# THE NO-CORE SHELL MODEL AS AN EFFECTIVE (FIELD) THEORY


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# WISDOM FROM STRASBOURG

observations. The first is that whatever the forces (hard or soft core, ancient or new) and the method of regularization (Brueckner $G$ matrix (Kahana *et al.*, 1969a; Kuo and Brown, 1966), Sussex direct extraction (Elliott *et al.*, 1968) or Jastrow correlations (Fiase *et al.*, 1988)) the effective matrix elements are *extraordinarily similar* (Pasquini and Zuker, 1978; Rutsgi *et al.*, 1971). The most recent results (Jiang *et al.*, 1989) amount to a vindication of the work of Kuo and Brown. We take this similarity to be the great strength of the realistic interactions, since it confers on them a model-independent status as direct links to the phase shifts.

Abzouzi, A., E. Caurier, and A. P. Zuker, 1991, Phys. Rev. Lett. **66**, 1134.

SM as EFT?
Here: NCSM as EFT

# Outline

- Why
- (Pionless) Effective (Field) Theory
- Life in the Harmonic Box
- Trapped Fermions
- Liberated Nucleons
- Conclusion & Outlook

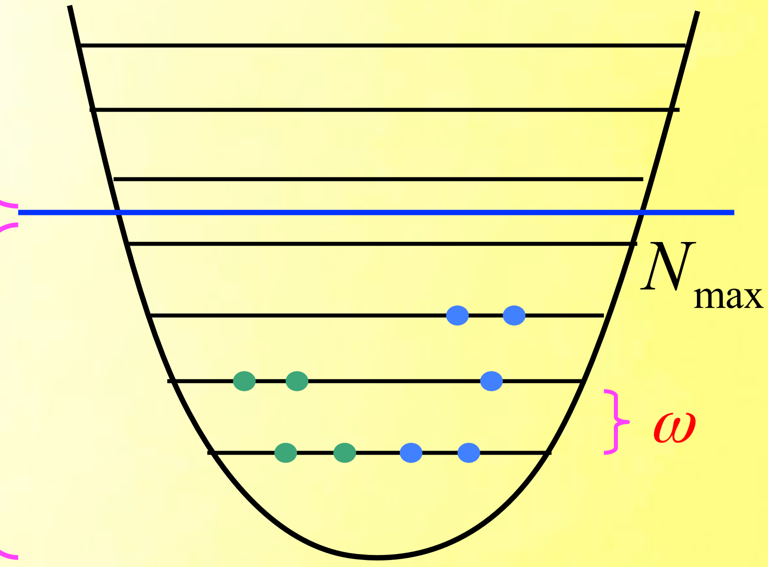
# A-body problem: Shell Model

"excluded space"

$$Q = 1 - P$$

"model space"

$$P = \sum_{n,l}^{2n+l \le N_{\max}} |nl\rangle\langle nl|$$




What are the "effective" interactions in the model space?

The "traditional" No-Core SM:

start with god-given (can be non-local!) potential, and run the RG in an HO basis

Barrett, Vary + Zhang '93

...

$$O_a \to P O_a^{\text{eff}} P = P O_a P + PHQ \frac{1}{E - QH_2Q} Q O_a P + \ldots$$

$$= O_a^{'} + O_{a+1}^{'} + \ldots + O_{A'}^{'} + \ldots + O_A^{'}$$

Feshbach projection

arbitrary truncation ("cluster approximation")

convergence:

$A' \to A$ for fixed $P$

$P \to 1$ for fixed $A'$

issues: systematic truncation error, consistent currents, *etc.*

**EFT addresses just these issues!**

# Facts of Life

- there is *always** an underlying theory

  all interactions among low-energy d.o.f.s allowed by symmetries

- there is *always** a "model space"

  renormalization-group invariance to tame arbitrary UV cutoff

**EFT©**

$Q : m = M$

$$T = T^{(\infty)}(Q) \sim N(M) \sum_{\nu=\nu_{\min}}^{\infty} \sum_{i} c_{\nu,i}(\Lambda) \left[\frac{Q}{M}\right]^{\nu} F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right)$$

$$\frac{\partial T}{\partial \Lambda} = 0$$

normalization — parameters — non-analytic, from quantum effects (loops)

"power counting"

truncate … $T = T^{(\bar{\nu})}\left\{1 + O\left(\frac{Q}{M}, \frac{Q}{\Lambda}\right)\right\}$ ➡ want… $\Lambda \gtrsim M$

there are *always** such errors

➡ Build interactions directly in NCSM model space!

* except, *maybe*, at the Planck scale

# Game Plan

$M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots$

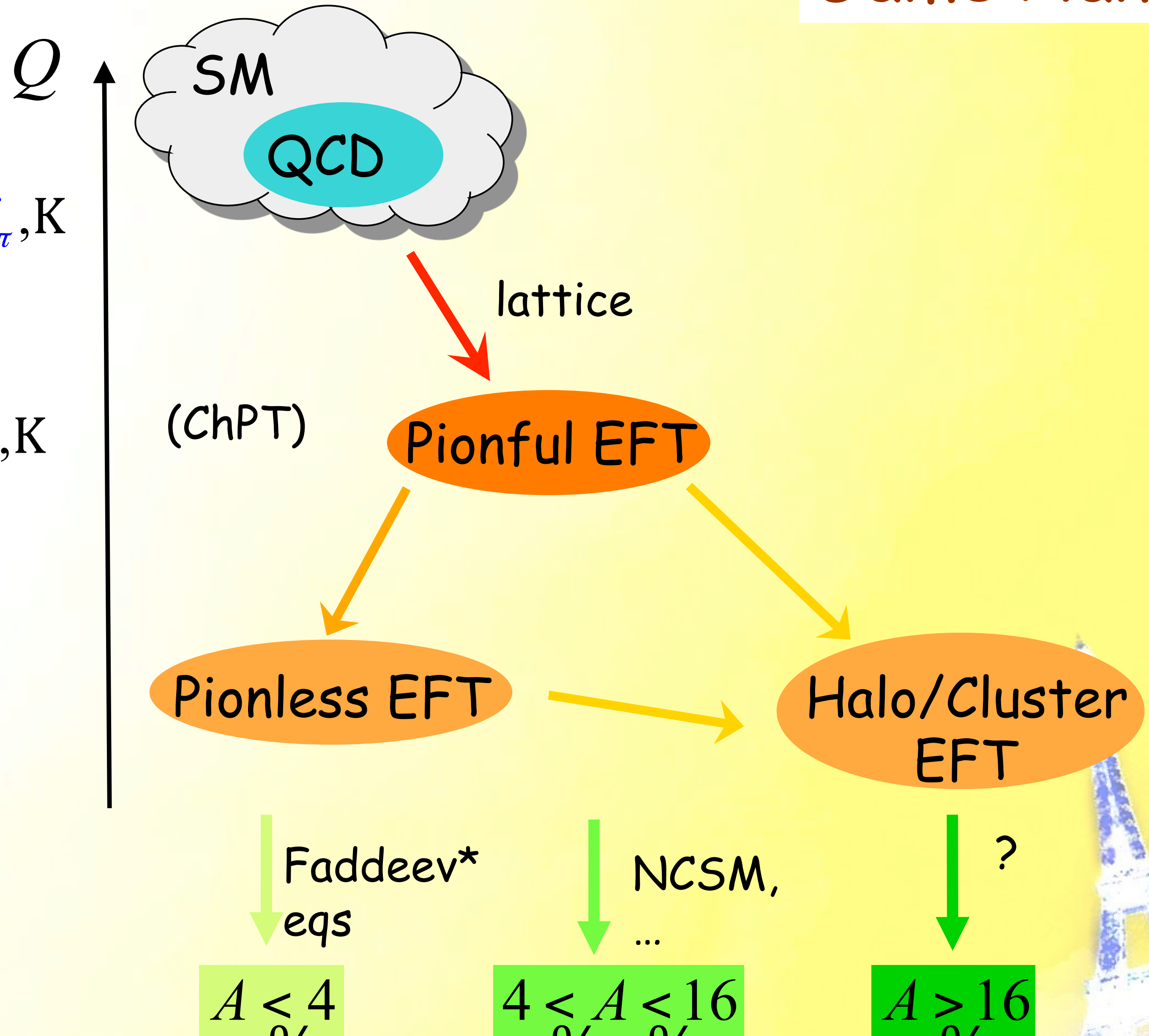
$\sim 1\,\text{GeV}$

$M_{nuc} \sim f_\pi, 1/r_{NN}, m_\pi, \ldots$

$\sim 100\,\text{MeV}$

$\aleph \sim 1/a_{NN}$

$\sim 30\,\text{MeV}$



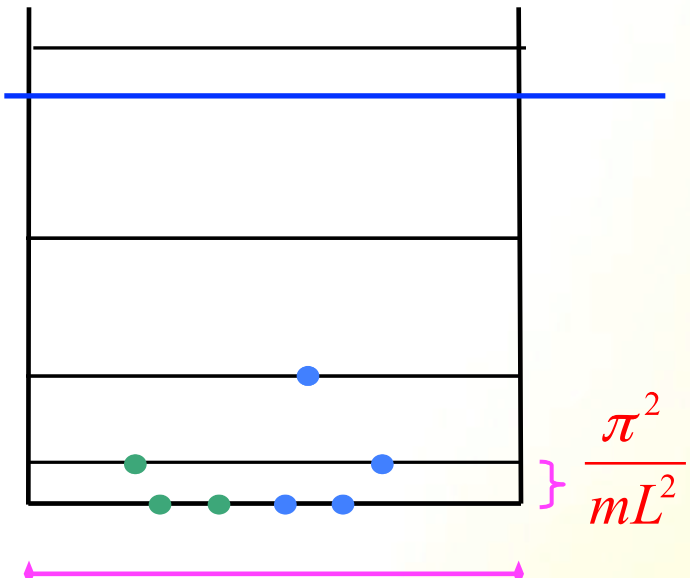
$A > 4$

# As A grows, given computational power limits number of accessible one-nucleon states

→ IR cutoff in addition to UV cutoff

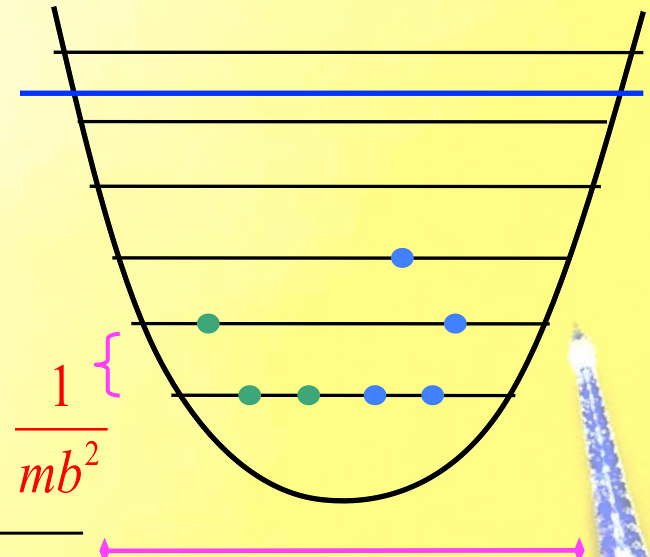
$\lambda$ momentum $\Lambda$

## Lattice Box



## Harmonic-Oscillator Box

"No-Core Shell Model"



