Universal parametrization of nucleon Form Factors

[Phys. Rev. D 85 (2012) 113004]

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GdR - QCD et Physique des Hadrons

Electromagnetic structure of hadrons: annihilation and scattering processes

IPN Orsay - October 3rd-5th, 2012

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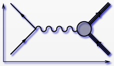
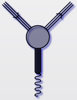
Agenda

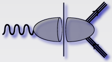


Nucleon Electromagnetic Form Factors

- Definition and properties

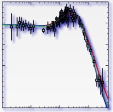


The Lomon-Gari-Krümpelmann model



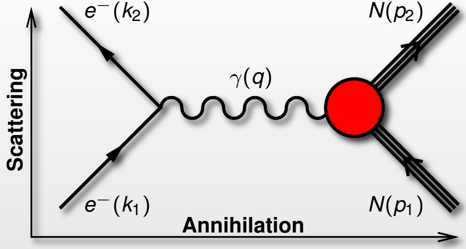
Time-like extension of the model

- Breit-Wigner regularization procedure



Data, fits, results and discussion

Dirac and Pauli Nucleon Form Factors



Scattering: $\boldsymbol{e^- N \to e^- N}$
Space-like kinematic region

$$q^2 = -2\omega_1\omega_2(1-\cos\theta_e) \leq 0$$

Annihilation: $\boldsymbol{e^+ e^- \leftrightarrow N\overline{N}}$
Time-like kinematic region

$$q^2 = 4\omega^2 > 0$$

Scattering amplitude in Born approximation

$$\mathcal{M} = \frac{1}{q^2}\,[e\,\overline{u}(k_2)\gamma_\mu u(k_1)]\,\underbrace{\left[e\,\overline{U}(p_2)\Gamma^\mu(p_1,p_2)U(p_1)\right]}_{\text{Nucleon EM 4-current: } J_N^\mu}$$

From Lorenz and gauge invariance

$$\Gamma^\mu(p_1,p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^N(q^2)$$

Dirac FF $F_1^N(q^2)$
- **Non-helicity flip**
- $F_1^N(0) = \mathcal{Q}_N$

Pauli FF $F_2^N(q^2)$
- **Helicity flip**
- $F_2^N(0) = \kappa_N$

$\mathcal{Q}_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

Sachs Nucleon Form Factors

Breit frame

No energy exchanged

$p_1 = (E, -\vec{q}/2)$

$p_2 = (E, \vec{q}/2)$

$q = (0, \vec{q})$

Nucleon EM 4-current

$$J_N^\mu = (J_N^0, \vec{J}_N) \quad \begin{cases} \rho_q = J_N^0 = e\left[F_1^N + \dfrac{q^2}{4M_N^2}F_2^N\right] \\ \vec{J}_N = e\,\overline{U}(p_2)\vec{\gamma}U(p_1)\,\left[F_1^N + F_2^N\right] \end{cases}$$

Sachs Nucleon Form Factors

$$G_E^N(q^2) = F_1^N(q^2) + \frac{q^2}{4M_N^2}F_2^N(q^2) \qquad G_M^N(q^2) = F_1^N(q^2) + F_2^N(q^2)$$

In the Breit frame G_E^N and G_M^N represent the **Fourier transforms** of **charge** and **magnetic moment spatial distributions** of the nucleon

Normalization at $q^2 = 0$

- $G_E^N(0) = \mathcal{Q}_N$
- $G_M^N(0) = \mu_N$

$\mu_N = \mathcal{Q}_N + \kappa_N$ is the nucleon magnetic moment

Isospin decomposition

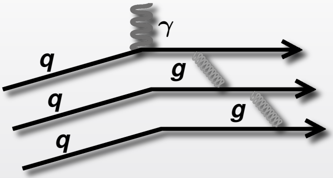
Isoscalar components

- $F_{1,2}^{is} = F_{1,2}^p + F_{1,2}^n$
- $G_{E,M}^{is} = G_{E,M}^p + G_{E,M}^n$

Isovector components

- $F_{1,2}^{iv} = F_{1,2}^p - F_{1,2}^n$
- $G_{E,M}^{iv} = G_{E,M}^p - G_{E,M}^n$

pQCD asymptotic behavior
Space-like region



- **pQCD**: as $q^2 \to -\infty$, $F_1(q^2)$ and $F_2(q^2)$ must follow counting rules
- Quarks exchange gluons to distribute momentum transferred by the photon

Dirac form factor F_1

- Non-helicity flip
- Two gluon propagators
- $F_1(q^2) \underset{q^2 \to -\infty}{\sim} (-\tilde{q}^2)^{-2}$

Pauli form factor F_2

- Helicity flip
- Two gluon propagators
- $F_2(q^2) \underset{q^2 \to -\infty}{\sim} (-\tilde{q}^2)^{-3}$

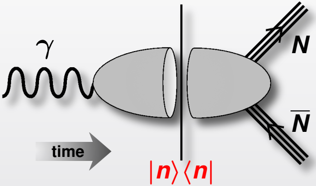
Sachs form factors G_E and G_M

- $G_{E,M}(q^2) \underset{q^2 \to -\infty}{\sim} (-\tilde{q}^2)^{-2}$
- Ratio: $\dfrac{G_E}{G_M} \underset{q^2 \to -\infty}{\sim}$ constant

