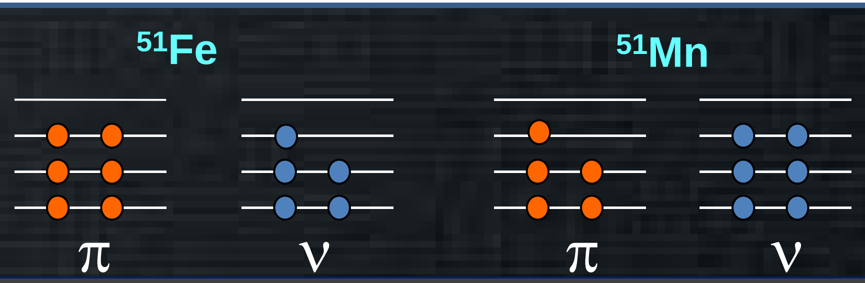
Define the operator

$$\mathbf{A}_\pi = \left[\left(a^+_\pi a^+_\pi\right)^{J=6}\left(a_\pi a_\pi\right)^{J=6}\right]^0$$

"Counts" the number of protons coupled to J=6

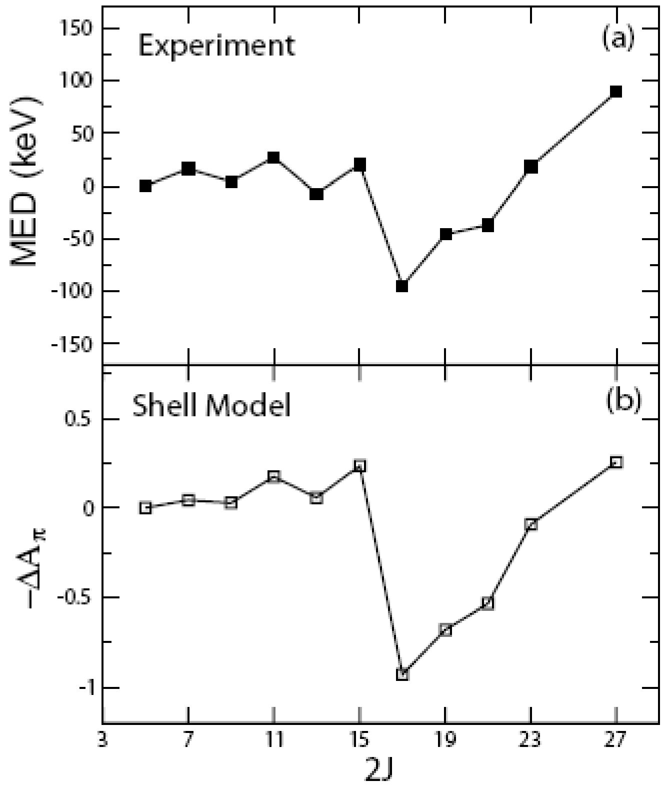
Calculate the difference of the expectation value in both mirror as a function of the angular momentum

$$\Delta\mathbf{A}_{\pi,J} = \langle \Phi_J | \mathbf{A}_\pi(Z_>) | \Phi_J \rangle - \langle \Phi'_J | \mathbf{A}_\pi(Z_<) | \Phi'_J \rangle$$



In ^{51}Fe (^{51}Mn) a pair of protons (neutrons) align first and at higher frequency align the neutrons (protons)

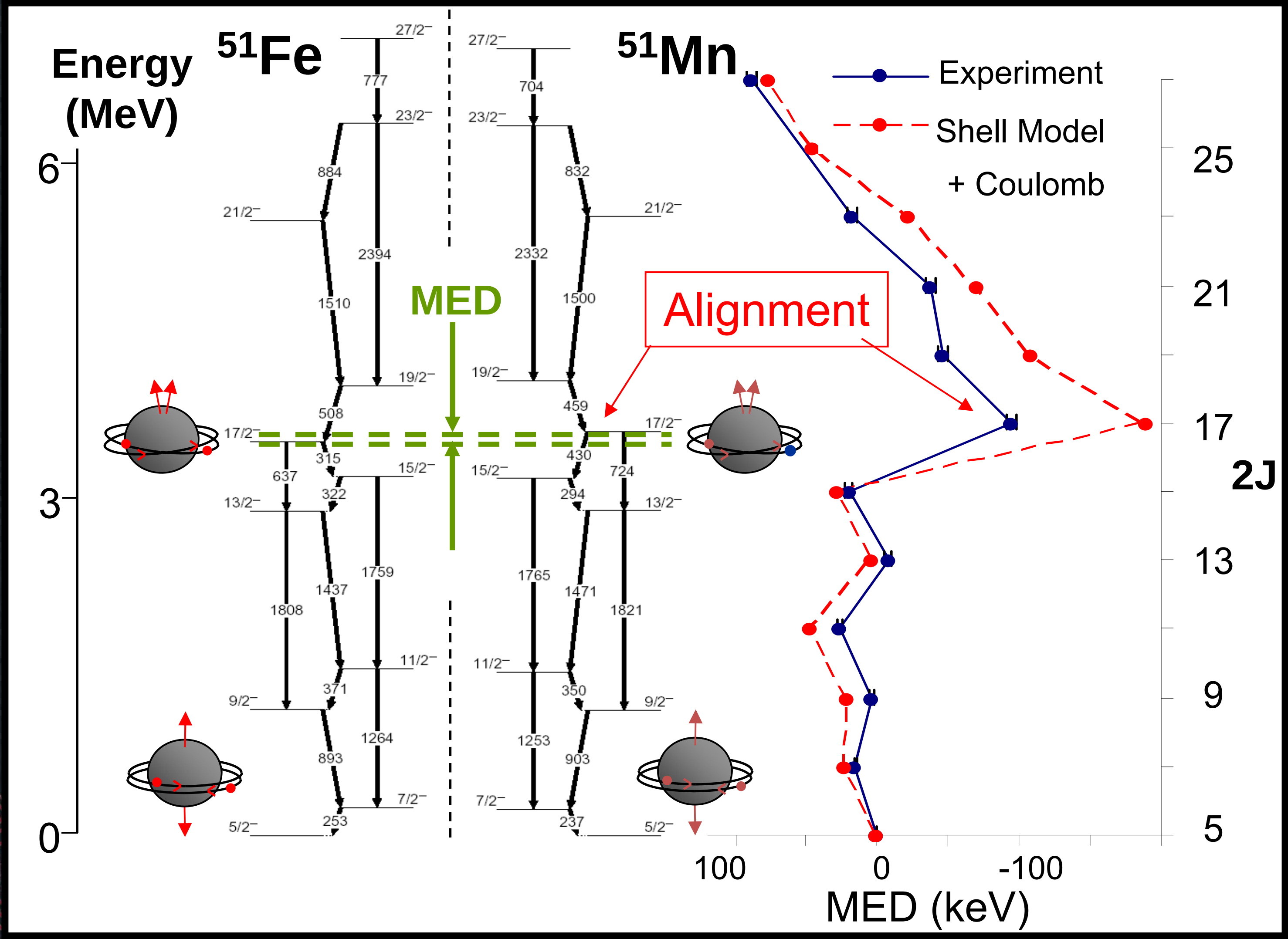
^{51}Fe-^{51}Mn



A. Poves and J. Sanchez-Solano in M.A.Bentley et al. PRC 62 (2000) 051303

MED and nucleon alignment

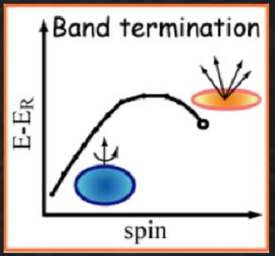
D.D. Warner, M.A. Bentley and P. Van Isacker,
Nature Physics 2 (2006) 311



Including monopole Coulomb effects

Can we do better?

When we “normalize” to the g.s. energy, these large effects vanish, however…
a small but important effect remains as a function of the angular momentum, and it is related to changes of the nuclear radius, or deformation (10’s of keV), and to single-particle effects.



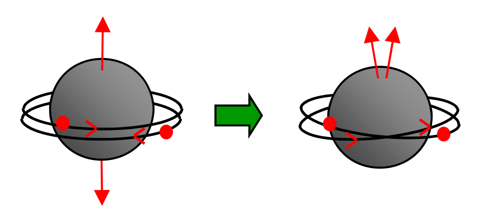
A.P. Zuker

Improving the description of Coulomb effects

$$\boxed{V_C = V_{CM} + V_{Cm}}$$

V_{CM} Multipole part of the Coulomb energy: →

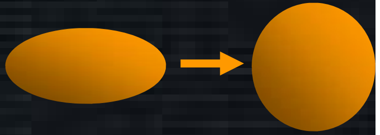
Between valence protons only



V_{cm} Monopole part of the Coulomb energy:

$$E_{Cr} = \frac{3}{5}\frac{e^2 Z(Z-1)}{R}$$

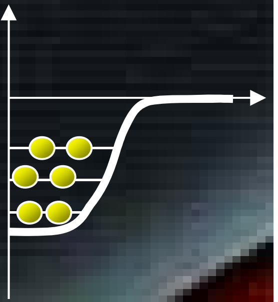
radial effect:
radius changes with J



L² term to account for shell effects

$$E_{Cll} = \frac{-4.5 Z_{cs}^{13/12}[2l(l+1)-N(N+3)]}{A^{1/3}(N+3/2)} keV$$

change the single-particle energies



electromagnetic LS term

$$E_{Cls} = (g_s - g_l)\frac{1}{4m_N^2 c^2}\left(\frac{1}{r}\frac{dV_C}{dr}\right)\mathbf{l.s}$$

A.P. Zuker

The radial term

Coulomb energy of a charged sphere:

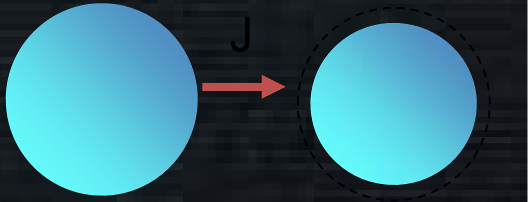
$$\boxed{E_C = \frac{3Z(Z-1)e^2}{5R_C}}$$

The difference between the energy of the ground states (CDE):

$$\boxed{T_z = \pm\frac{n}{2}}$$

$$\boxed{\Delta E_C(J=0) = E_C(Z_>) - E_C(Z_<) = \frac{3n(2Z_> - n)e^2}{5R_C}}$$

If R_C changes as a function of the angular momentum…



$$\Delta E_{Cr}(J) = \Delta E_C(J) - \Delta E_C(0) = \frac{3}{5}n(2Z_> - n)e^2\left(\frac{R_C(0) - R_C(J)}{R_C^2}\right)$$

$$= -\frac{3}{5}n(2Z_> - n)e^2\frac{\Delta R_C(J)}{R_C^2} = \boxed{nC\cdot\Delta R_C(J)}$$

Radial contribution to the MED

The radial effect with the shell model

The radial contribution can be calculated from the relative $p_{3/2}$ occupation number along the yrast band in the shell model framework:

$$\Delta\langle V_{Cr}\rangle_J = na_m \langle m_{p_{3/2}}\rangle_J = na_m \left\langle \frac{z_{p_{3/2}} + n_{p_{3/2}}}{2} \right\rangle_J$$

z and n are the number of protons and neutrons in the $p_{3/2}$ orbit, relative to the g.s. (J=0)

$$n = |N - Z| = 2T_z$$

a_m is not a free parameter but can be estimated experimental data:

The radial parameter amounts to a_m ~ 200 keV
the same for all MED studied so far (not only fp shell).

