Shell model calculations of β^- decay half-lives of r-process nuclei

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Content

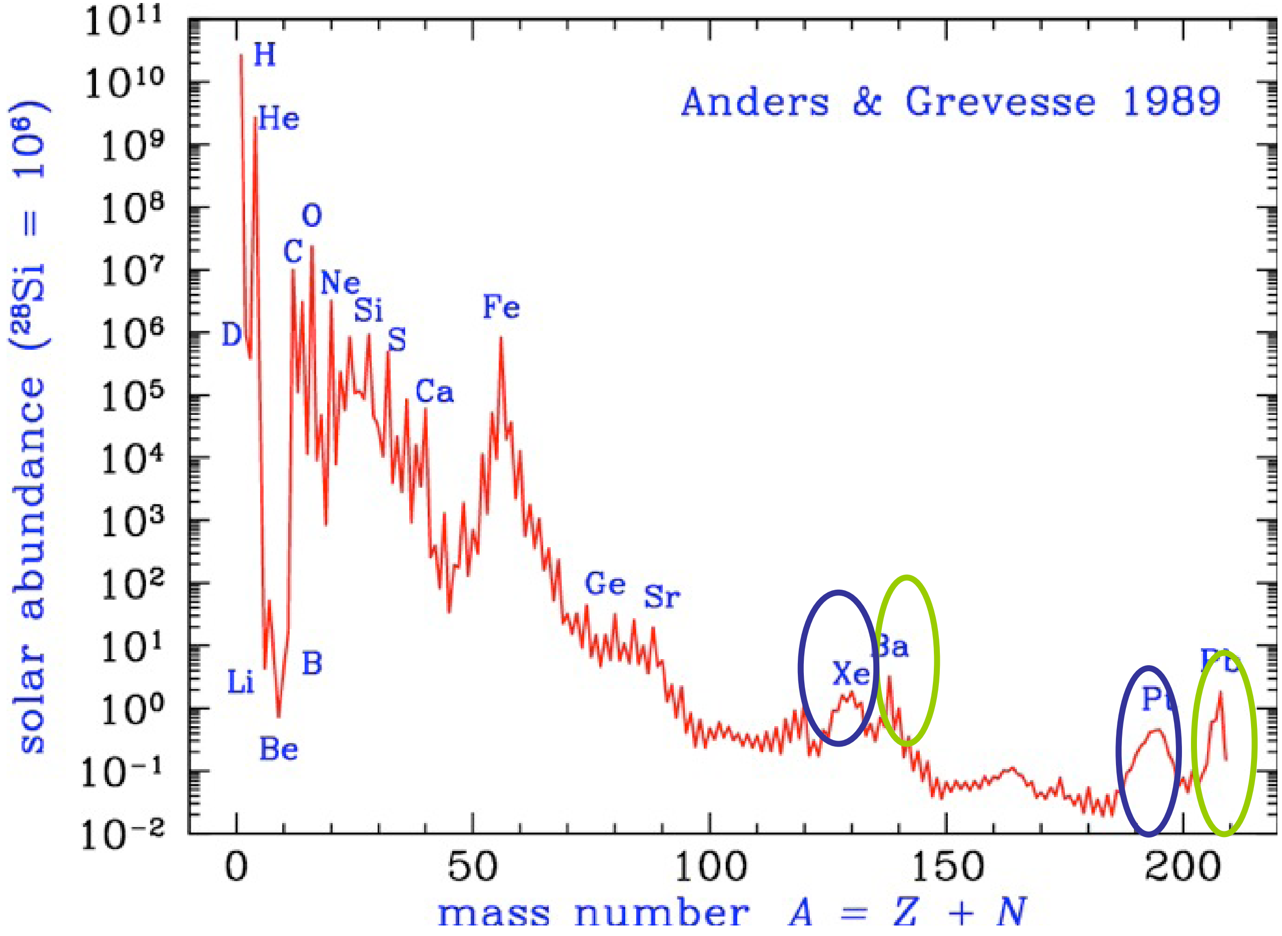
- Motivation
- Theory of Shell model and β^- decay
- Results of half-lives for r-process nuclei
- Conclusion and Outlook

Motivation

Nucleisynthesis of heavy elements: Neutron captures vs. beta-decay

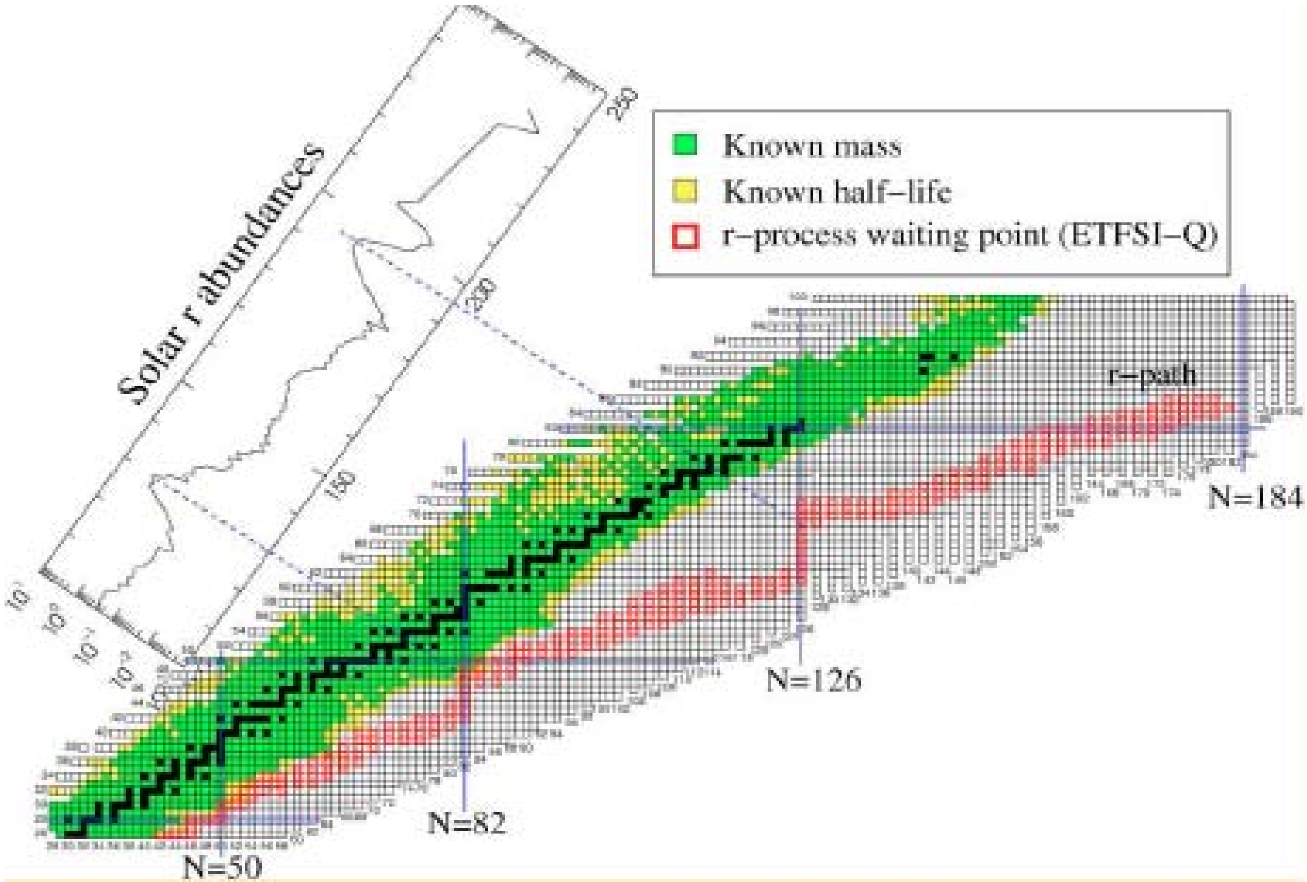
Hydrostatic burning : n-capture << beta decay ⟶