QCD correction

$$\tilde{q}^2 = q^2 \ln\left(\frac{-q^2}{\Lambda_{QCD}}\right)$$

Nucleon form factors
Time-like region ($q^2 > 0$)



- Crossing symmetry:

$$\langle N(p_2)|J^\mu|N(p_1)\rangle \to \langle \overline{N}(p_2)N(p_1)|J^\mu|0\rangle$$

- Form factors are **complex functions** of q^2

Cutkosky rule for Nucleons

$$\mathrm{Im}\langle \overline{N}(p_2)N(p_1)|J^\mu(0)|0\rangle \sim \sum_n \langle \overline{N}(p_2)N(p_1)|J^\mu(0)|n\rangle\langle n|J^\mu(0)|0\rangle \Rightarrow \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4m_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

Time-like asymptotic behavior

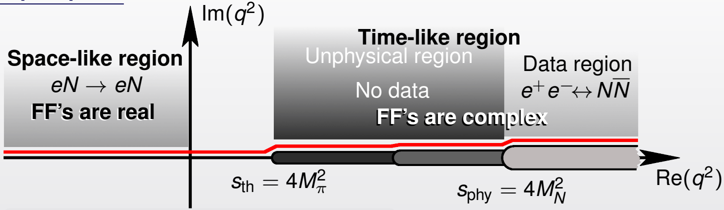
Phragmèn Lindelöf theorem:

If a function $f(z) \to a$ as $z \to \infty$ along a straight line, and $f(z) \to b$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \to a$ uniformly in this angle.

- $$\underbrace{\lim_{q^2\to-\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2\to+\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

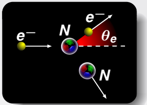
- $$G_{E,M} \underset{q^2\to+\infty}{\sim} (q^2)^{-2}$$ **real**

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

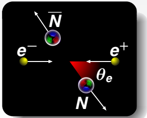
$$G_E(4M_N^2) = G_M(4M_N^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2\frac{\theta_e}{2}}{4\omega_1^3 \sin^4\frac{\theta_e}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta_e}{2}\right) G_M^2 \right] \frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_N^2}$$



Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2 \right]$$

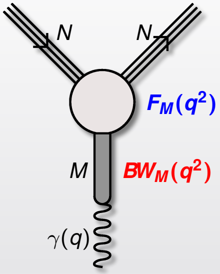
$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

The Lomon-Gari-Krünpelmann Model

The Model

The model is based on an idea of Iachello, Jackson and Landé, later developed by Gari and Krümpelmann, and Lomon. Nucleon EMFFs are parametrized using a **mixture** of Vector Meson Dominance (**VMD**) and **pQCD**.

- At low energy the coupling to the photons is described through vector meson exchange [VMD in terms of **propagators** $BW_M(q^2)$, $M = \rho, \omega, \phi, \rho', \omega'$].
- **Hadron/quark form factors** $F_M(q^2)$ at vector meson-nucleon (quark) vertices control transition to perturbative QCD at high momentum transfer.

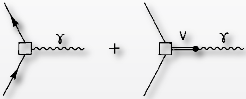


Analytic extension: space-like $\longrightarrow$ time-like

- **$BW_M(q^2)$ for broad mesons**:
  simple poles $\longrightarrow$ poles with finite energy-dependent widths
- **Dispersion relations**:
  rigorous analytic continuation of $BW_M(q^2)$ from time-like to space-like region

The **VMD** parametrization is for isospin components to single out different species of vector meson contributions

Isoscalar: ω, ω', ϕ Isovector: ρ, ρ'



Isospin components of Dirac and Pauli EMFFs

$$F_1^{\rm iv}(q^2)=\left[BW_\rho(q^2)+BW_{\rho'}(q^2)\right]F_1^\rho(q^2)+\left[1-BW_\rho(0)-BW_{\rho'}(0)\right]F_1^D(q^2)$$

$$F_2^{\rm iv}(q^2)=\left[\kappa_\rho BW_\rho(q^2)+\kappa_{\rho'}BW_{\rho'}(q^2)\right]F_2^\rho(q^2) +\left[\kappa_{\rm iv}-\kappa_\rho BW_\rho(0)-\kappa_{\rho'}BW_{\rho'}(0)\right]F_2^D(q^2)$$

$$F_1^{\rm is}(q^2)=\left[BW_\omega(q^2)+BW_{\omega'}(q^2)\right]F_1^\omega(q^2)+BW_\phi(q^2)F_1^\phi(q^2) +[1-BW_\omega(0)-BW_{\omega'}(0)]F_1^D(q^2)$$

$$F_2^{\rm is}(q^2)=\left[\kappa_\omega BW_\omega(q^2)+\kappa_{\omega'}BW_{\omega'}(q^2)\right]F_2^\omega(q^2)+\kappa_\phi BW_\phi(q^2)F_2^\phi(q^2) +[\kappa_{\rm is}-\kappa_\omega BW_\omega(0)-\kappa_{\omega'}BW_{\omega'}(0)]F_2^D(q^2)$$

