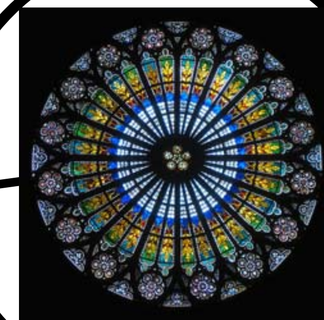
Nonmicroscopic and microscopic investigations of α condensate states in ^{12}C and ^{16}O.

Strasbourg 08/10/2012



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Dedicated to A.P. Zuker, PHD and HDR supervisor.

Basic concept of α clustering and related theoretical models

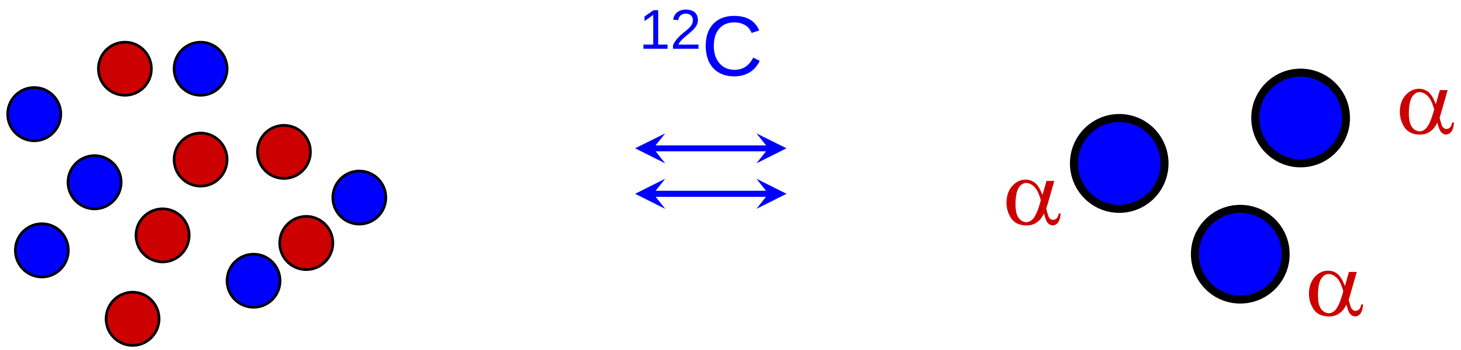
- Owing to its compactness and strong binding, the idea that the α particle tends to keep its own identity in light nuclei is at the basis of cluster models.

- According to the description of the α particle as an elementary particle or as a composite one, nonmicroscopic and microscopic theoretical frameworks have been developped.

- In nonmicroscopic approches, the α is treated as point like.

- In microscopic descriptions, the system is described at the nucleon level. Study of the wave functions described in adapted variational basis allows to point out clustering effect.

Microscopic approach ⟷ Nonmicroscopic approach



12 N = 12 Fermions

NN, … potentials

Antisymmetrization

3 α = 3 bosons

αα, …potentials

Symmetrization

- Investigations of the transition from descriptions in terms of A fermions toward those in terms of N bosons is interesting to point out states with strong clustering such as so-called condensate states, a typical example being the ^{12}C 0^+_2 Hoyle state.

- Cluster states are usually located near the thresholds of relevant cluster breakup.
- The physics of clustering necessitates in most cases a proper treatment of resonances.

- Unified theoretical frameworks.
- Difficulties to describe many body resonances.
- Study of reactions between the clusters possible.

Nonmicroscopic study of 3α bosonic states in the ^{12}C nucleus

R. Lazauskas, M. Dufour, PRC84, 064318 (2011)

➢For the nonmicroscopic approaches, one of the most important challenge is to replace forces acting between nucleons by phenomenological potentials between the α's.

➢Phenomenological $\alpha\alpha$ potentials chosen to be consistent with the non existence of ^{8}Be bound state and with experimental $\alpha\alpha$ phase shifts have been developped. They have also to simulate the Pauli Principle neglected in nonmicroscopic approaches.

•Local shalow Ali and Bodmer potential (1960): strong repulsive core to simulate the Pauli principle but underestimation of the ^{12}C ground state. $V\alpha\alpha\alpha$ are introduced.

•Others: Local deep Buck potential (forbidden states), …

- Our calculations have been performed with the non local fish-bone potential of Z. Papp and S. Moszkowski. (Modern Physics Letters B, 2008)

- Non locality: well adapted to describe composite systems.

- PM potential presents a rather sophiticated structure. It reproduces:

• The $E_{2\alpha}$ = 91.6 keV resonance energy in ^{8}Be.

• The L=0, 2, and 4, $\alpha\alpha$ phase shifts up to 20 MeV.

• The ground-state binding energy of ^{12}C.

• It also simulate the Pauli Principle. Pauli forbidden states of relative motion between the clusters are removed.

- Complemented by the Coulomb force between the α's.

No $V\alpha\alpha\alpha$ potential

➢The rigorous formalism of the Faddeev equations is used to solve the nonrelativistic three-body problem.

$$\Psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)}$$

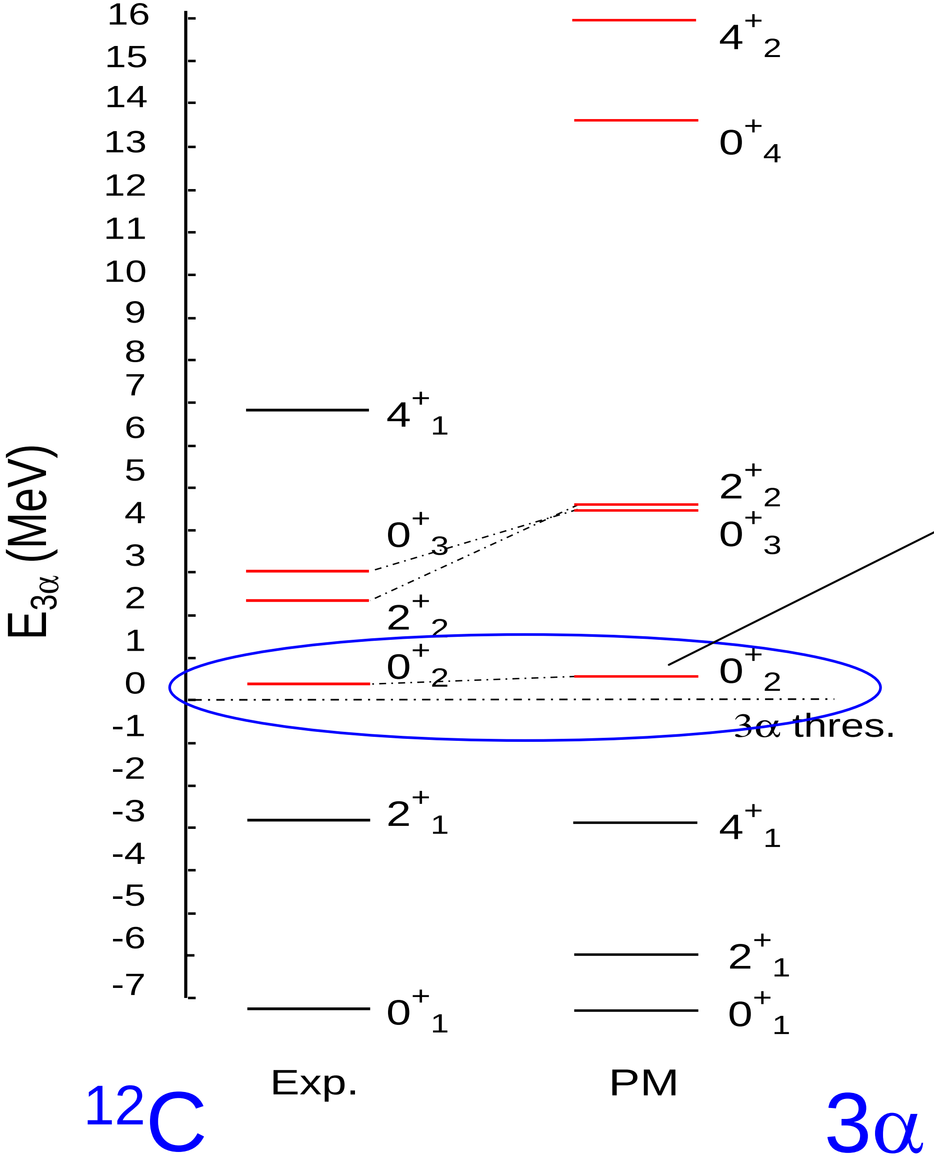
$$\begin{aligned}
(E - H_0 - V_{23}) \left|\psi^{(1)}\right\rangle &= V_{23}\left[\left|\psi^{(2)}\right\rangle + \left|\psi^{(3)}\right\rangle\right], \\
(E - H_0 - V_{31}) \left|\psi^{(2)}\right\rangle &= V_{31}\left[\left|\psi^{(3)}\right\rangle + \left|\psi^{(1)}\right\rangle\right], \\
(E - H_0 - V_{12}) \left|\psi^{(3)}\right\rangle &= V_{12}\left[\left|\psi^{(1)}\right\rangle + \left|\psi^{(2)}\right\rangle\right].
\end{aligned}$$

