Constituent Quark Models
and electromagnetic form factors

E. Santopinto
INFN
Genova

GDR-ph, Orsay, 2012

Outline of the talk

- The Model (hCQM)
- The helicity amplitudes
- The elastic e.m. form factors of the nucleon

The Model
(hCQM)

hypercentral **C**onstituent **Q**uark **M**odel

Hypercentral Constituent Quark Model
hCQM

free parameters fixed from the spectrum

Comment
The description of the spectrum is the first task of a model builder

Predictions for:
photocouplings
transition form factors
elastic from factors
....

describe data (if possible)
understand what is missing

LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains

a long range spin-independent confinement

a short range spin dependent term

Spin-independence $\longrightarrow$ SU(6) configurations

SU(6) configurations for three quark states

$$6 \times 6 \times 6 = \underset{A}{20} + \underset{M}{70} + \underset{M}{70} + \underset{S}{56}$$

Notation

(d, L^{π})

d = dim of SU(6) irrep
L = total orbital angular momentum
π = parity

THREE-QUARK WAVE FUNCTION

$$\Psi_{3q} \;=\; \theta_{colour} \times \chi_{spin} \times \phi_{iso} \times \psi_{space}$$

$SU(3)_c$ $SU(2)$ $SU(3)_f$ $O(3)$

SU(6) limit: (spin-independent interaction)

$SU(3)_c$ SU(6) O(3)

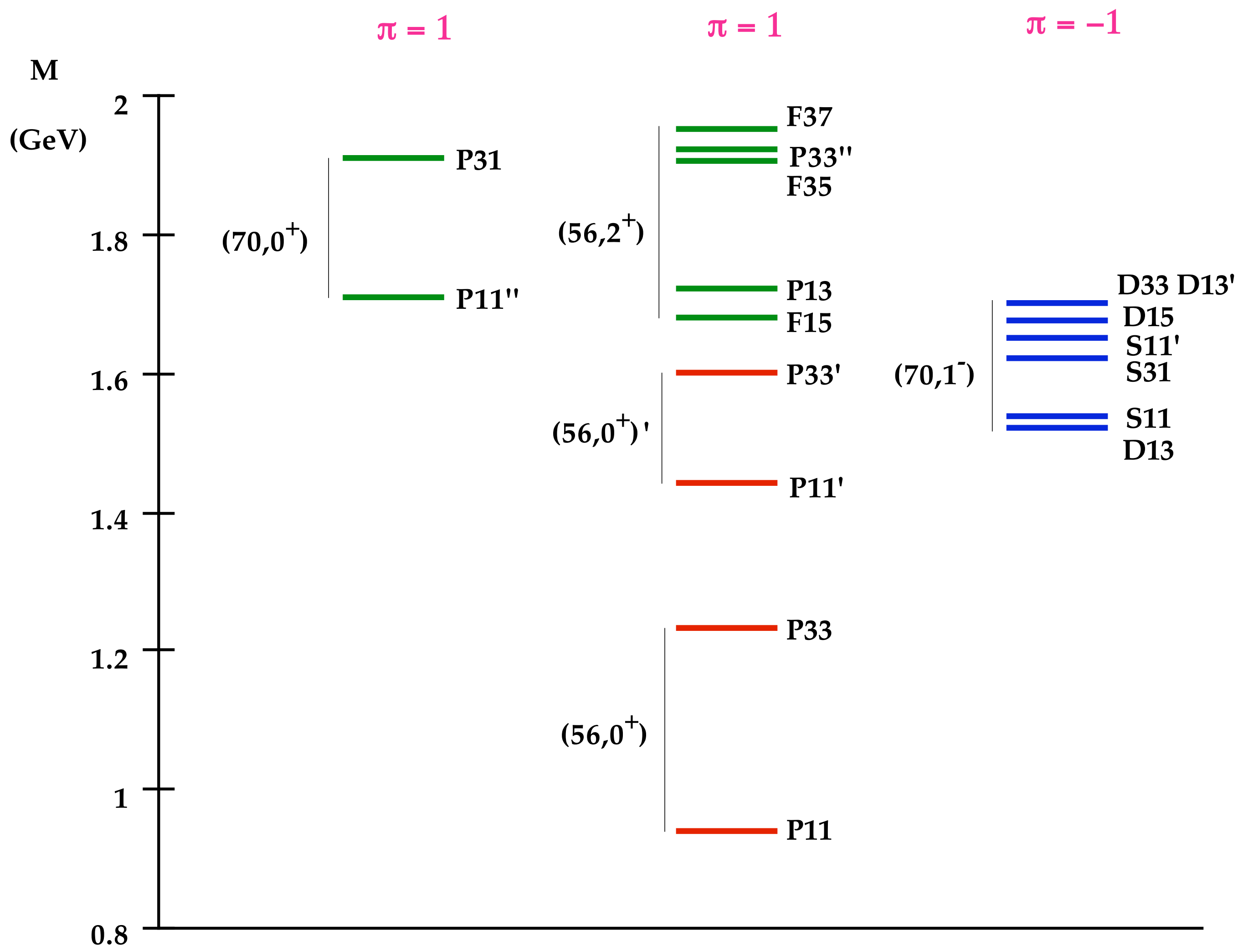
Permutation symmetry: $\boldsymbol{\Psi_{3q}}$ must be antisymmetric

θ_{colour} is a colour singlet => *A*

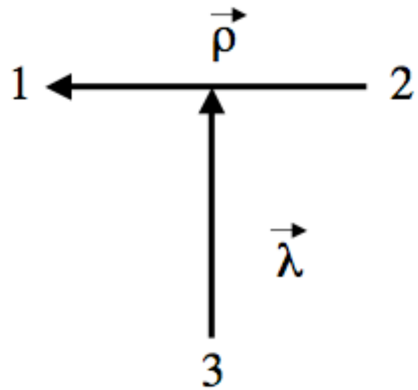
the rest must be symmetric

SU(6) & O(3) wf have the same symmetry (A, MS, MA, S)

PDG 4* & 3*


π = 1
π = 1
π = −1
M
(GeV)
2
1.8
1.6
1.4
1.2
1
0.8
P31
(70,0+)
P11''
F37
P33''
F35
(56,2+)
P13
F15
P33'
(56,0+)'
P11'
P33
(56,0+)
P11
D33 D13'
D15
S11'
S31
(70,1-)
S11
D13

Jacobi coordinates



Hyperspherical Coordinates

$$(\rho, \Omega_\rho, \lambda, \Omega_\lambda) \Rightarrow (x, t, \Omega_\rho, \Omega_\lambda)$$

$$x = \sqrt{\rho^2 + \lambda^2}$$ hyperradius

$$t = arctg\frac{\rho}{\lambda}$$ hyperangle

$$L^2(\Omega)Y_{[\gamma]}(\Omega) = -\gamma(\gamma+4)Y_{[\gamma]}(\Omega) \qquad \gamma = 2n + l_\rho + l_\lambda$$

$$L^2(\Omega) \Leftrightarrow C_2(O(6))$$

γ grand angular quantum number

$Y_{[\gamma]}(\Omega)$ Hyperspherical harmonics

$$\sum_{i<j} V(\mathbf{r}_{ij}) \approx V(x) + \ldots$$

$\gamma = 2n + l_\rho + l_\lambda$

Hasenfratz et al. 1980:
$\Sigma\, V(r_i,r_j)$ is approximately hypercentral

Hypercentral Hypothesis

$V = V(x)$

Factorization

$$\psi(x, t, \Omega_\rho, \Omega_\lambda) = \psi_{\nu\gamma}(x) \quad Y_{[\gamma, l_\rho, l_\lambda]}$$

("dynamics") ("geometry")

Only one differential equation in x (hyperradial equation)

Hypercentral Model

Phys. Lett. B, 1995

$V(x) = -\tau/x + \alpha\, x$

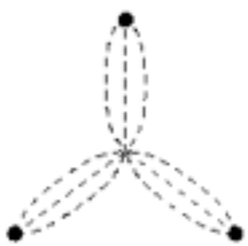
Hypercentral approximation of $V = - b/r + c\, r$

