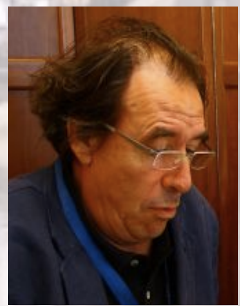
Weakly bound systems, continuum effects

Marek Płoszajczak
GANIL, Caen

CPZ-fest


Strasbourg, October 8-10, 2012

... A little bit of history ...

1981 was a good vendange in Strasbourg!

Physics Reports

Volume 70, Issue 4, April 1981, Pages 235–314

Theoretical spectroscopy and the fp shell

A. Poves†, A. Zuker

Laboratoire de Physique Nucléaire Théorique, C.R.N. Strasbourg, BP20, 67037 Strasbourg Cedex France

Physics Reports

Volume 71, Issue 3, May 1981, Pages 141–207

Quasiconfigurations and the theory of effective interactions

A Poves†, A Zuker

Laboratoire de Physique Nucléaire Théorique, C.R.N. Strasbourg, BP 20, 67037 Strasbourg Cedex, France

Ulisses and Flannagan Wake in the same year!

… somewhat earlier …

Monopole effect (anomaly) vs. multipole universality

E. Pasquini, PhD these, Report No. CRN/PT 76-14, Strasbourg 1976
E. Pasquini, A.P. Zuker, in Physics of Medium Light Nuclei, Florence, 1977
ed. by P. Blasi and R. Ricci (Editrice Compositrice, Bologna, 1978)
A. Poves, A.P. Zuker, Phys. Reports 70, 235 (1981)
E. Caurier, ANTOINE code (1989-2001)

Volume 109B, number 3 PHYSICS LETTERS 18 February 1982

THE TIME-DEPENDENT CLUSTER THEORY – APPLICATION TO THE α – α COLLISION

S. DROŻDŻ, J. OKOŁOWICZ and M. PŁOSZAJCZAK [1]
Institute of Nuclear Physics, PL-31-342 Krakow, Poland

Received 25 September 1981

Volume 109B, number 3 PHYSICS LETTERS 18 February 1982

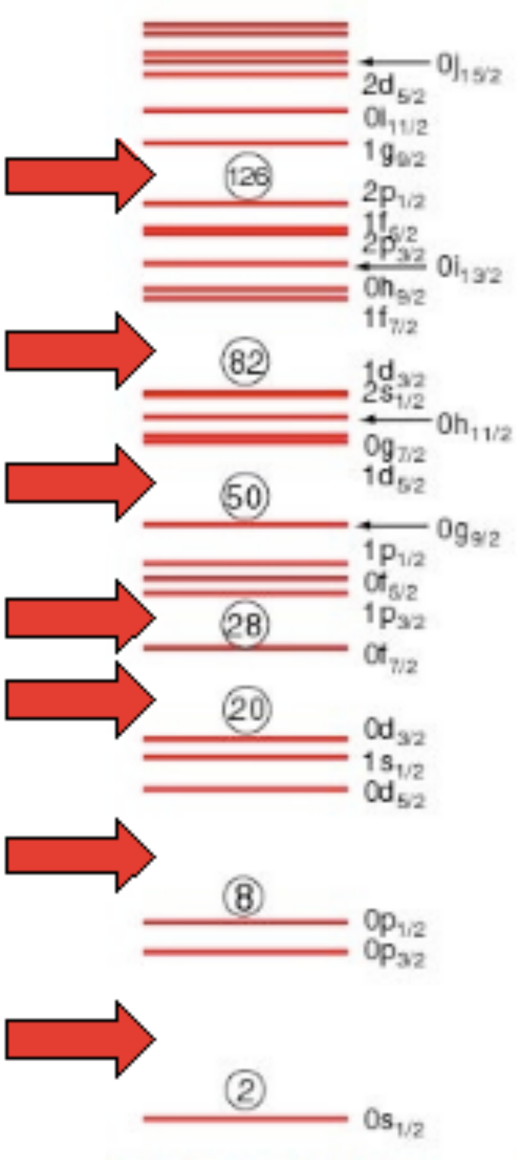
THE TIME DEPENDENT CLUSTER MODEL

E. CAURIER, B. GRAMMATICOS and T. SAMI
Laboratoire de Physique Nucléaire Théorique, CRN Strasbourg, France

Received 25 September 1981
Revised manuscript received 9 December 1981

one of the most popular many-body approach in nuclear physics

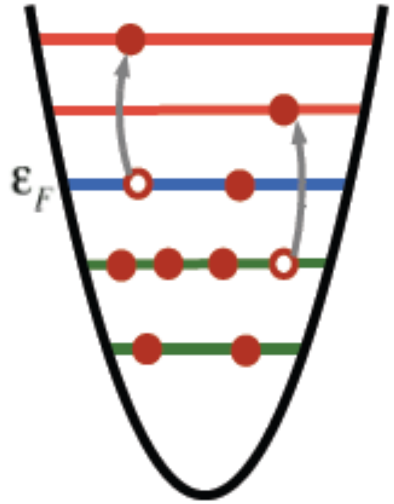
How it all began ...



Shell Model of Nuclei

Closed Quantum System

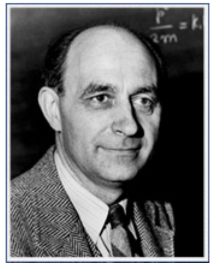
No coupling with decay channels



To what extent the change in boundary conditions at the nuclear surface due to Coulomb wave function distortion in the external region can explain relative displacement of states in mirror nuclei?

J.B. Ehrman (1950)

Role of boundary conditions in universal properties of reaction cross-sections at the threshold E.P. Wigner (1948)



Enrico Fermi

Maria Goeppert-Mayer

J. Hans D. Jensen

Eugene P. Wigner

A mutual interaction between nucleons is necessary to explain ground state spins of nuclei… but…

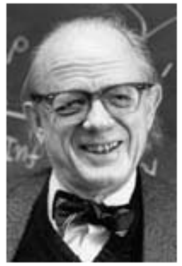
The bare interaction introduces short range correlations, i.e. admix into Shell Model wave functions states of high-lying configurations.

One can use Shell Model wave functions if effects of mixing of high lying states can be attributed to a change of the mutual interaction between valence nucleons

Recently, *ab initio* attempts starting from bare interactions between free nucleons.
Examples: Green's Function Monte Carlo method, No Core Shell Model,
No Core Gamow Shell Model, Coupled-Cluster approach, …

It is not clear how the simple and extremely successful Shell Model will emerge from these *ab initio* approaches.
Their results seem to show that the Shell Model could not possibly be a good approximation.

