On the Physical Meaning of Sachs Form Factors and on the Violation of the Dipole Dependence of G_E and G_M on Q^2

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The questions of how a dipole character of the dependence of the form factors G_E and G_M on the square of the momentum transfer to a proton, Q^2, arise and why a violation of this dependence occurs, which was first observed in a JLab polarization experiment, are investigated. The answers to these questions could be obtained owing to the use of the simplest QCD concepts of the proton structure and the results obtained by calculating the matrix elements of the proton current in the case of non-spin-flip and spin-flip transitions for protons in the diagonal spin basis (DSB), where the little Lorentz group common to the initial and final proton states is realized. In DSB, the form factors G_E and G_M are determined by the matrix elements $J_p^{\delta,\delta}$, and $J_p^{-\delta,\delta}$ of the proton current in the cases of non-spin-flip and spin-flip transitions for protons. In an arbitrary reference frame, the relations between these matrix elements and the form factors are $J_p^{\delta,\delta} \sim G_E$ and $J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M$ where $\tau = Q^2/4m^2$, with m being the proton mass. In considering the problem in question at the quark level, use is made of the model where the proton consists of three pointlike quarks having identical masses and where the respective matrix element of the proton current is the product of three quark-current amplitudes having the form $J_q^{\delta,\delta} \sim 1$ and $J_q^{-\delta,\delta} \sim \sqrt{\tau}$. It is shown that the aforementioned dipole dependence arises if the proton spin-flip is due to spin-flip for only one of the three quarks. As to violations of this dependence, they are caused by the contributions to $J_p^{\delta,\delta}$ from spin-flip transitions for two quarks or by the contribution to $J_p^{-\delta,\delta}$ from spin-flip transitions for all three quarks constituting the proton.

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1. INTRODUCTION

Experiments aimed at studying the proton form factors, the electric (G_E) and magnetic (G_M) ones, which are frequently referred to as the Sachs form factors, have been performed since the mid-1950s [1, 2] by using elastic electron-proton scattering. In the case of unpolarized electrons and protons, all experimental data on the behavior of the proton form factors were obtained by using the Rosenbluth formula [1] for the differential cross section for the reaction $ep \to ep$; that is,

$$\frac{d\sigma}{d\Omega_e} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_1^3 \sin^4(\theta_e/2)} \frac{1}{1+\tau} \left(G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right). \quad (1)$$

Here, $\tau = Q^2/4m^2$, $Q^2 = -q^2 = 4E_1E_2 \sin^2(\theta_e/2)$ is the square of the momentum transfer to the proton and m is the proton mass; E_1, E_2 and θ_e are, respectively, the initial-electron energy, the final-electron energy, and the electron scattering angle in the rest frame of the initial proton; the quantity ε is the degree of virtual photon linear polarization, $\varepsilon^{-1} = 1+2(1+\tau)\tan^2(\theta_e/2)$; and $\alpha = 1/137$ is the fine-structure constant. Expression (1) was obtained in the approximation of one-photon exchange. In deriving it, the electron mass was set to zero. With the aid of Rosenbluth's technique, it was found that the experimental dependences of G_E and G_M on Q^2 are well described up to 10 GeV^2 by the dipole-approximation expression

$$G_E = G_M/\mu = G_D(Q^2) \equiv (1 + Q^2/0.71)^{-2}, \quad (2)$$

where μ is the proton magnetic moment (μ=2.79).

In [3], Akhiezer and Rekalo proposed a method for measuring the ratio of the Sachs form factors. Their method relies on the phenomenon of polarization transfer from the longitudinally polarized initial electron to the final proton. They showed that the ratio of the degrees of longitudinal (P_l) and transverse (P_t) polarizations of the scattered proton has the form

$$\frac{P_l}{P_t} = -\frac{G_M}{G_E} \frac{E_1 + E_2}{2m} \tan\frac{\theta_e}{2}. \quad (3)$$

Precision experiments based on employing Eq. (3) were performed at JLab and were reported in [4, 5]. They showed that, in the range of $0.5 < Q^2 < 5.6$ GeV^2, there was a linear decrease in the ratio $\mu G_E/G_M$ with increasing Q^2:

$$\mu \frac{G_E}{G_M} = 1 - 0.13\,(Q^2 - 0.04). \quad (4)$$

This is at contradicts with data obtained with the aid of Rosenbluth's technique. According to those data, G_E and G_M approximately follow the dipole form up to

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the value of $Q^2 = 10$ GeV^2; concurrently, the approximate equality $\mu G_E/G_M \approx 1$ must hold. Repeated, more precise, measurements of the ratio $R = \mu G_E/G_M$ by Rosenbluth's method [6] only confirmed this contradiction, showing that the magnetic form factor did not differ within the errors from its counterpart obtained within Rosenbluth's technique and that the electric form factor fell short of the respective value in accordance with Eq. (4).

In order to resolve this contradiction, it was assumed that the discrepancy in question may be caused by disregarding, in the respective analysis, the contribution of two-photon exchange. There appeared a large number of articles devoted to this problem (see [7]; see also the review article of Arrington et al. [8] and references therein). At the present time, three experiments aimed at studying the contribution of two-photon exchange are known. These are an experiment at the VEPP-3 storage ring in Novosibirsk, the OLYMPUS experiment at the DORIS accelerator at DESY in Hamburg (Germany), and the EG5 CLAS experiment at JLab (USA).

In [9], we proposed a method for determining the Sachs form factors in the process $ep \to ep$ on the basis of measuring cross sections for spin-flip and non-spin-flip transitions for protons.

The objective of the present study is to show that the fundamental physical meaning of the form factors G_E and G_M is associated with their factorization in the matrix elements of the proton current that correspond to non-spin-flip and spin-flip transitions for protons. It is precisely this circumstance that explains the appearance of the squares of the Sachs form factors in Rosenbluth's cross section.

Yet another objective of this study is to show that the mechanism of one-photon exchange is sufficient for explaining the results of the polarization experiment at JLab and that violations of the dipole dependence at high values of Q^2 are due to the contribution of spin-flip quark-current amplitudes to non-spin-flip and spin-flip transitions for the proton.

2. ON THE PHYSICAL MEANING OF THE SACHS FORM FACTORS

Rosenbluth's cross section in the rest frame of the primary proton (1) has a compact form owing to the decomposition of G_E^2 and G_M^2. In text-books on particle physics, it is shown that the physical meaning of the form factors G_E and G_M is that, in the Breit frame of the initial and the final proton, they describe the distributions of the proton charge and magnetic moment, respectively; this means that, in the Breit frame, the matrix elements of the proton current for non-spin-flip and spin-flip transitions for the proton are expressed in terms of G_E and G_M, respectively. Moreover, the Sachs form factors are advantageous in view of the simplicity of expression (1).

The question of whether there is any physical meaning behind the decomposition of G_E^2 and G_M^2 in Rosenbluth's cross section was not raised and not discussed either in textbooks or in scientific literature published in the last 25 years. Nevertheless, it was shown many years ago in the article of Sikach [10] that the form factors G_E and G_M factorize in the diagonal spin basis (DSB) even at the level of amplitudes in calculating (in an arbitrary reference frame) the matrix elements of the proton current in the cases of non-spin-flip and spin-flip transitions for the proton.