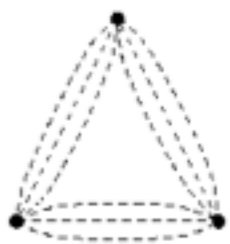
- QCD fundamental mechanism

3-body forces

Carlson et al, 1983
Capstick-Isgur 1986
hCQM 1995

- Flux tube model

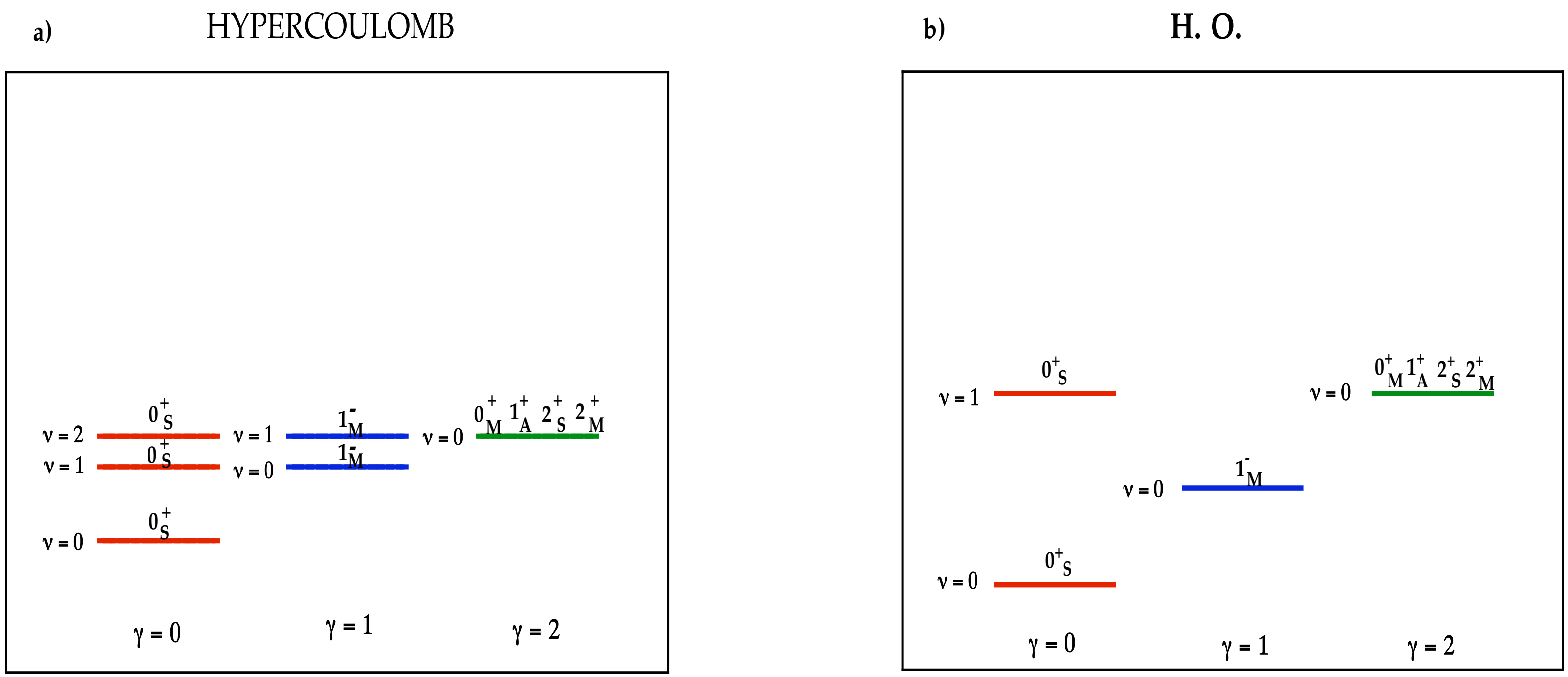




Two analytical solutions

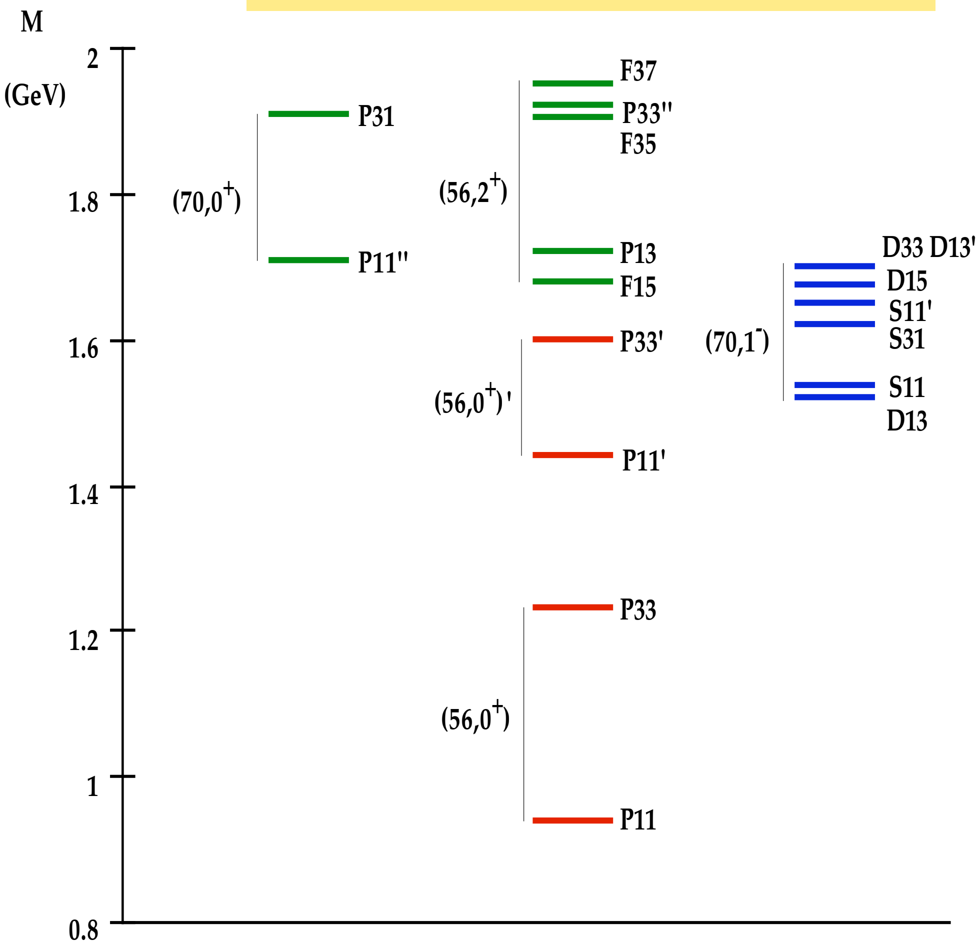
hyperCoulomb $\quad - \tau / x$

h. o. $\quad \Sigma_{i<j} 1/2\ k\ (r_i - r_j)^2 = 3/2\ k\ x^2$

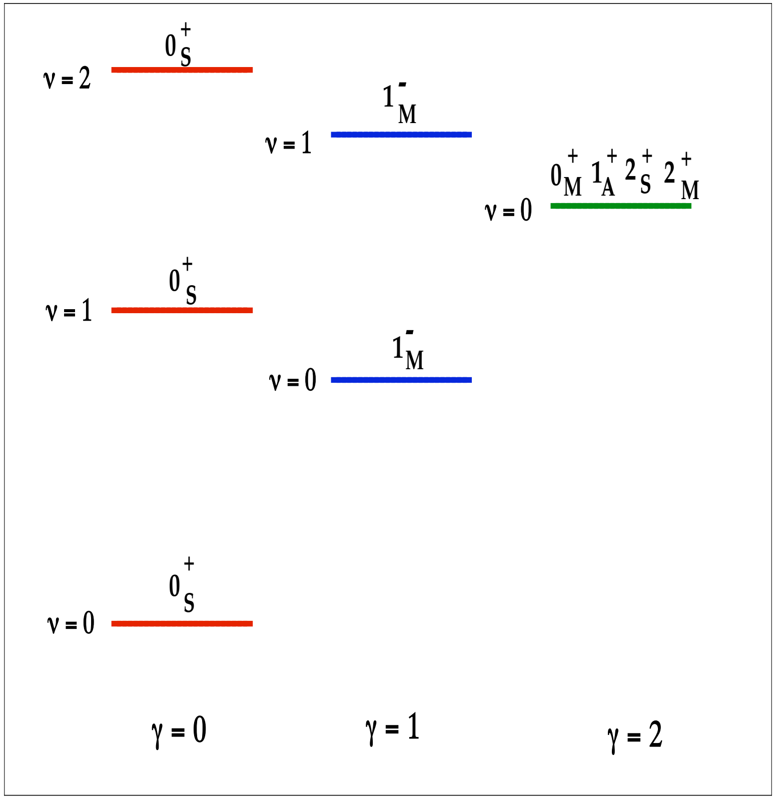


PDG 4* & 3*

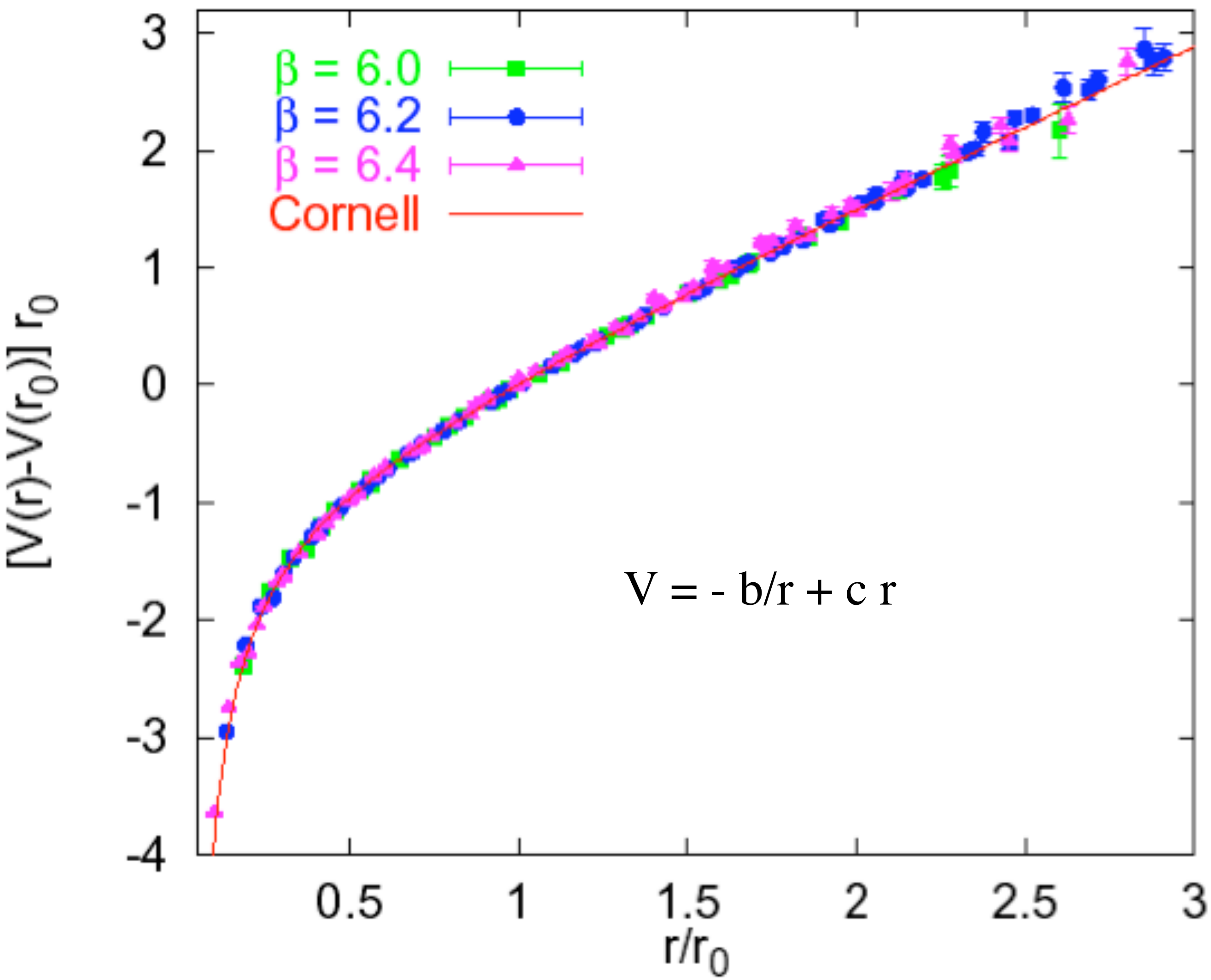
P = 1 P = 1 P = -1



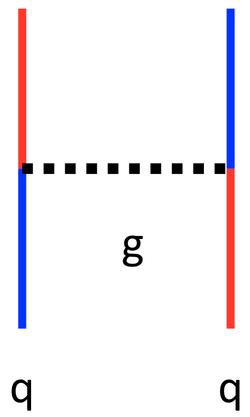
$V(x) = -\tau/x + \alpha x$



Quark-antiquark lattice potential   G.S. Bali Phys. Rep. 343, 1 (2001)



Introducing SU(6) violation



One Gluon Exchange

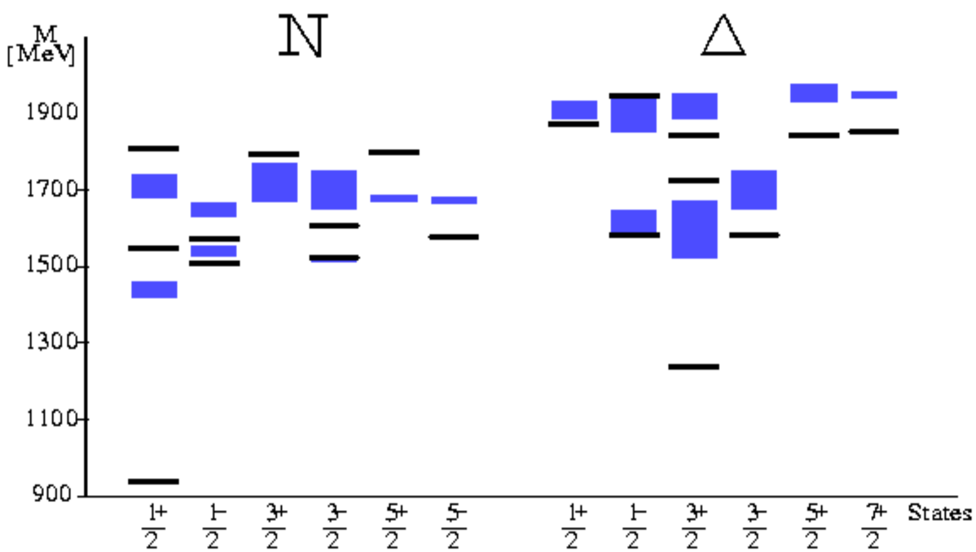
V_{OGE} = -a/r + Hyperfine interaction

Hypercentral Model (1)

$$H_{3q} = 3m + \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + V(x) + H_{hyp}$$

M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto, L. Tiator, Phys. Lett. B364 (1995), 231