energy

$$\frac{\pi^2}{mL^2} \qquad \frac{N^2\pi^2}{mL^2} \qquad \frac{\Lambda^2}{2m} \qquad \frac{\lambda^2}{2m} \qquad \frac{N_{\max}}{mb^2} \qquad \frac{1}{mb^2}$$

$$L = Na \qquad b = \sqrt{2/m\omega}$$

nuclear matter

few nucleons

Mueller *et al* '99

Lee *et al* '05

…

finite nuclei

few atoms

Stetcu *et al* '06

…

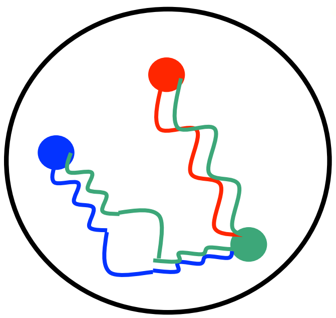
Stetcu *et al* '07

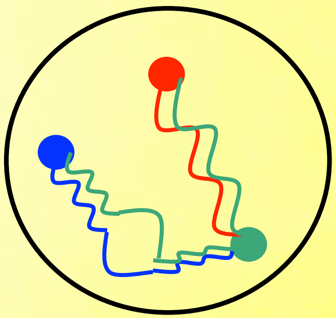
…

# Any EFT will do; for definiteness, pionless.

$r_{NN} : 1 \text{ fm}$

deuteron





$a_{NN} : 1/\aleph_{NN} \cong 4.5 \text{ fm}$

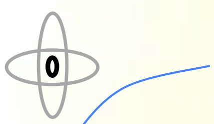
QCD: $SU(3)$ gauge theory of quarks

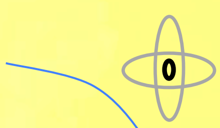
*cf.*

QED: $U(1)$ gauge theory of electrons and nuclei

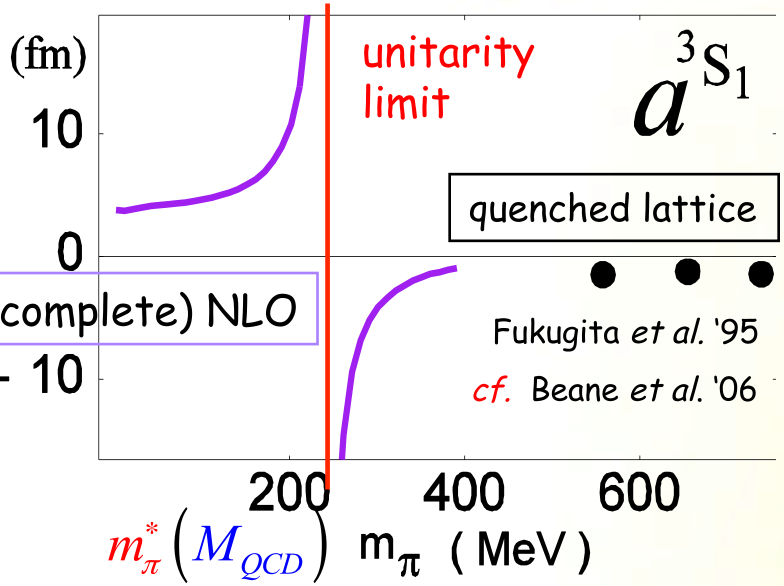
$r_{\text{He4He4}} : 5\,\overset{\circ}{\text{A}}$

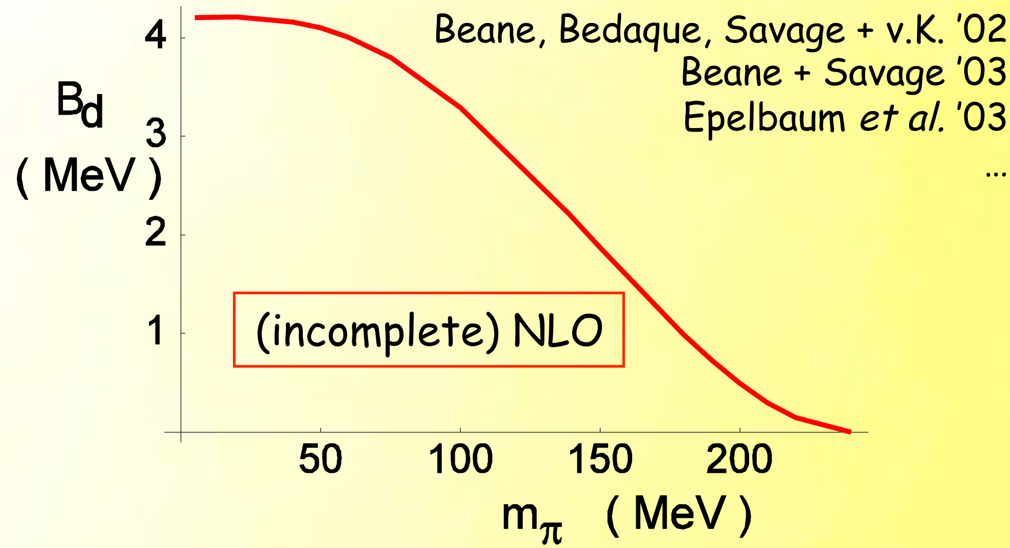
He4 dimer





$a_{\text{He4He4}} : 1/\aleph_{\text{He4He4}} \cong 125\,\overset{\circ}{\text{A}}$

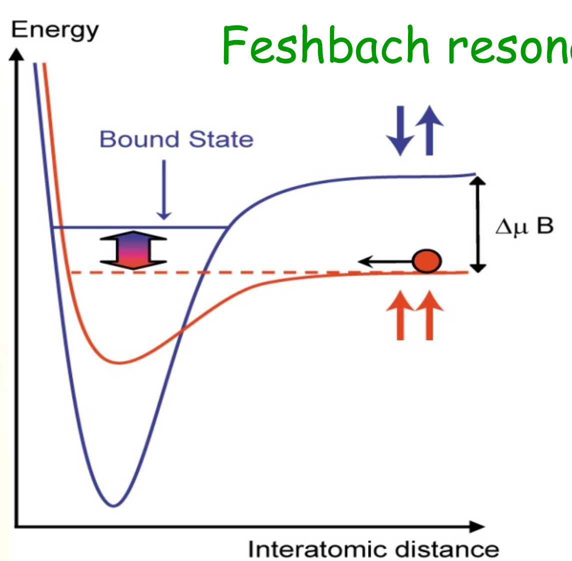


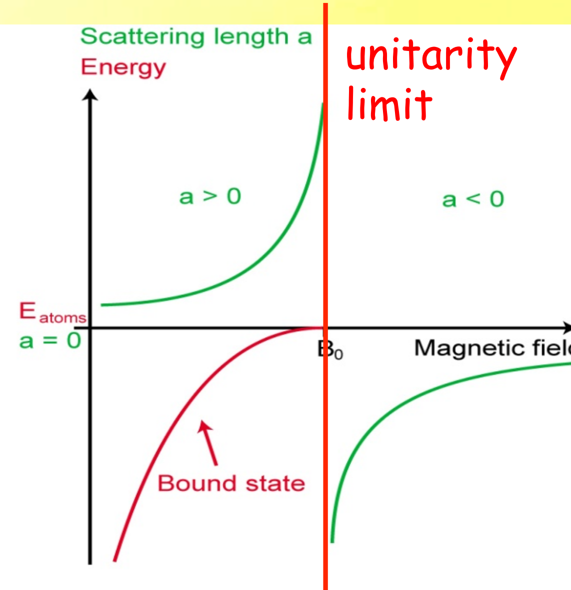




QCD near a Feshbach resonance in pion mass

Scale $\aleph$ : $\frac{m_\pi - m_\pi^*}{m_\pi^*} M_{nuc}$ emerges

*cf.* atoms as magnetic field varies




MIT webpage

$Q \sim \aleph = M_{nuc}$

**contact/pionless EFT**

- degrees of freedom: nucleons
- symmetries: Lorentz, $\not{P}$, $\not{T}$
- expansion in: $\dfrac{Q}{M_{nuc}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \ldots & \text{multipole} \end{cases}$ : $\frac{1}{3}$

Universality:
first orders apply also to atoms $M_{nuc} \to 1/l_{vdW}$ *where* $V(r) = -\dfrac{l_{vdW}^4}{2mr^6} + \ldots$

$$\mathcal{L}_{EFT} = N^+\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + C_0\, N^+N\, N^+N + D_0 N^+N\, N^+N\, N^+N$$
$$+ N^+\frac{\nabla^4}{8m_N{}^3}N + C_2\, N^+N\, N^+\nabla^2 N + \ldots$$

( omitting spin, isospin )

v.K. '97 '99
Kaplan, Savage + Wise '98
Gegelia '98
...

two-body sector ~
effective-range expansion

$$V_{ij} = \underbrace{C_0\delta^{(3)}\left(\vec r_i - \vec r_j\right)}_{\text{LO}} - \underbrace{2C_2\nabla^2\delta^{(3)}\left(\vec r_i - \vec r_j\right)}_{\text{NLO}} + \underbrace{4C_4\nabla^4\delta^{(3)}\left(\vec r_i - \vec r_j\right)}_{\text{NNLO}} + \ldots$$

