Recent advances in neutrinoless double beta decay with energy density functional methods

Tomás R. Rodríguez

TECHNISCHE UNIVERSITÄT DARMSTADT

Congratulations to Alfredo, Andrés, Etienne and all the people from Strasbourg-Madrid (SM) collaboration!!

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Outline

1. Introduction: 0νββ decay

2. Method: GCM+PNAMP

3. Results

4. Summary and Conclusions

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1. Introduction: $0\nu\beta\beta$ decay

Perrot's and Menéndez's talk!

2. Method: GCM+PNAMP

3. Results

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1. Introduction: 0νββ decay
2. Method: GCM+PNAMP
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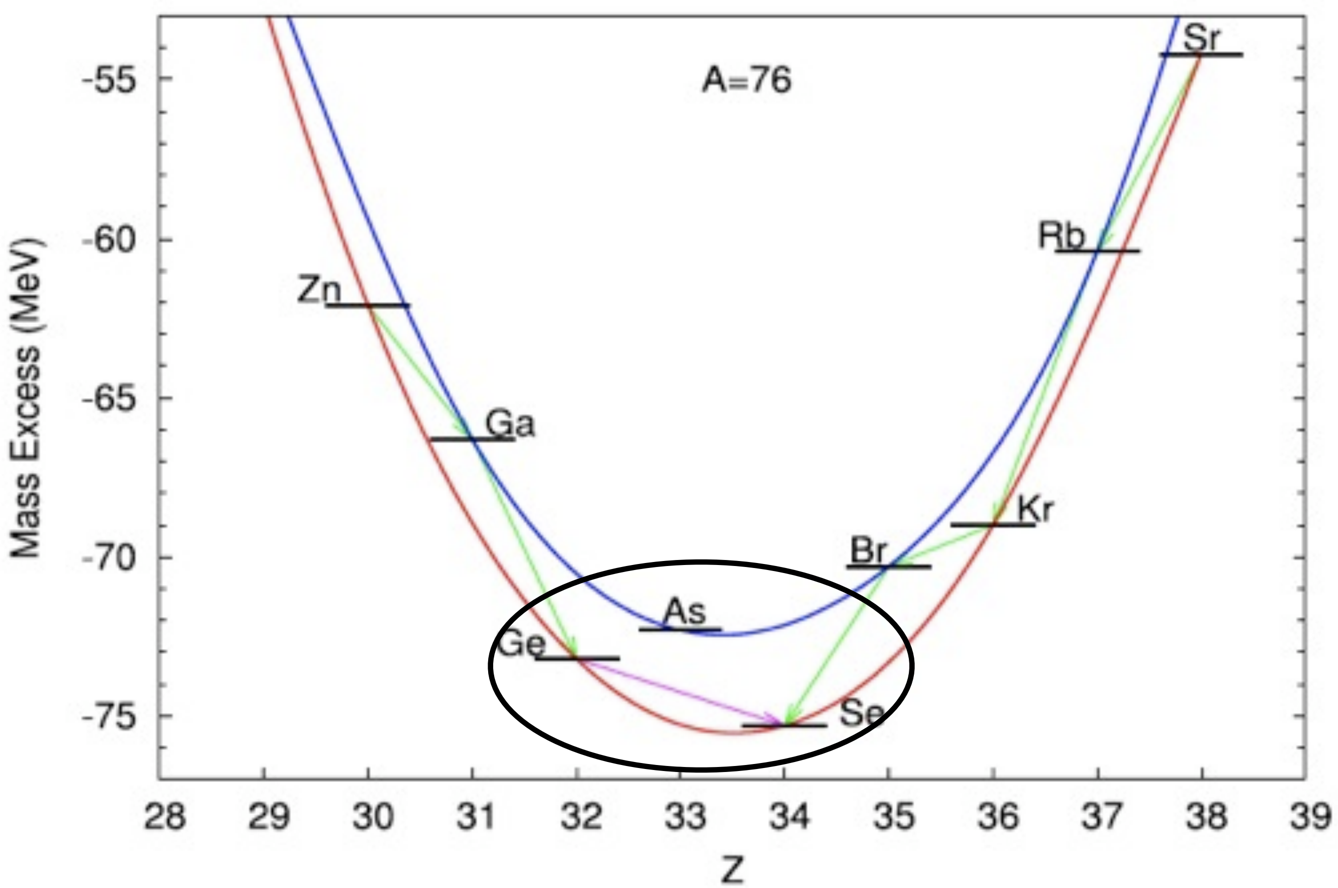
Perrot's and Menéndez's talk!

Robledo's talk!

Double beta decay

Process mediated by the weak interaction which occurs in those even-even nuclei where the single beta decay is energetically forbidden.



taken from J. Menéndez

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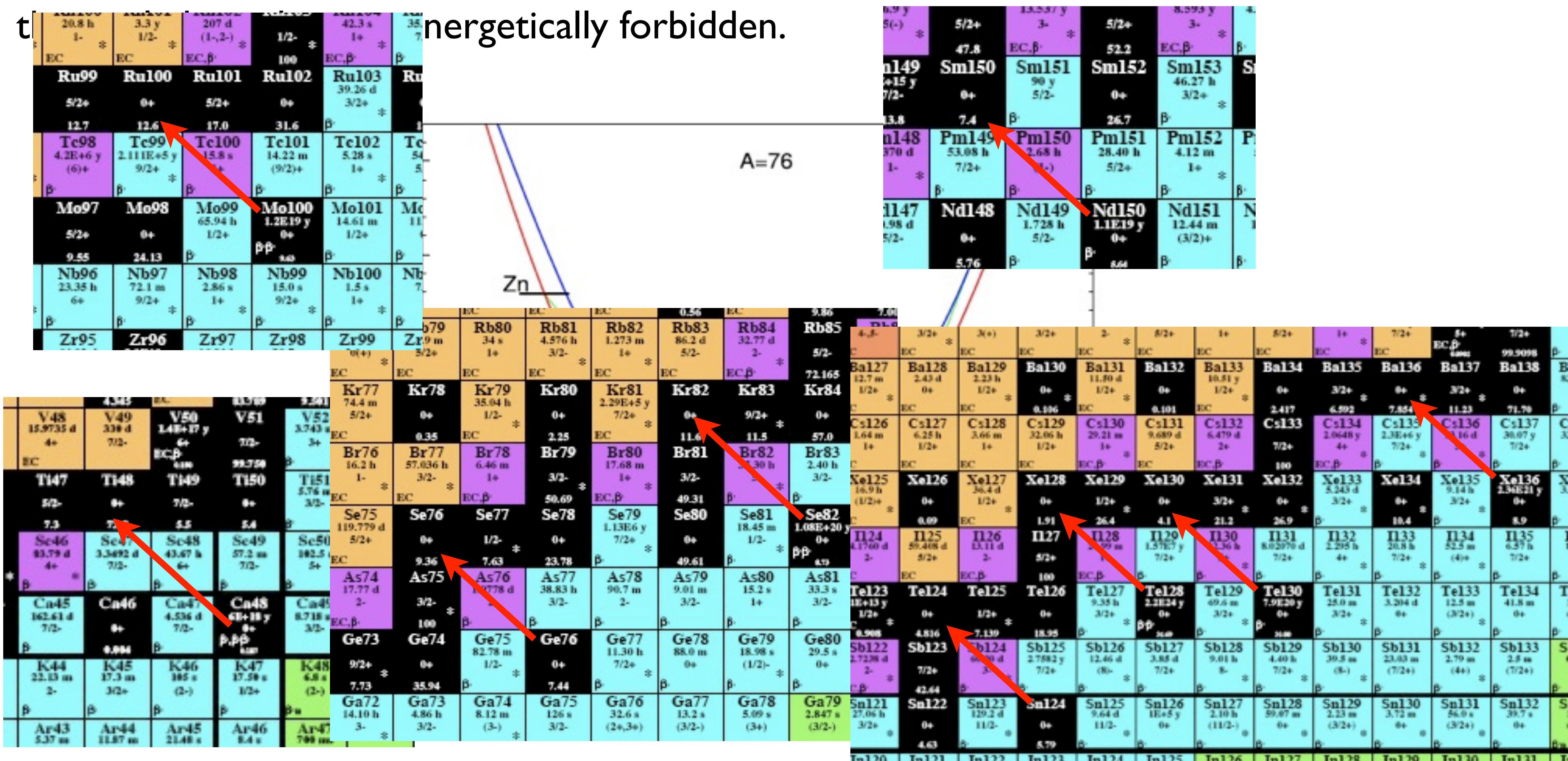

Two neutrino double beta decay $2\nu\beta\beta$

$$^{A}_{Z}X_{N} \Rightarrow ^{A}_{Z+2}Y_{N-2} + 2e^{-} + 2\bar{\nu}_{e}$$

- Conserves the leptonic number
- Compatible with massive or massless Dirac/Majorana neutrinos
- Experimentally observed ($T_{1/2}$~$10^{19\text{-}21}$ y)
- Within the Standard Model

Neutrinoless double beta decay $0\nu\beta\beta$

$$^{A}_{Z}X_{N} \Rightarrow ^{A}_{Z+2}Y_{N-2} + 2e^{-}$$

- Violates the leptonic number conservation
- Neutrinos are massive Majorana particles
- Experimentally not observed (yet?) ($T_{1/2}>10^{25}$ y)
- Beyond the Standard Model

Half-life neutrinoless double beta decay (Doi et al (1985))

$$\left(T_{1/2}^{0\nu\beta\beta}(0^+\rightarrow 0^+)\right)^{-1} = G_{01}\left|M^{0\nu\beta\beta}\right|^2\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2$$

light-neutrino exchange mechanism

- Kinematic phase space factor:

$$G_{01} = \frac{(Gg_A(0))^4 m_e^4}{64\pi^5 \ln 2}\int F_0(Z,\varepsilon_1)F_0(Z,\varepsilon_2)$$
$$\times p_1 p_2 \delta(\varepsilon_1+\varepsilon_2-E_f-E_i)d\varepsilon_1 d\varepsilon_2 d(\hat{\vec{p}}_1\cdot\hat{\vec{p}}_1)$$

- Effective neutrino mass:

$$\langle m_\nu\rangle = \sum_j U_{ej}^2 m_j$$

- Nuclear Matrix Element (NME):

$$M^{0\nu\beta\beta} = -\left(\frac{g_V(0)}{g_A(0)}\right)^2 M_F^{0\nu\beta\beta} + M_{GT}^{0\nu\beta\beta} - M_T^{0\nu\beta\beta}$$

Fermi Gamow-Teller Tensor

$$M^{0\nu\beta\beta} = -\left(\frac{g_V(0)}{g_A(0)}\right)^2 M_F^{0\nu\beta\beta} + M_{GT}^{0\nu\beta\beta} - M_T^{0\nu\beta\beta}$$

- Each term can be written as the expectation value of a transition operator acting on the initial al final states:

$$M_\xi^{0\nu\beta\beta} = \langle 0_f^+ | \hat{O}_\xi^{0\nu\beta\beta} | 0_i^+ \rangle$$

- Nuclear structure methods for calculating these NME:
    - Quasiparticle Random Phase Approximation in different versions: QRPA, RQRPA, SRQRPA. (Tübingen group, Jyüvaskylä group)
    - Interacting Shell Model -ISM- (Strasbourg-Madrid collaboration, Michigan)
    - Interacting Boson Model -IBM- (Yale group)
    - Projected Hartree-Fock-Bogoliubov -PHFB- (Lucknow-UNAM group)
    - Energy Density Functional

$$M^{0\nu\beta\beta} = -\left(\frac{g_V(0)}{g_A(0)}\right)^2 M_F^{0\nu\beta\beta} + M_{GT}^{0\nu\beta\beta} - M_T^{0\nu\beta\beta}$$

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- Nuclear structure methods for calculating these NME:

Different ways to deal with:
- Finding the best initial and final ground states.
- Handling the transition operator (inclusion of most relevant terms, corrections, approximations, etc.).

Some remarks about these methods:
- Calculations with limited single particle bases.
- Interactions fitted to the specific region (ISM) or to each nucleus individually (rest).
- Difficulties to include collective degrees of freedom.
- Problems with particle number conservation.

- **Effective nucleon-nucleon interaction:**

Gogny force (D1S-D1M) that is able to describe properly many phenomena along the whole nuclear chart.

$$V(1,2) = \sum_{i=1}^{2} e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_i^2}\left(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau\right)$$

$$+iW_0(\sigma_1+\sigma_2)\vec{k}\times\delta(\vec{r}_1-\vec{r}_2)\vec{k} + t_3(1+x_0P^\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left((\vec{r}_1+\vec{r}_2)/2\right)$$

$$+V_{\mathrm{Coulomb}}(\vec{r}_1,\vec{r}_2)$$

- **Method of solving the many-body problem:**

First step: Particle Number Projection (before the variation) of HFB-type wave functions.

Second step: Simultaneous **Particle Number and Angular Momentum Projection** (after the variation).

Third step: Configuration mixing within the framework of the **Generator Coordinate Method (GCM)**.

Determination of initial and final states (I)

Intrinsic state: Solve the PN-VAP equations with the Gogny D1S/D1M interaction

$|\Phi\rangle$ HFB states $\longrightarrow$ $\delta\left(E^{N,Z}\left[|\bar{\Phi}(q)\rangle\right]\right)_{|\bar{\Phi}\rangle=|\Phi\rangle}=0$

$$E^{N,Z}[|\Phi\rangle]=\frac{\langle\Phi|\hat{H}\hat{P}^N\hat{P}^Z|\Phi\rangle}{\langle\Phi|\hat{P}^N\hat{P}^Z|\Phi\rangle}+\varepsilon_{DD}^{N,Z}(|\Phi\rangle)-\lambda_q\langle\Phi|\hat{Q}|\Phi\rangle$$

Particle number and angular momentum projected state:

$$|IMK;NZ;q\rangle=\frac{2I+1}{8\pi^2}\int\mathcal{D}_{MK}^{I*}(\Omega)\hat{R}(\Omega)\hat{P}^N\hat{P}^Z|\Phi(q)\rangle d\Omega$$

General form (GCM state):

$$|IM;NZ\sigma\rangle=\sum_{Kq}f_{Kq}^{I;NZ,\sigma}|IMK;NZ;q\rangle$$

Hill-Wheeler-Griffin equation (GCM)

$$\sum_{K'q'}\left(\mathcal{H}_{KqK'q'}^{I;NZ}-E^{I;NZ;\sigma}\mathcal{N}_{KqK'q'}^{I;NZ}\right)f_{K'q'}^{I;NZ;\sigma}=0$$

$$\mathcal{N}_{KqK'q'}^{I;NZ}\equiv\langle IMK;NZ;q|IMK';NZ;q'\rangle$$

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$\longrightarrow$ generalized eigenvalue problem

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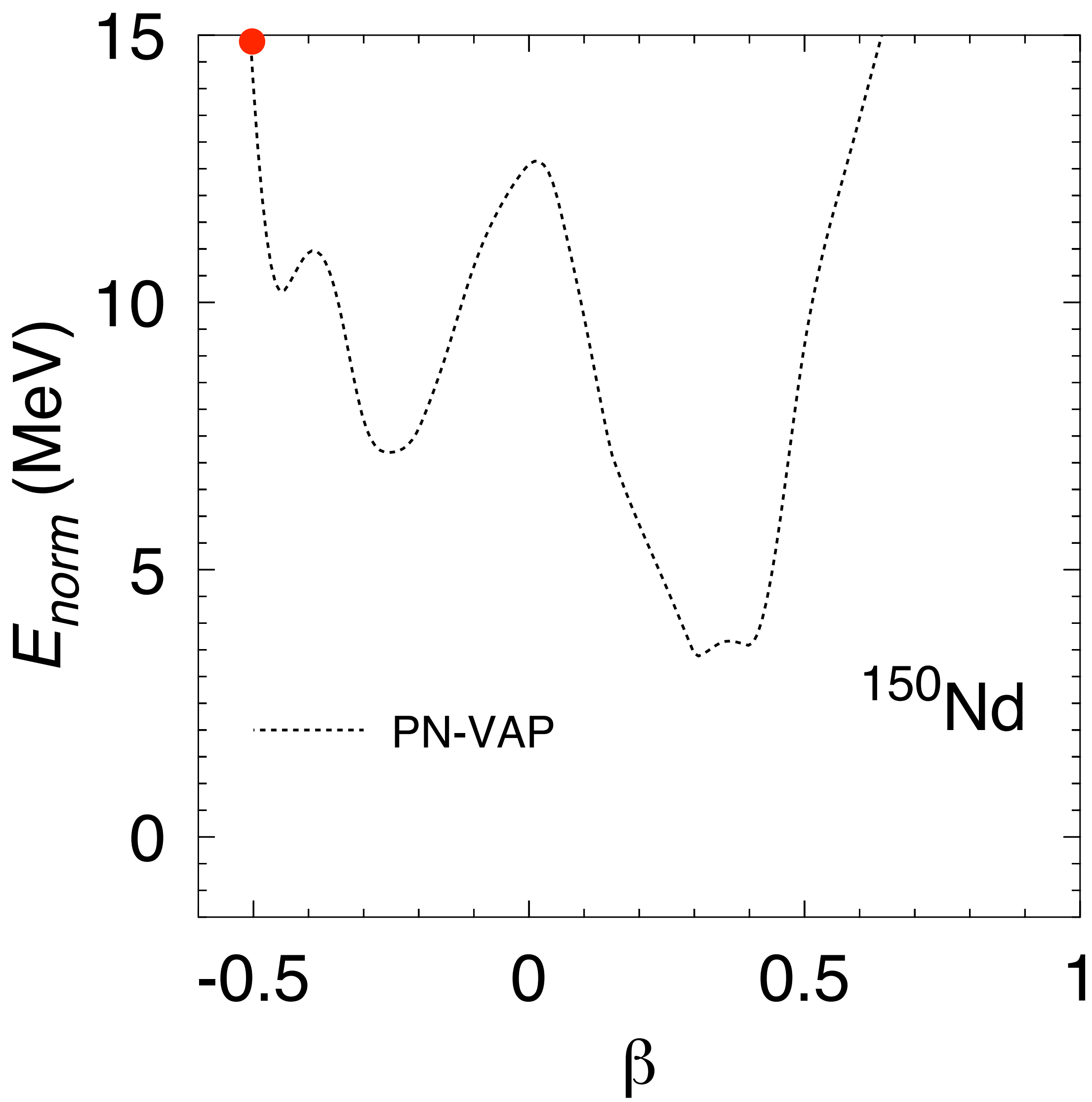