The ingredients...

$$F_i^{\rm I}(q^2) = \sum_{V_{\rm I}} \kappa_{V_{\rm I}}^{i-1} BW_{V_{\rm I}}(q^2) F_i^{V_{\rm I}}(q^2) + \left[\kappa_{\rm I}^{i-1} - \sum_{V_{\rm I}} \kappa_{V_{\rm I}}^{i-1} BW_{V_{\rm I}}(0) F_i^{V_{\rm I}}(0)\right] F_i^D(q^2)$$

$BW_{V_{\rm I}}(q^2)$

$$BW_{V_{\rm I}}(q^2) = \frac{g_{V_{\rm I}} M_{V_{\rm I}}^2}{f_{V_{\rm I}}} \times [V_{\rm I}\text{-propagator}] \begin{cases} (M_{V_{\rm I}}^2 - q^2)^{-1} & V_{\rm I} = \omega, \phi \\ \text{[analytic]} & V_{\rm I} = \rho, \rho', \omega' \end{cases}$$

Meson nucleon FFs

$V_I = \rho,\ \rho',\ \omega,\ \omega'$

$$F_i^{V_{\rm I}}(q^2) = f_i(q^2) = \frac{\Lambda_1^2}{\Lambda_1^2 - \widetilde{q}^2}\left(\frac{\Lambda_2^2}{\Lambda_2^2 - \widetilde{q}^2}\right)^i$$

$$F_1^{\phi}(q^2) = f_1(q^2)\left(\frac{\widetilde{q}^2}{\widetilde{q}^2 - \Lambda_1^2}\right)^{3/2}$$

$$F_2^{\phi}(q^2) = f_2(q^2)\left(\frac{\Lambda_1^2}{\mu_\phi^2}\,\frac{\widetilde{q}^2 - \mu_\phi^2}{\widetilde{q}^2 - \Lambda_1^2}\right)^{3/2}$$

Photon-valence quark coupling

$$F_i^D(q^2) = \frac{\Lambda_D^2}{\Lambda_D^2 - \widetilde{q}^2}\left(\frac{\Lambda_2^2}{\Lambda_2^2 - \widetilde{q}^2}\right)^i$$

QCD correction

$$\widetilde{q}^2 = q^2 \frac{\ln[(\Lambda_D^2 - q^2)/\Lambda_{\rm QCD}]}{\ln(\Lambda_D^2/\Lambda_{\rm QCD}^2)}$$

$\kappa_{V_{\rm I}}$ $\kappa_{\rm I}$

- $\kappa_{V_{\rm I}}$ is the ratio of tensor to vector $NNV_{\rm I}$-coupling at $q^2 = 0$
- Isospin anomalous magnetic moments: $\begin{cases} \kappa_{\rm is} = \kappa_p + \kappa_n \\ \kappa_{\rm iv} = \kappa_p - \kappa_n \end{cases}$

The ingredients. . .

$$F_i^I(q^2) = \sum_{V_I} \kappa_{V_I}^{i-1} BW_{V_I}(q^2) F_i^{V_I}(q^2) + \left[\kappa_I^{i-1} - \sum_{V_I} \kappa_{V_I}^{i-1} BW_{V_I}(0) F_i^{V_I}(0)\right] F_i^D(q^2)$$

$BW_{V_I}(q^2)$

$$BW_{V_I}(q^2) = \frac{g_{V_I} M_{V_I}^2}{f_{V_I}} \times [V_I\text{-propagator}] \begin{cases} (M_{V_I}^2 - q^2)^{-1} & V_I = \omega, \phi \\ [\text{analytic}] & V_I = \rho, \rho', \omega' \end{cases}$$

Meson nucleon FFs

$V_I = \rho,\ \rho',\ \omega,\ \omega'$

$$F_i^{V_I}(q^2) = f_i(q^2) = \frac{\Lambda_1^2}{\Lambda_1^2 - \tilde{q}^2}\left(\frac{\Lambda_2^2}{\Lambda_2^2 - \tilde{q}^2}\right)^i$$

$$F_1^{\phi}(q^2) = f_1(q^2)\left(\frac{\tilde{q}^2}{\tilde{q}^2 - \Lambda_1^2}\right)^{3/2}$$

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$$\tilde{q}^2 = q^2 \frac{\ln[(\Lambda_D^2 - q^2)/\Lambda_{QCD}]}{\ln(\Lambda_D^2/\Lambda_{QCD}^2)}$$

κ_{V_I} κ_I

- κ_{V_I} is the ratio of tensor to vector NNV_I-coupling at $q^2 = 0$
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The ingredients. . .