LO: $a_2$ NLO: $a_2, r_2$ NNLO: $a_2, r_2$

v.K. '97
Kaplan, Savage + Wise '98
Gegelia '98

$$V_{ijk} = \underbrace{D_0\delta^{(3)}\left(\vec r_i - \vec r_j\right)\delta^{(3)}\left(\vec r_j - \vec r_k\right)}_{\text{LO}} + \underbrace{D_2\nabla^2\delta^{(3)}\left(\vec r_i - \vec r_j\right)\delta^{(3)}\left(\vec r_j - \vec r_k\right)}_{\text{NNLO}} + \ldots$$

LO: $a_3$ NNLO: $a_3, r_3$

Bedaque, Hammer + v.K. '99
Hammer + Mehen '00
...

$$V_{ijkl} = \underbrace{E_0\delta^{(3)}\left(\vec r_i - \vec r_j\right)\delta^{(3)}\left(\vec r_j - \vec r_k\right)\delta^{(3)}\left(\vec r_k - \vec r_l\right)}_{\text{not LO}} + \ldots$$

not LO

Platter, Hammer + Meissner '04

# Untrapped nucleons

LO

$$H_A^{(0)} = \frac{1}{2m_N A}\sum_{[i<j]}\left(\vec{p}_i - \vec{p}_j\right)^2 + C_{0[0]}\sum_{[i<j]_0}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)$$

$S = 0$ pairs

$$+ C_{0[1]}\sum_{[i<j]_1}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + D_0\sum_{[i<j<k]}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)$$

$S = 1$ pairs

$S = 1/2$ triplets

EFT PC effectively justifies (modified) cluster approximation

$$H_A^{(0)}\psi_A^{(0)}\left(\vec{r}\right) = E_A^{(0)}\psi_A^{(0)}\left(\vec{r}\right)$$

Stetcu, Barrett +v.K., '07

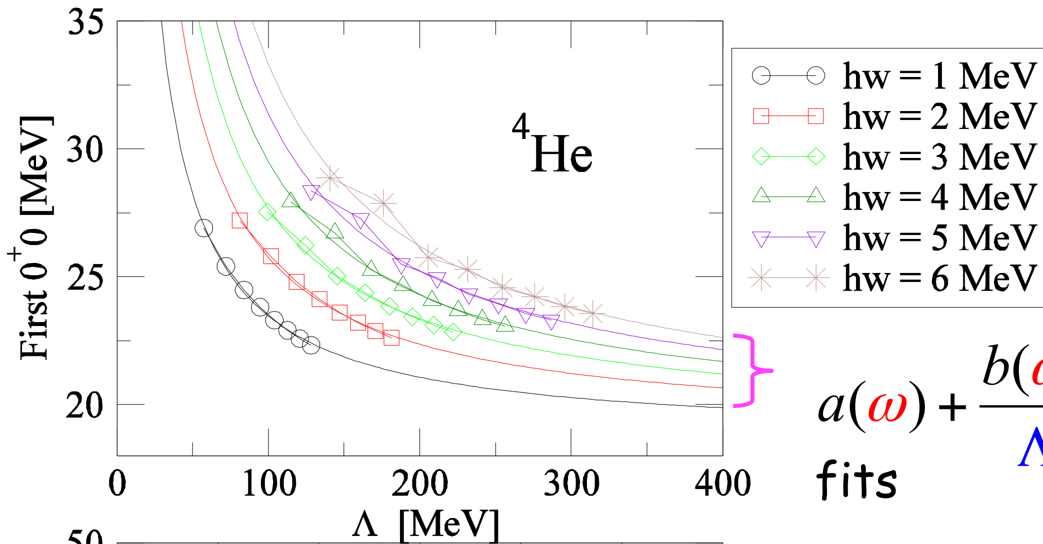
parameters fitted to d, t, a ground-state energies