- slow (s) process
- rapid (r) process



Explosive environment !
(n-capture >> beta decay)

About half of the elements heavier than Fe are produced by rapid neutron capture .

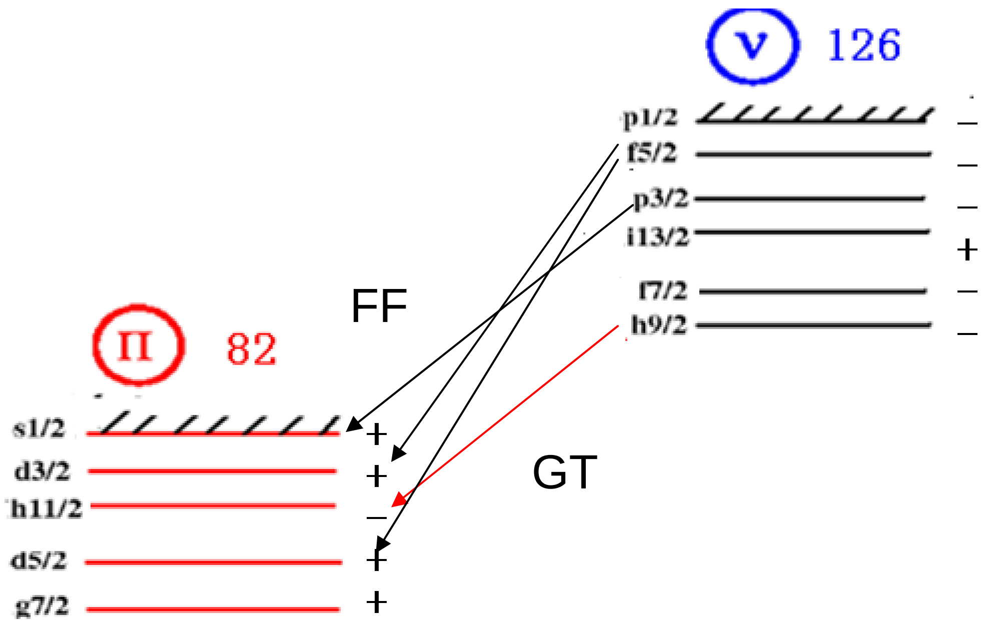


Our principal motivation is the astrophysical r-process.
Half-lives of N=50, 82, 126 are calculated:

1) They are particularly important at the r-process waiting points where abundances peaks are located. These half-lives determine the r-process time scale.

2) Nuclei there have shell closure and are taken as spherical, the calculations are tractable for the Shell model from the computational point of view.

The inclusion of the First Forbidden transitions:

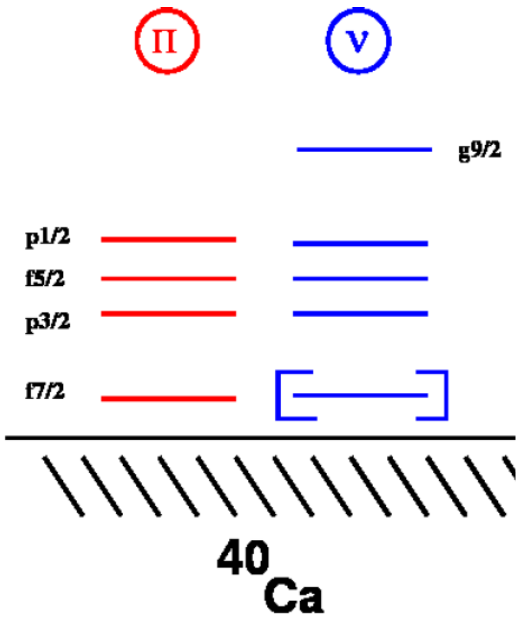


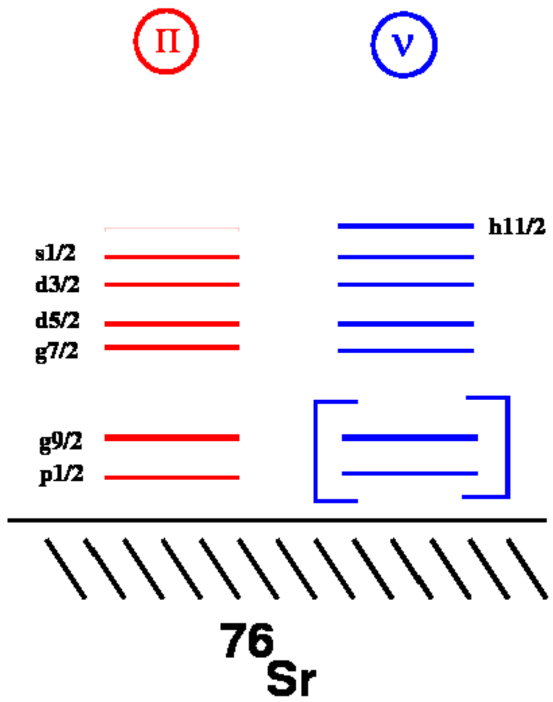
- Because for nuclei in these region, the protons and neutrons are in different shells and if you do beta decay, the parity will changed and hence the forbidden transition must be considered.