➢Calculation of bound, resonances and scattering states.

➢The Complex Scaling Method is used to compute the energy and width of the resonances.

Main Results …

Family of positive parity states (in red) which share common features with the very well reproduced Hoyle state.



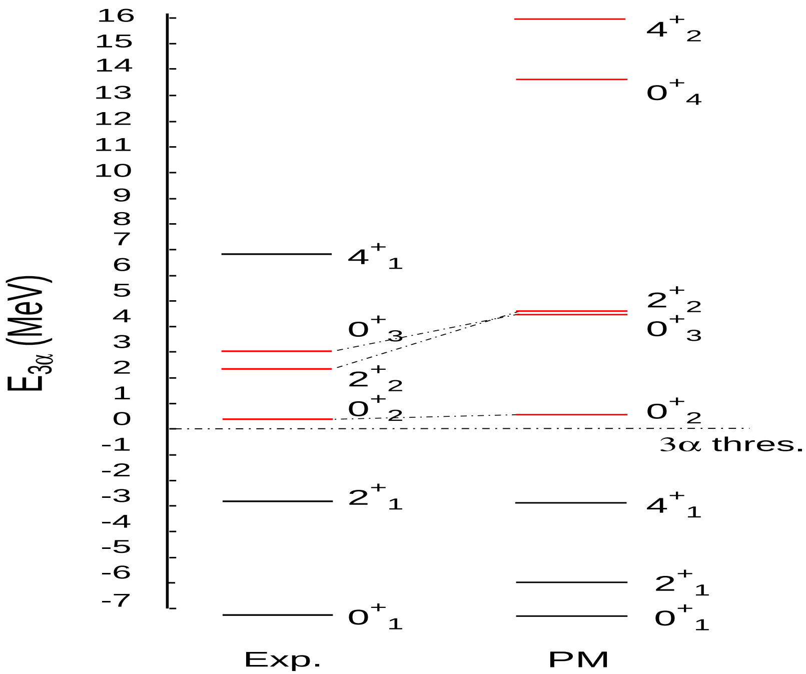
0^+_2, 0^+_3, 2^+_2, 0^+_4, 4^+_2

0^+_2 Hoyle State

$E_{3\alpha}^{Exp} = 0.3795$ MeV

$E_{3\alpha}^{PM} = 0.538$ MeV

Family of positive parity states: Common features



J^π	$[\ell_x, \ell_y]$
0_2^+	[0, 0] (0.96)
0_3^+	[2, 2] (1.0)
0_4^+	[4, 4] (0.82)
2_2^+	[2, 2] (0.94)
4_2^+	[4, 4] (0.80)

- Analysis of the Wave functions shows that respective wave functions are dominated by a single partial wave with $\ell_x = \ell_y$

ℓ_x and ℓ_y are partial angular momenta associated with Jacobi coordinate $\overrightarrow{x}$ and $\overrightarrow{y}$

J^π	$E_{3\alpha}^{expt}$	Γ^{expt}	$E_{3\alpha}^{PM}$	Γ^{PM}	$\sqrt{r^2}^{PM}$
0_2^+	0.3795	$(8.5 \pm 1.0) \times 10^{-6}$	0.538	$\leqslant 2.2 \times 10^{-3}$	3.98
0_3^+	≈ 3	≈ 3	4.42	0.85	3.58
2_2^+	2.33	0.6(1)	4.55	0.66	3.30
0_4^+			13.49	1.2	2.92
4_2^+			15.69	1.36	2.76

M. Freer *et al*., PRC 80, 041303 (2009)

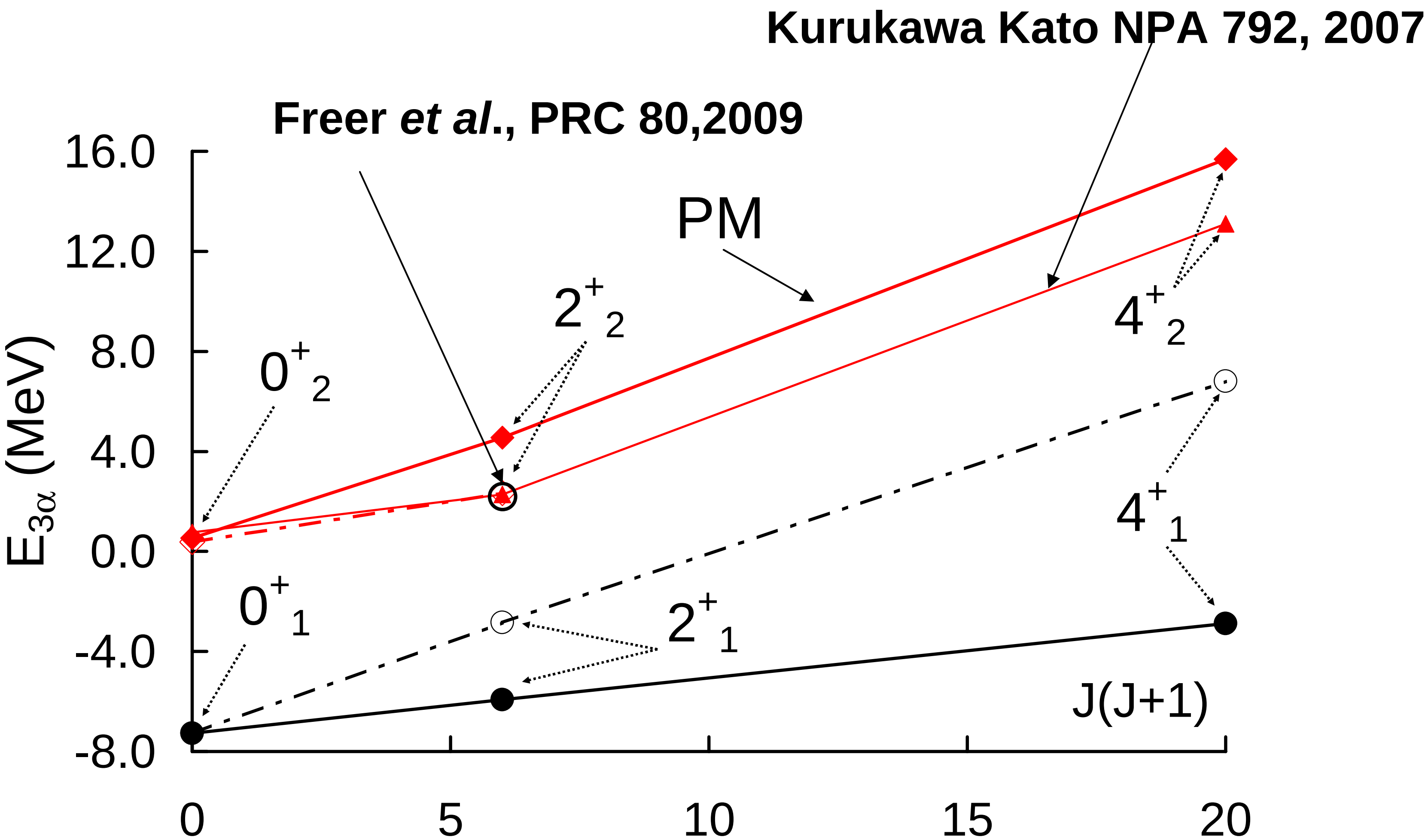
- rms radius (in fm) compatible with analysis of electron scattering data and other theoretical works.