2.1. Diagonal Spin Basis

In DSB, the spin 4-vectors s_1 and s_2 of fermions with 4-momenta q_1 (before the interaction) and q_2 (after it) have the form [10]

$$s_1 = -\frac{(v_1 v_2)v_1 - v_2}{\sqrt{(v_1 v_2)^2 - 1}}\ ,\ \ s_2 = \frac{(v_1 v_2)v_2 - v_1}{\sqrt{(v_1 v_2)^2 - 1}}\ , \tag{5}$$

where $v_1 = q_1/m$ and $v_2 = q_2/m$. Obviously, the spin 4-vectors in (5) satisfy ordinary conditions − that is, $s_1 q_1 = s_2 q_2 = 0$ and $s_1^2 = s_2^2 = -1$ − and are invariant under the transformations of the little group of Lorentz group (little Wigner group [11]) $L_{q_1 q_2}$ common to particles with 4-momenta q_1 and q_2: $L_{q_1 q_2} q_1 = q_1$ and $L_{q_1 q_2} q_2 = q_2$. We note that this group is isomorphic to the one-parameter subgroup of the rotational group $SO(3)$ with an axis whose direction is determined by the three-dimensional vector [12, 13]

$$\boldsymbol{a} = \boldsymbol{q}_1/q_{10} - \boldsymbol{q}_2/q_{20}\ . \tag{6}$$

For the two particles in question, the spin projections onto the direction specified by the vector in Eq. (6) simultaneously have specific values [12, 13], and the concept of non-spin-flip and spin-flip transitions acquires an absolute physical meaning.

The vector $\boldsymbol{a}$ in Eq. (6) is the difference of two three-dimensional vector, and the geometric image of the difference of two 3-vectors is a diagonal of the parallelogram spanned by these two vectors. This is the reason why the term “DSB” was introduced by academician F.I. Fedorov.

Let us consider the realization of DSB in the initial proton rest frame, where $q_1 = (q_{10}, \boldsymbol{q}_1) = (m, \boldsymbol{0})$. In this case for the vector $\boldsymbol{a}$ in Eq.(6) we have: $\boldsymbol{a} = \boldsymbol{n}_2 = \boldsymbol{q}_2/|\boldsymbol{q}_2|$; that is, the direction of final proton motion is a common direction onto which one projects the spins in question. Consequently, the polarization state of the final proton is a helicity state, while the spin 4-vectors s_1 and s_2 in (5) have the form

$$s_1 = (0, \boldsymbol{n}_2),\ s_2 = (|\boldsymbol{v}_2|, v_{20}\, \boldsymbol{n}_2), \tag{7}$$

that is, the axes of the spin projections $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ coincide with the direction of final-proton motion: $\boldsymbol{c}_1 = \boldsymbol{c}_2 = \boldsymbol{n}_2$.

The Breit frame, where $\boldsymbol{q}_2 = -\boldsymbol{q}_1$, is a particular case of the DSB.

2.2. Spin Operators and Calculation of Amplitudes for QED Processes in DSB

In DSB (5), the spin projection operators σ_1 and σ_2 for the initial and final Dirac particles with 4-momenta q_1 and q_2 coincide, as well as the respective raising and lowering spin operators $\sigma_1^{\pm\delta}$ and $\sigma_2^{\pm\delta}$, by virtue of the realization of the little Lorentz group $L_{q_1 q_2}$ in DSB and have the form [14, 15]

$$\sigma = \sigma_1 = \sigma_2 = \gamma^5 \hat{s_1}\hat{v_1} = \gamma^5 \hat{s_2}\hat{v_2} = \gamma^5 \hat{b}_0 \hat{b}_3,$$
$$\sigma^{\pm\delta} = \sigma_1^{\pm\delta} = \sigma_2^{\pm\delta} = -i/2\gamma^5 \hat{b}_{\pm\delta}, b_{\pm\delta} = b_1 \pm i\delta b_2, \quad (8)$$
$$\sigma u^\delta(q_i) = \delta u^\delta(q_i), \sigma^{\pm\delta} u^{\mp\delta}(q_i) = u^{\pm\delta}(q_i), \delta = \pm 1,$$

where $u^\delta(q_i) = u^\delta(q_i, s_i)$ are the bispinors of the initial and final states of the particles in DSB; $\hat{s}_1 = (s_1)_\mu \gamma^\mu$, γ^5, γ^μ are the Dirac matrixes.

In expressions (8), an orthonormalized basis of vectors b_A $(A = 0, 1, 2, 3)$,

$$(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa b_2^\sigma, \ (b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} q_1^\nu q_2^\kappa r^\sigma / \rho,$$
$$b_3 = q_- / \sqrt{-q_-^2}, \ b_0 = q_+ / \sqrt{q_+^2} \ , \quad (9)$$

was used to construct the respective spin operators. Here, $q_- = q_2 - q_1$, $q_+ = q_2 + q_1$, $\varepsilon_{\mu\nu\kappa\sigma}$ is the Levi-Civita tensor ($\varepsilon_{0123} = -1$), r is the participant-particle 4-momentum differing from q_1 and q_2, and ρ is determined from the normalization conditions $b_1^2 = b_2^2 = b_3^2 = -b_0^2 = -1$.

The matrix elements for QED processes have the form

$$M^{\pm\delta,\delta} = \overline{u}^{\pm\delta}(q_2) Q u^\delta(q_1) \ , \quad (10)$$

where Q is the interaction operator and $u^\delta(q_1)$ and $u^{\pm\delta}(q_2)$ are the bispinors of, respectively, the initial and the final state.

In the approach that we use, the calculation of matrix elements (amplitudes) that have the form (10) and which correspond to the fermion transition from the initial state $u^\delta(q_1)$ to the final state $u^{\pm\delta}(q_2)$ reduces to evaluating the trace of the product of Dirac operators [12, 14, 15]; that is,

$$M^{\pm\delta,\delta} = Tr(P_{21}^{\pm\delta,\delta} Q) \ , \ P_{21}^{\pm\delta,\delta} = u^\delta(q_1) \, \overline{u} \ ^{\pm\delta}(q_2) \ . \quad (11)$$

The explicit form of the operators $P_{21}^{\pm\delta,\delta}$ in DSB that correspond to non-spin-flip ($P_{21}^{\delta,\delta}$) and spin-flip ($P_{21}^{-\delta,\delta}$) transitions was obtained in [14, 15] and is given by

$$P_{21}^{\delta,\delta} = (\hat{q}_1 + m) \, \hat{b}_\delta \, \hat{b}_0 \, \hat{b}_\delta^* / 4 \ , \quad (12)$$
$$P_{21}^{-\delta,\delta} = \delta(\hat{q}_1 + m) \, \hat{b}_\delta \, \hat{b}_3 / 2 \ , \quad (13)$$

where $b_\delta^* = b_{-\delta} = b_1 - i\delta b_2$ and $b_\delta b_\delta^* = -2$.

2.3. Amplitudes of the Proton Current in DSB

In the Born approximation, the matrix element corresponding to the process of elastic electron - proton scattering,

$$e(p_1) + p\,(q_1, s_1) \rightarrow e(p_2) + p\,(q_2, s_2) \ , \quad (14)$$

has the form

$$M_{ep \rightarrow ep} = \overline{u}(p_2) \gamma^\mu u(p_1) \cdot \overline{u}(q_2) \Gamma_\mu(q^2) u(q_1) \, \frac{1}{q^2} \ , \quad (15)$$

$$\Gamma_\mu(q^2) = F_1 \gamma_\mu + \frac{F_2}{4M} (\hat{q}\gamma_\mu - \gamma_\mu \hat{q} \ ) \ , \quad (16)$$

where $u(p_i)$ and $u(q_i)$ are the bispinors of, respectively, the electrons and protons with 4-omenta p_i and q_i [accordingly, we have $p_i^2 = m_e^2$, $q_i^2 = m^2$, $\overline{u}(p_i) u(p_i) = 2m_e$, and $\overline{u}(q_i) u(q_i) = 2m \, (i = 1, 2)$]; F_1 and F_2 are, respectively, the Dirac and Pauli form factors; $q = q_2 - q_1$ is the 4-momentum transfer to the proton; and s_1 and s_2 are the polarization 4-vectors of, respectively, the initial and final protons.