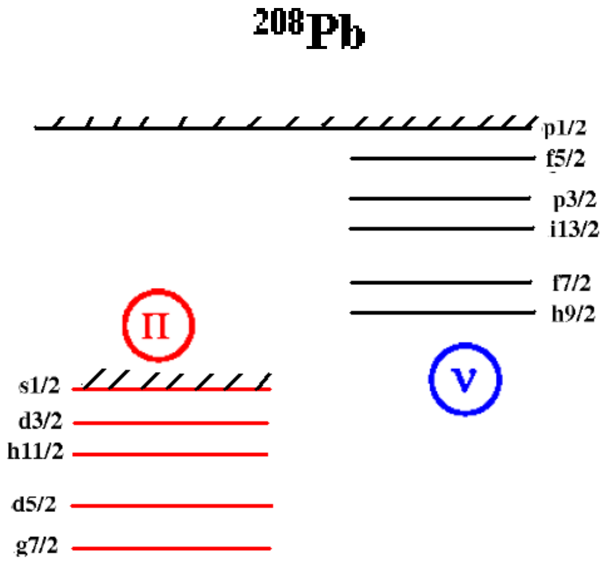
- Shell model code Nathan* is used to do the calculation:

*E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves and A. P. Zuker."The Shell Model as a Unified View of Nuclear Structure", Reviews of Modern Physics, 77 (2005) 427-488

Valence space in the calculation:







N=50: ^{48}Ca core

N=82: ^{88}Sr core

N=126: ^{208}Pb core

1. J.J. Cuenca-Garcia, et al., Eur. Phys J. A 34, (2007).
2. K. Sieja and F. Nowacki, Phys. Rev. C 81061303(R),(2010).
3. T.T.S. Kuo and G.H. Herling, U.S. Naval Research Laboratory Report No 2258, (1971)

Beta decay theory

- The half-life to a specific final state is given by:

$$ft = 6146\ s \qquad (1)$$

where f is the the phase factor:

$$f = \int_1^{W_0} C(W)F(Z,W)(W^2-1)^{1/2}W(W_0-W)^2dW \qquad (2)$$

C(W) is the shape factor and *F(Z;W)* is the Fermi function, which takes into account the distortion of the electron (positron) wave function due to Coulomb effects.

This is a GENERAL treatment, so it can be used for every kind of transition.

Shape factor:

- For Allowed transitions (which the angular momentum transfer equals zero), Shape factor dose not dependent on the electron energy and has the form:

$$C(W) = B(GT). \qquad (3)$$

Where The GT reduced transition probability is given by:

$$B(GT) = \left(\frac{g_A}{g_V}\right)^2 \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}_-^k || i \rangle^2}{2J_i + 1}, \qquad (4)$$

- For First forbidden transitions (Where the angular momentum transfer equals one), the shape factor can be written as:

$$C(W) = k + kaW + kb/W + kcW^2 \qquad (5)$$

Where coefficients k, ka, kb, kc are energy depend and are the combinations of the FF transition matrix elements as follows:

FF Matrix elements:

$$w = \lambda\sqrt{3}\frac{\langle f || \sum_k r_k \left[C_1^k \times \sigma^k\right]^0 t_-^k || i\rangle}{\sqrt{2J_i+1}},$$

$$w' = \lambda\sqrt{3}\frac{\langle f || \sum_k \frac{2}{3} r_k I(1,1,1,1,r_k) \left[C_1^k \times \sigma\right]^0 t_-^k || i\rangle}{\sqrt{2J_i+1}}$$

$$x = -\frac{\langle f || \sum_k r_k C_1^k t_-^k || i\rangle}{\sqrt{2J_i+1}},$$

$$x' = -\frac{\langle f || \sum_k \frac{2}{3} r_k I(1,1,1,1,r_k) C_1^k t_-^k || i\rangle}{\sqrt{2J_i+1}},$$

$$u = \lambda\sqrt{2}\frac{\langle f || \sum_k r_k \left[C_1^k \times \sigma^k\right]^1 t_-^k || i\rangle}{\sqrt{2J_i+1}},$$

$$u' = \lambda\sqrt{2}\frac{\langle f || \sum_k \frac{2}{3} r_k I(1,1,1,1,r_k) \left[C_1^k \times \sigma^k\right]^1 t_-^k || i\rangle}{\sqrt{2J_i+1}}$$

$$z = -2\lambda\frac{\langle f || \sum_k r_k \left[C_1^k \times \sigma\right]^2 t_-^k || i\rangle}{\sqrt{2J_i+1}},$$

$$\xi' v = -\frac{\lambda\sqrt{3}}{M}\frac{\langle f || \sum_k \left[\sigma^k \times \boldsymbol{\nabla}^k\right]^0 t_-^k || i\rangle}{\sqrt{2J_i+1}},$$

$$\xi' y = -\frac{1}{M}\frac{\langle f || \sum_k \boldsymbol{\nabla}^k t_-^k || i\rangle}{\sqrt{2J_i+1}},$$

With the Lanczos method, one can calculate the FF matrix elements distribution, and consequently to calculate the beta decay half life.

H. Behrens and W. Buhring, Nucl. Phys. **A** 162, 111(1971))

Average shape factor:

For allowed GT transitions, from Eq.(1) and (3), B(Gt） can be written as:

$$B(GT) = \frac{6146\ s}{f(0)t} \tag{6}$$

Where $f(0)$ has the form:

$$f_0 = \int_1^{W_0} F(Z,W)(W^2-1)^{1/2}W(W_0-W)^2 dW \tag{7}$$

To compare the FF forbidden transition with allowed transition, one can define the averaged shape factor:

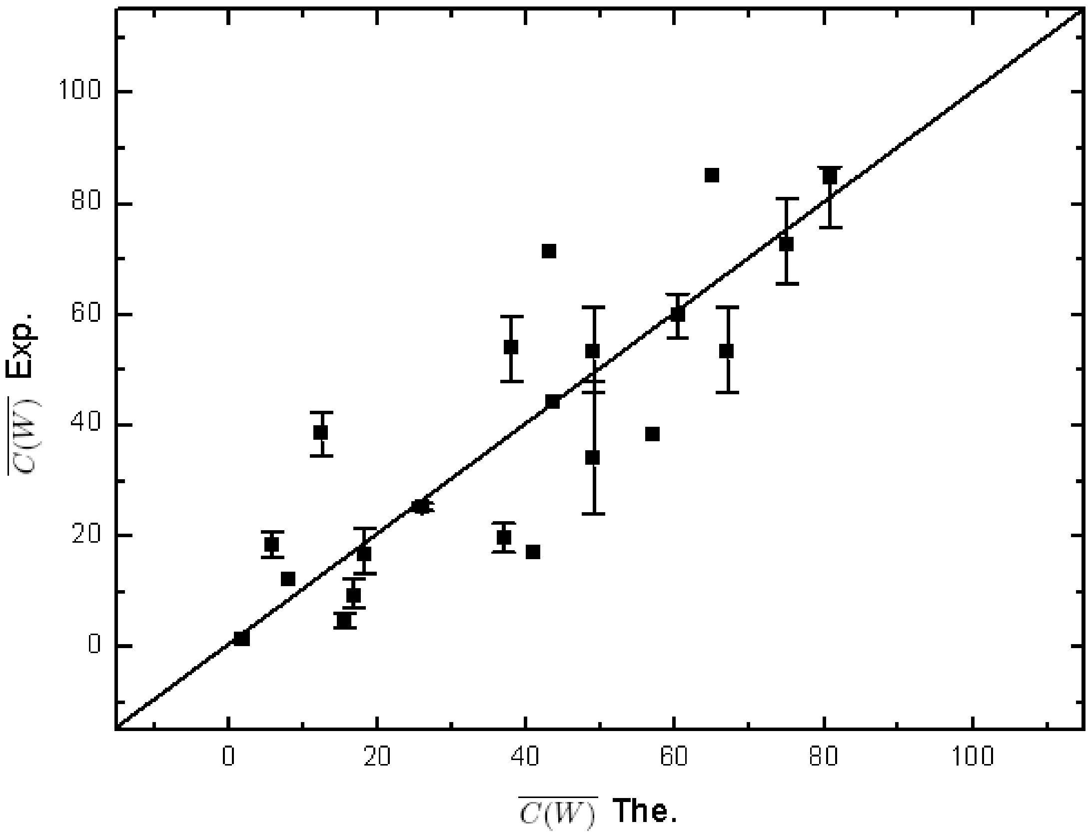
$$\overline{C(W)} = \frac{6146\ s}{f(0)t} = \frac{f}{f(0)} \tag{8}$$

Here f takes the form of Eq (2) and $f(0)$ is Eq. (7)

Quenching for different operators

- It is known that the theoretical GT strength is overestimated by a factor, which always called quenching factor. This factor can be seen as the percentage of wave function that lies at the configuration space.

- Typical quenching values for GT are around 0.7.



From the fitting, the quenching factors of FF are:

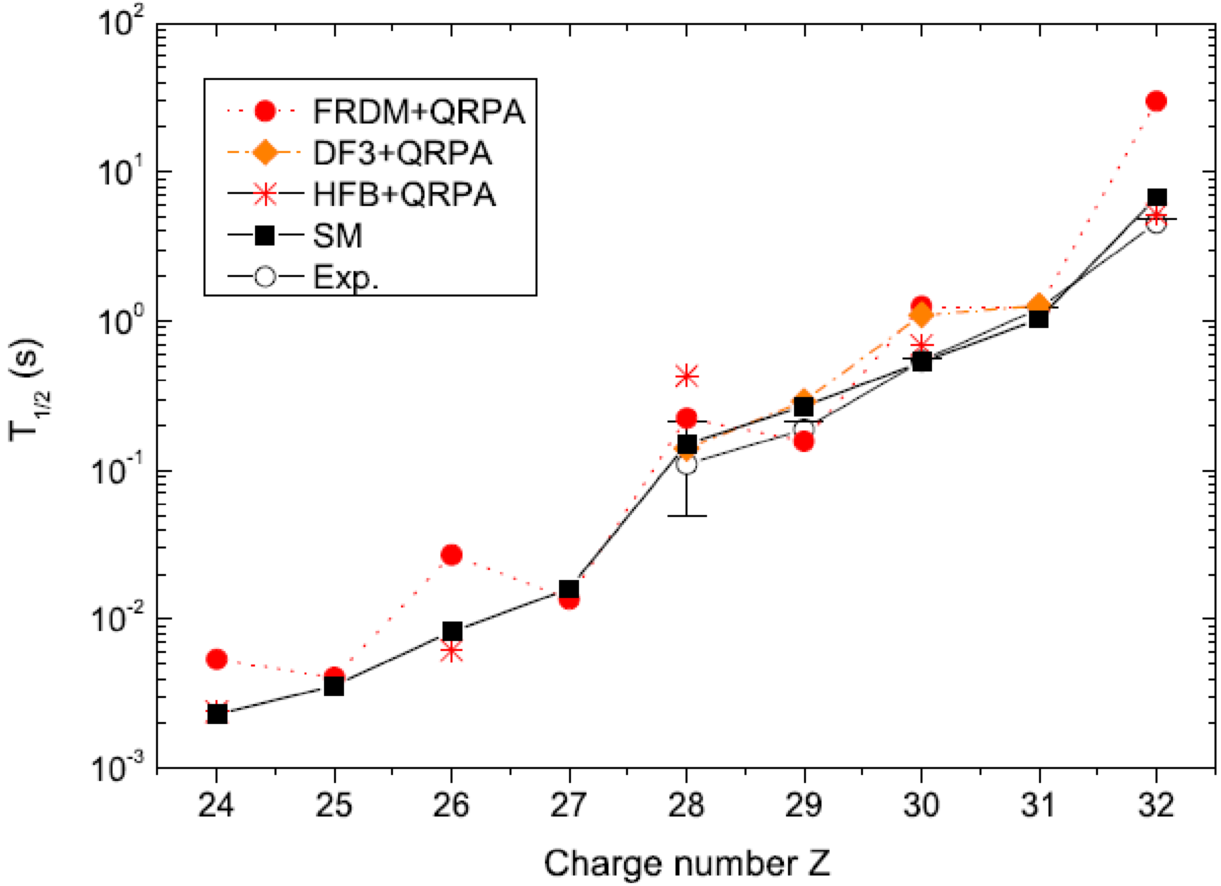
- w,w': 0.62; x,x': 0.51
- u,u': 0.36; z: 0.45
- $\xi'v$: 1.23; $\xi'y$: 0.51

After the quenching factors of FF are fixed, the quenching factors of GT are determined to be 0.66, which reproduces the half live of ^{130}Cd.