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$BW_{V_I}(q^2)$

$$BW_{V_I}(q^2) = \frac{g_{V_I} M_{V_I}^2}{f_{V_I}} \times [V_I\text{-propagator}] \begin{cases} (M_{V_I}^2 - q^2)^{-1} & V_I = \omega, \phi \\ [\text{analytic}] & V_I = \rho, \rho', \omega' \end{cases}$$

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$$F_2^{\phi}(q^2) = f_2(q^2) \left(\frac{\Lambda_1^2}{\mu_\phi^2} \frac{\tilde{q}^2 - \mu_\phi^2}{\tilde{q}^2 - \Lambda_1^2}\right)^{3/2}$$

Photon-valence quark coupling

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Photon-valence quark coupling

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$$F_i^{V_\mathrm{I}}(q^2) = f_i(q^2) = \frac{\Lambda_1^2}{\Lambda_1^2 - \tilde{q}^2}\left(\frac{\Lambda_2^2}{\Lambda_2^2 - \tilde{q}^2}\right)^i$$

$$F_1^{\phi}(q^2) = f_1(q^2)\left(\frac{\tilde{q}^2}{\tilde{q}^2 - \Lambda_1^2}\right)^{3/2}$$

$$F_2^{\phi}(q^2) = f_2(q^2)\left(\frac{\Lambda_1^2}{\mu_\phi^2}\,\frac{\tilde{q}^2 - \mu_\phi^2}{\tilde{q}^2 - \Lambda_1^2}\right)^{3/2}$$

Photon-valence quark coupling

$$F_i^D(q^2) = \frac{\Lambda_D^2}{\Lambda_D^2 - \tilde{q}^2}\left(\frac{\Lambda_2^2}{\Lambda_2^2 - \tilde{q}^2}\right)^i$$

QCD correction

$$\tilde{q}^2 = q^2 \frac{\ln[(\Lambda_D^2 - q^2)/\Lambda_{\mathrm{QCD}}]}{\ln(\Lambda_D^2/\Lambda_{\mathrm{QCD}}^2)}$$

κ_{V_I} κ_I

- κ_{V_I} is the ratio of tensor to vector NNV_I-coupling at $q^2 = 0$
- Isospin anomalous magnetic moments: $\begin{cases} \kappa_{\mathrm{is}} = \kappa_p + \kappa_n \\ \kappa_{\mathrm{iv}} = \kappa_p - \kappa_n \end{cases}$

Time-like extension of the model

Analytic Breit-Wigner formulas

Relativistic Breit-Wigner formula for an unstable particle with mass ***M*** and constant width **Γ**

$$BW(s) = \frac{1}{M^2 - s - i\Gamma M}$$

In the s-complex plane

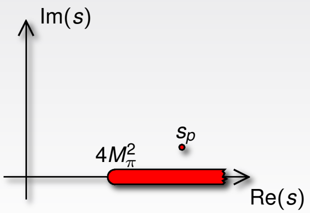
- Single pole
- No discontinuity cut

Energy-dependent widths are introduce considering decay rates extended to off shell particle masses. In the ρ meson case with $\boldsymbol{\Gamma(\rho \to \pi^+\pi^-) = \Gamma_\rho}$.

$$\Gamma(s) = \Gamma_0 \frac{M_\rho^2}{s} \left(\frac{s - 4M_\pi^2}{M_\rho^2 - 4M_\pi^2} \right)^{\frac{3}{2}}$$

$$BW(s) = \frac{s}{s(M_\rho^2 - s) - i\Gamma_0 M_\rho^3 \left(\frac{s-4M_\pi^2}{M_\rho^2-4M_\pi^2} \right)^{3/2}}$$

- Has the "required" discontinuity cut $(4M_\pi^2, \infty)$
- Maintains a complex pole $s_p \simeq M_\rho^2 + i\Gamma_\rho M_\rho$, slightly shifted w.r.t. the original position
- The power "3/2" in the denominator and the factor $1/s$ generate additional "physical" poles



Regularization of Breit-Wigner formulas

The BW formula with energy-dependent width has a set of N poles $\{z_j\}_{j=1}^{N}$ in the s-complex plane

- $P_N(s)$ is a suitable N degree polynomial
- β is a noninteger real number which defines the discontinuity cut

$$BW(s) = \frac{P_N(s)\prod_{j=1}^{N}(s-z_j)^{-1}}{M_\rho^2 - s - i\gamma(s-4M_\pi^2)^\beta}$$

To avoid unphysical divergences poles must be subtracted. BW formulas are **regularized** by adding counterparts that behave as the opposite of each pole.

Method #1

The subtraction can be done by hand...

$$\widetilde{BW}(s) = BW(s) - \sum_{k=1}^{N} \frac{P_N(z_k)\prod_{j=1,j\neq k}^{N}(z_k-z_j)^{-1}}{M_\rho^2 - z_k - i\gamma(z_k-4M_\pi^2)^\beta} \times \frac{1}{s-z_k}$$

- **Advantage**: easy to handle and to implement in codes
- **Drawback**: we need to know the pole positions

Regularization using dispersion relations

If $f(z)$ is an analytic function in the whole z complex plane with a real positive cut (s_0, ∞) and $f(z) = o(1/\ln|z|)$ as $z \to \infty$ then ....

$$f(z) = \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathrm{Im}\,[f(x)]\,dx}{x - z}$$

If the function $f(z)$ has also a finite number of isolated poles $\{z_j\}_{j=1}^{N}$, **residues** have to be considered.......................