The electromagnetic spin-orbit term

Analogous to the atomic case, the nuclear electromagnetic spin-orbit coupling has relativistic origin.

It results from the Larmor precession of the nucleons (protons and neutrons) in the nuclear electric field due to the intrinsic (spin) magnetic moments and the Thomas precession experienced by the protons due to their electric charge (orbital magnetic moment).

$$V_{Cls} = (g_s - g_l)\frac{1}{2m_N^2c^2}\left(\frac{1}{r}\frac{dV_C}{dr}\right)\vec{l}\cdot\vec{s}$$

(50 times smaller than the nuclear spin-orbit term)

$$E_{Cls} = (g_s - g_l)\frac{1}{2m_N^2c^2}\left(-\frac{Ze^2}{R}\right)\langle\vec{l}\cdot\vec{s}\rangle$$

$$\langle\vec{l}\cdot\vec{s}\rangle = -l \Leftrightarrow j = l + s$$

$$\langle\vec{l}\cdot\vec{s}\rangle = l + 1 \Leftrightarrow j = l - s$$

Effect depending on the orbit!!!

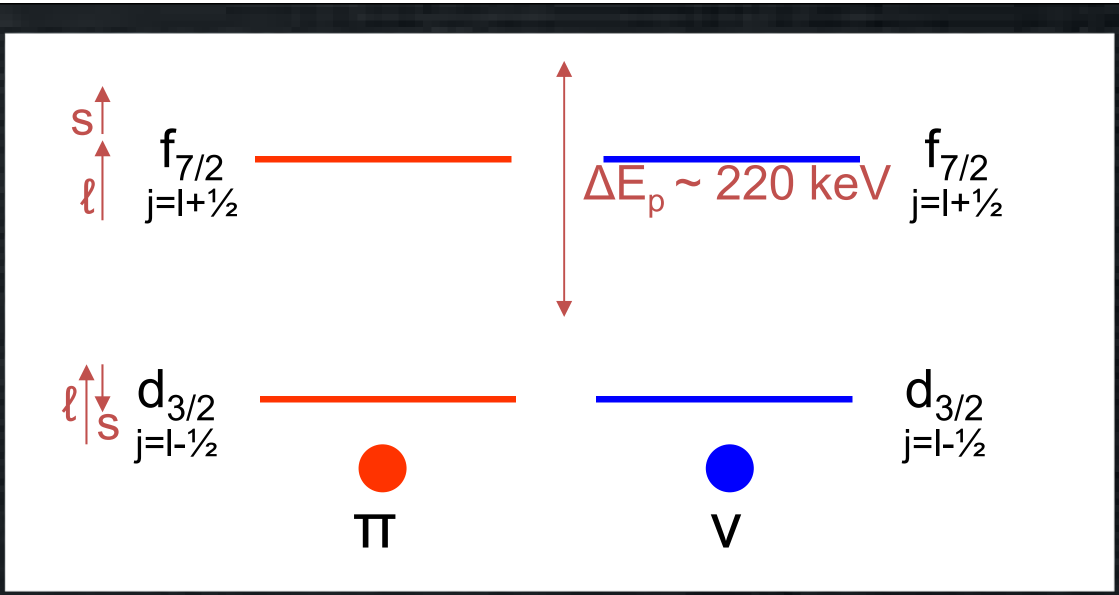
Electromagnetic spin-orbit effect

Acts differently on protons and neutrons:

$$g_s^\pi = +5.586, \quad g_\ell^\pi = 1$$
$$g_s^\nu = -3.828, \quad g_\ell^\nu = 0$$

The approximate values for the energy shifts result:

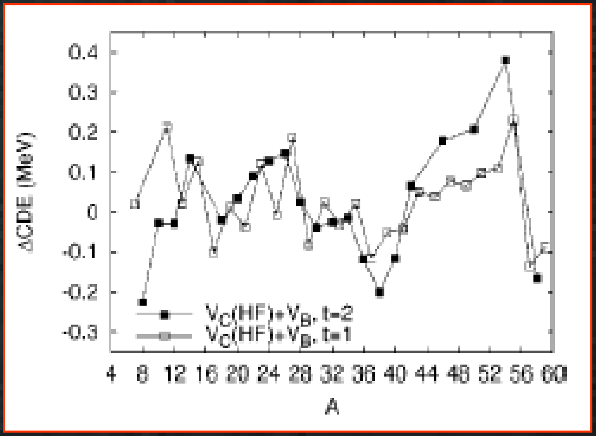
	$\pi, j = l + \frac{1}{2}$	$\pi, j = l - \frac{1}{2}$	$\nu, j = l + \frac{1}{2}$	$\nu, j = l - \frac{1}{2}$
E_{ls}	$-42(Z/A)l$	$+42(Z/A)(l+1)$	$+35(Z/A)l$	$-35(Z/A)(l+1)$



Its contribution to the MED becomes significant for configurations with a ***pure*** single-nucleon excitation to the $f_{7/2}$ shell: a proton excitation in one nucleus and a neutron excitation in its mirror

The orbital term

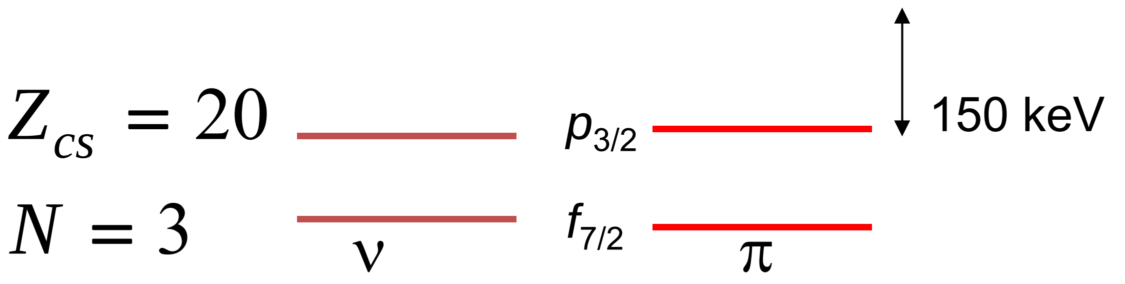
J. Duflo and A.P. Zuker, *Phys. Rev.* C 66 (2002) 051304(R)



The monopole Coulomb term accounts for shell effects. It changes the single-particle energy of the protons proportionally to the square of the orbital angular momentum.
For a proton in a main shell N above a closed shell Z_{cs} its value is deduced:

$$E_{Cll} = \frac{-4.5 Z_{cs}^{13/12}\left[2\ell(\ell+1) - N(N+3)\right]}{A^{1/3}\left(N+\frac{3}{2}\right)} [\text{keV}]$$

Eg. in the fp shell:



$Z_{cs} = 20$

$N = 3$

$p_{3/2}$

$f_{7/2}$

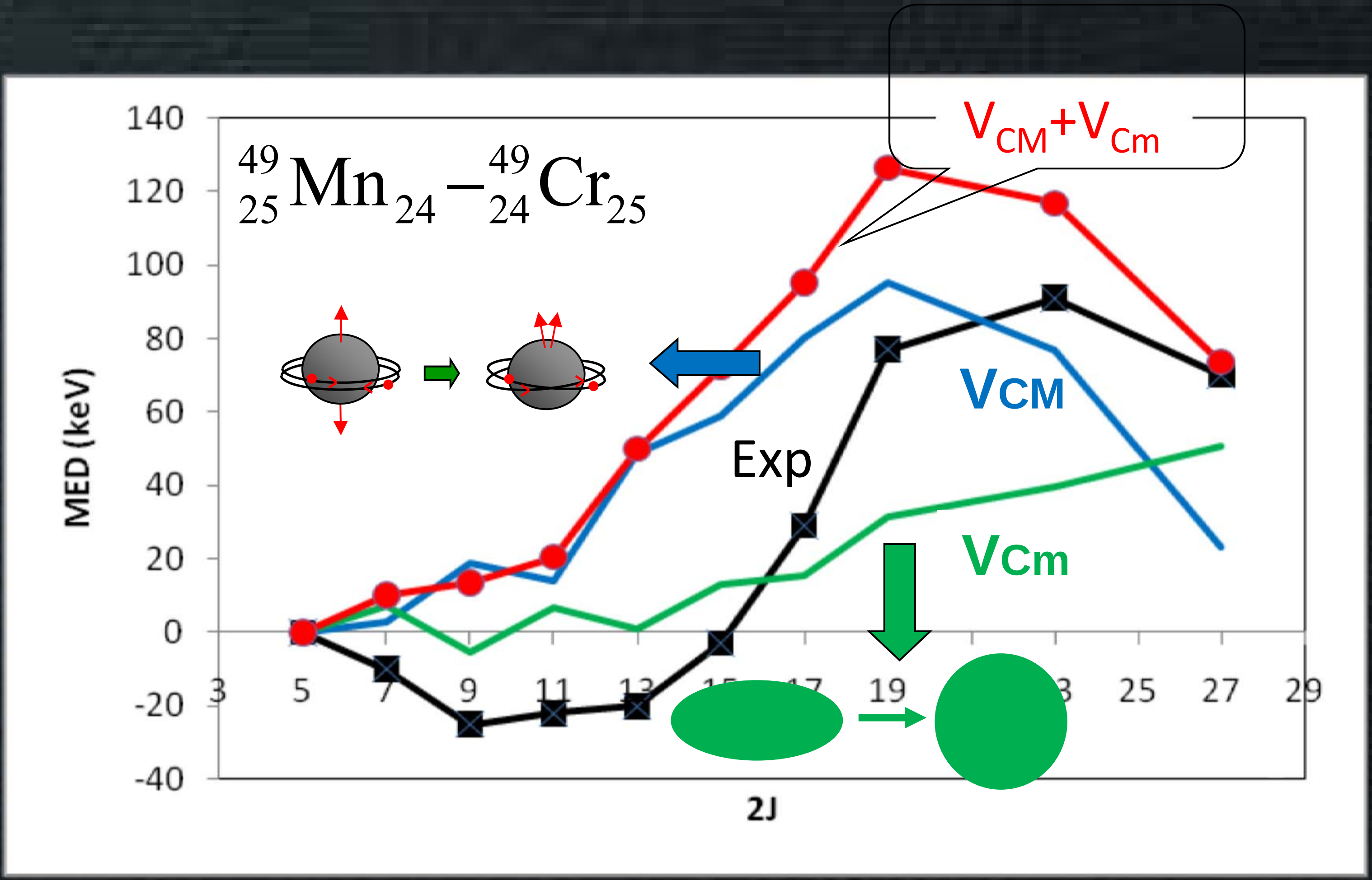
ν

π

150 keV

proton s.p. relative energy is increased by 150 keV

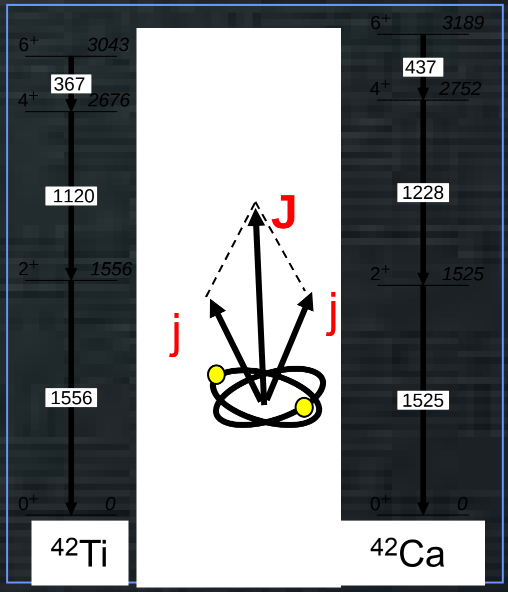
Are Coulomb corrections enough?