Results

Results for N=50

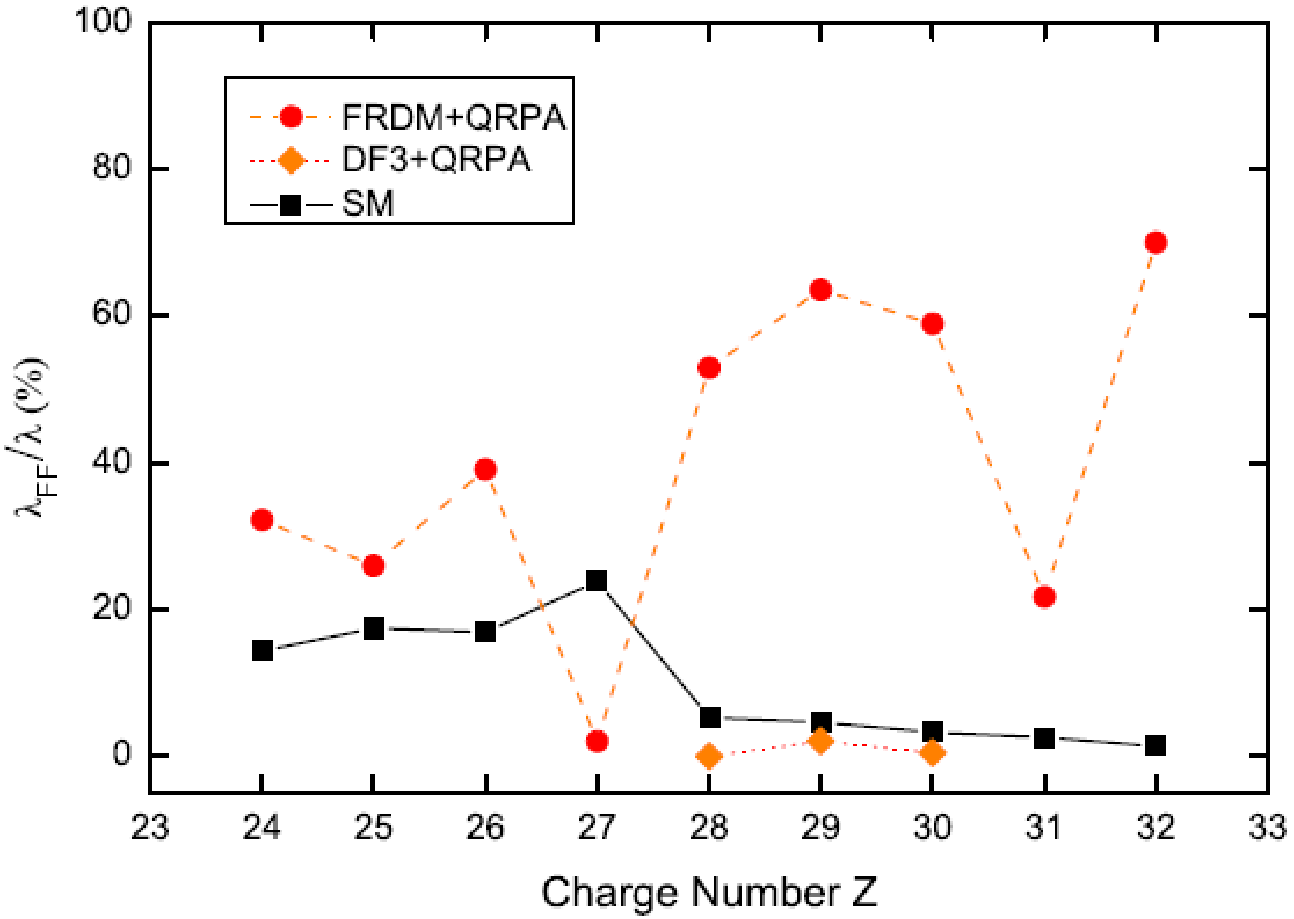
The half-lives of the *N* = 50 isotones.

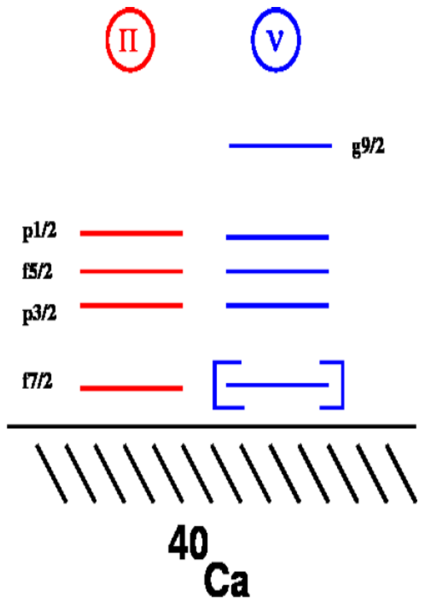


FRDM+QRPA:P. Moller et al, At. Data Nucl.Data Tables 66, (1997); Phys. Rev. C 67, (2003)
HFB+QRPA: J. Engel et al, Phys. Rev. C 60, 014302 (1999)
DF3+QRPA: I. N. Borzov et al., Phys. Rev. C 62, 035501(2000); Nucl. Phys. A 777, 645 (2006)