- 2^+_2: ~2 MeV higher as compared to the measure of Freer et *al*.

- 0^+_3: CSM not adapted to describe such a broad state.
  M. Itoh, measures compatible with 2 states (Debreccen 2012).

- 0^+_4, 4^+_2: no present experimental assignement.

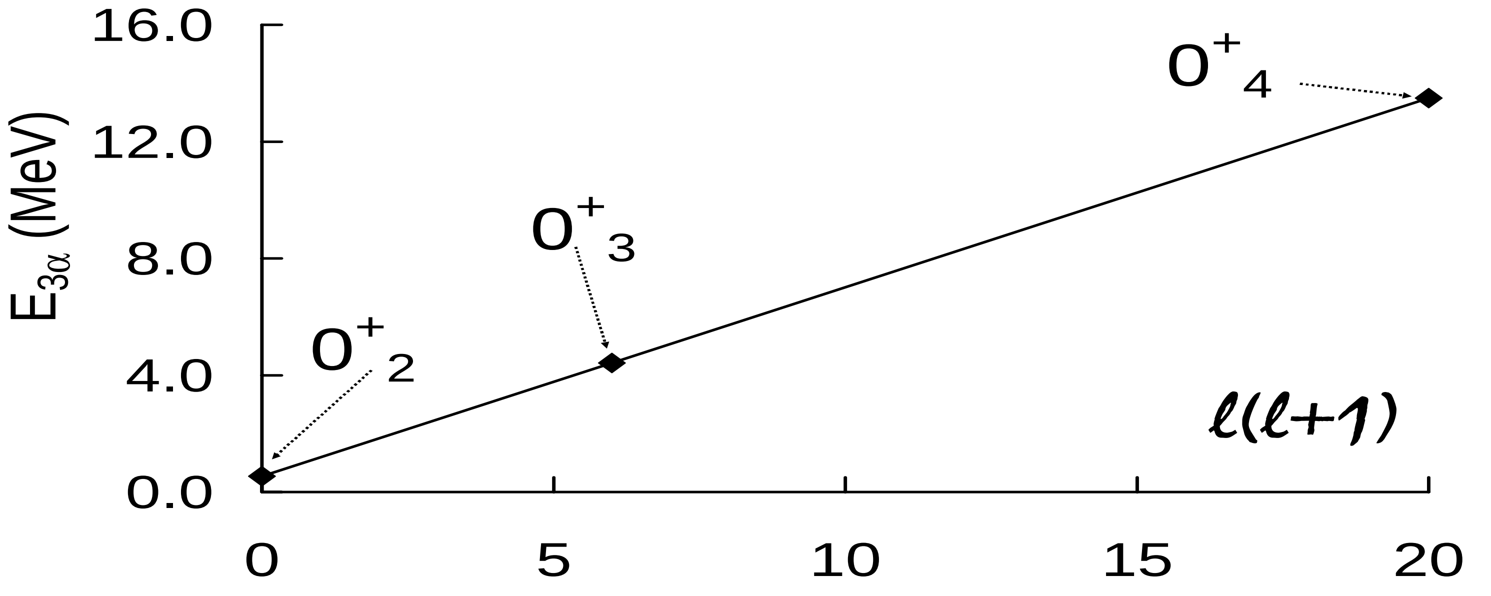
Hoyle-state rotational band (in red)



0^+_1: rms radius: exp. = 2.47 fm, Theo = 2.4 fm.

0^+_1, 2^+_1, 4^+_1 sequence keeps a rotational band structure but with a too-small rotational constant.

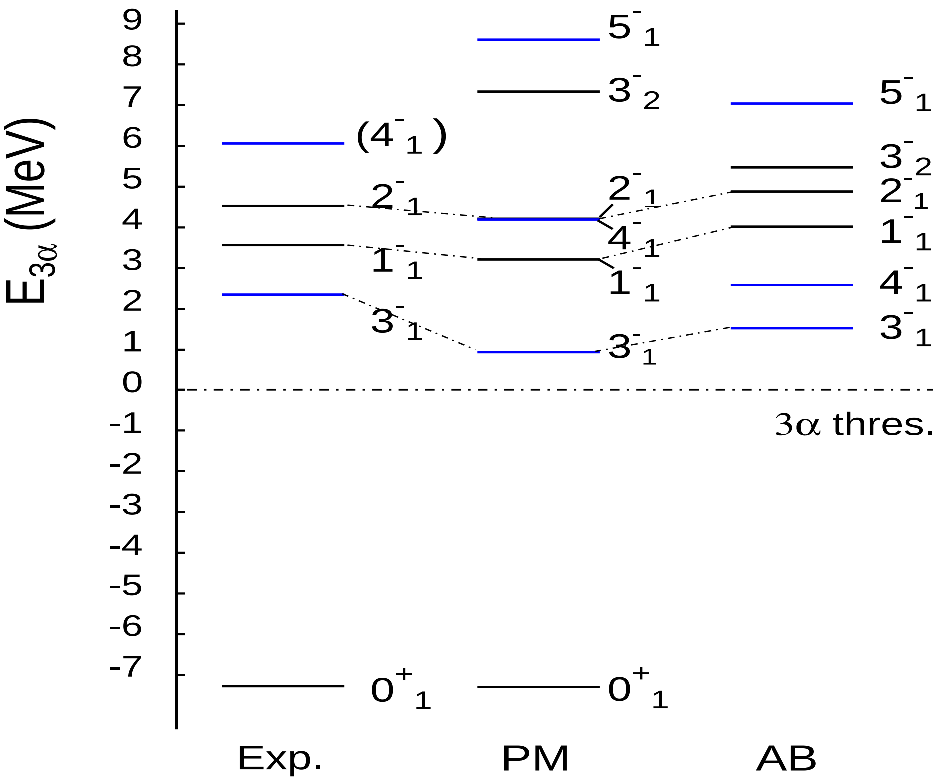
Kind of rotational band between the 0^+'s of the Hoyle family



The α-particle condensate in nuclei is a novel state described by a product state of a's, all with their c.o.m. in the lowest 0S orbit.
Yamada, Funaki, Horiuchi, Ropke, Schuck and Tohsaki, Clusters in Nuclei, Lecture Notes in Physics, Springer, Volume 2, Ed. C. Beck

- Idea of a set of condensate states differentiated by the relative angular momentum values.