The matrix elements of the proton current that correspond to non-spin-flip and spin-flip transitions for the proton are given by

$$(J_p^{\pm\delta,\delta})_\mu = \overline{u}^{\pm\delta}(q_2) \Gamma_\mu(q^2) u^\delta(q_1) \ . \quad (17)$$

With the aid of Eqs. (11) – (13), we can readily show that the matrix elements of the proton current in (17) that are calculated in DSB (5) have the form [10, 15]

$$(J_p^{\delta,\delta})_\mu = 2m \, G_E (b_0)_\mu \ , \quad (18)$$
$$(J_p^{-\delta,\delta})_\mu = -2m \, \delta \sqrt{\tau} G_M (b_\delta)_\mu \ , \quad (19)$$

$$G_E = F_1 + \frac{q^2}{4m^2} \, F_2 \ , \ G_M = F_1 + F_2 \ , \quad (20)$$

where G_E and G_M are the Sachs form factors and the quantities $\tau = Q^2/4m^2$, $Q^2 = -q^2$, $q = q_- = q_2 - q_1$, b_0, and b_δ were defined above.

We note that the amplitudes of the proton current in (18) and (19) satisfy the conditions of gauge invariance since, by virtue of the definitions of the 4-vectors b_0 and b_δ, the scalar products $b_0 q$ and $b_\delta q$ are equal to zero. Further, the matrix element $(J_p^{\delta,\delta})_\mu$ of the proton current in (18) for the non-spin-flip transition for the proton is expressed in terms of the 4-vector b_0. This matrix element corresponds to the exchange of a virtual photon that has a scalar polarization ($b_0^2 = 1$) and which therefore cannot carry away a spin moment. At the same time, the matrix element $(J_p^{-\delta,\delta})_\mu$ in (19) for the spin-flip transition for the proton is expressed in terms of the complex 4-vector b_δ. It corresponds to the exchange of a virtual photon having a circular polarization vector ($b_\delta^2 = 0, b_\delta b_\delta^* = -2$) and carrying away a spin moment, with the result that there occurs proton spin-flip. Thus, our analysis of expressions (18) and (19) obtained for the matrix elements in question leads to the conclusion that these expressions are fully adequate to the physical picture of the phenomena being considered. It follows that the electric and magnetic form factors G_E and G_M acquire a fundamental physical meaning owing to their factorization in the matrix elements of the proton current for non-spin-flip and spin-flip transitions for the proton. It is precisely because of the factorization of G_E and G_M in the amplitudes in Eqs. (18) and (19) that the contributions to Rosenbluth's cross section for non-spin-flip and spin-flip transitions for the proton are controlled by the terms containing G_E^2 and G_M^2, respectively.

In the case of pointlike particles having a mass m_0, the amplitudes for their currents have the form

$$(J_q^{\delta,\delta})_\mu = 2\,m_0\,(b_0)_\mu\,, \qquad (21)$$

$$(J_q^{-\delta,\delta})_\mu = -2\,m_0\,\delta\sqrt{\tau_0}\,(b_\delta)_\mu\,, \tau_0 = Q_q^2/4m_0^2\,. \quad (22)$$

In the ultrarelativistic massless case, only spin-flip transitions [see Eqs. (19) and (22)] contribute to the cross section for the process being considered, since the amplitudes in (18) and (21) vanish. At first glance, this conclusion contradicts the well-known fact that, in the ultrarelativistic limit, only processes in which the particle helicity is conserved survive at high energy; that is, only amplitudes corresponding to non-helicity-flip transitions do not vanish in the massless limit. Such processes are frequently referred to as non-spin-flip processes. However, this terminology is quite uncertain since the particles involved have different directions of motion before and after the interaction event. Moreover, it is erroneous since, in non-helicity-flip processes, the spins of the particles are in fact flipped at high energies. There is no contradiction here since, in DSB, the initial state for ultrarelativistic particles is a helicity state, while the final state has a negative helicity [15] (see Eqs. (A7) and (A8)), with the result that

$$M^{-\delta,\delta} = M^{-(-\lambda),\lambda} = M^{\lambda,\lambda},\, M^{\delta,\delta} = M^{-\lambda,\lambda} = 0\,. \quad (23)$$

We note that, in addition to the representation in (16) for $\Gamma_\mu(q^2)$, the following equivalent representation is used in the literature for this quantity:

$$\Gamma_\mu(q^2) = G_M\gamma_\mu - \frac{(q_1+q_2)_\mu}{2m}\,F_2\,. \qquad (24)$$

On the basis of explicit form (16) and (24) for $\Gamma_\mu(q^2)$, in the literature it is likely just starting with the paper of Lepage and Brodsky [17] stated that the Dirac (Pauli) form factor F_1 (F_2) corresponds to helicity-non-flip (helicity-flip) transitions of the proton, respectively. In fact, it is the form factor G_E (G_M) rather than F_2 (F_1) [see Eq. (18), (19), (23)] that is responsible for helicity-flip (helicity-non-flip) transitions at high q_1 and q_2.

We also note that in the literature sometimes there is no clear understanding of the physical meaning of the quantity ε in formula (1). So in [18] written that the quantity ε is a measure (degree) of the longitudinal polarization of the virtual photon. In fact ε is the degree of linear polarization of the virtual photon (see [3, 19]).

3. ON THE VIOLATION OF THE DIPOLE CHARACTER OF THE Q^2 DEPENDENCE OF G_E AND G_M

Since $|b_0| = 1$ and $|b_\delta b_\delta^*| = 2$, the Q^2 dependence of the absolute values of the matrix elements of the proton (18) and pointlike-particle ($J_q^{\pm\delta,\delta}$) currents can readily be obtained from Eqs. (18), (19), (21), and (22). The results are

$$J_p^{\delta,\delta} \sim\, 2\,m\,G_E, J_p^{-\delta,\delta} \sim\, 2m\sqrt{\tau}\,G_M\,, \qquad (25)$$

$$J_q^{\delta,\delta} \sim 2\,m_0\,,\; J_q^{-\delta,\delta} \sim 2m_0\sqrt{\tau_0}\,. \qquad (26)$$

We note that the factorization of $2m$ in expressions (18), (19), (21), (22), (25), and (26) is due to normalizing the particle bispinors by the condition $\bar{u}_i u_i = 2m_i$. In performing further calculations, it is more convenient to employ the normalization condition $\bar{u}_i u_i = 1$. Instead of expressions (25) and (26), we will then use the expressions

$$J_p^{\delta,\delta} \sim\, G_E, J_p^{-\delta,\delta} \sim\, \sqrt{\tau}\,G_M\,, \qquad (27)$$

$$J_q^{\delta,\delta} \sim 1\,,\;\; J_q^{-\delta,\delta} \sim \sqrt{\tau_0}\,. \qquad (28)$$

Relations (27) and (28) make it possible to show how there arise the dipole dependence of G_E and G_M on Q^2 and its violations observed in the aforementioned JLab experiment. In considering the problem at the quark level, we will employ, for this purpose, a model where the proton consists of three pointlike quarks having the same mass m_0 and where the respective matrix element of the proton current is the product of three quark-current amplitudes having the form $J_q^{\delta,\delta} \sim 1$ and $J_q^{-\delta,\delta} \sim \sqrt{\tau_0}$. Below, we will show that the dipole dependence arises at relatively moderate values of the momentum transfer squared, in which case non-spin-flip quark-current amplitudes are dominant. As Q^2 grows, the spin-flip quark-current amplitudes begin making a significant contribution to expressions (18) and (19), and this ultimately leads to the dependence in (4).

There are two possibilities for a proton non-spin-flip transition: (i) none of the three quarks undergoes a spin-flip transition, and (ii) two quarks undergo a spin-flip transition, while the third does not. We denote the number of such ways as $n_q^{\delta,\delta} = [0,2]$ in accordance with the number of quarks involved in a spin-flip process (none or two).