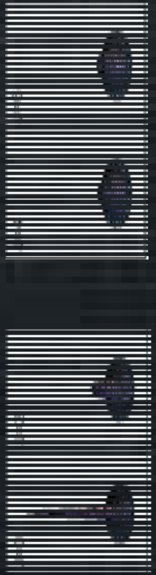


Another term of "nuclear" nature is needed, but it has to be big!

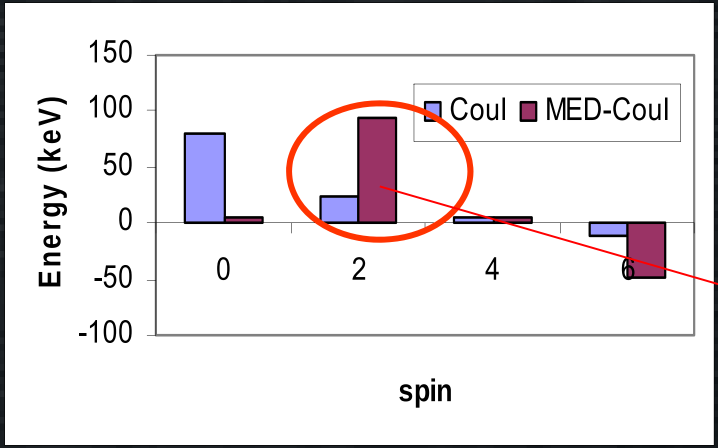
Looking for an empirical interaction

Yrast spectra of the T=1 ^{42}Ti and ^{42}Ca





Assume that the configurations of these states are pure $(f_{7/2})^2$



Coul: calculated
MED: measured
(monopole subtracted)

J=2 anomaly

This suggests that the role of the missing isospin non conserving term is at least as important as the Coulomb potential in the observed MED

Simple ansatz for the application to nuclei in the pf shell:

$$V_B^{pf} = V_B\left[\pi f_{7/2}^{\,2}\right]^{J=2} = 100\,\mathrm{keV}$$

A. P. Zuker et al., PRL 89, 142502 (2002)

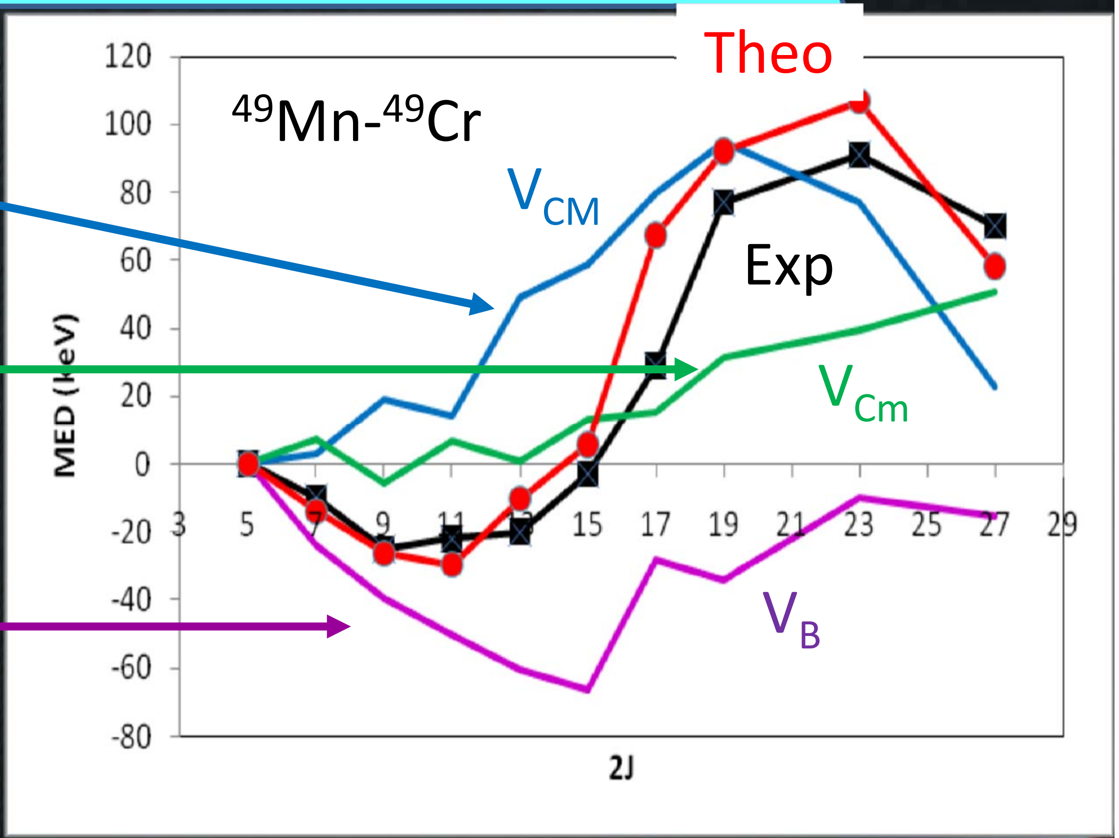
Calculating the MED with SM

$$\mathrm{MED}_J = \langle \mathrm{VCM} \rangle_J + \langle \mathrm{VCm} \rangle_J + \langle \mathrm{VB} \rangle_J$$

VCM: gives information on the nucleon alignment or recoupling

VCm: gives information on changes in the nuclear radius

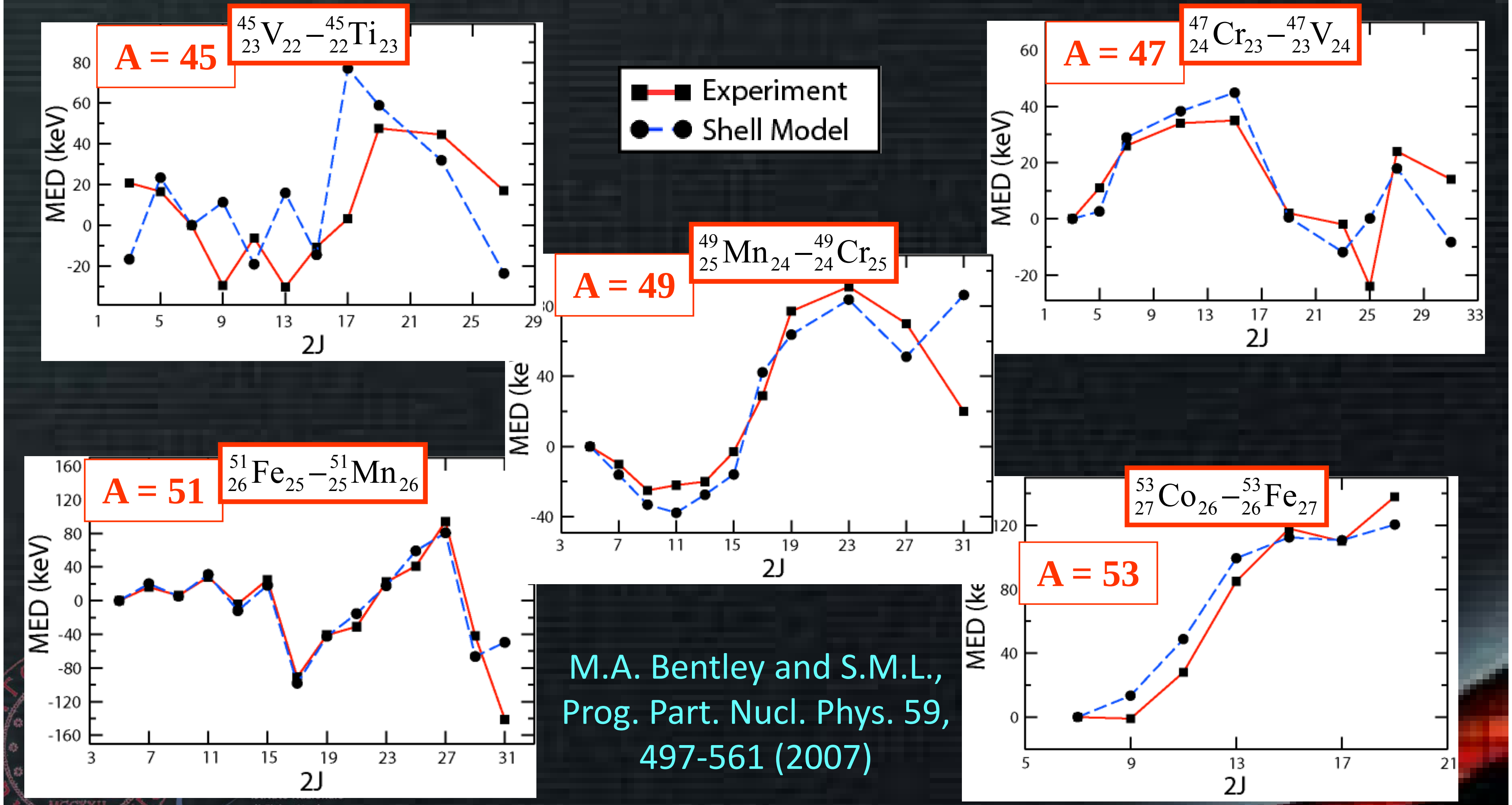
Important contribution from the VB term: $V(\pi[f_{7/2}f_{7/2}]_2)$=100 keV of the same order as the Coulomb contributions!



A. P. Zuker et al., PRL 89, 142502 (2002)

MED in T=1/2 states

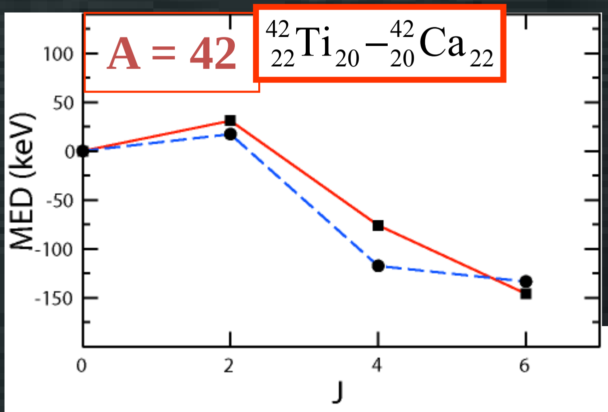
Very good quantitative description of data without free parameters

