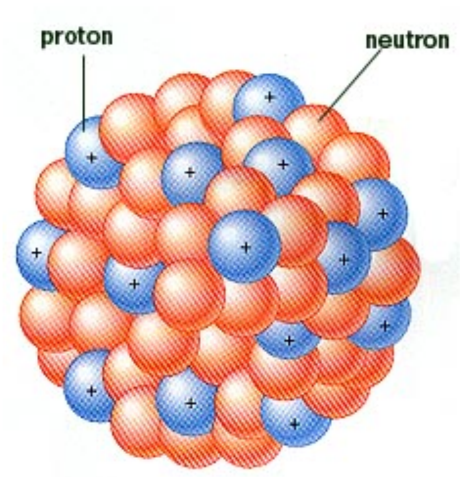
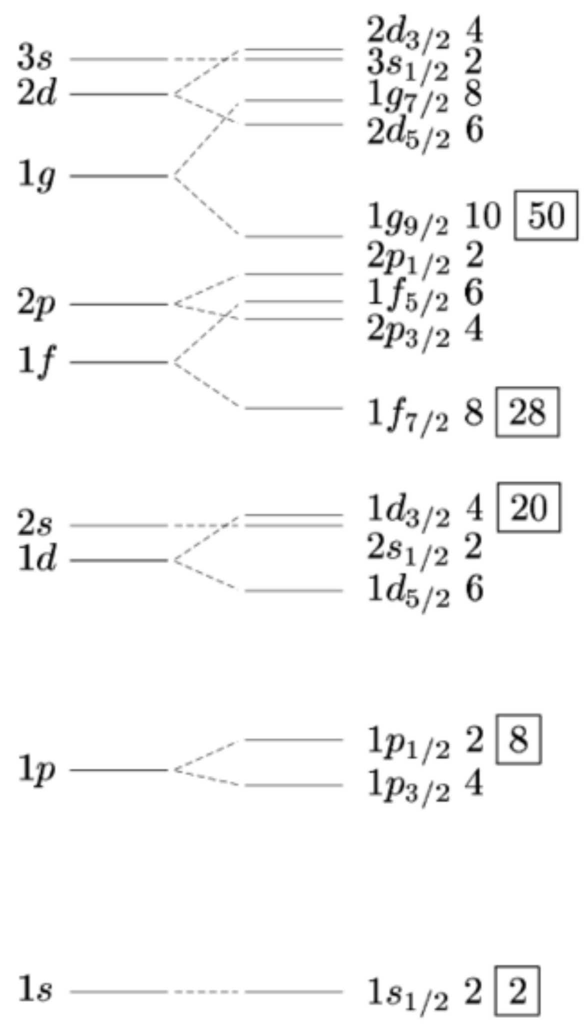
V. Velázquez, FC-UNAM

COMPLEXITY, SCALE INVARIANCE AND SENSITIVITY IN THE NUCLEAR SHELL MODEL.

- 1. Motivation
- 2. Complexity
- 3. Scale invariance
- 4. Sensitivity

Atomic nucleus is in principle a complex system.





- There is an inherent interest in the mechanism that describes the transition from quantum mechanics to the classical mechanics.

- What properties loss the system in this transition?

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NUCLEAR
PHYSICS A

Sensitivity and chaos in quantum systems ★

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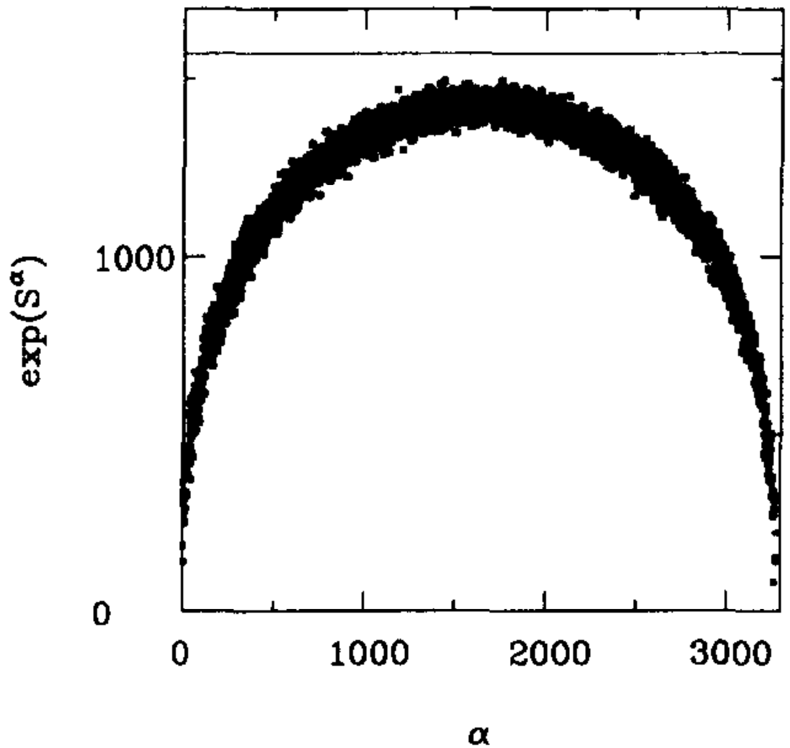
Towards the end of the seventies, Casati and collaborators focussed their attention on the investigation of the quantum mechanics of a classically chaotic system [9]. They chose the kicked quantum pendulum as their system of interest since its classical version is chaotic and its quantum propagator can be computed analytically over one cycle of the driving force. On the basis of the occurrence of classical chaos in this system, complicated quantum behavior was expected, if not the analog of classical deterministic chaos with exponential sensitivity in the wave functions. But instead of instabilities and chaos, the kicked pendulum showed orderly behavior and no sensitivity of the wavefunctions. Even more puzzling: the energy of the quantum kicked pendulum turned out to be bounded whereas the energy of its classical counterpart exhibits a diffusive growth due to the underlying classical chaoticity of the motion. This landmark result meant that chaos in the wavefunctions of a quantum system tends to be suppressed by quantum interference effects. Similar observations were reported for other quantum systems [8,10,11].

The degree of complexity of each state |α> can be measure of several ways. For example the information entropy for 12 particles in sd shell $J^pT=2^+\,0$



Information entropy, chaos and complexity of the shell model eigenvector, Vladimir Zelevinsky, Mihai Horoi, B. Alex Brown, PLB 350, (1995), 141-146.

Where the state and the entropy are:

$$|\alpha\rangle = \sum_k c_k^\alpha |k\rangle$$

$$S^\alpha = -\sum_k |c_k^\alpha|^2 \ln |c_k^\alpha|^2$$

The principal property of a quantum state, is its linearity thats originates the spectral descomposition

$$\left|\Psi\right\rangle = \sum_n c_n \left|u_n\right\rangle$$

If we want to know the *n*th energy of such state, the result is dependent of the mixing degree of the wave function. If we have pure states:

$$H_n\left|\Psi\right\rangle = E_n \sum_n c_n \left|u_n\right\rangle$$

$$= H_n\left|u_n\right\rangle = E_n\left|u_n\right\rangle$$

… but, if the wave function is a mixed state on the basis,

$$H_n \left| \Psi \right\rangle = \bar{E}_n \left| \Psi \right\rangle$$

where, the energy is now an average of the energies of a subset of the basis. The deviation of the central value comes from the mixing procecess in the formation of such state. How big is this subset, give us an idea of how big is the mixing status. This mixing comes from the nature of the interactions between themselves.

The most used classical limits are:

Thermal	➔	Bunched	➔	Bose-Einstein	$1/f^2$
-Poisson	➔	Random	➔	Poisson	$1/f^0$

In order to study the transition from quantum to chaos, we select the ^{48}Ca nucleus, that has been very investigated in quantum chaos*
We have 1627 $|3_i^+\rangle$ states; i=1,1627.
Antoine* solve the Scrhödinger equation for 8 particles over a core of ^{40}Ca, using the interaction

$$\hat{H} = \hat{H}_m + \hat{H}_M$$

In this work we use the following two body interactions.

$$\hat{H} = \hat{H}_m + \hat{H}_{KB3}$$

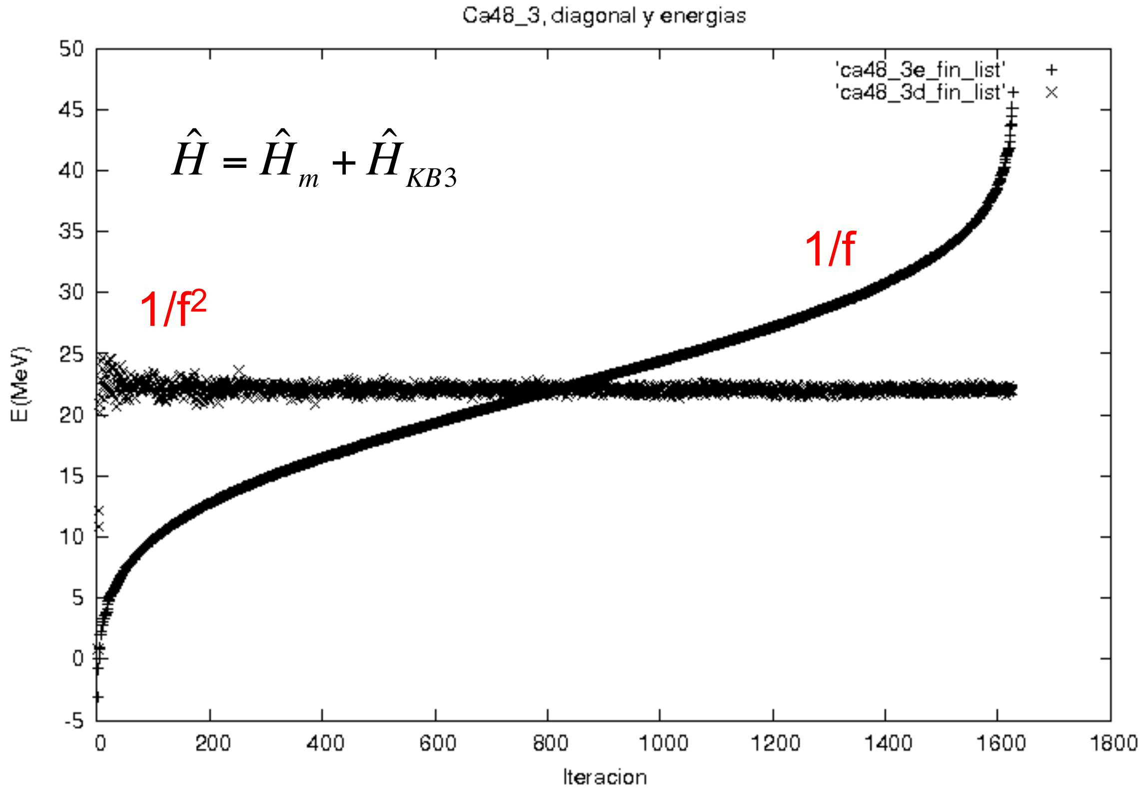
$$\hat{H} = \hat{H}_m - \chi \hat{Q} \bullet \hat{Q}$$

$$\hat{H} = \hat{H}_m - \chi \hat{Q} \bullet \hat{Q} - g \hat{P} \bullet \hat{P}$$

$$\hat{H} = \hat{H}_m + \hat{H}_{TBRE}$$

Quantum chaos in A=45-50 atomic nuclei, E. Caurier, J.M.G. Gómez, V.R. Manfredi L. Salasnich, Physics Letters B 365 (1996) 7-11.
* Antoine code. E. Caurier Strasbourg, 1989.

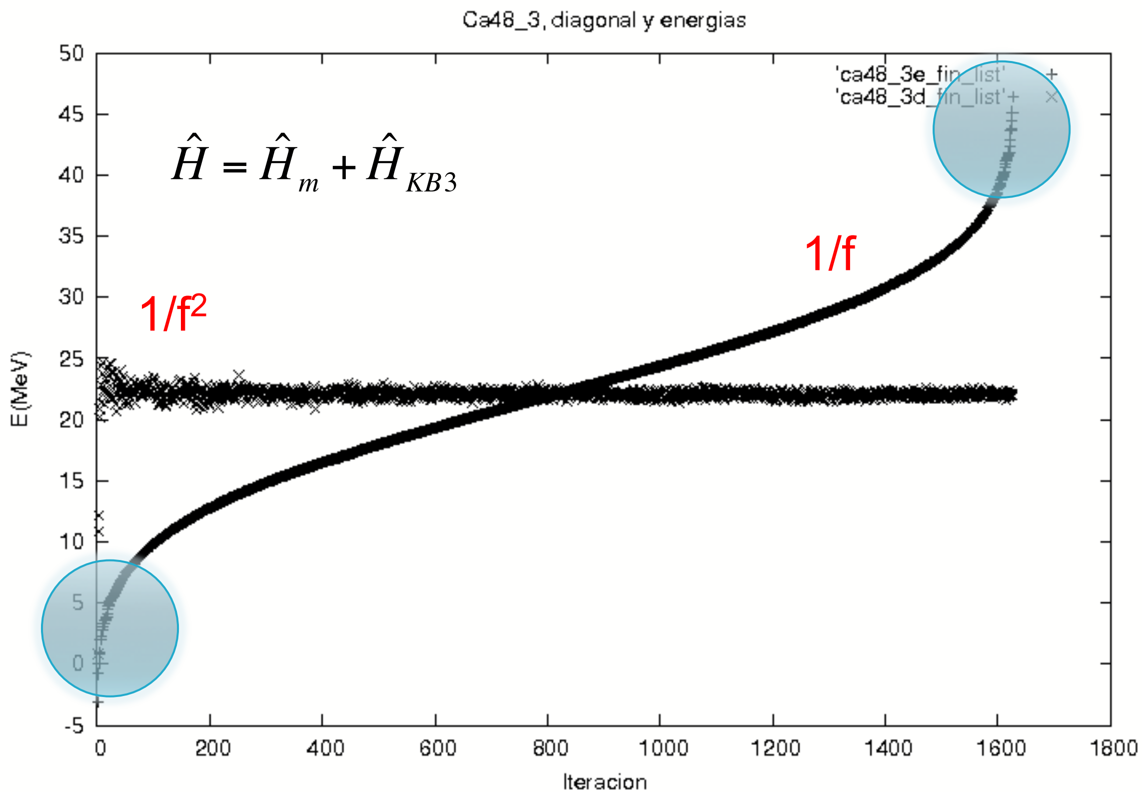
In each energy sequence we use the power spectrum tool*



