Hadron electromagnetic form factors in Time-like region

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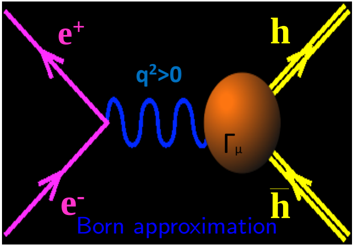
Table of contents

Time-like electromagnetic processes of hadrons

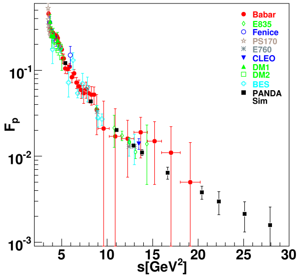


- Γ_μ is parametrized in terms of 2S+1 FFs.
- Optical theorem requires imaginary part for $q^2 > 4m_\pi^2$
  - Spin 0 (π, K) $\rightarrow$ one FF .
  - Spin 1/2 (nucleons) $\rightarrow$ 2 FFs : G_E and G_M.
    - Differential cross section $\rightarrow$ modulus of 2 FFs.
    - polarization observables $\rightarrow$ relative phase.
  - Spin 1 (Deuteron, $\rho - meson$) $\rightarrow$ 3 FFs : G_C, G_M and G_Q.

Time-like Form Factors : Data

- Proton ($|G_M|$; $|G_E| = |G_M|$ or $|G_E| = 0$):

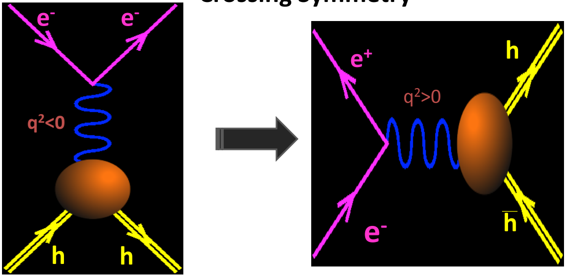
$$e^+ + e^- \leftrightarrow \bar{p} + p$$



- Neutron ($|G_M|$)
  - 4 points measured by FENICE experiment ($e^+ + e^- \to \bar{n} + n$)
- Deuteron and ρ-meson
  - no data in TL region.

Time-like Form Factors : Models

Crossing Symmetry

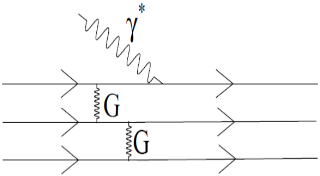


FFs are analytical functions of q^2

- Most of FFs models are constructed to describe and Fit SL data .
- Time-like FFs (q^2 positive) can be obtained by analytic continuation from the Space-like FFs (q^2 negative)
- At high q^2, pQCD gives predictions for q^2-dependence.

pQCD parametrization of proton FFs

SPACE-LIKE REGION
$(Q^2 = -q^2)$



- quark counting rules :

$$G_{E,M}(Q^2) \sim 1/(Q^2)^2$$

- pQCD :

$$G_{E,M}(Q^2) \sim \frac{\alpha_s^2(Q^2)}{(Q^2)^2}$$

- Asymptotic freedom, beta function :

$$\alpha_s = \frac{1}{b \ln(Q^2/\Lambda^2)}, \quad b = \frac{11 - 2/3N_f}{4\pi}, \quad Q^2 > \Lambda^2.$$

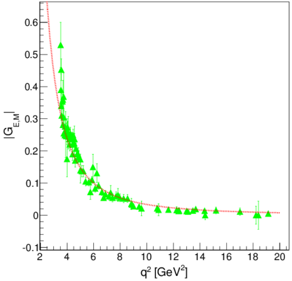
pQCD parametrization of proton FFs

Time-like region

- Analytical extension : $\ln(Q^2/\Lambda^2) \to \ln(q^2/\Lambda^2) - i\pi,\ q^2 > \Lambda^2.$

$$G_{E,M} = \frac{A}{(q^2)^2[\ln(q^2/\Lambda^2) - i\pi]^2},\ \Lambda = 0.3\ GeV,\ A = 96.2\ GeV^4$$

$$G_E(4M_p^2) = G_M(4M_p^2)$$



Vector Meson Dominance formalism

- VMD model describes interaction of photon with hadron (γHH vertex) through exchange of vectors mesons (ω, ϕ, ρ).
- VMD parametrization:

$$F(Q^2) = \Sigma_\nu A_\nu \frac{m_\nu^2}{m_\nu^2 + Q^2}$$

- small number of parameters with physical meaning (masses, coupling constants,...).
- can be extended to TL region.