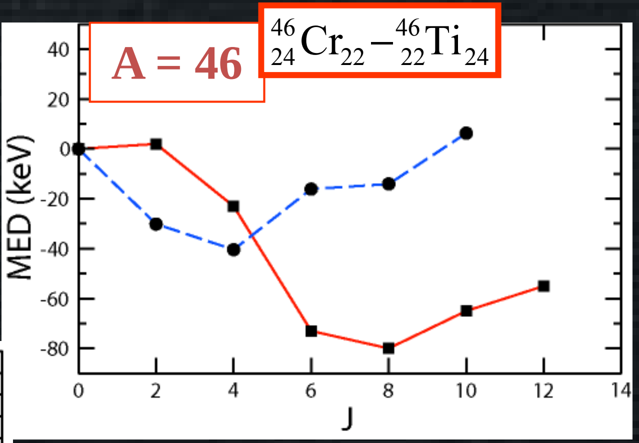


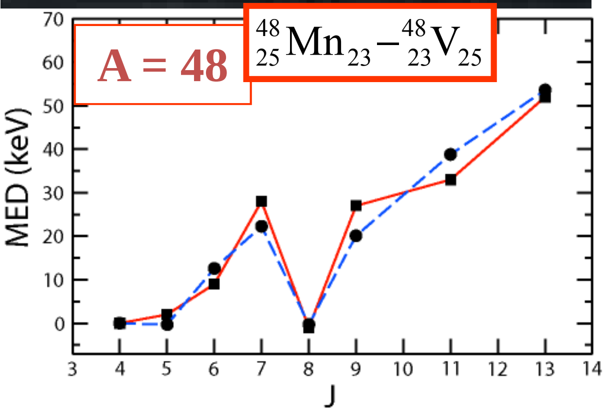
M.A. Bentley and S.M.L., Prog. Part. Nucl. Phys. 59, 497-561 (2007)

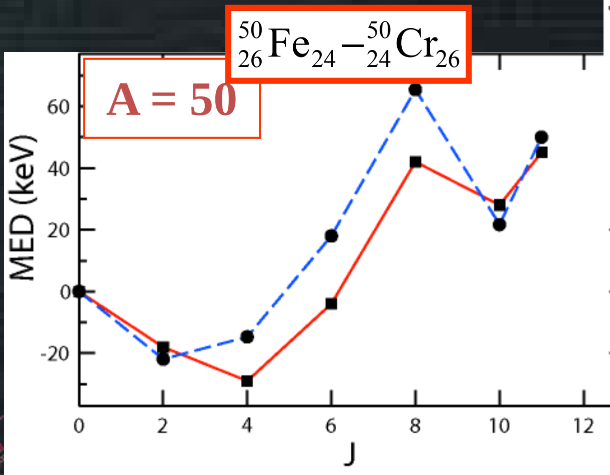
Mirror Energy Differences in T=1 states

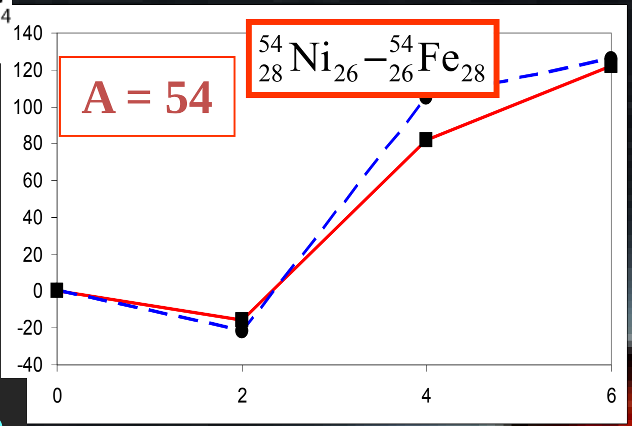
■—■ Experiment
●- -● Shell Model











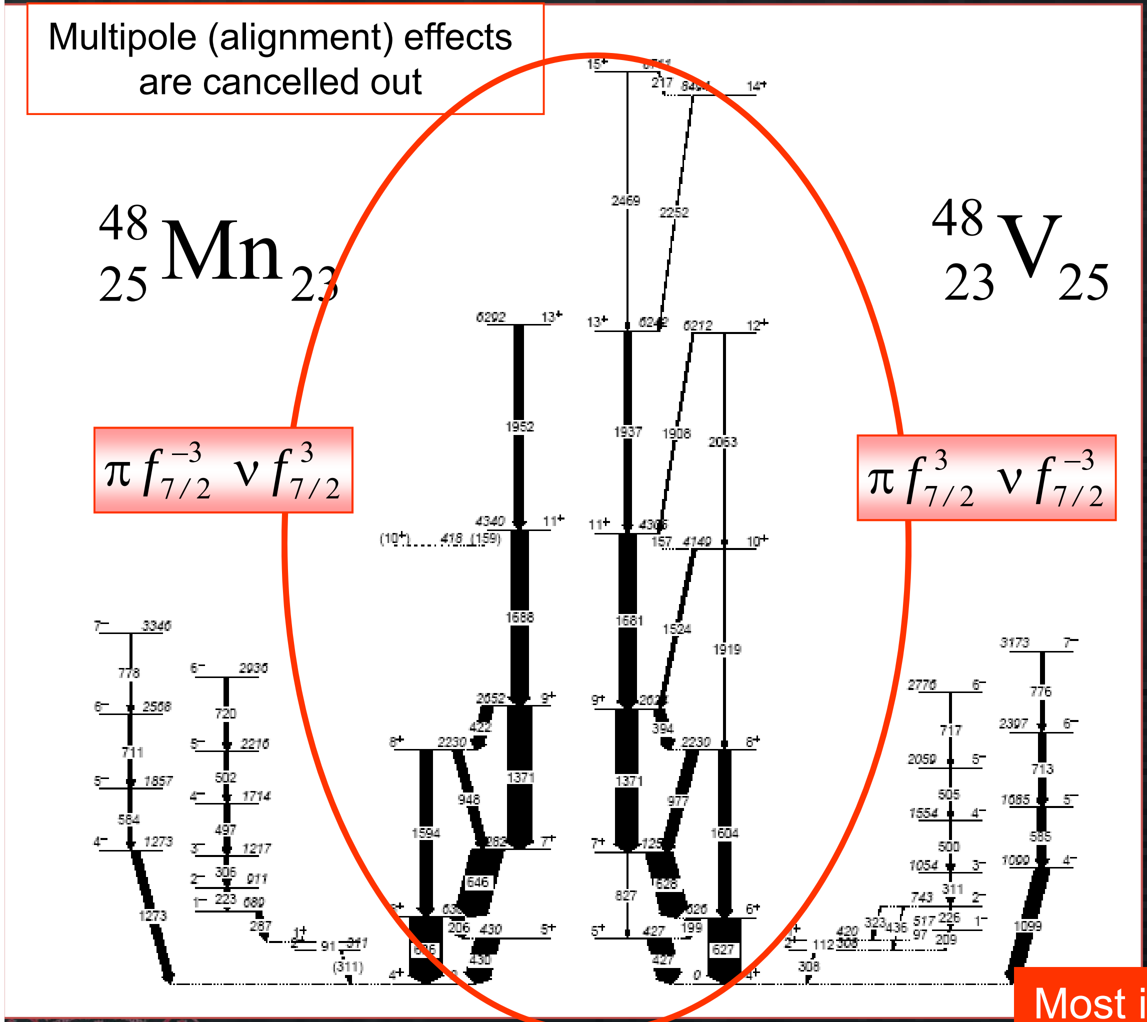
M.A. Bentley and SML, Prog. Part. Nucl. Phys. 59, 497-561 (2007)

Same parameterization for the whole $f_{7/2}$ shell!

Some illustrative examples

Evidence of the monopole radial effect

Multipole (alignment) effects are cancelled out



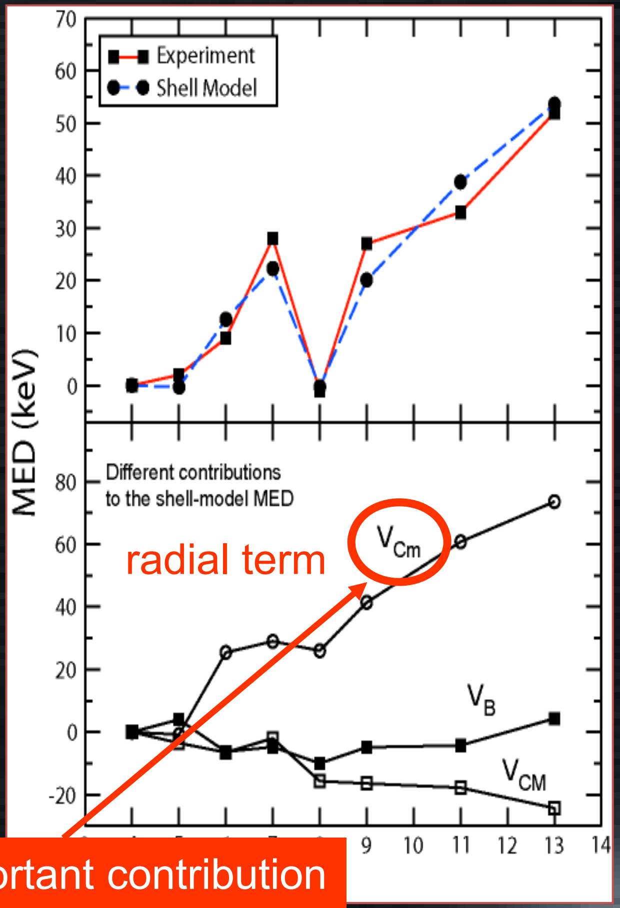
$^{48}_{25}\mathrm{Mn}_{23}$

$^{48}_{23}\mathrm{V}_{25}$

$\pi f_{7/2}^{-3}\ \nu f_{7/2}^{3}$

$\pi f_{7/2}^{3}\ \nu f_{7/2}^{-3}$

M.A. Bentley et al.,
PRL 97, 132501 (2006)

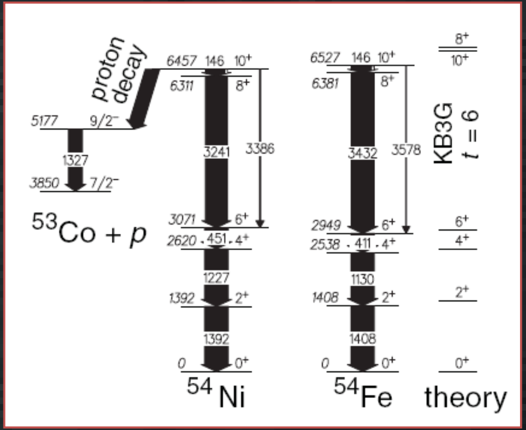


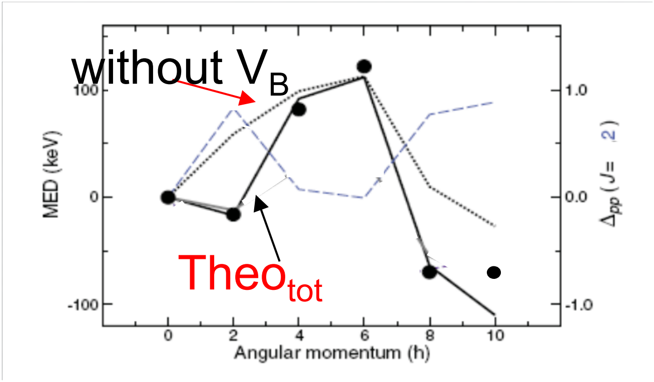
Most important contribution

The nucleus changes shape
towards band termination

VB in the T=1 and T=3/2 mirrors

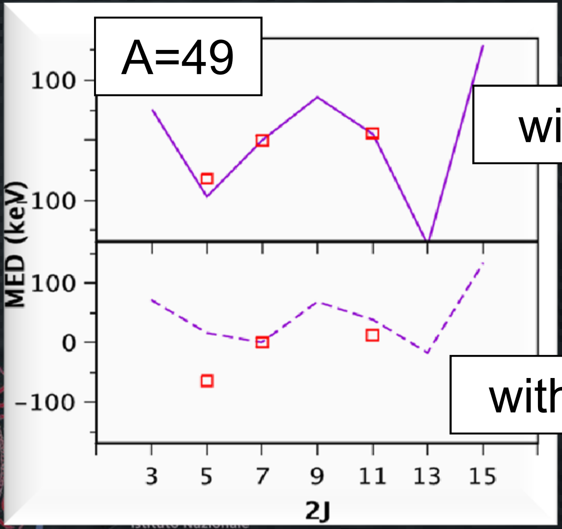
J=2 anomaly

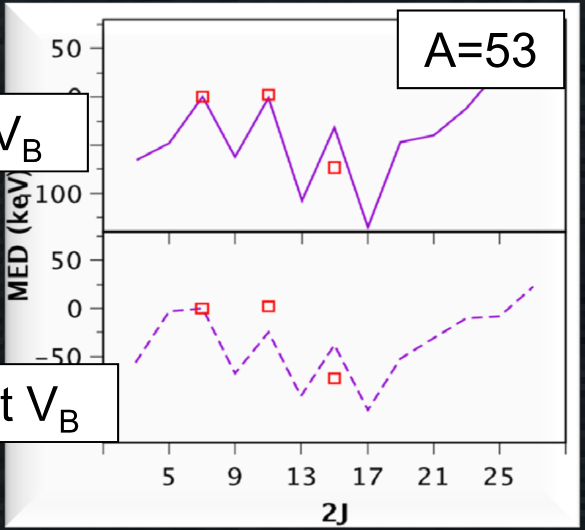




A.Gadea et al., PRL 97, 152501 (2006)