ANNALS OF PHYSICS: **19**, 287–313 (1962)



A Unified Theory of Nuclear Reactions. II*

Herman Feshbach

Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

The principal device employed, as in part I, is the projection operator which selects the open channel components of the wave function...

A unified approach to nuclear structure and reactions

C. Mahaux, H.A. Weidenmüller, *Shell Model Approach to Nuclear Reactions (1969)*
H.W.Bartz et al, Nucl. Phys. A275 (1977) 111
K. Bennaceur et al, Nucl. Phys. A651 (1999) 289
J. Rotureau et al, Nucl. Phys. A767 (2006) 13
…

Open QS solution in Q :

$$\mathcal{H}_{QQ}^{\text{eff}}|\Psi_\alpha\rangle = \mathcal{E}_\alpha(\text{E},\text{V}_0)|\Psi_\alpha\rangle$$

$$\langle\Psi_{\tilde{\alpha}}|\mathcal{H}_{QQ}^{\text{eff}} = \mathcal{E}_\alpha^*(\text{E},\text{V}_0)\langle\Psi_{\tilde{\alpha}}|$$

$$\langle\Psi_{\tilde{\alpha}}|\Psi_\beta\rangle = \delta_{\alpha\beta}$$

$$\mathcal{H}_{QQ}^{eff}(\text{E}) = \text{H}_{QQ} + \text{H}_{QP}\frac{1}{\text{E} - \text{H}_{PP}}\text{H}_{PQ}$$

$$\Psi_\alpha = \sum_i b_{\alpha i}\Phi_i^{(\text{SM})} \Rightarrow \Psi_E^c \sim \sum_\alpha c_\alpha \Psi_\alpha$$

For bound states: $\mathcal{E}_\alpha(E)$ is real and $\mathcal{E}_\alpha(E) = E$
For unbound states: physical resonances ≡ poles of S-matrix

PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano
National Bureau of Standards, Washington, D. C.
(Received July 14, 1961)

The actual stationary states may be represented as superpositions of states of different configurations which are "mixed" by the "configuration interaction," i.e., by terms of the Hamiltonian that are disregarded in the independent-particle approximation. The effects of configuration interaction are particularly conspicuous at energy levels above the lowest ionization threshold, where states of different configurations coincide in energy exactly since at least some of them belong to a continuous spectrum. The mixing of a configuration belonging to a discrete spectrum with continuous spectrum configurations gives rise to the phenomenon of *autoionization*. The exact coincidence of the energies of different configurations makes the ordinary perturbation theory inadequate, so that special procedures are required for the treatment of autoionization and of related phenomena.

U. Fano

The Hilbert space includes bound and scattering states → Resonance spectrum is discarded as unphysical

PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano
National Bureau of Standards, Washington, D. C.
(Received July 14, 1961)

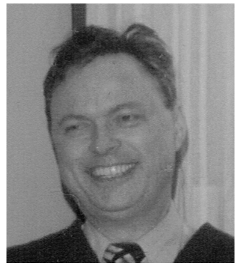
The achievement of this goal took ~40 years and required the development of:

U. Fano

- New mathematical concepts: Rigged Hilbert Space (≥1964),…

I.M. Gelfand

- Generalized completeness relation including s.p. bound states, resonances, and scattering states (~1968)



T. Berggren

- New many-body framework(s): Gamow Shell Model (2002),…

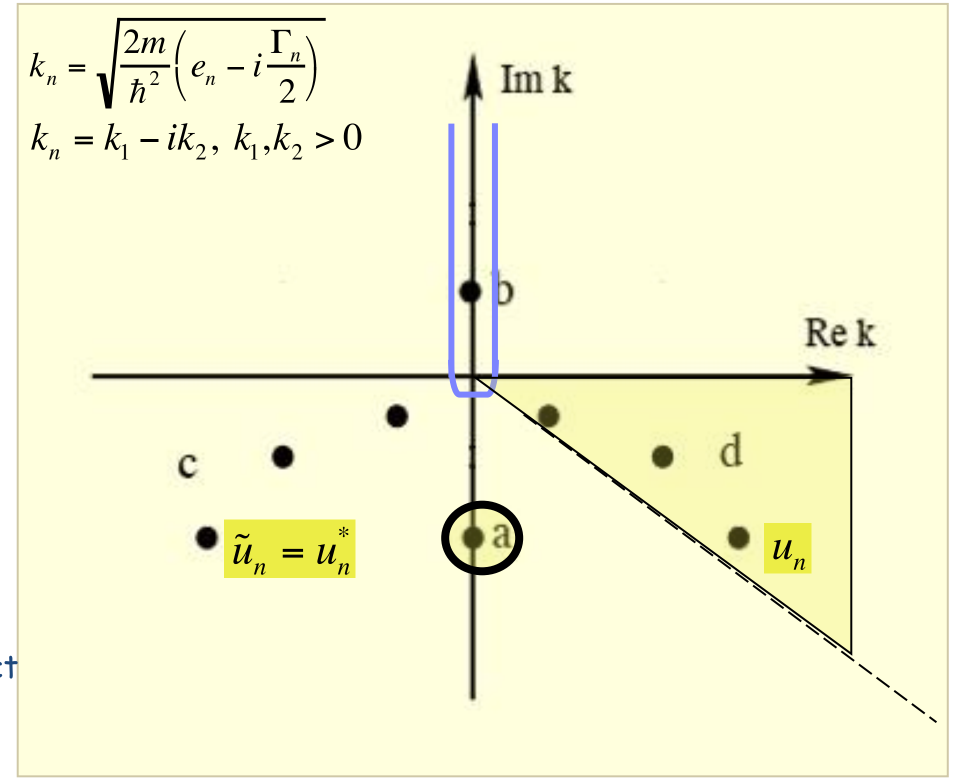
Weakly bound/unbound states

- Configuration interaction approach -

$$i\hbar\frac{\partial}{\partial t}\Phi(r,t)=\hat{H}\Phi(r,t)\ ;\quad \Phi(r,t)=\tau(t)\Psi(r)$$

$$\hat{H}\Psi=\left(e-i\frac{\Gamma}{2}\right)\Psi \quad \rightarrow \quad \tau(t)=\exp\left(-i\left(e-i\frac{\Gamma}{2}\right)\right)$$

$$\Psi(0,k)=0\ ,\quad \begin{cases}\Psi(\vec{r},k)\underset{r\to\infty}{\rightarrow} O_l(kr)\\ \Psi(\vec{r},k)\underset{r\to\infty}{\rightarrow} I_l(kr)+O_l(kr)\end{cases}$$



Only bound states are integrable!