Results for N=50

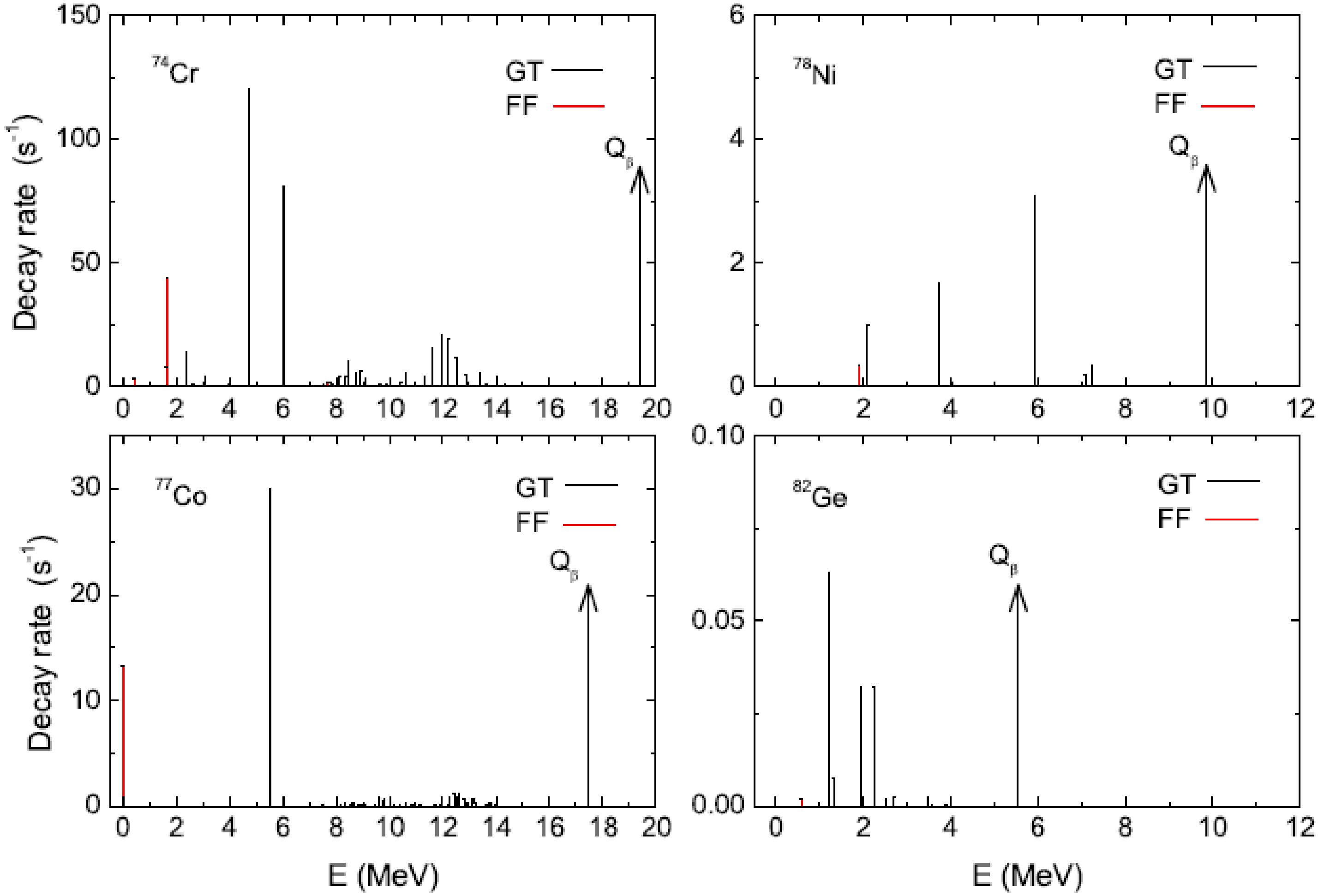
The effect of FF transitions to the half-lives.





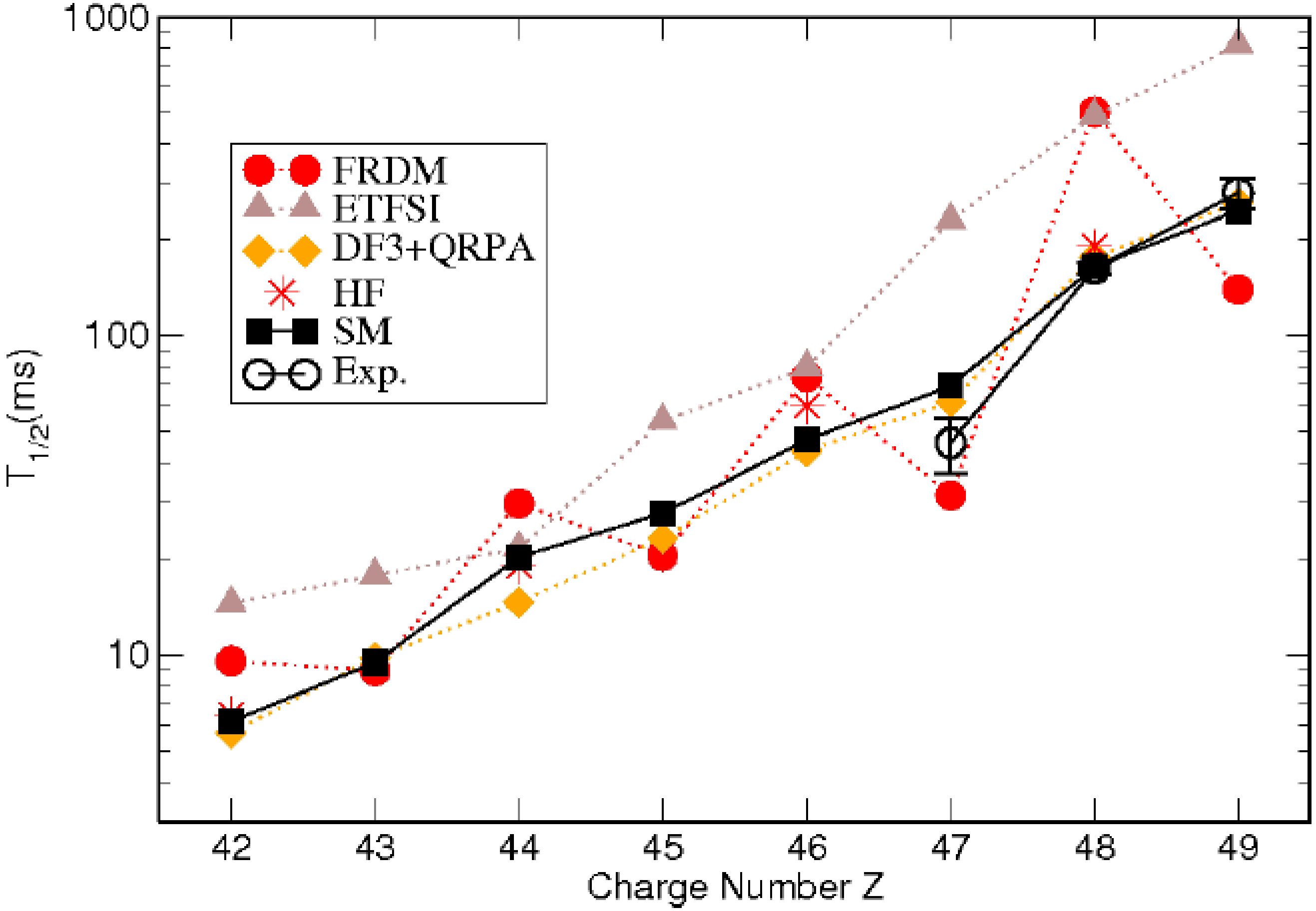
Results for N=50

The decay rates of GT transition and FF transition.