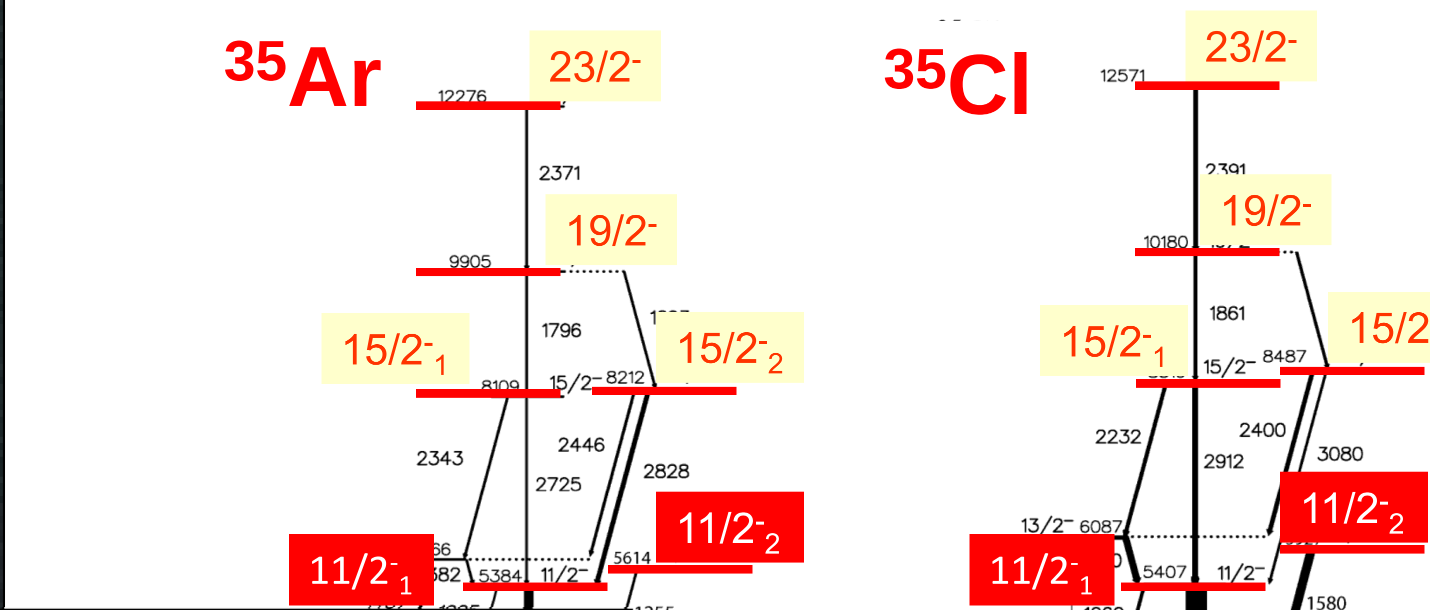
D. Rudolph et al., PRC **78**, 021301(R) (2008)

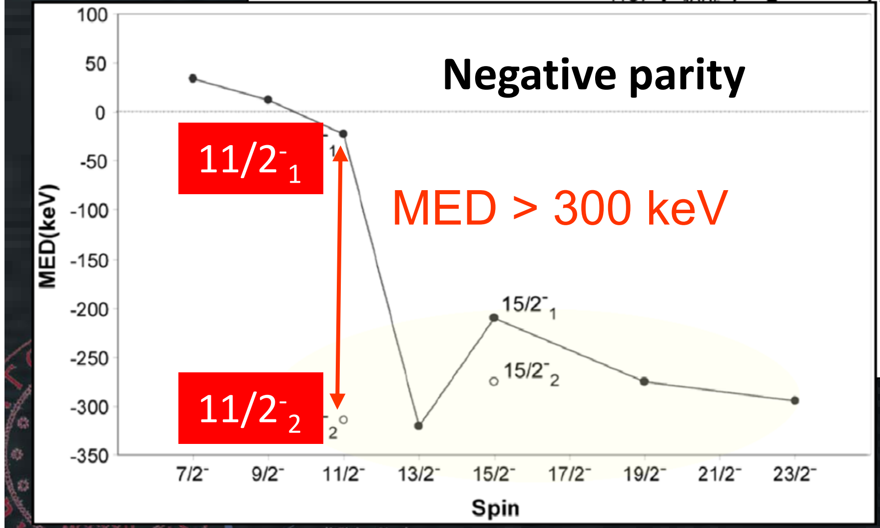




J.R. Brown et al.
PRC 80, 011306(R) 2009

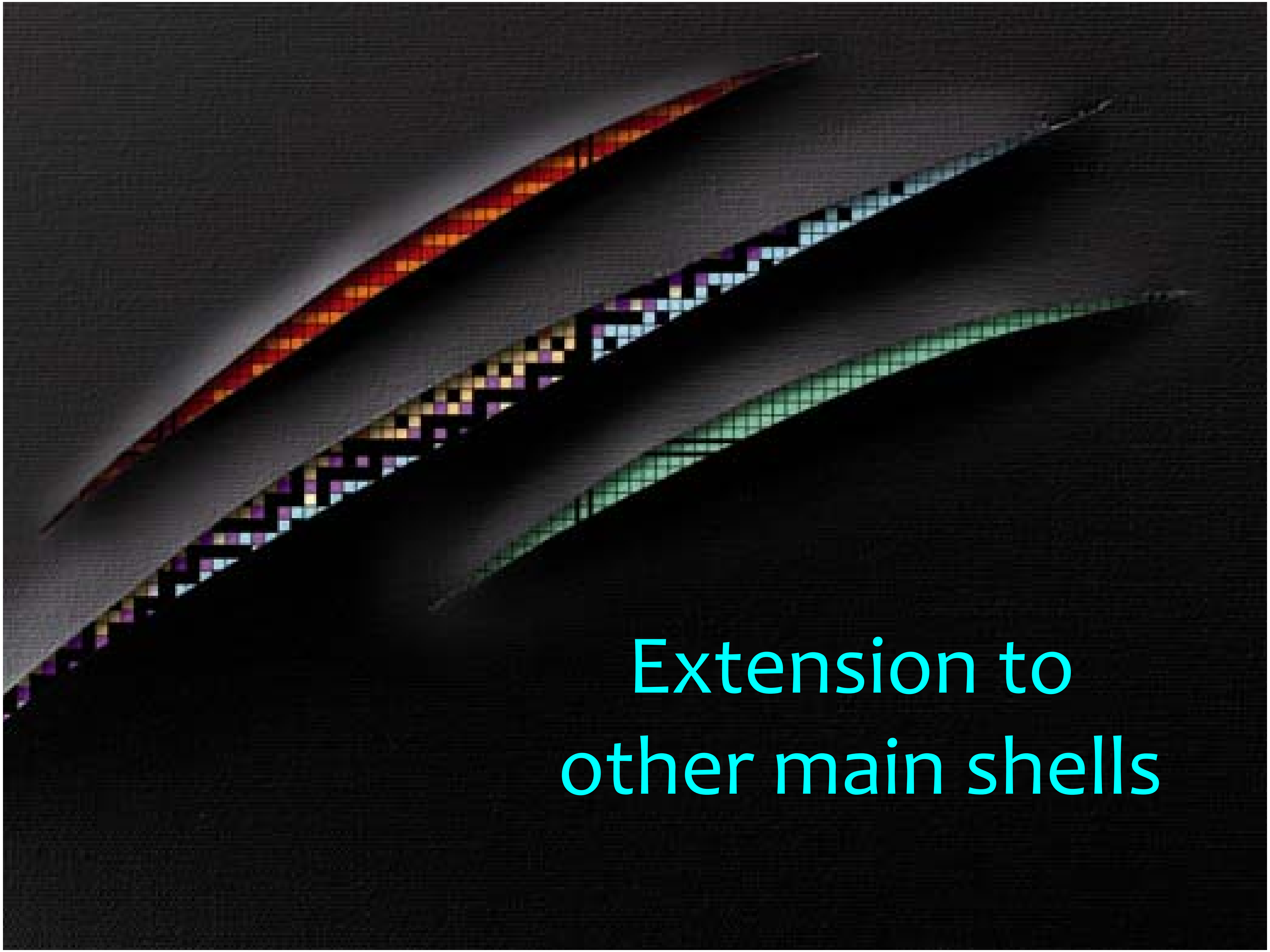
The electromagnetic spin-orbit effect: disentangling configurations





From the MED experimental values we can identify those states with configurations of pure proton (neutron) excitation to the $f_{7/2}$ shell.

F. Della Vedova et al.,
Phys.Rev. C 75, 034317 (2007)



Extension to other main shells

Understanding the ISB term VB

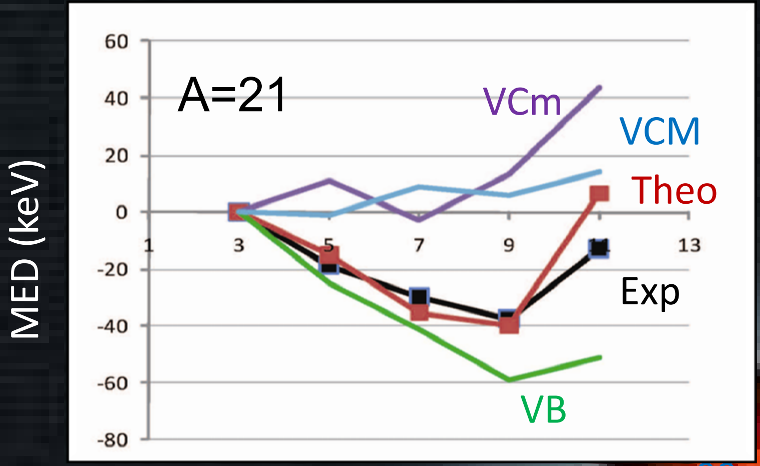
Can we understand the origin of this term from the N-N interaction?

Is the ISB term a general feature or is it just confined to the $f_{7/2}$ shell?