Negative-parity resonances are also well reproduced



In blue: States corresponding to the K=3^-_1 rotational band.

- Bosonic nature of the wave functions.
- Interpretation in terms of condensate impossible.

Nonmicroscopic investigation summary

R. Lazauskas, M. Dufour, PRC84, 064318 (2011)

- The non local Papp and Moszkovski potential appears to be an efficient tool to study the 3 α system without $V\alpha\alpha\alpha$ potential.

- Confirmation of the 0^+_2 state as a 3 α dilute gas with zero angular-momentum values between the α's.

- We get other states which share common features with the Hoyle state and which are differentiated by the relative angular momentum values.

Calculations of the 4 α system within the same framework, in progress …

Microscopic study of condensate states in the ^{16}O nucleus with a $^{12}C + \alpha$ model.

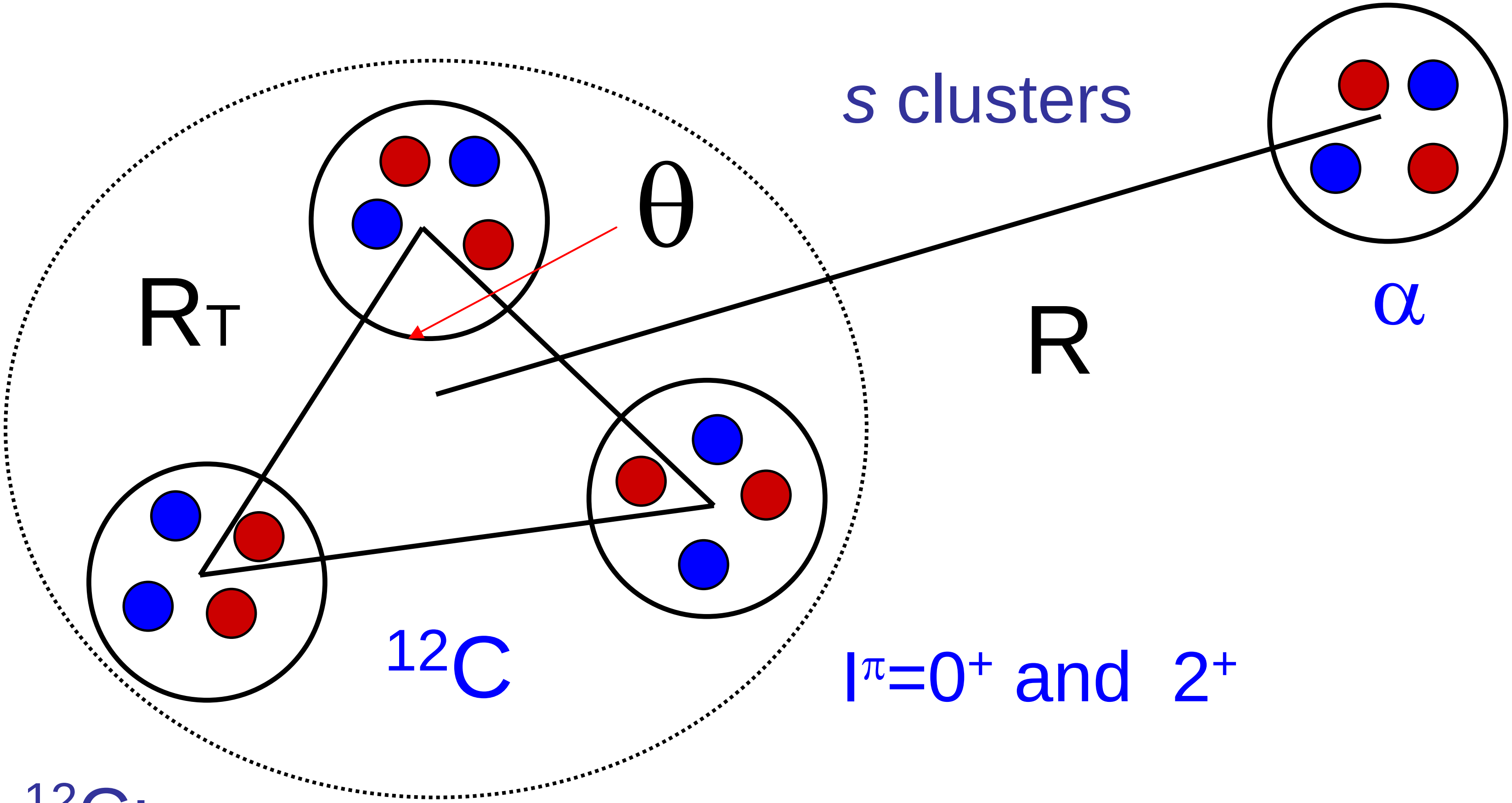
M.Dufour, P. Descouvemont

- The Schrödinger equation of the 16-Nucleon system is variationaly solved with the Generator Coordinate Method (GCM) combined with the Microscopic R-Matrix method (MRM).

- This framework ensures a good asymptotic behaviour of the wave functions.

P. Descouvemont, D. Baye, Rep. Prog., Phys. 73 (2010) 036301.
P. Descouvemont, M. Dufour, Microscopic Cluster Models, Lecture Notes in Physics, Springer T2 (2011) (Ed. C. Beck).

16 Nucleon wave functions fully antisymmetrized



^{12}C:

- Isosceles triangles described by 2 Generator coordinates.
- To mix isosceles triangles to optimize the $0^{+}_{2} - 0^{+}_{1}$ gap.

Physical idea to identify a 4α condensate state in such a microscopic approach:

- The 0^+_2 Hoyle state in ^{12}C is a 3 α states.
- An equivalent 4 α state in ^{16}O is expected to be a resonance above the 4 α threshold.
- Its microscopic wave function is expected to have a dominant component on the $^{12}C(0^+_2)+\alpha$ channel.
- To identify ^{16}O condensate by analysing the $^{12}C(0^+_2)+\alpha$ channel.

Our theoretical framework is fully adapted to perform such analysis.

Resonating Group Method (RGM) partial WF

$$\Psi^{JM\pi} = \sum_{c\ell I} \Psi^{JM\pi}_{c\ell I} = \sum_{c\ell I} \mathcal{A} \; g^{J\pi}_{c\ell I}(\rho) \; \varphi^{J\pi}_{c\ell I}$$

- $\mathcal{A}$ is the A-nucleon antisymmetrizor.
- $g^{J\pi}_{c\ell I}(\rho)$ are the radial functions.
- $\varphi^{J\pi}_{c\ell I} = \left[[\phi^{I_1}_c \otimes \phi^{I_2}_c]^I \otimes Y_\ell(\Omega_\rho) \right]^{JM}$, are the channel WFs.
- $\phi^{I_1}_c$ and $\phi^{I_2}_c$ are the internal WFs of (1)=^{12}C and (2)=α.
- RGM and GCM are equivalent.
- In the GCM, the $g^{J\pi}_{c\ell I}(\rho)$ functions are expanded over Gaussian functions centered at different locations called the generator coordinates.