predicted 4He excited, 6Li ground energies

# LO

Stetcu, Barrett +v.K., '07

$N_{max} \leq 16$

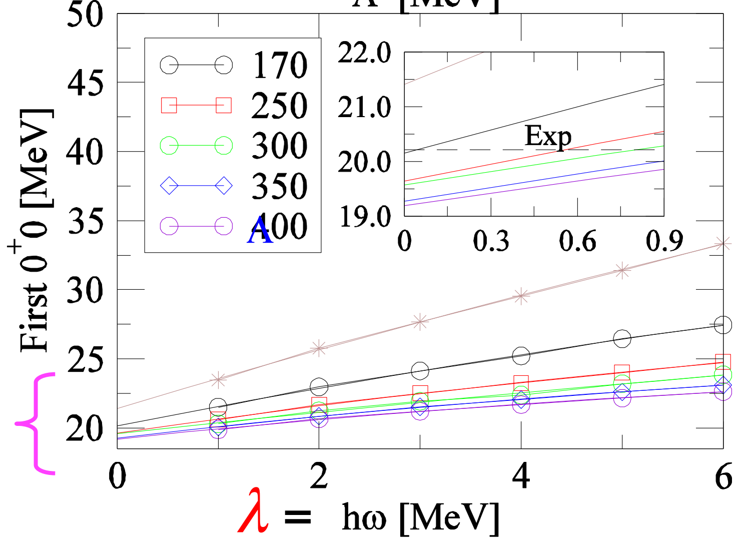



$a(\omega) + \frac{b(\omega)}{\Lambda}$

fits

$\alpha + \beta\omega + \gamma\omega^2$

fits

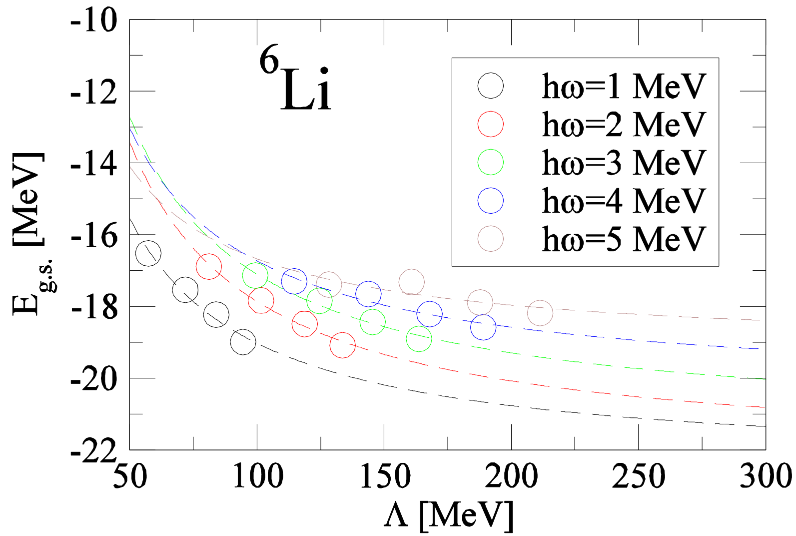



**works within ~10% !**

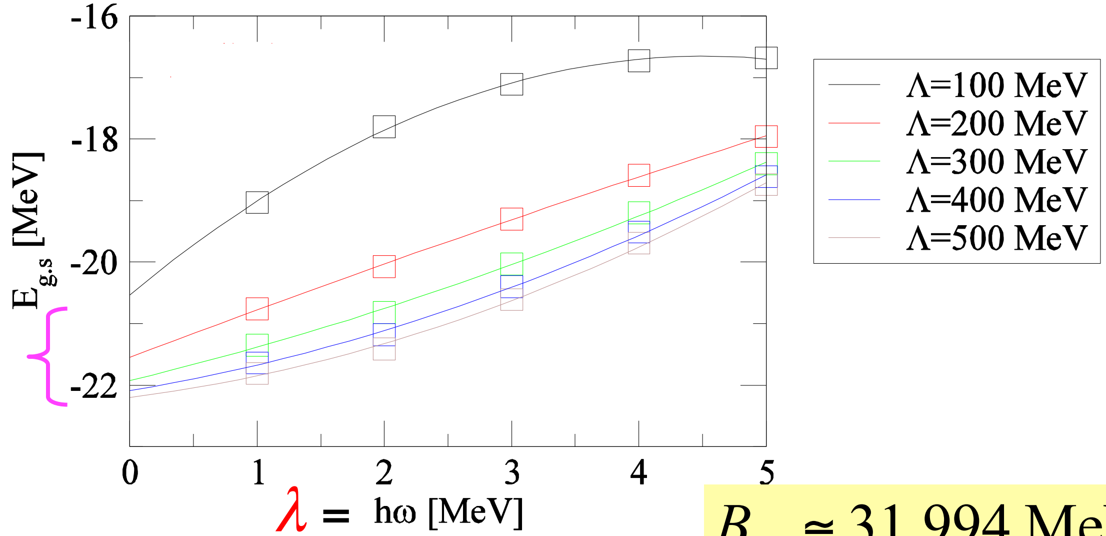
# LO

Stetcu, Barrett +v.K., '07

$N_{max} \le 8$




$a(\omega) + \dfrac{b(\omega)}{\Lambda}$ fits



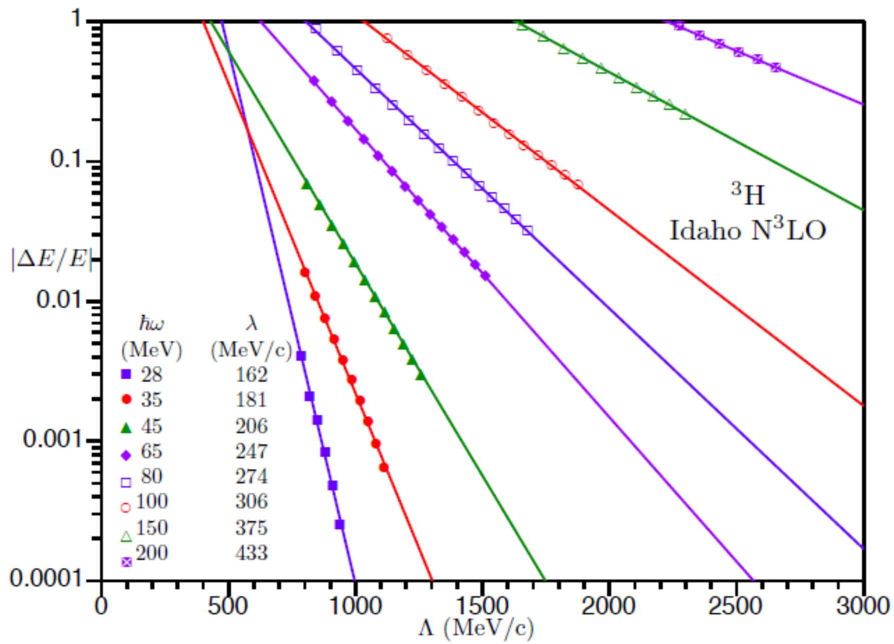

$\alpha + \beta\omega + \gamma\omega^2$ fits

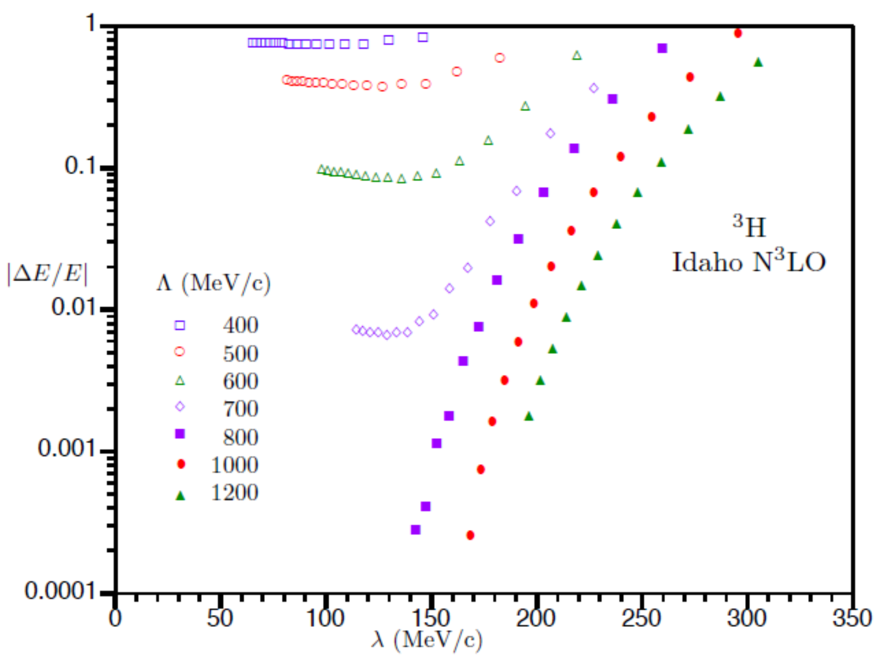
$B_{gs} \cong 31.994$ MeV (exp)

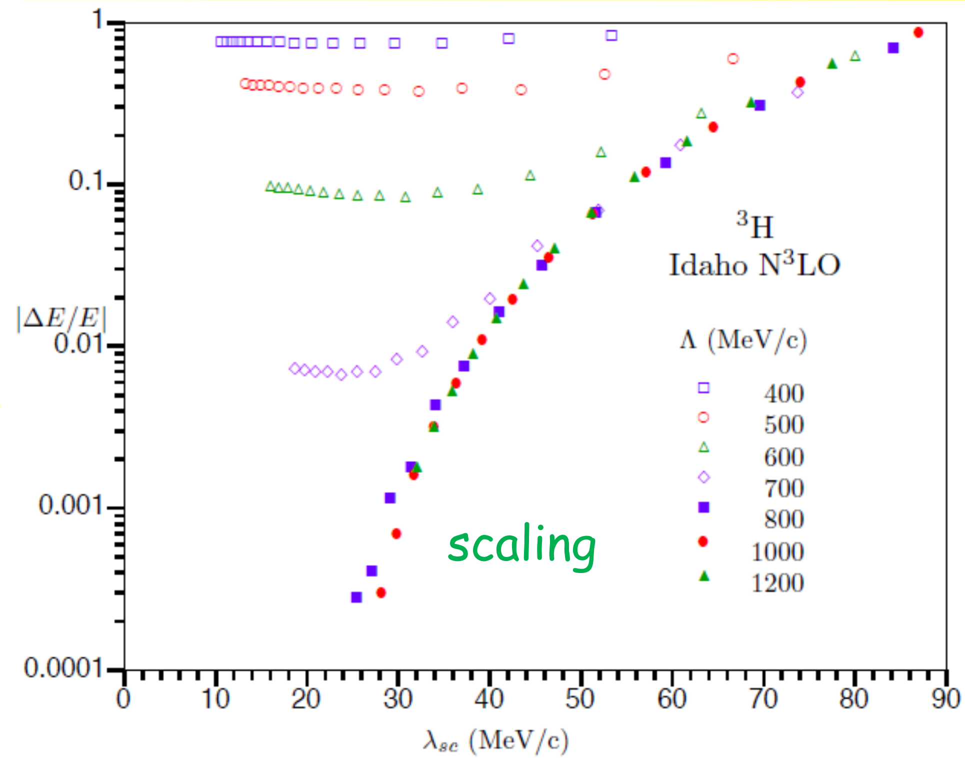
works within ~30%

Coon, Avetian, Kruse, Maris, Vary + v.K., '12

# Extrapolations in a HO basis




$$= \frac{\lambda^2}{\Lambda}$$

# Untrapped nucleons

**LO**

$$H_A^{(0)} = \frac{1}{2m_N A}\sum_{[i<j]}\left(\vec{p}_i - \vec{p}_j\right)^2 + C_{0[0]}\sum_{[i<j]_0}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)$$

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$S = 1$ pairs $\qquad$ $S = 1/2$ triplets

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Stetcu, Barrett +v.K., '07

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predicted 4He excited, 6Li ground energies

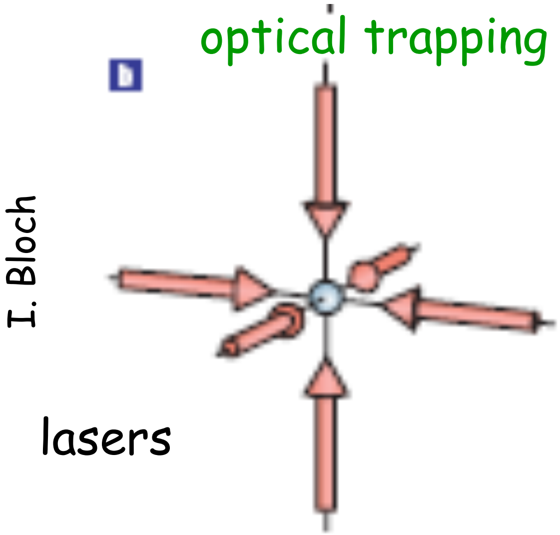
works within ~30%

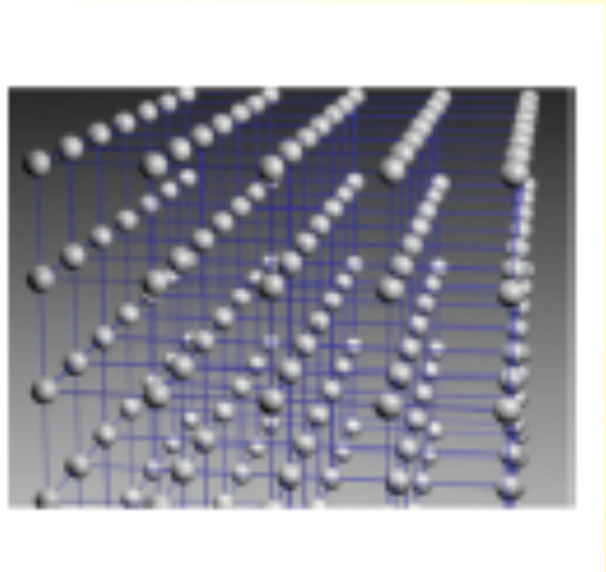
but parameters proliferate: *e.g.*, at NLO two more 2-body parameters

can we fit them to scattering data? $\quad C_{2[0]}(\Lambda),\ C_{2[1]}(\Lambda)$

Yes, trap them!

# Trapped fermions



$$V(\vec{r}) \propto \alpha(\omega_L)\left|\vec{E}(\vec{r})\right|^2$$

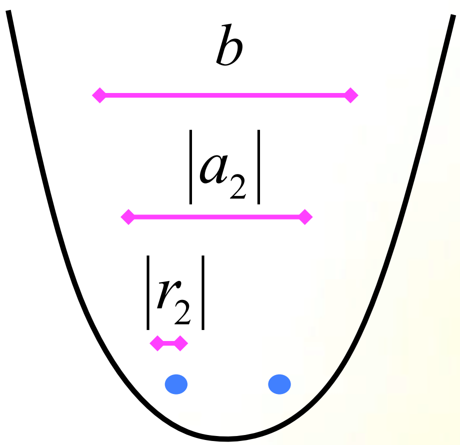
$$\propto \sum_i \sin^2(k_L r_i)$$

$$\approx k_L^2 \vec{r}^{\,2}$$

standing waves

low-tunneling regime

(band insulator)




$\frac{b}{|r_2|}$ ? 1 universal behavior

$\frac{b}{|a_2|}$ $\to \infty$ untrapped limit

$<$ 1 % significant trap effects

$\to 0$ only low-energy scale given by *b*

some semi-analytical results known

test our method

Rotureau, Stetcu, Barrett + v.K., '09

# Life in the Box

$$H_A = \frac{\omega}{2}\left\{\sum_{i=1}^{A}\left[\frac{1}{2}b^2 p_i^2 + 2\frac{r_i^2}{b^2}\right] + 2\mu_2 b^2 V\left(\left\{\vec{r}_i - \vec{r}_j\right\}\right)\right\} = H_A^{(cm)} + H_A^{(rel)}$$

two-body reduced mass $\mu_2 = m/2$

$S$ waves only in LO

**LO** $$H_A^{(0)}\left|\psi_A^{(0)}\right\rangle = E_A^{(0)}\left|\psi_A^{(0)}\right\rangle$$

**NLO** $$E_A^{(1)} = \left\langle\psi_A^{(0)}\right|V_A^{(1)}\left|\psi_A^{(0)}\right\rangle$$

**NNLO** $$E_A^{(2)} = \left\langle\psi_A^{(0)}\right|V_2^{(2)}\left|\psi_A^{(0)}\right\rangle + \frac{1}{2}\left\{\left\langle\psi_A^{(0)}\right|V_2^{(1)}\left|\psi_A^{(1)}\right\rangle + \left\langle\psi_A^{(1)}\right|V_2^{(1)}\left|\psi_A^{(0)}\right\rangle\right\}$$

etc.