Spin 1/2

Protons and neutrons

VMD parametrization of spin 1/2 FFs

Space like

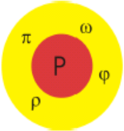
Iachello, Jackson and Lande (Physics letters 43 B 1973)

$$F_1^s(Q^2) = \frac{g(Q^2)}{2}\left[(1-\beta_\omega-\beta_\phi)+\beta_\omega\frac{\mu_\omega^2}{\mu_\omega^2+Q^2}+\beta_\phi\frac{\mu_\phi^2}{\mu_\phi^2+Q^2}\right],$$

$$F_1^v(Q^2) = \frac{g(Q^2)}{2}\left[(1-\beta_\rho)+\beta_\rho\frac{\mu_\rho^2+8\Gamma_\rho\mu_\pi/\pi}{(\mu_\rho^2+Q^2)+(4\mu_\pi^2+Q^2)\Gamma_\rho\alpha(Q^2)/\mu_\pi}\right],$$

$$F_2^s(Q^2) = \frac{g(Q^2)}{2}\left[(\mu_p+\mu_n-1-\alpha_\phi)\frac{\mu_\omega^2}{\mu_\omega^2+Q^2}+\alpha_\phi\frac{\mu_\phi^2}{\mu_\phi^2+Q^2}\right],$$

$$F_2^v(Q^2) = \frac{g(Q^2)}{2}\left[(\mu_p-\mu_n-1)\frac{\mu_\rho^2+8\Gamma_\rho\mu_\pi/\pi}{(\mu_\rho^2+Q^2)+(4\mu_\pi^2+Q^2)\Gamma_\rho\alpha(Q^2)/\mu_\pi}\right],$$



Time like

Iachello and Wan PRC 69 (2004)

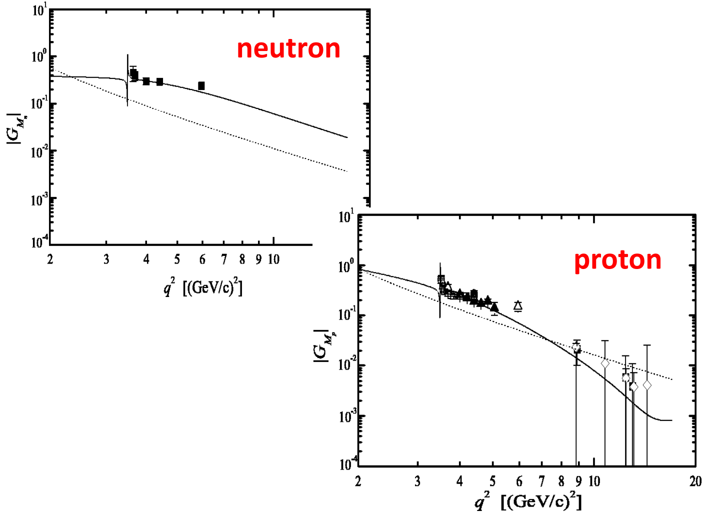
$$g(q^2)=\frac{1}{(1-\gamma e^{i\theta}q^2)^2}$$

$$\frac{m_\rho^2}{m_\rho^2-q^2}\rightarrow\frac{m_\rho^2+8\Gamma_\rho m_\pi/\pi}{m_\rho^2-q^2+(4m_\pi^2-q^2)\Gamma_\rho\alpha(q^2)/m_\pi+i\Gamma_\rho 4m_\pi\beta(q^2)}$$

VMD parametrization of spin 1/2 FFs

Good agreement with both proton and neutron TL data

Iachello and Wan, PRC 69 (2004)