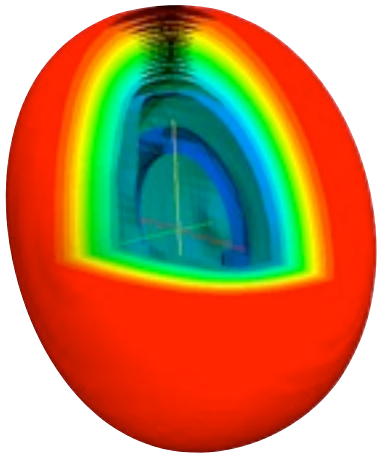

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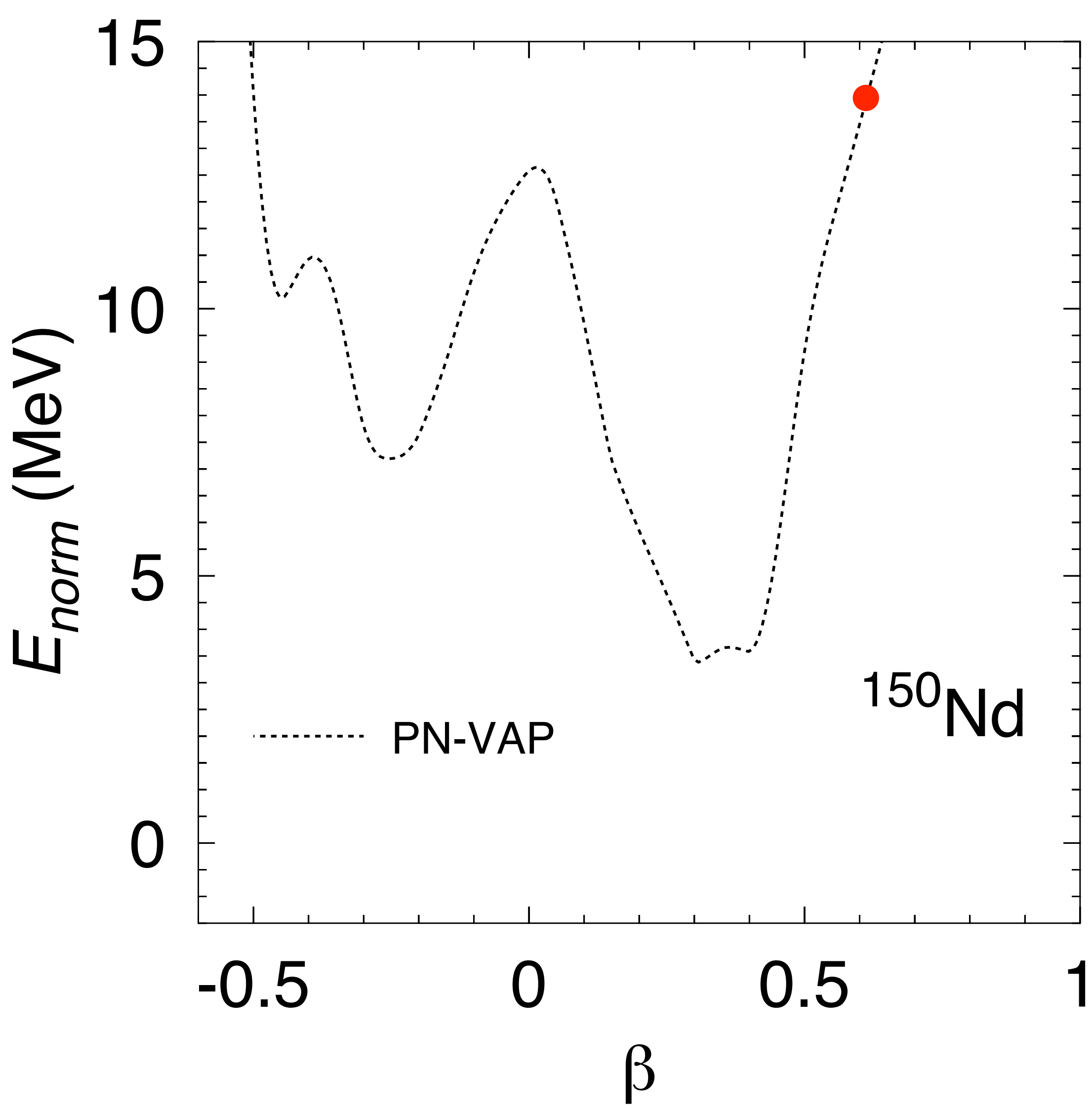

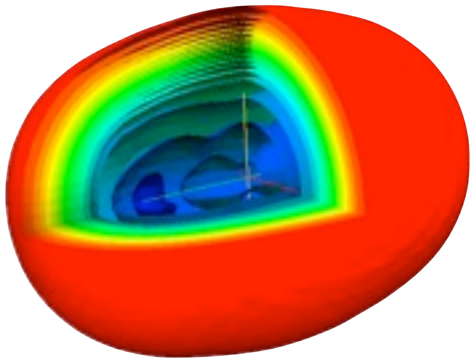

Particle number projection

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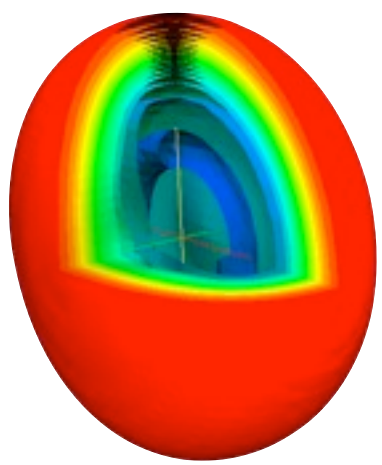

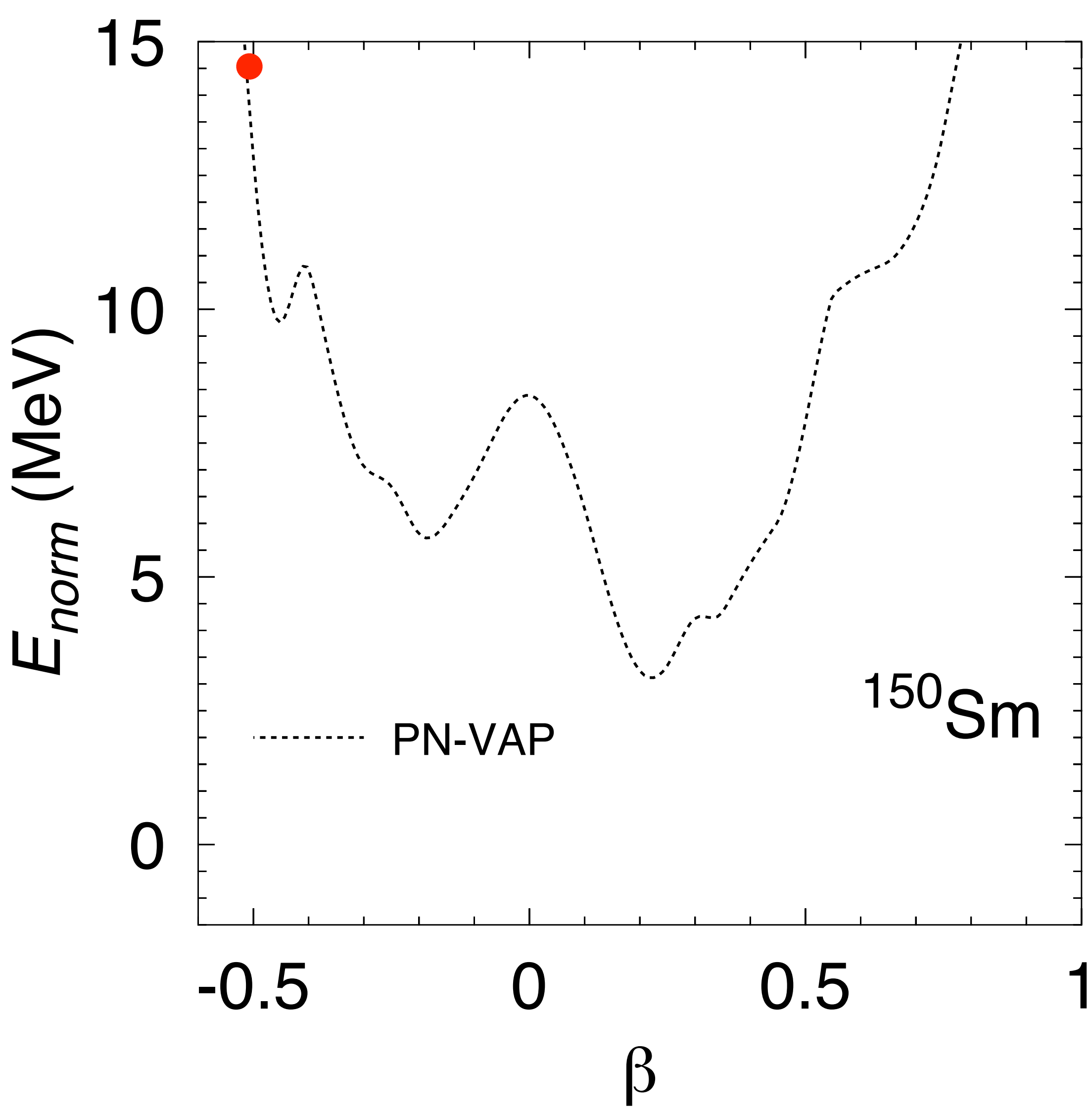

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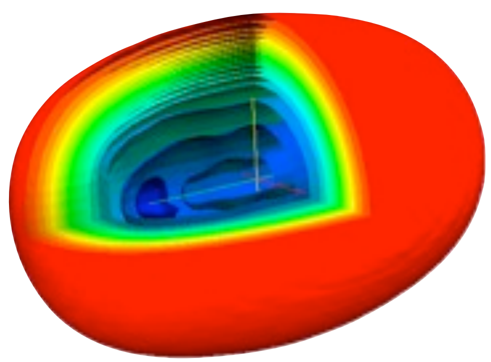

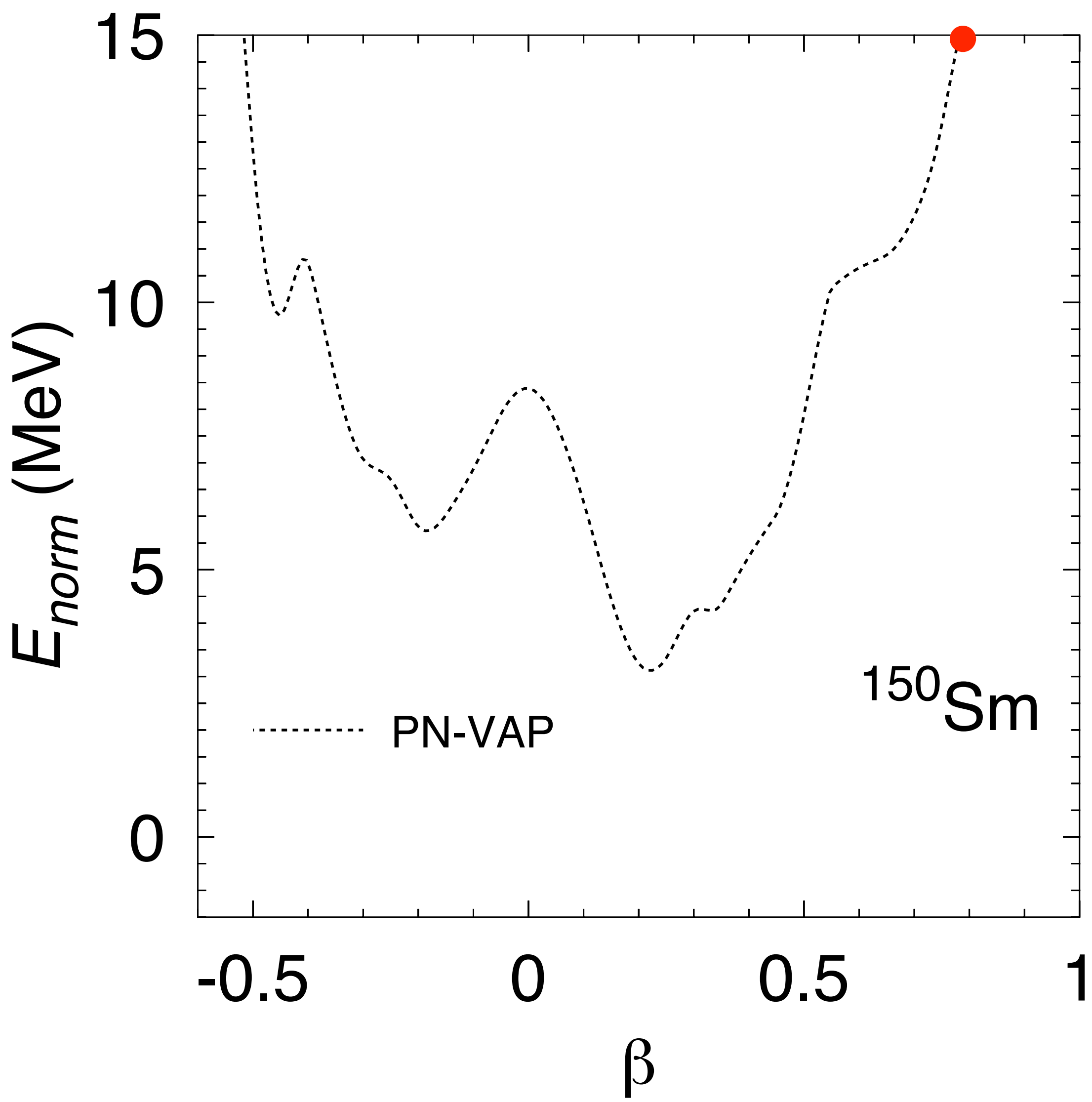

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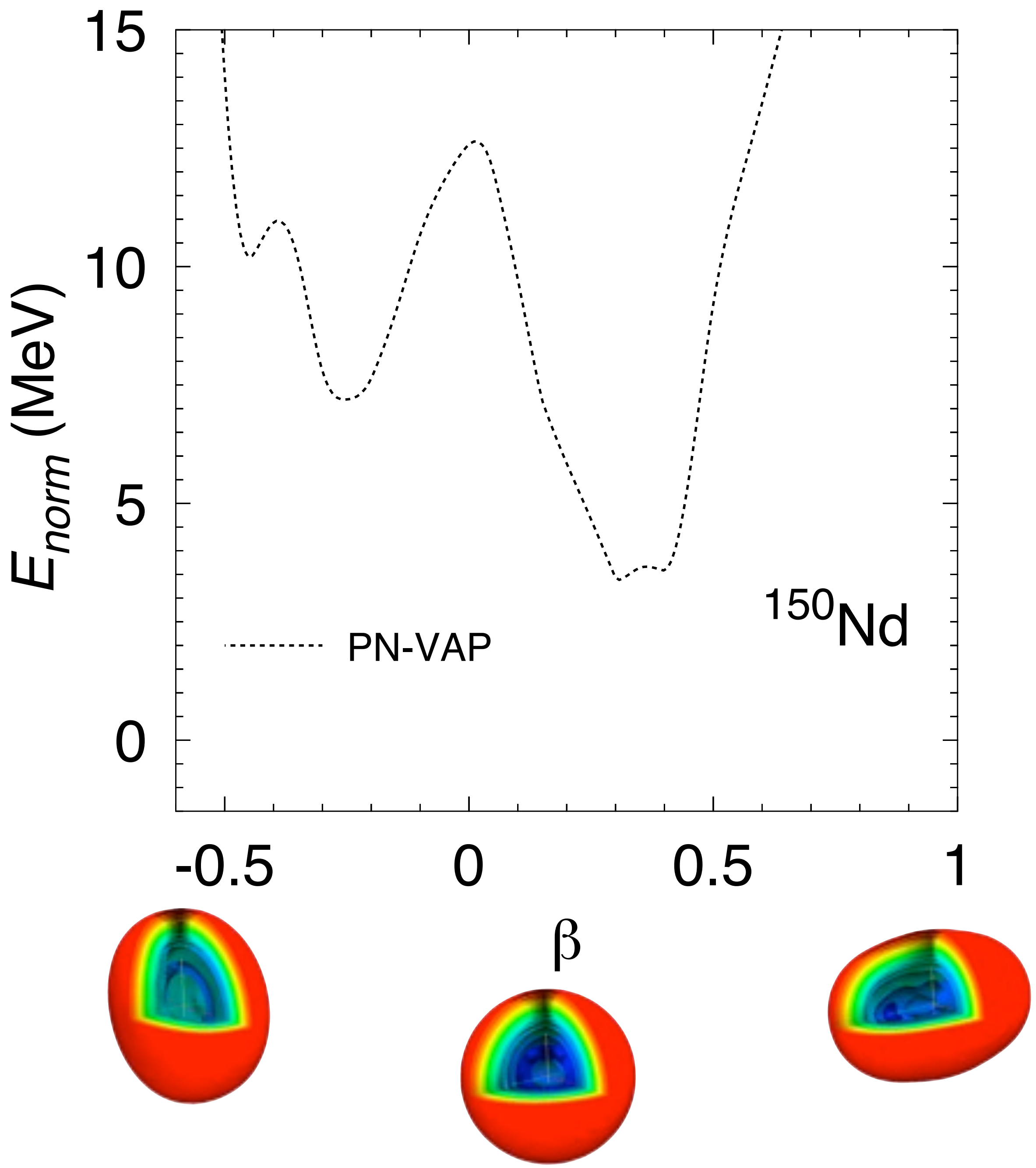

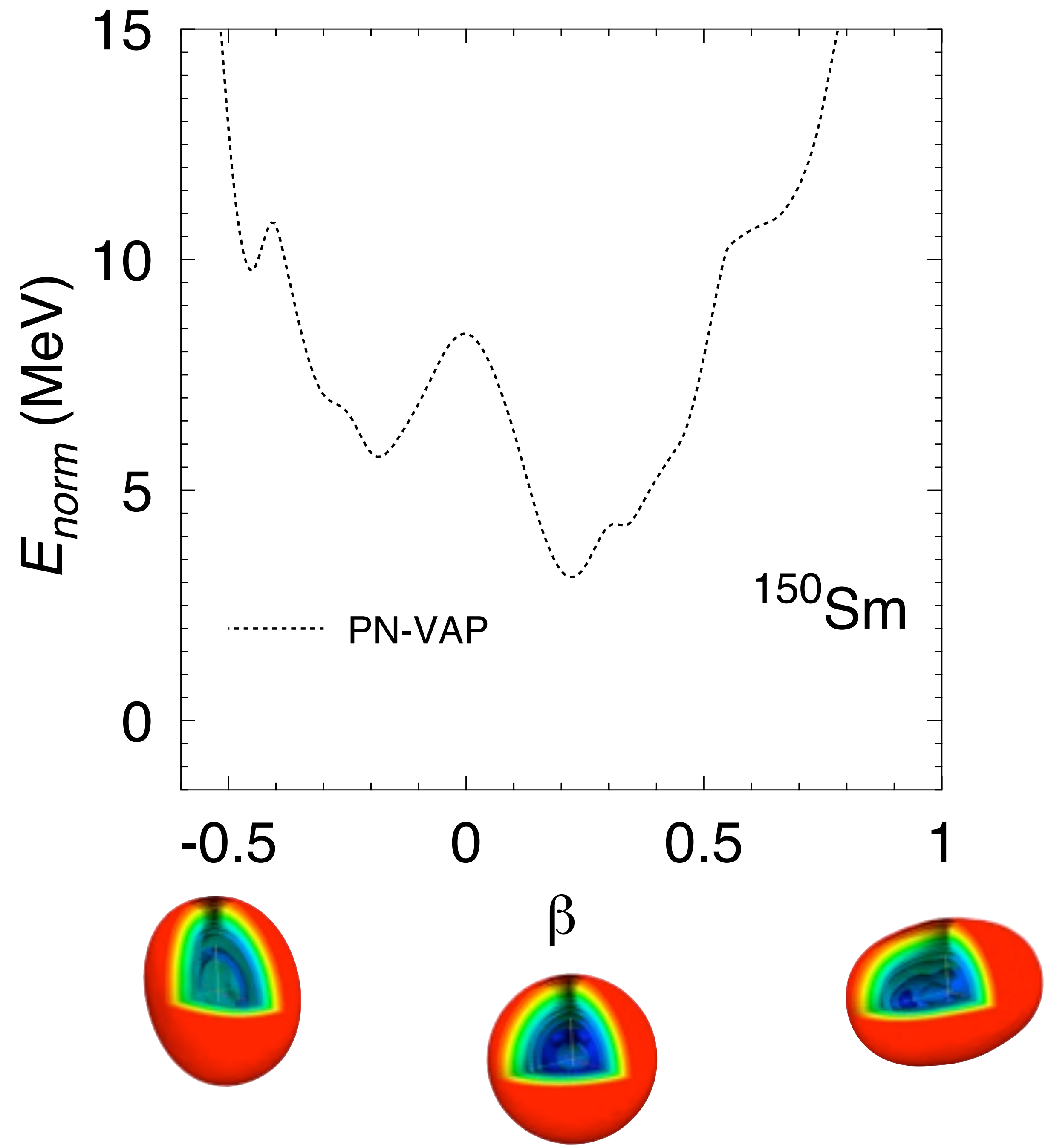

Determination of initial and final states (II)