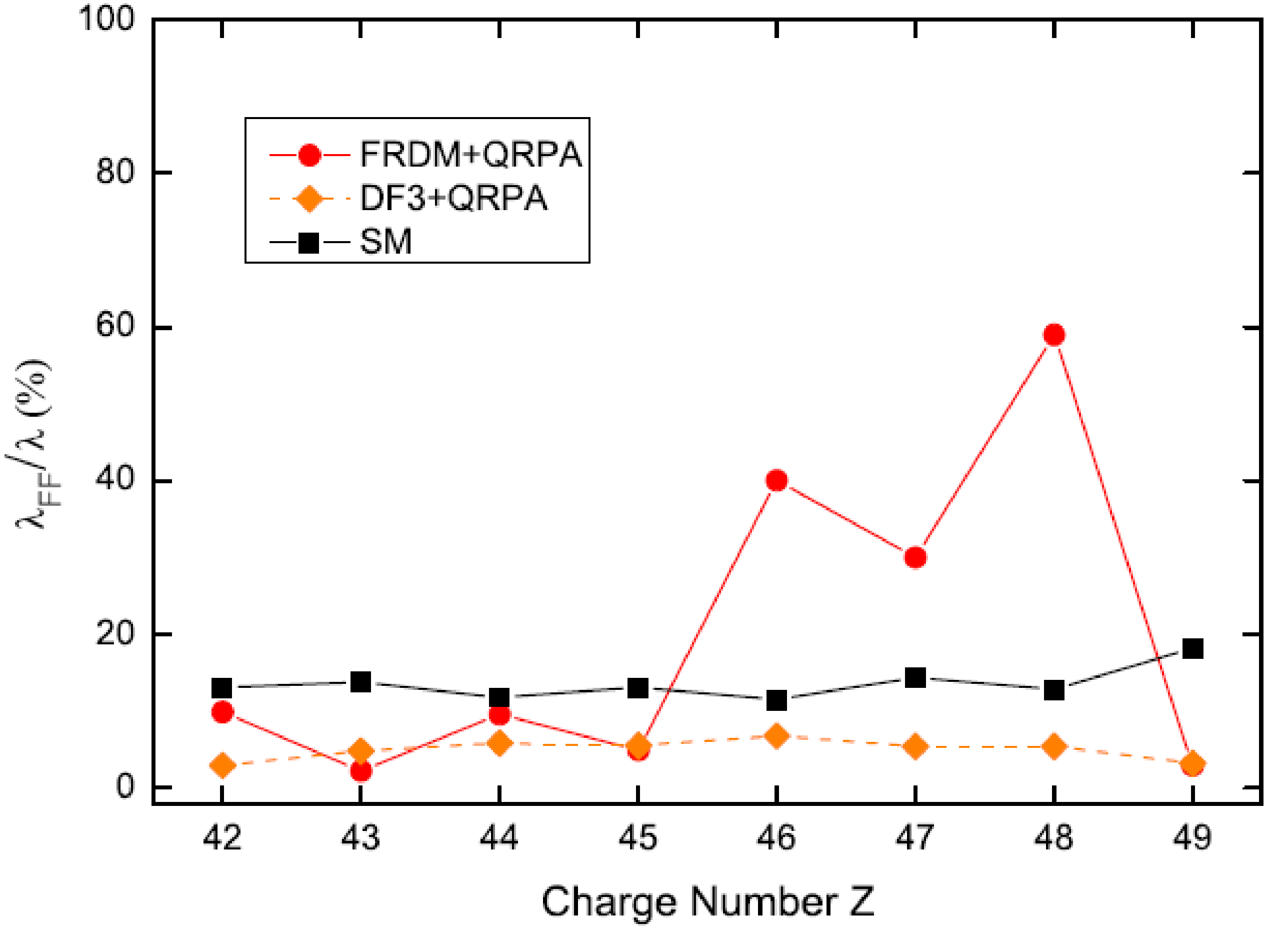
Results for N=82

Half-lives of the N = 82 isotones are compared with different theoretical model calculations.