Euclidean inner product → Rigged Hilbert Space inner product

$$\langle u_n|u_n\rangle=\int_0^\infty dr u_n^*(r)u_n(r) \longrightarrow \langle\tilde{u}_n|u_n\rangle=\int_0^\infty dr\tilde{u}_n^*(r)u_n(r)$$

Rigged Hilbert Space (RHS) is the natural setting of Quantum Mechanics in which resonance spectrum, Dirac bra-ket formalism (and Heisenberg uncertainty relations) have place

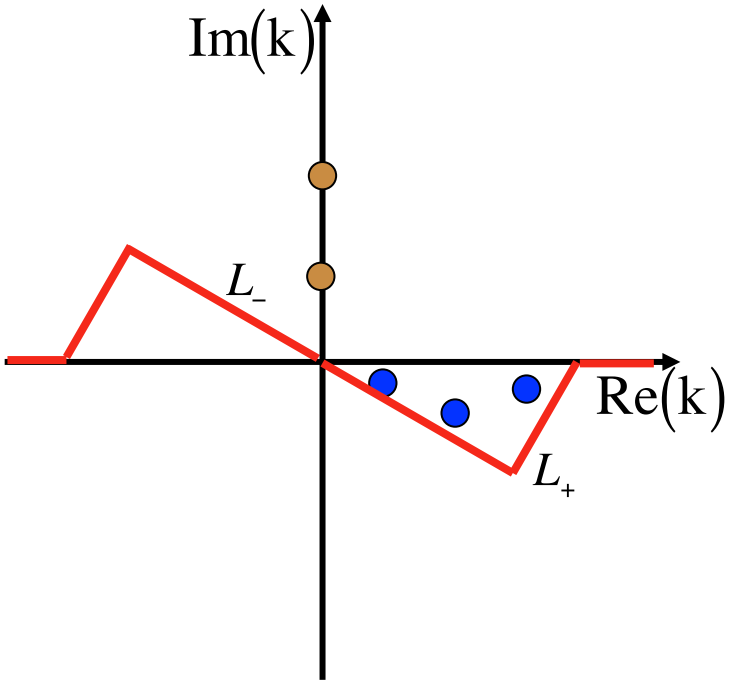
→

I.M. Gel'fand and N. J. Vilenkin. *Generalized Functions*, vol. 4: *Some Applications of Harmonic Analysis. Rigged Hilbert Spaces*, Academic Press, New York, 1964

G. Ludwig, *Foundation of Quantum Mechanics*, Vol. I and II, Springer-Verlag, New York, 1983

Completeness relation

T. Berggren, Nucl. Phys. A109, 265 (1968)



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1 \quad ; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

bound states resonances

non-resonant continuum

$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle\langle\widetilde{SD}_k| \cong 1$$

$$H \rightarrow [H]_{ij} = [H]_{ji}$$

complex-symmetric eigenvalue problem for hermitian Hamiltonian

Gamow Shell Model

N. Michel et al, PRL 89 (2002) 042502; PRC 79, 014304 (2009)
R. Id Betan et al, PRL 89 (2002) 042501

+

Density Matrix Renormalization Group method

J. Rotureau et al., PRL 97, 110603 (2007); PRC 79, 014304 (2009)

Other applications:

Continuum Coupled Cluster approach

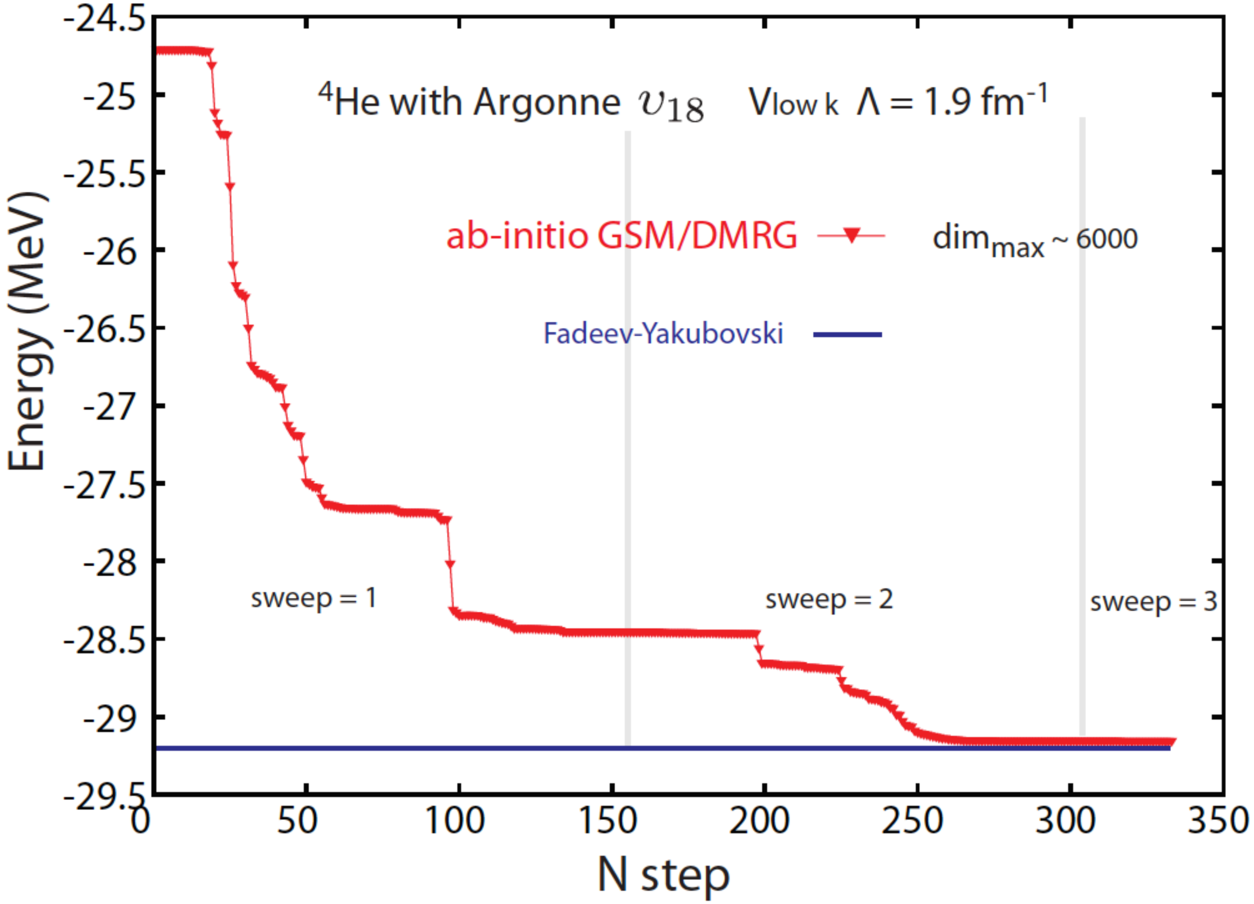
G. Hagen et al, PLB 656, 169 (2007)