$$f(z) + 2\pi i \sum_{j=1}^{N} \mathrm{Res}\left[\frac{f(z')}{z' - z}, z_j\right] = \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathrm{Im}\,[f(x)]\,dx}{x - z}$$

Writing

$$f(z) = \phi(z)\prod_{j=1}^{N}\frac{1}{z - z_j}$$

$\phi(z)$ is the pole-free part of $f(z)$

$$f(z) + \sum_{k=1}^{N}\frac{\phi(z_k)}{z_k - z}\prod_{j\neq k}^{N}\frac{1}{z_k - z_j} = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{Im}\,[f(x)]\,dx}{x - z}$$

Method #2

$$\widetilde{BW}(s) = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}\,[BW(s')]\,ds'}{s' - s}$$

- **Advantage**: we do not need to know the coordinates of the poles
- **Drawback**: we have to compute integrals over infinite intervals

Fit, results and discussion

The fitting procedure

Space-like region

Quantity (Q_i)	G_M^p	G_E^p	G_M^n	G_E^n	$\frac{\mu_p G_E^p}{G_M^p}$	$\frac{\mu_n G_E^n}{G_M^n}$
n. of points (N_i)	68	36	65	14	25	13

Time-like

$\lvert G_{\rm eff}^p \rvert$	$\lvert G_{\rm eff}^n \rvert$
81(43)	5

Global χ^2

$$\chi^2 = \sum_{i=1}^{9} \tau_i \cdot \chi_i^2$$

Contribution of the data set: $\{q_k^2, v_k^i, \delta v_k^i; N_i\}$

$$\chi_i^2 = \sum_{k=1}^{N_i} \left(\frac{Q_i(q_k^2) - v_k^i}{\delta v_k^i} \right)^2$$

13 Free parameters

$\Lambda_1, \Lambda_2, \Lambda_D$
Parametrize hadronic FFs and control the transition from non perturbative to perturbative QCD

Five pairs $(\kappa_{V_l}, g_{V_l}/f_{V_l})$ of vector meson anomalous magnetic momenta and couplings with:
$V_l = \rho, \rho', \omega, \omega', \phi$

- Masses and widths of all vector mesons are fixed to the PDG values
- For the QCD scale two values have been considered: $\Lambda_{\rm QCD} = 0.15,\ 0.10$ GeV

Energy-dependent widths are used only for broad resonances: ρ, ρ', ω'
Two different analytic structures have been used

- **Case=s**: $\Gamma_s(s) = \Gamma_0 \frac{M^2}{s} \left(\frac{s - s_{\rm th}}{M^2 - s_{\rm th}} \right)^{3/2}$
- **Case=1**: $\Gamma_1(s) = \Gamma_0 \left(\frac{s - s_{\rm th}}{M^2 - s_{\rm th}} \right)^{3/2}$

Resonances are assumed to decay predominantly into a two-body channel of mass $s_{\rm th}^{1/2}$

Contrary to other experiments, the ***BaBar*** data on $|G_{\rm eff}^p(q^2)|$, in the time-like region, have been obtained by studying the angular distribution of the the radiative process

$$e^+e^- \to e^+e^-\gamma_{\rm init} \to p\bar{p}\gamma_{\rm init}$$

The initial state radiation, in particular kinematic regions, could entail additional corrections [PRD84, 017301] that are not taken into account by the ***BaBar*** collaboration

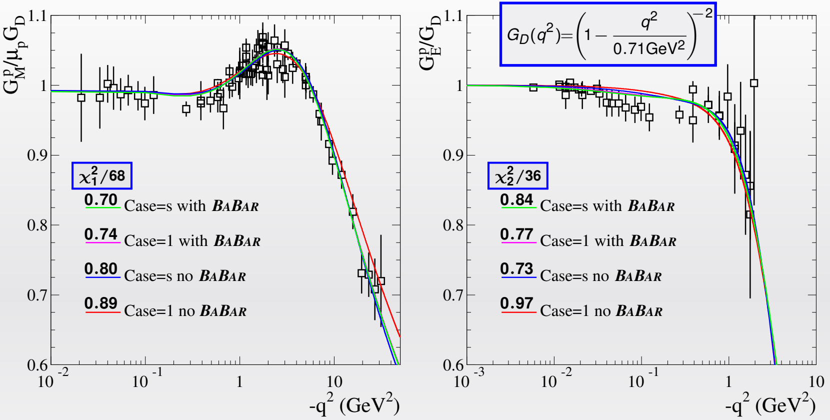
As a consequence we considered the possibility of not including these data in the fit

Four cases

- **swB:** Case=s - with *BaBar*
- **snB:** Case=s - no *BaBar*
- **1wB:** Case=1 - with *BaBar*
- **1nB:** Case=1 - no *BaBar*