Microscopic R Matrix method (MRM)

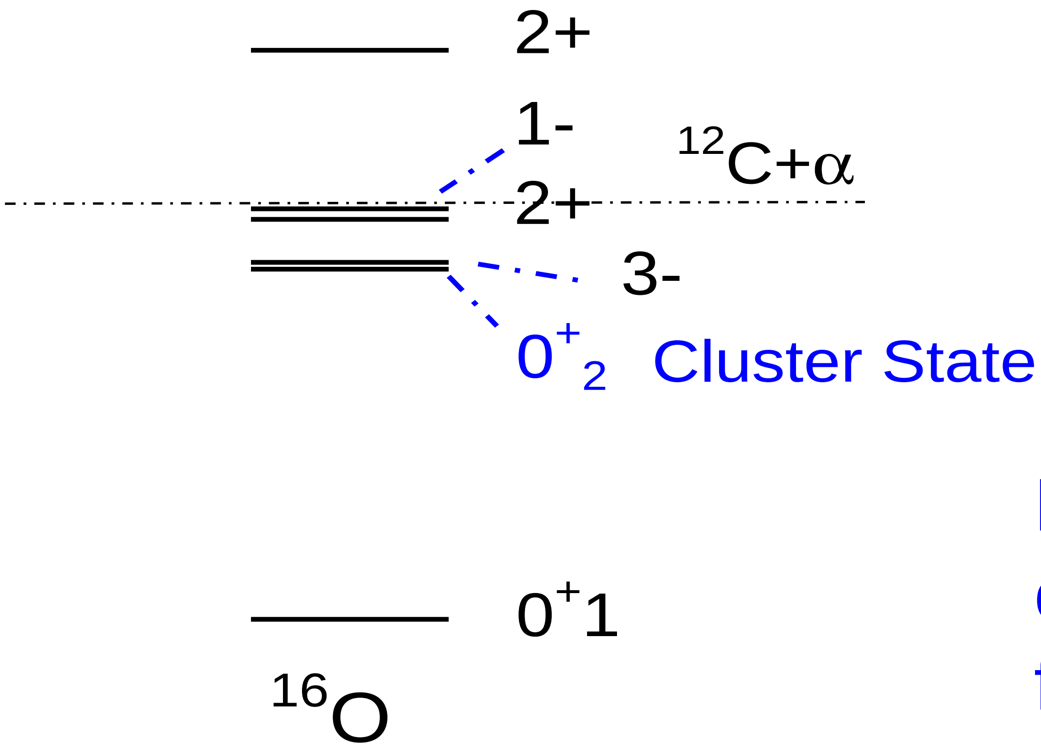
- GCM basis functions have a Gaussian asymptotic behaviour and cannot directly describe scattering states.
- The wrong Gaussian behaviour of the GCM functions is corrected by using the MRM.
- Partial reduced widths and phase shifts in each channel of reaction can be calculated.

$\theta^2 = a^3 g(a)^2/3$ (a=channel radius)

$U=e^{2i\delta}$ (Collision matrix)

➢ Calculation of bound, resonance and scattering states.

Calculations performed with Volkov V2 and Minnesota effective interactions complemented by the Coulomb interaction rigorously treated.

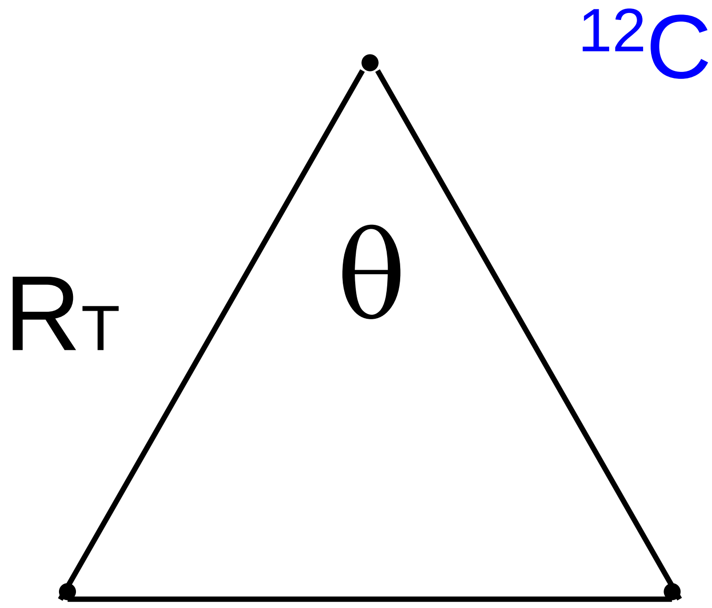


Popular forces well adapted to such variational basis.

Both forces contain only one parameter fitted on the 0^+_2.

Conditions of calculation:

- Seven ^{12}C isosceles triangles are selected.