Necessary conditions for such studies:
- good amount of data
- good shell model description of the structure

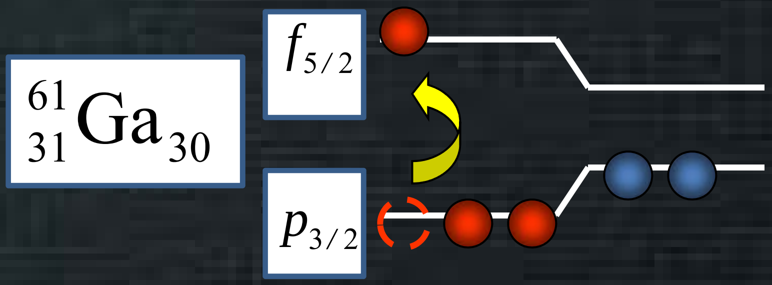
A similar VB term as in the $f_{7/2}$ shell is needed to reproduce these data in the sd shell

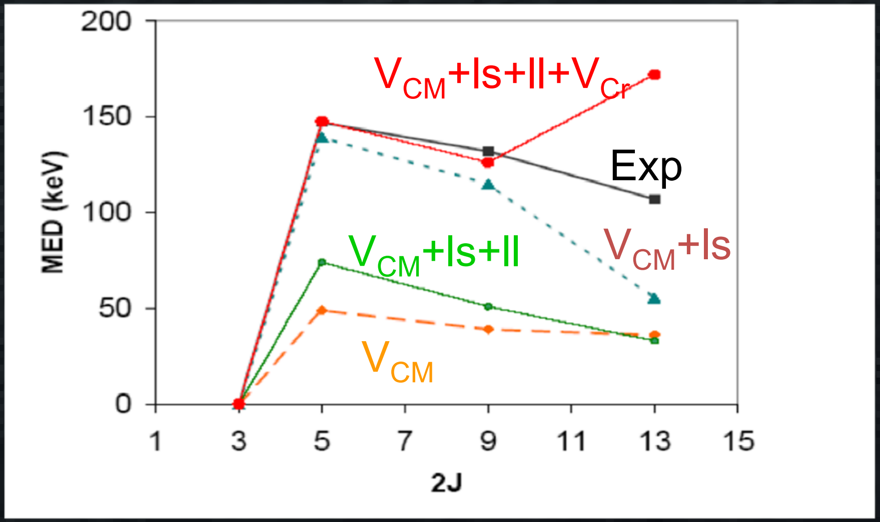
This suggests that the ISB term has a general character



Exploring the fp shell

The mirror pair ^{61}Ga – ^{61}Zn

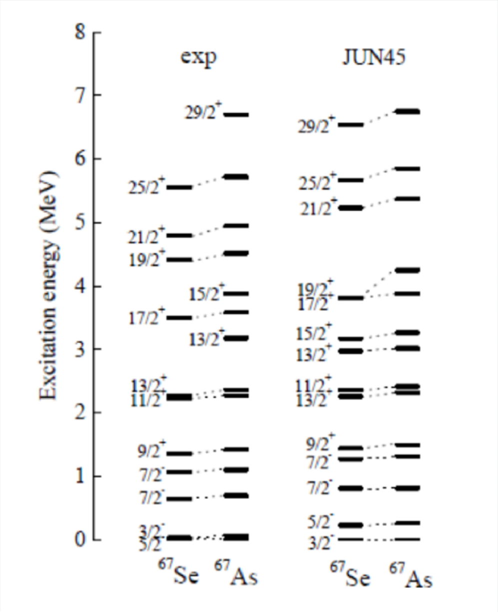


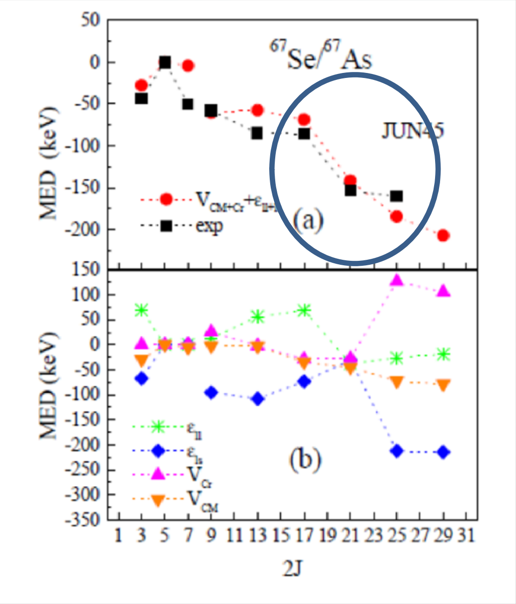


M.A. Bentley and SML,
Prog. Part. Nucl. Phys. 59,
497-561 (2007)

The mirror pair ^{67}Se – ^{67}As

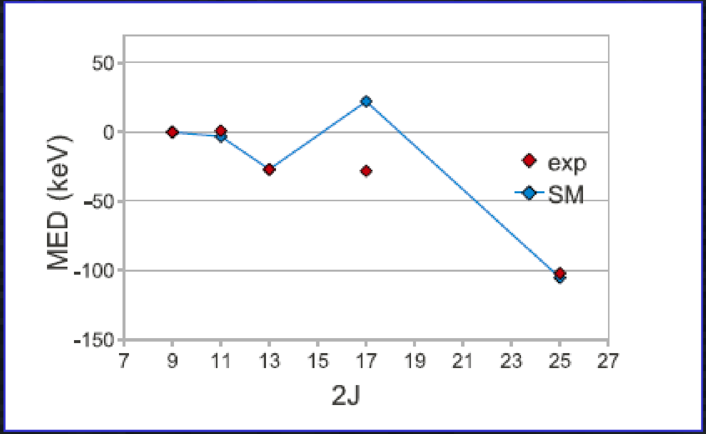
fpg shell



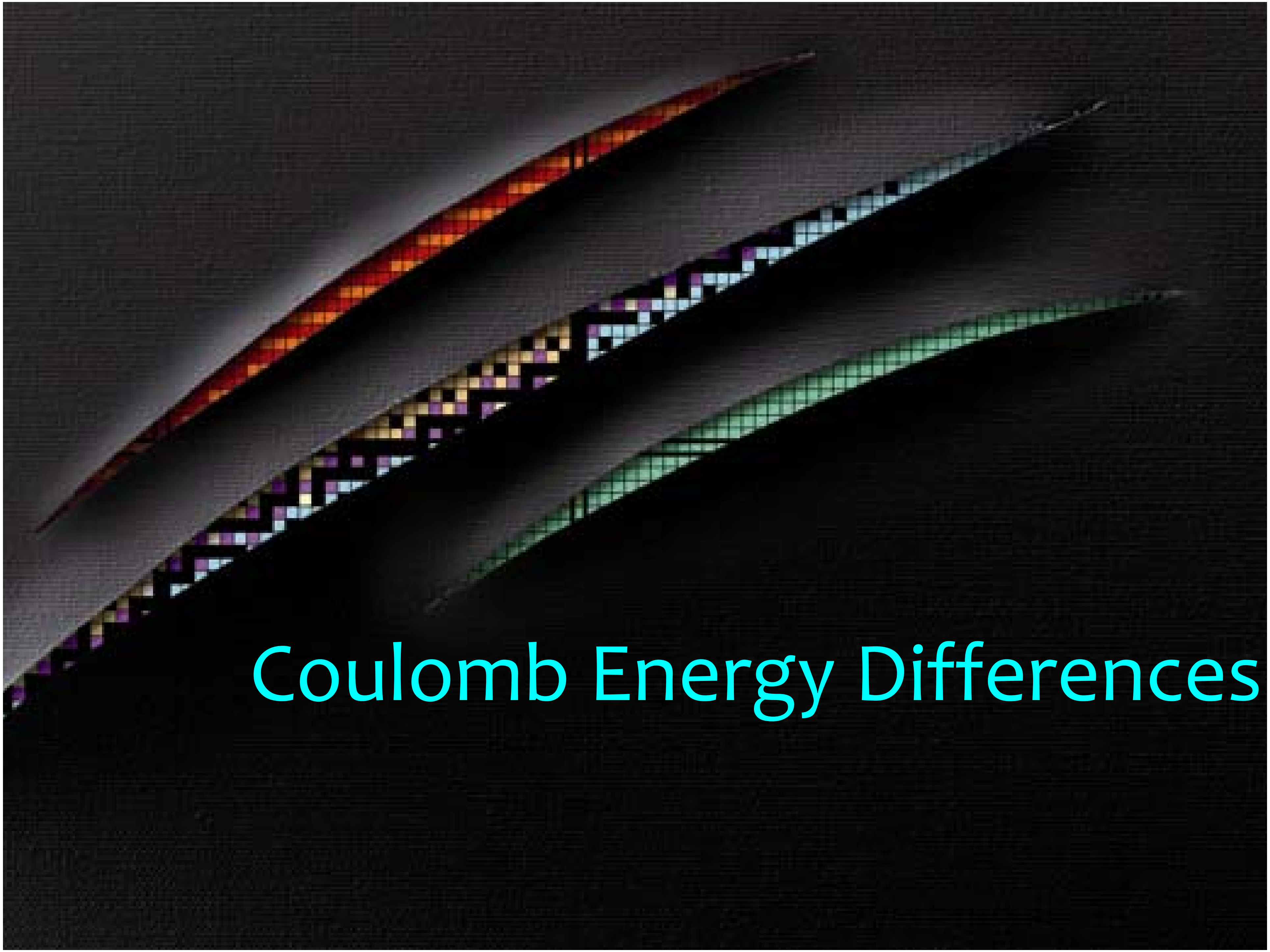


Shape coexistence causes the crossing or mixing of levels of the same spin.
A careful look at the 21/2$^+$ wfs shows different structures in the IAS!
Important to have a good shell model description.

Data: R. Orlandi et al.,
PRL 103, 052501 (2009)
Calc. K. Kaneko et al.,
PRC 82, 061301, 2010



A. Boso, Thesis,
B. Padova 2011



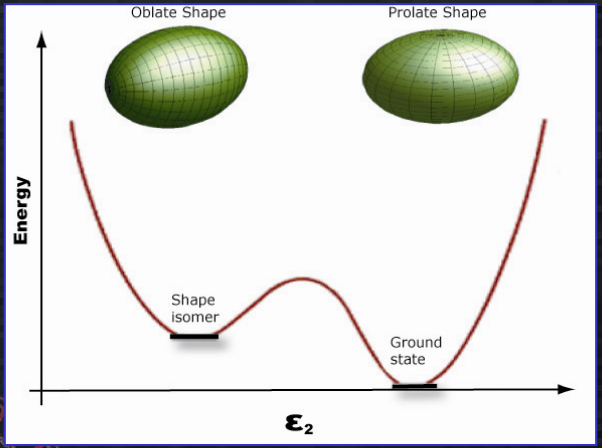
Coulomb Energy Differences

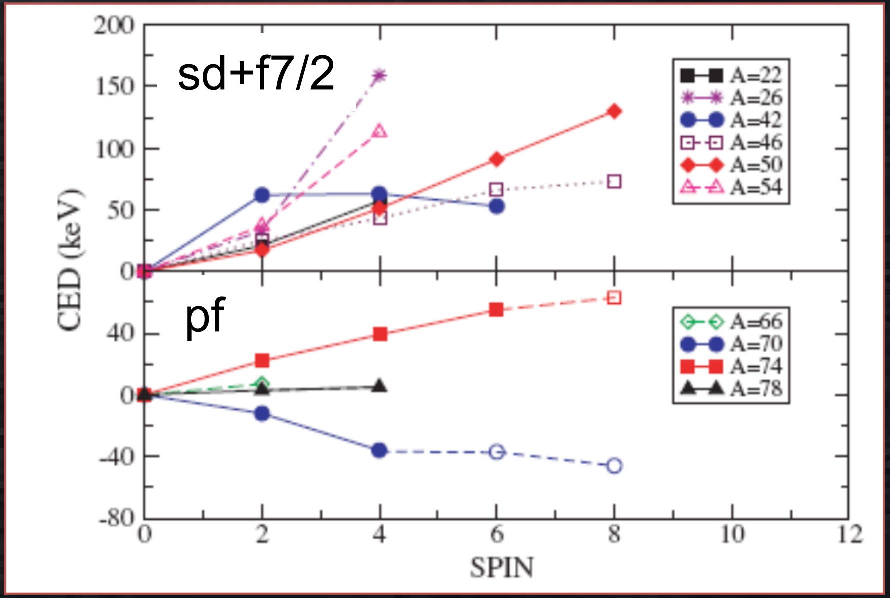
Isospin symmetry and shape coexistence

$$CED(J)=E_J(N=Z)-E_J\,(N=Z+2)$$

fpg shell

Large shell gaps exist at both oblate *and* prolate shapes for $N = Z = 34$ and 36