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$\longrightarrow$ generalized eigenvalue problem

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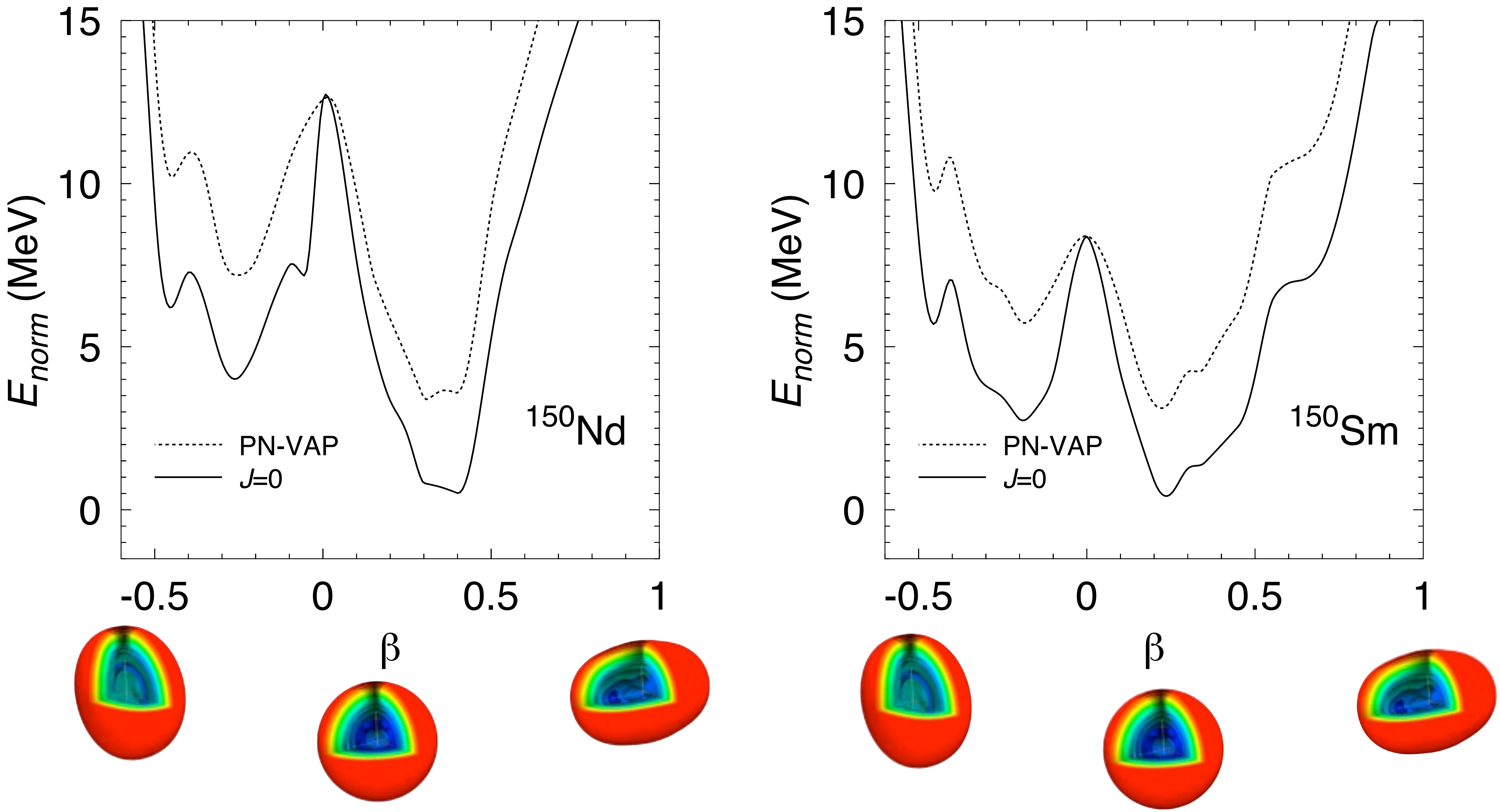

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Solving HWG equation:

1. Diagonalization of the norm overlap:

$$\sum_{K'q'}\mathcal{N}_{KqK'q'}^{I;NZ}u_{K'q';\Lambda}^{I;NZ}=n_{\Lambda}^{I;NZ}u_{Kq;\Lambda}^{I;NZ}$$

2. Natural basis:

$$|\Lambda^{IM;NZ}\rangle=\sum_{Kq}\frac{u_{Kq;\Lambda}^{I;NZ}}{\sqrt{n_{\Lambda}^{I;NZ}}}|IMK;NZ;q\rangle\ ;n_{\Lambda}^{I;NZ}/n_{max}^{I,NZ}>\zeta$$

3. Normal eigenvalue problem:

$$\sum_{\Lambda'}\langle\Lambda^{I;NZ}|\hat{H}|\Lambda'^{I;NZ}\rangle G_{\Lambda'}^{I;NZ;\sigma}=E^{I;NZ;\sigma}G_{\Lambda}^{I;NZ;\sigma}$$

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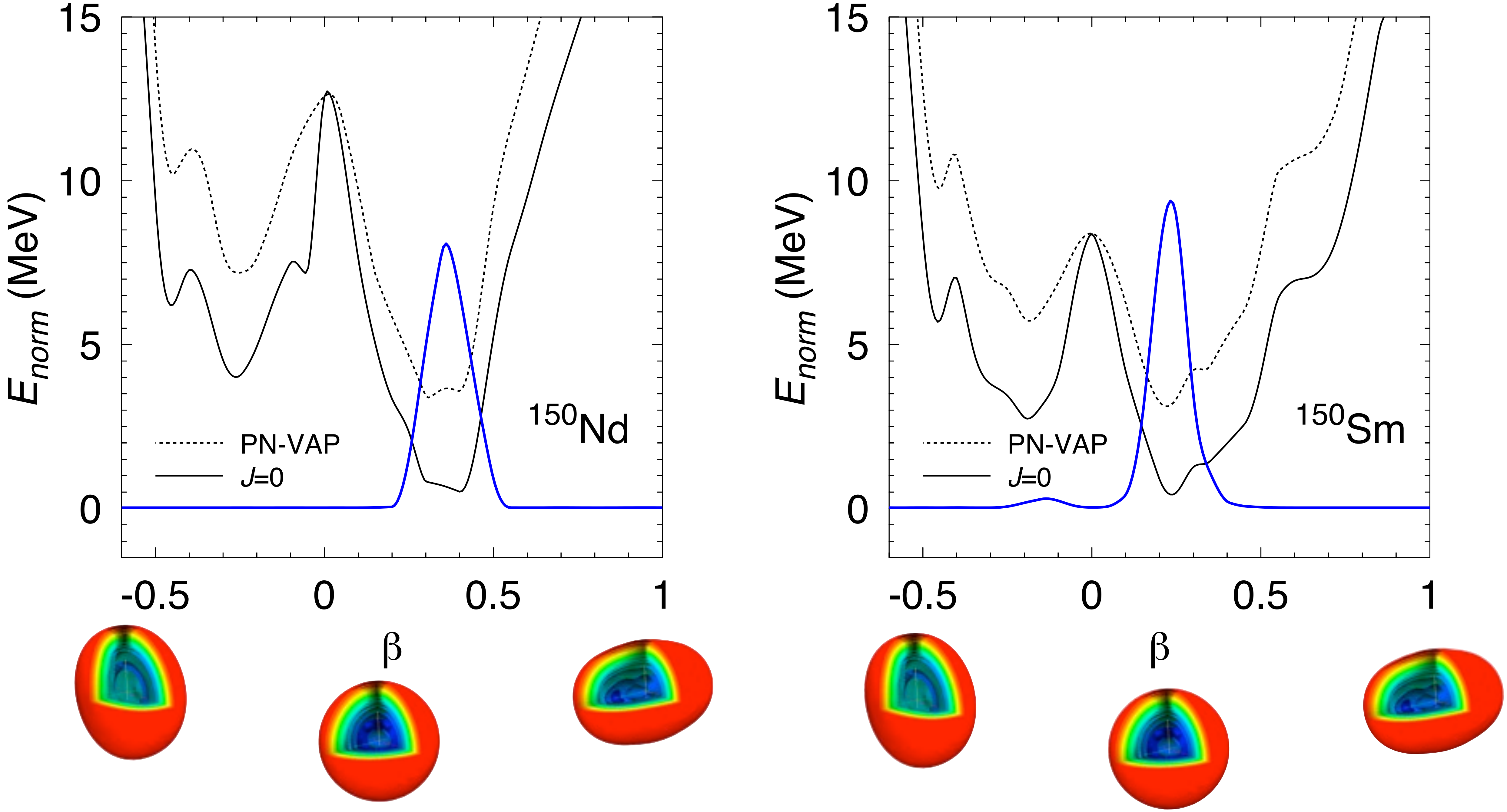
Configuration (shape) mixing

Determination of initial and final states (& III)



Transitions

1. Axial states $K=0$
2. Angular momentum $I=0$
3. Quadrupole deformations $q=q_{20}$

$\longrightarrow$

$$|0;N_iZ_i;\sigma\rangle = \sum_{\Lambda_i} G_{\Lambda_i}^{0;N_iZ_i;\sigma}|\Lambda_i^{0;N_iZ_i}\rangle$$

$$|0;N_fZ_f;\sigma\rangle = \sum_{\Lambda_f} G_{\Lambda_f}^{0;N_fZ_f;\sigma}|\Lambda_f^{0;N_fZ_f}\rangle$$

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TRANSITIONS:

$$M^{0\nu\beta\beta}_{\xi} = \langle 0^+_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0^+_i\rangle = \langle 0;N_fZ_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0;N_iZ_i\rangle =$$

$$\sum_{\Lambda_f\Lambda_i}\left(G^{0;N_fZ_f}_{\Lambda_f}\right)^*\langle\Lambda_f^{0;N_fZ_f}|\hat{O}^{0\nu\beta\beta}_{\xi}|\Lambda_i^{0;N_iZ_i}\rangle G^{0;N_iZ_i}_{\Lambda_i} = \sum_{q_iq_f;\Lambda_f\Lambda_i}$$

$$\left(\frac{u^{0;N_fZ_f}_{q_f,\Lambda_f}}{\sqrt{n^{0;N_fZ_f}_{\Lambda_f}}}\right)^*\left(G^{0;N_fZ_f}_{\Lambda_f}\right)^*\langle 0;N_fZ_f;q_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0;N_iZ_i;q_i\rangle\left(G^{0;N_iZ_i}_{\Lambda_i}\right)\left(\frac{u^{0;N_iZ_i}_{q_i,\Lambda_i}}{\sqrt{n^{0;N_iZ_i}_{\Lambda_i}}}\right)$$

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$$\left(\frac{u^{0;N_fZ_f}_{q_f,\Lambda_f}}{\sqrt{n^{0;N_fZ_f}_{\Lambda_f}}}\right)^*\left(G^{0;N_fZ_f}_{\Lambda_f}\right)^*\langle 0;N_fZ_f;q_f|\hat{O}^{0\nu\beta\beta}_{\xi}|0;N_iZ_i;q_i\rangle\left(G^{0;N_iZ_i}_{\Lambda_i}\right)\left(\frac{u^{0;N_iZ_i}_{q_i,\Lambda_i}}{\sqrt{n^{0;N_iZ_i}_{\Lambda_i}}}\right)$$

Matrix elements of the double beta transition operators between particle number and angular momentum projected states

Neutrinoless double beta decay candidates

440 *T.R. Rodríguez, G. Martinez-Pinedo / Progress in Particle and Nuclear Physics 66 (2011) 436–440*

Table 1
Masses, rms charge radii and total Gamow–Teller strengths $S_{-(+)}$ for mother (granddaughter) calculated with Gogny D1S GCM+PNAMP functional compared to experimental values. Theoretical values for $S_{+/-}$ are quenched by a factor $(0.74)^2$.

Isotope	BEth (MeV)	BE^{exp} (MeV) [27]	Rth (fm)	R^{exp}(fm) [28]	$S^{theo}_{-/+}$	$S^{exp}_{-/+}$
^{48}Ca	420.623	415.991	3.465	3.473	13.55	(14.4 ± 2.2 [29])
^{48}Ti	423.597	418.699	3.557	3.591	1.99	(1.9 ± 0.5 [29])
^{76}Ge	664.204	661.598	4.024	4.081	20.97	(19.89 [30])
^{76}Se	664.949	662.072	4.074	4.139	1.49	(1.45 ± 0.07 [31])
^{82}Se	716.794	712.842	4.100	4.139	23.56	(21.91 [30])
^{82}Kr	717.859	714.273	4.130	4.192	1.24	
^{96}Zr	829.432	828.995	4.298	4.349	27.63	
^{96}Mo	833.793	830.778	4.319	4.384	2.56	(0.29 ± 0.08 [32])
^{100}Mo	861.526	860.457	4.372	4.445	27.87	(26.69 [30])
^{100}Ru	864.875	861.927	4.388	4.453	2.48	
^{116}Cd	988.469	987.440	4.556	4.628	34.30	(32.70 [30])
^{116}Sn	991.079	988.684	4.567	4.626	2.61	($1.09^{+0.13}_{-0.57}$ [33])
^{124}Sn	1051.668	1049.96	4.622	4.675	40.65	
^{124}Te	1051.562	1050.69	4.664	4.717	1.63	
^{128}Te	1082.257	1081.44	4.686	4.735	40.48	(40.08 [30])
^{128}Xe	1080.996	1080.74	4.723	4.775	1.45	
^{130}Te	1096.627	1095.94	4.695	4.742	43.57	(45.90 [30])
^{130}Xe	1097.245	1096.91	4.732	4.783	1.19	
^{136}Xe	1143.333	1141.88	4.756	4.799	46.71	
^{136}Ba	1143.202	1142.77	4.786	4.832	0.96	
^{150}Nd	1234.512	1237.45	5.034	5.041	50.32	
^{150}Sm	1235.936	1239.25	5.041	5.040	1.45	

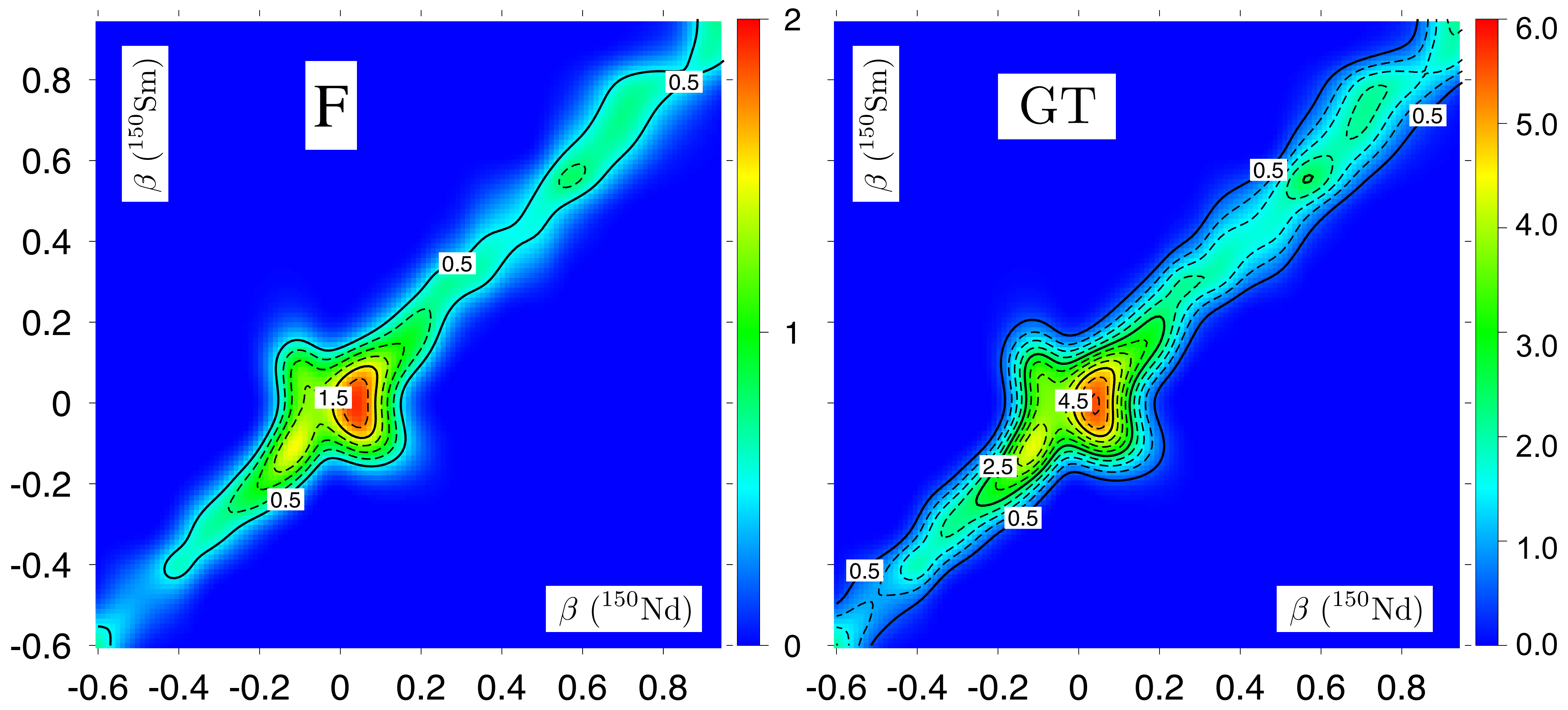
Good agreement between experimental and theoretical *Q*-values, radii and total strength (quenched)

NME: deformation and mixing

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_\xi^{0\nu\beta\beta} | 0; N_i Z_i; q_i\rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f\rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i\rangle}}$$