Relaño A, Gómez J M G, Molina R A, Retamosa J and Faleiro E 2002 Phys. Rev. Lett. 89 244102

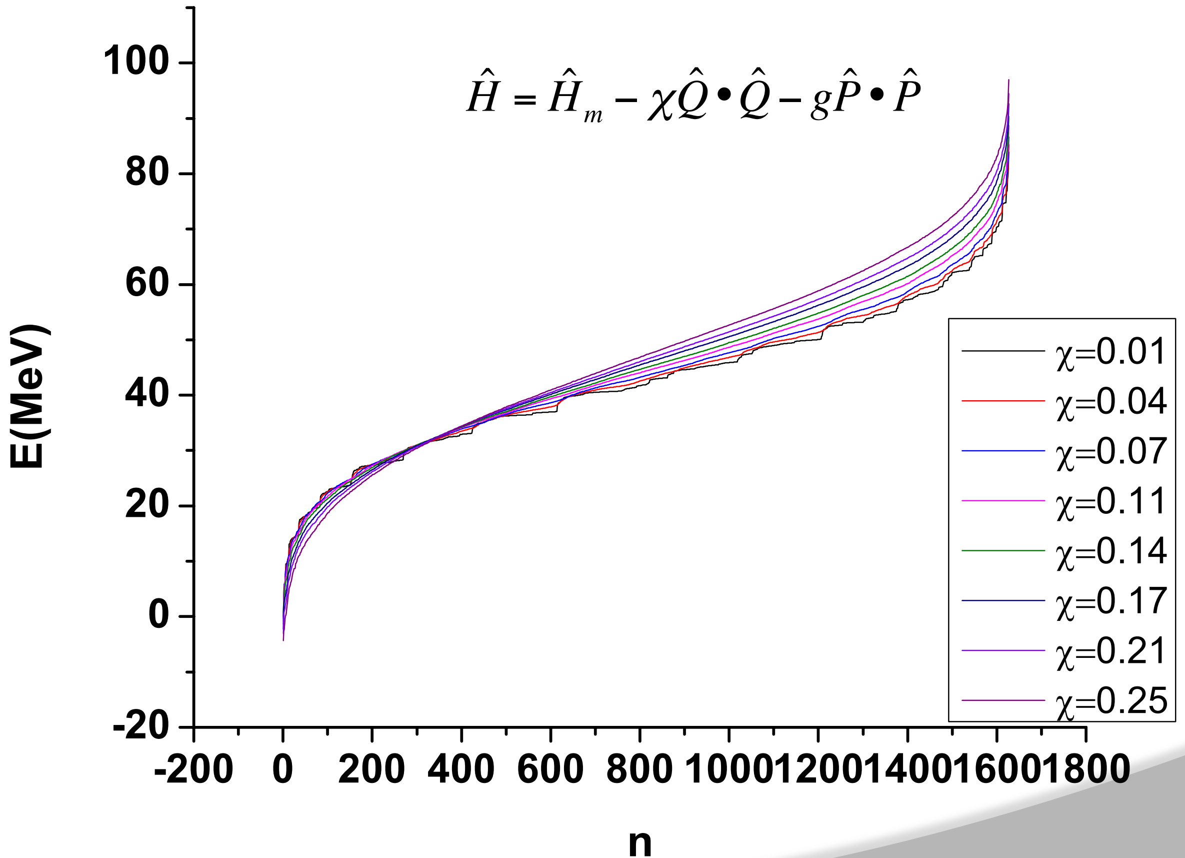
Faleiro E, Gómez J M G, Molina R A, Munñoz L, Relaño A and Retamosa J 2004 Phys. Rev. Lett. 93 244101.

The most of nuclear energies have 1/f noise. The diagonal matrix elements follows a $1/f^2$ noise.



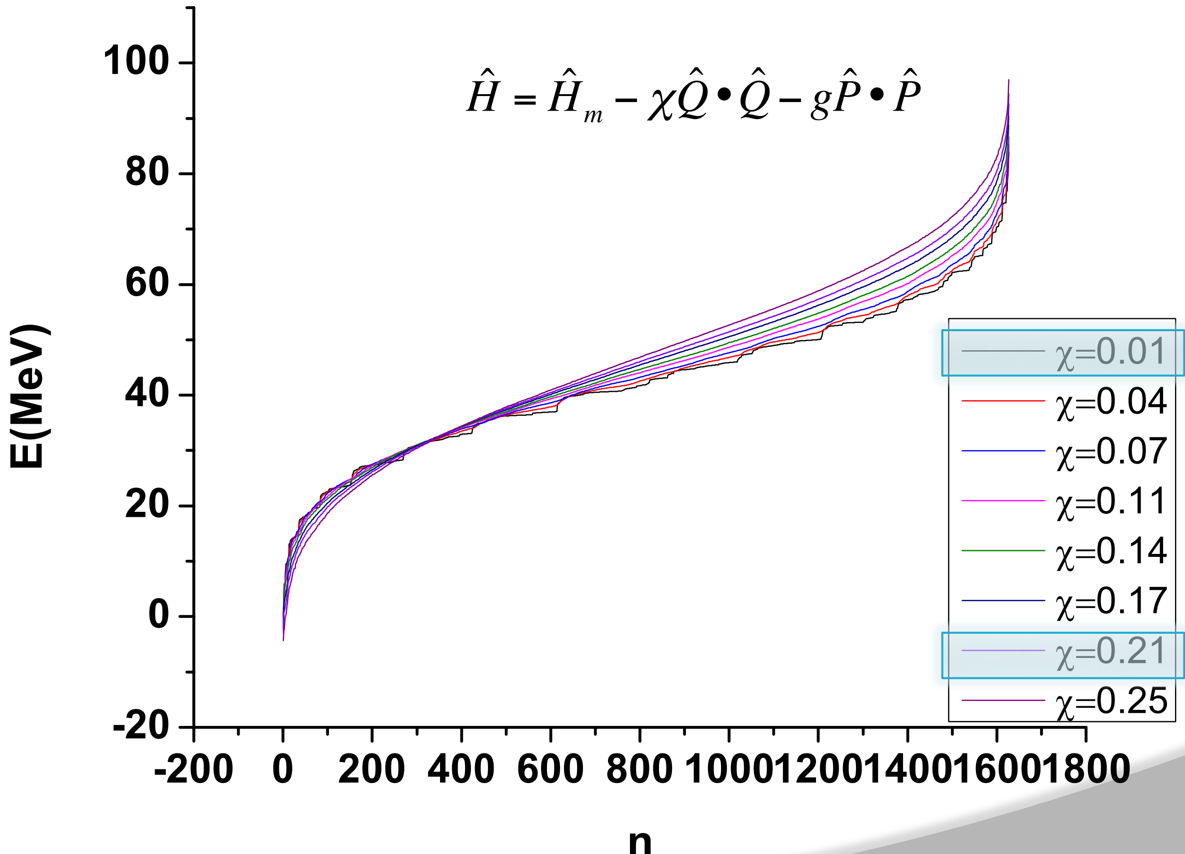
n

For the goal of obtain information about the transition we change the KB3 interaction for one schematic: g=0.46



Complexity

For the goal of obtain information about the transition we change the KB3 Interaction for one schematic: g=0.46

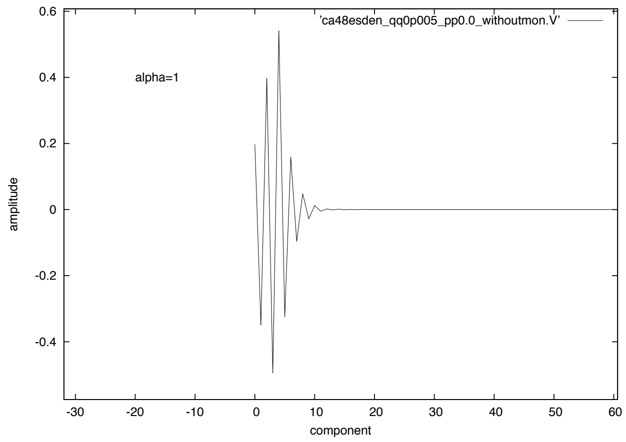


Comparing the wavefunction of leves at the extremes with levels in the center of the sequence (in the Lanczos basis), for the the case most similar to the diagonal matrix elements:

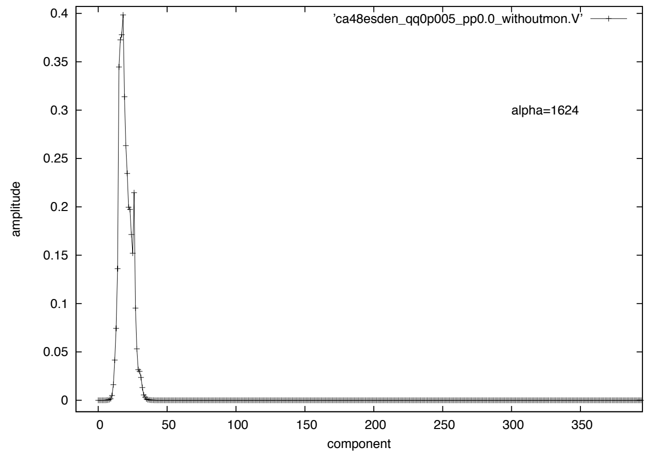
$$\hat{H} = \hat{H}_m - \chi \hat{Q} \bullet \hat{Q} - g \hat{P} \bullet \hat{P}$$

$$with \chi = 0.005, g = 0.0$$

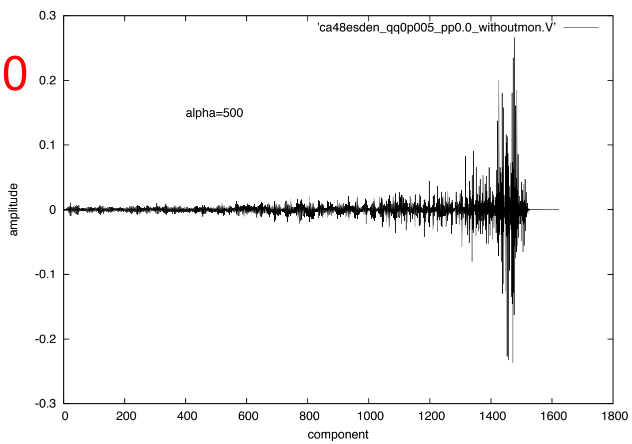
1



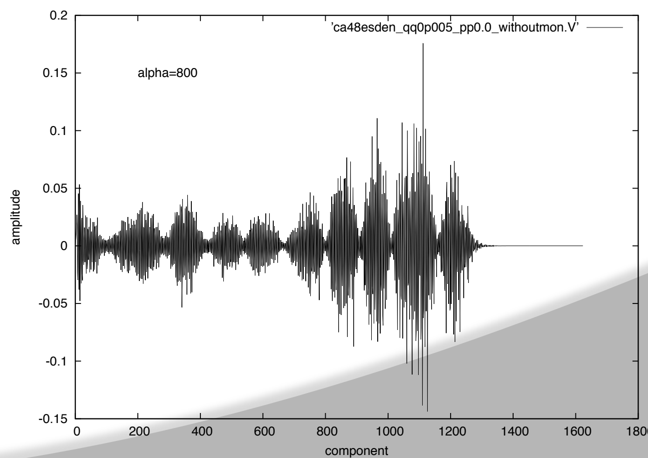
1627



500



800

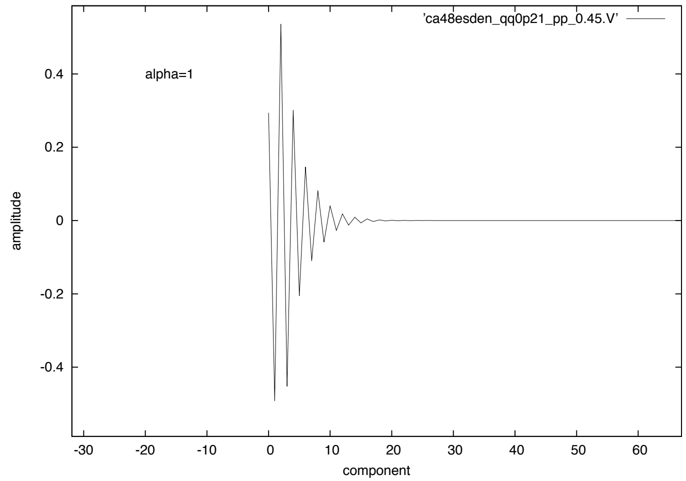


Comparing the wavefunction of energy leves at the extremes with those in the center of the sequence (in the Lanczos basis), for the the case :