Proton space-like G_E^p and G_M^p

Data collection: PRC82,045211



The charge radius

$$\tilde{r}_p \equiv \sqrt{\langle r^2 \rangle_p} = \left[\frac{1}{6}\frac{dG_E^p(q^2)}{dq^2}\right]^{1/2}_{q^2=0}$$

swB $\tilde{r}_p = 0.876$ fm

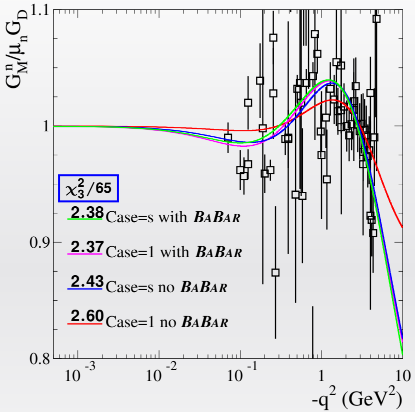
1wB $\tilde{r}_p = 0.863$ fm

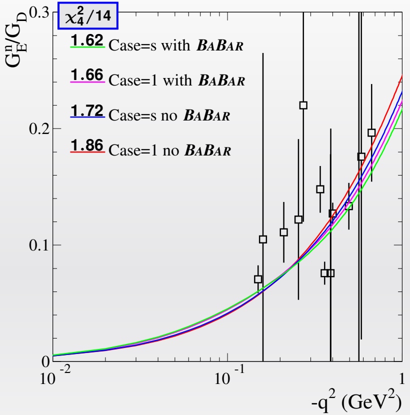
snB $\tilde{r}_p = 0.834$ fm

1nB $\tilde{r}_p = 0.819$ fm

Neutron space-like G_E^n and G_M^n

Data collection: PRC82,045211





The mean square charge radius

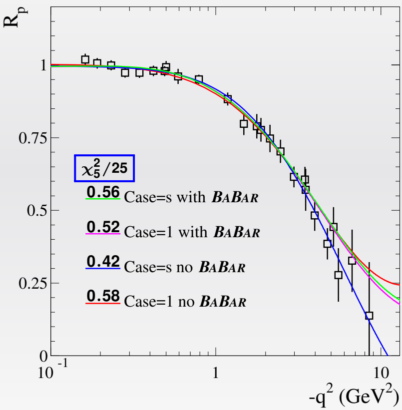
$$\langle r_n^2 \rangle = \frac{1}{6}\frac{dG_E^n(q^2)}{dq^2}\bigg|_{q^2=0}$$

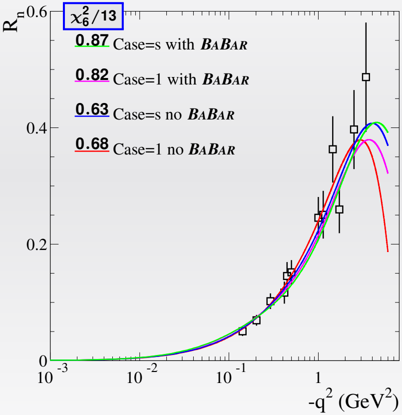
swB $\langle r_n^2 \rangle = -0.117$ fm^2
1wB $\langle r_n^2 \rangle = -0.117$ fm^2
snB $\langle r_n^2 \rangle = -0.112$ fm^2
1nB $\langle r_n^2 \rangle = -0.112$ fm^2

Space-like ratios

$R_p = \mu_p G_E^p / G_M^p$ and $R_n = \mu_n G_M^n / G_M^n$

Data collection: PRC82,045211





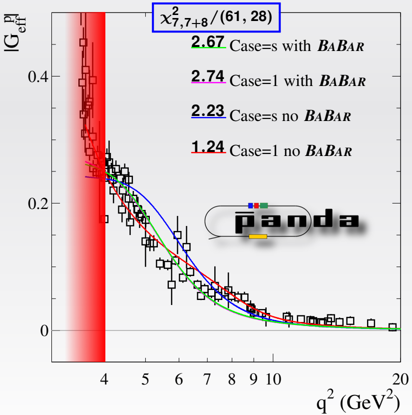
Time-like "effective" FFs: $|G^p_{\rm eff}|$ and $|G^n_{\rm eff}|$

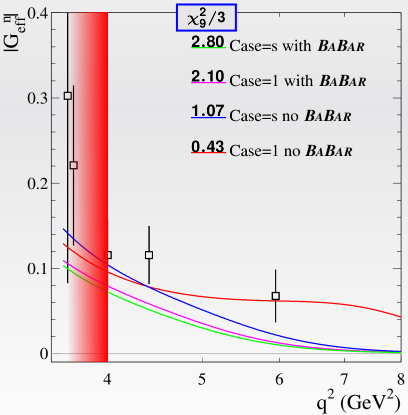
From experiments

$$|G^N_{\rm eff}| = \sqrt{\frac{\sigma(e^+e^- \to N\bar{N})}{\frac{4\pi\alpha^2}{3q^2}\sqrt{1-\frac{4M_N^2}{q^2}}\left(1+\frac{2M_N^2}{q^2}\right)}}$$

