

# Nucleon-Delta electromagnetic form factors

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In collaboration with [M. T. Peña](#)

Electromagnetic structure of hadrons: annihilation and scattering  
processes - GDR-PH-QCD, Meeting Groupe 2

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- 1 Spectator quark model
- 2  $\gamma^* N \rightarrow \Delta$  reaction
  - Form factors
  - Dalitz decay
- 3 Nucleon form factors in timelike region
- 4 Conclusions

# Motivation and goals

- Study the reaction  $\gamma^* N \rightarrow R$  (or  $R \rightarrow \gamma^* N$ )
  - spacelike region ( $q^2 < 0$ )
  - timelike region ( $q^2 > 0$ )

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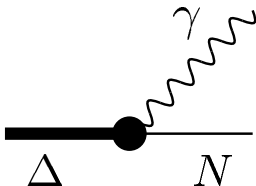
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Framework: quark model  $\oplus$  effective pion cloud

# Kinematics ( $\gamma^* N \rightarrow \Delta$ )



$\Delta$  rest frame

$$P_{\Delta} = (W, 0, 0, 0); \quad P_N = (E_N, 0, 0, -|\mathbf{q}|); \quad q = (\omega, 0, 0, |\mathbf{q}|)$$

Timelike  $q^2 > 0$

$$\omega = \frac{W^2 - M^2 + q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W + M)^2 - q^2][(W - M)^2 - q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 - q^2}{2W}$$

Spacelike  $Q^2 > 0$

$$\omega = \frac{W^2 - M^2 - Q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W + M) + Q^2][(W - M)^2 + Q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 + Q^2}{2W}$$

TL:  $q^2 \leq (W - M)^2$

# $\gamma^* N \rightarrow \Delta$ transition form factors

$\gamma N \rightarrow \Delta$ :

$$J^\mu = \bar{u}_\beta(P_+) \left[ G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu} \right] \gamma_5 u(P_-)$$

$u_\beta$  Rarita-Schwinger spinor      3 independent form factors

$$q_\mu J^\mu = 0 \Rightarrow G_4 = (M_\Delta + M_N) G_1 + \frac{1}{2} (M_\Delta^2 - M_N^2) G_2 - Q^2 G_3$$

$$G_M^* = \frac{M_N}{3(M_N + M_\Delta)} \left\{ [(3M_\Delta + M_N)(M_\Delta + M_N) + Q^2] \frac{G_1}{M_\Delta} + (M_\Delta^2 - M_N^2) G_2 - 2Q^2 G_3 \right\}$$

$$G_E^* = \frac{M_N}{3(M_N + M_\Delta)} \left\{ (M_\Delta^2 - M_N^2 - Q^2) \frac{G_1}{M_\Delta} (M_\Delta^2 - M_N^2) G_2 - 2Q^2 G_3 \right\}$$

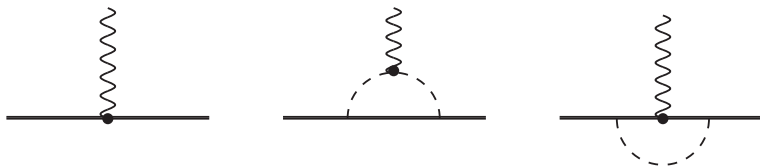
$$G_C^* = \frac{M_N}{3(M_N + M_\Delta)} \left\{ 4M_\Delta G_1 + (3M_\Delta^2 + M_N^2 + Q^2) G_2 + 2(M_\Delta^2 - M_N^2 - Q^2) G_3 \right\}$$

Dalitz decay

$$|G_T(Q^2; M_\Delta)|^2 = |G_M^*(Q^2; M_\Delta)|^2 + 3|G_E^*(Q^2; M_\Delta)|^2 - \frac{Q^2}{2M_\Delta^2} |G_C^*(Q^2; M_\Delta)|^2$$

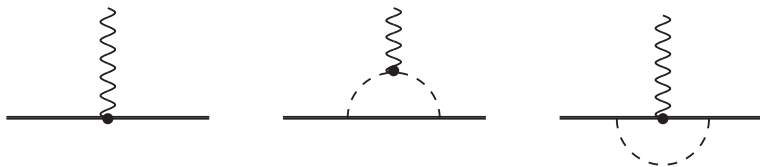


# $\gamma^* N \rightarrow \Delta$ : electromagnetic interaction



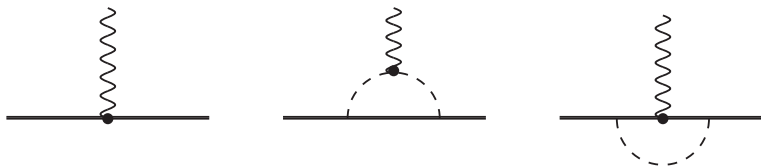
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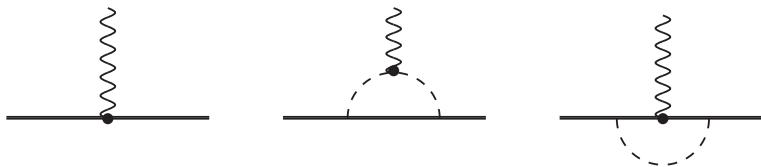
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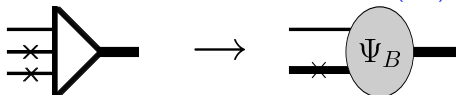
separation is model dependent ...

but dependence can be reduced using lattice QCD data

[for large  $m_\pi$  only  $G_X^\pi \approx 0$ ]