A=150

T.R.R., Martínez-Pinedo, PRL 2010



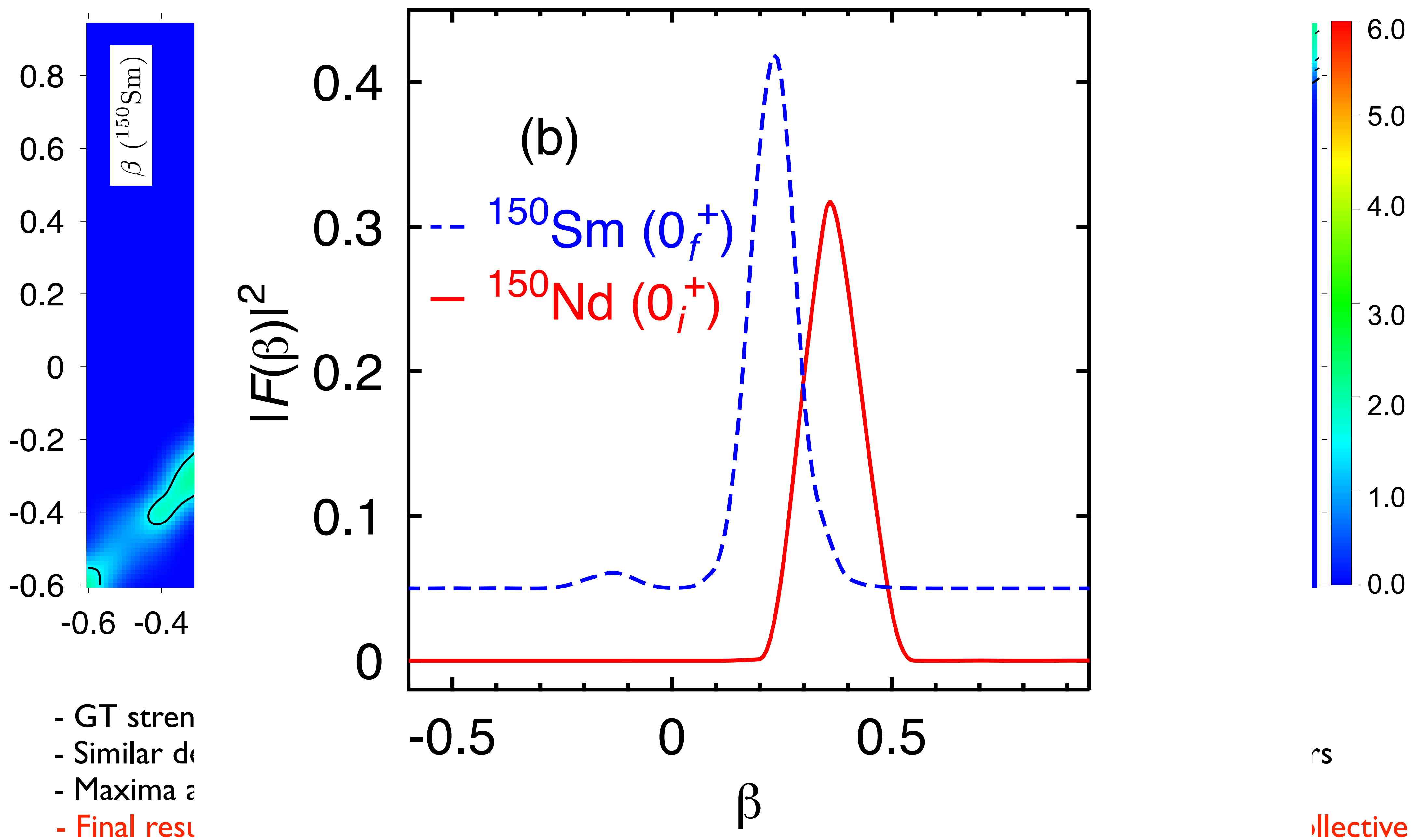
- GT strength greater than Fermi.
- Similar deformation between mother and granddaughter is favored by the transition operators
- Maxima are found close to sphericity although some other local maxima are found

NME: deformation and mixing

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}^{0\nu\beta\beta}_{\xi} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

A=150

T.R.R., Martínez-Pinedo, PRL 2010



- GT stren
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- Maxima a
- Final resu ...llective states with... ...

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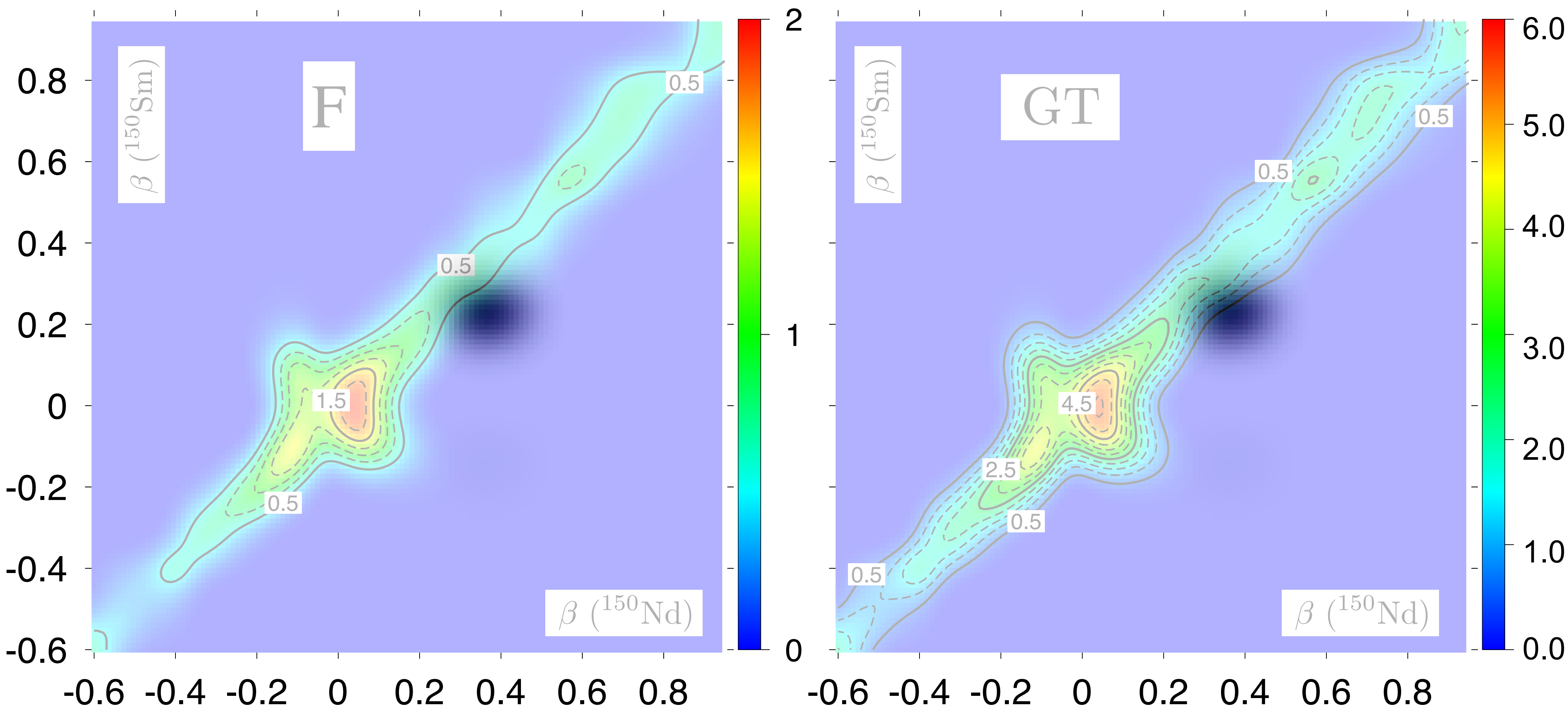
NME: deformation and mixing

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_{\xi}^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

A=150

T.R.R., Martínez-Pinedo, PRL 2010



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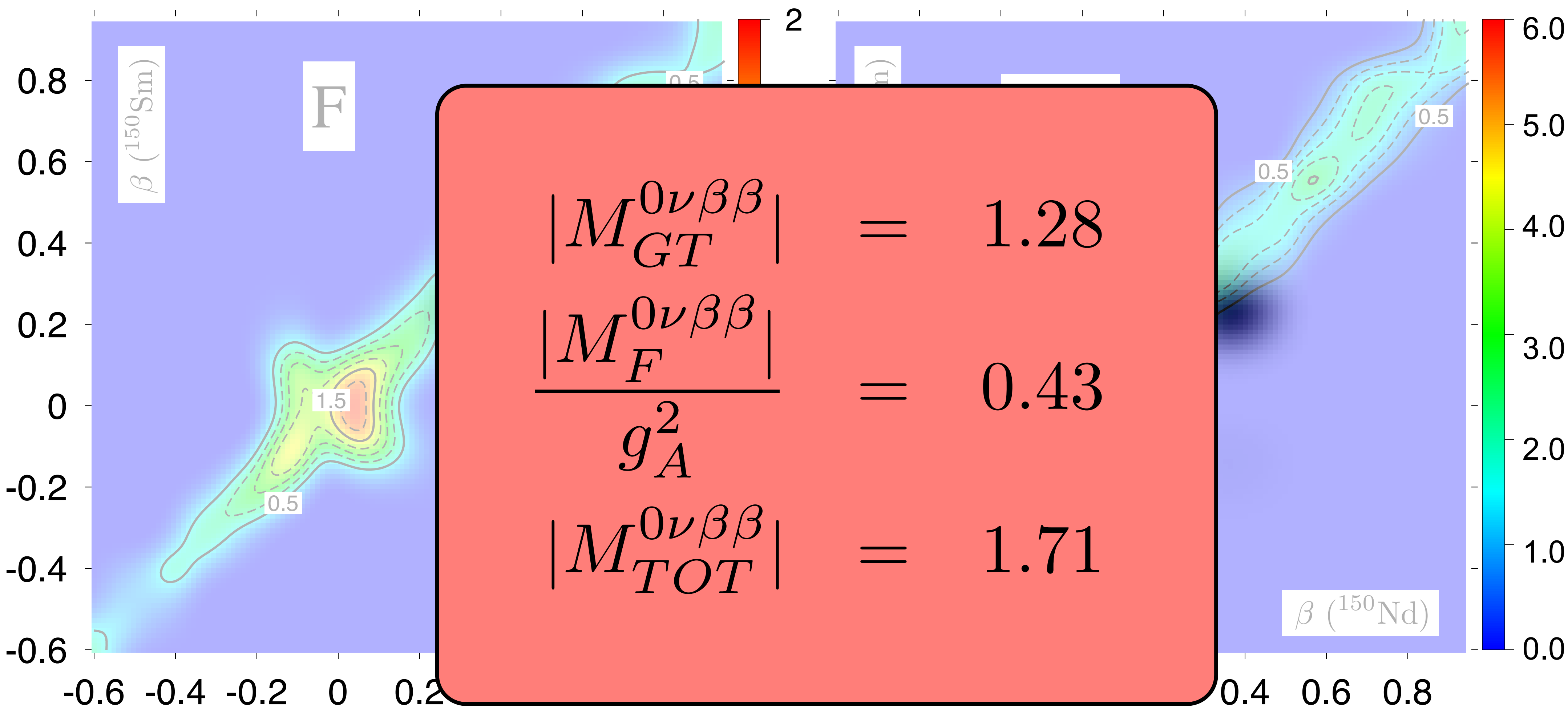
$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_{\xi}^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

A=150

T.R.R., Martínez-Pinedo, PRL 2010



$$|M_{GT}^{0\nu\beta\beta}| = 1.28$$

$$\frac{|M_{F}^{0\nu\beta\beta}|}{g_A^2} = 0.43$$

$$|M_{TOT}^{0\nu\beta\beta}| = 1.71$$

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NME: deformation and mixing

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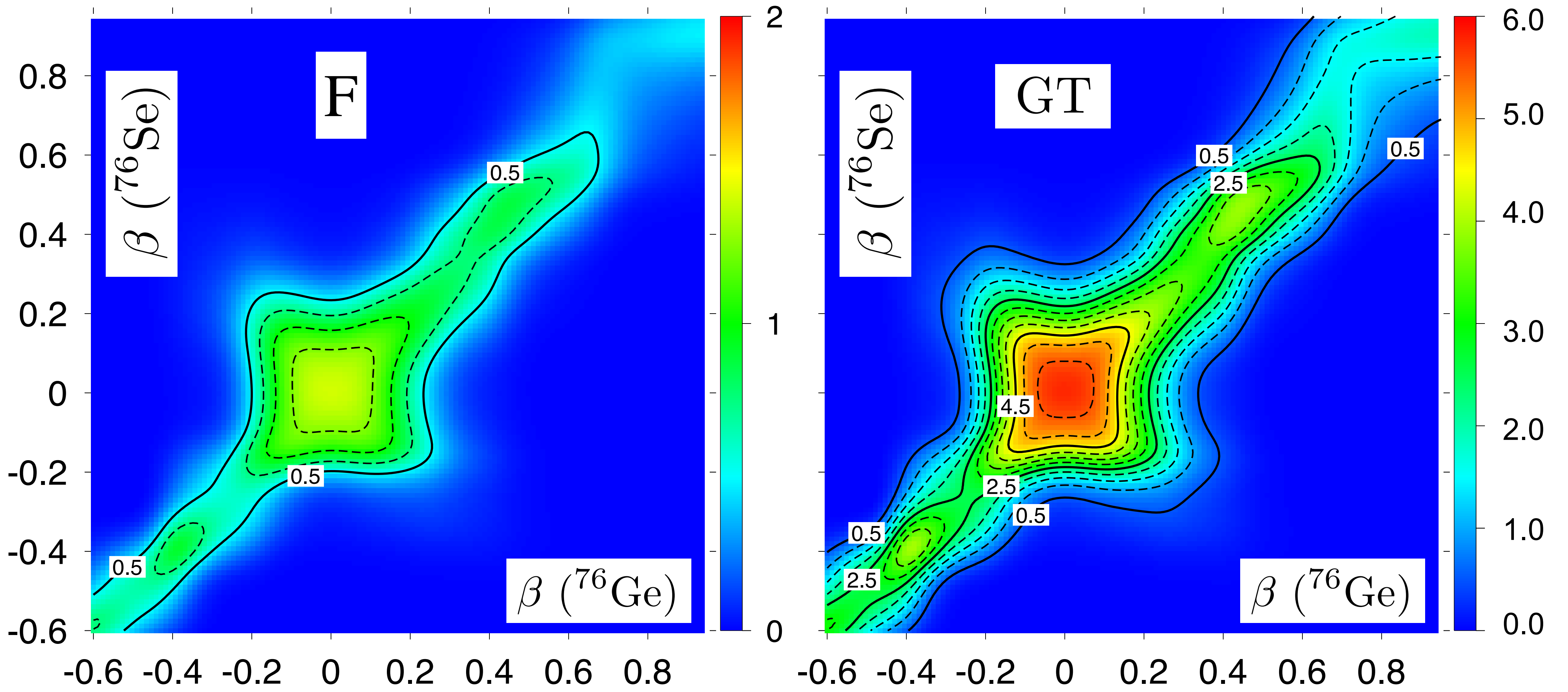
$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}^{0\nu\beta\beta}_{\xi} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

A=76



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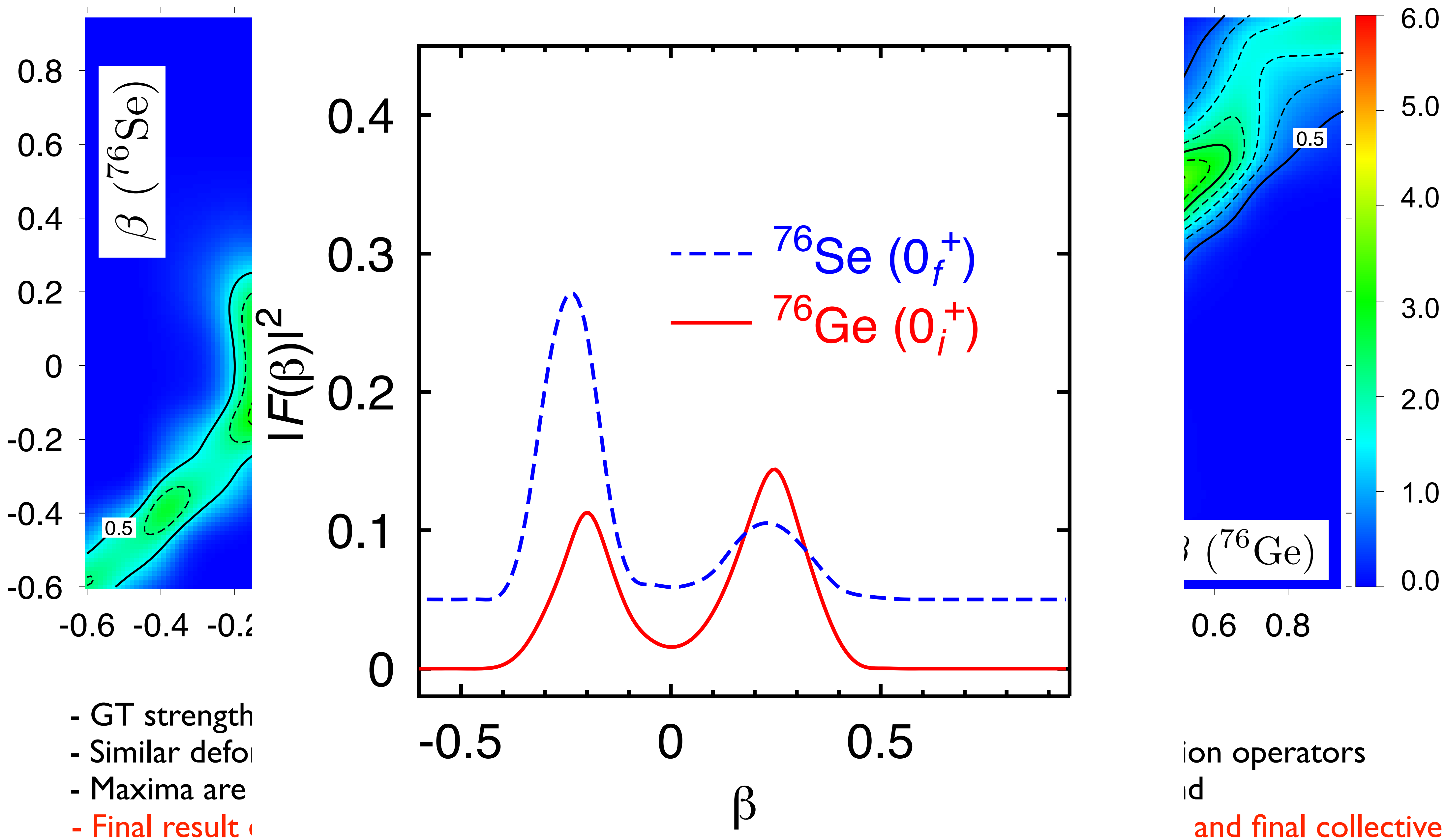
$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_{\xi}^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