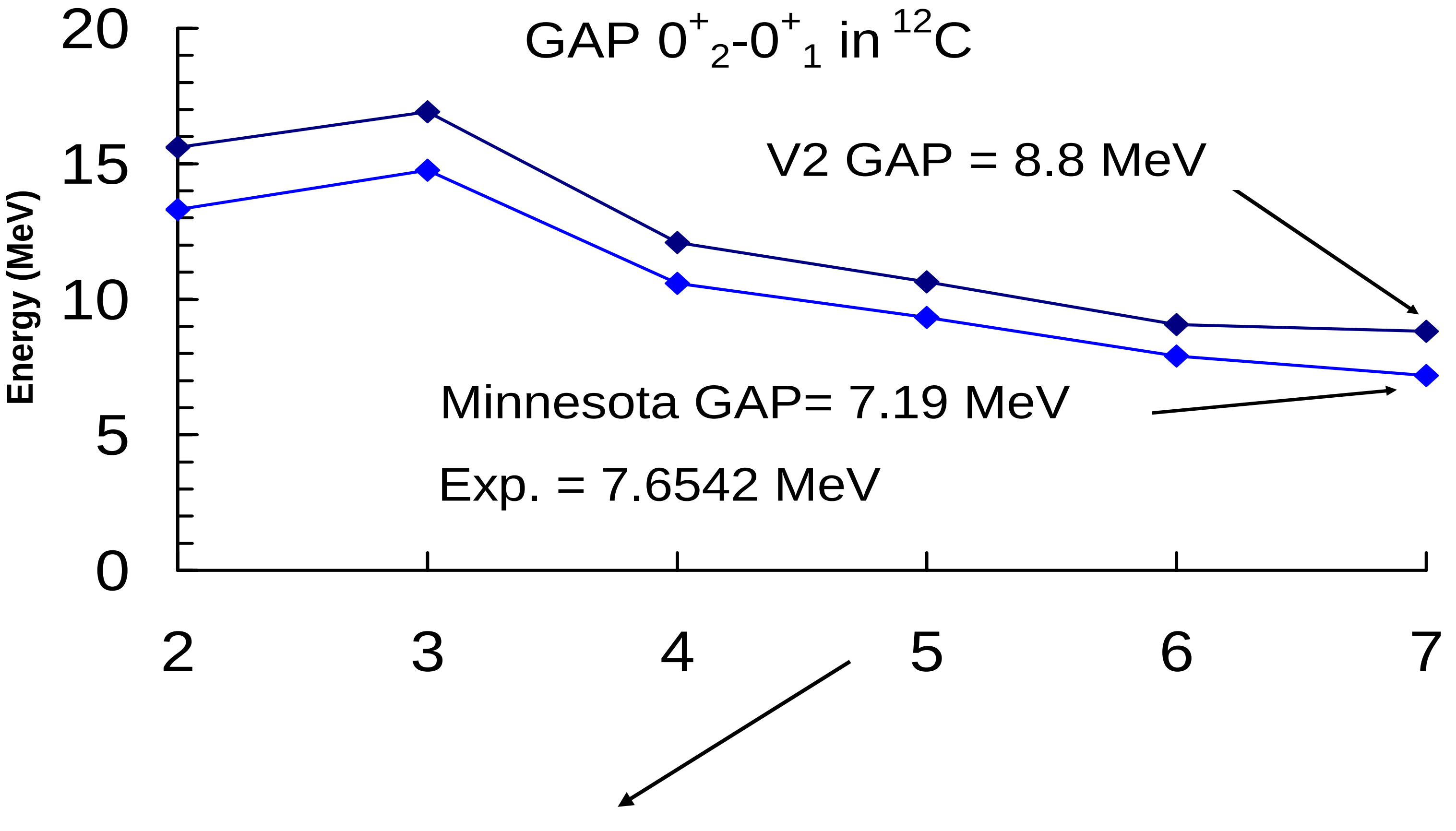


	R_T	Θ
T1	2.06	28.07
T2	3.00	100.00
T3	4.00	30.00
T4	5.00	45.00
T5	6.00	30.00
T6	4.00	110.00
T7	1.00	60.00

- Way to proceed: we search the T which minimizes the 0^+_1 GS, then we choose the second T, which after diagonalization minimizes GS and 0^+_2-0^+_1 gap, then the third T, etc.

- Only one step in the full 16 nucleon calculation.

GAP evolution as a function of the number of triangles included in the variational basis



Means with 2 T, 3 T, …, 7 T.

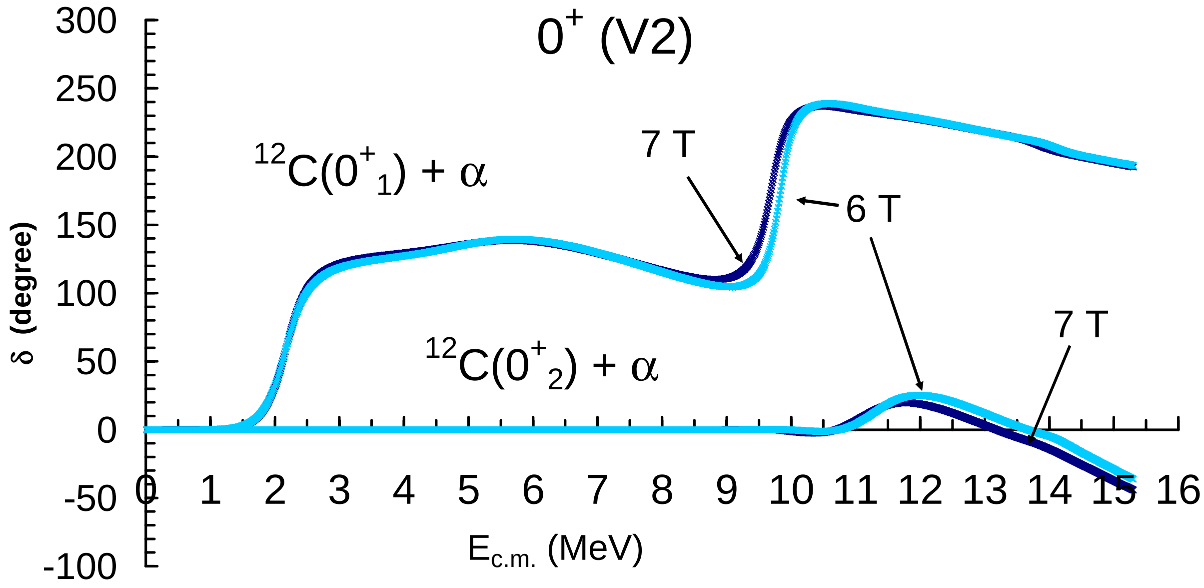
Necessity and Difficulty to treat many-body resonances

	BS approx.	MRM $E_{c.m.}$	Γ
1	-14.045	-14.045	
2	-1.112	-1.112	
3	2.257	2.200	0.46
4	3.686	3.818	1.86
5	5.973	6.369	1.96
6	6.635	6.859	3.71
7	9.060	9.724	0.37
8	9.705	10.137	4.33
9	11.083	11.498	1.45
10	11.676	13.897	6.76
11	13.810	13.997	0.92
12	14.226	15.701	4.68

- How to decipher between physical and non physical ones?

Phase shift analysis of the 0^+ resonances (V2)

Stable results

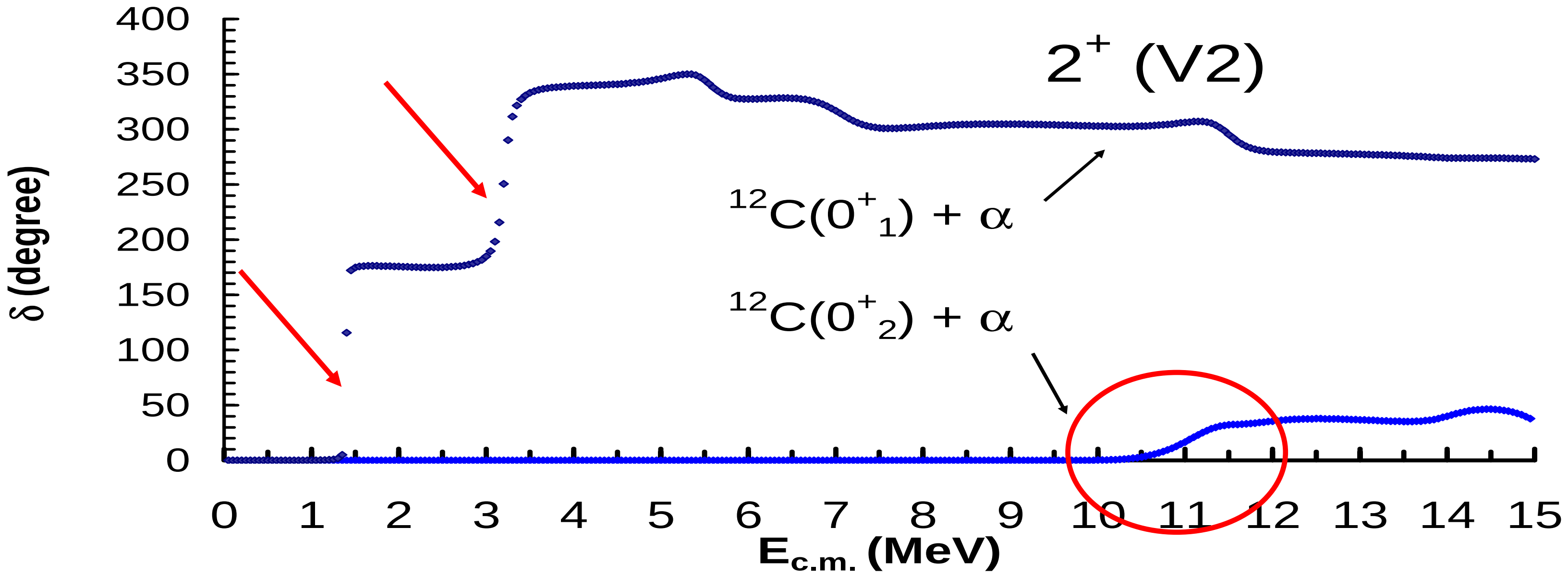


E_{cm}	Γ (MeV)
2.20	0.46
3.82	1.86
6.37	1.96
6.86	3.71
9.72	0.37
10.14	4.33
11.50	1.45
13.87	6.76
14.00	0.91
15.70	4.68

- Iterative methods based on Breit Wigner approximation give several broad resonances.

- 3 physical resonances.
Among them, two are dominant in the $^{12}C(0^+_2)+\alpha$ channel.