# Spectator QM: wave functions

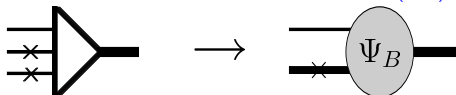
- Baryons as a  $qqq$  system F. Gross, GR and M. T. Peña: PRC 77, 015202 (2008); PRD 85, 093005 (2012)



[spectator framework] system with 2 on-shell  $q$  and a off-shell quark  
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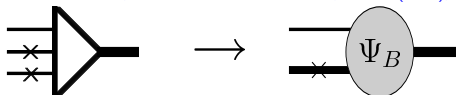


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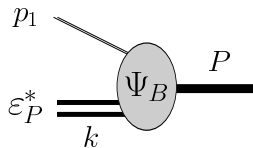


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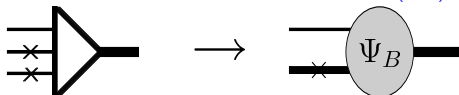
$$B = \text{diquark} \oplus \text{quark}$$

$$\Psi_B = \sum (\text{color}) \otimes (\text{flavor}) \otimes (\text{spin}) \otimes (\text{orbital}) \otimes \underbrace{\psi_B(P, k)}_{\text{radial}}$$



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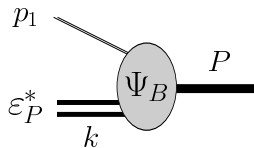


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- Phenomenological radial wf  $\psi_B$  (momentum scale parameters)



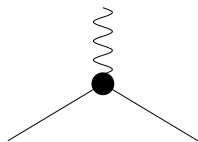
# Spectator QM: quark current (VMD at quark level)

- **Constituent quarks** (quark form factors)

$$j_I^\mu = \left[ \frac{1}{6} f_{1+} + \frac{1}{2} f_{1-} \tau_3 \right] \gamma^\mu + \left[ \frac{1}{6} f_{2+} + \frac{1}{2} f_{2-} \tau_3 \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$$

Quarks with **anomalous** magnetic moments  $\kappa_u, \kappa_d$

quark-antiquark  
 $\oplus$  gluon dressing

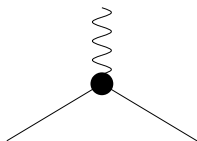


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- **Vector meson dominance parameterization:** PRC77 015202 (2008)

$$f_{1\pm} = \lambda_q + (1 - \lambda_q) \frac{m_v^2}{m_v^2 + Q^2} + c_\pm \frac{M_h^2 Q^2}{(M_h^2 + Q^2)^2}$$

$$f_{2\pm} = \kappa_\pm \left\{ d_\pm \frac{m_v^2}{m_v^2 + Q^2} + (1 - d_\pm) \frac{M_h^2}{M_h^2 + Q^2} \right\}$$

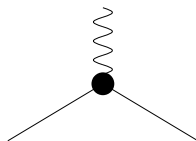
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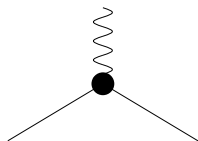
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The diagram shows the decomposition of the quark current form factor  $f_{1\pm}$  into a contact term (green dot) and two meson exchange terms (green loops). The first loop has a pole at  $m_v^2$  and the second at  $M_h^2$ . The corresponding mathematical expressions are:

$$f_{1\pm} = \lambda_q + (1 - \lambda_q) \frac{m_v^2}{m_v^2 + Q^2} + c_{\pm} \frac{M_h^2 Q^2}{(M_h^2 + Q^2)^2}$$

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- Parametrization can be generalized for  $\neq$  regimes: lattice, nuclear med.
- Timelike regime:  $Q^2 = -q^2$ ;  $m_\rho \rightarrow m_\rho - i\frac{\Gamma_\rho}{2}$

# Spectator QM: Transition currents ( $\gamma N \rightarrow N^*$ )

Quark current  $j_I^\mu \oplus$  Baryon wave function  $\Psi_B \Rightarrow J^\mu$

Transition current  $J^\mu$  in **spectator formalism**

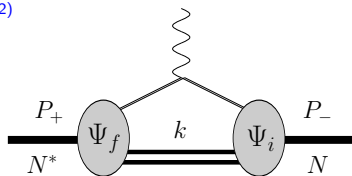
Gross et al PR 186 (1969); PRC 45, 2094 (1992)

Gross, GR and Peña, PRC77 015202 (2008); PRD D85, 093005 (2012)

**Relativistic impulse approximation:**

$$J^\mu = 3 \sum_\lambda \int_k \bar{\Psi}_f(P_+, k) j_I^\mu \Psi_i(P_-, k)$$

$$q = P_+ - P_-, \quad P = \frac{1}{2}(P_+ + P_-), \quad Q^2 = -q^2$$



**diquark on-shell**

# Spacelike: $\gamma^* N \rightarrow \Delta, G_M^*, S$ -states

- Model with S-wave for the nucleon ( $N$ ) and  $\Delta$ :  $G_E^* = G_C^* = 0$   
GR, MT Peña and F Gross EPJ A36, 329 (2008)

$$G_M^B(Q^2) = \frac{8}{3\sqrt{3}} \frac{M}{M+W} f_v \int_k \psi_\Delta \psi_N,$$

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- Quark degrees of freedom insufficient to explain data !!**

$$\int_k \psi_\Delta \psi_N \Big|_{Q^2=0} \leq 1 \Rightarrow G_M^B(0) \leq 2.07;$$