## $A=2$

Stetcu, Barrett, Vary + v.K. '08
Rotureau, Stetcu, Barrett + v.K. '10

**LO** $H_2^{(0)}\left|\psi_2^{(0)}\right\rangle = E_2^{(0)}\left|\psi_2^{(0)}\right\rangle$

$$\Rightarrow \frac{2\pi b}{\mu_2 C_0^{(0)}\left(N_{2\max},\omega\right)} = -\frac{2}{\pi^{1/2}}\sum_{n=0}^{N_{2\max}/2}\frac{L_n^{(1/2)}(0)}{2n+3/2-\left(E_2^{(0)}/\omega\right)}$$

input one $\frac{E_2^{(0)}}{\omega} = \frac{E_2^{(0)}}{\omega}\left(\frac{b}{a_2}\right)$ ⇨ determine $C_0^{(0)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

*e.g.* lowest level

**NLO** $E_2^{(1)} = \left\langle\psi_2^{(0)}\right|V_2^{(1)}\left|\psi_2^{(0)}\right\rangle = \ldots$

input second level ⇨ determine $C_2^{(1)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

**NNLO** $E_2^{(2)} = \left\langle\psi_2^{(0)}\right|V_2^{(2)}\left|\psi_2^{(0)}\right\rangle + \frac{1}{2}\left\{\left\langle\psi_2^{(0)}\right|V_2^{(1)}\left|\psi_2^{(1)}\right\rangle + \left\langle\psi_2^{(1)}\right|V_2^{(1)}\left|\psi_2^{(0)}\right\rangle\right\} = \ldots$

input third level ⇨ determine $C_4^{(2)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

*etc.*

Rotureau, Stetcu, Barrett + v.K., '09

# Where do levels come from?

$$N_{2\max} \to \infty$$

$$\psi_2(0 < r = b) \propto \frac{1}{r}\left\{1 - 2\frac{\Gamma\left(\frac{3}{4} - \frac{E_2}{2\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E_2}{2\omega}\right)}\frac{r}{b} + O\left(\frac{r^2}{b^2}\right)\right\}$$

$$= \left[1 - \mu a_2 r_2 E + K\right]\frac{r}{a_2}$$

$$\Rightarrow \frac{\Gamma\left(\frac{3}{4} - \frac{E_2}{2\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E_2}{2\omega}\right)} = \frac{b}{2a_2}\left\{1 - \frac{a_2 r_2}{b^2}\frac{E_2}{\omega} + K\right\}$$

LO

NLO, NNLO

Busch *et al.* '98
Blume + Greene '02
Block + Holthaus '02
Bolda, Tiesinga + Julienne '02
...

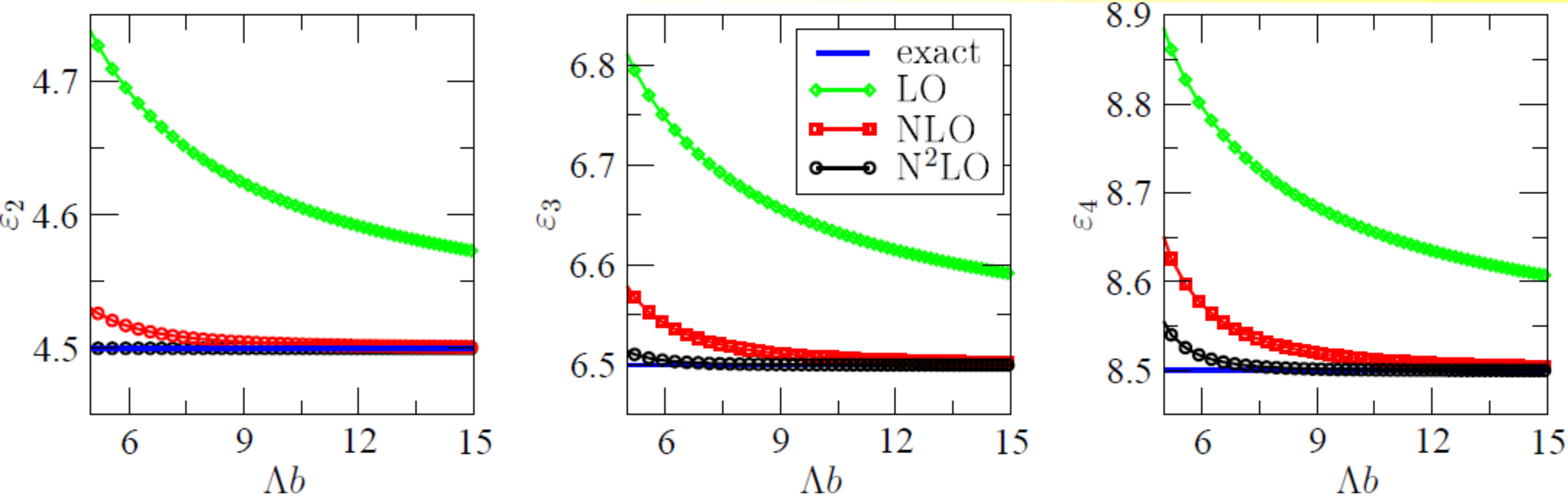
$$\frac{b}{a_2} \to \infty$$

$$\frac{E_{2,0}}{\omega} = -\frac{b^2}{a_2^2} + K$$ untrapped bound state

$$\frac{E_{2,n}}{\omega} = -\frac{1}{2} + 2n + K \quad (n = 1, 2, K)$$ scattering states

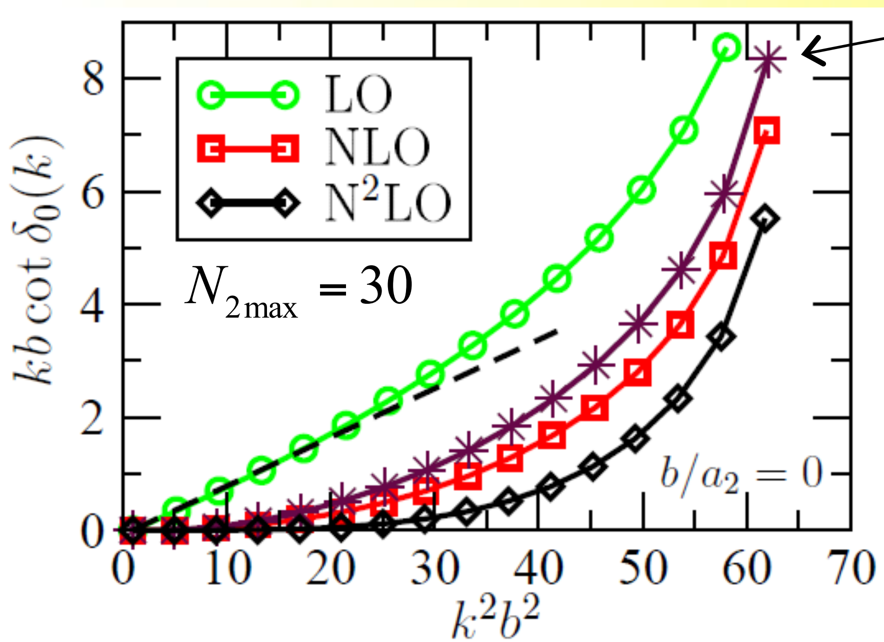
at unitarity

Stetcu, Barrett, Vary + v.K. '08
Rotureau, Stetcu, Barrett + v.K. '10




$$\varepsilon_n \equiv E_{2n}/\omega$$

*cf.* Luu
*et al* . '10




NNLO Hamiltonian
fully diagonalized:
worse than NLO!

Rotureau, Stetcu, Barrett, Birse + v.K. '10

$A \geq 3$ include few-body forces

$$N_{A\max} \geq N_{2\max} \quad \begin{cases} 1)\ N_{A\max}\ ?\ N_{2\max} \Rightarrow E_A = E_A\left(N_{2\max}, \omega\right) \\ 2)\ N_{2\max}\ ?\ 1 \end{cases}$$

$\frac{b}{a_2} \to \infty$

- lowest states: free-space bound states
  binding energy info $B_{A,0} = -E_{A,0}, K$
- other states: scattering states
  phase-shift info, for example:

$$\frac{\Gamma\left(\frac{3}{4} - \frac{E_{A,n} - E_{A-1,0}}{2\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E_{A,n} - E_{A-1,0}}{2\omega}\right)} = -\sqrt{\frac{E_{A,n} - E_{A-1,0}}{2\omega}} \cot \delta_{1,A-1} \left(\frac{2}{b}\sqrt{\frac{E_{A,n} - E_{A-1,0}}{2\omega}}\right)$$

*S*-wave phase shift for particle/lighter b.s. scattering

# Trapped two-component fermions: $S = 1/2$

$$V = \sum_{[i<j]_0} \left\{ C_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) - 2C_2 \nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + 4C_4 \nabla^4 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) \right\}$$

$S = 0$ pairs

S wave only in LOs

**up to NNLO**

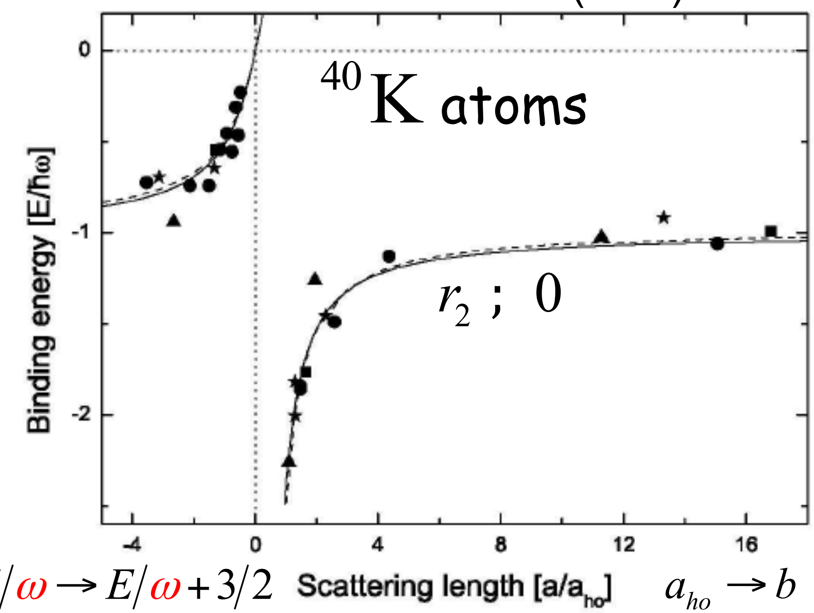
no { $S = 1$ two-body force; three-body force in LOs }

+ HO is physics

$A = 2$ fit to data *e.g.*

$A \geq 3$ no fit

Stoeferle et al (ETH) '06




$$\frac{b}{a_2} \to -\infty \quad \frac{E_A}{\omega} = \text{filling of HO shells}$$