A=76



- GT strength
- Similar defo
- Maxima are
- Final result
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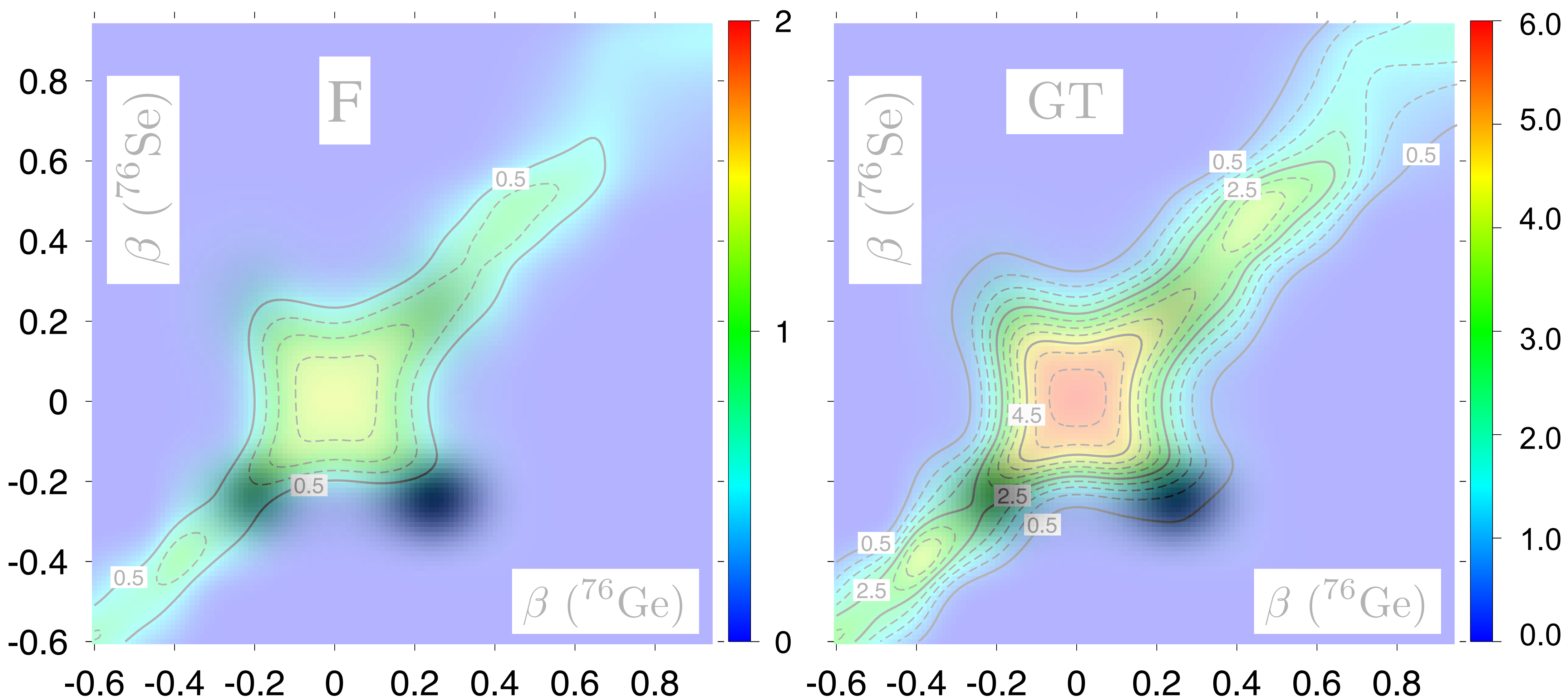
$$\frac{\langle 0;N_fZ_f;q_f|\hat{O}_\xi^{0\nu\beta\beta}|0;N_iZ_i;q_i\rangle}{\sqrt{\langle 0;N_fZ_f;q_f|0;N_fZ_f;q_f\rangle\langle 0;N_iZ_i;q_i|0;N_iZ_i;q_i\rangle}}$$

A=76



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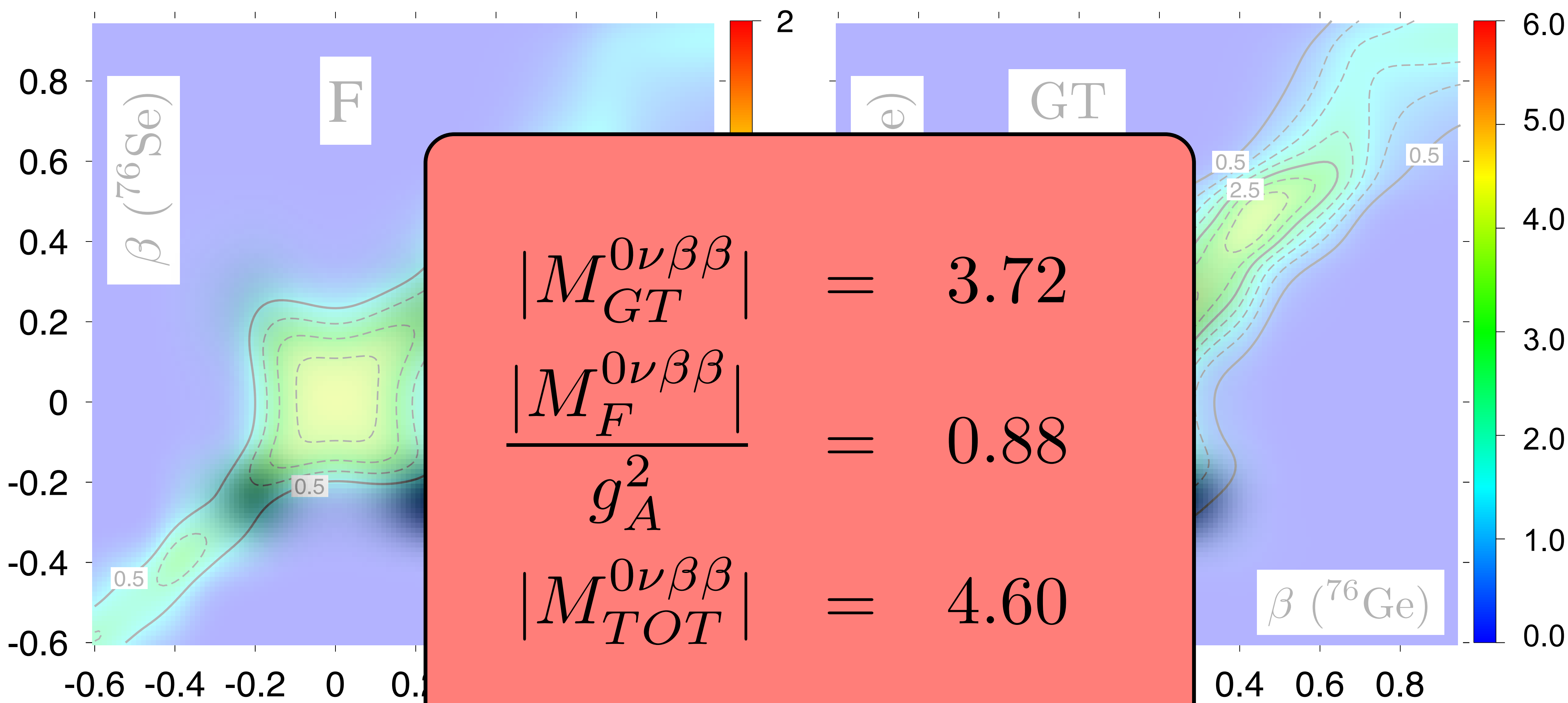

$$|M^{0\nu\beta\beta}_{GT}| = 3.72$$

$$\frac{|M^{0\nu\beta\beta}_{F}|}{g_A^2} = 0.88$$

$$|M^{0\nu\beta\beta}_{TOT}| = 4.60$$

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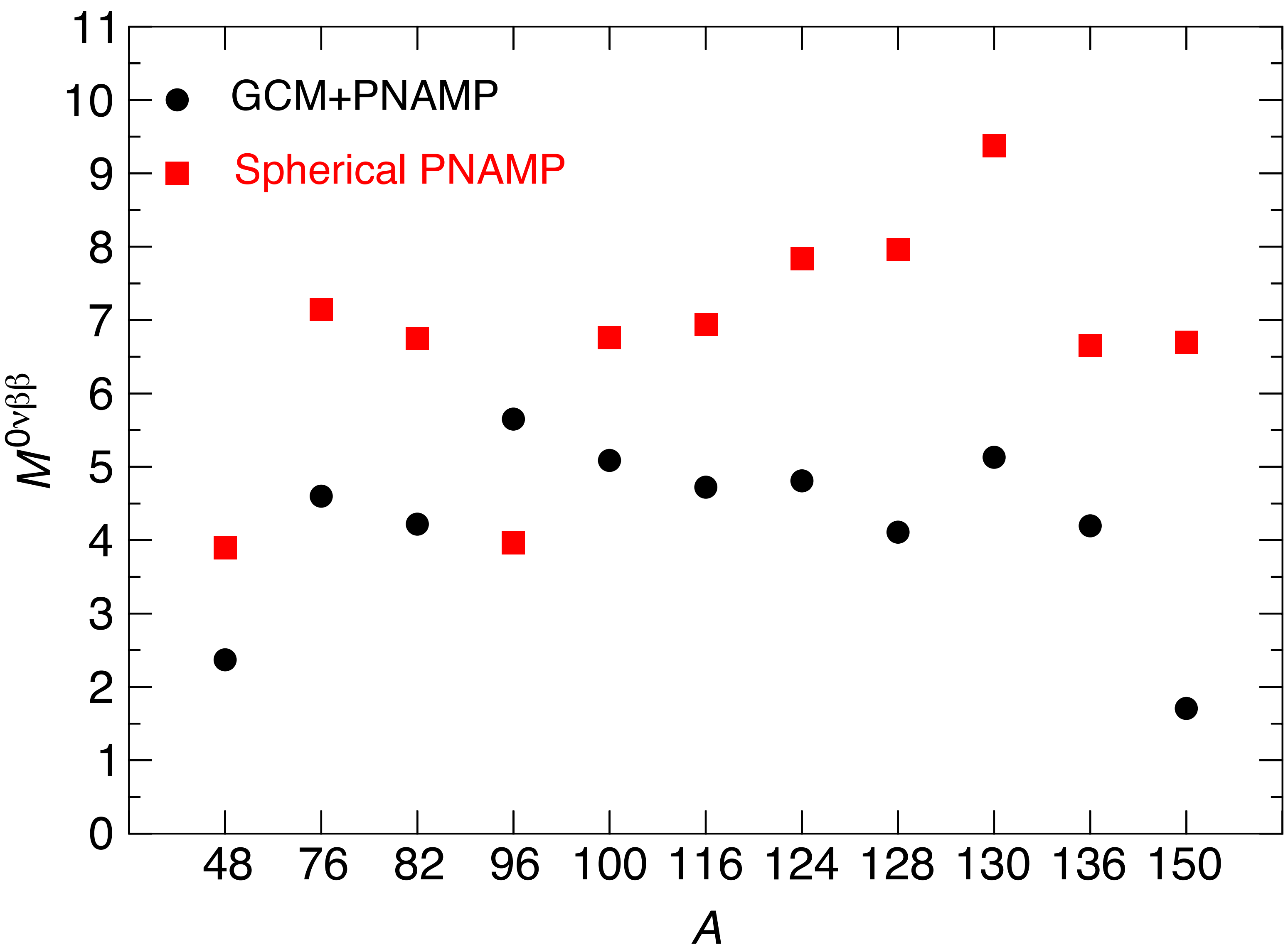
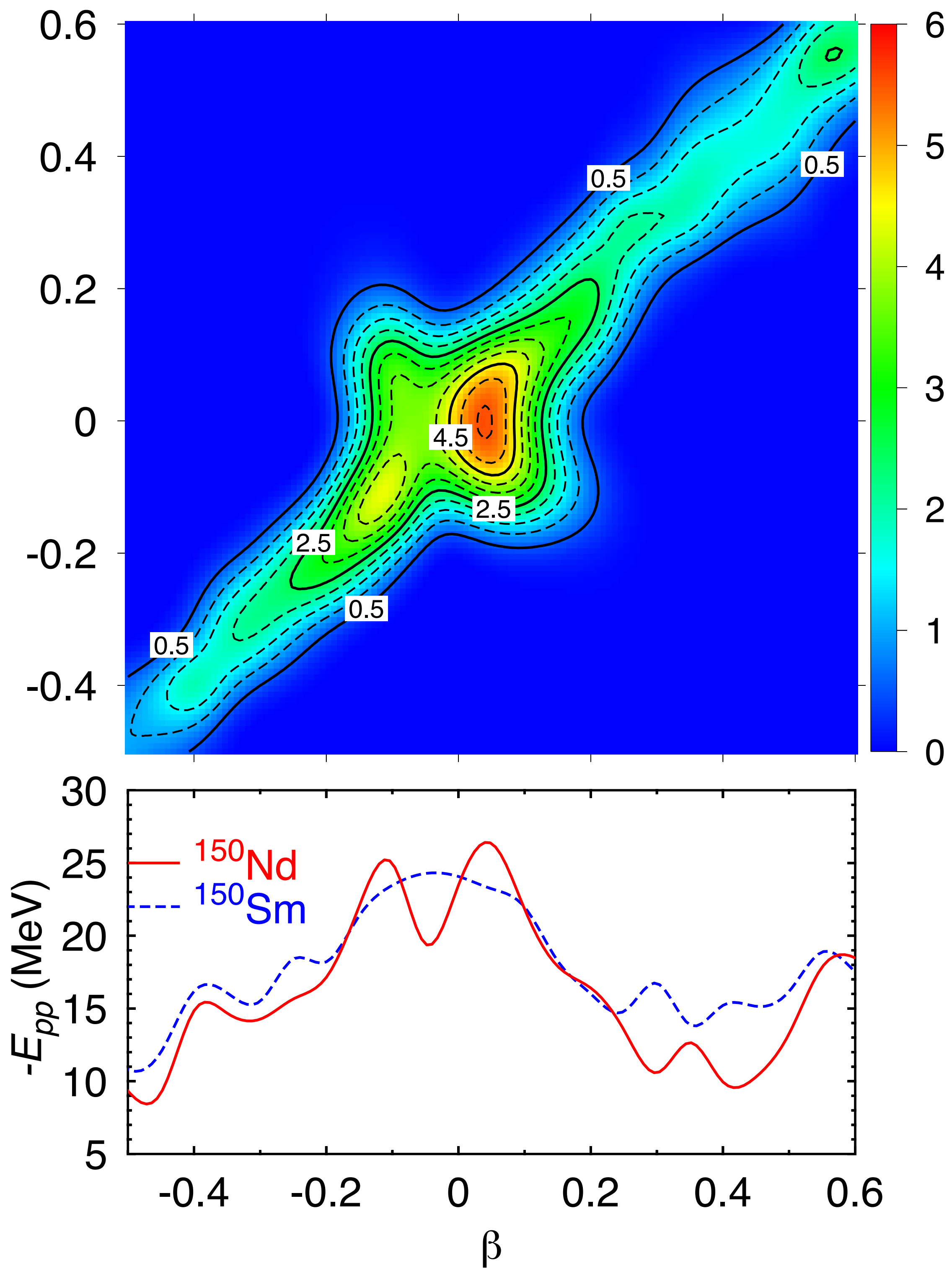


Noticeable difference in the NME if only intrinsic spherical configurations are considered without configuration mixing.

NME: Pairing



- The structure is related to the pairing energy (particle-particle) of the nuclei involved in the transition

- Maxima of the strength correspond to maxima in pairing energy

- In agreement with seniority arguments (increasing seniority decreases NME)

- Pairing energy and NME involve similar Wick theorem's contractions in this formalism

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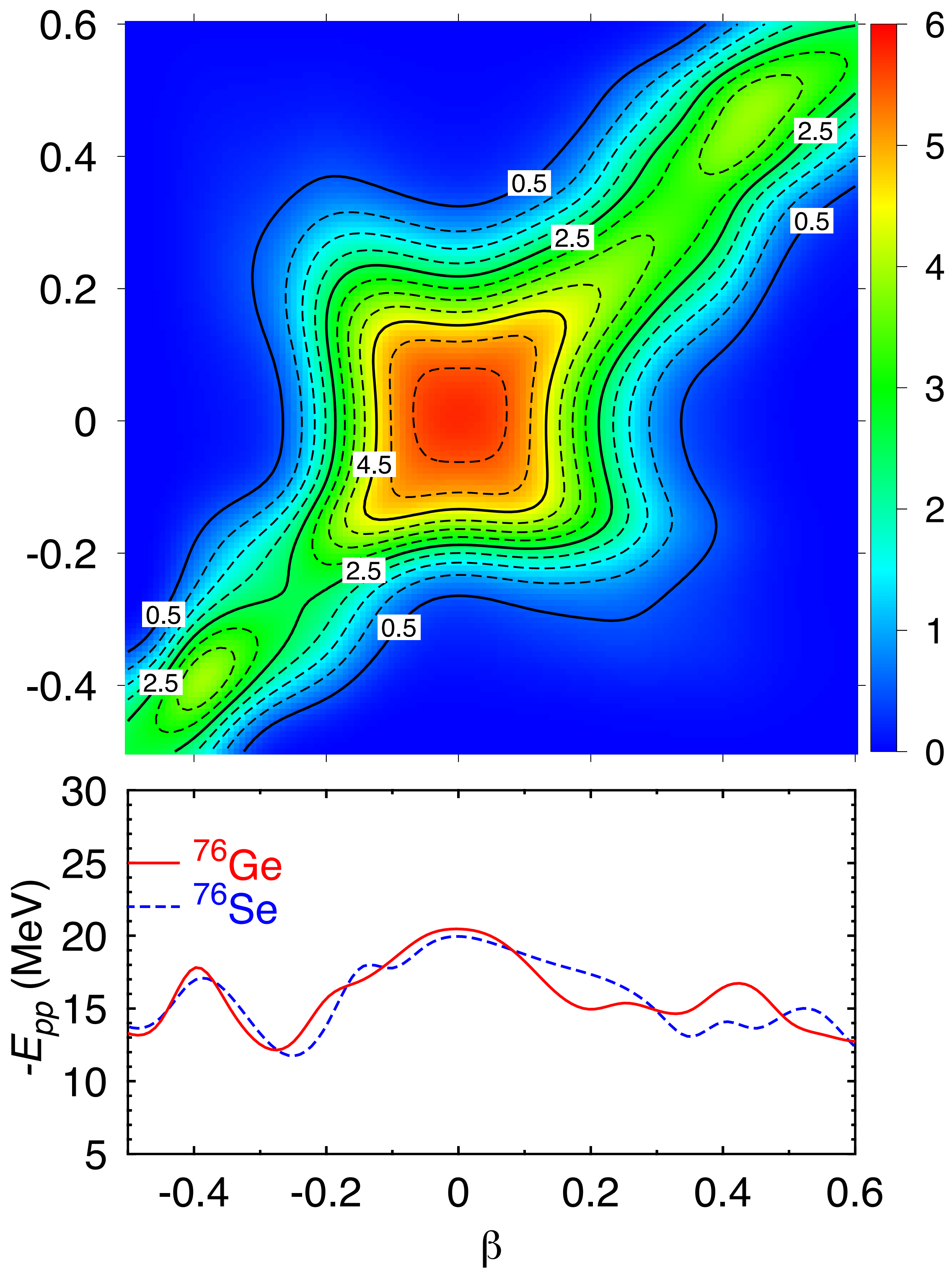