Phase shift analysis of the 2^+ resonances (V2)



- 2^+ resonances at 11.31 MeV, Γ=0.9 MeV and at 11.96 MeV, Γ= 1.8 MeV as a possible member of a rotational band based on the previous Hoyle state candidates.

- One possible 4^+ resonance at 13.74 MeV, Γ=1.5 MeV.

Reduced α widths (in %) for 0^+ candidate (V2)

$0^+ E_{c.m.}$	I, ℓ Channel	θ_c
9.72 MeV		
	$I = O_1^+ \ \ell = 0$ $I = O_2^+ \ \ell = 0$ $I = 2_1^+ \ \ell = 0$	6.4×10^{-2} 6.8×10^{-2} 2.4×10^{-2}
11.49 MeV		
	$I = O_1^+ \ \ell = 0$ $I = O_2^+ \ \ell = 0$ $I = 2_1^+ \ \ell = 2$	1.5×10^{-2} 7.7×10^{-1} 4.1×10^{-2}

- The 2^+_1-0^+_1 gap in ^{12}C is overestimated of ~ 1.5 MeV.
- The $I=2^+$ channel cannot be neglected in our results.
- Analysis in progress.

- Band of Chevallier *et. al.* PR, 1967 (extract).
- *Remeasurement: Notre-Dame (M. Freer-Conf. Debreccen 2012.)*

	$E^{Exp.}$	Γ	E^{GCM}	Γ^{GCM}	Ref[a]
0+	9.59		9.72	0.37	9.59
0+			11.50	1.45	
2+	9.79	0.370	11.31	0.9	10.00
2+	10.01	0.260	11.96	1.8	
4+	10.89	0.02	12.65	0.9	11.0
4+			13.74	1.48	
4+			14.68	1.11	

In MeV

$^{12}C+\alpha = {}^{8}Be+{}^{8}Be$

Compatible with a 4 α state interpretation

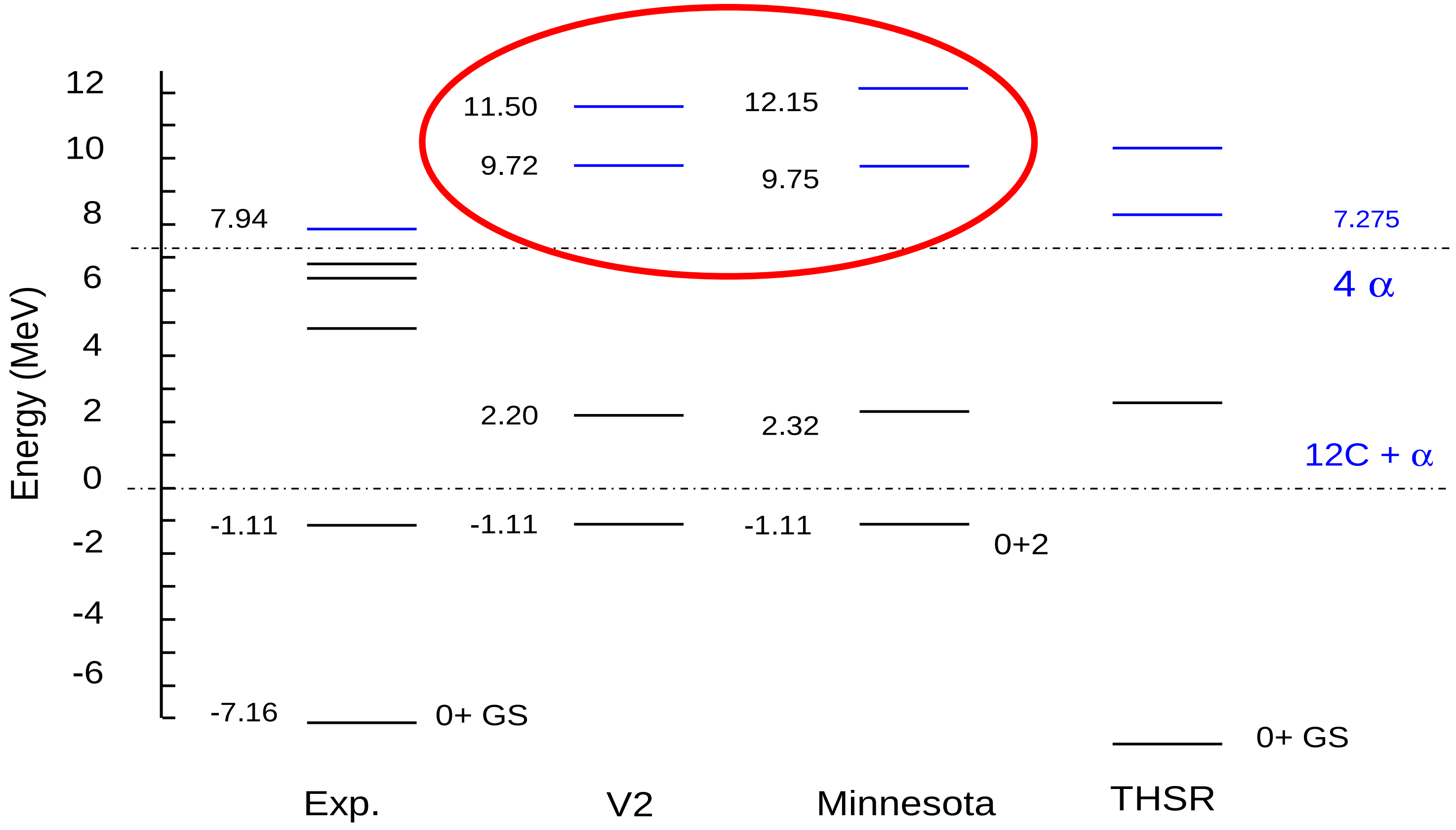
Ref[a]: Abgrall, Baron, Caurier and Monsonego,

Physics Letters 1967, 8p-8h rotational band in 16O

Intrinsic state: Axially symmetric 8p-8h deformed states

0^+ (no measured) fitted, Bound state approximation.

0^+ ^{16}O spectrum



- Two 0^+ with dominant componant in the ^{12}C(0^+_2) + α channel
- Agreement with THSR WF, Funaki *et al*., PRC82,024312 (2010)

(THSR =Tohsaki, Horiuchi, Schuck, and Ropke + CSM)

Microscopic investigation summary

Preliminary results:

- Two 0^+ candidates as possible condensate.
- At least, one 2^+ and one 4^+
- Calculation of the phase shifts are necessary to identify true physical states.

Conclusion:

- Both nonmicroscopic and microscopic approaches can be used to point out states with strong clustering such as so-called condensate states.