Proton spin flip can also proceed in two ways: (i) one quark undergoes a spin-flip transition, while the other two do not, and (ii) all three quarks undergo a spin-flip transition. We denote the number of such ways by $n_q^{-\delta,\delta} = [1,3]$ in accordance with the number of quarks involved in a spin-flip process (one or three). Thus, there are in all four combinations to be considered:

$$n_q^{\delta,\delta} \times n_q^{-\delta,\delta} = (0,1) + (0,3) + (2,1) + (2,3)\,. \qquad (29)$$

Of these, the first, (0, 1), corresponds to the dipole dependence of the form factors G_E and G_M on Q^2, in which case none of the quarks reverses a spin upon the proton non-spin-flip transition (the first number in parentheses is zero); at the same time, the proton spin-flip is due to the spin-flip for only one quark (the second number in parentheses is equal to unity).

We obtain $G_E/G_M \sim 1$ for the (0, 1) and (2, 3) sets in (29), $Q^2G_E/G_M \sim 4m^2$ for the (0, 3) set, and $Q^2G_M/G_E \sim 4m^2$ for the (2, 1) set.

3.1. Dipole Dependence of the Form Factors G_E and G_M on Q^2, $G_E/G_M \sim 1$

In order to show how there arises the dipole dependence in the behavior of the Sachs form factors, we will make use of the above expressions (27) and (28) and rely on a model where a proton consists of three pointlike quarks having identical masses and where the proton-current amplitude is the product of three quark-current amplitudes. It is convenient to represent this conceptual framework in the form of the following diagrams:

$$J_d^{\delta,\delta} = \begin{matrix} + & \to\to * \to\to\to\to & + \\ - & \to\to\to * \to\to\to & - \\ + & \to\to\to\to * \to\to & + \end{matrix} \quad \text{non-spin-flip}, \quad (30)$$

$$J_d^{-\delta,\delta} = \begin{matrix} + & \to\to * \to\to\to\to & - \\ - & \to\to\to * \to\to\to & - \\ + & \to\to\to\to * \to\to & + \end{matrix} \quad \text{spin-flip}. \quad (31)$$

The diagram in Eq. (30) corresponds to a proton non-spin-flip transition for the case where there is no spin flip for any of the three quarks. It follows that, in this case, the matrix element of the proton current must be proportional to G_E [see Eq. (27)]. As a result, we have

$$J_d^{\delta,\delta} \sim G_E \sim 1 \times 1 \times 1 \times \frac{1}{Q^4}, \quad (32)$$

where the factors of unity correspond to non-spin-flip transitions [see Eq. (28)] for three pointlike quarks of mass m_0 and Q^4 arises in the denominator owing to two gluon propagators. From here, we obtain

$$G_E \sim \frac{1}{Q^4}. \quad (33)$$

The diagram in Eq. (31) corresponds to the transition where spin-flip occurs for the up quark but does not occur for the two down quarks; in summary, this corresponds to the proton spin-flip transition. According to Eqs. (27), the matrix element of the proton current must be proportional to $\sqrt{\tau}\, G_M$ in this case. As a result, we have

$$J_d^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau_0} \times 1 \times 1 \times \frac{1}{Q^4}. \quad (34)$$

whence we obtain

$$G_M \sim \frac{\sqrt{\tau_0}}{\sqrt{\tau}}\, \frac{1}{Q^4}. \quad (35)$$

The factor $\sqrt{\tau_0}$ on the right-hand side of Eq. (34) corresponds to the spin-flip transition for the up quark, while the two factors of 1 correspond to the non-spin-flip transition for the down quarks; two gluon propagators yield Q^4 in the denominators on the right-hand sides of (34) and (35). In order to calculate the ratio $\sqrt{\tau_0}/\sqrt{\tau}$ in Eq. (35), we assume that the mass of each quark is 1/3 of the proton mass and that the momentum transfer to each quark is 1/3 of the momentum transfer to the proton. This leads to the equality $\sqrt{\tau_0}/\sqrt{\tau} = 1$. As a result, we arrive at

$$G_E \sim \frac{1}{Q^4},\, G_M \sim \frac{1}{Q^4},\, \frac{G_E}{G_M} \sim 1. \quad (36)$$

Thus, the dipole dependence in the behavior of the form factors arises owing to the contribution to $J_p^{\pm\delta,\delta}$ from only one quark spin-flip transition upon proton interaction with a virtual photon. This dependence is valid at low Q^2, in which case quark non-spin-flip transitions are dominant. Below, we everywhere set $\sqrt{\tau_0} = \sqrt{\tau}$.

3.2. Dependence $G_E/G_M \sim 4m^2/Q^2$

For the dipole dependence to be violated, it is necessary that the number of quarks undergoing a spin-flip transition not be minimal [it is minimal in the case of (0, 1) set]. Here, we consider the (0, 3) set, in which case spin-flip transitions for all three quarks contribute to $J_p^{-\delta,\delta}$. This may occur only in the case where momentum transfers to the proton are high. In order to prove this, we write equalities similar to (32) and (34); that is,

$$J_p^{\delta,\delta} \sim G_E \sim 1 \times 1 \times 1 \times \frac{1}{Q^4}, \quad (37)$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} \times \frac{1}{Q^4}. \quad (38)$$

From here, we obtain

$$G_E \sim \frac{1}{Q^4},\ G_M \sim \frac{\tau}{Q^4},\ \frac{G_E}{G_M} \sim \frac{1}{\tau} \sim \frac{4m^2}{Q^2}, \quad (39)$$

$$Q^2\, \frac{G_E}{G_M} \sim 4m^2 = const. \quad (40)$$

It follows that, for $Q^2 > 4m^2$, the ratio G_E/G_M becomes smaller than unity. This is one possible way of violation of the dipole dependence in question. It is due to the occurrence of the spin-flip process for all three quarks. At the same time, the dependence that we obtained differs from the dependence observed at JLab and presented in Eq. (4).

3.3. The case $G_E/G_M \sim 1$, but both G_E and G_M behavior deviate from the dipole

Let us consider the (2,3) spin combination in set (29). It is generated by spin-flip transitions for two quarks in the case of the contribution to $J_p^{\delta,\delta}$ and by spin-flip transitions for all three quarks in the case of the contribution to $J_p^{-\delta,\delta}$. In the case being considered, we have

$$J_p^{\delta,\delta} \sim G_E \sim \sqrt{\tau} \times \sqrt{\tau} \times 1 \times \frac{1}{Q^4} , \quad (41)$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} \times \frac{1}{Q^4} . \quad (42)$$

Whence we obtain

$$G_E \sim \frac{\tau}{Q^4} , \quad G_M \sim \frac{\tau}{Q^4} , \quad \frac{G_E}{G_M} \sim 1 . \quad (43)$$

Therefore, the form factor ratio G_E/G_M behaves in just the same way as in the dipole model. However, the dependence $G_E \sim 1/(4m^2Q^2)$ and the dependence $G_M \sim 1/(4m^2Q^2)$ are not of the dipole character ($G_E \sim 1/Q^4$ and $G_M \sim 1/Q^4$), such dependences not being observed in the experiment.

3.4. Dependence $G_E/G_M \sim Q^2/4m^2$

Let us now consider the (2,1) spin combination in set (29). It is generated by spin-flip transitions for two quarks in the case of the contribution to $J_p^{\delta,\delta}$ and by spin-flip transitions for only one quark in the case of the contribution to $J_p^{-\delta,\delta}$. Following the same line of reasoning as above, one can readily show that, for the (2,1) set, G_E and G_M have the form

$$G_E \sim \frac{\tau}{Q^4} , \; G_M \sim \frac{1}{Q^4} , \quad (44)$$

that is, the ratio G_E/G_M behaves as

$$\frac{G_E}{G_M} \sim \tau \sim \frac{Q^2}{4m^2} , \; Q^2 \, \frac{G_M}{G_E} \sim 4m^2 = const . \quad (45)$$

3.5. Spin Parametrization for G_E/G_M

The non-spin-flip and spin-flip proton-current amplitudes ($J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$, respectively) can be represented as the linear combinations

$$J_p^{\delta,\delta} \;=\; \alpha_0 \, J_q^{\delta,\delta} J_q^{-\delta,-\delta} J_q^{\delta,\delta} + \alpha_2 \, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{\delta,\delta}, \quad (46)$$

$$J_p^{-\delta,\delta} \;=\; \beta_1 J_q^{-\delta,\delta} J_q^{\delta,\delta} J_q^{-\delta,-\delta} + \beta_3 \, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{-\delta,\delta}, \quad (47)$$

where the coefficients α_0, α_2, β_1 and β_3 have a clear physical meaning and their indices determine the number of quarks undergoing spin-flip transitions and contributing to proton non-spin-flip and spin-flip transitions. With the aid of Eqs. (46) and (47), one can readily obtain a general expression for the ratio G_E/G_M. The result is

$$\frac{G_E}{G_M} = \frac{\alpha_0 + \alpha_2 \, \tau}{\beta_1 + \beta_3 \, \tau} . \quad (48)$$

This expression may serve as a basis for constructing a spin parametrization and fits to experimental data obtained by measuring the ratio G_E/G_M.