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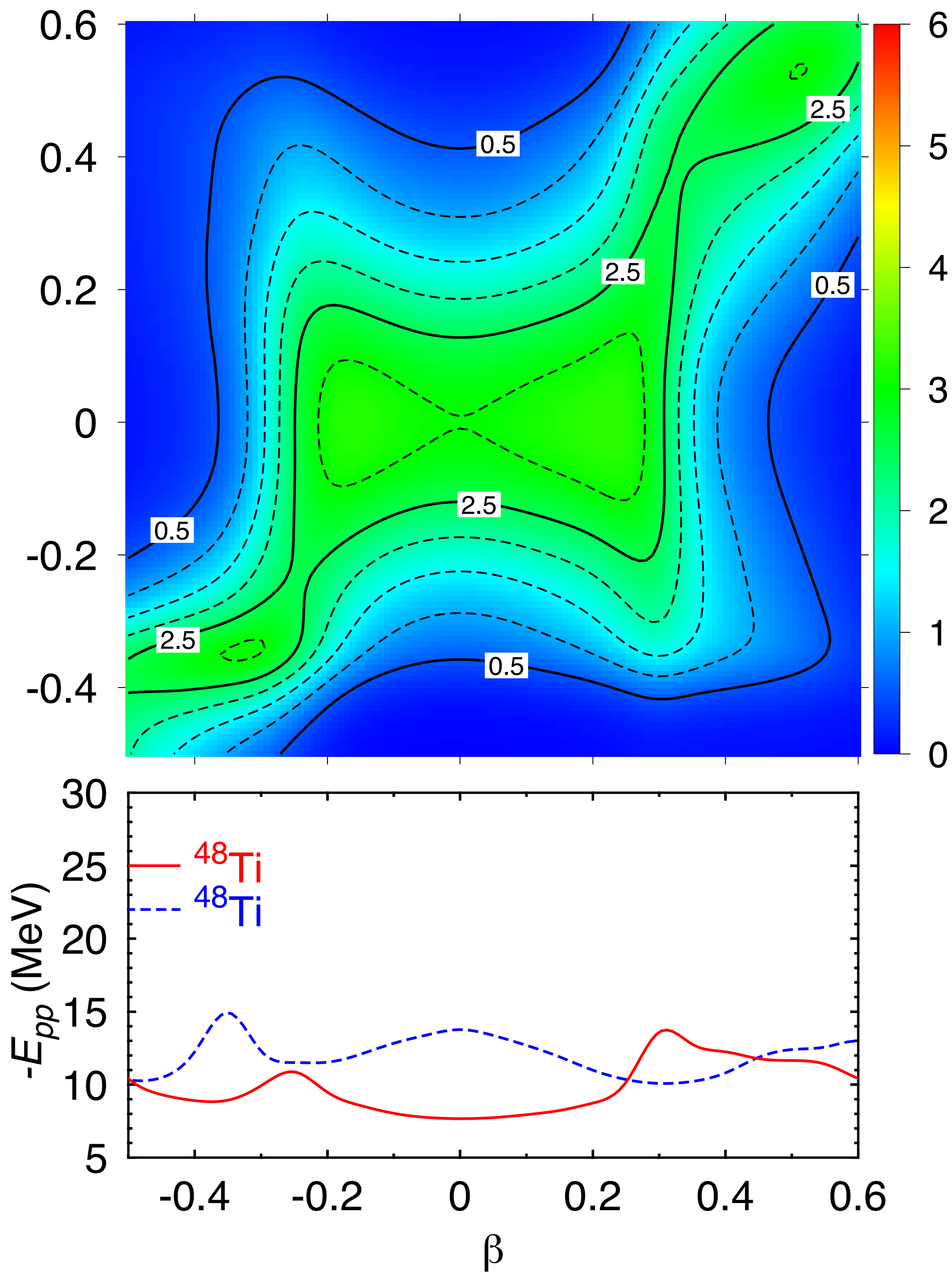

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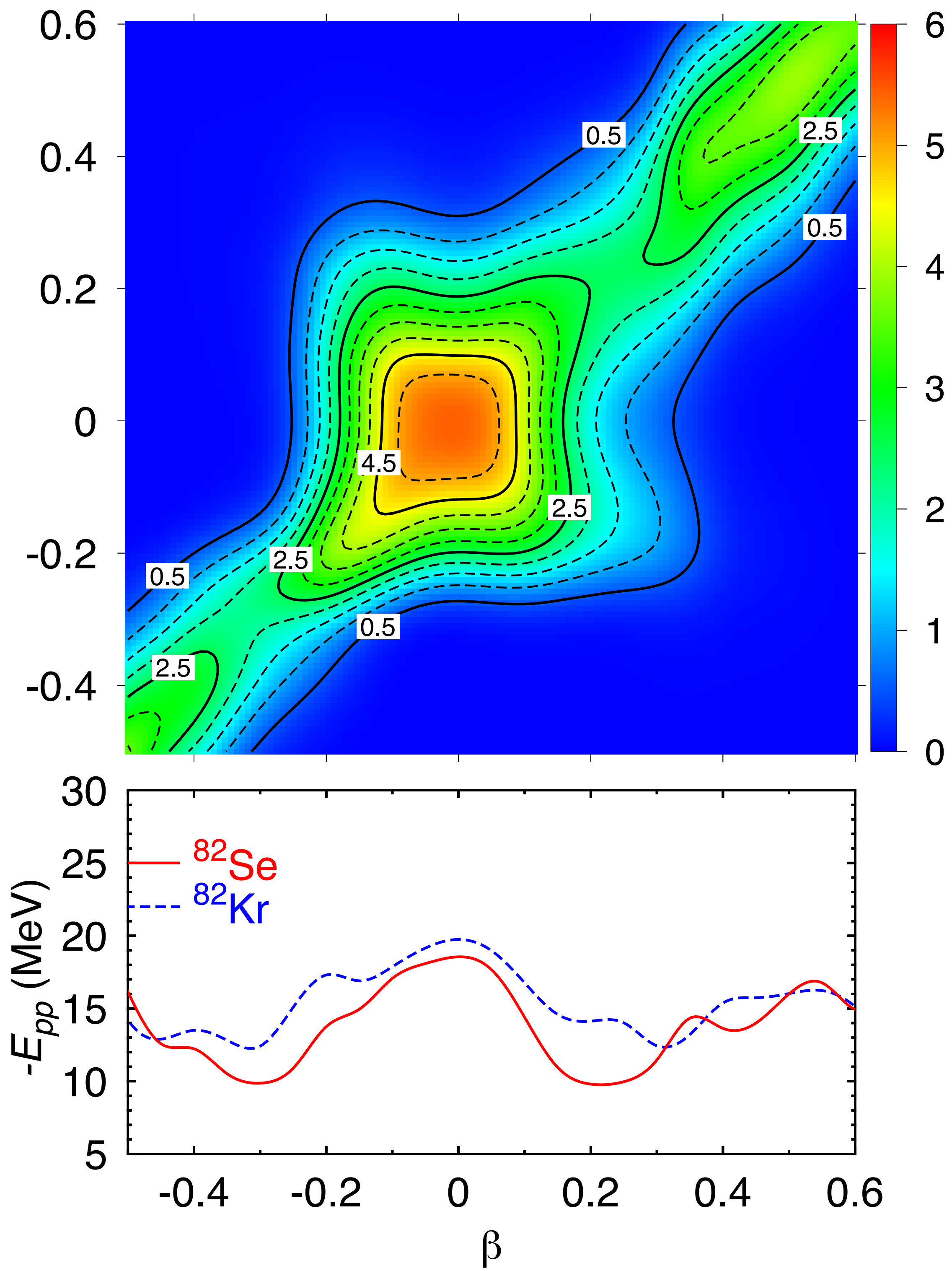

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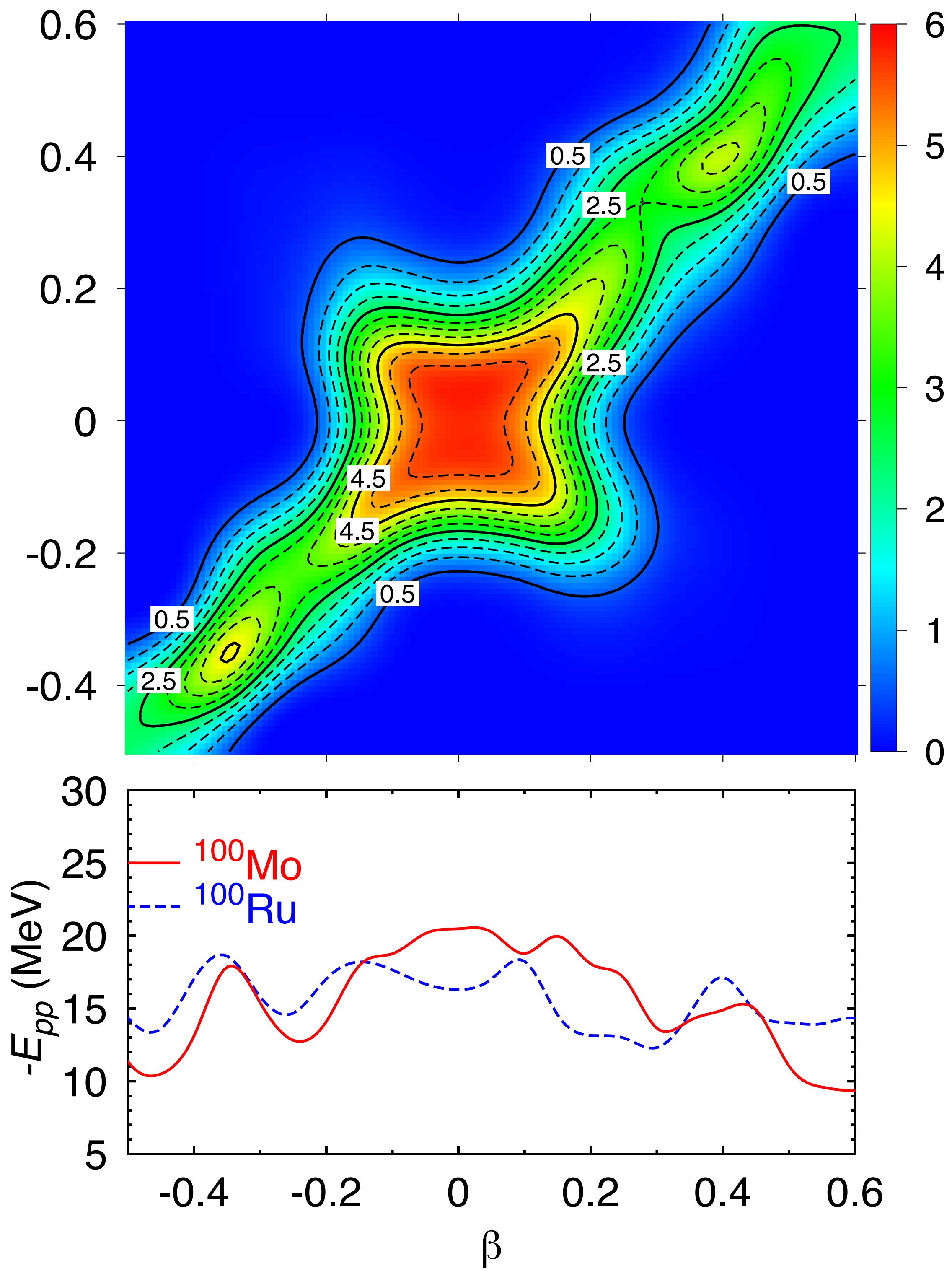

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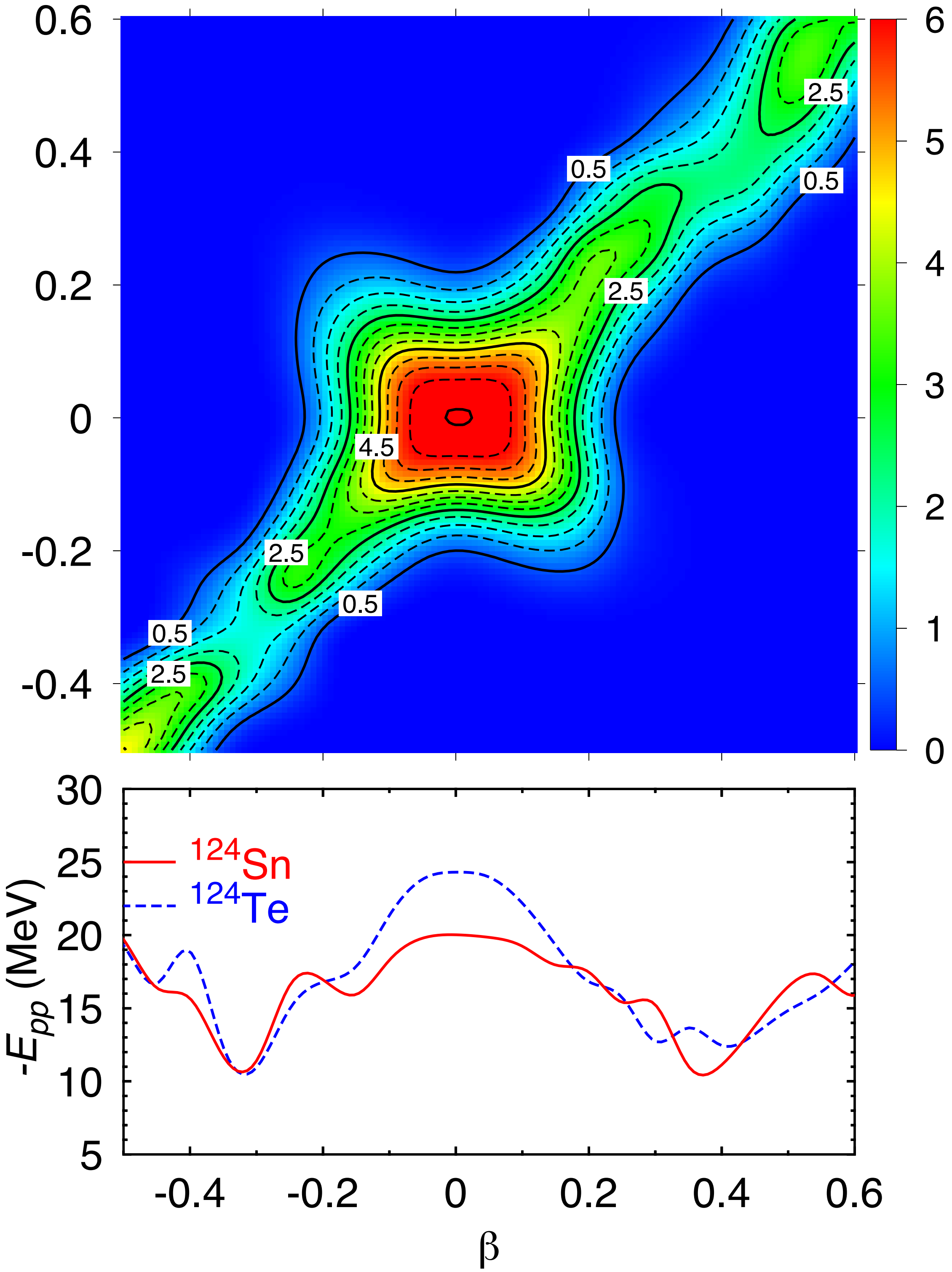

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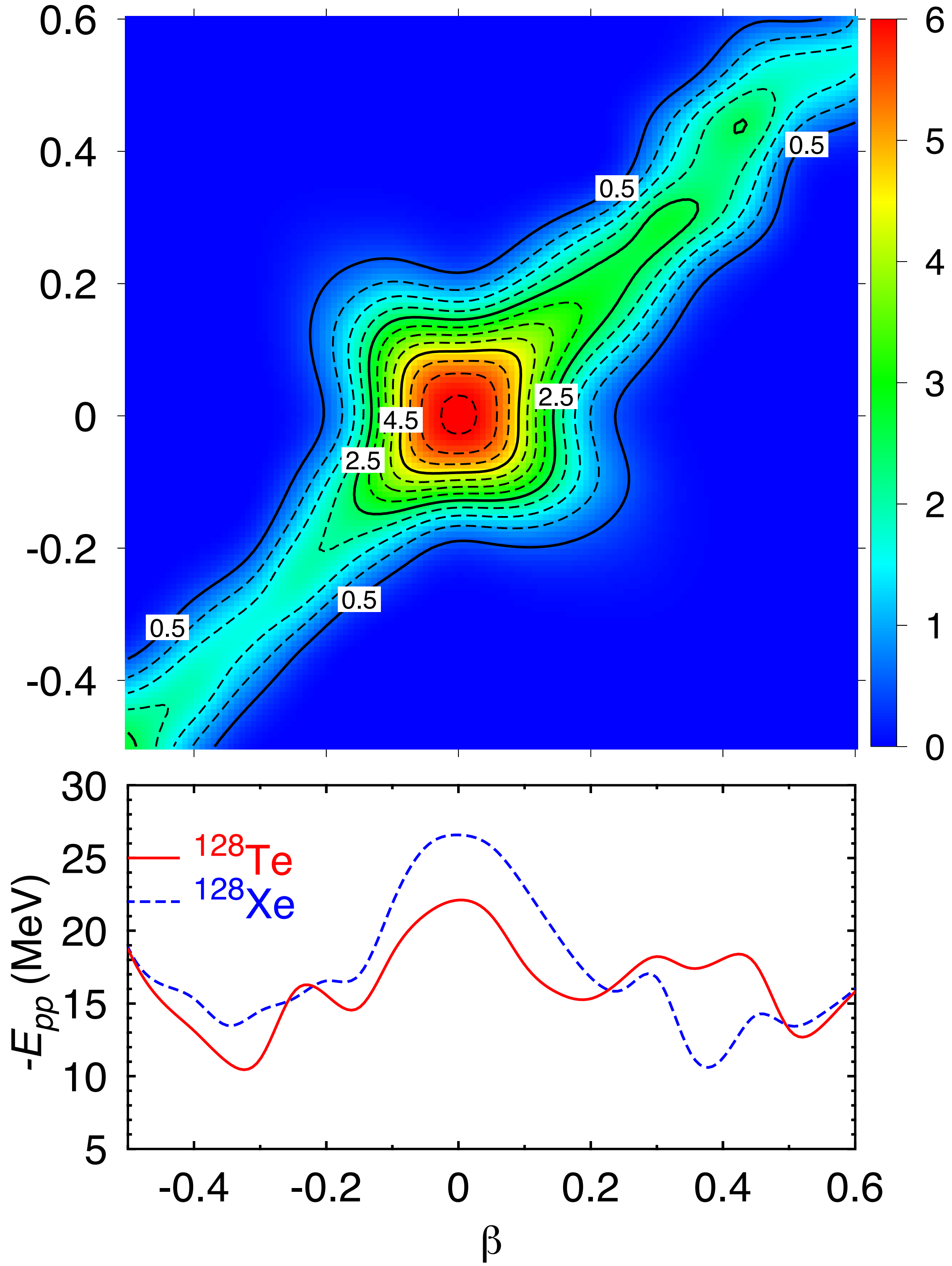