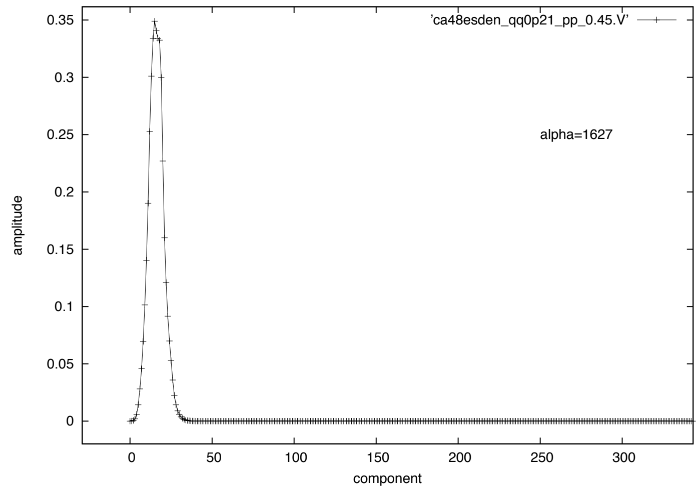
$$\hat{H} = \hat{H}_m - \chi \hat{Q} \bullet \hat{Q} - g \hat{P} \bullet \hat{P}$$

$$with\, \chi = 0.21, g = 0.46$$

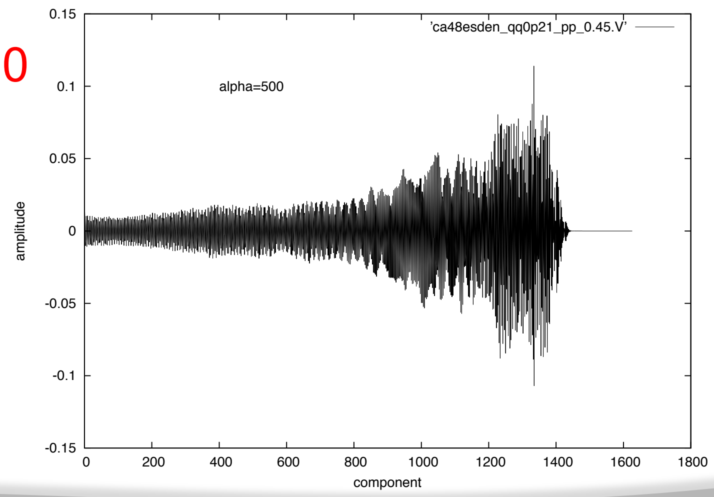
1



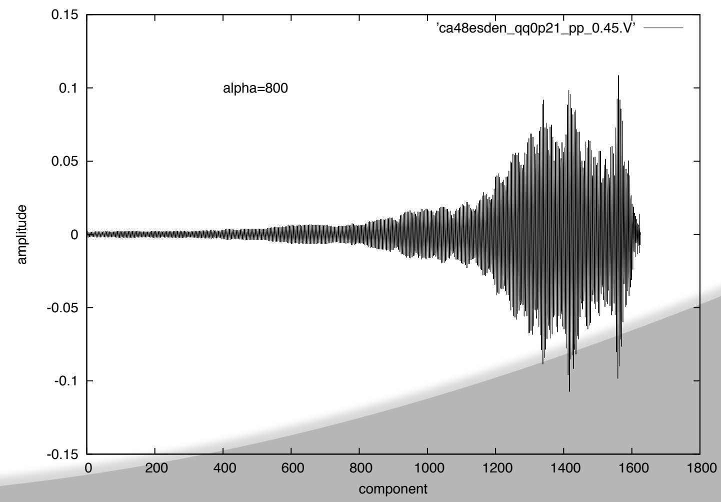
1627



500



800



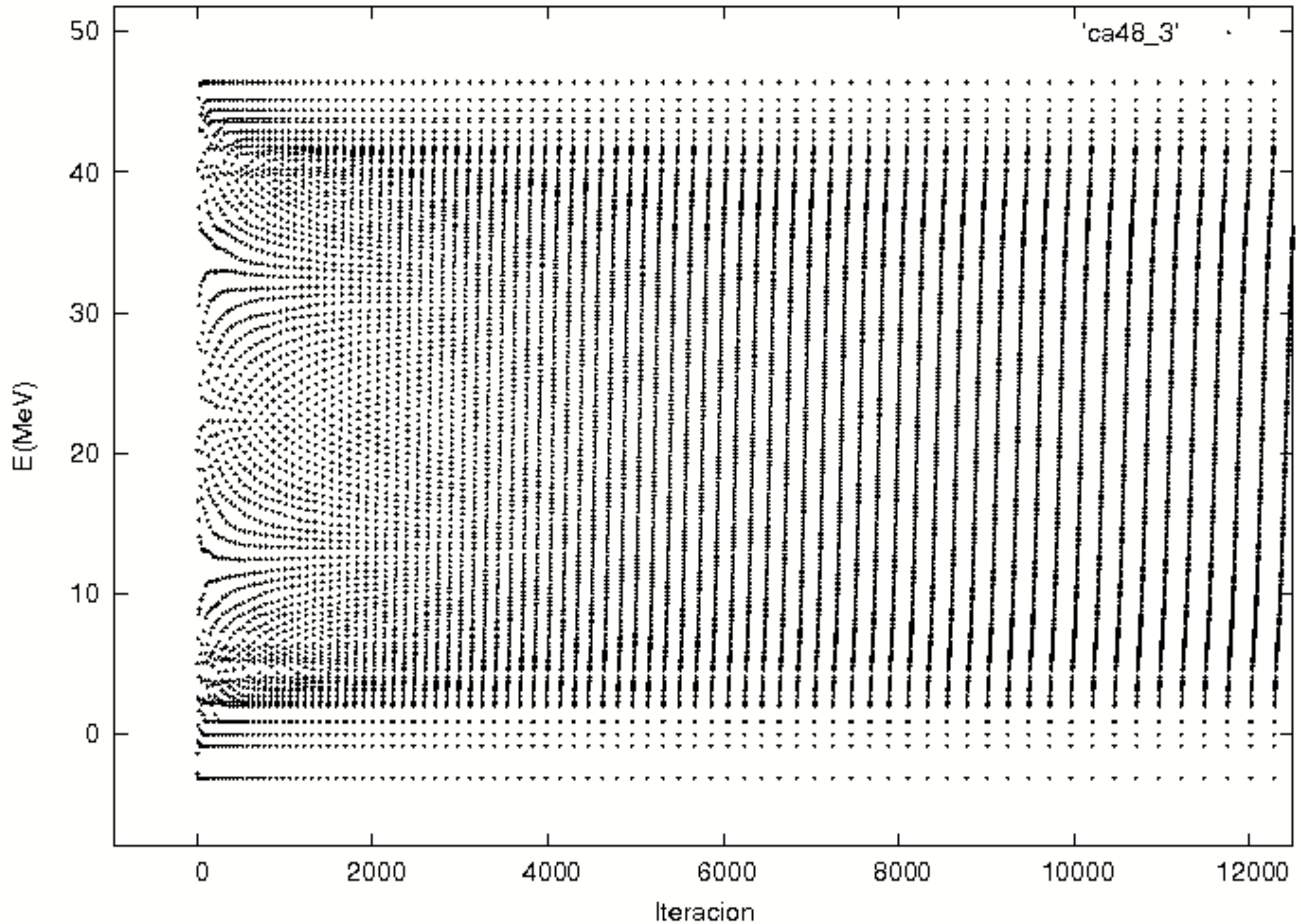
The quantity of mixed states are bigger in the case with stronger quadrupole interaction in comparison with the weak one. However the extreme energy levels has changed slowly. We could say then, in the level sequency, we have two statistical behaviors:

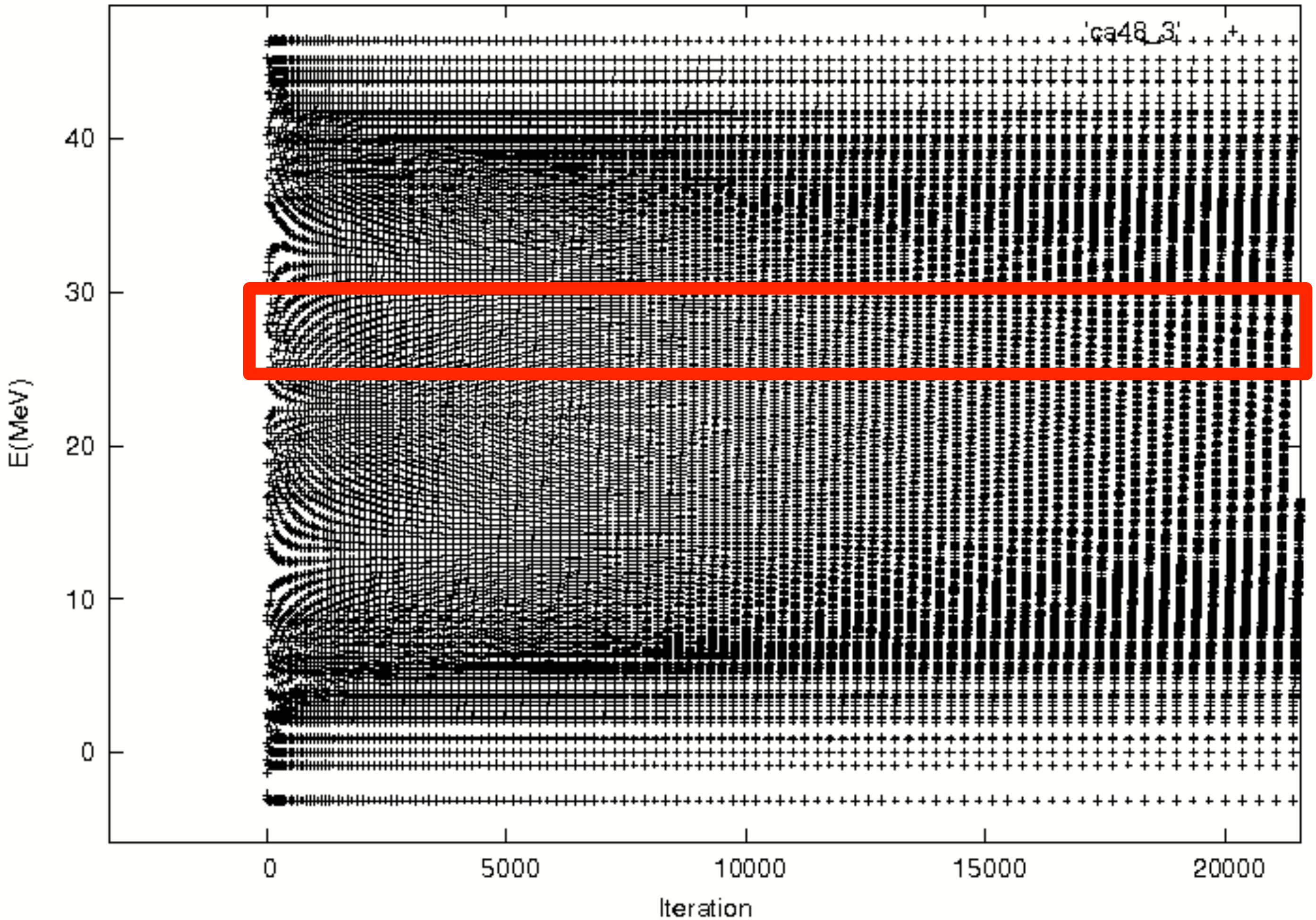
Complexity: high statistical mix

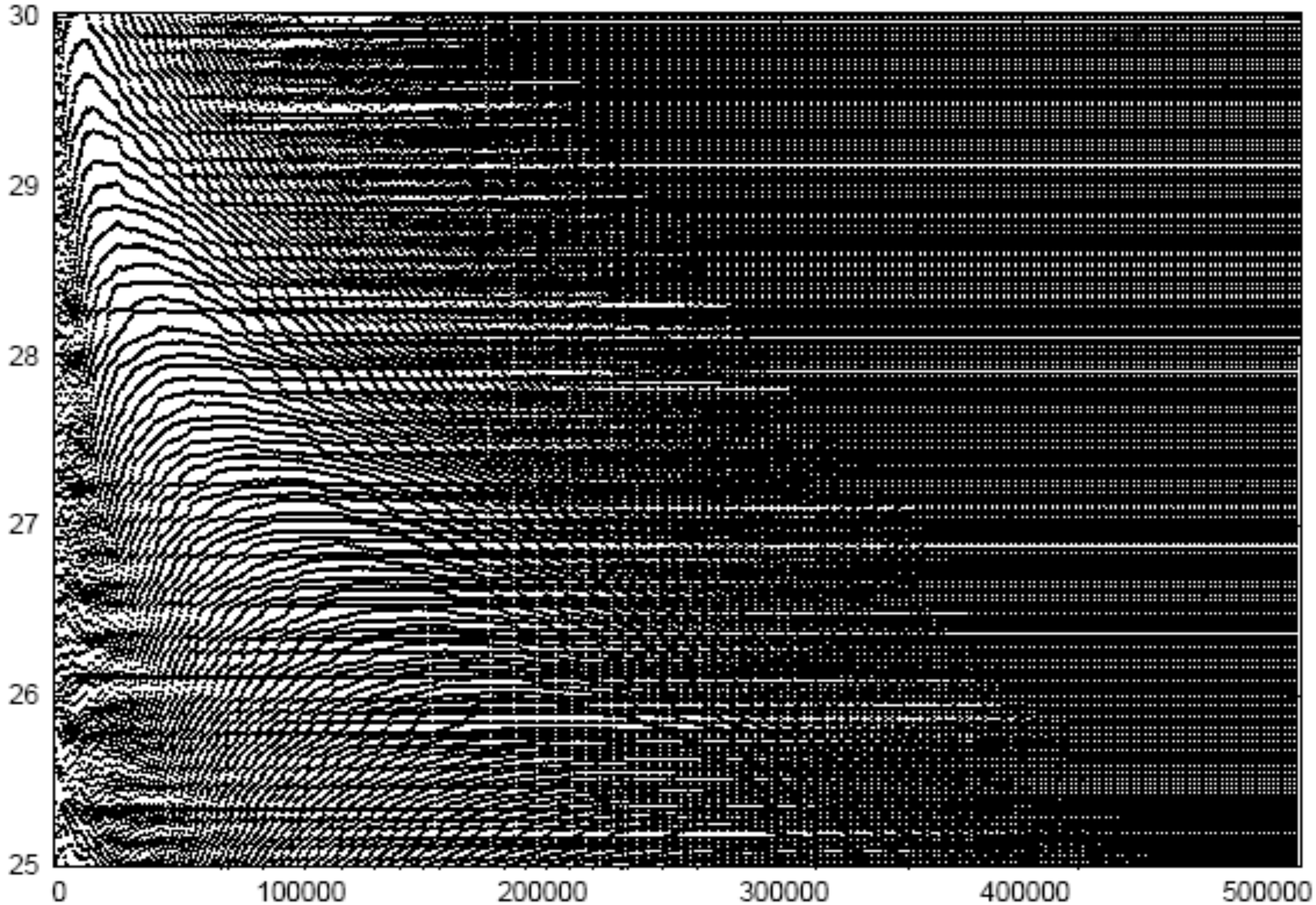
Order: low statistical mix

We can to see what happen in the convergence processes in the Antoine calculation.The problem of obtain both eigenvalues and eigenvectors is solved in Antoine through the diagonalization of a tridiagonal matrix using the iterative Lanczos method. In each iteration Antoine obtain one eigenvalue and one eigenvector. When we put all energies together, we have:

Antoine use Lanczos for the diagonalization processes. This is a iterative method. We can to get one sight about the mechanism of interaction associated with the chaos