- $V(x) = -\frac{\tau}{x} + \alpha x;$ $\quad H_{hyp} = A\left[\sum_{i<j} V^S(\mathbf{r}_i, \mathbf{r}_j) \;\; \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \text{tensor}\right]$
- 3 parameters $\tau \; \alpha \; A \leftarrow$ fixed to the spectrum, $m = \frac{M}{3}$



$\tau = 4.59$

$\alpha = 1.61 \text{ fm}^{-1}$

$A \Leftarrow (N - \Delta)$

$$x = \sqrt{\rho^2 + \lambda^2}$$

hyperradius

Results (predictions)
with the Hypercentral Constituent
Quark Model

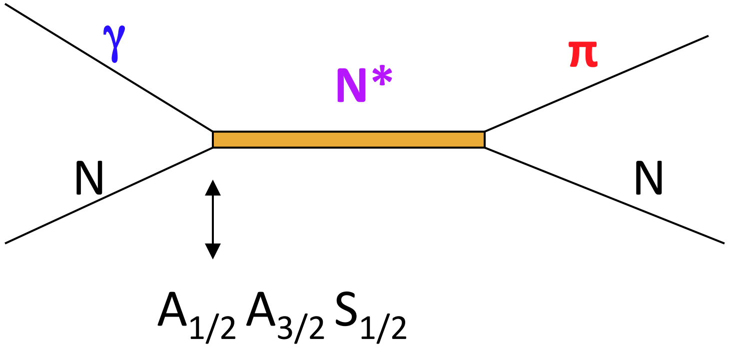
for

- Helicity amplitudes
- Elastic nucleon form factors

The helicity amplitudes

HELICITY AMPLITUDES

Extracted from electroproduction of mesons



Definition

$A_{1/2} = \langle N^*\ J_z = 1/2 \mid H^T_{em} \mid N\ J_z = -1/2 \rangle$ §

$A_{3/2} = \langle N^*\ J_z = 3/2 \mid H^T_{em} \mid N\ J_z = 1/2 \rangle$ §

$S_{1/2} = \langle N^*\ J_z = 1/2 \mid H^L_{em} \mid N\ J_z = 1/2 \rangle$

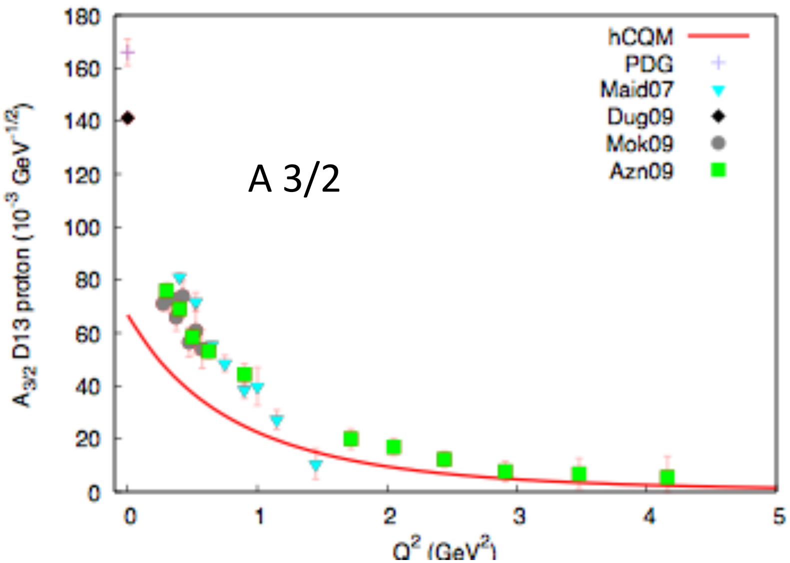
N, N^* nucleon and resonance as 3q states

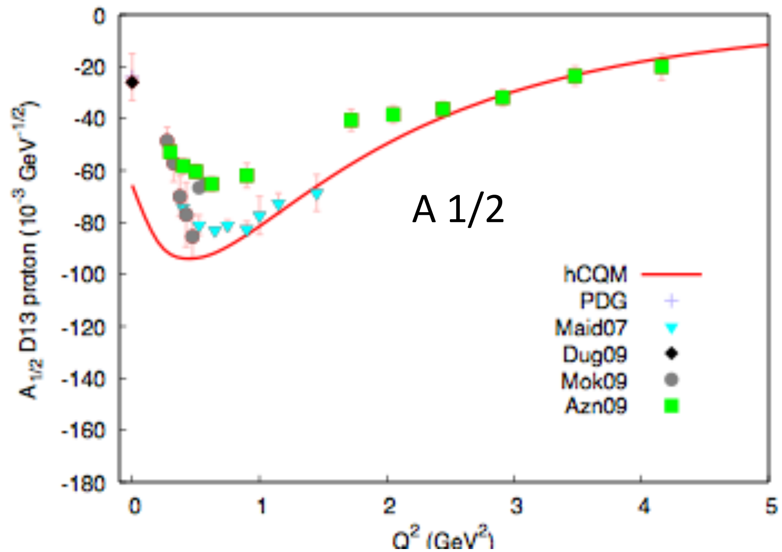
H^T_{em} H^l_{em} model transition operator