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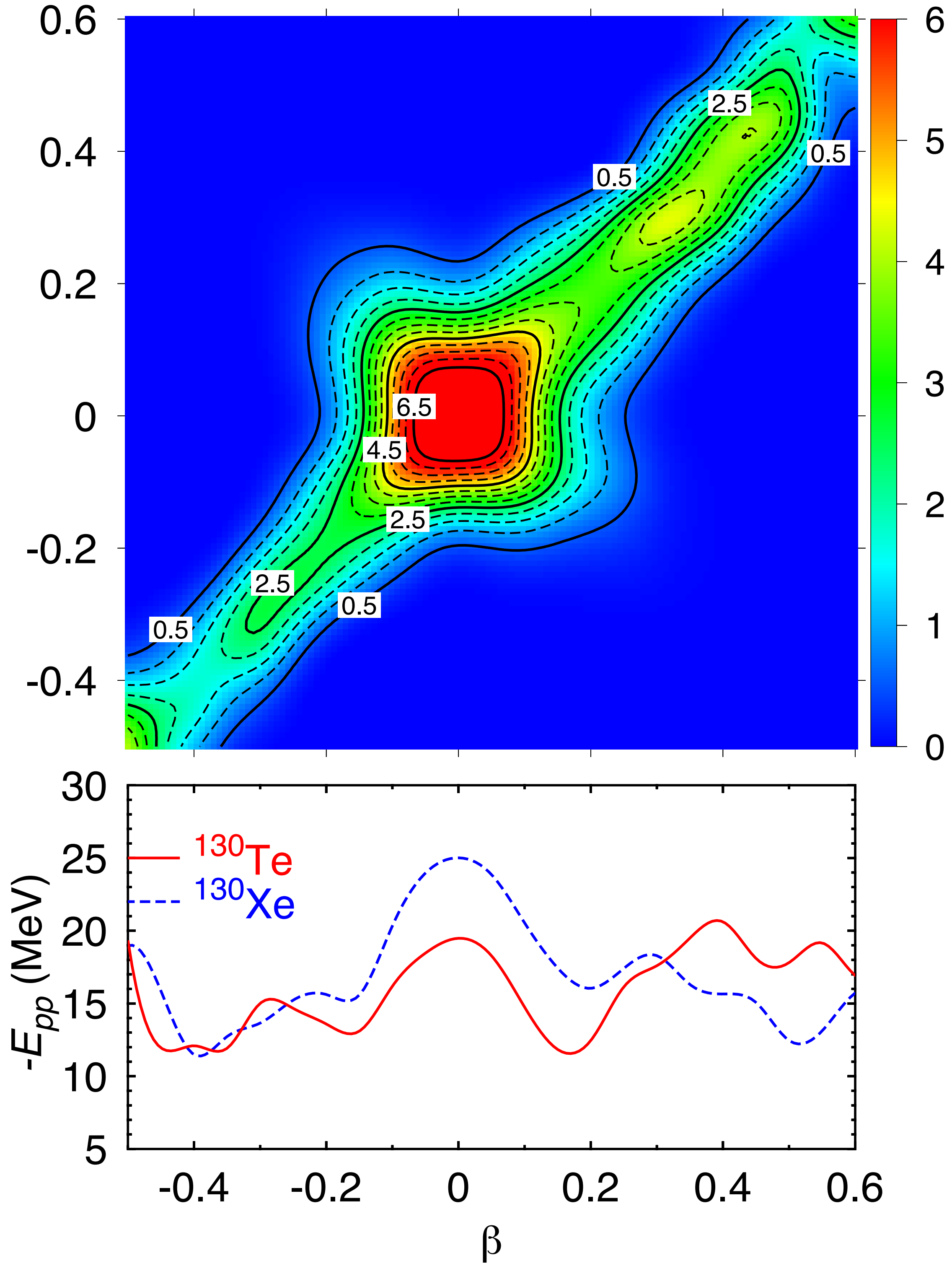

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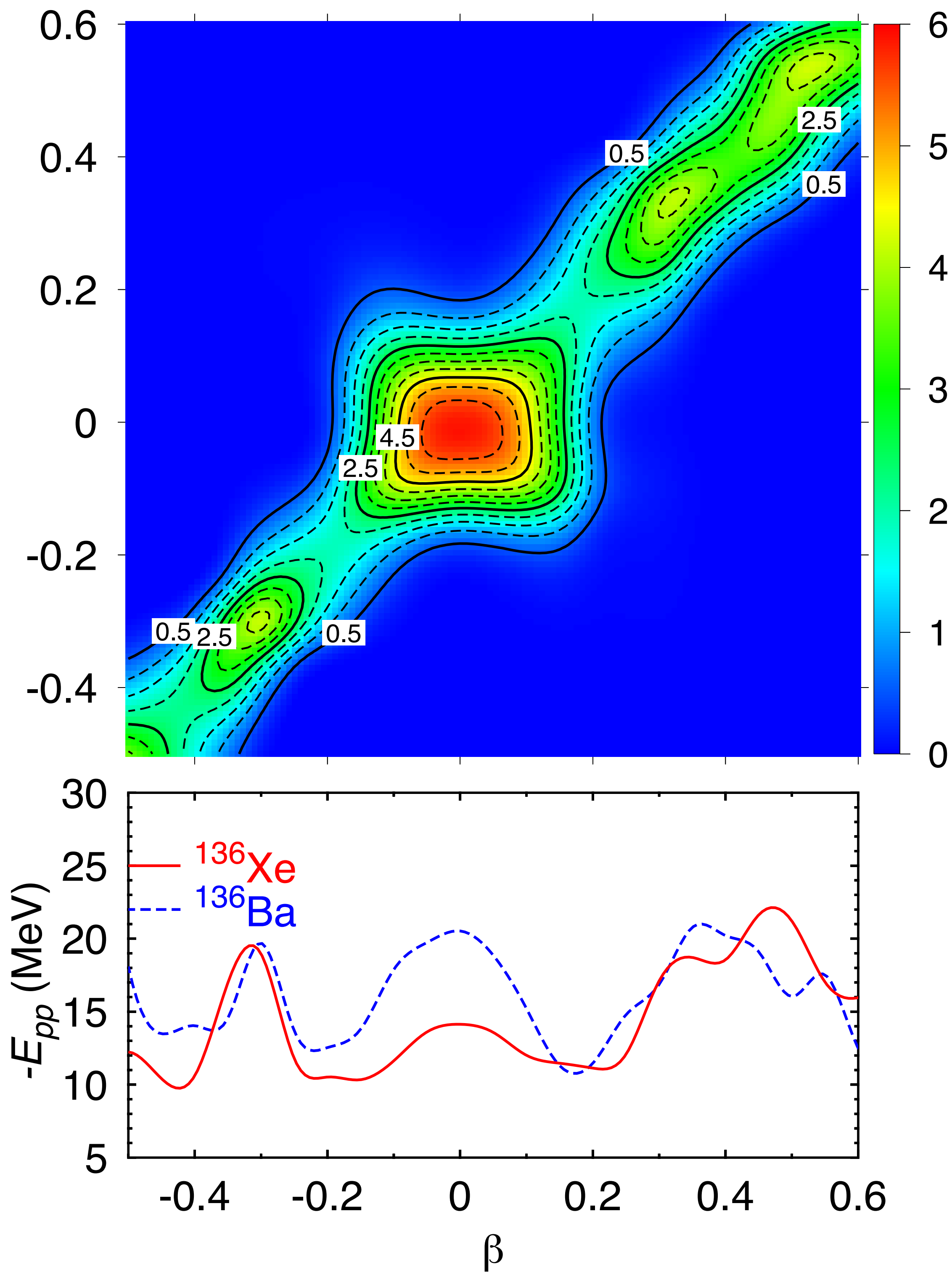

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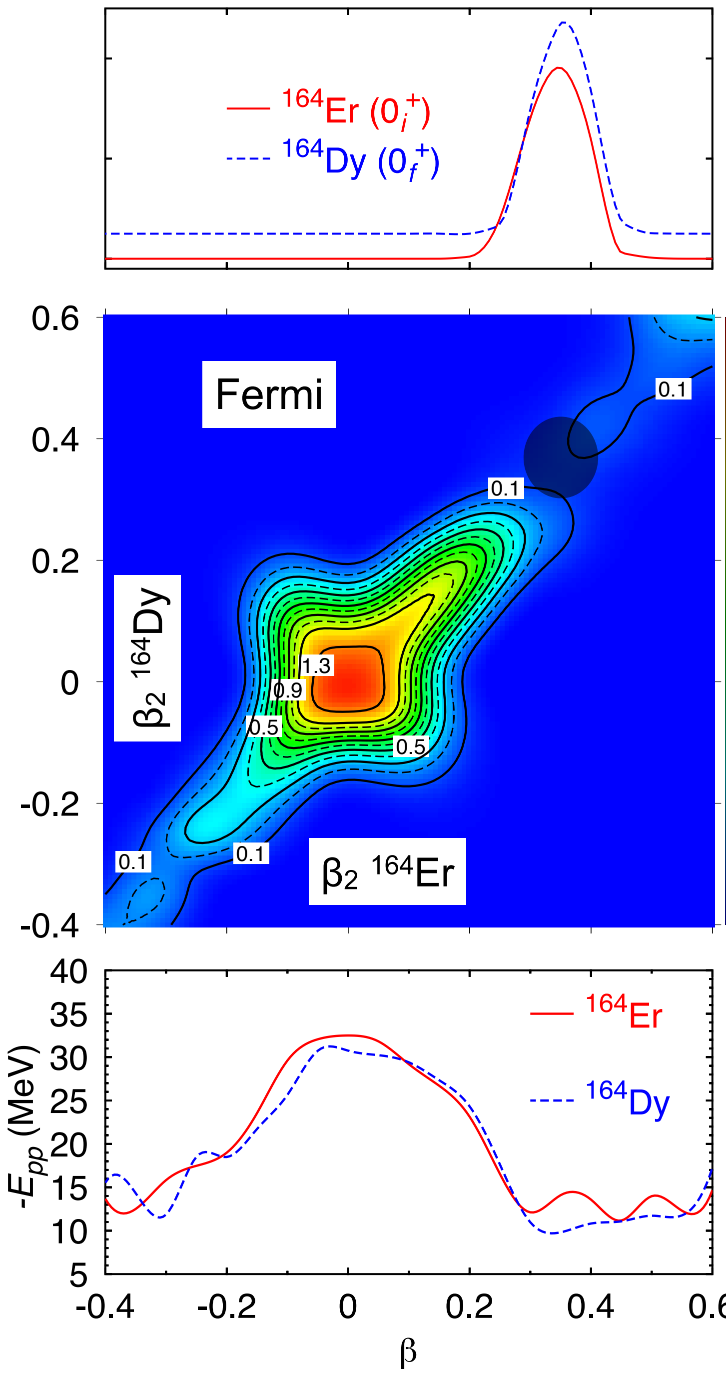
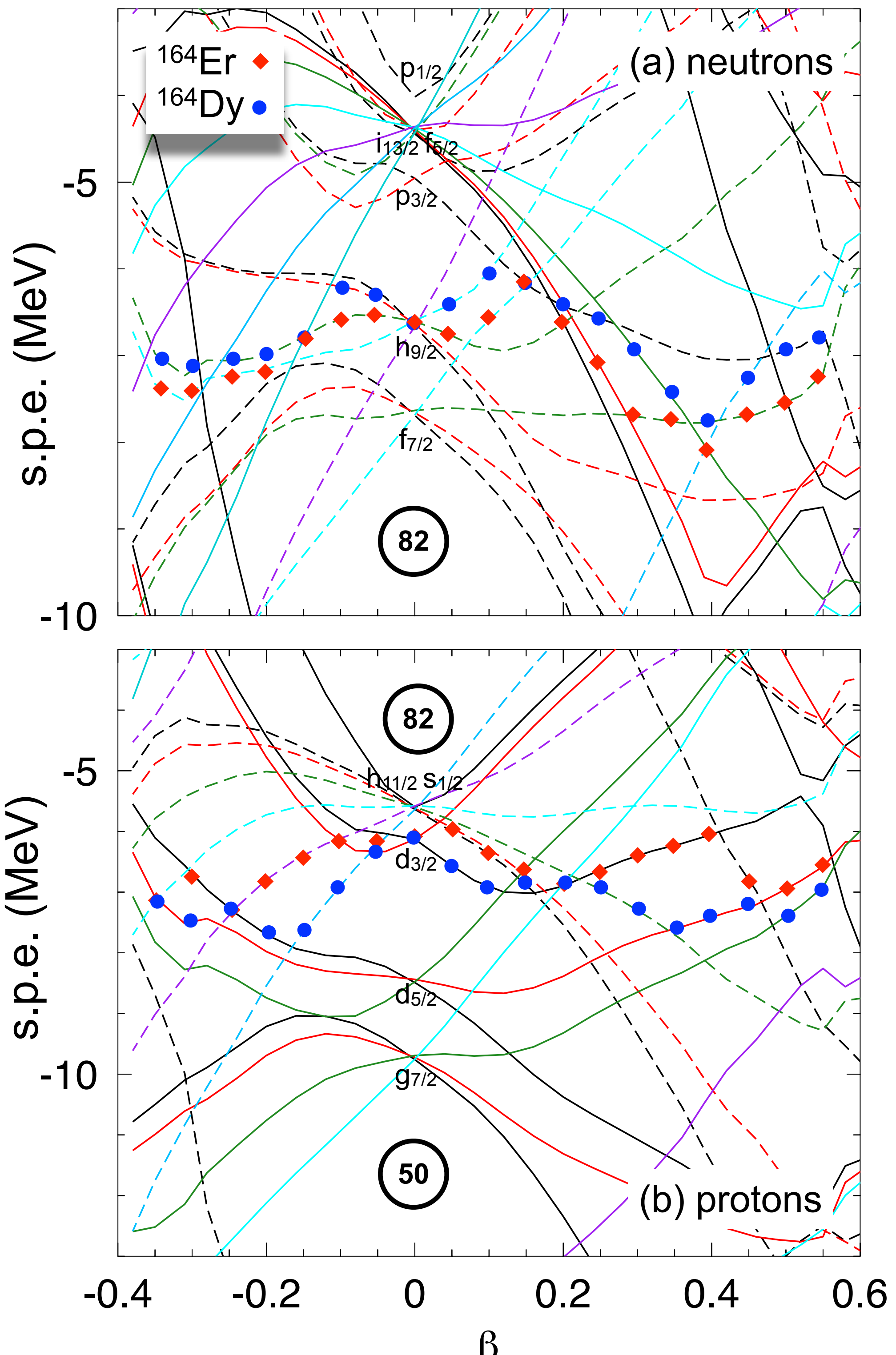
NME: Pairing

T.R.R., G. Martiínez-Pinedo, PRC 2012

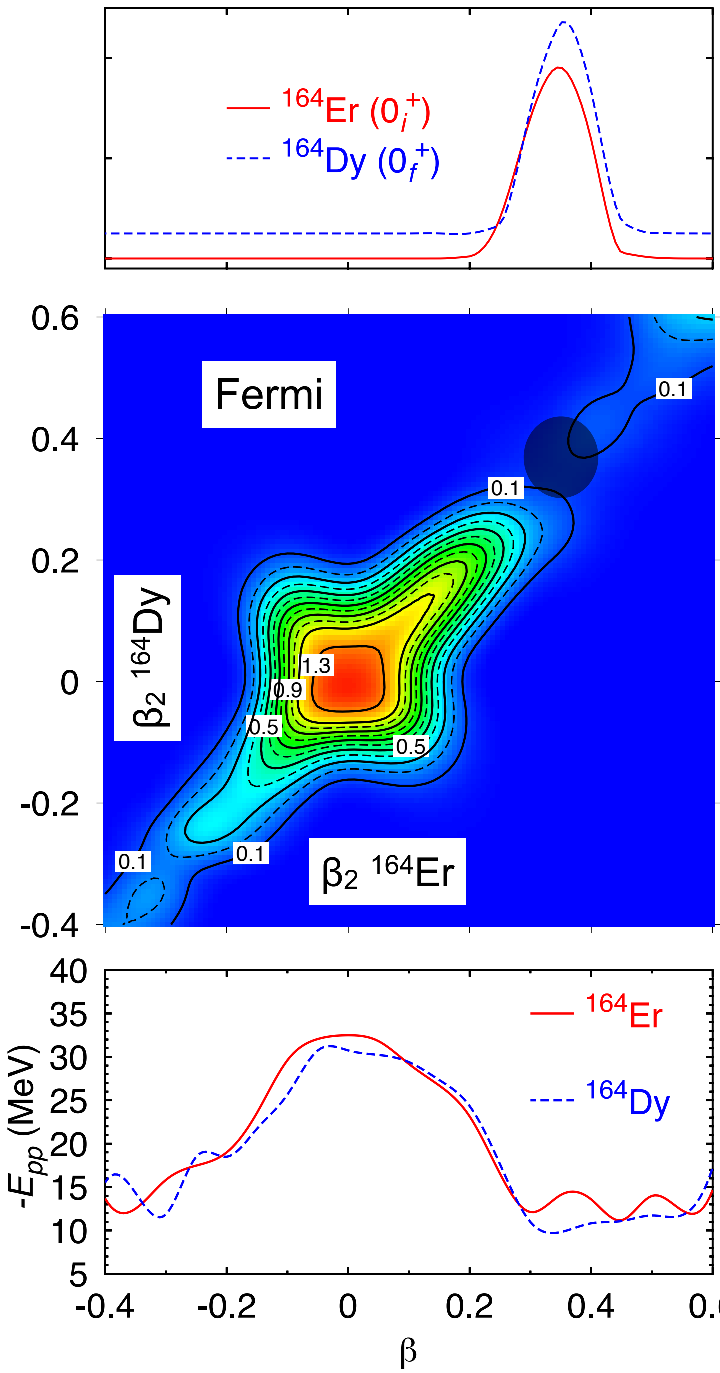


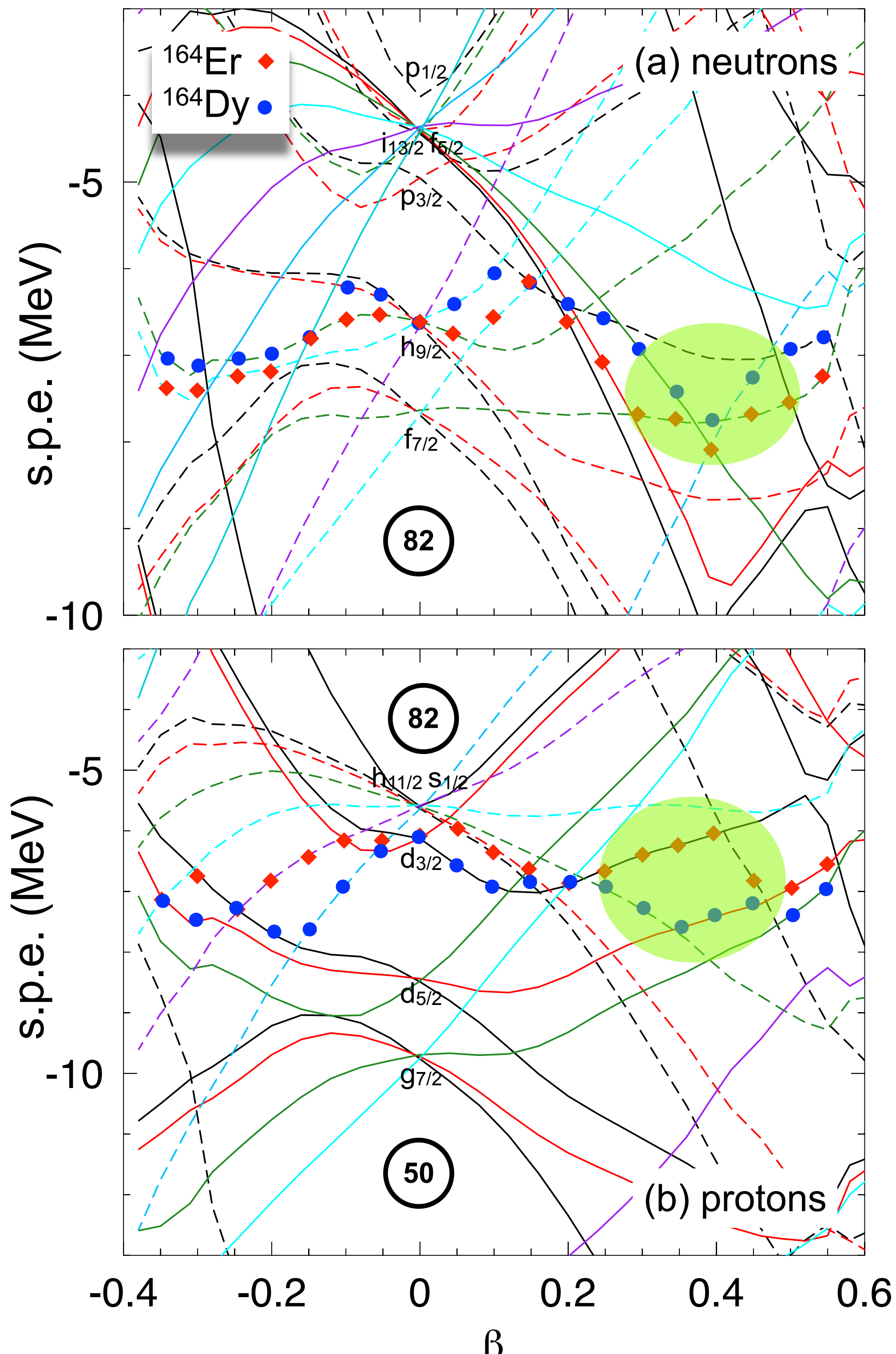


- The minima (maxima) of the NME as a function of deformation are related to areas with low (high) level density around the Fermi level of the initial and final states.