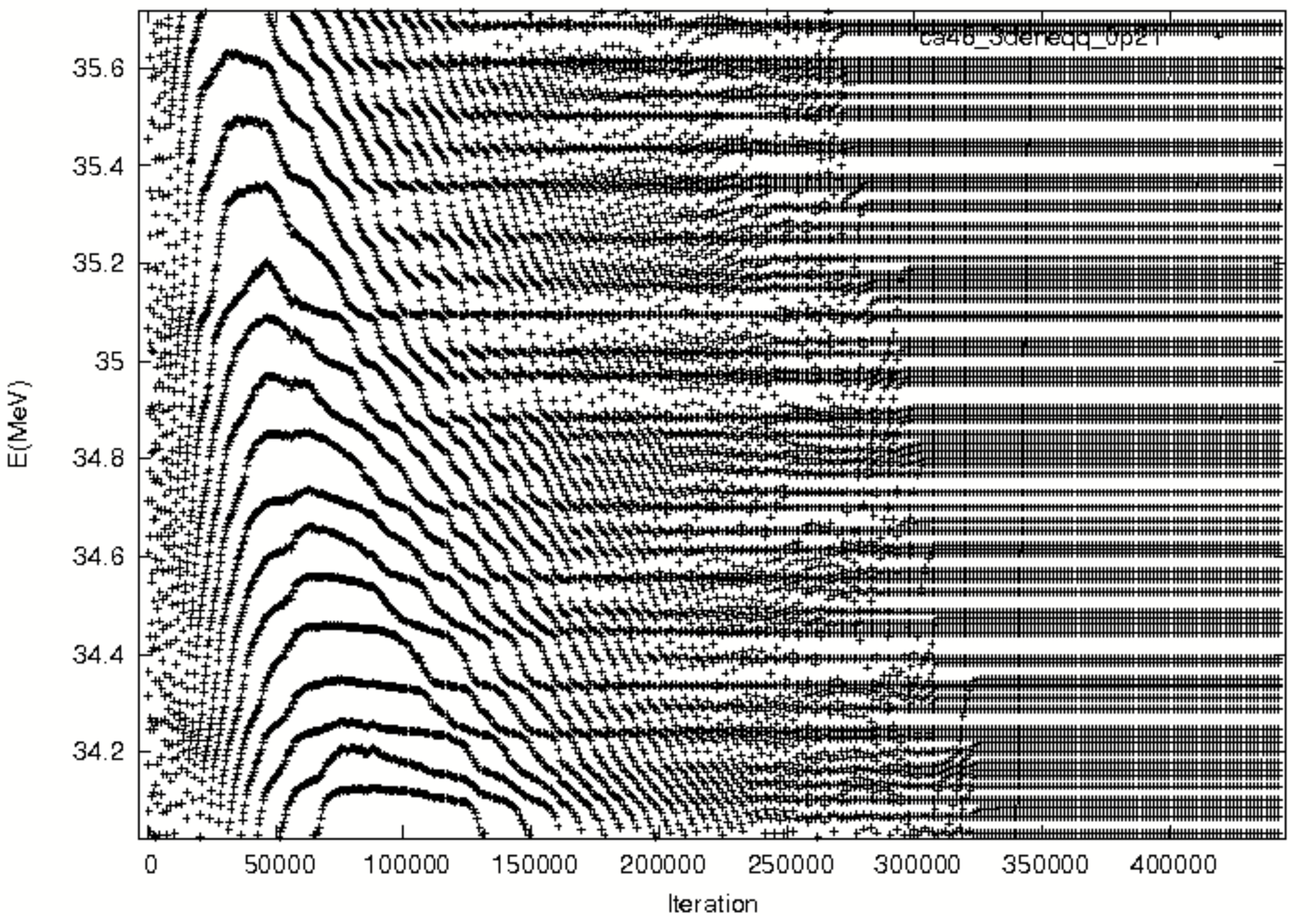



'ca48_3'
40
30
20
10
0
E(MeV)
0
5000
10000
15000
20000
Iteration

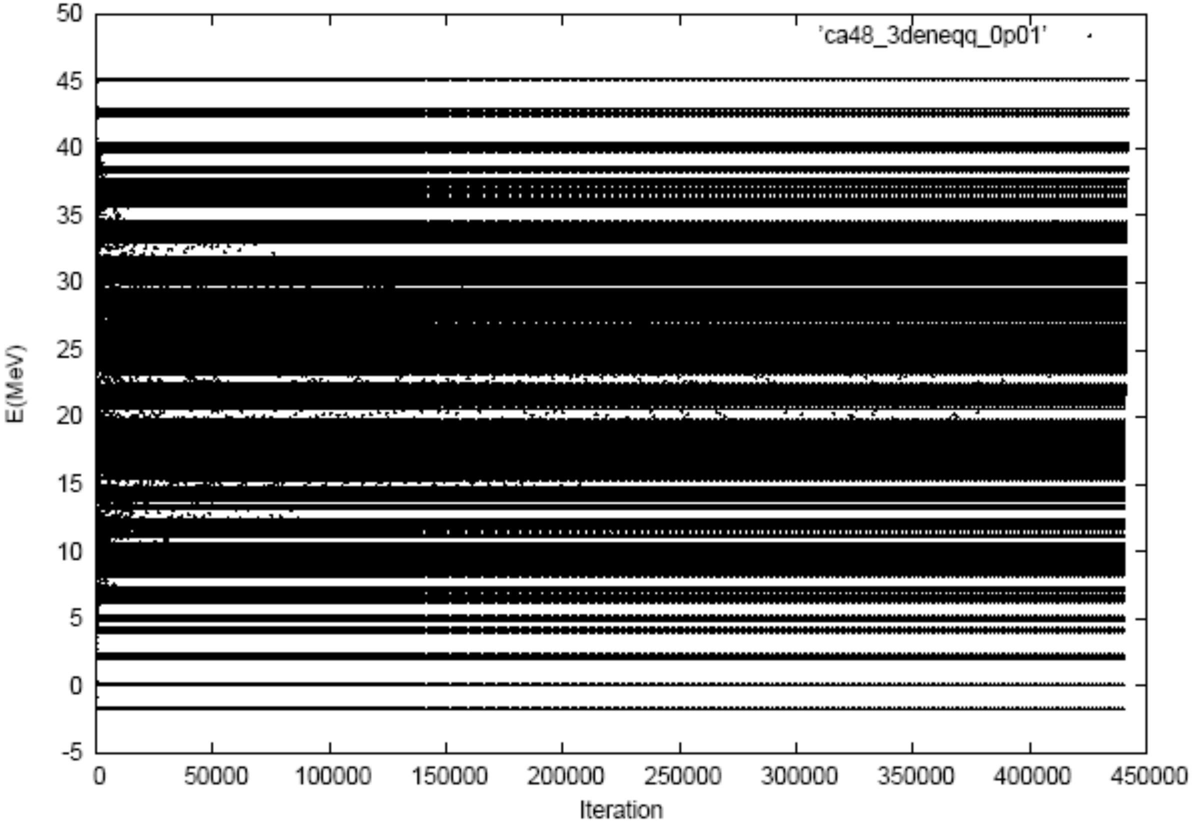


30
29
28
27
26
25
0
100000
200000
300000
400000
500000

Another detail. The quadrupole interaction mix states of wide regions. This detail come from χ=0.21

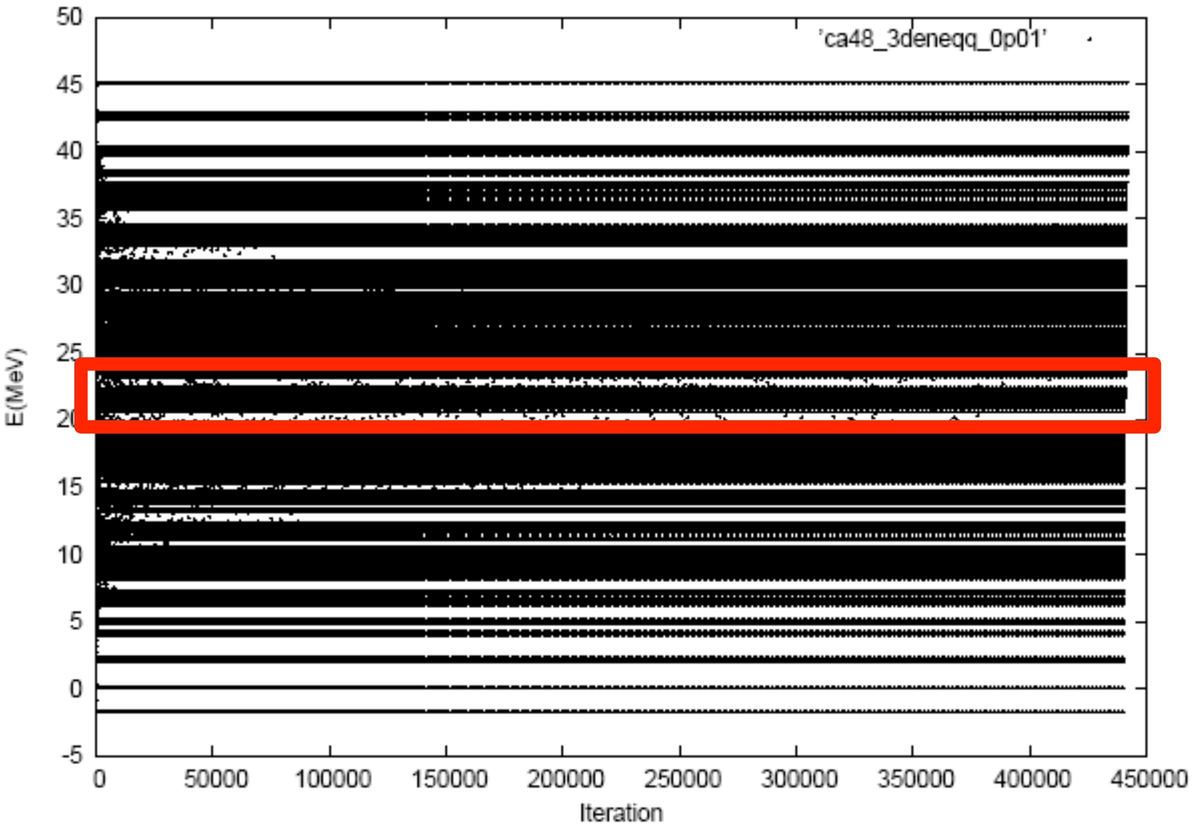


For a weak QQ interaction we have:



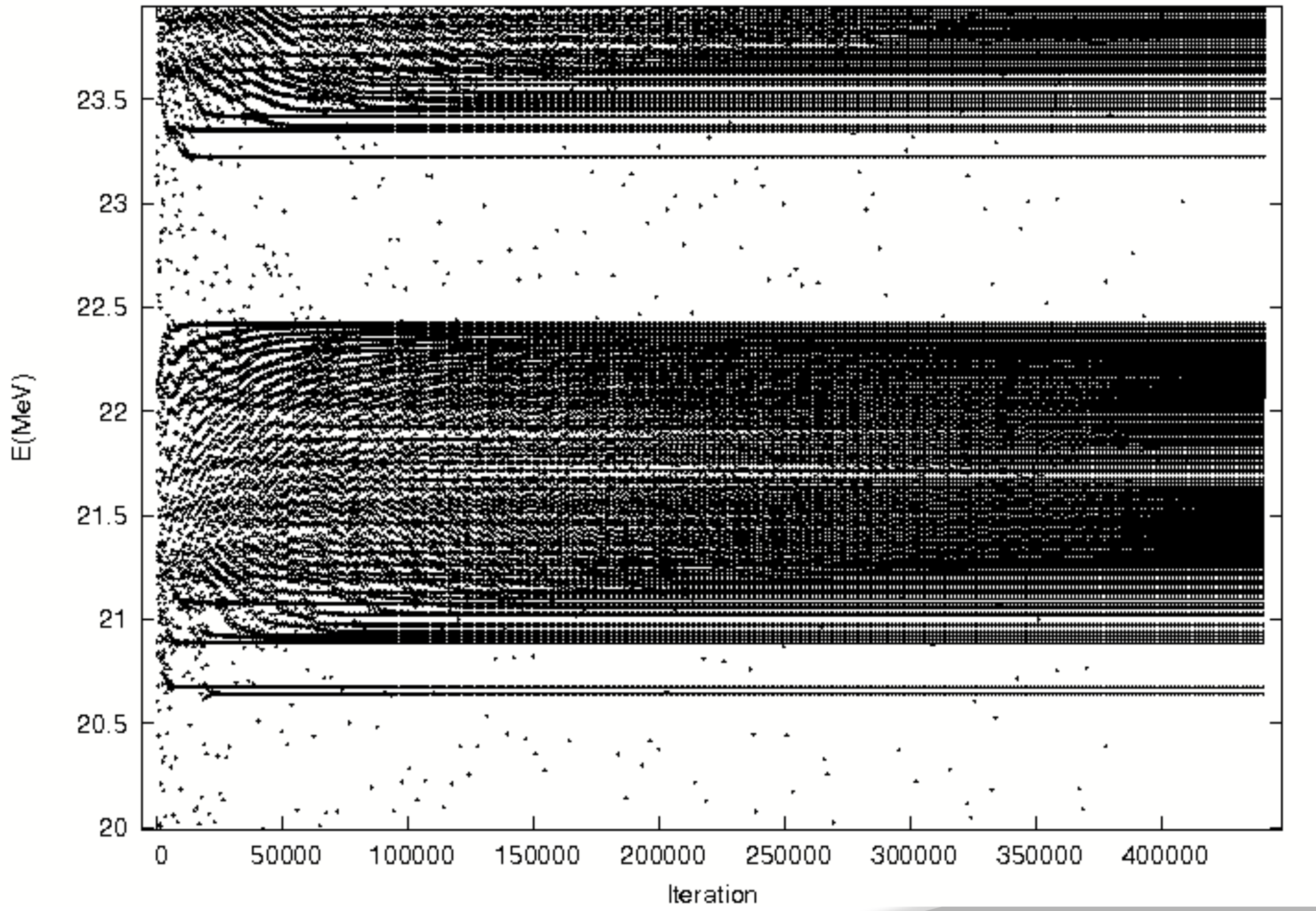
For a weak QQ interaction we have:

$$\chi = 0.01$$

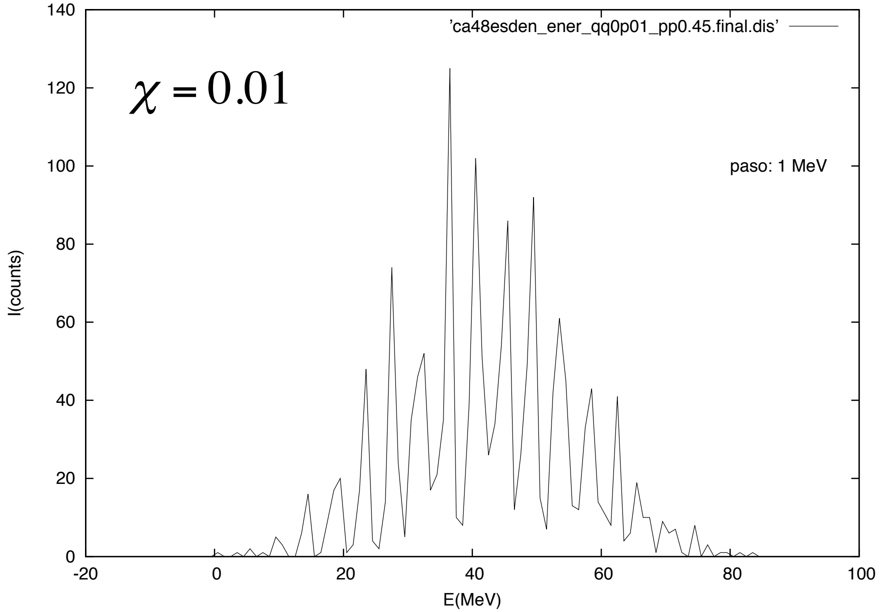


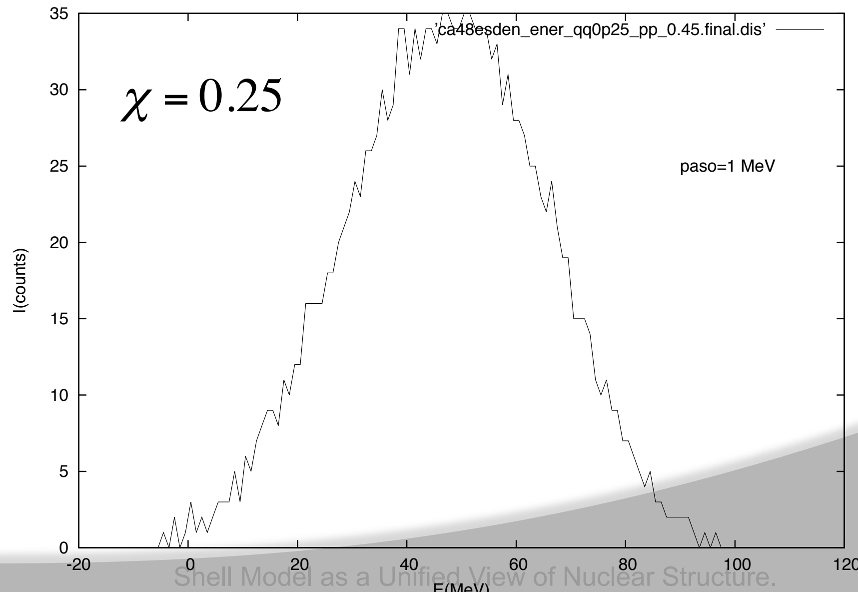
The interaction QQ is there, but acts locally.

$$\chi = 0.01$$



The comparison between the two distribution of energy levels with weak and stronger quadrupole interaction shows:

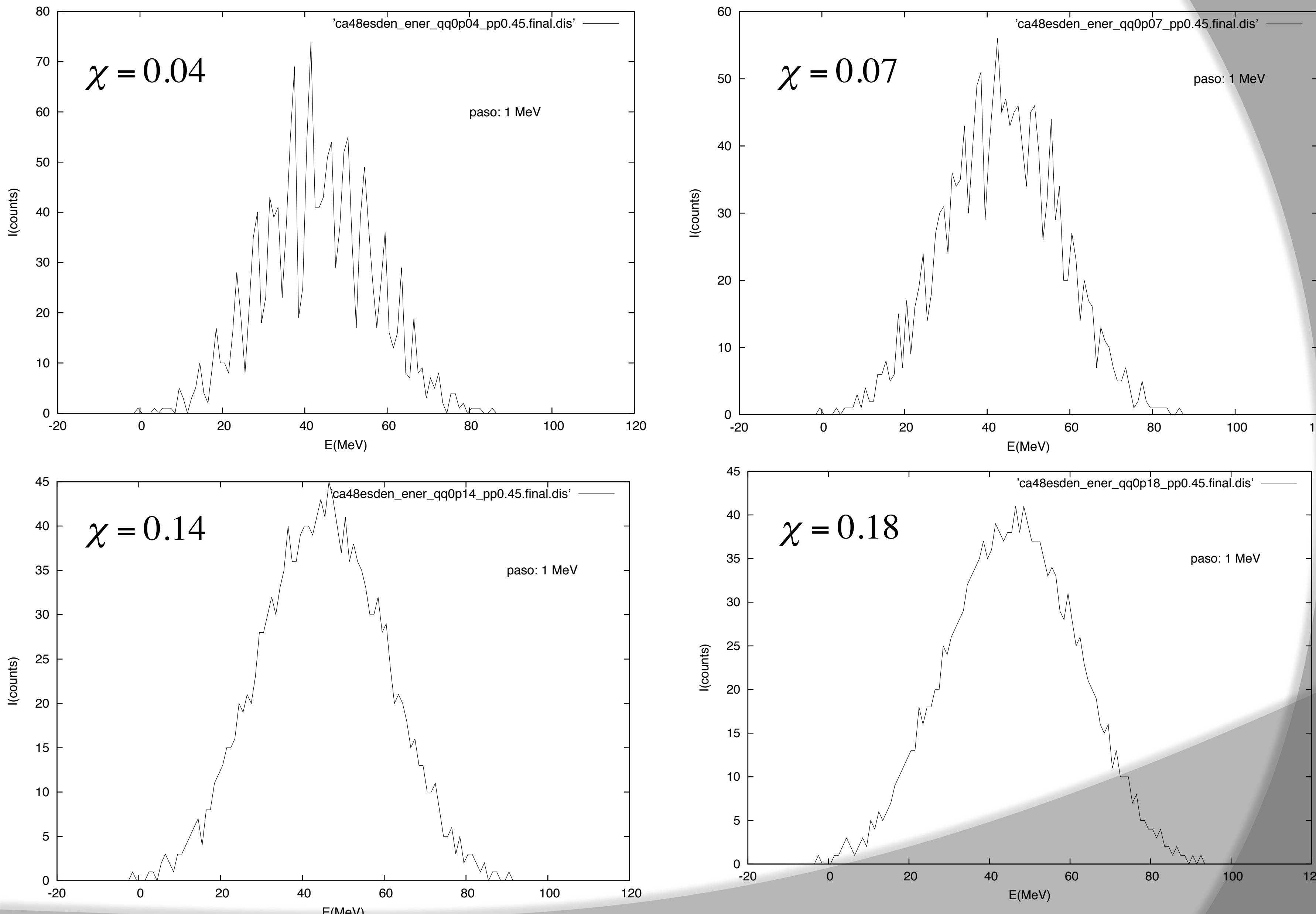




As we are seen, the energy distribution is fractioned when we use a weak QQ interaction.

…However, this fractioned interaction reminds us an interference pattern. In a constrained quantum system each particle (with or without mass) can be represented by standing waves when the wavelenght accomplish the border conditions. This is the origin of the gaps. Each state can develop Interference with itself. When the state is a pure state. The QQ interaction get the complexity.

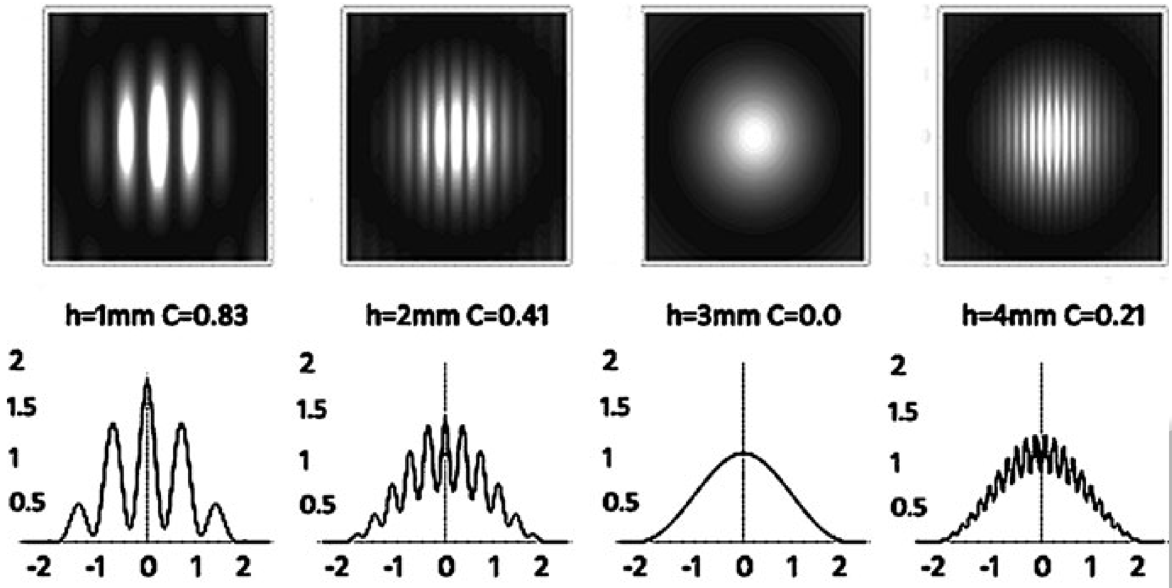
If we see the intermediate distributions:



$\chi = 0.04$
'ca48esden_ener_qq0p04_pp0.45.final.dis'
paso: 1 MeV
I(counts)
E(MeV)
$\chi = 0.07$
'ca48esden_ener_qq0p07_pp0.45.final.dis'
paso: 1 MeV
I(counts)
E(MeV)
$\chi = 0.14$
'ca48esden_ener_qq0p14_pp0.45.final.dis'
paso: 1 MeV
I(counts)
E(MeV)
$\chi = 0.18$
'ca48esden_ener_qq0p18_pp0.45.final.dis'
paso: 1 MeV
I(counts)
E(MeV)

Its possible also, to obtain the correlation degree, using the visibility.

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \propto C$$



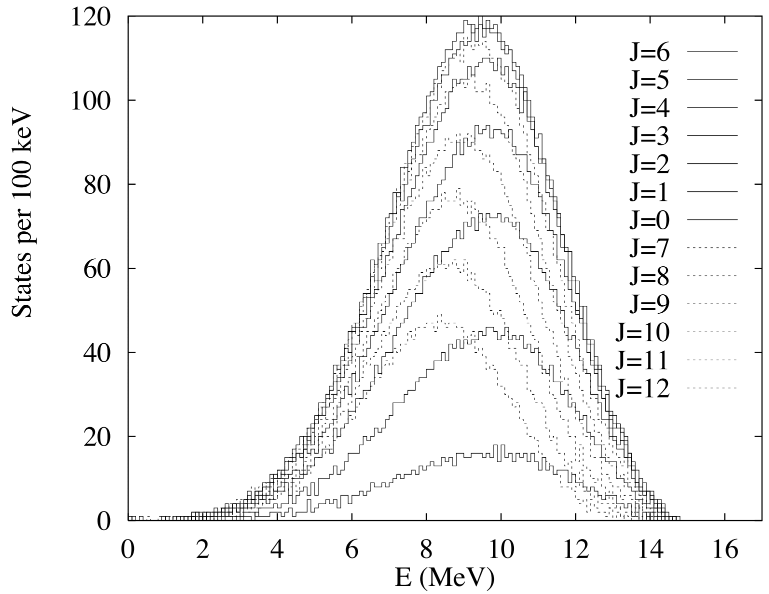


FIG. 5. Level densities for $(fpgd)^8$, $J = 0, 12$. The legend indicates the order in which ρ_J appear as d_J increases (full lines) or decreases (dashes)

Canonical form of Hamiltonian matrices, A. P. Zuker, L. Waha Ndeuna, F. Nowacki, E. Caurier, Phys. Rev C, 02130R, (2001) We can to conclude that the nonintegrable interaction destroys this interference,remaining some fluctuations.

Are these fluctuations related with the quantum fluctuations? Because the energy series have 1/f noise its possible that the energy distributions too.

The 1/f noise have still information. The question is, information about what? Could be about of:
-The chaotic processes, like the complex dynamics (billiards with protons and neutrons, etc.)
-The information of an almost lost coherence.
-?

A system which state is now a statistical mix of the basis state, can be considered classical because are loss its principal quantum properties. This state cannot to have self interference or entanglement

This behavior in the distribution of states remember us the interference of the light with different spectral width: from coherent to thermal statistical.

Quantum pure states can produce energy gaps or interference among identical states and self-interference.