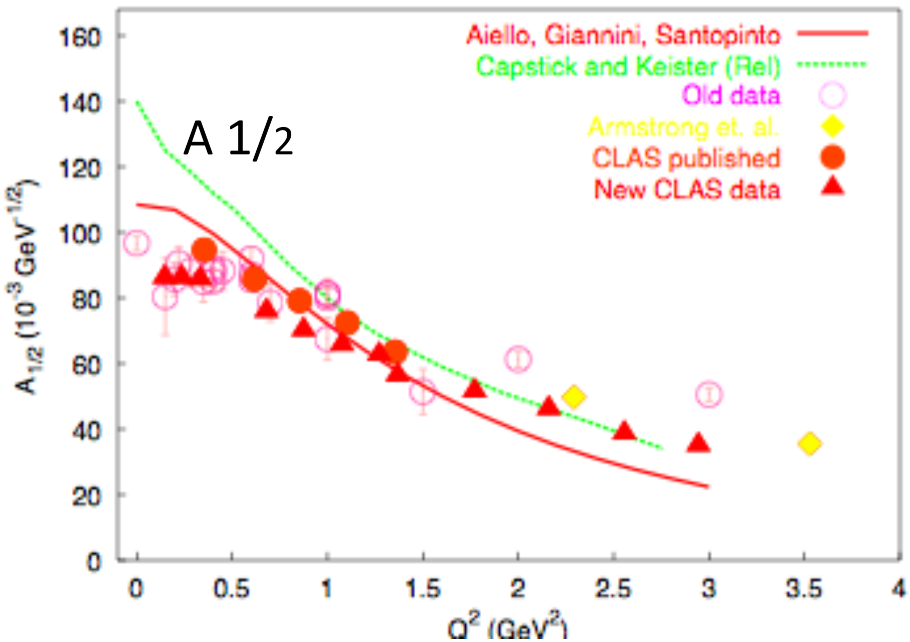
§ results for the negative parity resonances:
M. Aiello, M.Giannini, E. Santopinto J. Phys. G24, 753 (1998)

Systematic predictions for transverse and longitudinal amplitudes
E. Santopinto, M.Giannini, submitted to PR C

D13 transition amplitudes







S11 transition amplitudes

- The hCQM seems to provide realistic three-quark wave functions

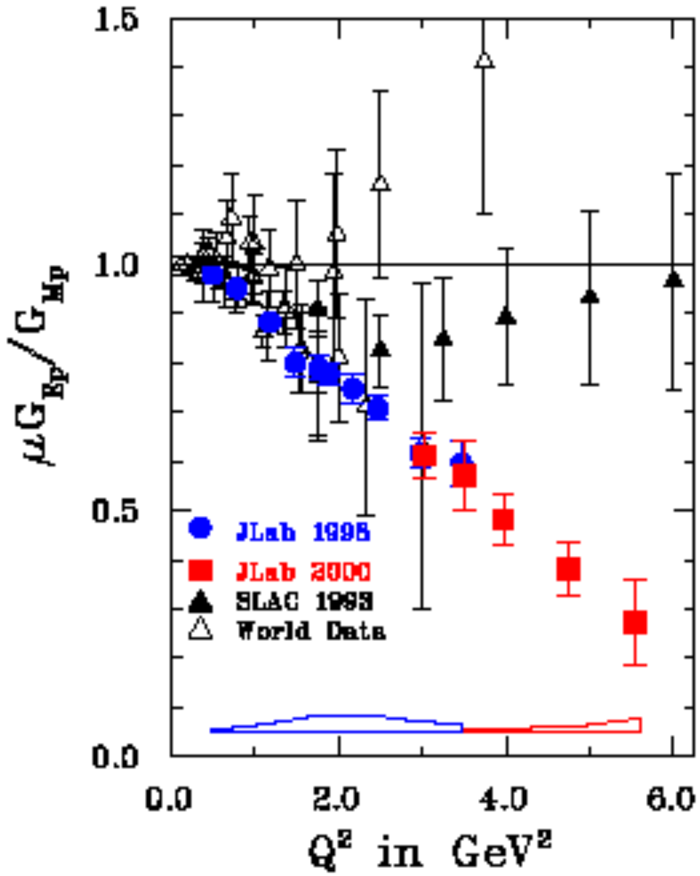
Solvable model

$V(x) = -\tau/x + \alpha\, x$ linear term treated as a perturbation

- energy levels expressed analytically
- unperturbed wf given by the 1/x term

E. Santopinto, F. Iachello, M.Giannini, Eur. Phys. J. A **1**, 307 (1998)

The nucleon elastic form factors



- elastic scattering of polarized electrons on polarized protons

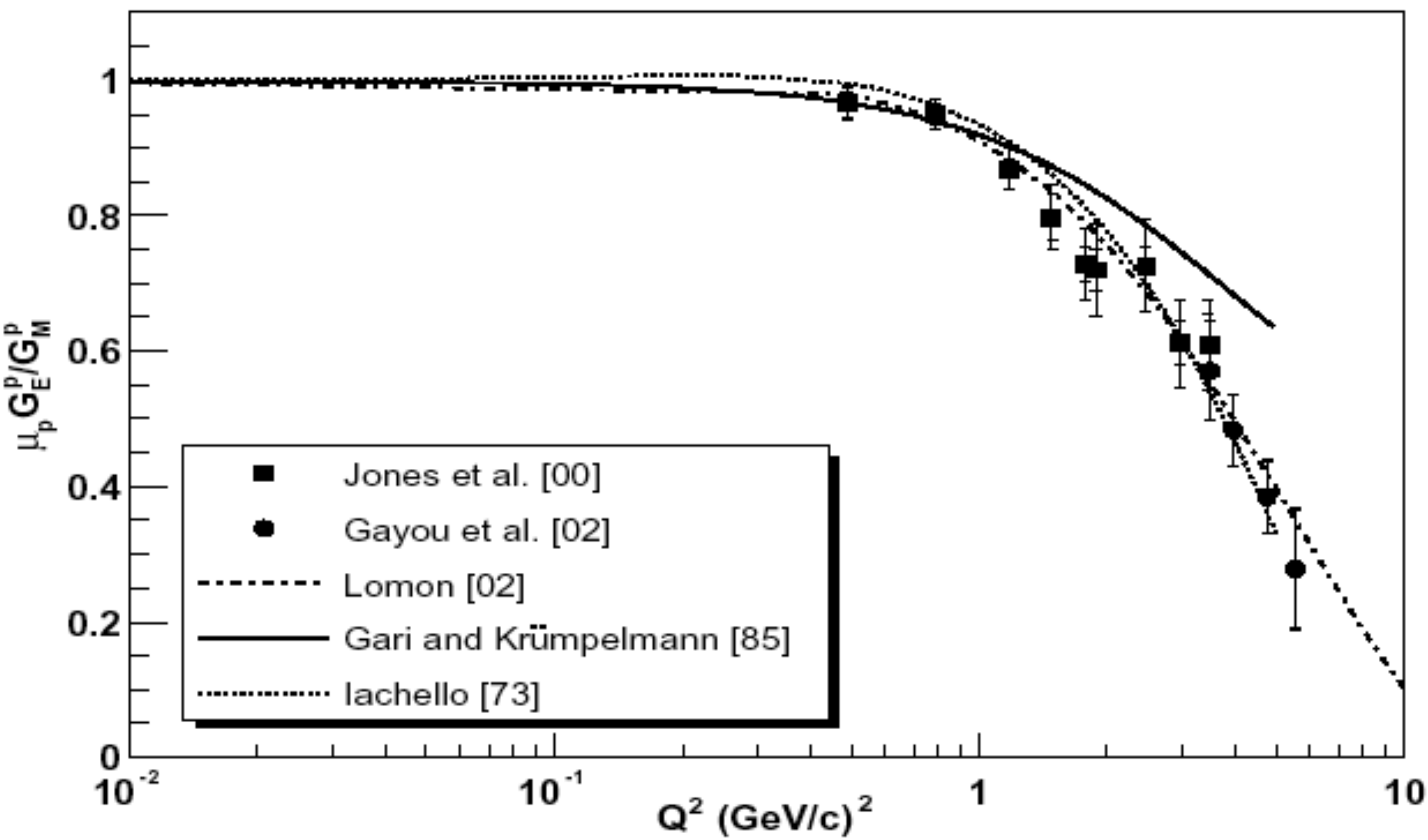
- measurement of polarizations asymmetry gives directly the ratio G^p_E/G^p_M

- discrepancy with Rosenbluth data (?)

- linear and strong decrease

- pointing towards a zero (!)

- new data (jan 2010) seem to confirm the behaviour



1
0.8
0.6
0.4
0.2
0
$\mu_p G_E^p/G_M^p$
10^{-2}
10^{-1}
1
10
Q^2 (GeV/c)^2
Jones et al. [00]
Gayou et al. [02]
Lomon [02]
Gari and Krümpelmann [85]
Iachello [73]

RELATIVITY

Various levels

- relativistic kinetic energy
- Lorentz boosts
- Relativistic dynamics
- quark-antiquark pair effects (meson cloud)
- relativistic equations (BS, DS)

Point Form Relativistic Dynamics

Point Form is one of the Relativistic Hamiltonian Dynamics
for a fixed number of particles (Dirac)

Construction of a representation of the Poincaré generators
P_μ (tetramomentum), J_k (angular momenta), K_i (boosts)
obeying the Poincaré group commutation relations
in particular

$$[P_k , K_i] = i \delta_{kj} H$$

Three forms:
Light (LF), Instant (IF), Point (PF)
Differ in the number and type of (interaction) free generators