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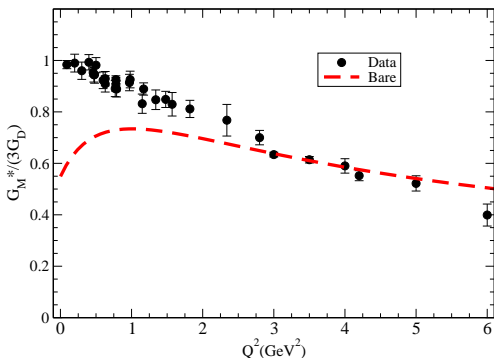
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Classical problem:

Sato and Lee, PRC 63, 055201 (2001); Kamalov et al, PRC 64, 032201 (2001)

# Spacelike: $G_M^*$ (valence)

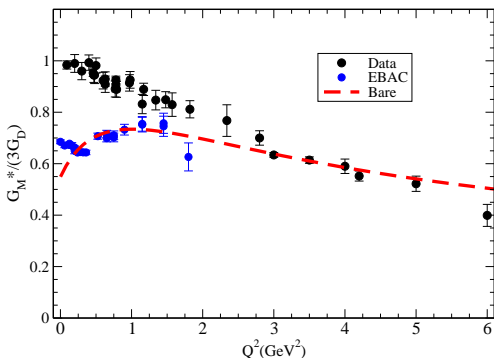
GR and MT Peña PRD 80, 013008 (2009) -  $G_D = \left(1 + \frac{Q^2}{0.71}\right)^{-2}$



- Valence quark model underestimates the **data** ...

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- Valence quark model  $\approx$  EBAC/Jlab (pion cloud removed)

# Spacelike: $G_M^*$ (pion cloud)

Including pion cloud ...

GR, M. T. Peña and F. Gross, EPJA 36, 329 (2008)

$$G_M^* = G_M^B + G_M^\pi$$

large  $Q^2$  (pQCD):  $G_M^\pi \propto 1/Q^8$

Phenomenologic parametrization of the pion cloud component

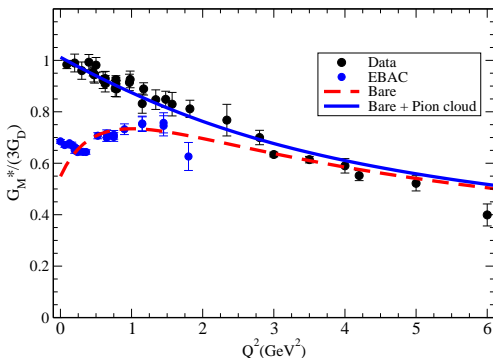
$$G_M^\pi = \lambda_\pi (3G_D) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2} \right)^2$$

$\lambda_\pi$ : strength of pion cloud (% of  $\pi$  cloud at  $Q^2 = 0$ )  $\lambda_\pi = 0.44$

$G_D = \left( \frac{\Lambda_N^2}{\Lambda_N^2 + Q^2} \right)^2$ ,  $\Lambda_N^2 = 0.71 \text{ GeV}^2$ : dipole form factor

$\Lambda_\pi^2$ : pion cloud cutoff ( $\approx 1.5 \text{ GeV}^2$ )

GR and MT Peña PRD 80, 013008 (2009)



- Bare  $\approx$  EBAC model  $\oplus G_M^\pi = \lambda_\pi \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2} \right)^2 (3G_D)$   $\frac{G_M^B(0)}{3G_D} \leq 0.7$

# Spacelike: $\Delta$ with D states

$\Delta$  wave function: EPJA 36, 329 (2008); PRD 78, 114017 (2008); PRD 80, 013008 (2009)

$$\Psi_{\Delta} = N [\Psi_S + a\Psi_{D3} + b\Psi_{D1}]$$

$$S = 0 \oplus \frac{3}{2}, \quad D3 = 2 \oplus \frac{3}{2}, \quad D1 = 2 \oplus \frac{1}{2}$$

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EBAC: Diaz et al PRC 75, 015205 (2007) Fixes  $G_M^{\pi}$
- **D-states:**  $G_E^*, G_C^*$  fitted to lattice data (only valence quark effects)  
PRD 80, 013008 (2009) D-states  $\approx 1\%$



GR and MT Peña, JPG 36, 115011 (2009)

- Quark current (VMD):  $j_I^\mu = j_1 \gamma^\mu + j_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$   
 $j_I^\mu(M_N; m_\rho, M_h = 2M_N) \rightarrow j_I^\mu(M_N^{\text{latt}}; m_\rho^{\text{latt}}, 2M_N^{\text{latt}})$

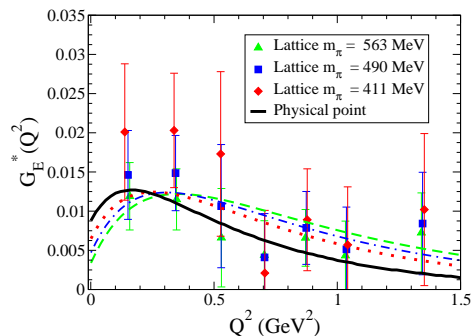
- Wave functions:

$$\Psi_B(\{M_B\}) \rightarrow \Psi_B(\{M_B^{\text{latt}}\})$$

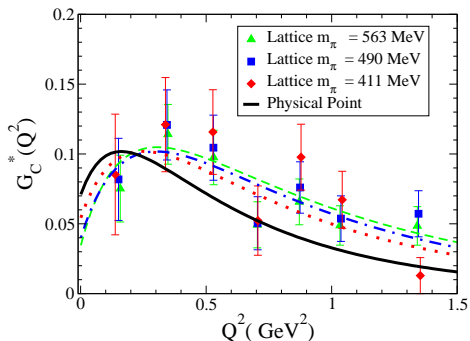
$\Rightarrow$  Implicit  $m_\pi$  dependence in  $G_X$  [Form factors]