No-Core Gamow Shell Model

G. Papadimitriou, J. Rotureau, N. Michel, M.P. (2012)

Example: ^{4}He – NCGSM against Fadeev-Yakubovsky

G. Papadimitriou, J. Rotureau, N. Michel, M.P. (2012)



- 2 neutrons
- 2 protons
- Pole space A:0s1/2 (p/n)
- Continuum space B:
  - p3/2,p1/2,s1/2 real energy continua
  - d5/2-d3/2, f5/2-f7/2, g7/2-g9/2 — H.O states
- 156 s.p. states total

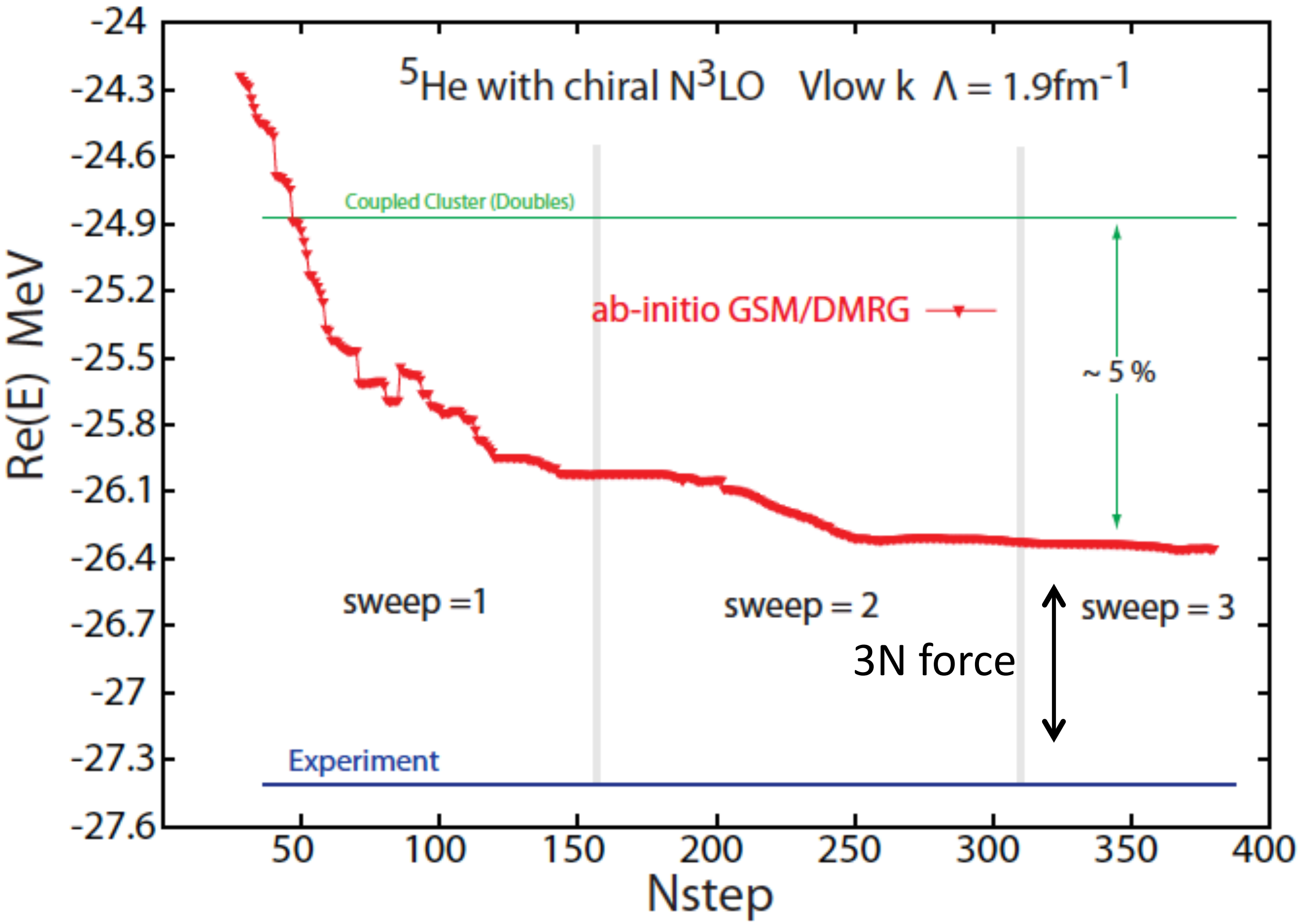
Dim for direct diagon: 119,864,088

$E_{ab\text{-}initio}$ = -29.15 MeV

E_{FY} = -29.19 MeV

Example: ^{5}He ground state energy with chiral N^3LO

G. Papadimitriou, J. Rotureau, N. Michel, M.P. (2012)

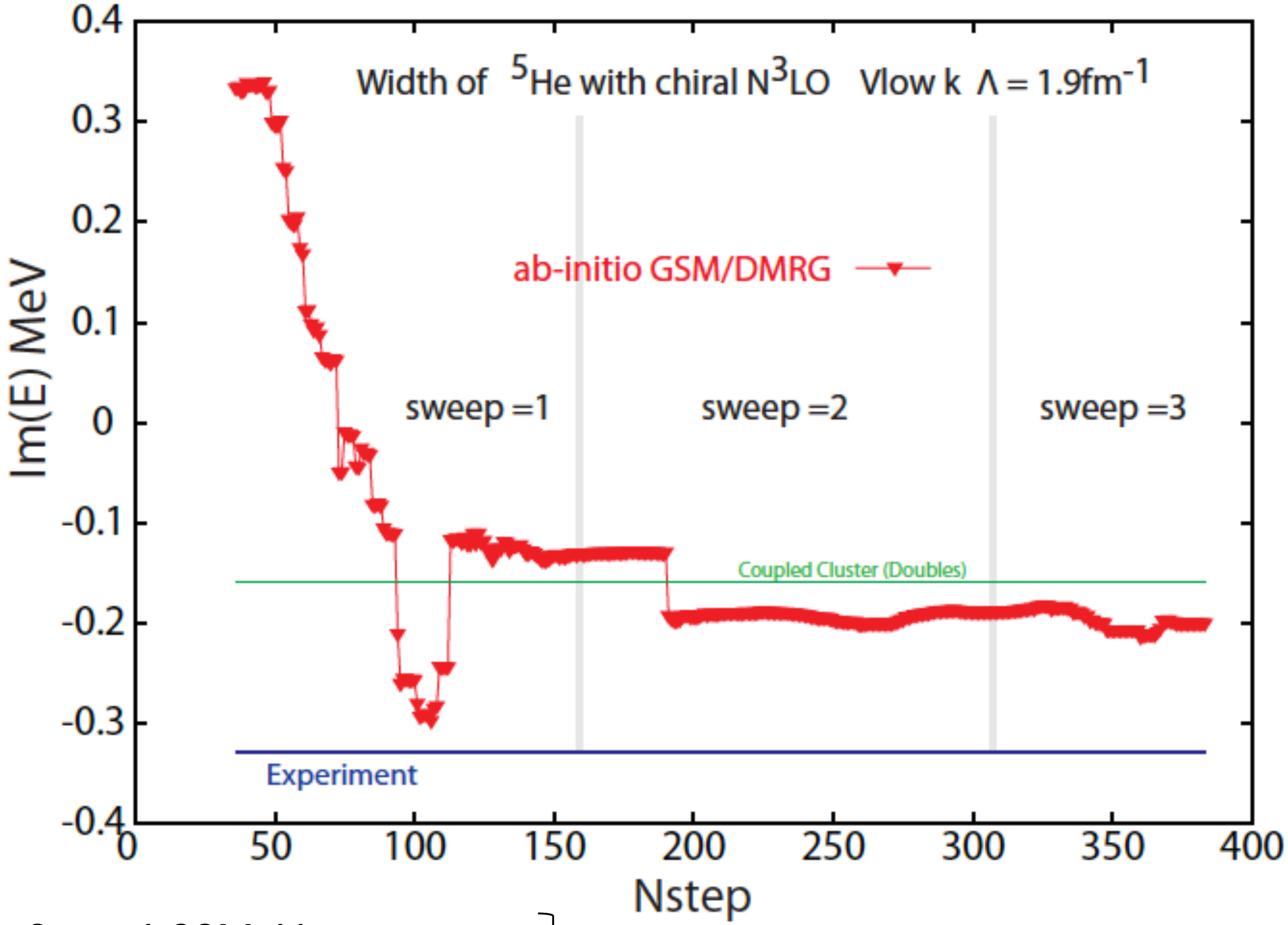


- 3 neutrons
- 2 protons
- Pole space A:0s1/2 (p/n) + 0p3/2 n resonant state