Point form: P_μ interaction dependent

J_k and K_i free

Mass operator $M = M_0 + M_I$

$$M_0 = \Sigma_i \sqrt{\vec{\mathbf{p}}_i^{\,2} + m^2}$$

$$\boxed{\Sigma_i\, \vec{\mathbf{p}}_i = 0}$$

$\vec{\mathbf{P}}_i$ undergo the same Wigner rotation -> M_0 is invariant

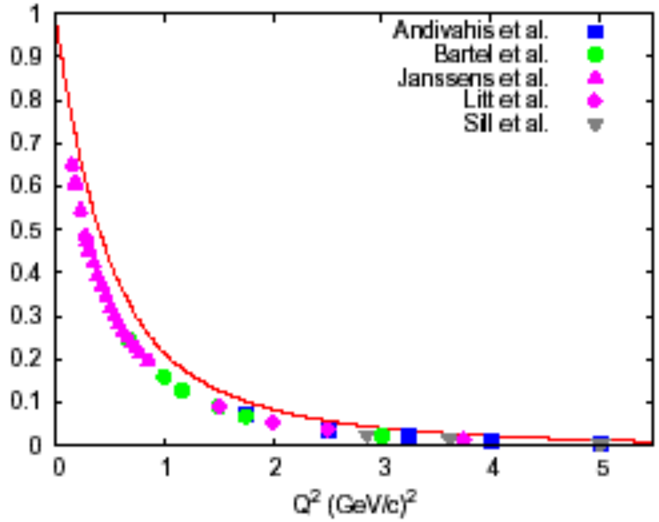
Similar reasoning for the hyperradius

Moving three-quark states are obtained through (interaction free) Lorentz boosts (velocity states)

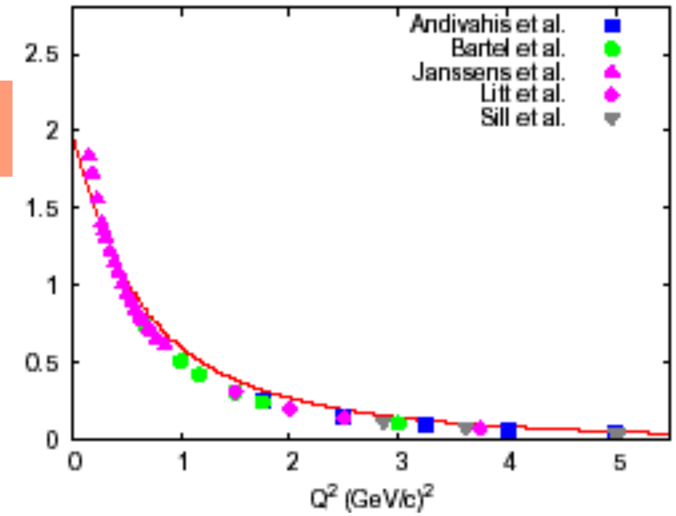
Calculated values!

- Boosts to initial and final states
- Expansion of current to any order
- Conserved current

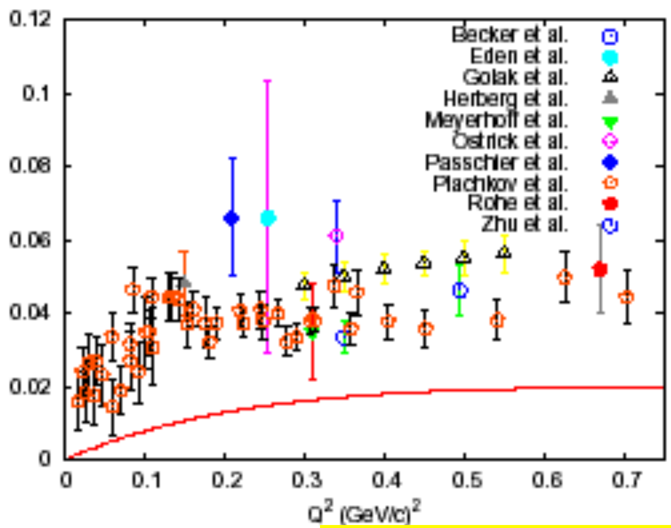
G_E^p



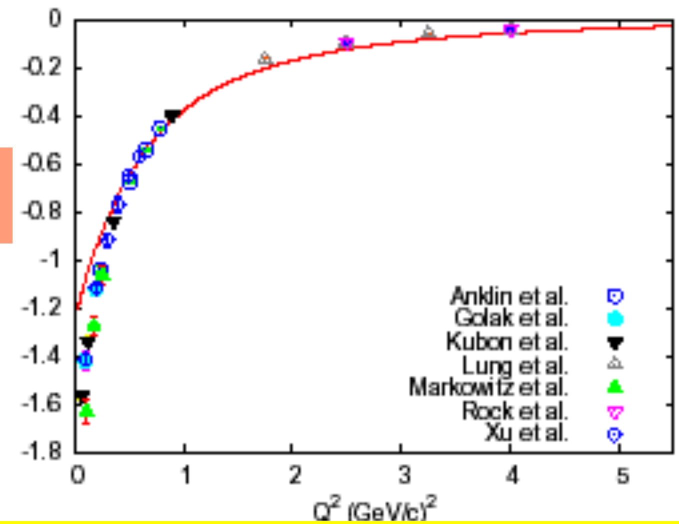
G_M^p



G_E^n

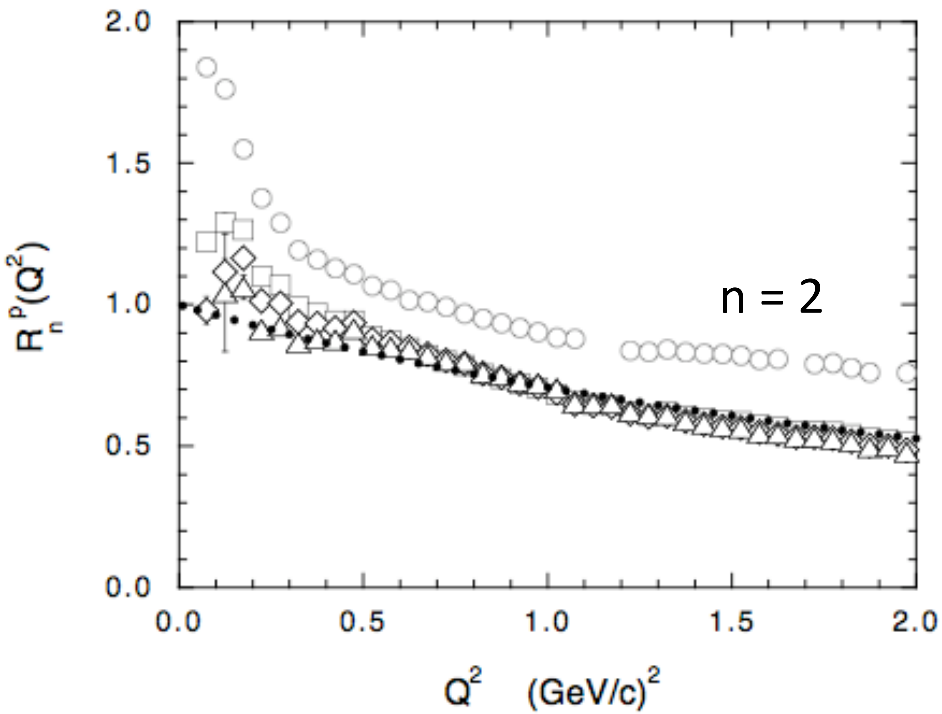


G_M^n



De Sanctis, Giannini, Santopintoi, Vassallo Phys. Rev. C76, 062201 (2007)c

Further support 2



Ratio between
proton Nachtmann moments &
CQ distribution

Bloom-Gilman duality

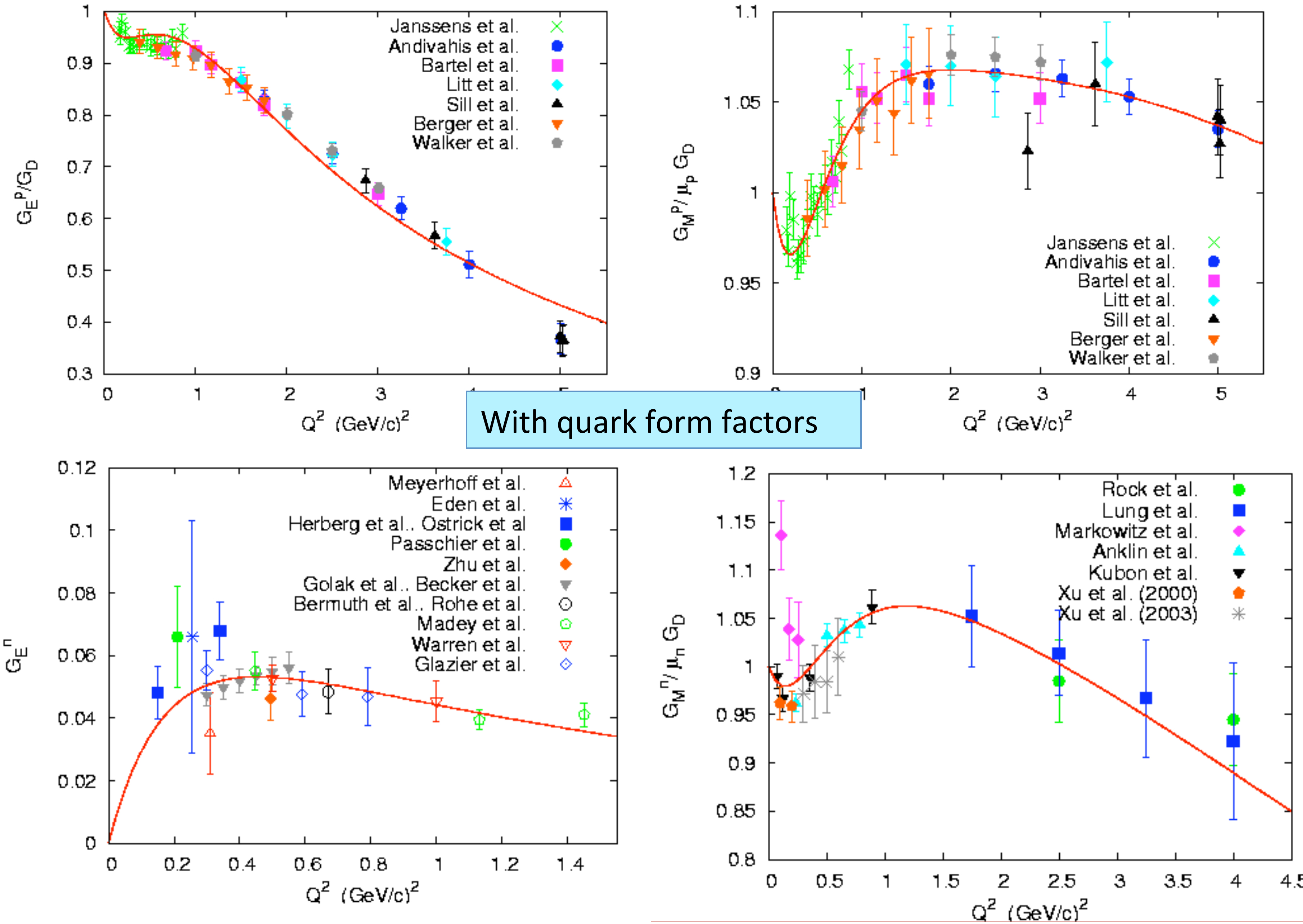
Inelastic proton scattering as elastic scattering on CQ