NME: Pairing

T.R.R., G. Martiínez-Pinedo, PRC 2012





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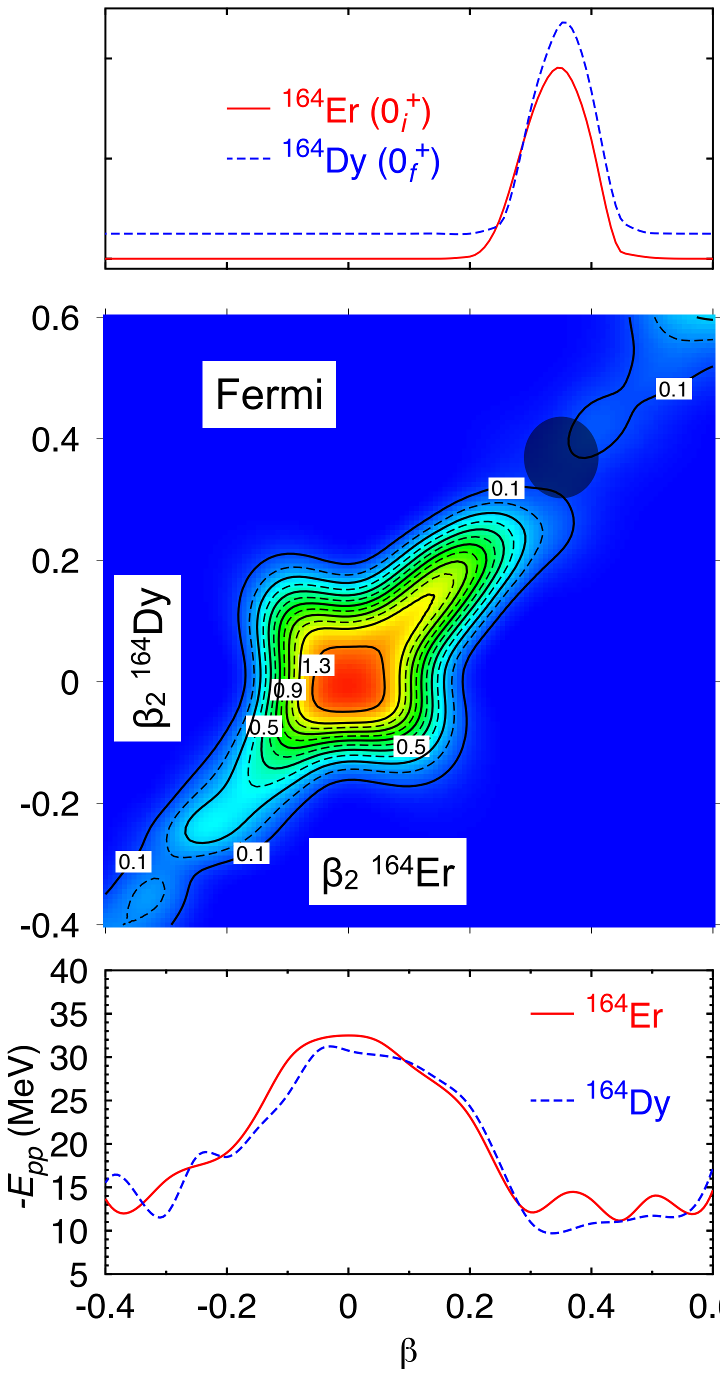
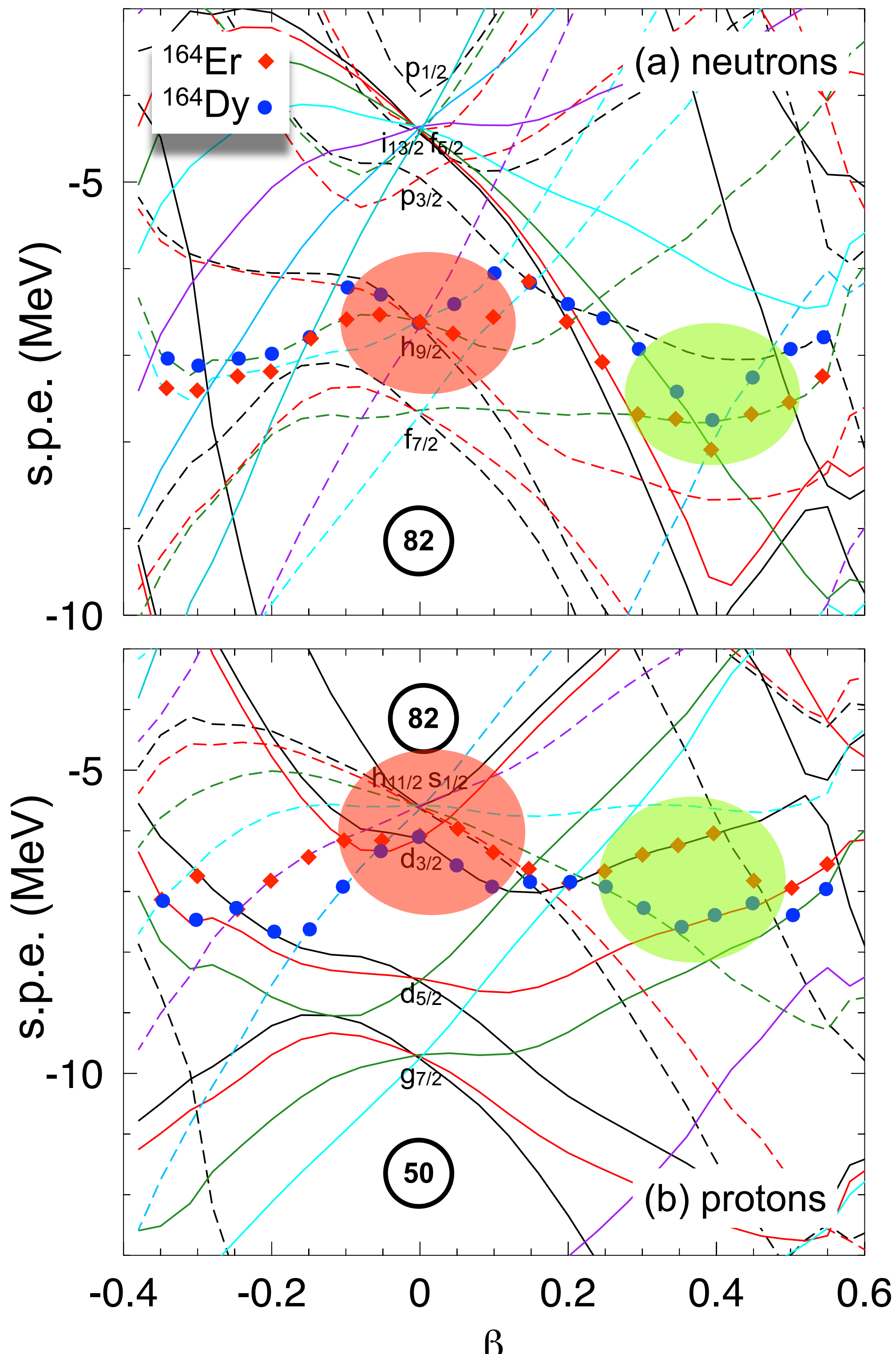




- The minima (maxima) of the NME as a function of deformation are related to areas with low (high) level density around the Fermi level of the initial and final states.

NME: Summary of the results

Gogny D1S parametrization

A	48	76	82	96	100	116	124	128	130	136	150	152	164	180
$M^{0\nu}$	2.37	4.60	4.22	5.65	5.08	4.72	4.81	4.11	5.13	4.20	1.71	1.07	0.64	0.58
$T_{1/2}$ (y)	28.5 $\times10^{23}$	76.9 $\times10^{23}$	20.8 $\times10^{23}$	5.48 $\times10^{23}$	8.64 $\times10^{23}$	9.24 $\times10^{23}$	16.2 $\times10^{23}$	343.1 $\times10^{23}$	8.84 $\times10^{23}$	12.7 $\times10^{23}$	16.5 $\times10^{23}$	4.2 $\times10^{31}$	1.3 $\times10^{36}$	1.6 $\times10^{34}$

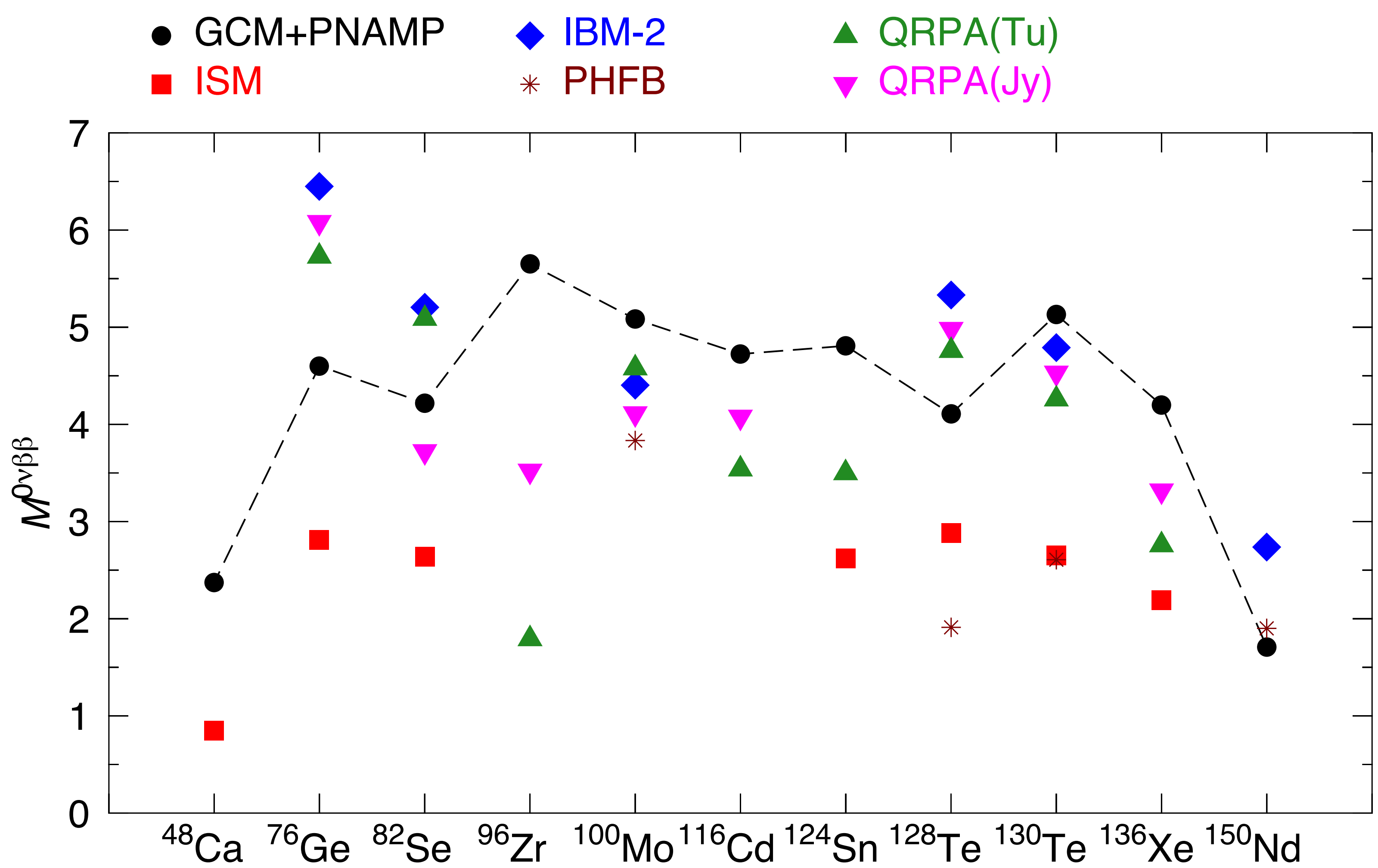
Gogny D1M parametrization

A	48	76	82	96	100	116	124	128	130	136	150	152	164	180
$M^{0\nu}$	2.43	4.64	4.28	5.70	5.19	4.83	4.71	3.98	5.07	4.29	1.36	0.89	0.50	0.38
$T_{1/2}$ (y)	27.1 $\times10^{23}$	75.6 $\times10^{23}$	20.2 $\times10^{23}$	5.38 $\times10^{23}$	8.28 $\times10^{23}$	8.82 $\times10^{23}$	16.9 $\times10^{23}$	365.8 $\times10^{23}$	9.05 $\times10^{23}$	12.2 $\times10^{23}$	26.1 $\times10^{23}$	6.2 $\times10^{31}$	2.1 $\times10^{36}$	3.8 $\times10^{34}$

double beta decay (columns 48–150) double electron capture (columns 152–180)

NME: Summary of the results

T.R.R., Martínez-Pinedo, PRL 2010



QRPA (Jy):)M. Kortelainen, J. Suhonen, PRC 75, 051303(R) (2007) and PRC 76, 024315 (2007)

QRPA(Tu): F. Simkovic et al., PRC 77, 045503 (2008)

ISM: J. Menendez et al., PRL 100, 52503 (2008)

IBM-2:J. Barea, F. Iachello, PRC 77, 045503 (2008)

PHFB: K. Chaturvedi et al. PRC 78, 054302 (2008)

- Higher values than the ones predicted by ISM calculations (larger valence space, lower seniority components).
- For *A*=76, 82, 128, 150 we predict smaller values than the ones given by QRPA and/or IBM while for *A*=96, 100, 116, 124, 130, 136 larger values are obtained.
- Consistent results with the rest of the models. Notice that we are using the same interaction for all the nuclei.
- Further studies are needed to understand what is missing in the different models.

NME: Summary of the results

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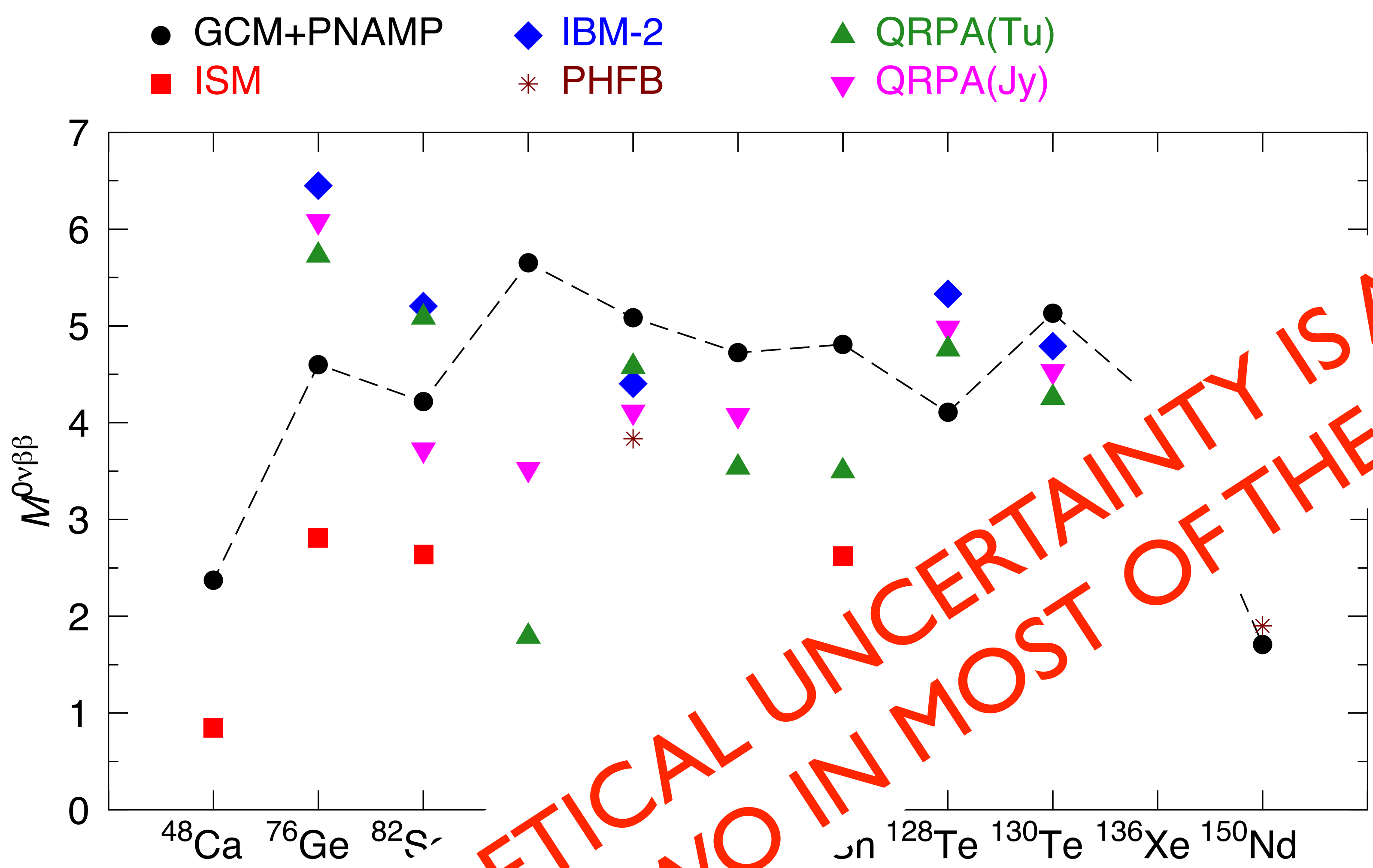
T.R.R., Martínez-Pinedo, PRL 2010



THEORETICAL UNCERTAINTY IS A FACTOR TWO IN MOST OF THE CASES

QRPA (Jy): M. Ko... en, J. Suhonen, ... 51303(R) ... PRC 76, ...

... Simkovic ... PRC 77, 045503 ... 08)

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- Further studies are needed to understand what is missing in the different models.

Summary and conclusions

• Energy density functional methods allow the inclusion of deformation of the initial and final states in the calculation of NME for neutrinoless double beta decay and double electron capture.

• Equal deformation of initial and final states is favored by the transition operator. Spherical configurations are more favored than deformed ones.

• NME values are strongly correlated to the pairing energies of initial and final states.

• NMEs are much larger for the double beta decay candidates than for double electron capture ones (deformation).

Outlook

- Calculations in the pf-shell.

- Other degrees of freedom should be also explored (pairing vibrations, octupole deformations, triaxiality, explicit quasiparticle excitations...)

- Isospin symmetry breaking and restoration.

- Occupation numbers.

Acknowledgments

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A. Poves (UAM-Madrid)
C. Smorra (Universität Heidelberg)