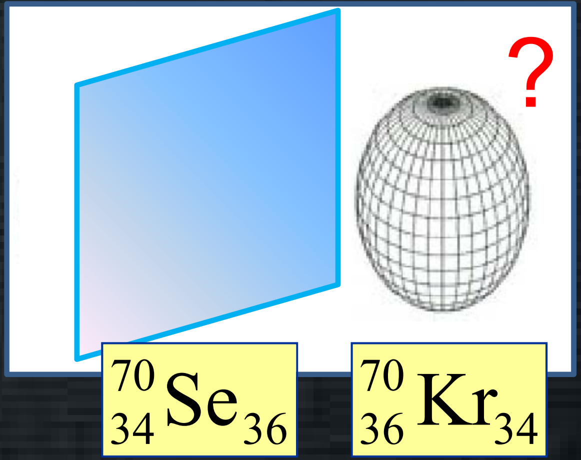


Negative values of the CED have been associated to changes in the Coulomb energy due to shape changes as a function of J

CED: The case of the non-identical twins?

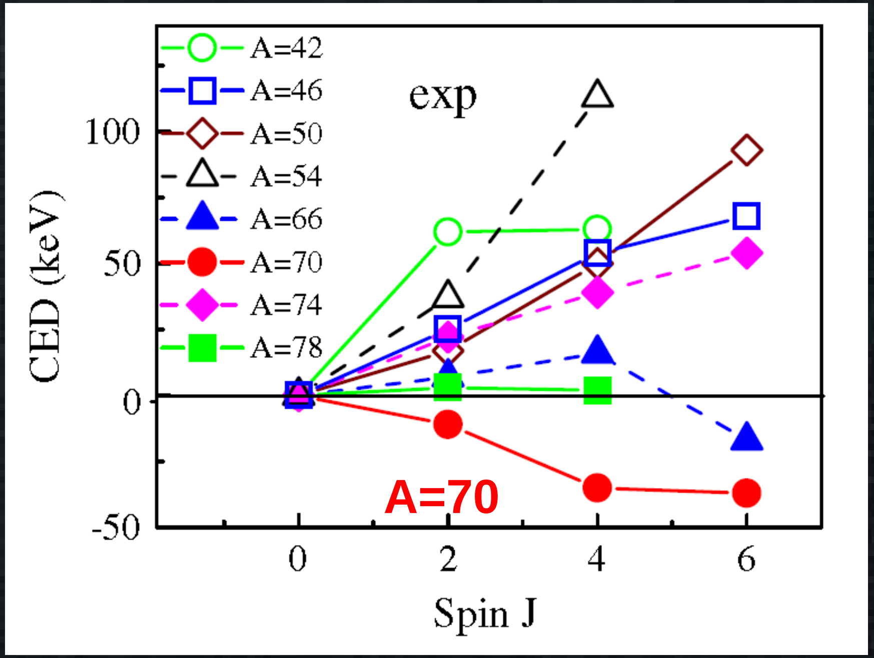
$$^{70}_{35}\mathrm{Br}_{35} - ^{70}_{34}\mathrm{Se}_{36}, T = 1$$

MED very sensitive to structural changes



$^{70}_{34}\mathrm{Se}_{36}$ $^{70}_{36}\mathrm{Kr}_{34}$

J. Ljungvall et al.,
PRL 100, 102502 (2008)



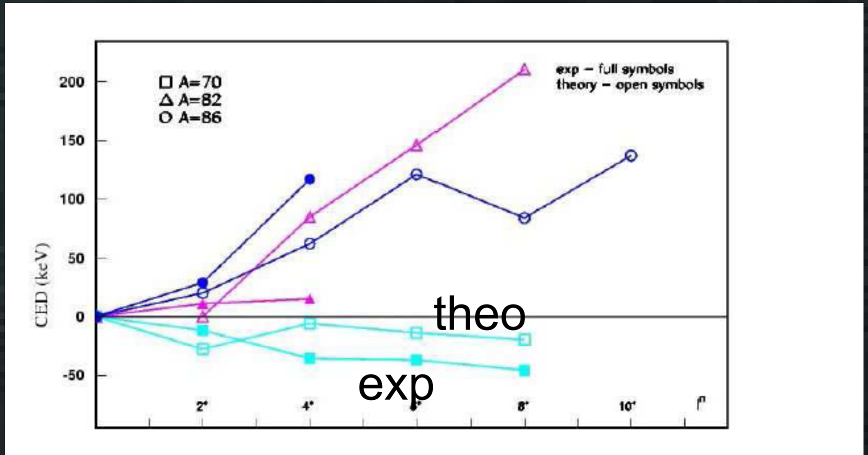
from K. Kaneko at al., PRL 109, 092504 (2012)

Modifications in the Fermi surface induced by isospin-breaking interactions can cause rapid changes in the nuclear shape that may lead to different shapes in the ground-state configurations of nuclei belonging to the same isospin multiplet.

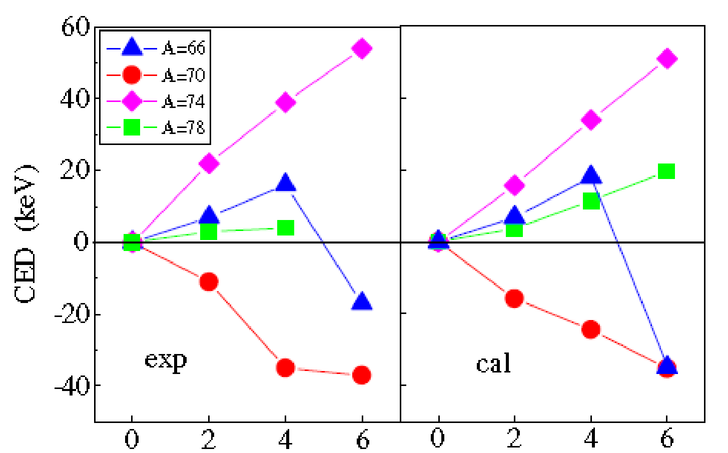
CED: BMF vs SM in A=70

Beyond mean field (Vampire) CED based on different shape mixing (axially symmetric) of the analog states.

Shell model calculations following Zuker et al, not considered the Vcm (radial) term.



A. Petrovici, J. Phys. G 37,064036 (2010)

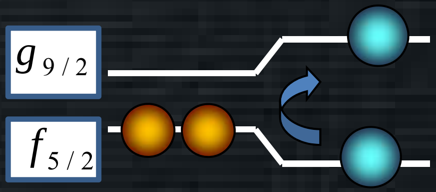


K. Kaneko at al., PRL 109, 092504 (2012)

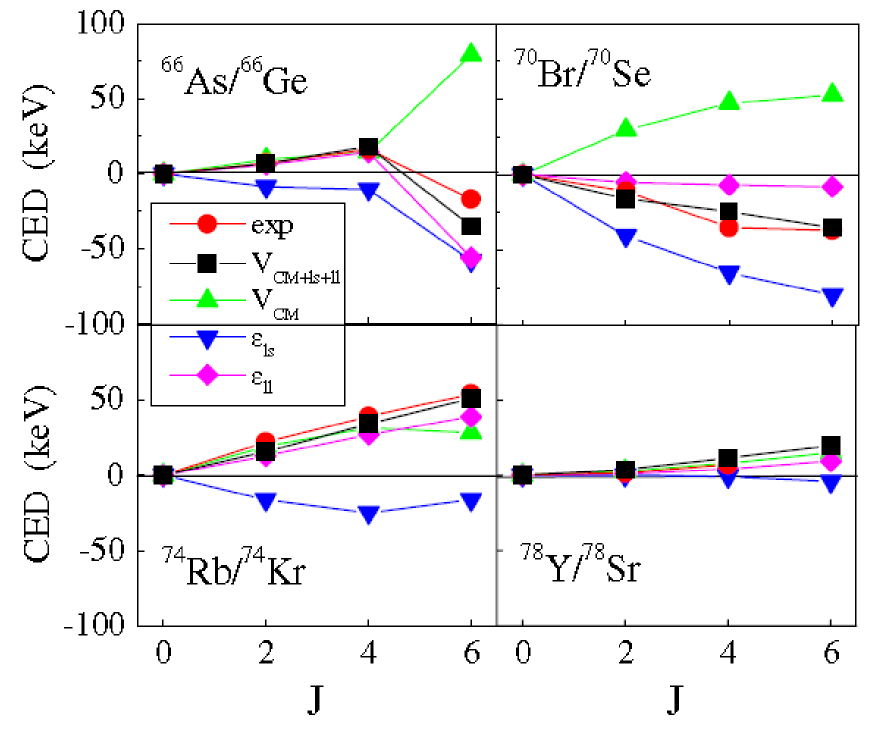
Negative values due to different shapes

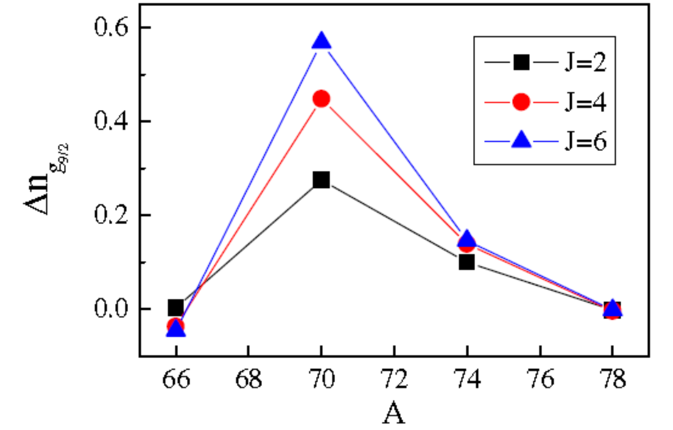
Negative values due to the spin-orbit term