- For large  $m_\pi$ :  $G_X^\pi \approx 0$
- $G_X$  given only by  $G_X^B$  (valence quark)
- $G_X^B$  can be compared with lattice (or fitted)

Fit to lattice QCD data (bare contribution) -small D-state  
 Alexandrou et al, PRD, 77, 085012 (2008)

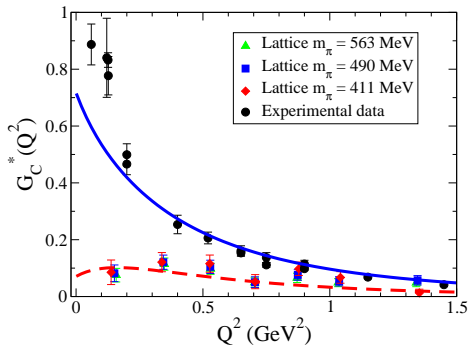
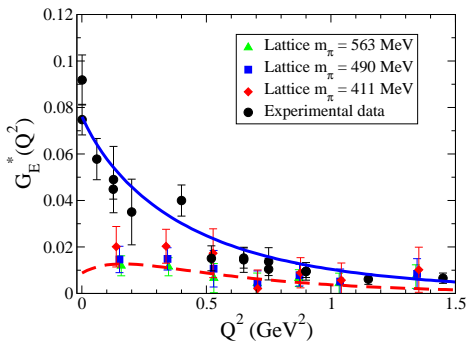


D3 state: 0.72%



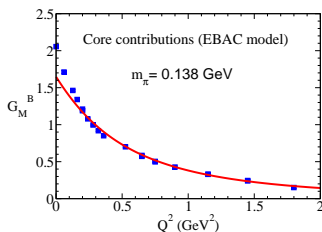
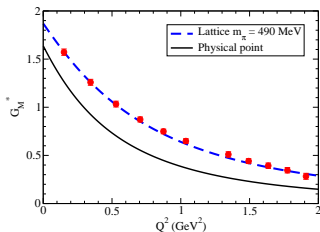
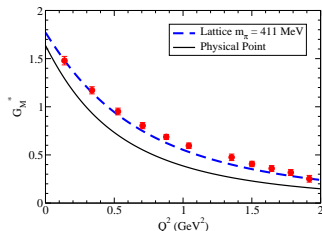
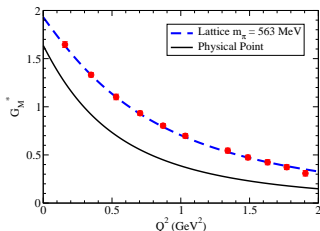
D1 state: 0.72%

# Spacelike: $G_E^*(Q^2)$ , $G_C^*(Q^2)$ (bare + pion cloud) †



Small valence quark contributions (physical limit)  
 Important pion cloud contributions (Large  $N_c$ ; no parameters)  
 GR, MT Peña PRD 80, 013008 (2009)

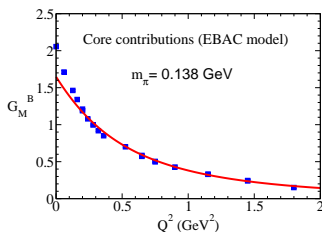
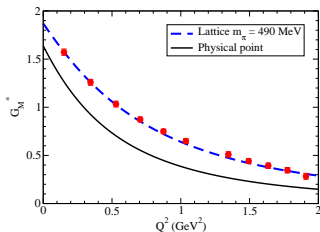
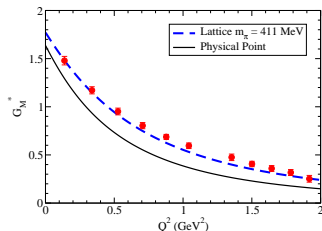
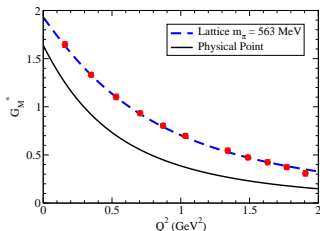
# Spacelike: $G_M^*(Q^2)$ on lattice [PRD 80, 013008 (2009)]



Lattice: Alexandrou et al, PRD 77, 085012 (2008)

⊕ EBAC: J. Diaz et al, PRC 75, 015205 (2007)

# Spacelike: $G_M^*(Q^2)$ on lattice [PRD 80, 013008 (2009)]



Fit to EBAC; Lattice: S-state  $\oplus$   $\mathcal{O}(D\text{-states})$

$\Rightarrow$ :  $G_M^*(\text{lattice})$  is a prediction (neglecting D-states)

# Spacelike $\gamma^* N \rightarrow \Delta$ : Summary

- Model developed for the **nucleon** and  **$\Delta$  on-shell** systems
- **Valence quark** contributions
  - constrained by **EBAC results** (baryons's core)
  - consistent with **lattice data**  
for different baryon masses (function of  $m_\pi$ )
- **S-state is the dominate effect**  $G_E^* = G_C^* = 0$
- **Pion cloud contribution** constrained by physical data:  
 $G_M^\pi(Q^2; W) = G_M^\pi(Q^2; M_\Delta)$  (independent of  $M$ )

- Valence quark model applied for  $q^2 = -Q^2$  and  $M_\Delta \rightarrow W$ ;

$$\frac{m_\rho^2}{m_\rho^2 - q^2} \rightarrow \frac{m_\rho^2}{m_\rho^2 - q^2 - im_\rho\Gamma_\rho}$$

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- Include  $\rho$ -width ( $2\pi$  cut): Gounaris and Sakurai, PRL 21, 244 (1968)

$$\Gamma_\rho(q^2) = \Gamma_\rho^0 \left( \frac{q^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2} \frac{m_\rho}{q} \theta(q^2 - 4m_\pi^2)$$



# Timelike: Extension for timelike [PRD 85, 113014 (2012)]

- Valence quark model applied for  $q^2 = -Q^2$  and  $M_\Delta \rightarrow W$ ;

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- Pion cloud?