(approximate) scaling function $\longrightarrow$ square of CQ ff

$F(Q^2) = 1/(1+ 1/6\ r_{CQ}^{\ 2}\ Q^2)$

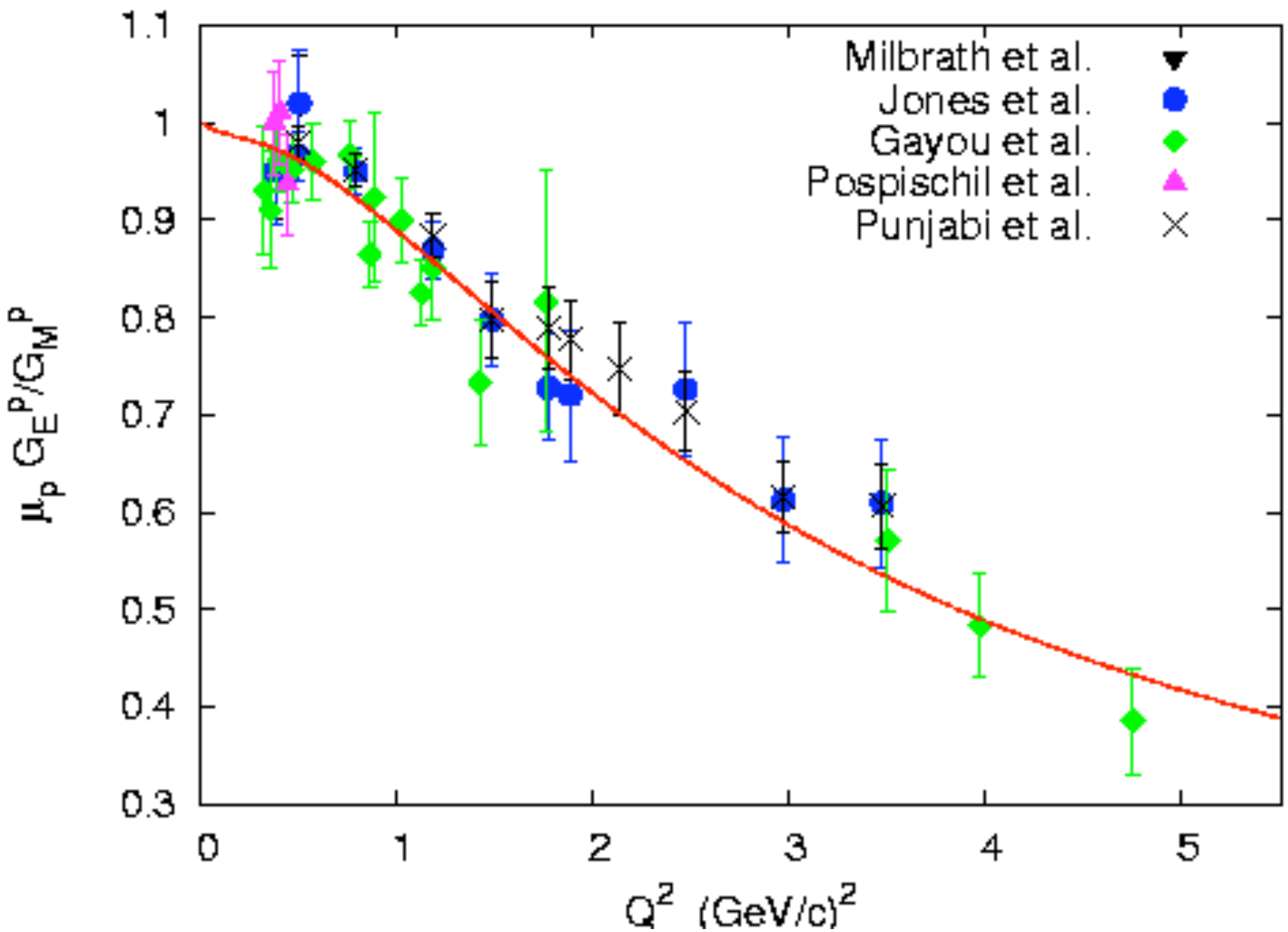
$r_{CQ} \cong$ **0.2 fm**

Ricco et al., PR **D67,** 094004 (2003)



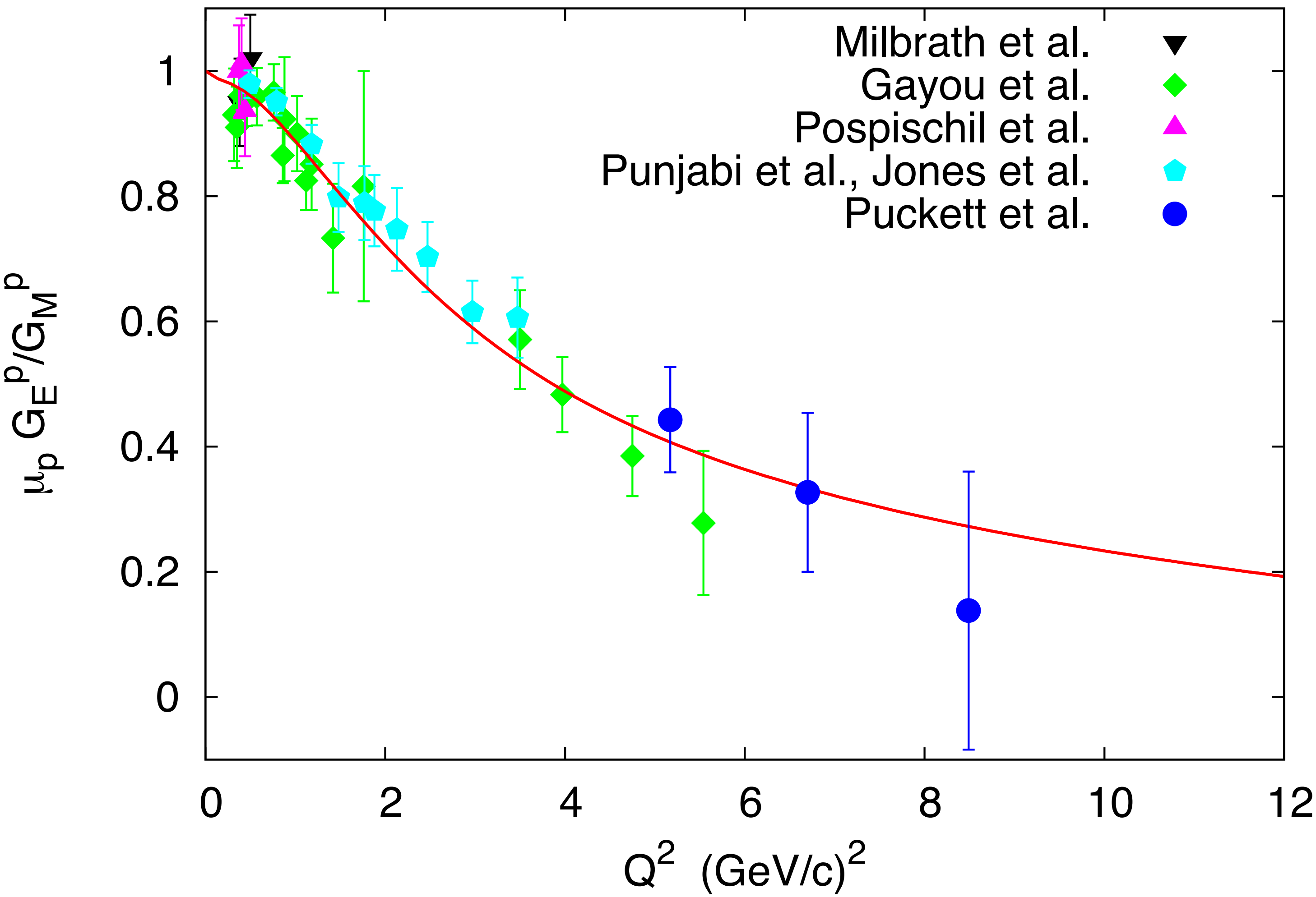
With quark form factors

De Sanctis, Giannini, Santopintoi, Vassallo Phys. Rev. C76, 062201 (2007)