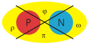


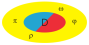
Spin 1

Deuteron

VMD parametrization of deuteron FFs

C. Adamuscin, G.I. Gakh and E.T-G. PRC 73, 045204 (2006)





- One hard core with surrounding meson cloud.

$$G_i(Q^2) = N_i g_i(Q^2) F_i(Q^2), \quad i = c, q, m$$

  - Isoscalar vector mesons (ω, ϕ)

$$F_i(Q^2) = 1 - \alpha_i - \beta_i + \alpha_i \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_i \frac{m_\phi^2}{m_\phi^2 + Q^2}$$

  - Intrinsic structure

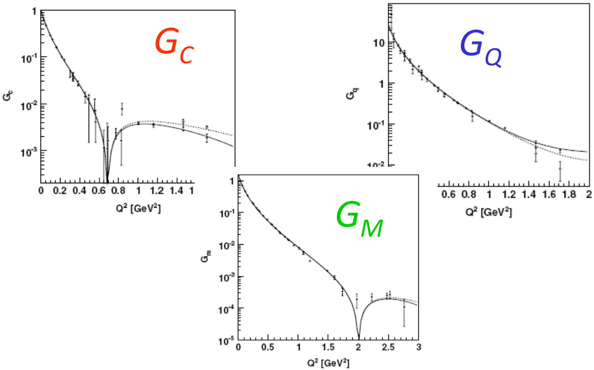
$$g_i(Q^2) = 1/[1 + \gamma_i Q^2]^{\delta_i}$$

  - N_i's : correspond to the normalisation static properties:

$$G_C(0) = 1, G_Q(0) = M_d^2 Q_d^2 = 25.83, G_M(0) = 1.714$$

VMD parametrization of deuteron FFs

*C. Adamuscin, G.I. Gakh and E.T-G. PRC **73**, 045204 (2006)*



Due to the quadratic dependence of FFs on the cross section and polarization observables in ed elastic scattering, there are 2 sets of experimental points for G_C and G_Q.

Spin 1

ρ meson

Model for ρ meson FFs

C. Adamuscin, G.I. Gakh and E.T-G. Phys.Rev. C75 065202 (2007)

Space-like region

$$G_C(q^2) = \frac{G_C(0)(A+Bq^2)m_C^2}{(m_C^2 - q^2)^2}$$
$$G_M(q^2) = \frac{G_M(0)m_M^4}{(m_M^2 - q^2)^2}$$
$$G_Q(q^2) = \frac{G_Q(0)m_M^4}{(m_Q^2 - q^2)^2}$$

Time-like region

$$G_C(q^2) = \frac{G_C(0)(A+Bq^2)m_C^2}{(m_C^2 - q^2 - im_C\Gamma_C)^2}$$
$$G_M(q^2) = \frac{G_M(0)m_M^4}{(m_M^2 - q^2 - im_M\Gamma_M)^2}$$
$$G_Q(q^2) = \frac{G_Q(0)m_M^4}{(m_Q^2 - q^2 - im_Q\Gamma_Q)^2}$$

- Parameters (A, B, m_i) are fixed based on theoretical (covariant and light front) calculation in SL. PRC 55, 2043 (1997)
- A=1, B=0.33 from the node of $G_C(q^2 = -3GeV^2)$

Helicity amplitudes measurements by BaBar

PHYSICAL REVIEW D **78,** 071103(R) (2008)

Observation of $e^+e^- \to \rho^+\rho^-$ near $\sqrt{s} = 10.58$ GeV

$$|F_{00}|^2 : |F_{10}|^2 : |F_{11}|^2 = 0.51 \pm 0.14(stat) \pm 0.07(syst)$$
$$0.10 \pm 0.04(stat) \pm 0.01(syst)$$
$$0.04 \pm 0.03(stat) \pm 0.01(syst)$$

$$|F_{00}|^2 + 4|F_{10}|^2 + 2|F_{11}|^2 = 0.01$$

To fit the data, we need a model for the 3 FFs

Helicity amplitudes of $\gamma^* \to \rho^+\rho^-$

- Matrix element :

$$M = \epsilon\cdot(p_1-p_2)[-G_1(q^2)U_1^*\cdot U_2^* + \frac{G_3(q^2)}{M^2}(U_1^*\cdot q U_2^*\cdot q - \frac{q^2}{2}U_1^*\cdot U_2^*)] - G_2(q^2)(\epsilon\cdot U_1^* U_2^*\cdot q - \epsilon\cdot U_2^* U_1^*\cdot q),$$