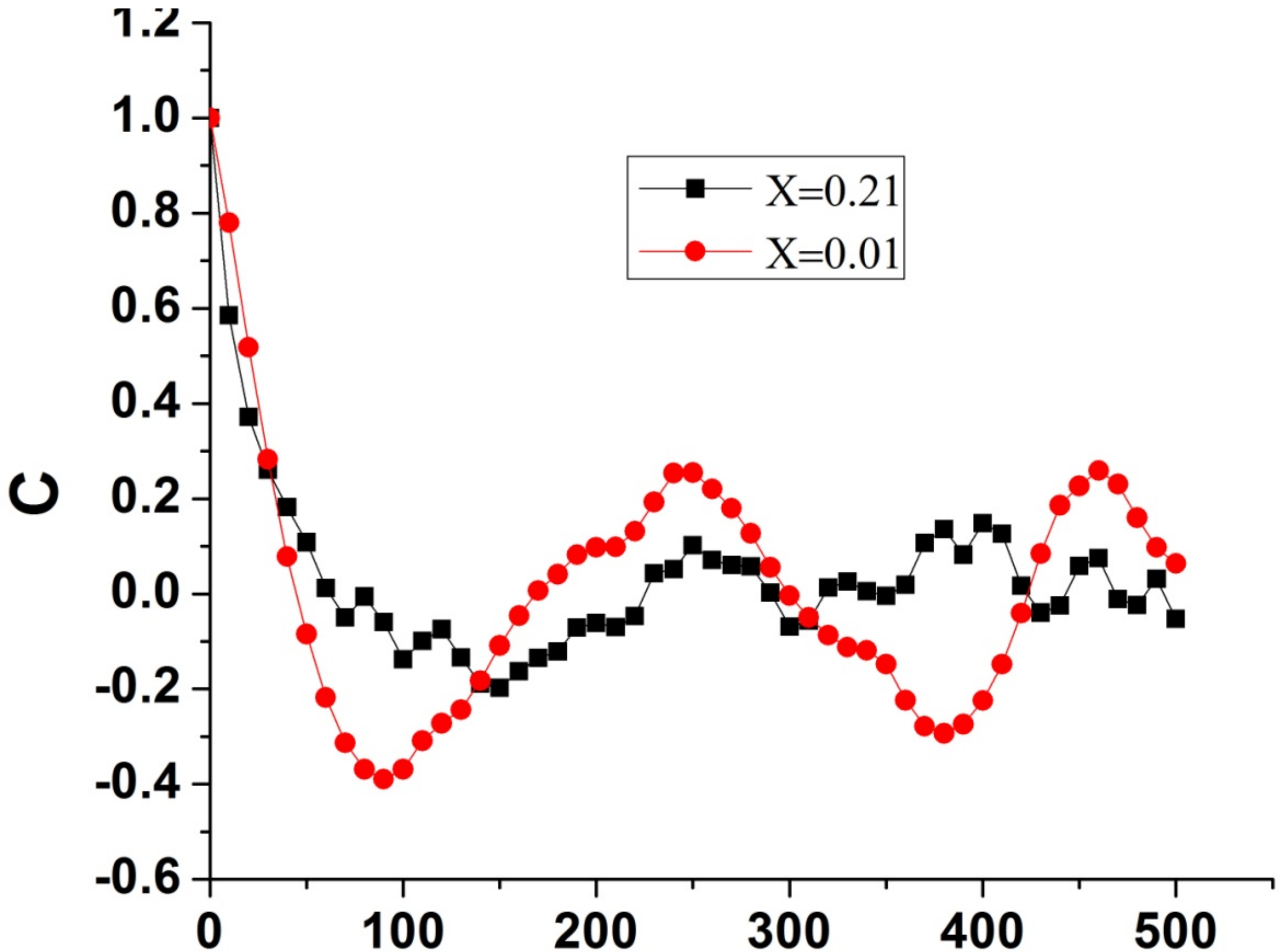
In the frame of this comparison, the 1/f noise or chaos come from the memory of the lossed coherence of pure states. Then, we can think about of certain coherence length :

$$L_c \propto \frac{1}{\Delta\omega_E}$$

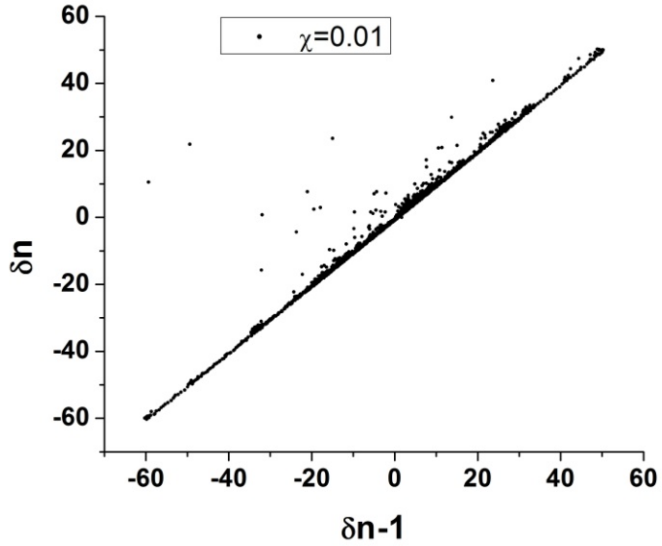
where $\Delta\omega_E$ is the spectral width, proportional to the number of basis states involves in the fluctuations of the mean value of E.

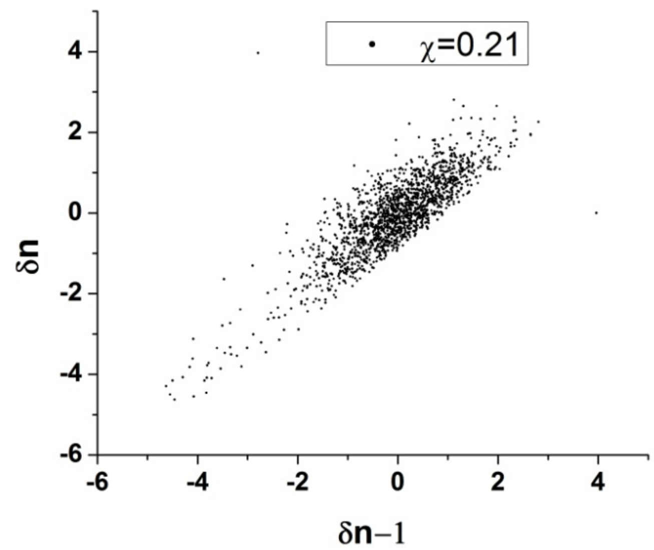
We take the auto-correlation function of one energy serie with itself displaced n positions.

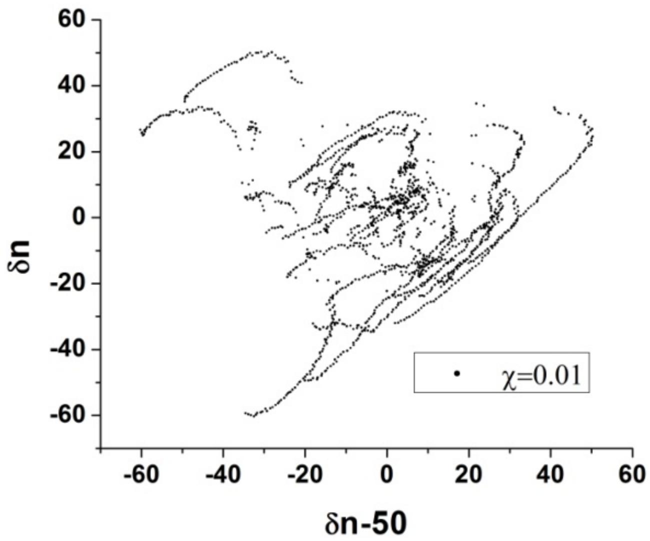
$$C(E_i, E_j) = \frac{\sigma(E_i, E_j)}{\sigma(E_i)\sigma(E_j)}$$

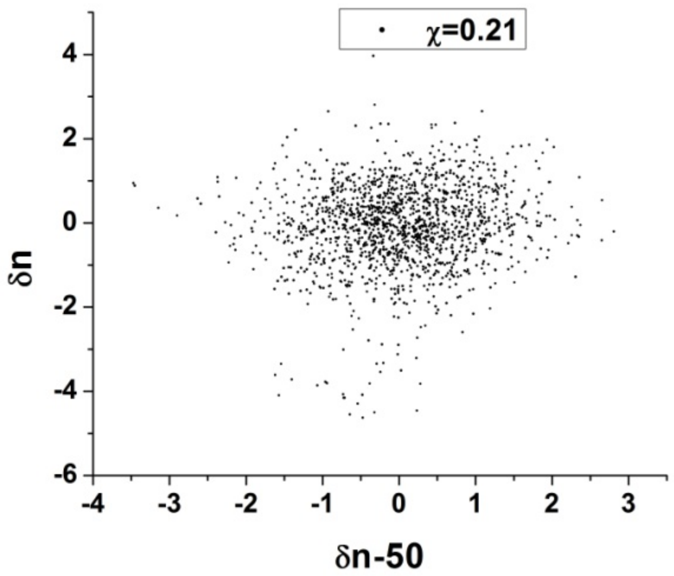


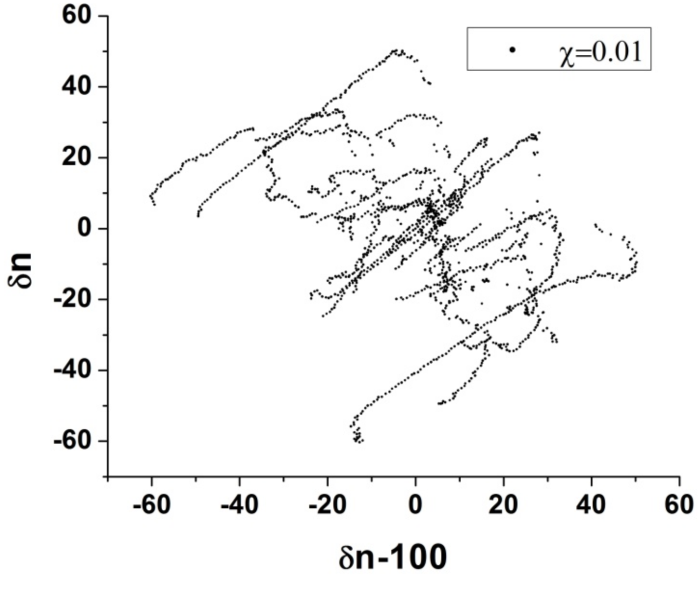
C
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
0
100
200
300
400
500
X=0.21
X=0.01


χ=0.01
δn
δn-1
60
40
20
0
-20
-40
-60
-60 -40 -20 0 20 40 60

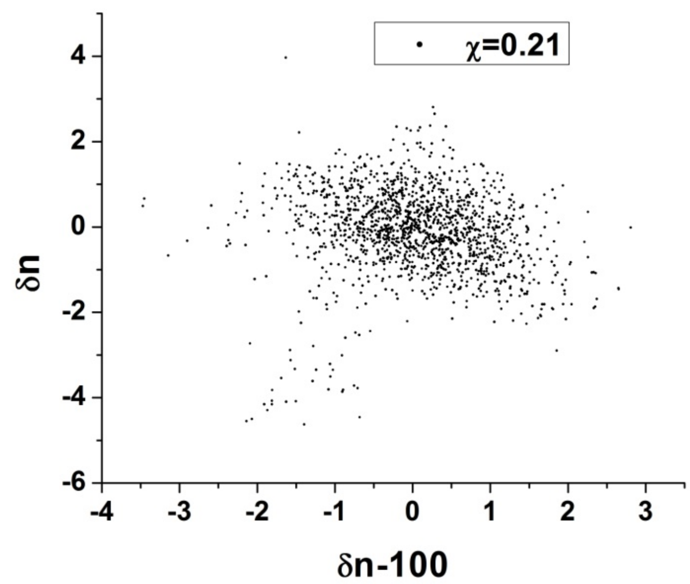

χ=0.21
δn
δn−1
4
2
0
-2
-4
-6
-6 -4 -2 0 2 4


60
40
20
0
-20
-40
-60
δn
-60 -40 -20 0 20 40 60
δn-50
χ=0.01


4
2
0
-2
-4
-6
δn
-4 -3 -2 -1 0 1 2 3
δn-50
χ=0.21

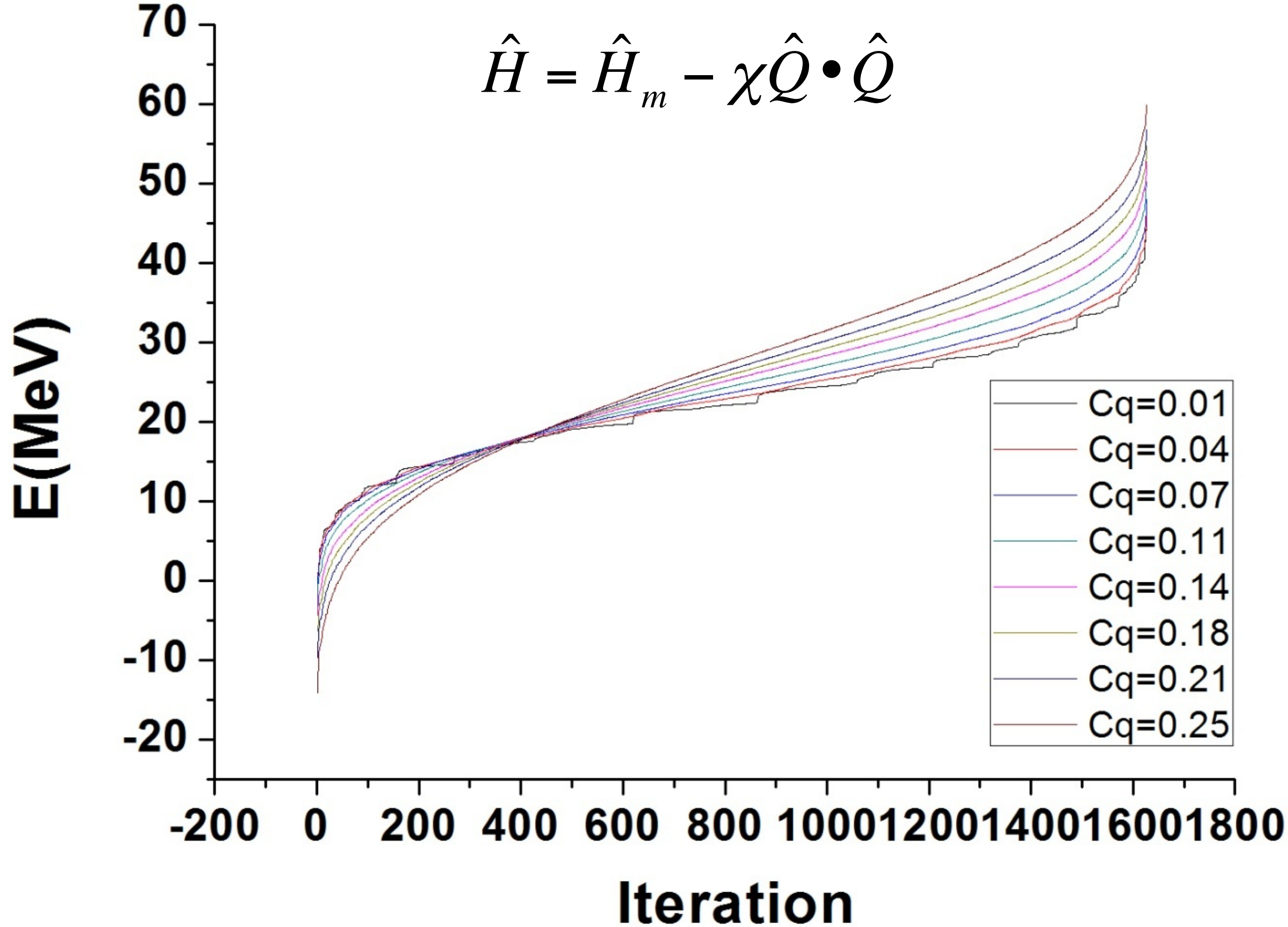


χ=0.01
δn
δn-100



χ=0.21
δn
δn-100

$$\hat{H} = \hat{H}_m - \chi \hat{Q} \bullet \hat{Q}$$



Sensitivity

PHYSICAL REVIEW E **84**, 016224 (2011)