Because of the requirement that the dipole dependence hold for $\tau \ll 1$, the coefficients α_0 and β_1 in Eq. (48) must obviously be close to unity: $\alpha_0 \sim 1$ and $\beta_1 \sim 1$. With allowance for this comment, we expand the right-hand side of (48) in a power series for $\tau \ll 1$. As a result, we arrive at the law of a linear decrease in the ratio G_E/G_M as Q^2 increases; this law agrees with the law in (4) established experimentally in [5]:

$$\frac{G_E}{G_M} \sim 1 - \frac{(\beta_3 - \alpha_2)}{4m^2} \, Q^2 . \quad (49)$$

Thus, the measurement of the ratio G_E/G_M provides valuable insights into the proton and to determine the number of its quarks whose spins were reversed.

Conclusion

The questions of how a dipole character of the dependence of the form factors G_E and G_M on the square of the momentum transfer to a proton, Q^2, arise and why a violation of this dependence occurs, which was first observed in a JLab polarization experiment, are investigated. The answers to these questions could be obtained owing to the use of the simplest QCD concepts of the proton structure and the results obtained by calculating the matrix elements of the proton current in the case of non-spin-flip and spin-flip transitions for protons in the diagonal spin basis (DSB), where the little Lorentz group common to the initial and final proton states is realized. In DSB, the form factors G_E and G_M are determined by the matrix elements $J_p^{\delta,\delta}$, and $J_p^{-\delta,\delta}$ of the proton current in the cases of non-spin-flip and spin-flip transitions for protons. In an arbitrary reference frame, the relations between these matrix elements and the form factors are $J_p^{\delta,\delta} \sim G_E$ and $J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M$ where $\tau = Q^2/4m^2$, with m being the proton mass. In considering the problem in question at the quark level, use is made of the model where the proton consists of three pointlike quarks having identical masses and where the respective matrix element of the proton current is the product of three quark-current amplitudes having the form $J_q^{\delta,\delta} \sim 1$ and $J_q^{-\delta,\delta} \sim \sqrt{\tau}$. It is shown that the aforementioned dipole dependence arises if the proton spin-flip is due to spin-flip for only one of the three quarks. As to violations of this dependence, they are caused by the contributions to $J_p^{\delta,\delta}$ from spin-flip transitions for two quarks or by the contribution to $J_p^{-\delta,\delta}$ from spin-flip transitions for all three quarks constituting the proton.

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Appendix A: Calculation of QED matrix elements in the DSB

Introduction

In DSB the little Wigner group [11] common for the initial and final states, is being realized [12, 13]. This brings the spin operators of *in*- and *out*-particles to coincidence and makes it possible to separate the interactions with and without change in the spin states of the particles involved in the reaction in the covariant form and, thus, to trace the dynamics of the spin interaction. The spin states of massless particles in the DSB coincides up to a sign with the helicity basis [14, 15]; in this case, the DSB formalism is equivalent to the CALKUL group method [16]. In contrast to methods of CALKUL-group etc, the developed approach is valid both for massive fermions and for massless ones. There occur no problems with accounting for spin-flip amplitudes in it. No auxiliary vectors are to be introduced in DSB. Just 4-momenta of particles participating in reaction are required in it to construct the mathematical apparatus for amplitude calculation.

In the DSB, Wigner rotations, which are purely kinematical in nature, are separated from the amplitudes. This leads to maximal simplification of the mathematical structure of the matrix elements in the DSB, and the resulting expressions give the truest reflection of the physical essential of spin phenomena.

In the used by us Bogush-Fedorov covariant approach [13] the calculation of matrix elements of the form (10) reduces to evaluating the trace:

$$M^{\pm\delta,\delta}=Tr(P_{21}^{\pm\delta,\delta}Q)\ ,\ P_{21}^{\pm\delta,\delta}=u^{\delta}(q_1)\,\overline{u}^{\,\pm\delta}(q_2)\ . \quad \text{(A1)}$$

To construction of the operators $P_{21}^{\pm\delta,\delta}$ we need to know

- the projection operators of the particle states: $\tau_1^{\delta}=u^{\delta}(q_1)\,\overline{u}^{\delta}(q_1)$ and $\tau_2^{\delta}=u^{\delta}(q_2)\,\overline{u}^{\delta}(q_2)$;
- the operator T_{21} (and its inverse operator T_{12}, $T_{12}=T_{21}^{-1}$, $T_{21}T_{12}=1$) for the transition from the initial to the final state without spin-flip: $u^{\delta}(q_2)=T_{21}u^{\delta}(q_1)$, $u^{\delta}(q_1)=T_{12}u^{\delta}(q_2)$, $\overline{u}^{\delta}(q_2)=\overline{u}^{\delta}(q_1)T_{12}$;
- the raising and lowering spin operators in the case of transitions with spin flip. They given by Eq.(8).

1. The projection operators of particles with spin 1/2 in the DSB

Let us consider the projection operators of particles with spin 1/2, $\tau_i^{\delta}=u^{\delta}(q_i)\,\overline{u}^{\delta}(q_i)$ [20]:

$$\tau_i^{\delta}=1/2(\hat{q}_i+m)(1-\delta\gamma^5\hat{s}_i)\ , \quad \text{(A2)}$$

where q_i and s_i are 4-momenta and spin 4-vectors with $q_i^2=m^2$ and $s_i^2=-1$, $q_is_i=0, i=(1,2)$. In the DSB (5) the operators τ_i^{δ} (A2) have the form [14, 15]:

$$\begin{aligned}\tau_1^{\delta}\ &=\ 1/2\,[m+\xi_+\hat{b}_0-\xi_-\hat{b}_3+ &\text{(A3)}\\ &+\ \delta\gamma^5\,(\xi_-\hat{b}_0-\xi_+\hat{b}_3-m\hat{b}_3\hat{b}_0)]\,,\\ \tau_2^{\delta}\ &=\ 1/2\,[m+\xi_+\hat{b}_0+\xi_-\hat{b}_3- &\text{(A4)}\\ &-\ \delta\gamma^5(\xi_-\hat{b}_0+\xi_+\hat{b}_3+m\hat{b}_3\hat{b}_0)]\,.\end{aligned}$$

Here 4-vectors b_0, b_3 and q_+, q_- are defined by Eq. (9), $\xi_{\pm}=\sqrt{\pm q_{\pm}^2}/2$. Owing to (8), the spin parts of the projection operators for particles 1 and 2 in the DSB can be made identical, and so we have [15]:

$$\tau_i^{\delta}=-1/4\,(\hat{q}_i+m)\,\hat{b}_{\delta}\,\hat{b}_{\delta}^{*}\ , \quad \text{(A5)}$$

where $b_{\delta}^{*}=b_{-\delta}=b_1-i\delta b_2$ and $b_{\delta}b_{\delta}^{*}=-2$. Here 4-vectors b_1, b_2 are defined by Eq. (9).