ϵ, U_1, U_2 are the polarization vectors of γ^*, ρ^+, ρ^-

- helicity amplitudes:

$$F_{\lambda_1\lambda_2} = M^{\lambda}_{\lambda_1\lambda_2} = M(\epsilon\to\epsilon^{(\lambda)}, U_1\to U_1^{(\lambda_1)}, U_2\to U_2^{(\lambda_2)}),\ \lambda=\lambda_1-\lambda_2$$

  - spin one photon : $F_{1-1} = F_{-11} = 0$
  - symmetry properties : $F_{-1-1} = F_{11},\ F_{10} = F_{01} = F_{-10} = F_{0-1}$

$$F_{00},\quad F_{10},\quad F_{11}$$

Helicity amplitudes of $\gamma^* \to \rho^+\rho^-$

Helicity amplitudes are combination of the three complex FFs

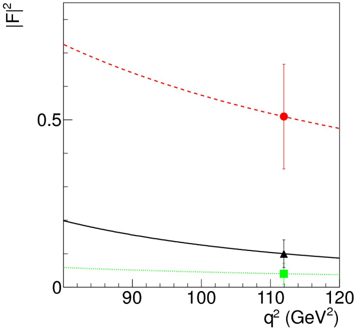
$$
\begin{aligned}
F_{00} &= -\frac{\sqrt{q^2-4M^2}}{2M^2}[q^2(G_1+G_2+G_3)-2M^2G_1],\\
F_{11} &= \sqrt{q^2-4M^2}(G_1+2\tau G_3),\\
F_{10} &= -\sqrt{\tau(q^2-4M^2)}G_2, \quad \tau=\frac{q^2}{4M^2}.
\end{aligned}
$$

Total cross section:

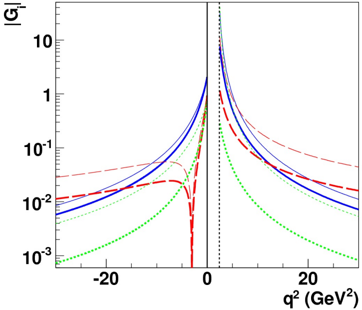
$$
\sigma(e^+e^-\to\rho^+\rho^-)=\frac{\pi\alpha^2\beta^3}{3q^2}\frac{1}{4M^2(\tau-1)}(|F_{00}|^2+4|F_{10}|^2+2|F_{11}|^2)
$$

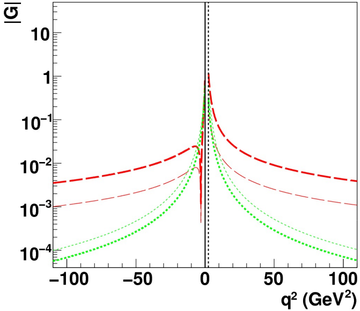
Fit the data

Ref.	m_C (GeV)	m_M (GeV)	m_Q (GeV)
[3]	1.34 ± 2	1.42 ± 0.5	1.51 ± 0.1
This work (I)	$1.05^{+0.05}_{-0.09}$	$1.28^{+0.06}_{-0.08}$	$0.97^{+0.02}_{-0.01}$
This work (II)	$0.77^{+0.05}_{-0.02}$	$1.28^{+0.06}_{-0.08}$	$1.12^{+0.05}_{-0.08}$



Absolute value of ρ-meson FFs: $|G_M|$, $|G_C|$ and $|G_Q|$





Conclusions

- Models based on vector meson dominance give useful parametrization in SL and TL regions for spin 1/2 hadrons.

- The VMD parametrization can reproduce the unique measurement on the spin one TL FFs given in terms of helicity amplitudes.

- PANDA experiment will provide a powerful test of the existent models of the nucleon FFs in a wide q^2 range.