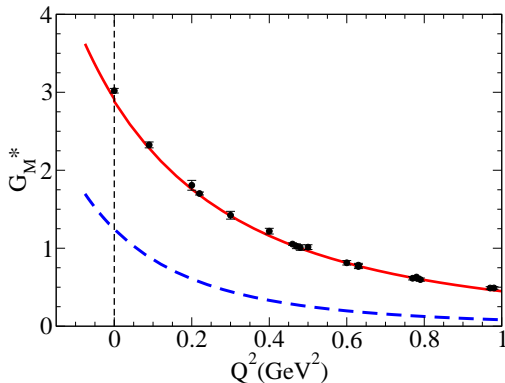
$$G_M^\pi(q^2) = \lambda_\pi (3G_D) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

$$G_D(q^2) = \left( \frac{\Lambda_N^2}{\Lambda_N^2 - q^2 - i\Gamma_N \Lambda_N} \right)^2, \quad \Gamma_N \equiv \Gamma_\rho$$

Naive model (unphysical pole  $q^2 = 0.71 \text{ GeV}^2$ ): **model 1**

Timelike:  $G_M^*$  for  $W = M_\Delta$  (Real part)

**S-state approximation** [ $G_E^* = G_C^* = 0$ ]  $Q^2 \geq -(W - M)^2$



— — — Bare; — Total

# Timelike: Extension for timelike (2) - Model 1

- Why use  $G_D$  for  $q^2 < 0$  ?

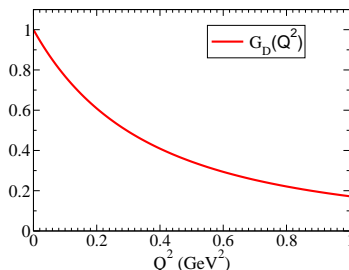
$$G_M^\pi(q^2) = \lambda_\pi(3G_D) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

# Timelike: Extension for timelike (2) - Model 1

- Why use  $G_D$  for  $q^2 < 0$  ?

$$G_M^\pi(q^2) = \lambda_\pi(3G_D) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

- $G_D$  included for  $q^2 < 0$ , to provide an extra falloff

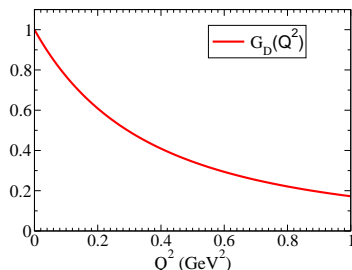


# Timelike: Extension for timelike (2) - Model 1

- Why use  $G_D$  for  $q^2 < 0$  ?

$$G_M^\pi(q^2) = \lambda_\pi(3G_D) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

- $G_D$  included for  $q^2 < 0$ , to provide an extra falloff



- Unphysical singularity on  $G_D$

## Timelike: Extension for timelike (3)

- Alternative function:  $\rho$ -propagator with pion cloud

F. Iachello and Q. Wan, PRC 69, 055204 (2004) for  $q^2 \gg 4m_\pi^2$ :

$$F_\rho(Q^2) = \frac{m_\rho^2}{m_\rho^2 + Q^2 + \frac{1}{\pi} \frac{\Gamma_\rho^0}{m_\pi} Q^2 \log \frac{Q^2}{m_\pi^2}}$$

$\Gamma_\rho^0 = \Gamma_\rho(m_\rho^2)$  decay constant

# Timelike: Extension for timelike (3)

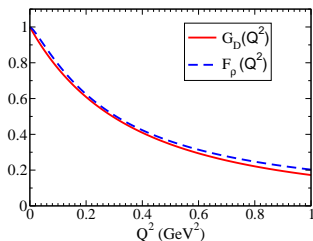
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$\Gamma_\rho^0 = \Gamma_\rho(m_\rho^2)$  decay constant

- $F_\rho$  provide an **extra falloff** for  $q^2 < 0$



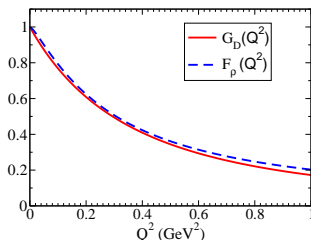
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$\Gamma_\rho^0 = \Gamma_\rho(m_\rho^2)$  decay constant

- $F_\rho$  provide an **extra falloff** for  $q^2 < 0$



- Include explicit effect of **pion cloud dressing**



- Model 2: ( $\simeq$  Model 1 in SL)

$$G_M^\pi(q^2) = \lambda_\pi (3F_\rho) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

# Timelike: Extension for timelike (4) - Model 2

- Model 2: ( $\simeq$  Model 1 in SL)

$$G_M^\pi(q^2) = \lambda_\pi (3F_\rho) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

- For  $q^2 > 0$

$$F_\rho(q^2) = \frac{m_\rho^2}{m_\rho^2 - q^2 - \frac{1}{\pi} \frac{\Gamma_\rho^0}{m_\pi} q^2 \log \frac{q^2}{m_\pi^2} + i \frac{\Gamma_\rho^0}{m_\pi} q^2}$$

- Model 2: ( $\simeq$  Model 1 in SL)

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- Simulates pion cloud dressing (include in  $F_\rho$ )