- Continuum space B:
  - p3/2 complex continuum
  - p1/2-s1/2 real continua
  - d5/2-d3/2, f5/2-f7/2, g7/2-g9/2 } H.O states
- 157 s.p. states total

Dim for direct diagon: 3×10^9

DMRG dim ~ 10^5

Example: ^{5}He ground state width with chiral $N^{3}LO$



S_{1n} = -1.20MeV

S_{1n} (exp) = -0.89 MeV

- Unbound character of ^{5}He reproduced
- Good agreement of the width with experiment

Future plans:

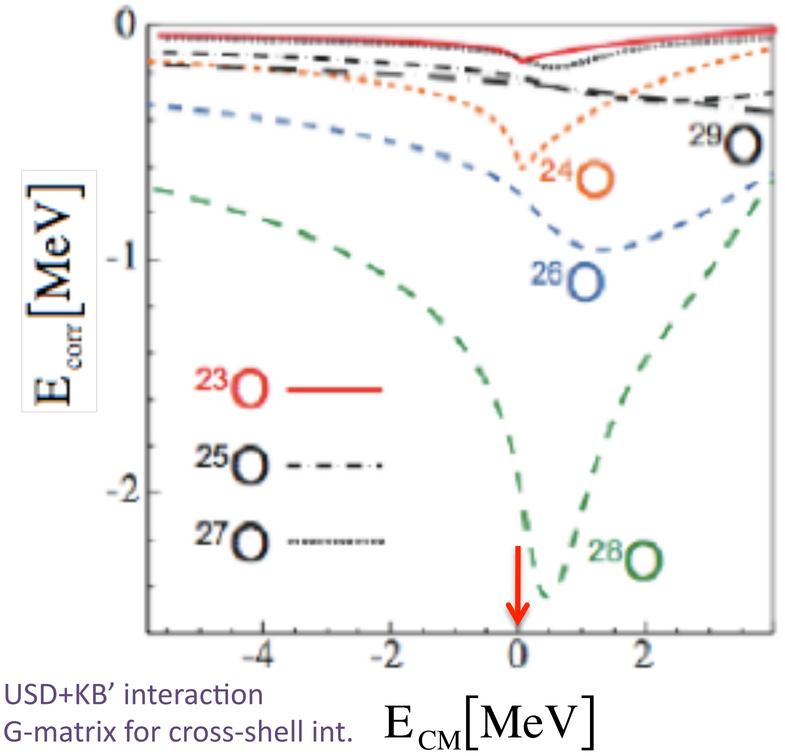
- heavy Hydrogen isotopes: ^{3}H, ^{4}H, ^{5}H, ^{6}H, ^{7}H
- Borromean halo systems: ^{4}He, ^{5}He, ^{6}He, ^{7}He, ^{8}He

How does the continuum works?

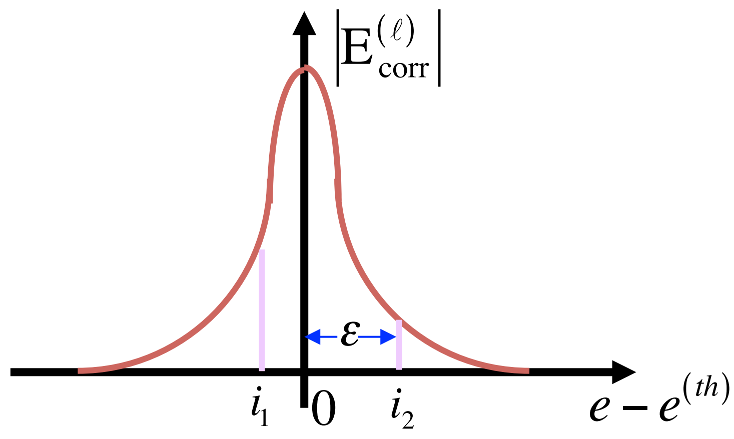
Continuum coupling correlation energy is of the same order as the pairing correlation energy

?