Jacobi coordinates

$$\overrightarrow{x_{ij}} = \overrightarrow{r_j} - \overrightarrow{r_i}$$

$$\overrightarrow{y_{ij}} = \frac{2}{\sqrt{3}} \left[\overrightarrow{r_k} - \frac{1}{2}\left(\overrightarrow{r_i} + \overrightarrow{r_j}\right)\right].$$

$$\widetilde{\psi}_{ij,k}(\overrightarrow{x_{ij}}, \overrightarrow{y_{ij}}) = \sum_{\ell_x,\ell_y} \frac{\psi^{LM}_{\ell_x \ell_y}(x_{ij}, y_{ij})}{x_{ij} y_{ij}} \left[Y_{\ell_x}(\widehat{x_{ij}}) \otimes Y_{\ell_y}(\widehat{y_{ij}})\right]_{LM},$$

ℓ_x and ℓ_y are partial angular momenta associated with Jacobi coordinate $\overrightarrow{x}$ and $\overrightarrow{y}$

$$A_{\ell_x,\ell_y} =$$

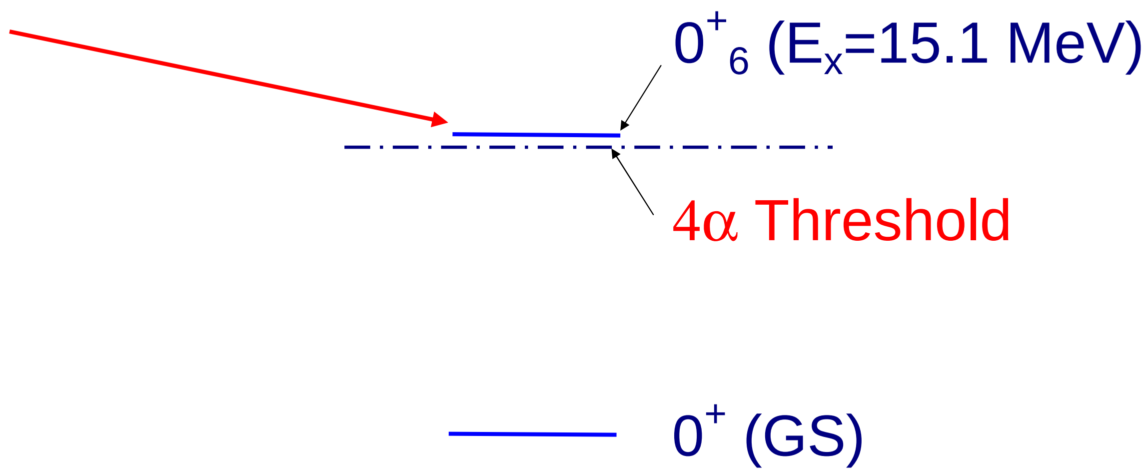
$$\frac{3}{N} \oint \oint \widetilde{\psi}_{ij,k}^{\ell_x,\ell_y}(\overrightarrow{x_{ij}}, \overrightarrow{y_{ij}}) \widetilde{\Psi}(\overrightarrow{x_{ij}}, \overrightarrow{y_{ij}}) d^3 x_{ij} d^3 y_{ij}, \qquad (1)$$

where $\widetilde{\psi}_{ij,k}^{\ell_x,\ell_y}(\overrightarrow{x_{ij}}, \overrightarrow{y_{ij}})$ is given by

$$\widetilde{\psi}_{ij,k}^{\ell_x,\ell_y}(\overrightarrow{x_{ij}}, \overrightarrow{y_{ij}}) = \frac{\psi_{\ell_x \ell_y}^{LM}(x_{ij}, y_{ij})}{x_{ij} y_{ij}} \left[Y_{\ell_x}(\widehat{x_{ij}}) \otimes Y_{\ell_y}(\widehat{y_{ij}}) \right]_{LM}. \qquad (2)$$

Next step: ^{16}O Alpha Condensates

^{16}O: 4 α states



Funaki, Yamada, Horiuchi, Ropke, Schuck and Tohsaki, Phys. Rev. Letters 101, 082502 (2008)

Microscopic Hamiltonian

$$\mathcal{H} = \sum_{i}^{A} T_i + \sum_{i<j=1}^{A} \left(V_{ij}^{NN} + V_{ij}^{SO} + V_{ij}^{Coul} \right)$$

- Central part: combination of N_g Gaussian form factors

$$V_{ij}^{NN}(r) = \sum_{k=1}^{N_g} V_{0k} \exp(-(r/a_k)^2)\,(w_k - m_k P_{ij}^{\sigma} P_{ij}^{\tau} + b_k P_{ij}^{\sigma} - h_k P_{ij}^{\tau}).$$

  - Volkov, Minnesota forces - One free parameter
  - Extended Volkov Interaction - Two free parameters[2]
- V_{ij}^{SO}, Spin-Orbit force - One free parameter
- V_{ij}^{Coul}, Coulomb force - Exactly treated

[2] M.Dufour and P. Descouvemont, Nucl. Phys. **A726**, 53 (2003).

Nuclear Alpha-Particle Condensates

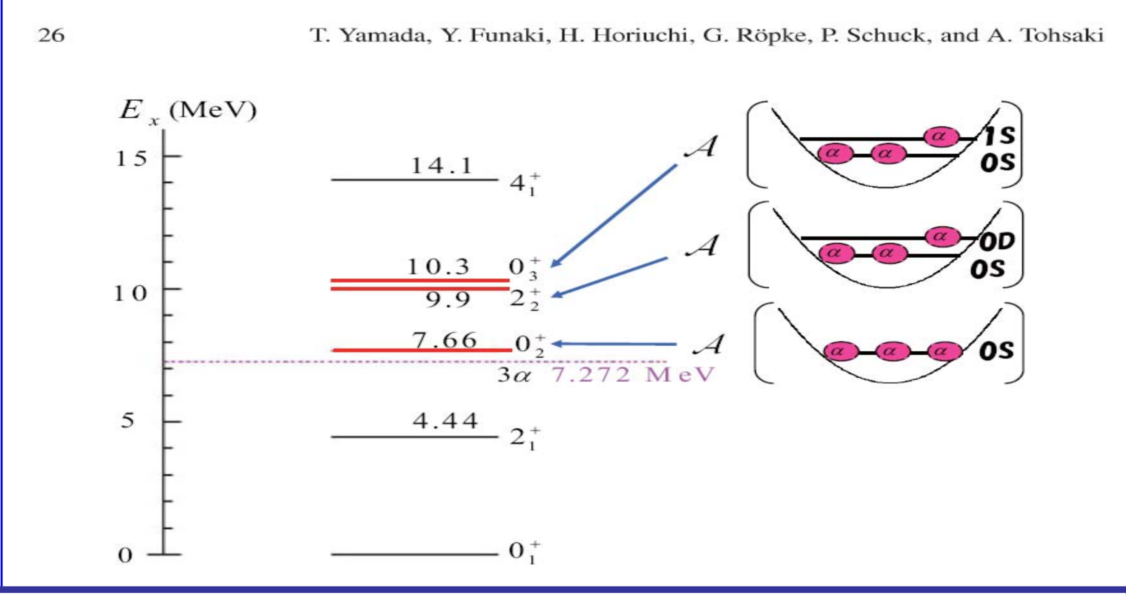
The α-particle condensate in nuclei is a novel state described by a product state of α's, all with their c.o.m. in the lowest 0S orbit.

Yamada, Funaki, Horiuchi, Ropke, Schuck and Tohsaki, Clusters in Nuclei,

Lecture Notes in Physics, Springer, Volume 2, Ed. C. Beck

Hoyle state :

- Plays a crucial role in Nuclear astrophysics (3 α process)
- 3-α condensate



Many works (microscopic and non microscopic):

- ^{8}Be+α states,(GCM), **Descouvemont, Baye,Phys. Rev. C36, 54 (1987)**
- **AMD, FMD, NoCore Shell Model (No), OCM (Yamada, Schuck), hyperspherical formalism, …**

Theoretical Framework Summary

- Unified description of bound, resonant and scattering states
- Exact treatment of antisymmetrization: the Pauli principle is exactly treated.
- Rigorous center of mass separation.
- The quantum numbers associated with the colliding nuclei are restored.
- Exact asymptotic behaviour of the WF’s.
- Once the interaction is fixed, the results are parameter free.
- Cluster approximation – The GCM variational basis is finite
- Effective interactions.
- No systematic, heavy framework.