## Timelike: Extension for timelike (4) - Model 2

- Model 2: ( $\simeq$  Model 1 in SL)

$$G_M^\pi(q^2) = \lambda_\pi (3F_\rho) \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

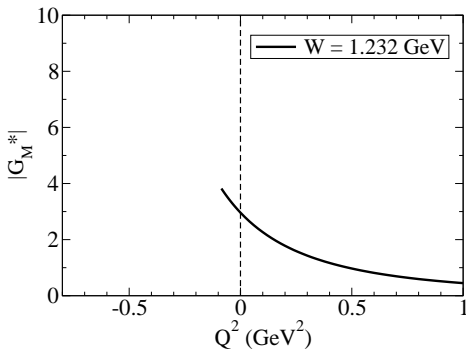
- For  $q^2 > 0$

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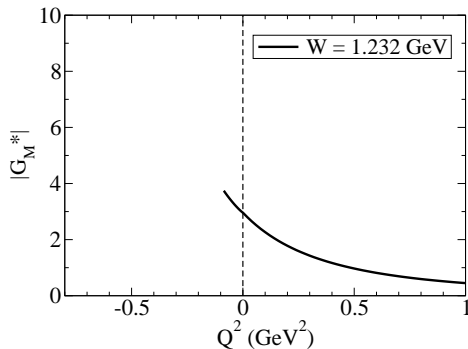
- Simulates pion cloud dressing (include in  $F_\rho$ )
- **No unphysical singularities**

Models defined only for  $Q^2 \geq -(W - M)^2$

Model 1

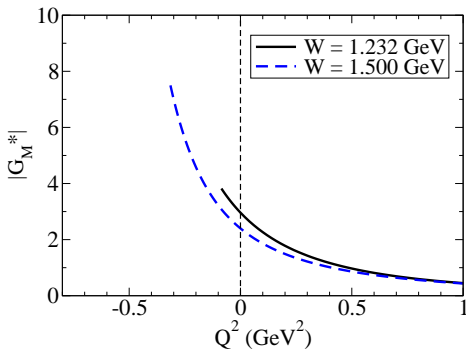


Model 2

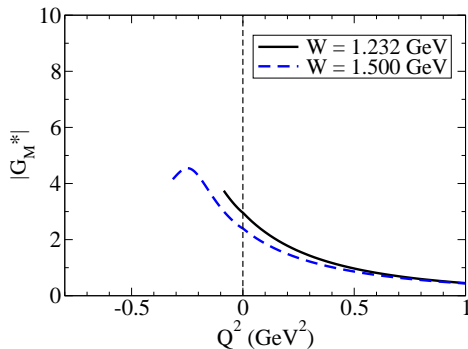


Models defined only for  $Q^2 \geq -(W - M)^2$

Model 1

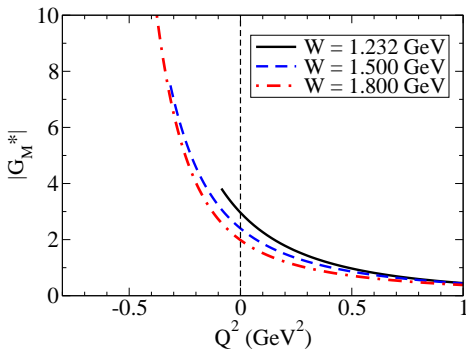


Model 2

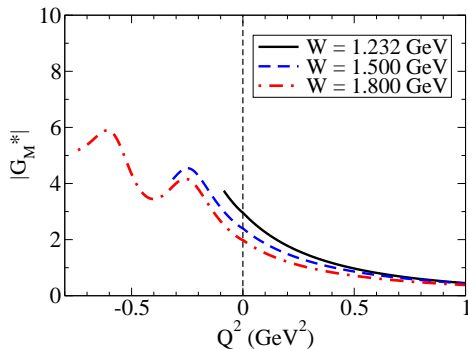


Models defined only for  $Q^2 \geq -(W - M)^2$

## Model 1

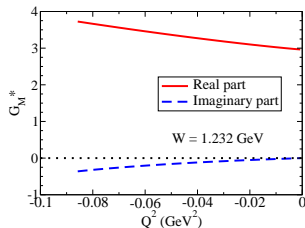
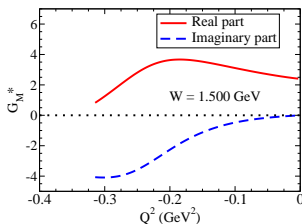
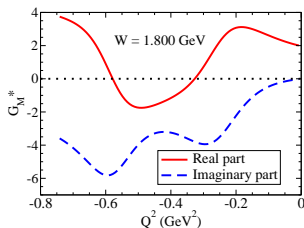


## Model 2



# Timelike: Form factors $G_M^*$ - Model 2 (1)

Model for  $Q^2 \geq -(W - M)^2$  GR and M. T. Peña, PRD 85, 113014 (2012)



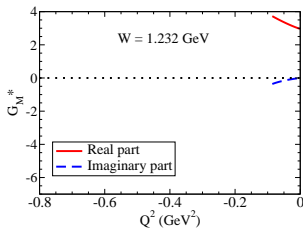
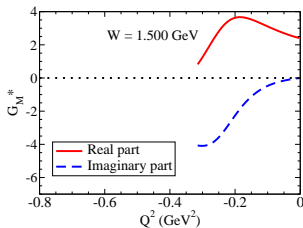
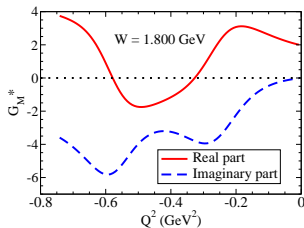
$$G_M^*(Q^2; W) = \underbrace{G_M^B(Q^2; W)}_{\text{VMD}} + \underbrace{G_M^\pi(Q^2)}_{\propto F_\rho}$$

$$G_M^B(Q^2; W) = \frac{8}{3\sqrt{3}} \frac{M}{M+W} f_v \int_k \psi_\Delta \psi_N, \quad G_M^\pi(Q^2) = 3\lambda_\pi F_\rho \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$