Instability of SM eigenstates at the channel threshold?



USD+KB' interaction
G-matrix for cross-shell int.
WB continuum coupling



Admixture of many-body continuum states with $E > E_{th}$

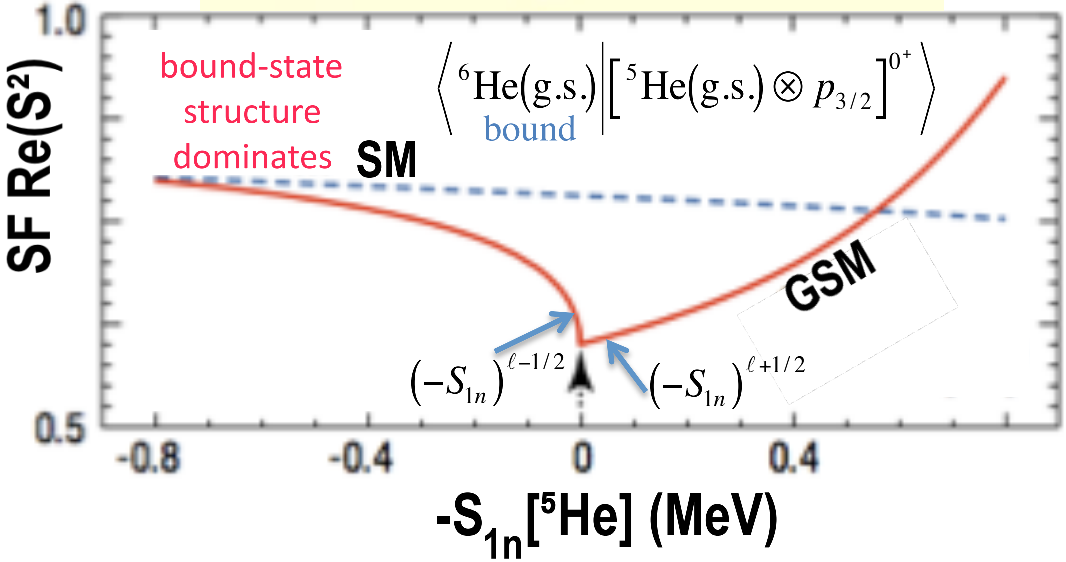
What is the nature of this phenomenon?

The interplay between Hermitian and anti-Hermitian couplings is a source of collective effects

- resonance trapping — P. Kleinwächter, I. Rotter, PRC 32, 1742 (1985)
- super-radiance phenomenon — N. Auerbach, V.G. Zelevinsky, Rep. Prog. Phys. 74, 106301 (2011)
- modification of spectral fluctuations — Y.V. Fyodorov, B.A. Khoruzhenko, PRL 83, 65 (1999)
- multichannel coupling effects in reaction cross-sections and shell occupancies — N. Michel, W. Nazarewicz, M.P., PRC 75, 031301 (2007)
- clustering — J. Okolowicz, M.P., W. Nazarewicz, arXiv:1202.6290

Example:

$$S^2 \equiv \int u_{\ell j}^2(r)dr = \sum_{\mathcal{B}} \langle \widetilde{\Psi_A^{J_A}} || a_{\ell j}^{+}(\mathcal{B}) || \Psi_{A-1}^{J_{A-1}} \rangle^2$$



Analogy with the Wigner threshold phenomenon for reaction cross-sections

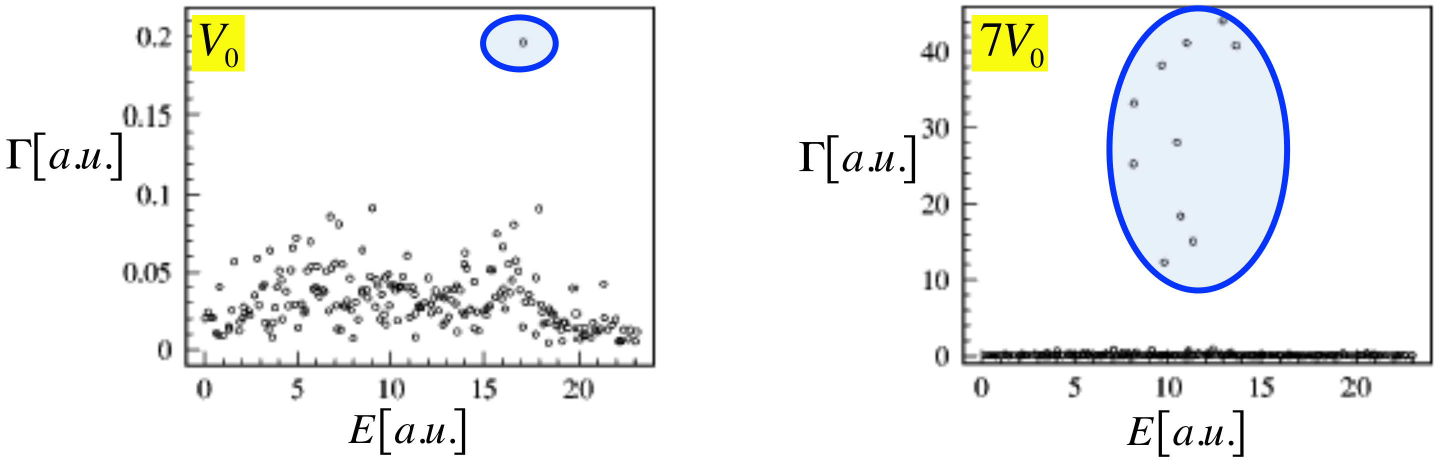
E.P. Wigner, PR 73, 1002 (1948)

$$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1} \quad \Leftrightarrow \quad (-S_n)^{\ell-1/2} \text{ for } S_n < 0$$

$$X(a,b)Y : \sigma_\ell \sim k^{2\ell+1} \quad \Leftrightarrow \quad (-S_n)^{\ell+1/2} \text{ for } S_n > 0$$

Example: Segregation of time scales in the continuum

$J^{\pi} = 0^{+}, T = 0$ states in ^{24}Mg , 10 channels



- The number of broad states is limited by the number of decay channels
- The majority of CSM states are relatively narrow