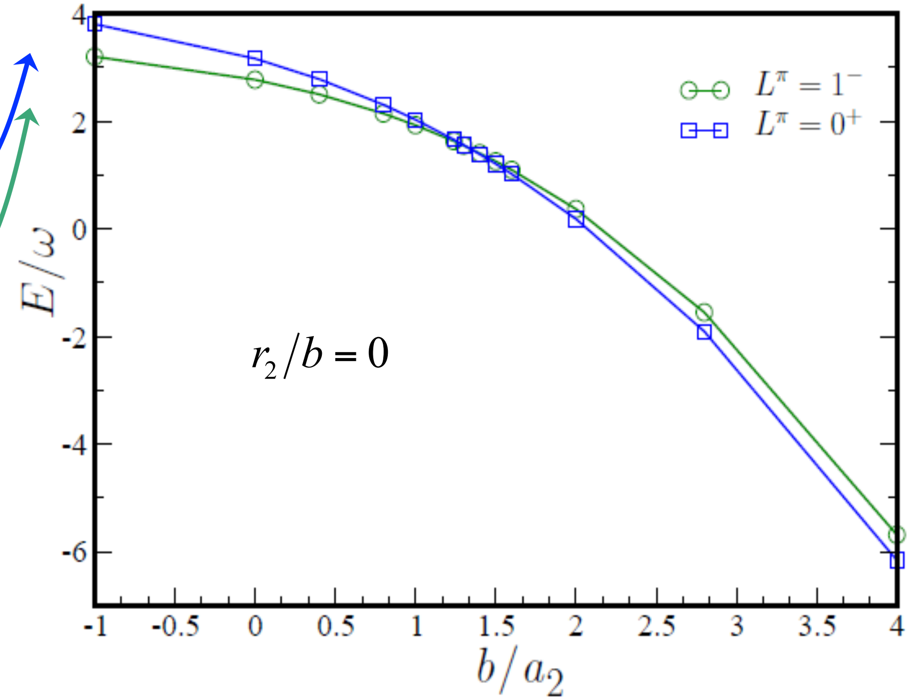
$$\frac{b}{|a_2|} \to 0 \quad \frac{E_A}{\omega} = \varepsilon_A\left(N_{2\max}\right)$$ (independent of ω since *b* only scale)

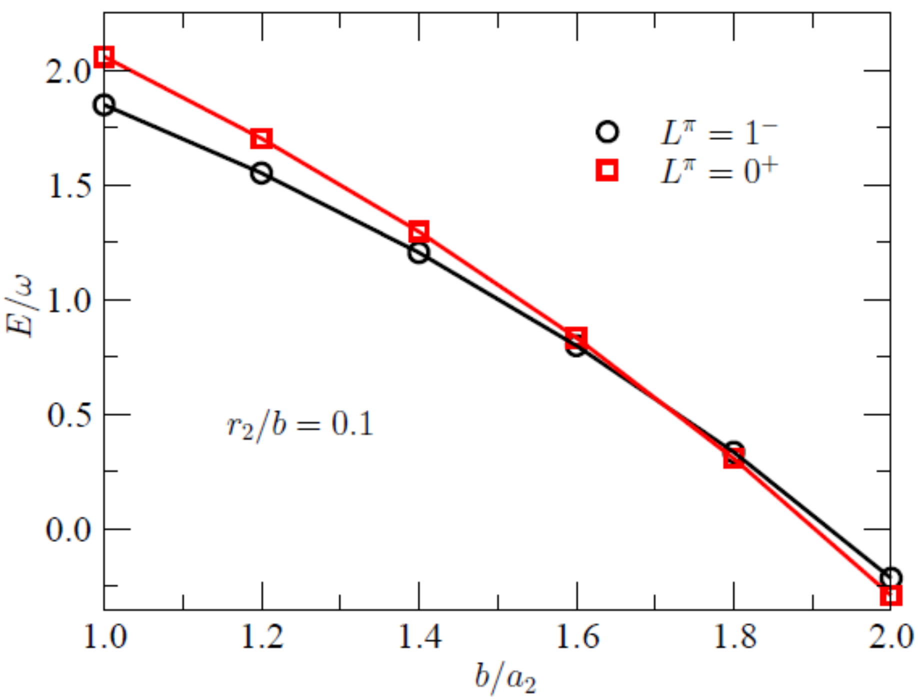
Stetcu, Barrett, Vary + v.K., '07
Kerstner + Duan '07
Rotureau, Stetcu, Barrett, Birse + v.K. '10

$$\frac{E_3}{\omega} \to \begin{cases} 5 & 1S2P \\ 4 & 2S1P \end{cases}$$

$A = 3$

inversion of g.s. parity!




$$\frac{E_3}{\omega} \approx -\frac{b^2}{a_2^2}$$

$(\text{atom+dimer})_{S\text{ wave}}$

NNLO

cliparts.com

Rotureau, Stetcu, Barrett, Birse + v.K. '10 *cf.* von Stecher, Blume + Greene '08

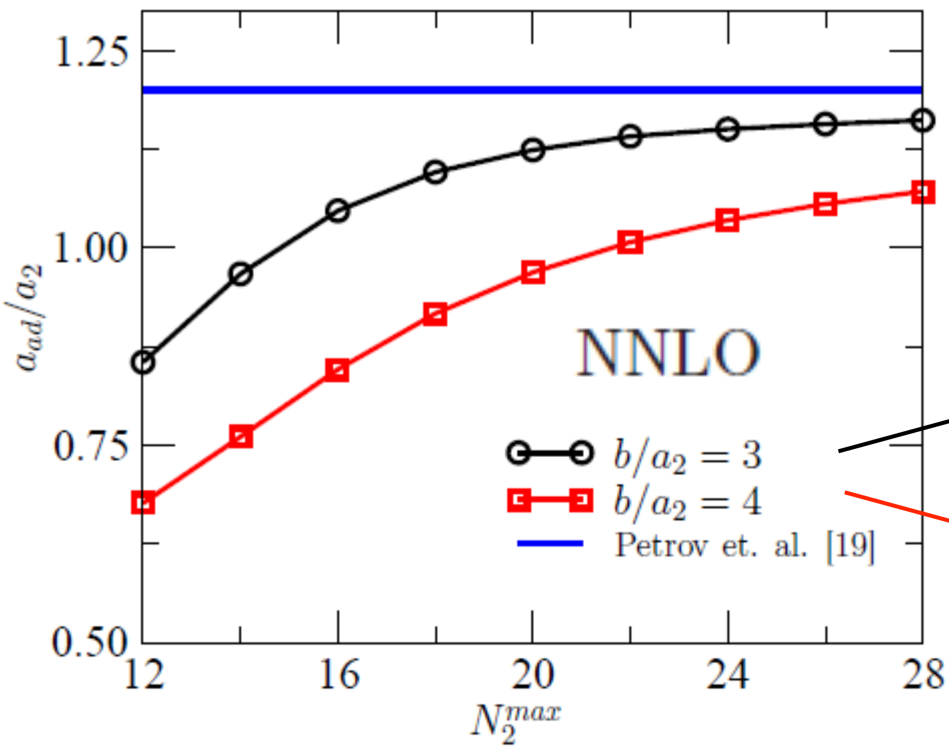
$$\frac{\Gamma(3/4-(E_{3;n}-E_{2;0})/2\omega)}{\Gamma(1/4-(E_{3;n}-E_{2;0})/2\omega)} = \frac{b'}{2a_{ad}} - \frac{r_{ad}}{2b'}\frac{E_{3;n}-E_{2;0}}{\omega} + \dots$$

3-body energy above dimer g.s.

$$b' = \frac{1}{\sqrt{\mu_{ad}\omega}}$$

$A = 3$

use two levels, eliminate $r_{ad}$:




better precision at smaller cutoffs

better dimer inside trap

# Liberated nucleons

add $\begin{cases} S=1 \text{ two-body force} \\ \text{three-body force in LOs} \end{cases}$

+ HO is <u>not</u> physics

$$V=\sum_{S=0,1}\sum_{[i<j]_S}\left\{C_{0[S]}\,\delta^{(3)}\left(\vec{r}_i-\vec{r}_j\right)-2C_{2[S]}\nabla^2\delta^{(3)}\left(\vec{r}_i-\vec{r}_j\right)\right\}$$

$$+D_0\sum_{[i<j<k]}\delta^{(3)}\left(\vec{r}_i-\vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j-\vec{r}_k\right)$$