The role of the electormagnetic *l.s* term

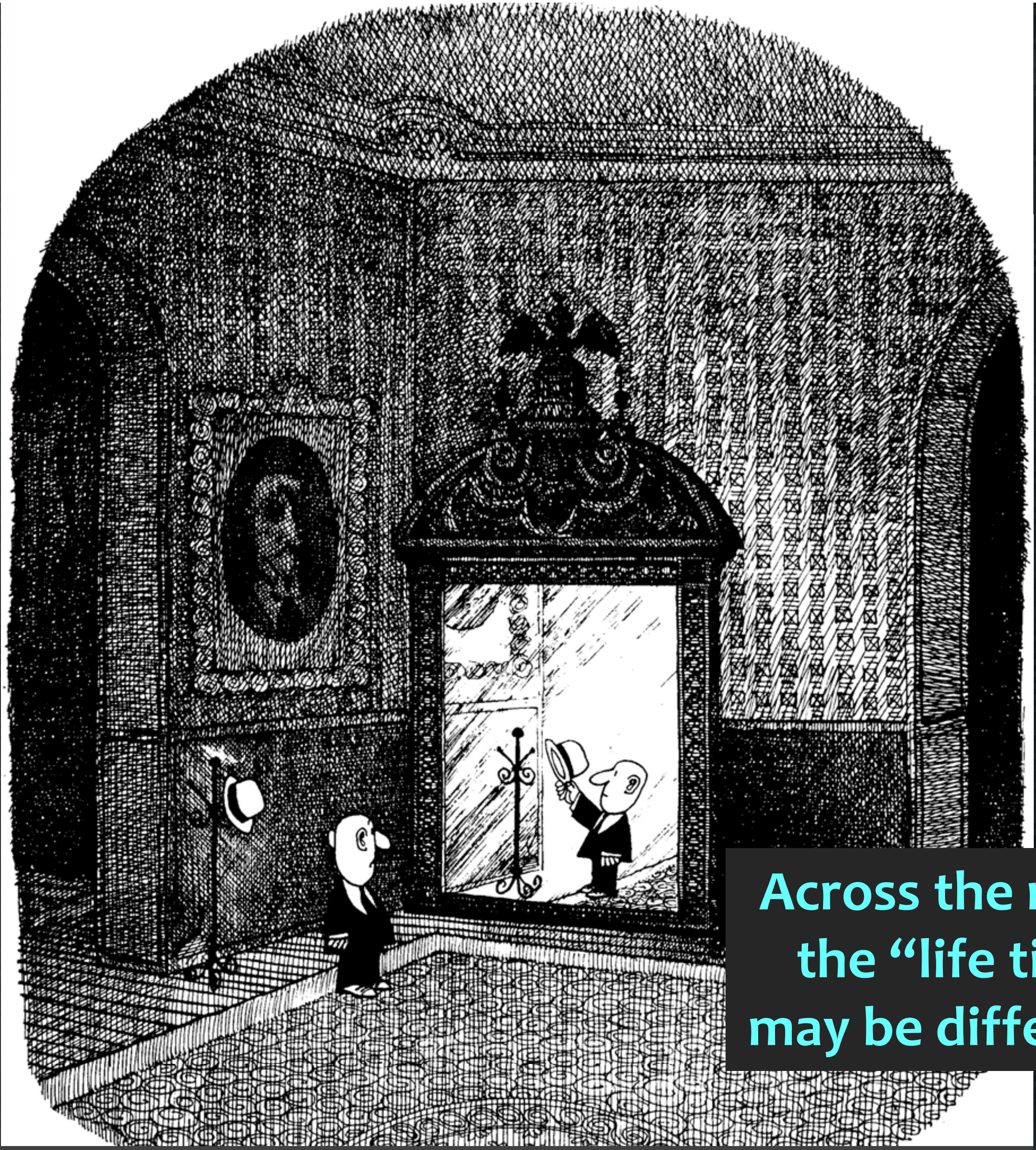


$$\Delta n^J_{g_{9/2}} = n^J_{g_{9/2}} - n^{J=0}_{g_{9/2}}$$





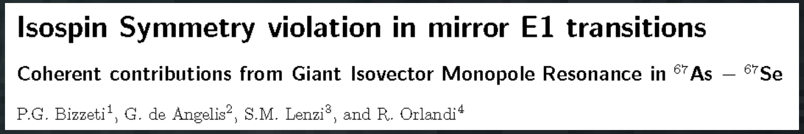
K. Kaneko at al., PRL 109, 092504 (2012)



Across the mirror
the "life time"
may be different...

Differences in the mirror B(E1) strength

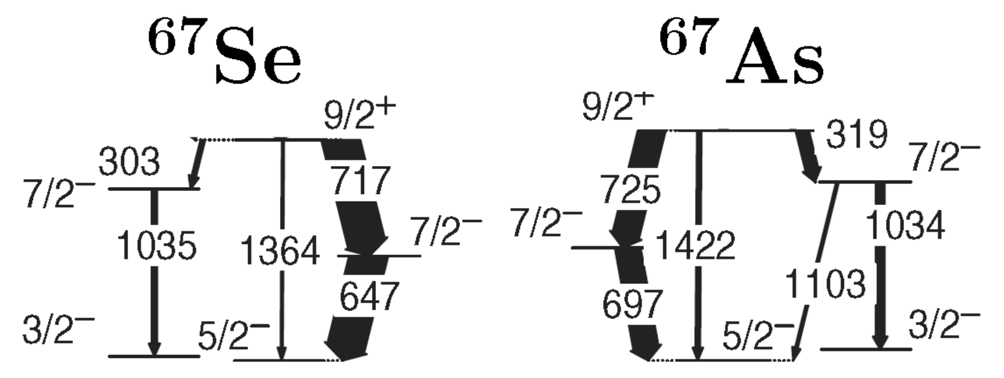
Isospin mixing in mirror pairs



Isospin Symmetry violation in mirror E1 transitions

Coherent contributions from Giant Isovector Monopole Resonance in ^{67}As – ^{67}Se

P.G. Bizzeti[1], G. de Angelis[2], S.M. Lenzi[3], and R. Orlandi[4]

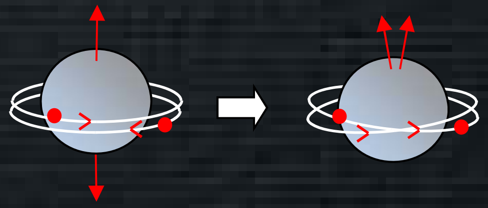


The coherent effect of Coulomb-induced mixing with the high-lying Giant Isovector Monopole Resonance is proposed as the most probable process to produce large asymmetry in the E1 transitions, with comparatively small effect on the other properties of the parent and daughter level.

Summary

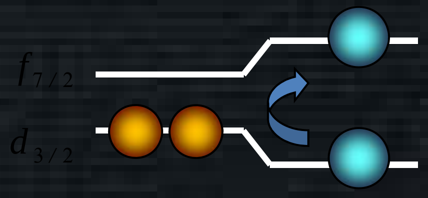
These studies allow to learn about:

- Mechanism of nucleon alignment at the backbending



- Evolution of the radii along a rotational band

- Importance of the single-particle effects:
  - test interactions and basis
  - information on the configurations



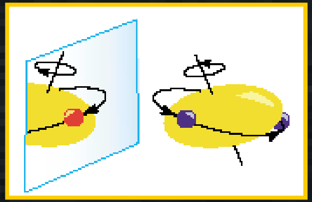
- Evidence of isospin-non-conserving terms in the nuclear interaction

J=2 anomaly

The same parameterization seems to hold in other shells which indicates a general character of this model

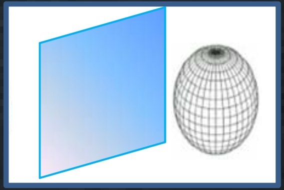
Outlook

Proton-rich nuclei present several interesting properties and phenomena that can give information on the charge symmetry and independence of the nuclear interaction



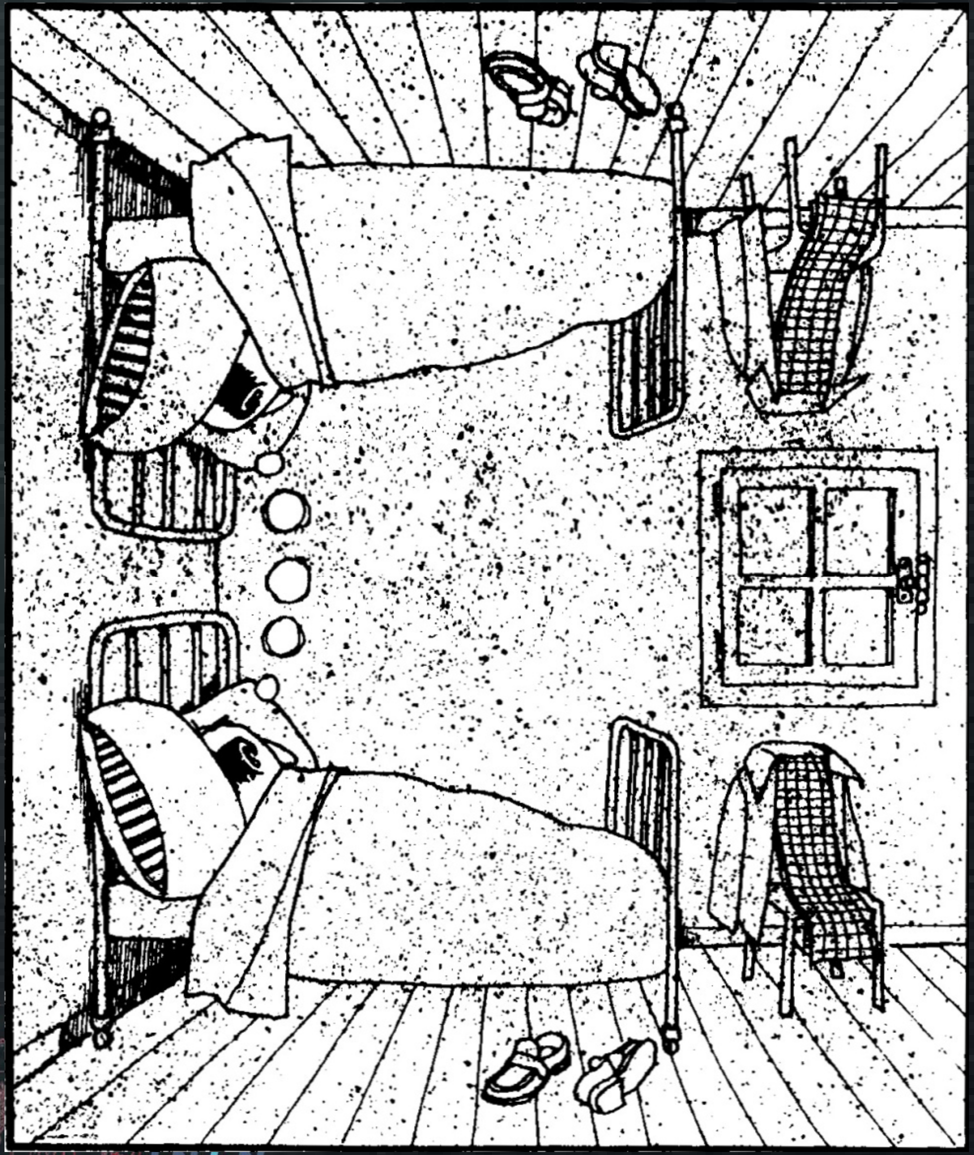
→ probe the ISB terms in other shell model spaces

Measure $T_z < 0$ members of the multiplets and study MED and Triplet energy differences.



→Measure transition probabilities and electromagnetic moments to probe shape coexistence and dynamical symmetries

→ Understand the origin of the VB term or J= 2 anomaly



Thanks Andres!

Thanks Alfredo!

Thanks Etienne!

Thanks to the Strasbourg-Madrid collaboration!

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di Fisica Nucleare