Fitting function

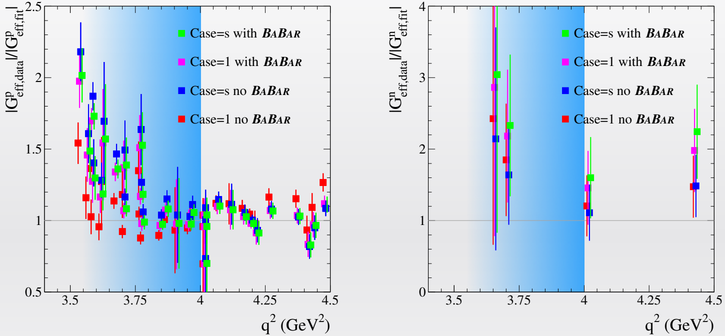
$$|G^N_{\rm eff}| = \sqrt{\frac{|G^N_M(q^2)|^2 + \frac{2M_N^2}{q^2}|G^N_E(q^2)|^2}{1+\frac{2M_N^2}{q^2}}}$$





Time-like threshold behavior

$$|G_{\rm eff}^N| = \sqrt{\sigma(e^+e^- \to N\overline{N})}\left[\frac{4\pi\alpha^2}{3q^2}\sqrt{1-4M_N^2/q^2}\left(1+2M_N^2/q^2\right)\right]^{-1/2}$$

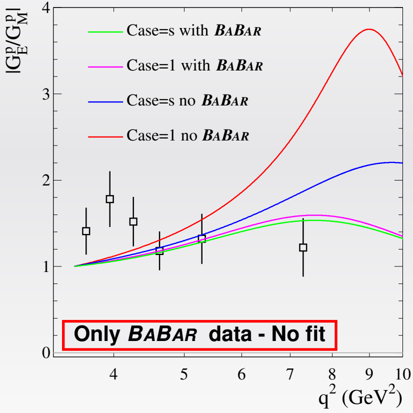


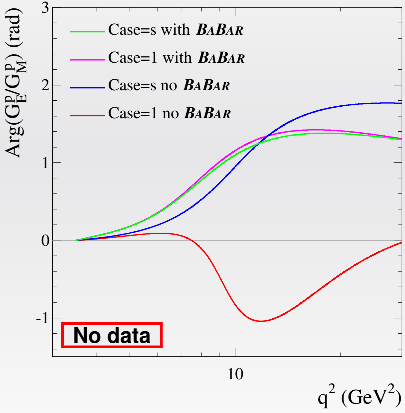
- As a consequence of near-threshold **flat cross sections**, effective nucleon EMFFs have a steep enhancement when $q^2 \to (2M_N)^2$
- Such flat cross sections are in contrast with the expectation in case of smooth EMFFs ($\sigma \propto \beta_N$)
- To extract Born cross sections and hence $|G_{\rm eff}^N|$ in the threshold region, data have to be corrected for **Coulomb** as well as **strong** effects

To avoid ambiguities due to the not well known form and interplay of these threshold corrections, **time-like data below $q^2 = 4$ GeV^2 have not been considered**

Prediction for the time-like ratio G_E^p/G_M^p

Data: PRD73,012005



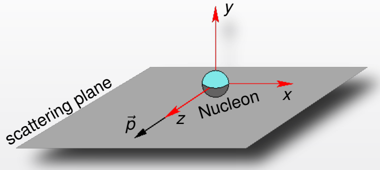


Polarization formulae in the time-like region

The ratio $R_N(q^2)$ is complex for $q^2 \geq s_{\rm th}$

$$R_N(q^2) = \mu_p \frac{G_E^N(q^2)}{G_M^N(q^2)} = |R_N(q^2)| e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

$$\mathcal{P}_y^N = -\frac{\sin(2\theta)|R_N|\sin(\rho)}{D_N\sqrt{\tau}} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \equiv \mathcal{A}_y$$

Does not depend on P_e

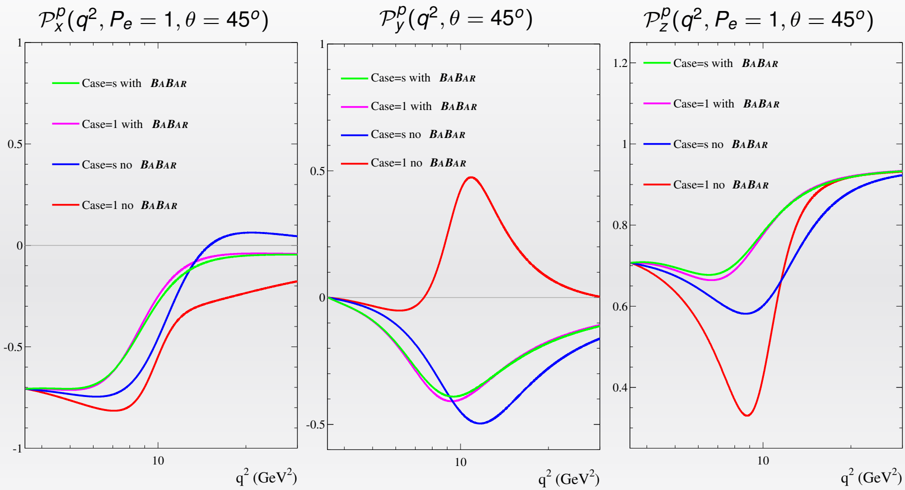
$$\mathcal{P}_x^N = -P_e \frac{2\sin(2\theta)|R_N|\cos(\rho)}{D_N\sqrt{\tau}}$$

$$\mathcal{P}_z^N = P_e \frac{2\cos(\theta)}{D_N}$$

Does not depend on ρ

$$D_N = \frac{1 + \cos^2\theta + \frac{1}{\tau}|R_N|^2 \sin^2\theta}{\mu_N}, \qquad \tau = \frac{q^2}{4M_N^2}, \qquad P_e = \text{electron polarization}$$