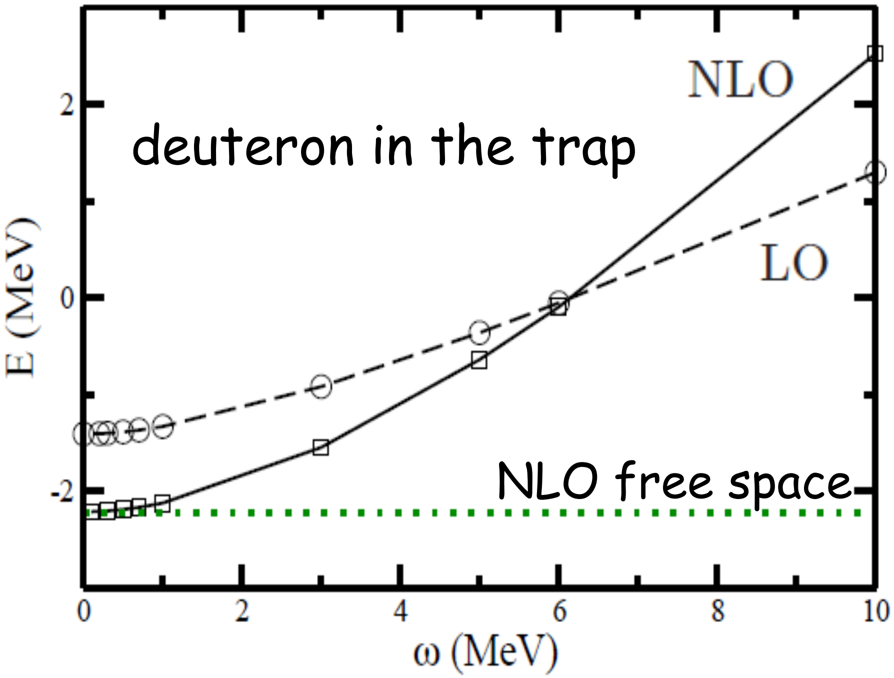
single parameter

$S=1/2$ triplets

*S* wave only

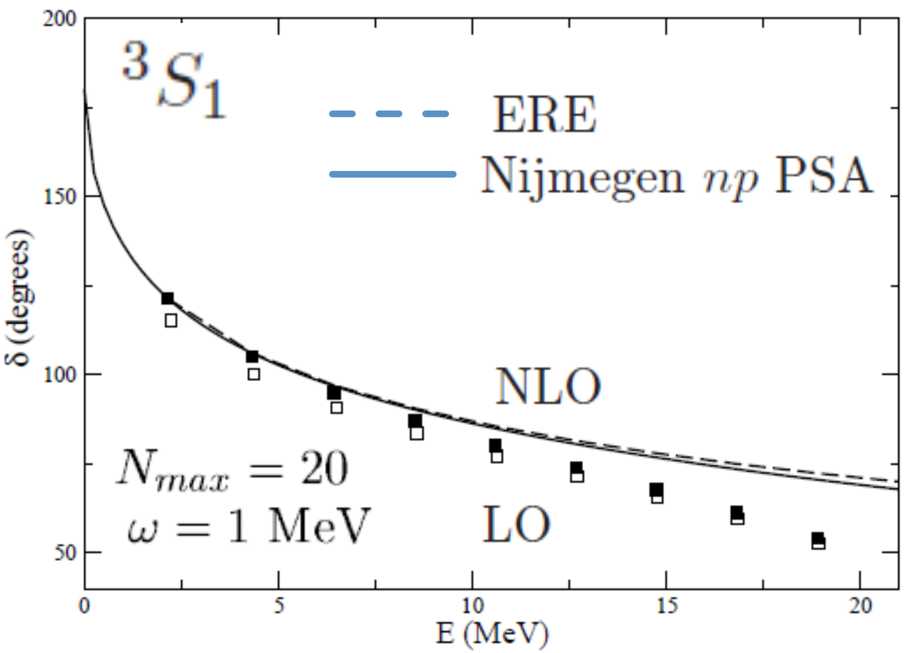
up to NLO

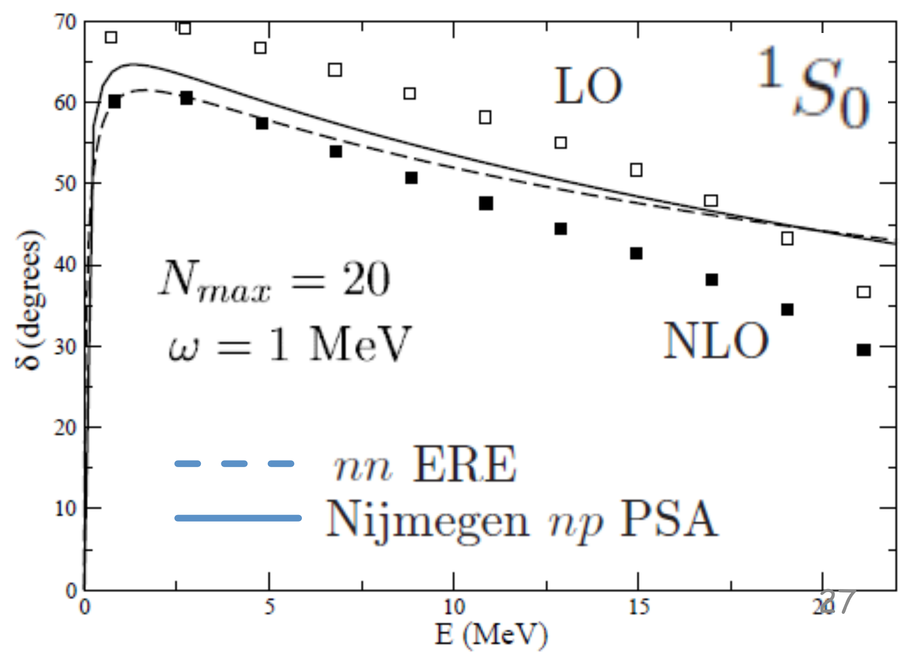
Rotureau, Stetcu, Barrett + v.K. '11



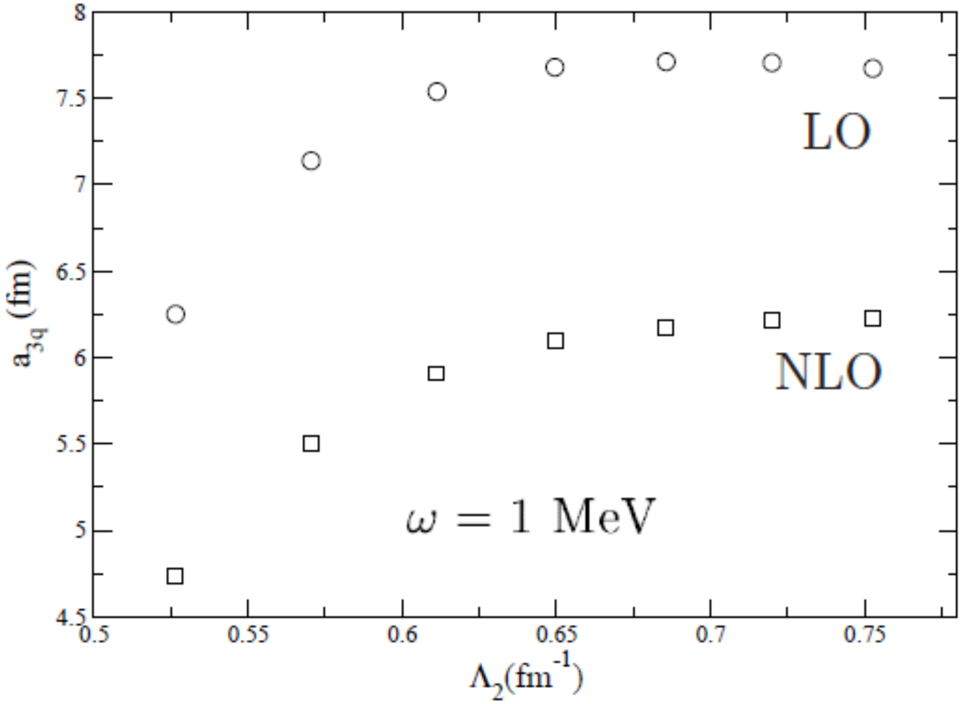

$A = 2$

NLO

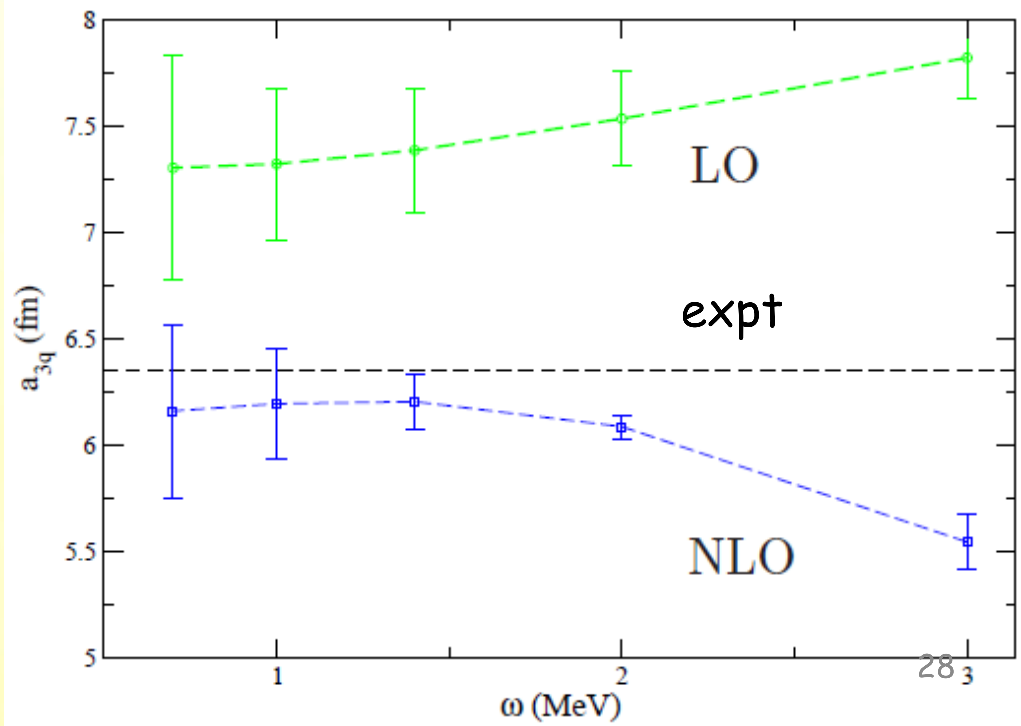


Rotureau, Stetcu, Barrett + v.K. '11




$$\frac{1}{a_{3q}} = \frac{1}{a_{3q}(\infty)} + \frac{\alpha_1}{\Lambda_2^{p_1}} + \frac{\alpha_2}{\Lambda_2^{p_2}}$$