# Timelike: Form factors $G_M^*$ - Model 2 (2)

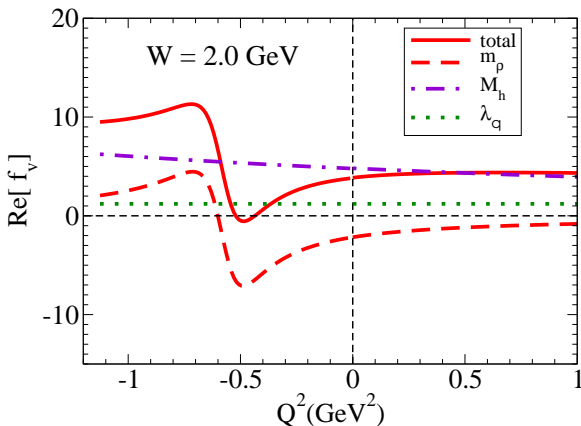
Model for  $Q^2 \geq -(W - M)^2$  GR and M. T. Peña, PRD 85, 113014 (2012)



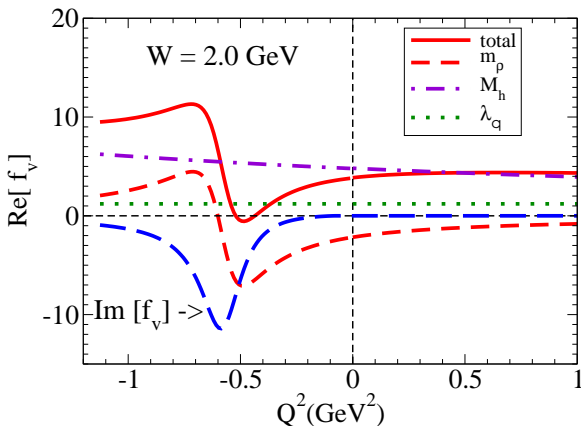
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$$G_M^B(Q^2) = \frac{8}{3\sqrt{3}} \frac{M}{M+W} f_v \int_k \psi_\Delta \psi_N \quad f_v = f_{1-} + \frac{W+M}{2M} f_{2-}$$

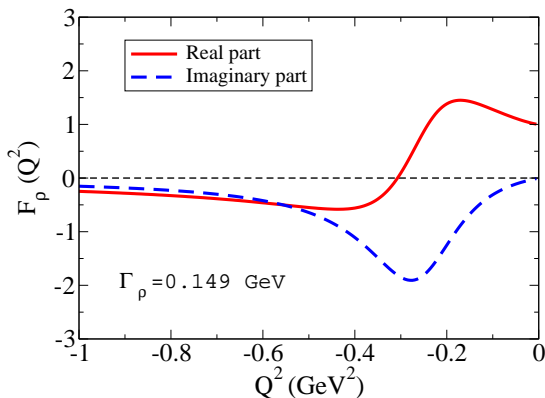


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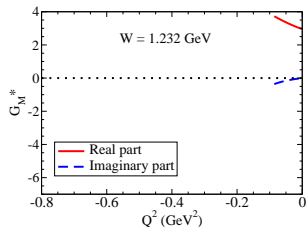
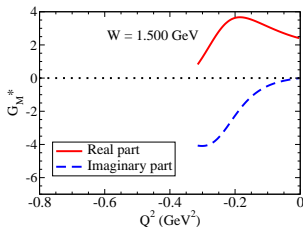
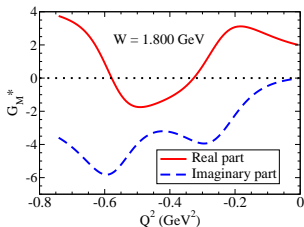
# Timelike: Pion cloud structure; $F_\rho$

$$G_M^\pi(Q^2) = 3\lambda_\pi F_\rho \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2 \quad F_\rho(q^2) = \frac{m_\rho^2}{m_\rho^2 - q^2 - \frac{1}{\pi} \frac{\Gamma_\rho^0}{m_\pi} q^2 \log \frac{q^2}{m_\pi^2} + i \frac{\Gamma_\rho^0}{m_\pi} q^2}$$



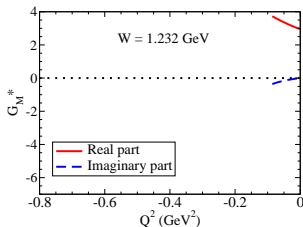
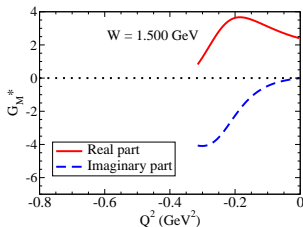
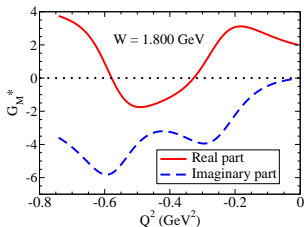
# Timelike: Form factors $G_M^*$ - Model 2

GR and M. T. Peña, PRD 85, 113014 (2012)