In the massless case the projection operators τ_1^{δ} and τ_2^{δ} (A3) and (A4) take the form [14, 15]:

$$\tau_1^{\delta}=\hat{q}_1\,(1-\delta\gamma^5)/2\ ,\tau_2^{\delta}=\hat{q}_2\,(1+\delta\gamma^5)/2\ . \quad \text{(A6)}$$

It is easy to show that the operators τ_1^{δ} and τ_2^{δ} (A6) satisfy the relations:

$$\begin{aligned}\gamma^5\tau_1^{\delta}\ &=\ \delta\,\tau_1^{\delta}\ ,\quad \gamma^5\tau_2^{\delta}=-\delta\,\tau_2^{\delta}\ , &\text{(A7)}\\ \tau_1^{\delta}\,\gamma^5\ &=\ -\delta\,\tau_1^{\delta}\ ,\ \tau_2^{\delta}\,\gamma^5=\delta\,\tau_2^{\delta}\ , &\text{(A8)}\end{aligned}$$

which imply that in the massless case the initial state is a helicity state, and the final state has negative helicity.

2. The operator T_{21} for the transition from the initial to the final state without spin-flip

The bispinors of the initial and final states of the particles, $u^{\delta}(q_1)$ and $u^{\delta}(q_2)$, can be related to each other by using the transition operators T_{21} and $T_{12}=T_{21}^{-1}$ [14, 15]:

$$u^{\delta}(q_2)=T_{21}\,u^{\delta}(q_1)\ ,\overline{u}^{\delta}(q_2)=\overline{u}^{\delta}(q_1)\,T_{12}\ , \quad \text{(A9)}$$

which in the DSB have the form [14, 15]:

$$T_{21}=\frac{1+\hat{v}_2\hat{v}_1}{\sqrt{2(v_1v_2+1)}}\ ,\ T_{12}=\frac{1+\hat{v}_1\hat{v}_2}{\sqrt{2(v_1v_2+1)}}\ , \quad \text{(A10)}$$

where $v_i=q_i/m$. Note that the Dirac equation can be used to reduce the transition operators T_{21} and T_{12} (A10) to the same form [14, 15]:

$$T_{21}=T_{12}=\hat{b}_0\ . \quad \text{(A11)}$$

3. The construction of operators $P_{21}^{\pm\delta,\delta}=u^{\delta}(q_1)\,\overline{u}^{\,\pm\delta}(q_2)$

In the papers [14, 15] we have constructed the operators $P_{21}^{\pm\delta,\delta}=u^{\delta}(q_1)\,\overline{u}^{\,\pm\delta}(q_2)$ (11) used to calculate the

DSB amplitudes in the case of transitions without and with spin-flip. They can be easy evaluated by the next way:

$$P_{21}^{\delta,\delta} = u^\delta(q_1)\,\overline{u}^\delta(q_2) = u^\delta(q_1)\,\overline{u}^\delta(q_1)T_{12} = \tau_1^\delta\,T_{12},$$
$$P_{21}^{-\delta,\delta} = u^\delta(q_1)\,\overline{u}^{\,-\delta}(q_2) = \sigma^{+\delta}u^{-\delta}(q_1)\,\overline{u}^{\,-\delta}(q_2) = = \sigma^{+\delta}P_{21}^{-\delta,-\delta}. \quad \text{(A12)}$$

The operators $P_{21}^{\pm\delta,\delta}$ (A12) determine the structure of the spin dependence of the matrix elements (10) in the case of transitions without spin-flip ($M^{\delta,\delta}$) and with spin-flip ($M^{-\delta,\delta}$). Their explicit form in the DSB can easily be obtained by using Eqs. (8), (A3), (A4), and (A11):

$$P_{31}^{\delta,\delta} = [\,\xi_+ + m\,\hat{b}_0 - \xi_-\hat{b}_3\,\hat{b}_0 + + \delta\gamma^5(\xi_- - m\,\hat{b}_3 - \xi_+\hat{b}_3\,\hat{b}_0)]/2, \quad \text{(A13)}$$
$$P_{31}^{-\delta,\delta} = -\delta\,(\,\xi_- + m\,\hat{b}_3 + \xi_+\,\delta\,\gamma^5\,)\,\hat{b}_\delta/2\,. \quad \text{(A14)}$$

Equations (A13) and (A14) can be used to calculate the matrix elements, both with and without spin-flip, for arbitrary Q. In particular, if the interaction operator reduces to the form

$$Q = \hat{A}_1 + \gamma^5\,\hat{A}_2\,, \quad \text{(A15)}$$

where A_1 and A_2 are any 4-vectors, then for the matrix elements (10) we will have:

$$M^{\delta,\delta} = 2m\,(A_1 b_0 + \delta\,A_2 b_3\,)\,, \quad \text{(A16)}$$
$$M^{-\delta,\delta} = 2\,[-\delta\xi_-\,(A_1 b_\delta) + \xi_+\,(A_2 b_\delta)]\,. \quad \text{(A17)}$$

Equations (A13) and (A14) can be written more compactly by using the operators (A5) and (A11), and also the expressions [14, 15]:

$$\hat{b}_3\hat{b}_0\hat{b}_\delta = -\delta\gamma^5\hat{b}_\delta, \gamma^5\hat{b}_\delta\hat{b}_0 = \delta\hat{b}_3\hat{b}_\delta, \gamma^5\hat{b}_\delta\hat{b}_3 = \delta\hat{b}_0\hat{b}_\delta\,. \quad \text{(A18)}$$

As a result, for the operators $P_{21}^{\pm\delta,\delta}$ we have [14, 15]:

$$P_{21}^{\delta,\delta} = (\hat{q}_1 + m)\,\hat{b}_\delta\,\hat{b}_0\,\hat{b}_\delta^*/4\,, \quad \text{(A19)}$$
$$P_{21}^{-\delta,\delta} = \delta\,(\hat{q}_1 + m)\,\hat{b}_\delta\,\hat{b}_3/2\,. \quad \text{(A20)}$$

In the massless case ($q_1^2 = q_2^2 = 0$) the operators $P_{21}^{\pm\delta,\delta}$ in (A13) and (A14) take the form [14, 15]:

$$P_{21}^{\delta,\delta} = \xi(1 + \delta\gamma^5)(1 + \hat{b}_0\hat{b}_3)/2\,, \quad \text{(A21)}$$
$$P_{31}^{-\delta,\delta} = -\delta\xi(1 + \delta\gamma^5)\,\hat{b}_\delta/2\,, \quad \text{(A22)}$$

where $\xi = \xi_+ = \xi_- = \sqrt{q_1 q_2/2}$.

Appendix B: Standard method for calculation $ep \to ep$ cross sections

The cross section (1) can be represented as the sum of the cross sections without spin-flip ($\sigma^{\delta,\delta}$) and with spin-flip ($-\sigma^{\delta,\delta}$) of the initial proton:

$$\frac{d\sigma}{d\Omega} = \kappa\left(G_E^2 + \frac{\tau}{\varepsilon}\,G_M^2\right) = \kappa\,(\sigma^{\delta,\delta} + \sigma^{-\delta,\delta}), \quad \text{(B1)}$$
$$\sigma^{\delta,\delta} = G_E^2,\;\; \sigma^{-\delta,\delta} = \frac{\tau}{\varepsilon}\,G_M^2\,. \quad \text{(B2)}$$

where κ is the factor in front of the parentheses in Eq. (1). At the same time the axes of the spin projections $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ should be coincide with the direction of final-proton motion: $\boldsymbol{c}_1 = \boldsymbol{c}_2 = \boldsymbol{n}_2$ and the spin 4-vectors s_1 and s_2 for initial and final protons must have the form

$$s_1 = (0, \boldsymbol{n}_2),\; s_2 = (|\boldsymbol{v}_2|, v_{20}\,\boldsymbol{n}_2)\,. \quad \text{(B3)}$$

The terms $\sigma^{\delta,\delta}$ and $\sigma^{-\delta,\delta}$ in Eq. (B1), (B2) are the cross sections without and with the spin-flip for the case when the initial and final protons are fully polarized in the direction of the motion of the final proton. For the case when $\vec{c}_1 = \vec{n}_2$ and $\vec{c}_2 = \vec{n}_2$ we have $\sigma^{\delta,\delta}$ and for the case when $\vec{c}_1 = \vec{n}_2$ and $\vec{c}_2 = -\vec{n}_2$ we have $\sigma^{-\delta,\delta}$.