J. Okolowicz et al., Phys. Reports 374 (2003) 271

Example: Charge radii and neutron correlations in halo nuclei: ^{6}He and ^{8}He

G. Papadimitriou et al, PRC 84, 051304(R) (2011)

Translationally invariant Hamiltonian:

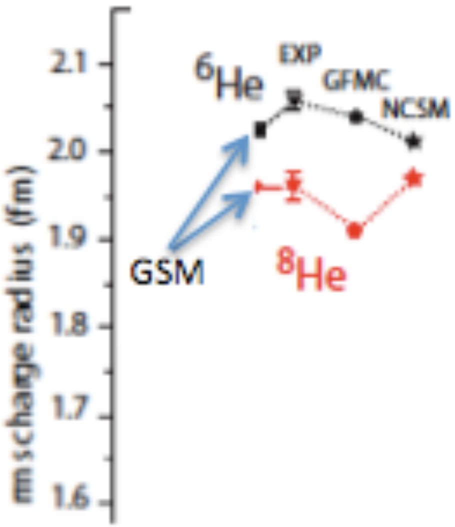
$$H = \sum_{i=1}^{A_v} \left(\frac{\mathbf{p}_i^2}{2\mu} + U_i \right) + \sum_{i<j}^{A_v} \left(V_{ij} + \frac{\mathbf{p}_i \mathbf{p}_j}{A_c} \right)$$

"Recoil" term

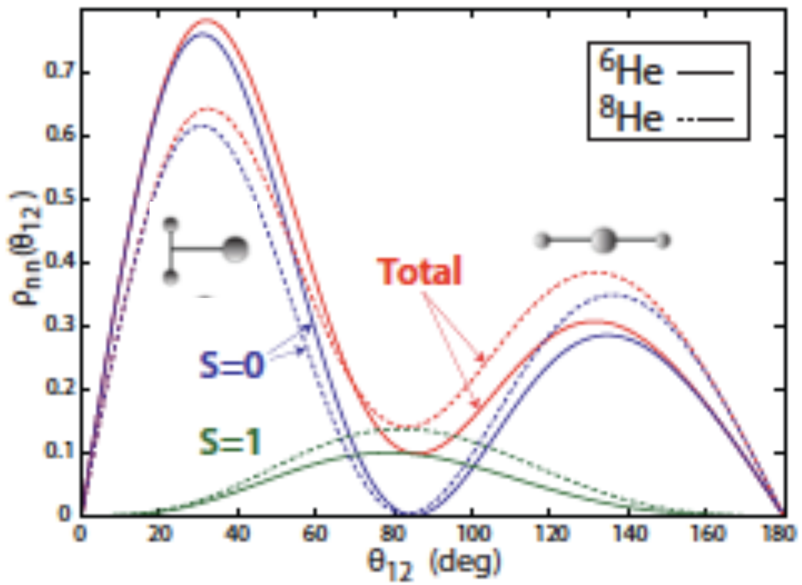
U - '^{5}He' WS potential with s.o.
V – finite-range Minnesota int.

GSM Hamiltonian reproduces the energetics in the helium isotopic chain:

$$S_{1n}, S_{2n}, 2^+\left({}^6\text{He}\right), 3/2^-\left({}^7\text{He}\right), \ldots$$



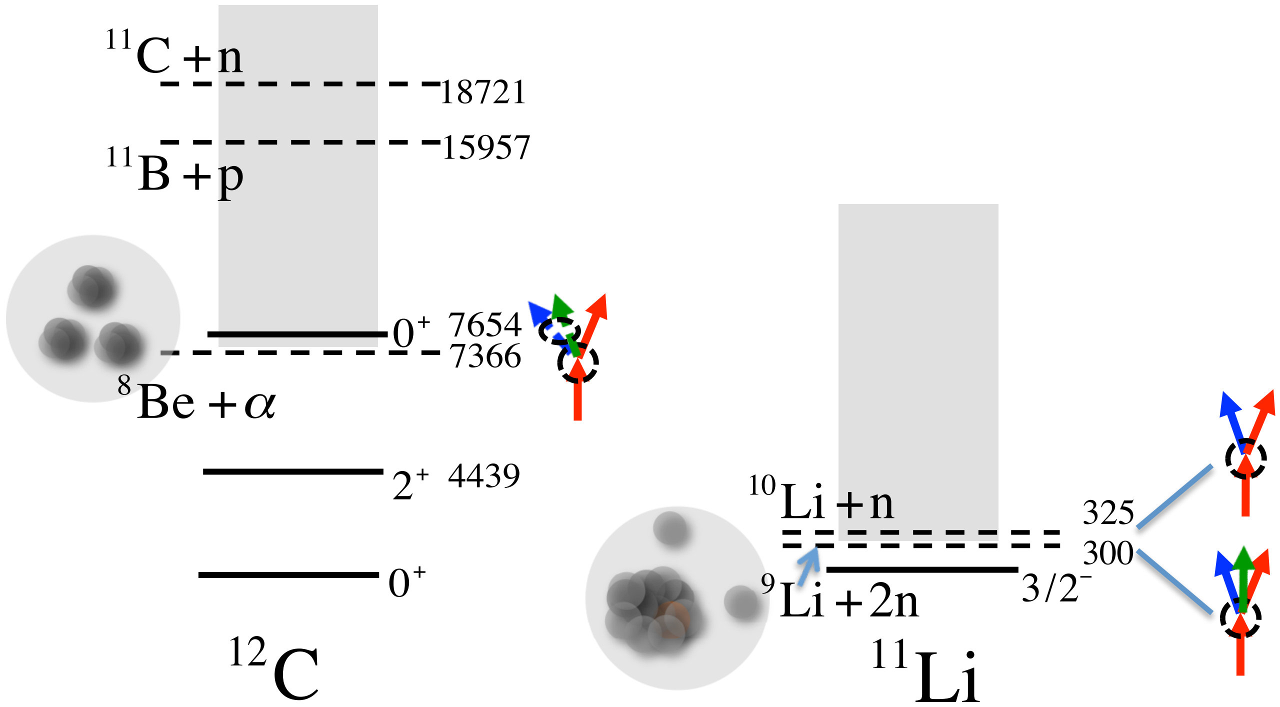
$$S_{2n}\left({}^6\text{He}\right) < S_{2n}\left({}^8\text{He}\right)$$



Reduction of the charge radius in ^{8}He is due to a reduction of S=0 'dineutron configuration' which is strongly enhanced by the coupling to the continuum.