Criticality and long-range correlations in time series in classical and quantum systems

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[2] *Grand Accélérateur National dĺons Lourds, BP F-55027, Caen Cedex 5, France*
[3] *Instituto de Geriatría, Periférico Sur No. 2767, Col. San Jerónimo Lídice, Del. Magdalena Contreras, 10200 México D.F., Mexico*
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[5] *Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, 180 00 Prague, Czech Republic*
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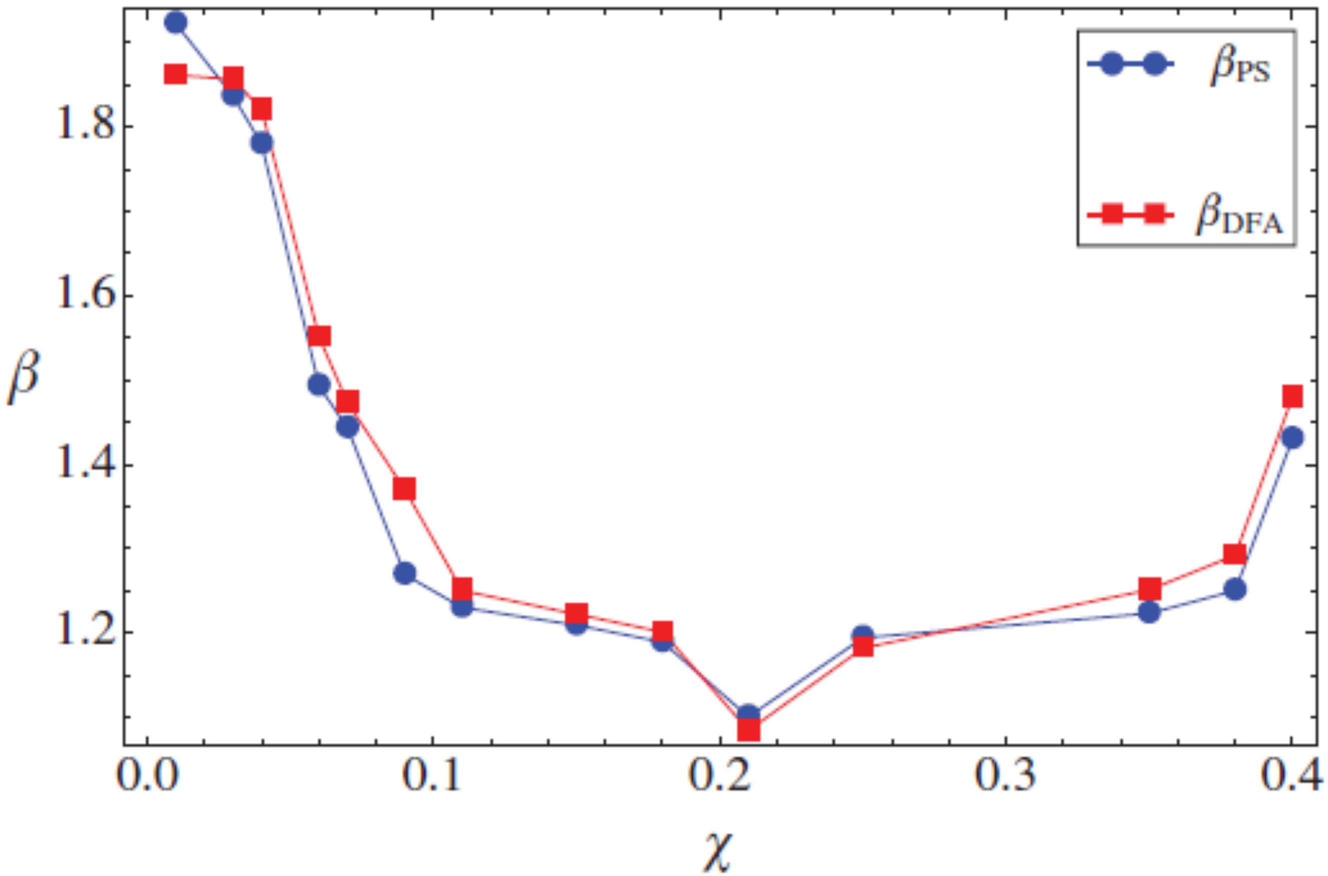
We present arguments which indicate that a transitional state in between two different regimes implies the occurrence of $1/f$ time series and that this property is generic in both classical and quantum systems. Our study focuses on two particular examples: the one-dimensional module-1 logistic map and nuclear excitation spectra obtained with a schematic shell-model Hamiltonian. We suggest that a transitional point is characterized by the long-range correlations implied by $1/f$ time series. We apply a Fourier spectral analysis and the detrended fluctuation analysis method to study the fluctuations to each system.

DOI: 10.1103/PhysRevE.84.016224 PACS number(s): 05.45.Tp, 02.50.−r, 05.40.Ca



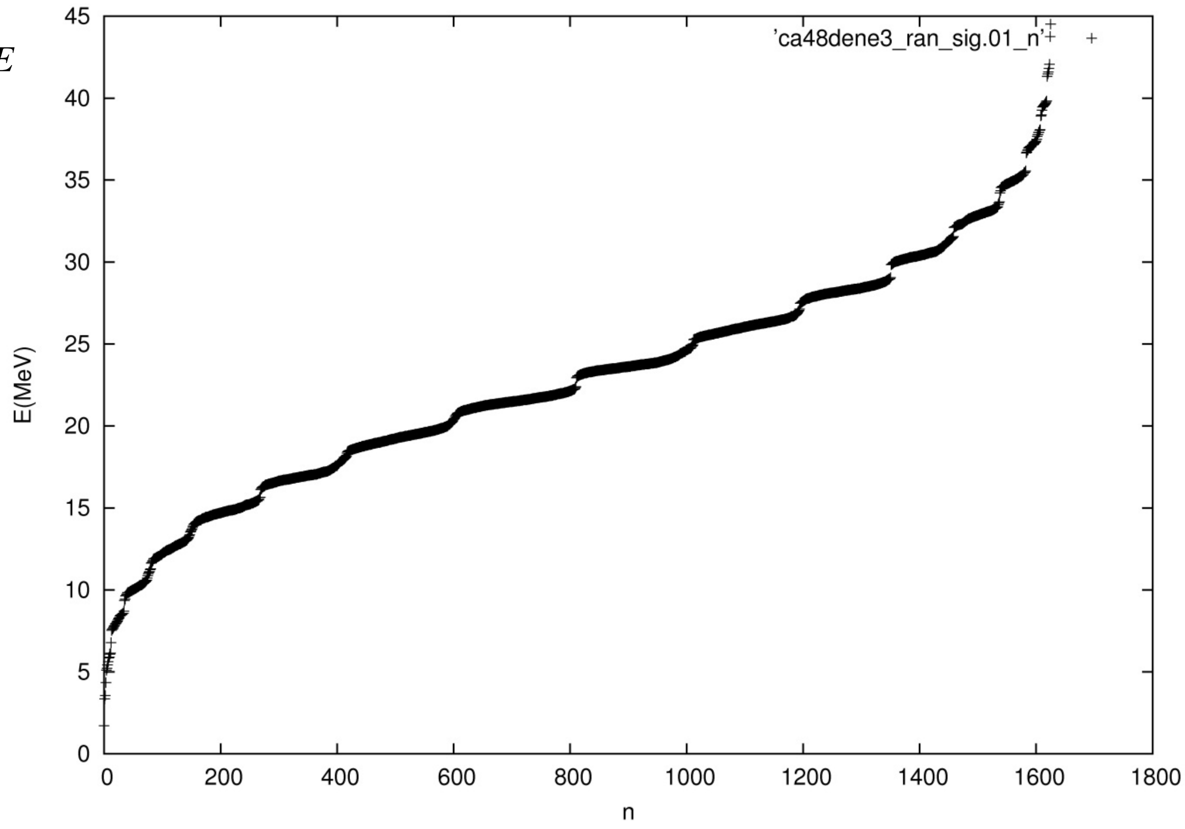
β_{PS}
β_{DFA}
1.8
1.6
1.4
1.2
β
0.0
0.1
0.2
0.3
0.4
χ