Let us remind that the general form for spin 4-vectors s_1 and s_2 for protons with 4-momentum q_1, q_2 is:

$$s_i = (s_{0i}, \boldsymbol{s}_i), s_{0i} = \boldsymbol{v}_i\,\boldsymbol{c}_i, \boldsymbol{s}_i = \boldsymbol{c}_i + \frac{(\boldsymbol{c}_i\boldsymbol{v}_i)\,\boldsymbol{v}_i}{1 + v_{0i}}, \quad \text{(B4)}$$

where $v_i = (v_{0i}, \boldsymbol{v}_i) = q_i/m, i = 1, 2$.

To prove the relation (B1), (B2) there are two additional ways:

- Using the standard method calculation for QED processes cross sections [20].
- With help of book [21] by F. Halzen and A. Martin "Quarks and leptons. An Introductory Course in Modern Particle Physics", EXERCISE 8.7, Page 178, 1984 (in English); Page 214, 1987 (in Russian).

1. Standard method for calculation $ep \to ep$ cross sections

Evaluation of the cross section for the process $ep \to ep$ reduces to the calculation of the square modulus of the matrix element (15) for this process:

$$\sigma \sim |M_{ep\to ep}|^2 = |\overline{u}(p_2)\gamma^\mu u(p_1)\cdot\overline{u}(q_2)\Gamma_\mu(q^2)u(q_1)|^2\,.$$

In the standard method [20] this calculation of σ with taken into account the polarization of initial in final protons reduces to determination of product of lepton ($L^{\mu\nu}$) and proton ($W_{\mu\nu}$) tensors

$$\sigma_{s_1,s_2} \sim L^{\mu\nu}W_{\mu\nu}, \quad \text{(B5)}$$
$$L^{\mu\nu} = 2\cdot\mathrm{Tr}(\tau_2^e\gamma^\mu\tau_1^e\gamma^\nu), \quad \text{(B6)}$$
$$W_{\mu\nu} = \mathrm{Tr}(\tau_2^p\,\Gamma_\mu\tau_1^p\,\overline{\Gamma}_\nu)\,, \quad \text{(B7)}$$

with

$$\tau_1^e = \frac{1}{2}(\hat{p_1} + m_e),\; \tau_2^e = \frac{1}{2}(\hat{p_2} + m_e)\,,$$
$$\tau_1^p = \frac{1}{2}(\hat{q_1} + m)(1 - \delta_1\gamma_5\hat{s}_1)\,,$$
$$\tau_2^p = \frac{1}{2}(\hat{q_2} + m)(1 - \delta_2\gamma_5\hat{s}_2)\,.$$

Lepton tensor $L^{\mu\nu}$ (B6) have the form

$$L^{\mu\nu} = 2\,(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) + q^2 g^{\mu\nu}\,. \qquad \text{(B8)}$$

Tensor $L^{\mu\nu}$ in terms $p_+ = p_2 + p_1; p_- = p_2 - p_1$ have a form

$$L_2^{\mu\nu} \equiv L^{\mu\nu} = p_+^\mu p_+^\nu - p_-^\mu p_-^\nu + q^2 g^{\mu\nu}\,. \qquad \text{(B9)}$$

In this equation the term $p_-^\mu p_-^\nu$ can be safely omitted as far as it do not contribute to the cross section of process (B5). It is the consequence of the gauge invariance of QED amplitudes. As a result for the lepton tensor we obtain a new, compact expression

$$L_c^{\mu\nu} \equiv L^{\mu\nu} = p_+^\mu p_+^\nu + q^2 g^{\mu\nu}\,. \qquad \text{(B10)}$$

Using the representation (24) for $\Gamma_\mu(q^2)$ and the definition of Dirac formfactor in terms of the Sachs ones

$$F_1 = \frac{G_E + \tau G_M}{1+\tau} = \frac{4m^2}{q_+^2}\,(G_E + \tau G_M)\,, \qquad \text{(B11)}$$

we obtain for tensor $W_{\mu\nu}$

$$W_{\mu\nu} \equiv W_{\mu\nu}^{\delta_1\delta_2} = \frac{1+\delta_1\delta_2}{2}\,W_{\mu\nu}^{\delta,\delta} + \frac{1-\delta_1\delta_2}{2}\,W_{\mu\nu}^{-\delta,\delta}, \qquad \text{(B12)}$$

with

$$W_{\mu\nu}^{\delta,\delta} = \frac{4m^2 G_E^2}{q_+^2}\,(q_+)_\mu (q_+)_\nu\,, \qquad \text{(B13)}$$

$$W_{\mu\nu}^{-\delta,\delta} = \frac{4m^2\tau G_M^2}{q_+^2}\,\{(q_+)_\mu(q_+)_\nu - q_+^2 g_{\mu\nu} + \qquad \text{(B14)}$$
$$+(q_-)_\mu(q_-)_\nu\, q_+^2/q_-^2 - 4i\delta\varepsilon_{\mu\nu\rho\sigma} q_-^\rho q_+^\sigma \sqrt{q_+^2}/\sqrt{-q_-^2}\,\}\,,$$

where we as well can omit the term $(q_-)_\mu(q_-)_\nu$.

Note that for the case of unpolarized leptons (initial and the scattered) the asymmetry part of the tensor $W_{\mu\nu}^{-\delta,\delta}$ (or the imaginary part of it) in (B14) as well do not contribute to the cross section of process $ep \to ep$. So for tensors $W_{\mu\nu}^{\delta,\delta}$ and $W_{\mu\nu}^{-\delta,\delta}$, which corresponds to the cases with spin-flip an without spin-flip, for the unpolarized leptons we have

$$W_{\mu\nu}^{\delta,\delta} = \frac{4m^2 G_E^2}{q_+^2}\,(q_+)_\mu(q_+)_\nu\,, \qquad \text{(B15)}$$

$$W_{\mu\nu}^{-\delta,\delta} = \frac{4m^2\tau G_M^2}{q_+^2}\,\{(q_+)_\mu(q_+)_\nu - q_+^2 g_{\mu\nu}\}\,. \qquad \text{(B16)}$$

Forming the product of leptonic tensor (B10) and the proton one (B12) with (B15), (B16)) we obtain:

$$\sigma_{s_1,s_2} = \frac{(1+\delta_1\delta_2)}{2} W_{ep\to ep}^{\delta,\delta} + \frac{(1-\delta_1\delta_2)}{2}\,W_{ep\to ep}^{-\delta,\delta}, \qquad \text{(B17)}$$

$$W_{ep\to ep}^{\delta,\delta} = \frac{4m^2 G_E^2}{q_+^2}\,[(p_+q_+)^2 + q_+^2 q_-^2]\,, \qquad \text{(B18)}$$

$$W_{ep\to ep}^{-\delta,\delta} = \frac{4m^2\tau G_M^2}{q_+^2}[(p_+q_+)^2 - q_+^2(q_-^2 + 4m_e^2)]\,. \qquad \text{(B19)}$$

With the help of the matrix elements of the proton current (18), (19) calculation probability of the process $ep \to ep$ can be reduced to calculation of the trivial trace:

$$|\,T\,|^2 = \frac{4m^2}{q^4}\,\frac{1}{8}\sum_\delta Tr(G_E^2(\hat p_2 + m_e)\hat b_0(\hat p_1 + m_e)\hat b_0 +$$
$$+\,\tau\,G_M^2(\hat p_2 + m_e)\hat b_\delta(\hat p_1 + m_e)\hat b_\delta^*)\,.$$

The expression for $|T|^2$ leads to the cross section, which coincides with result in [20]:

$$d\sigma = \frac{\alpha^2 do}{4w^2}\,\frac{1}{1+\tau}\,(\,G_E^2\,Y_I + \tau\,G_M^2\,Y_{II}\,)\,\frac{1}{q^4}\,, \qquad \text{(B20)}$$
$$Y_I = (p_+q_+)^2 + q_+^2 q_-^2\,,\; Y_{II} = (p_+q_+)^2 - q_+^2(q_-^2 + 4m_e^2)\,.$$

Thus, the differential cross section for the $ep \to ep$ process naturally splits into the sum of two terms containing only the squares of the Sachs form factors and corresponding to the contribution of transition without ($\sim G_E^2$) and with ($\sim G_M^2$) proton spin-flip.