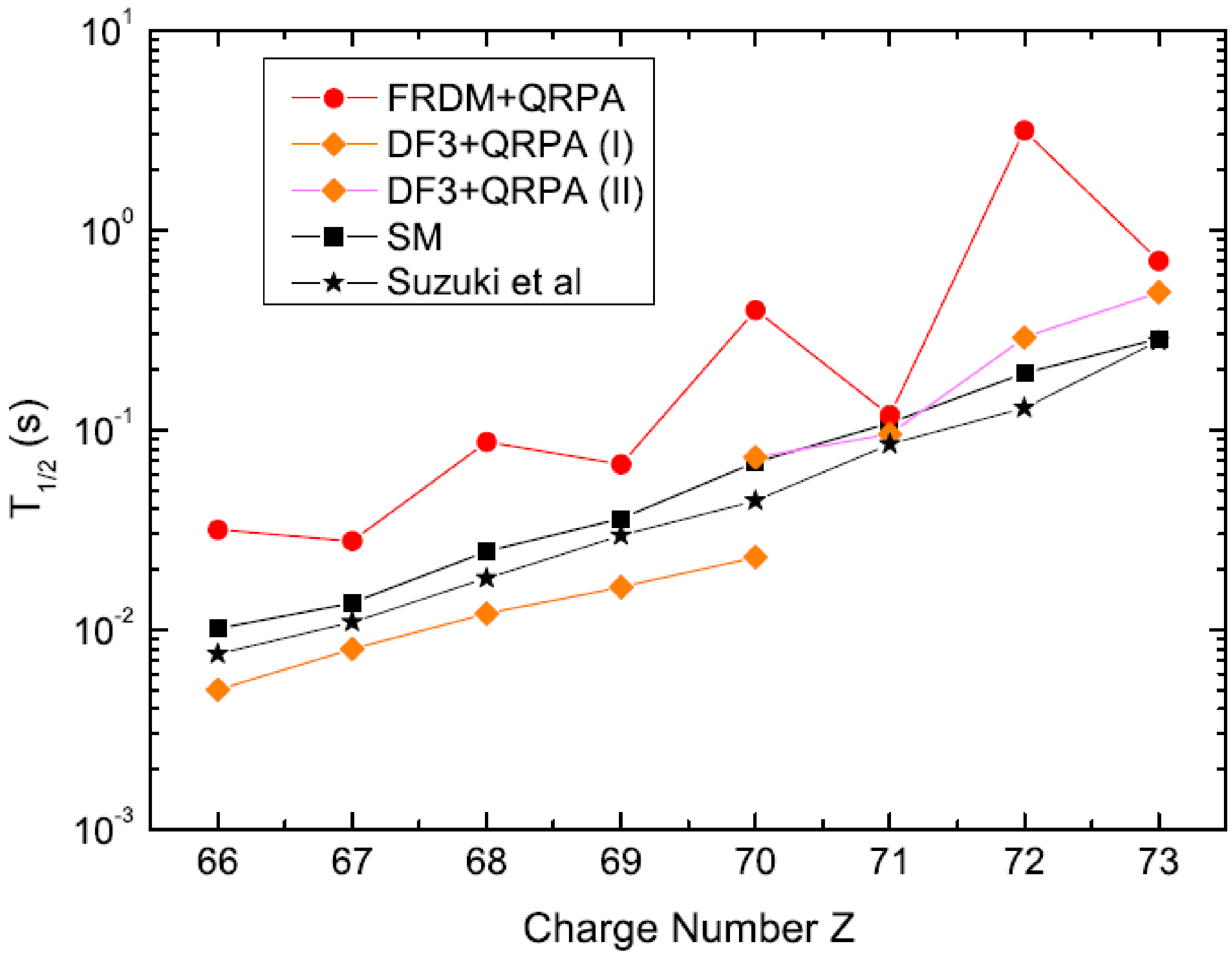
Results for N=82

Contribution of the first-forbidden transitions of the N= 82 isotones expressed in %.

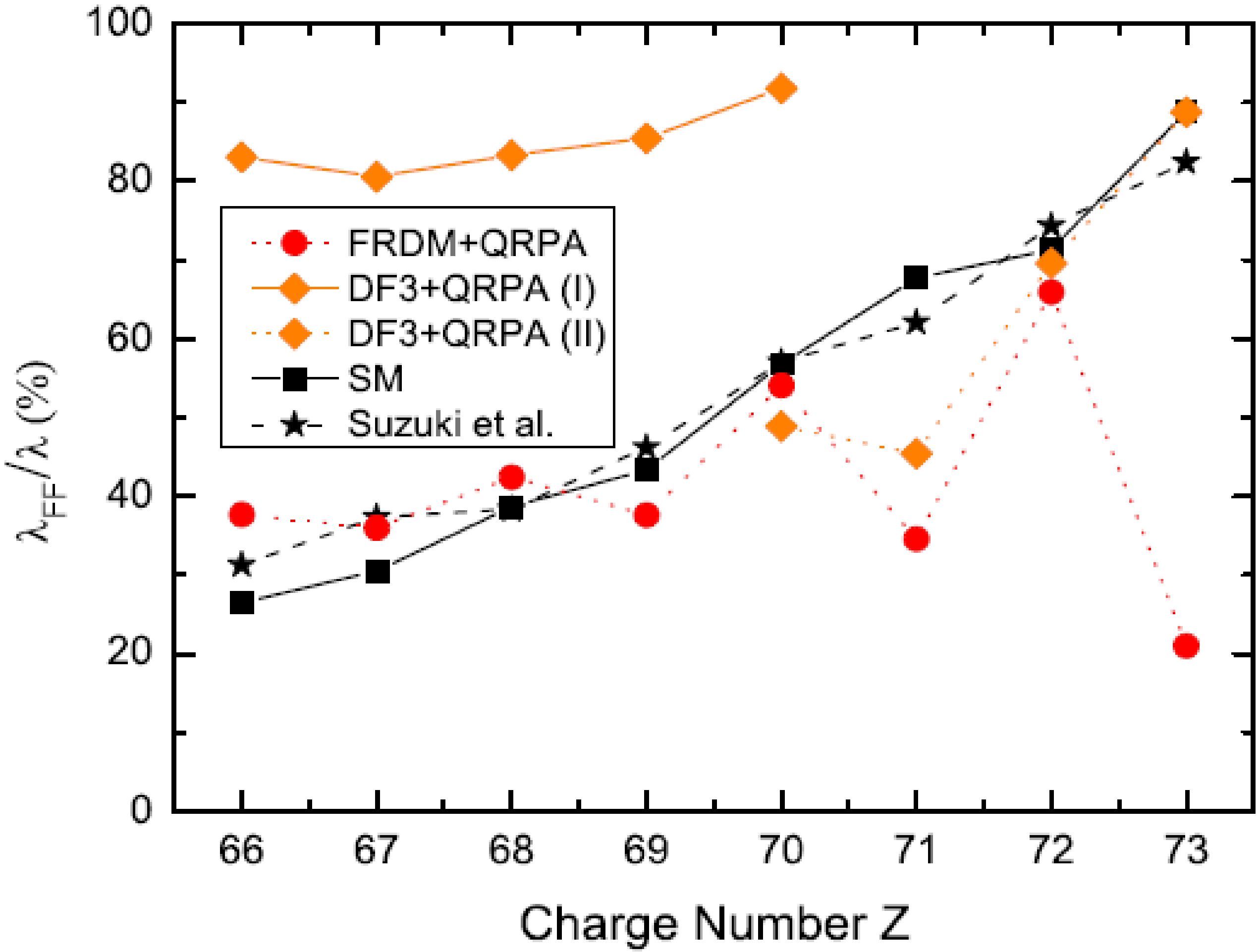


Results for N=126

Comparison of half-lives of the $N = 126$ isotones



Contribution of the first-forbidden transitions of the N= 126 isotones expressed in %



Conclusions and outlook

Conclusions:

- We have made calculations of beta decay half lives, which the FF transition are included. for the R process waiting point nuclei around N=50,82 and 126.
- The contribution from FF transitions are important, especially for nuclei around N=126 region, and FF must be included in the half-lives of R-process nuclei.

Outlook:

- To do more systematic calculations for other nuclei
- To incorporate the rates with FF transitions into nucleisynthesis network calculations.

Thank you !