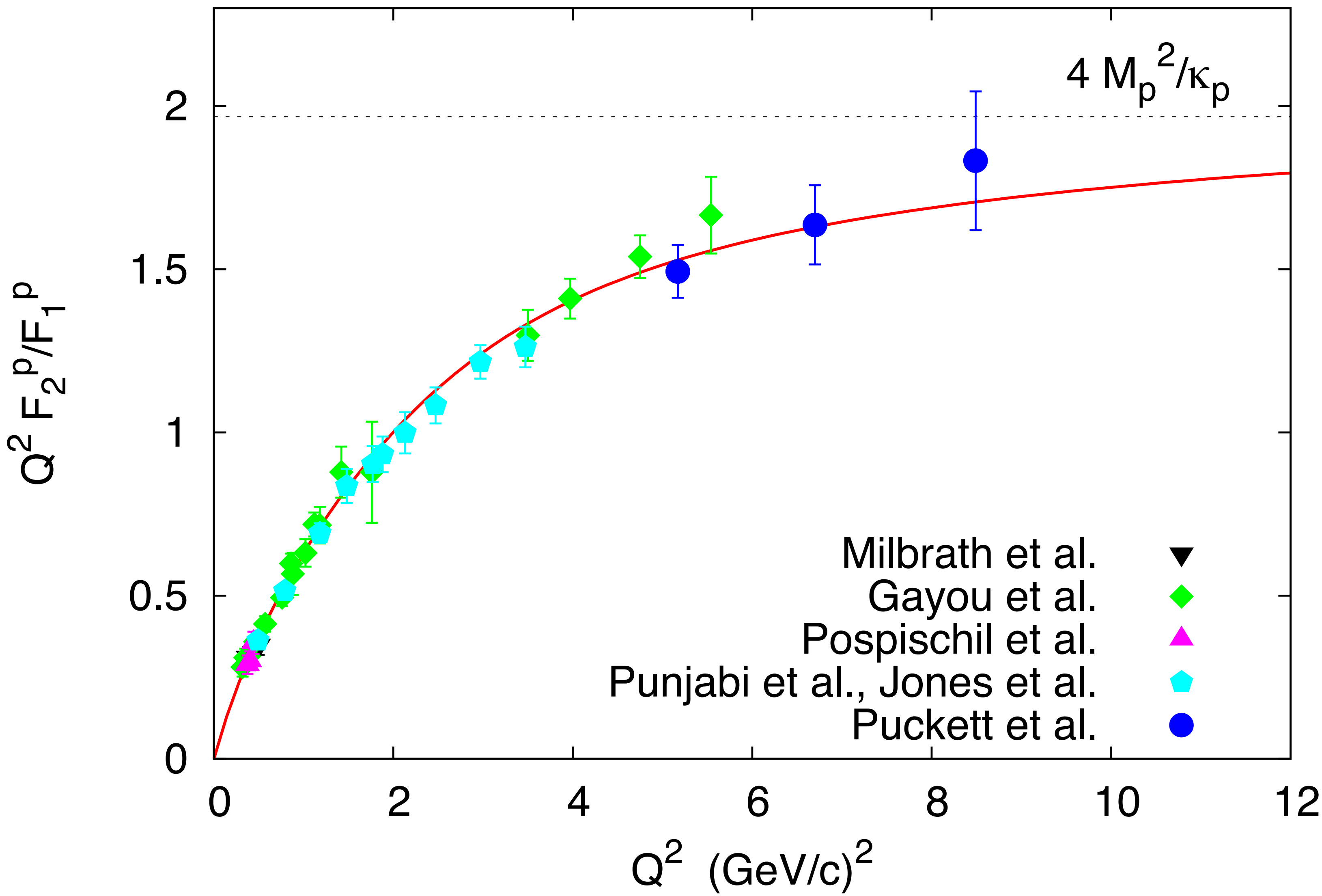


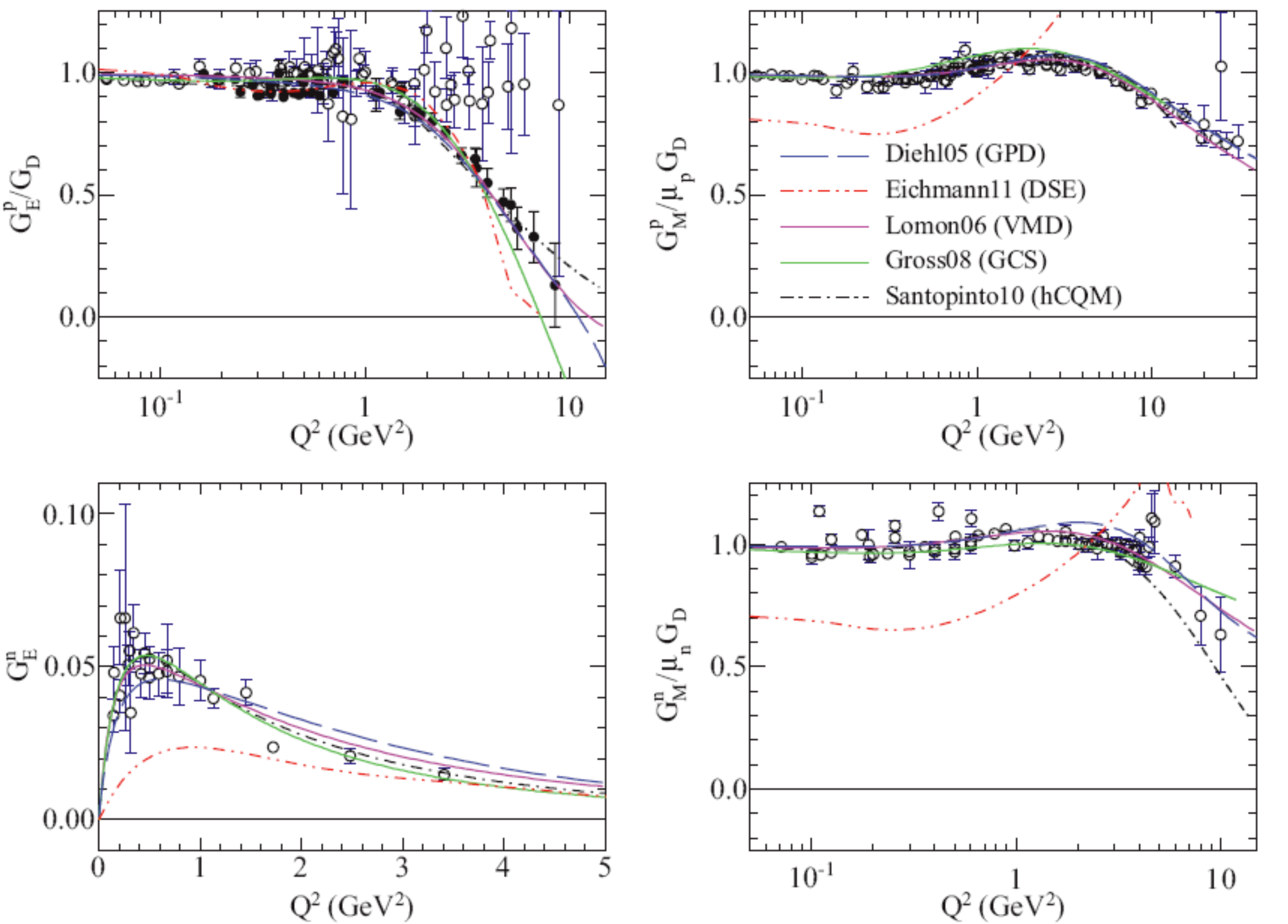
De Sanctis, Giannini, Santopintoi, Vassallo Phys. Rev. C76, 062201 (2007)

E. Santopinto,A. Vassallo, M. M. Giannini, and M. De Sanctis, Phys. Rev. C 82, 065204 (2010)



E. Santopinto, A. Vassallo, M. M. Giannini, and M. De Sanctis, Phys. Rev. C 82, 065204 (2010)




G_E^p/G_D
$G_M^p/\mu_p G_D$
G_E^n
$G_M^n/\mu_n G_D$
Q^2 (GeV^2)
Diehl05 (GPD)
Eichmann11 (DSE)
Lomon06 (VMD)
Gross08 (GCS)
Santopinto10 (hCQM)

Conclusions

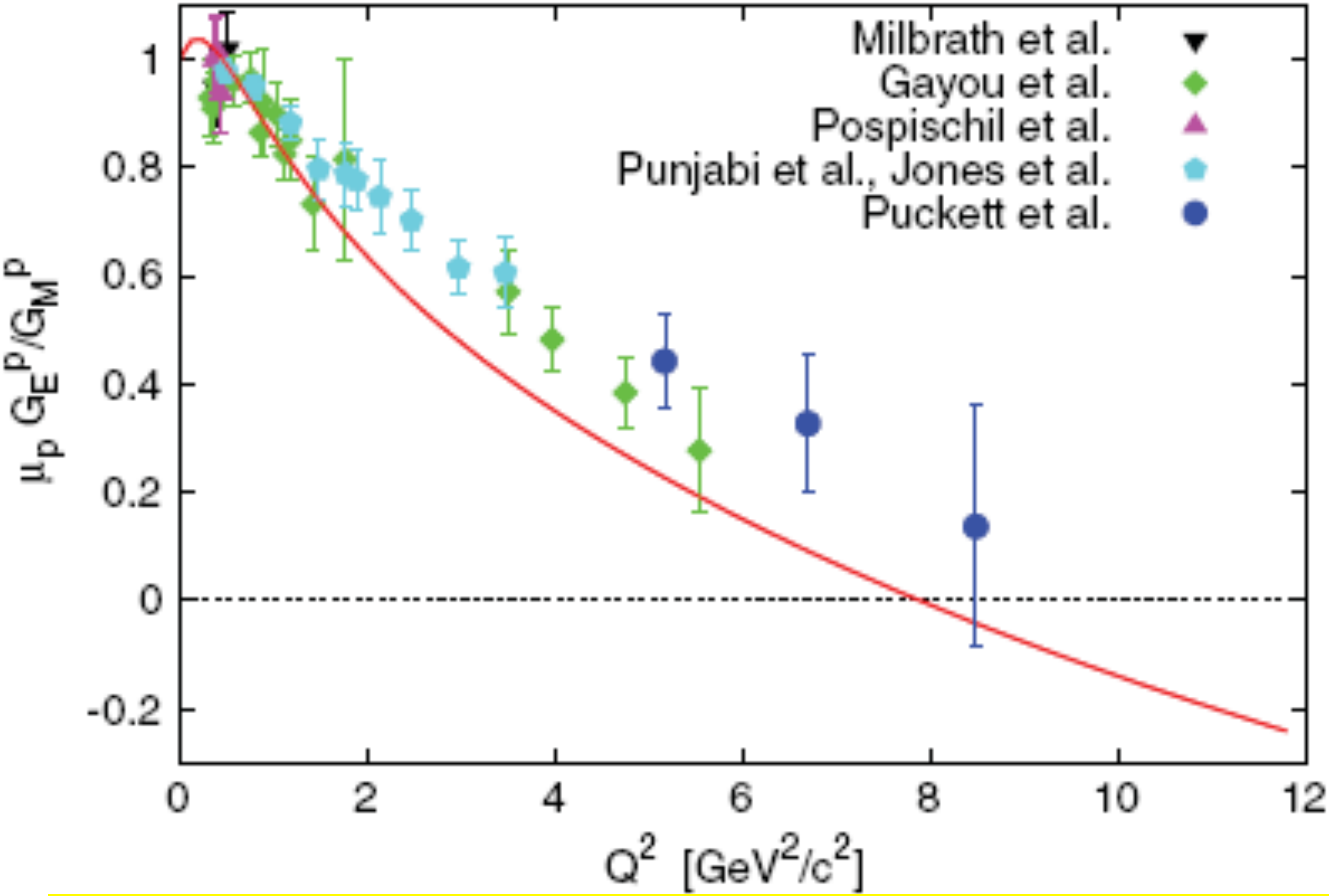
- CQM provide a good systematic frame for baryon studies
- description of e.m. properties (specially N-N* transitions)
- possibility of understanding missing mechanisms
- quark antiquark pairs effects