In the paper [9] based on the use of the expression (B17) a new method of measuring of the Sachs form factors was suggested. It was shown that they can be determined separately and independently by direct measurements of the cross sections without and with spin-flip of the initial proton, which should be at rest and fully polarized in the direction of the motion of the scattered proton.

Using the matrix elements of the proton current in DSB (18), (19) for the proton tensor $W_{\mu\nu}^{\delta_1,\delta_2}$ one can construct an another equivalent and compact expression:

$$W_{\mu\nu}^{\delta_1,\delta_2} = 4m^2\left[\frac{(1+\delta_1\delta_2)}{2}\,G_E^2(b_0)_\mu(b_0)_\nu + \right.$$
$$\left. + \frac{(1-\delta_1\delta_2)}{2}\,\tau_p\,G_M^2(b_\delta)_\mu(b_\delta^*)_\nu\right]. \qquad \text{(B21)}$$

For the leptonic tensor in the case of electrons can be written similar expression:

$$L_{\mu\nu}^{\delta_{e_1}\delta_{e_2}} = 4m_e^2\left[\frac{(1+\delta_{e_1}\delta_{e_2})}{2}\,(a_0)_\mu(a_0)_\nu + \right.$$
$$\left. + \frac{(1-\delta_{e_1}\delta_{e_2})}{2}\,\tau_e\,(a_{\delta_e})_\mu(a_{\delta_e}^*)_\nu\right], \qquad \text{(B22)}$$

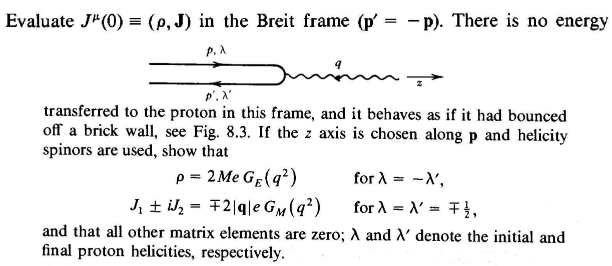
with orthonormal basis of 4-vectors $a_A\,(A = 0,1,2,3)$, constructed from 4-momenta of the electrons:

$$(a_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} a_0^\nu a_3^\kappa a_2^\sigma,\; (a_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} p_1^\nu p_2^\kappa q_1^\sigma/\rho,$$
$$a_3 = p_-/\sqrt{-p_-^2},\; a_0 = p_+/\sqrt{p_+^2}\,. \qquad \text{(B23)}$$

Here, $p_- = p_2 - p_1$, $p_+ = p_2 + p_1$, and ρ is determined from the normalization conditions $a_1^2 = a_2^2 = a_3^2 = -a_0^2 = -1$, $a_{\delta_e} = a_1 + i\delta_e a_2, a_{\delta_e}^* = a_1 - i\delta_e a_2$.

Appendix C: An alternative method of calculation of the spin-flip and non-flip proton current matrix elements

To prove the correctness of the results obtained in the DSB for the proton current matrix elements (18), (19) we propose to consider here Exercise 8.7 at page 178 from book of F. Halzen and A. Martin [21] (Fig. 8.3 also extracted from this book and show at Figure 1). In this exercise one suggests to consider the matrix elements of the proton current in the Breit reference frame and show that the proton transition with helicity-flip (without helicity-flip) are determined by only the Sachs electric formfactor G_E (magnetic form factor G_M).


Evaluate $J^\mu(0) \equiv (\rho, \mathbf{J})$ in the Breit frame ($\mathbf{p}' = -\mathbf{p}$). There is no energy

transferred to the proton in this frame, and it behaves as if it had bounced off a brick wall, see Fig. 8.3. If the z axis is chosen along $\mathbf{p}$ and helicity spinors are used, show that

$$\rho = 2Me\,G_E(q^2) \qquad \text{for } \lambda = -\lambda',$$
$$J_1 \pm iJ_2 = \mp 2|\mathbf{q}|e\,G_M(q^2) \qquad \text{for } \lambda = \lambda' = \mp\tfrac{1}{2},$$

and that all other matrix elements are zero; λ and λ' denote the initial and final proton helicities, respectively.

FIG. 1: **Exercise 8.7 at page 178 from book of F. Halzen and A. Martin [21].**

From this picture, we see that in the Breit-system a transition with (without) a change in the sign of helicity is the transition without (with) spin-flip of the proton:

$$J_\mu^{-\lambda,\lambda} = J_\mu^{\delta,\delta} = 2\,e\,M\,G_E\,(b_0)_\mu\,, \tag{C1}$$
$$J_\mu^{\lambda,\lambda} = J_\mu^{-\delta,\delta} = -2e\,\delta|\boldsymbol{q}|\,G_M\,(b_\delta)_\mu\,, \tag{C2}$$
$$|\boldsymbol{q}| = \sqrt{Q^2},$$

where

$$b_0 = (1,0,0,0), b_1 = (0,1,0,0), b_2 = (0,0,1,0), \tag{C3}$$
$$b_3 = (0,0,0,1), b_\delta = b_1 + i\delta b_2, \delta = \pm 1\,.$$

Below we will dropped the factor e in matrix elements and denote by the letter m of the proton mass:

$$J_\mu^{\delta,\delta} = 2\,m\,G_E\,(b_0)_\mu\,, J_\mu^{-\delta,\delta} = -2\,m\delta\sqrt{\tau}\,G_M\,(b_\delta)_\mu. \tag{C4}$$

In the Breit system where $q_1 = (q_0, -\boldsymbol{q}), q_2 = (q_0, \boldsymbol{q})$ and the spin states of the initial and final protons are helicity, so they spin four-vectors s_1 s_2 have the form:

$$s_1 = (-|\boldsymbol{v}|, v_0\boldsymbol{n}_2),\ s_2 = (|\boldsymbol{v}|, v_0\boldsymbol{n}_2)\,, \boldsymbol{n}_2 = \boldsymbol{q_2}/|\boldsymbol{q}_2|\,. \tag{C5}$$

Let us make transition from Breit system to an arbitrary reference frame. For this purpose we need to write the four basic vectors b_A (C3) in the covariant form. We will construct 4-vectors b_A through the 4-momenta of participating in the reaction particles. The unit 4-vectors b_0 and b_3 can be written as the normalized per unit the sum and difference between the momenta of final and initial protons:

$$b_0 = \frac{q_+}{\sqrt{q_+^2}},\ b_3 = \frac{q_-}{\sqrt{-q_-^2}}\ , \tag{C6}$$
$$(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa b_2^\sigma, (b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa p_1^\sigma/\rho\,, \tag{C7}$$
$$q_+ = q_1 + q_2 = (2q_0, 0, 0, 0), \Rightarrow b_0 = (1,0,0,0),$$
$$q_- = q_2 - q_1 = (0,0,0,2q), \Rightarrow b_3 = (0,0,0,1),$$

The matrix elements of the proton current (C4) by using (C6), (C7) coincide with results (18), (19) in DSB and are valid in arbitrary reference frame.

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