$^4a_3 = 6.35 \pm 0.02$ fm

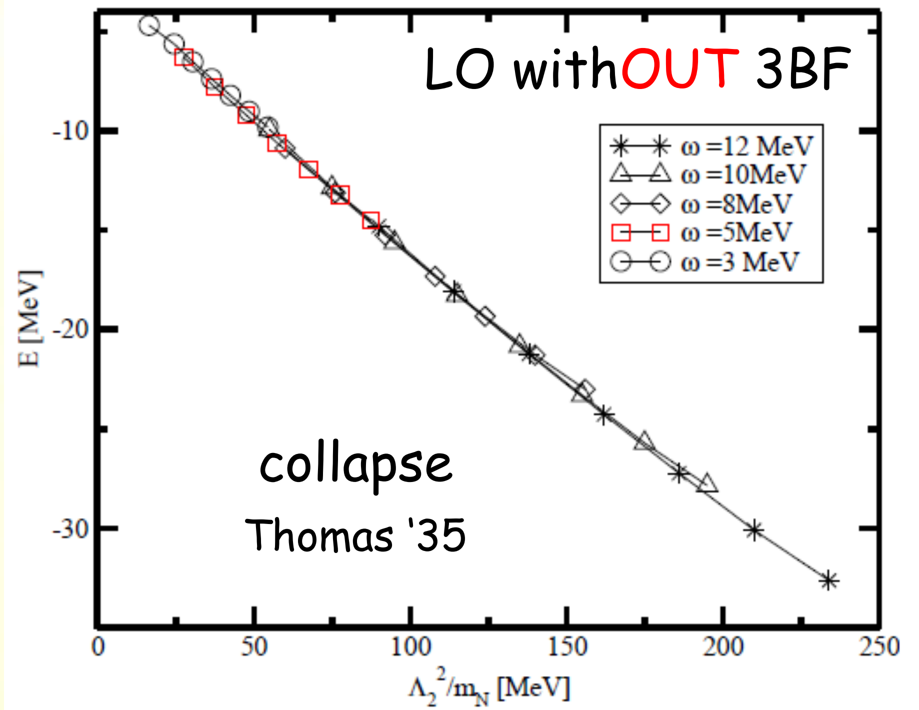
Dilg *et al.* '71

cf. NNLO $^4a_3 = 6.33 \pm 0.10$ fm

Bedaque, Hammer + v.K. '98

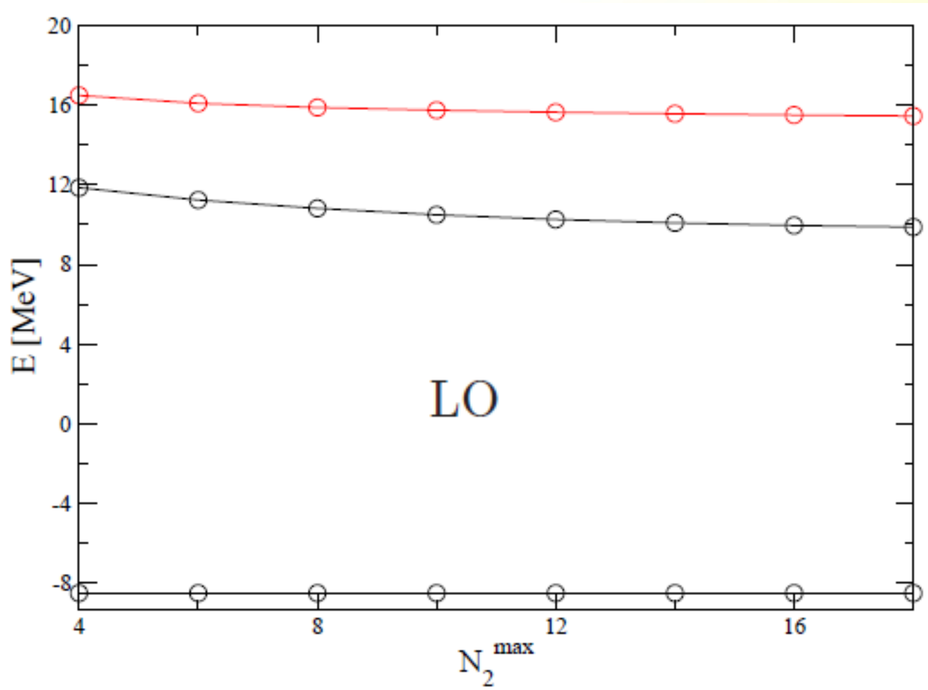
Rotureau, Stetcu, Barrett + v.K. '11

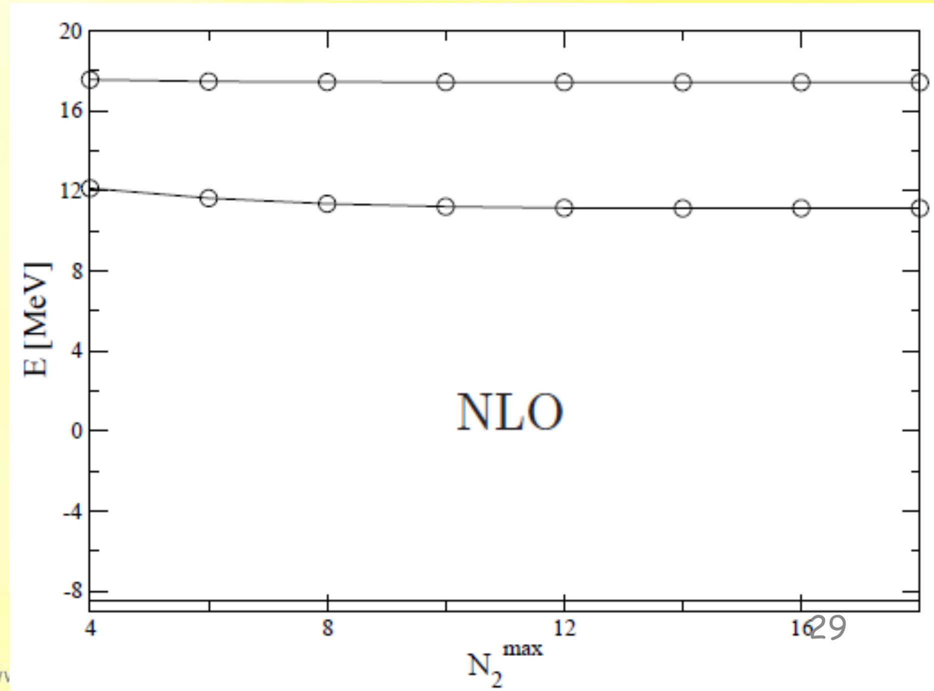
$I = 1/2,\ J^{\pi} = 1/2^{+}$



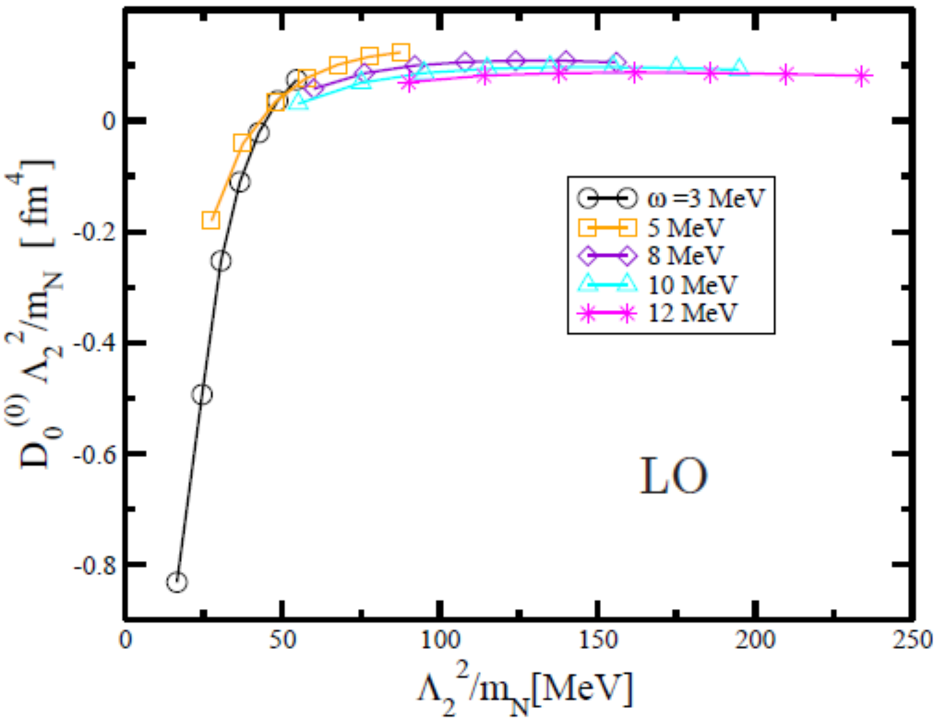

similar for bosons
Tölle, Hammer + Metsch '10

fit 3BF to triton BE



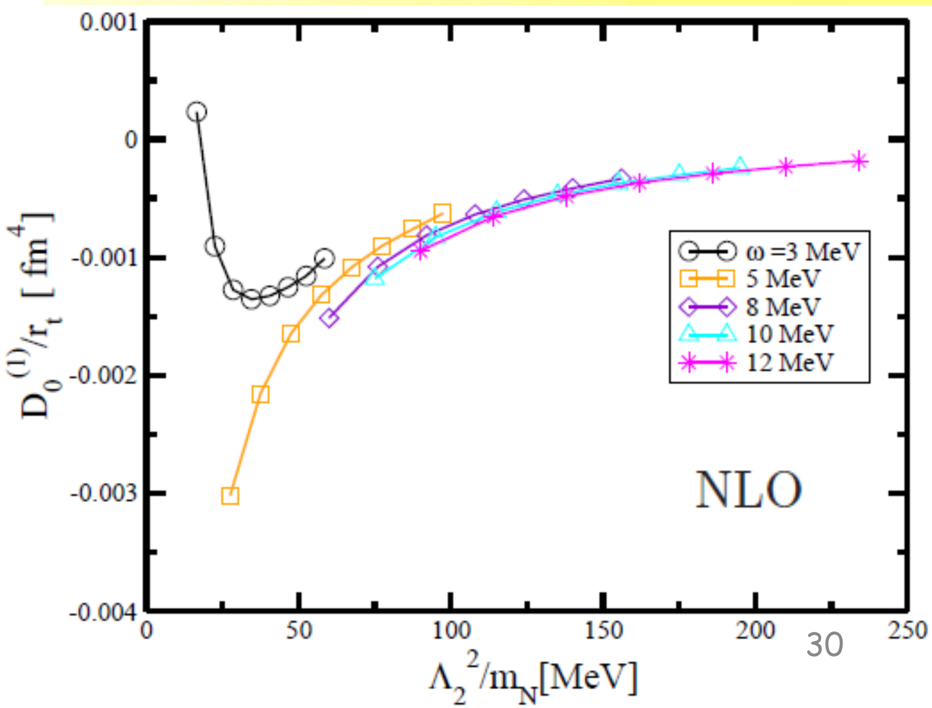
Rotureau, Stetcu, Barrett + v.K. '11




fit 3BF to triton BE

cf. limit cycle

Bedaque, Hammer + v.K. '99



# Conclusion & Outlook

- ✓ EFT can be solved in HO basis with scattering input
- ✓ Nucleons with pionless EFT in HO similar to trapped atoms near a Feshbach resonance
- ✓ Convergence improves with increasing order
- ✓ Few-body binding energies and scattering parameters can be calculated
- ✓ More extensive calculations with more nucleons and in pionful EFT are needed

ALFREDO, ANDRÉS AND ETIENNE,
THANKS FOR THE INSPIRATION!

I HOPE TO REPAY ONE DAY
WITH A FORMULATION OF
THE SM AS AN EFT.