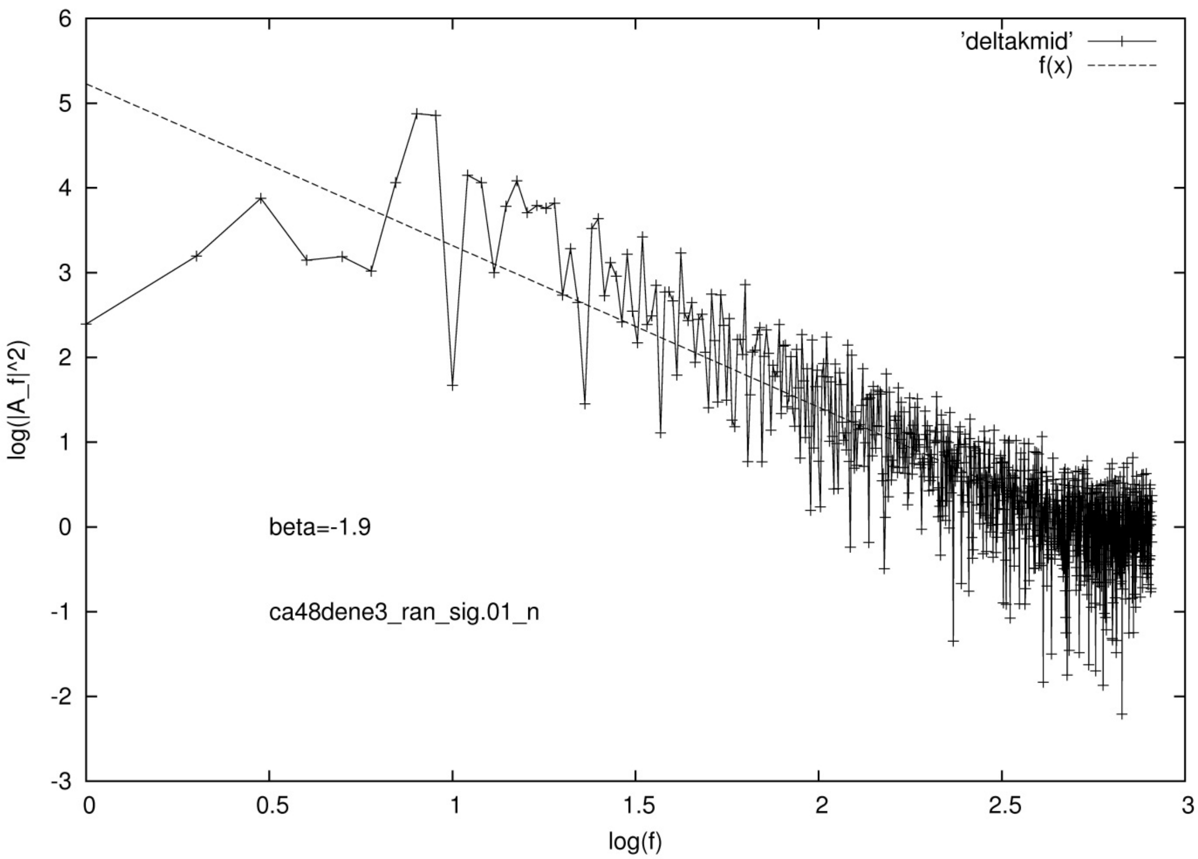
Scale Invariance

$$\hat{H} = \hat{H}_m + \hat{H}_{TBRE}$$

$$\sigma = 0.01$$

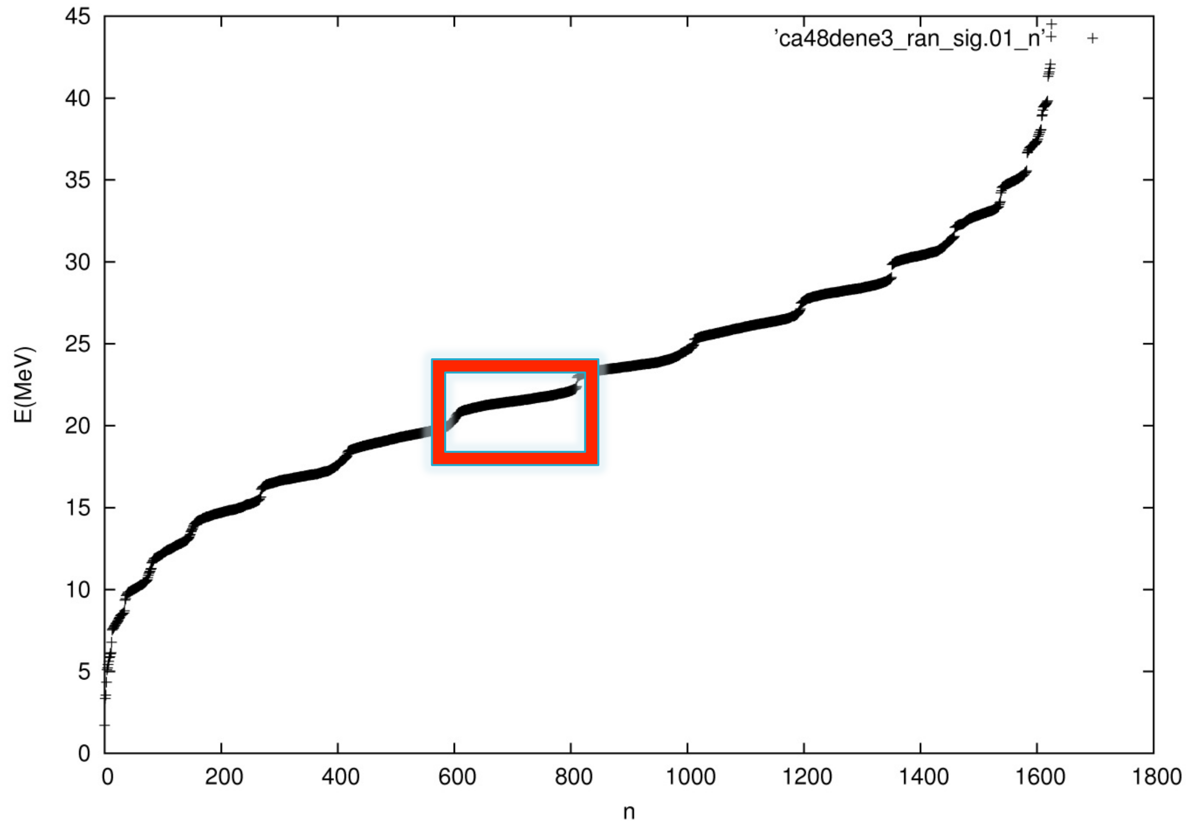
$$\sigma_{KB3} = 0.6$$



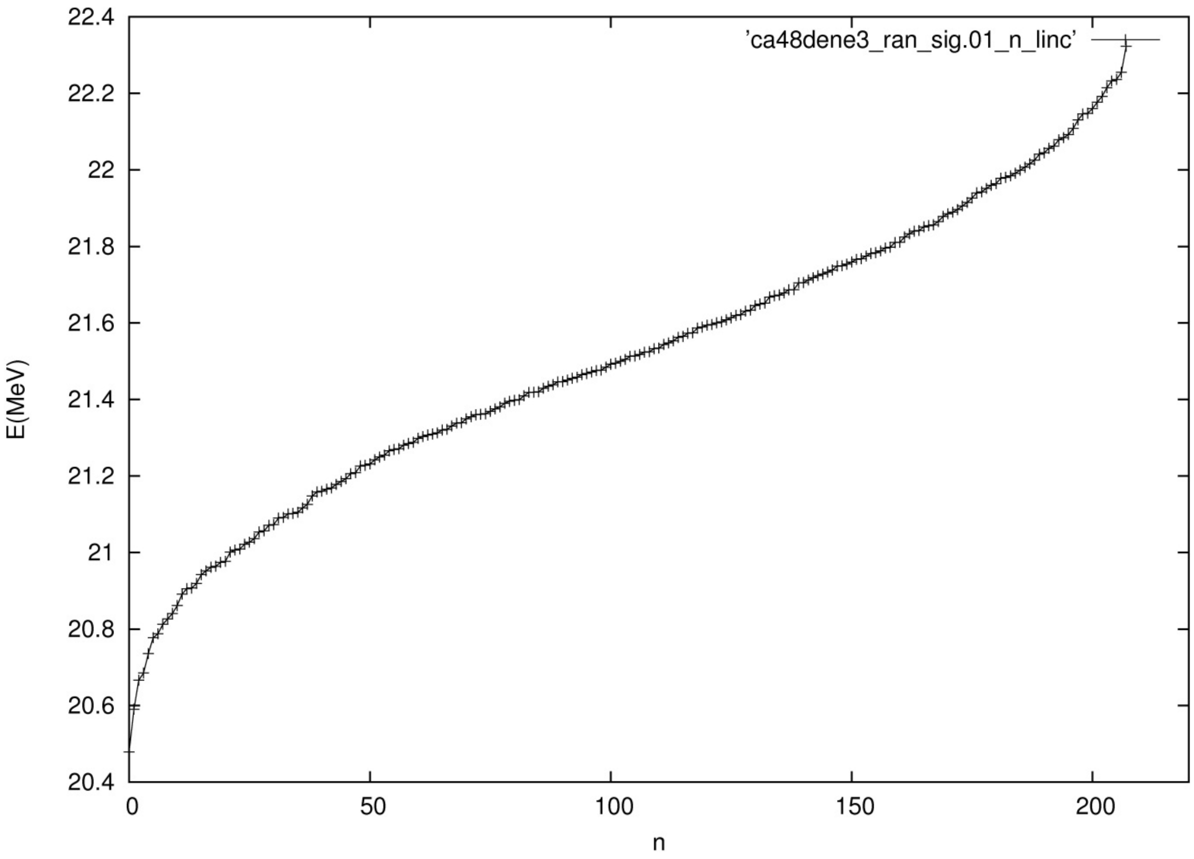

'deltakmid'
f(x)
beta=-1.9
ca48dene3_ran_sig.01_n
log(|A_f|^2)
log(f)
6
5
4
3
2
1
0
-1
-2
-3
0
0.5
1
1.5
2
2.5
3

$\beta = -1.9$

$\sigma = 0.01$

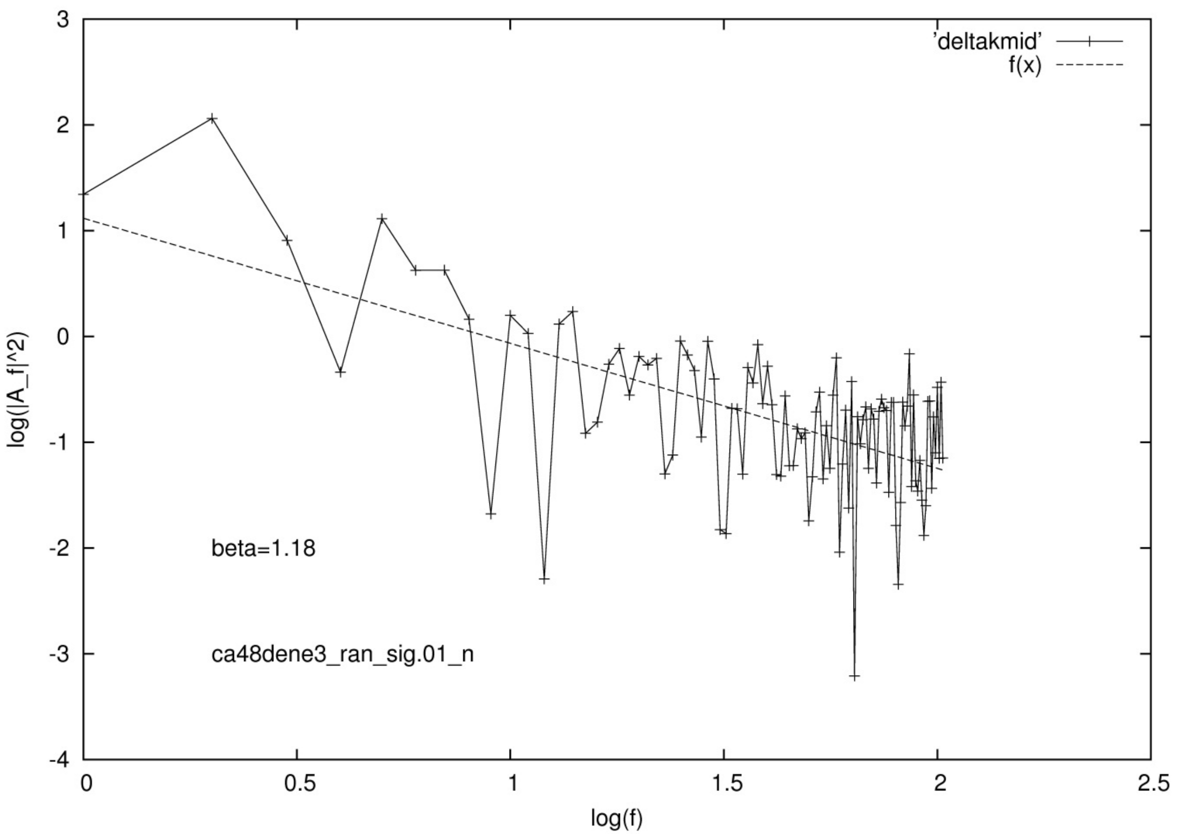

'ca48dene3_ran_sig.01_n'
E(MeV)
n
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
1800

$\sigma = 0.01$



'ca48dene3_ran_sig.01_n_linc'
22.4
22.2
22
21.8
21.6
21.4
21.2
21
20.8
20.6
20.4
E(MeV)
0
50
100
150
200
n

$\sigma = 0.01$



β=−*1.18*

CONCLUSIONS

-In shell model calculations for ^{48}Ca (3^+), we have two statistical limits: quantum and chaos.

-The transition from quantum to chaos comes from the loss of the quantum coherence.

-The transitions comes from the configurations mixing originated by the non-integrable interactions.

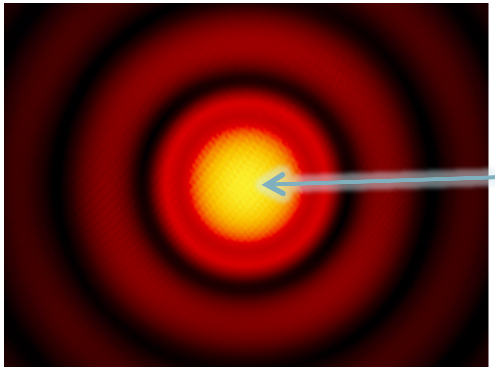
-The memory of this quantum coherence can be reinterpreted as as the origin of chaotic fluctuations.

-If you have a non integrable interaction, then it will be appears at Some scale, showing 1/f noise.

Dedicated to Andrés, Etienne and Alfredo.

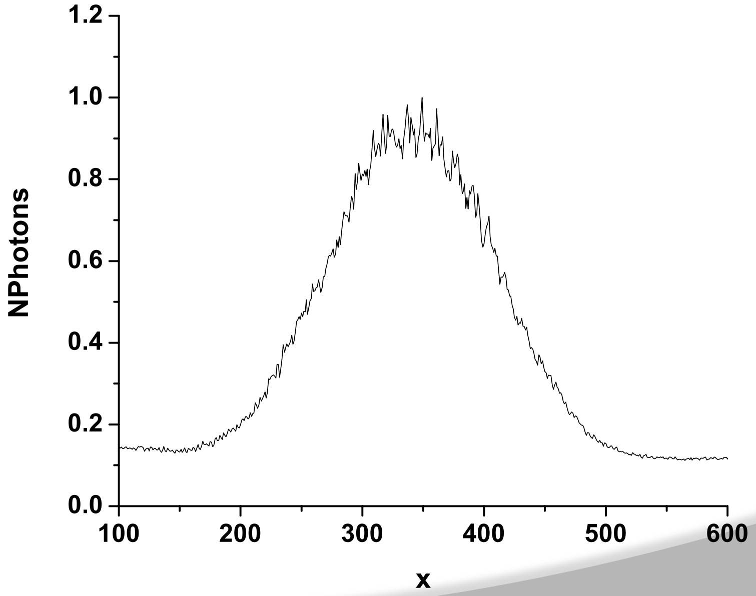
- Emmanuel Landa
- Alejandro Frank
- Rubén Fossion
- Andrés P. Zuker

Finally, an experiment. We propose one experiment with light where the correlation is almost visible. A laser He-Ne cross a pin-hole. The diffraction pattern is:

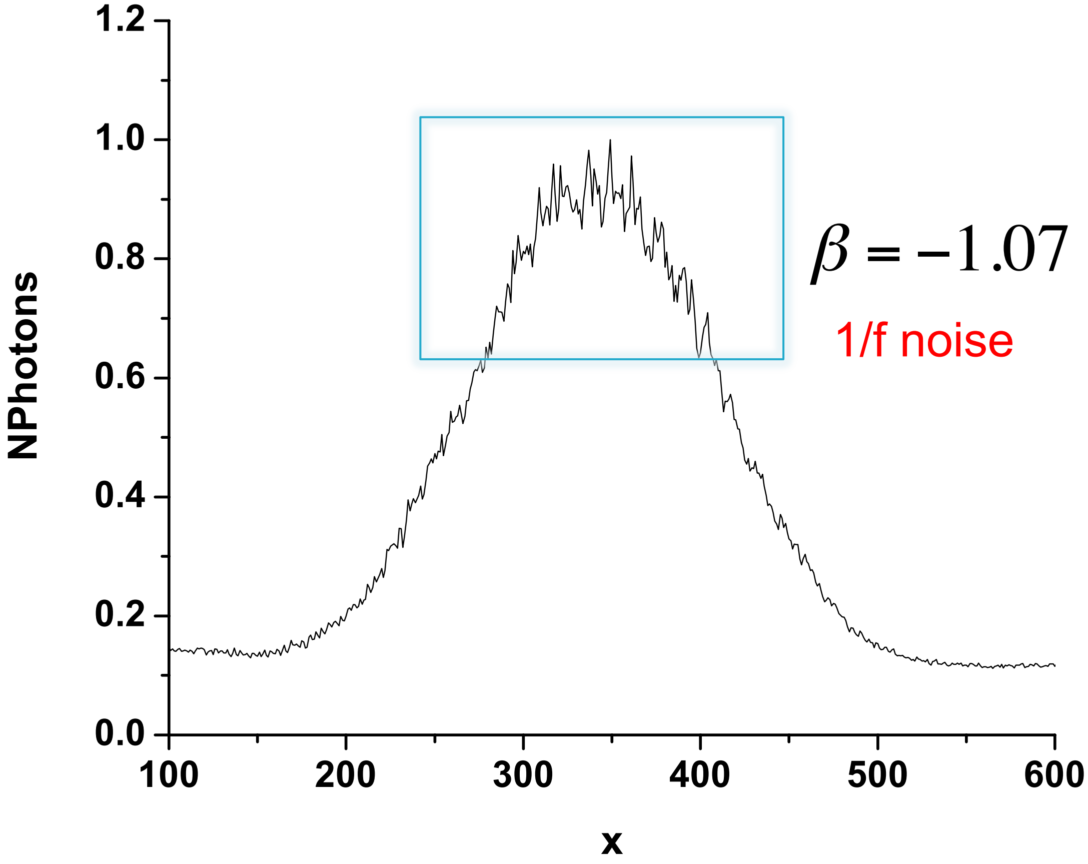


Incoherent zone?

A detector is moved along of the diffraction pattern counting photons.



The fluctuations can be interpreted as the memory of the spatial coherence.



$\beta = -1.07$

1/f noise