Example: Nuclear clustering

J. Okolowicz, W. Nazarewicz, M.P., arXiv:1202.6290

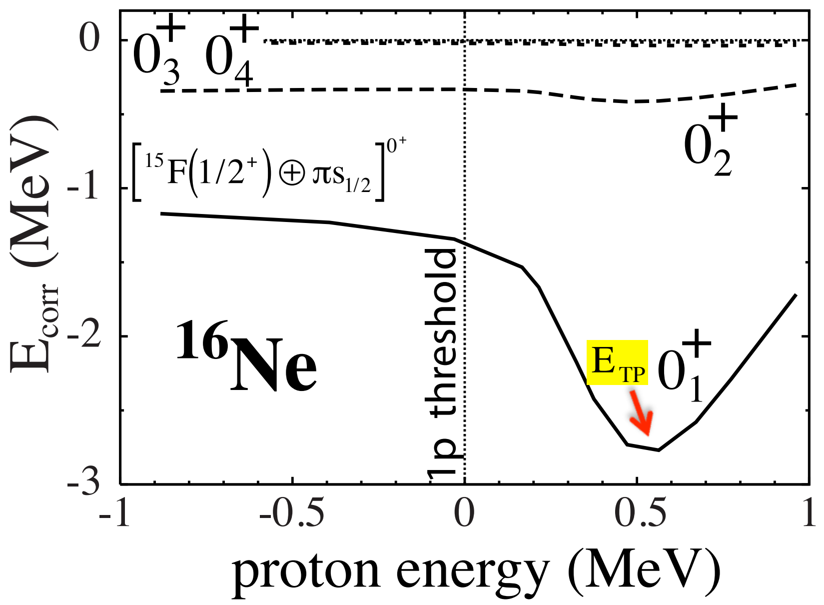


Specific:

Energetic order of emission thresholds and absence of stable cluster entirely composed of like nucleons

Generic:

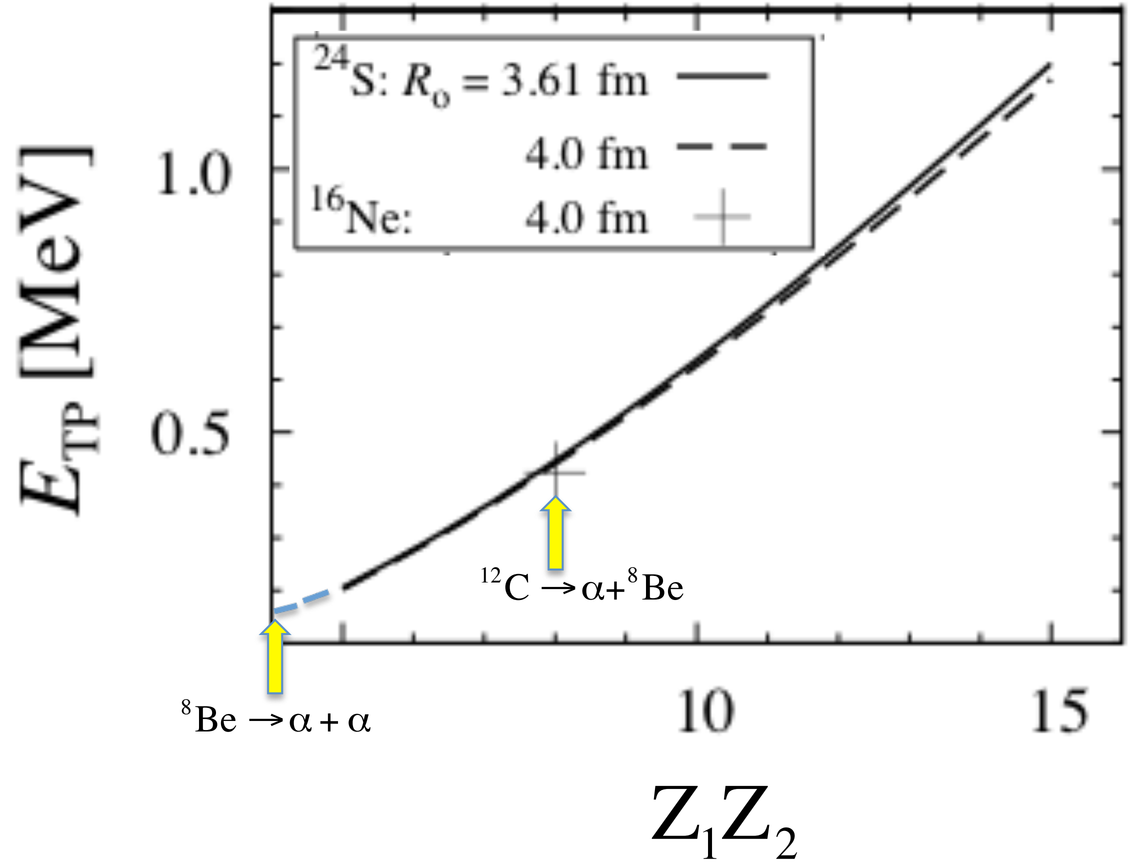
Correlations in the near-threshold states depend on the nature of the nearest branching point



The maximum continuum coupling point E_{TP} are determined by the interplay between the Coulomb (centrifugal) interaction and the continuum coupling

Interaction through the continuum leads to the collectivization of SM eigenstates and formation of the (aligned) CSM eigenstate which couples strongly to the decay channel and, hence, carries many of its characteristics.

Universality of the collective mixing of eigenstates via the continuum



$^{12}C(0_2^+) \cong 290$keV

$^{8}Be(0_1^+) = 97$keV

For a given value of Z_1Z_2, the maximum continuum coupling energy depends weakly on the nature of the charged particle decay channel and the parameters of the potential.

Outlook

1. The non-resonant continuum is essential for the spectroscopy of weakly bound nuclei:
   - mixing of Shell Model states through the particle continuum, modification of the effective interaction and NN correlations
   - collective phenomena: clustering, resonance trapping, super-radiance, multichannel coupling effects in reaction cross-sections and shell occupancies, modification of spectral fluctuations
   - breaking of the isospin symmetry
   - energy shifts, coalescence of eigenvalues, …

2. The clustering is the generic near-threshold phenomenon in open quantum system which does not originate from any particular property of forces or any dynamical symmetry of the many-body problem.
   - Nuclear clustering is a consequence of the collective coupling of SM states via the decay channel and may appear in the narrow energy window around the point of maximum continuum coupling.

All the best for you!