Prediction for the proton polarization



Summary of χ^2 contributions

	Q_i	N_i	minimum χ_i^2			
			case=s With BaBar	case=1 With BaBar	case=s No BaBar	case=1 No BaBar
space-like	G_M^p	68	48.7	50.1	54.6	60.8
	G_E^p	36	30.4	27.6	26.2	35.0
	G_M^n	65	154.6	154.2	158.2	167.0
	G_E^n	14	22.7	23.2	24.1	26.0
	$\mu_p G_E^p / G_M^p$	25	13.9	12.9	10.6	14.4
	$\mu_n G_E^n / G_M^n$	13	11.3	10.7	8.2	8.9
time-like	$\lvert G_{\text{eff}}^p \rvert$	61 (28)	162.5	166.7	62.2	35.0
	$\lvert G_{\text{eff}}^n \rvert$	3	8.4	6.3	3.2	0.3
	Total	285(252)	452.5	451.7	347.3	347.4
	Normalized χ^2		1.663	1.661	1.453	1.454

Best values of fit parameters and constants

Parameter	case = s With *BABAR*	case = 1 With *BABAR*	case = s No *BABAR*	case = 1 No *BABAR*
g_ρ/f_ρ	2.766	2.410	0.9029	0.4181
κ_ρ	-1.194	-1.084	0.8267	0.6885
M_ρ (GeV)	0.7755 (fixed)			
Γ_ρ (GeV)	0.1491 (fixed)			
g_ω/f_ω	-1.057	-1.043	-0.2308	-0.4894
κ_ω	-3.240	-3.317	-9.859	-1.398
M_ω (GeV)	0.78263 (fixed)			
g_ϕ/f_ϕ	0.1871	0.1445	-0.0131	-0.1156
κ_ϕ	-2.004	-3.045	37.218	-0.2613
M_ϕ (GeV)	1.019 (fixed)			
μ_ϕ (GeV)	20.0 (fixed)			
$g_{\omega'}/f_{\omega'}$	2.015	1.974	1.265	1.649
$\kappa_{\omega'}$	-2.053	-2.010	-2.044	-0.6712
$M_{\omega'}$ (GeV)	1.425 (fixed)			
$\Gamma_{\omega'}$ (GeV)	0.215 (fixed)			
$g_{\rho'}/f_{\rho'}$	-3.475	-3.274	-0.8730	-0.0369
$\kappa_{\rho'}$	-1.657	-1.724	-2.832	-104.35
$M_{\rho'}$ (GeV)	1.465 (fixed)			
$\Gamma_{\rho'}$ (GeV)	0.400 (fixed)			
Λ_1 (GeV)	0.4801	0.5000	0.6474	0.6446
Λ_2 (GeV)	3.0536	3.0562	3.0872	3.6719
Λ_D (GeV)	0.7263	0.7416	0.8573	0.8967
Λ_{QCD} (GeV)	0.150			0.100

Discussion

Time-like extension of the Lomon-Gari-Krümpelmann model

- Breit-Wigner formulas describing broad intermediate vector mesons have been modified including energy-dependent widths in two scenarios: $\Gamma_1(q^2)$, minimal alteration, and $\Gamma_s(q^2)$ derived from relativistic perturbation theory
- A **regularization procedure** has been defined to remove unwanted poles and so to fulfill the analyticity requirements in the whole q^2 complex plane

Fit results

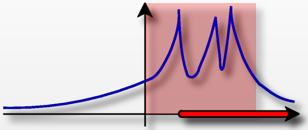
- The improvements in the BW formulas do not affect the space-like fit quality
- The **simultaneous space-like and time-like** fit is satisfactory
- The χ^2 contributions from each space-like data set are almost **unchanged** between **case=1** and **case=*s***
- The quality of the fit is **poorer when *BABAR* data are included**

Predictions

- The knowledge of the analytic structure of EMFFs allows us to make predictions on **polarization observables**, EMFF **phases** and **"pure" moduli**
- Measurements of such observables would be effective in **discriminating among the different models and parametrizations [BESIII, PANDA, ...]**

BACK-UP SLIDES

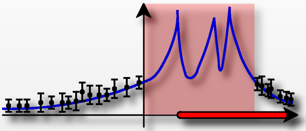
Key points



EMFFs are **analytic functions** in the whole q^2 complex plane

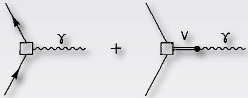
EMFFs are well defined in both space-like and time-like regions

$\Rightarrow$



All parametrizations **must** possess this property

They should be able to describe scattering and annihilation data



VMD-based models can be easily analytically extended to all real values of q^2

They have amplitudes with the required complex structure

Low-$|q^2|$ data are described as superposition of resonance tails

In the **time-like region**, where these tails are complex, data are even more sensitive to the considered VMD contributions