unquenching: important break through

Calculations for the time like form factors are underway.

Ratio $\mu_p G_E^p/G_M^p$

*De Sanctis, Ferretti, Santopinto, Vassallo, Phys. Rev. C **84**, 055201 (2011)*



Interacting Quark Diquark model , *E. Santopinto, Phys. Rev. C **72**, 022201(R) (2005)*

Axial form factor of the nucleon

Adamuscìn, Tomasi, Santopinto, Bijker,Phys. Rev. C 78, 035201 (2008)

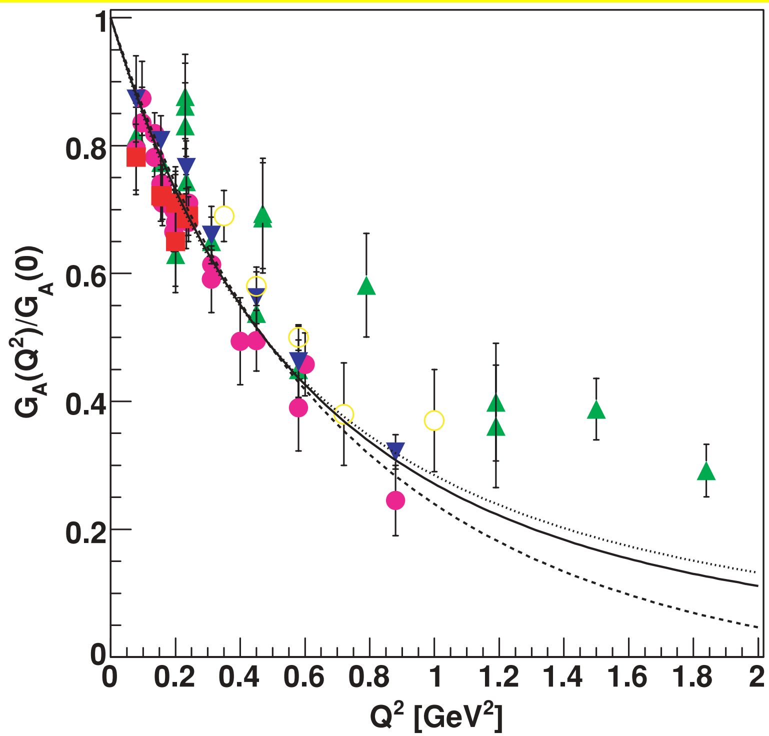


FIG. 1. (Color online) Comparison between theoretical and experimental values of the axial form factor of the nucleon $G_A(Q^2)$ as a function of Q^2. The theoretical values are calculated in the two-component model using Eq. (7) with $\alpha = 1.57$ and $\gamma = 0.25$ GeV^{-2} [12] (dashed line), and $\alpha = 0.95$ and $\gamma = 0.515$ GeV^{-2} [13] (solid line), and the dipole form of Eq. (1) with $M_A = 1.069$ GeV (dotted line). The experimental values were extracted according different models: PCAC [25] (pink, solid circles), FPV (red, solid squares) [26], SP (green, solid triangles) [24], DR (blue, trianglesdown) [27], Δ (yellow, open circles) [28].

Adamuscìn, Tomasi, Santopinto, Bijker, Phys. Rev. C 78, 035201 (2008)

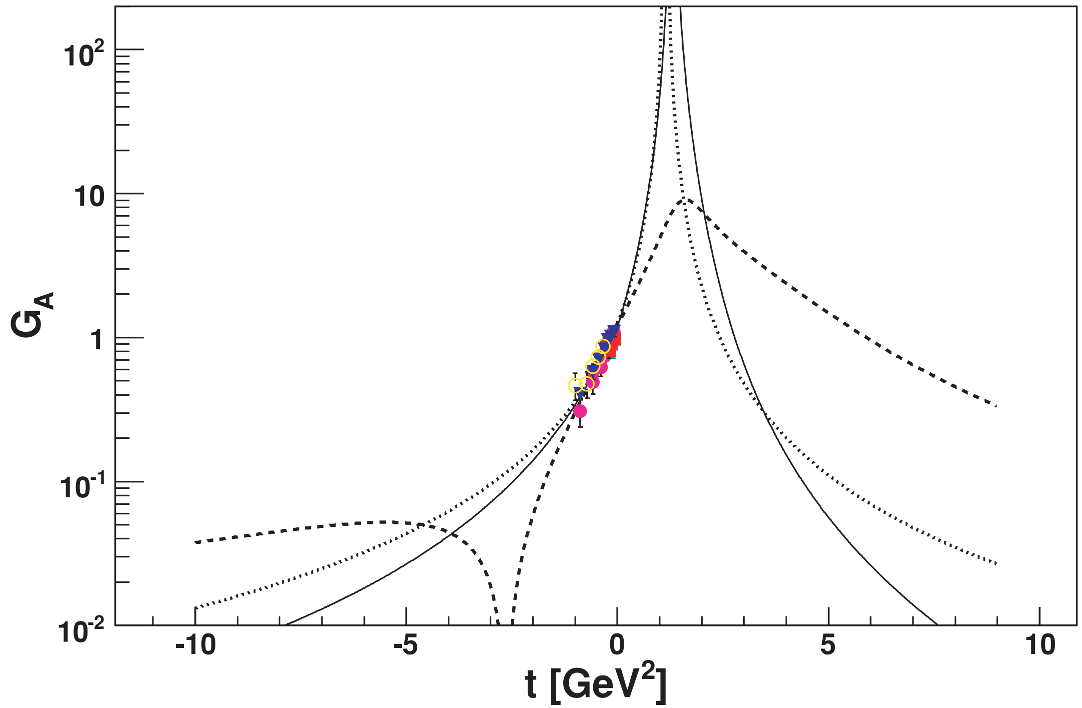


FIG. 2. (Color online) As Fig. 1, but for the absolute value of the axial form factor $|G_A(t)|$ in the space-like ($t < 0$) and time-like ($t > 0$) regions. In the time-like region, $\delta = 0.925$ for IJL [14] and $\delta = 0.397$ for BI [13].

III. TWO-COMPONENT MODEL

In the two-component model [12,13], the axial nucleon form factor is described as

$$G_A(Q^2) = G_A(0) g(Q^2) \left[1 - \alpha + \alpha \frac{m_A^2}{m_A^2 + Q^2} \right],$$
$$g(Q^2) = \left(1 + \gamma Q^2\right)^{-2}, \tag{2}$$

with $Q^2 > 0$ in the space-like region. $g(Q^2)$ denotes the coupling to the intrinsic structure (three valence quarks) of the nucleon, and m_A is the mass of the lowest axial meson $a_1(1260)$ with quantum numbers $I^G(J^{PC}) = 1^-(1^{++})$ and $m_A = 1.230$ GeV. We note that, unlike other studies, in which m_A is a parameter, here it corresponds to the mass of the axial meson $a_1(1260)$. In the present case, γ is taken from previous studies of the electromagnetic form factors of the nucleon [12,13]. Therefore, α is the only fitting parameter.

Time like region

$$G_A(t) = G_A(0)g(t)\left[1-\alpha+\alpha\frac{m_A^2\left(m_A^2-t+im_A\Gamma_A\right)}{\left(m_A^2-t\right)^2+(m_A\Gamma_A)^2}\right],$$

with

$$g(t) = \left(1-e^{i\delta}\gamma t\right)^{-2}.$$

It can be measured with Experiments at FAIR and BES3

suggested through the reactions $N\bar{p} \to \gamma^* N\pi$ and the crossed channels [18–20]. The cross section related to these processes is large and such experiments may be performed in future colliders, such as FAIR (Germany), BES3 (China), DANAE (Italy). Such experiments also seem to be encouraged by our finding of a non-negligible time-like axial form factor, at least up to a few GeV^2, as shown in Fig. 2.