# Timelike: Form factors $G_M^*$ - Model 2

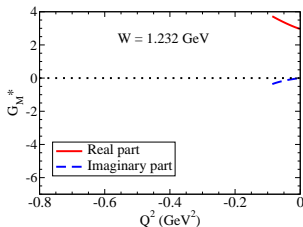
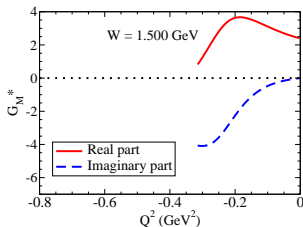
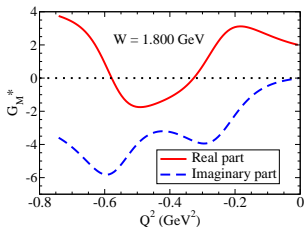
GR and M. T. Peña, PRD 85, 113014 (2012)



- Dominance of  $Re(G_M^*)$  for small  $q^2$

# Timelike: Form factors $G_M^*$ - Model 2

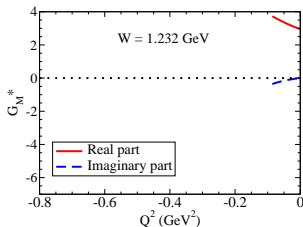
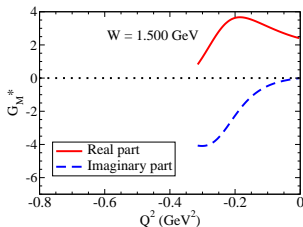
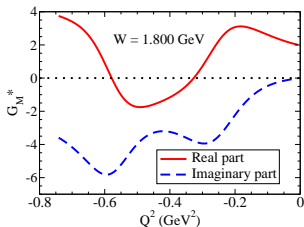
GR and M. T. Peña, PRD 85, 113014 (2012)



- Dominance of  $Re(G_M^*)$  for small  $q^2$
- Peak for  $Im(G_M^*)$  near  $q^2 \simeq 0.3 \text{ GeV}^2$  (pion cloud)

# Timelike: Form factors $G_M^*$ - Model 2

GR and M. T. Peña, PRD 85, 113014 (2012)



- Dominance of  $Re(G_M^*)$  for small  $q^2$
- Peak for  $Im(G_M^*)$  near  $q^2 \simeq 0.3 \text{ GeV}^2$  (pion cloud)
- Peak for  $Im(G_M^*)$  near  $q^2 \simeq m_\rho^2$  (valence quark)



# Dalitz decay: $g_{\Delta}(W)$ and $\Gamma(W)$

G. Wolf et al., NPA 517, 615 (1990)

- Calculation of **Breit-Wigner** mass distribution of  $\Delta(1232)$

$$g_{\Delta}(W) = A \frac{W^2 \Gamma_{tot}(W)}{(W^2 - M_{\Delta}^2)^2 + W^2 [\Gamma_{tot}(W)]^2}$$

$$\Gamma_{tot}(W) = \Gamma_{\pi N}(W) + \Gamma_{\gamma N}(W) + \Gamma_{e^+e^-N}(W)$$

# Dalitz decay: $g_{\Delta}(W)$ and $\Gamma(W)$

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$$\Gamma_{tot}(W) = \Gamma_{\pi N}(W) + \Gamma_{\gamma N}(W) + \Gamma_{e^+e^-N}(W)$$

- Dominant process  $\Delta \rightarrow \pi N$   $\left[ \nu(W) = \frac{\beta^2}{\beta^2 + q_{\pi}^2(W)} \right]$

$$\Gamma_{\pi N}(W) = \frac{M_{\Delta}}{W} \left( \frac{q_{\pi}(W)}{q_{\pi}(M_{\Delta})} \right)^3 \left( \frac{\nu(W)}{\nu(M_{\Delta})} \right)^2 \Gamma_{\pi N}^0$$

# Dalitz decay: $g_{\Delta}(W)$ and $\Gamma(W)$

G. Wolf et al., NPA 517, 615 (1990)

- Calculation of **Breit-Wigner** mass distribution of  $\Delta(1232)$

$$g_{\Delta}(W) = A \frac{W^2 \Gamma_{tot}(W)}{(W^2 - M_{\Delta}^2)^2 + W^2 [\Gamma_{tot}(W)]^2}$$

$$\Gamma_{tot}(W) = \Gamma_{\pi N}(W) + \Gamma_{\gamma N}(W) + \Gamma_{e^+e^-N}(W)$$

- Dominant process  $\Delta \rightarrow \pi N$   $\left[ \nu(W) = \frac{\beta^2}{\beta^2 + q_{\pi}^2(W)} \right]$

$$\Gamma_{\pi N}(W) = \frac{M_{\Delta}}{W} \left( \frac{q_{\pi}(W)}{q_{\pi}(M_{\Delta})} \right)^3 \left( \frac{\nu(W)}{\nu(M_{\Delta})} \right)^2 \Gamma_{\pi N}^0$$

- $\Gamma_{\gamma N}$  and  $\Gamma_{e^+e^-N}$  calculated using the model for  $\Delta \rightarrow \gamma^* N$

# Dalitz decay: $\Gamma_{\gamma N}(W)$ and $\Gamma_{e^+e^-N}(W)$

- Width function  $\Gamma_{\gamma^*N}(q; W)$  with  $q = \sqrt{q^2}$   
 $y_{\pm} = (W \pm M)^2 - q^2$

F. Dohrmann et al, EPJA 45, 401 (2010)  $\alpha \simeq 1/137$

$$\Gamma_{\gamma^*N}(q; W) = \frac{\alpha}{16} \frac{(W + M)^2}{M^2 W^3} \sqrt{y_+ y_-} |G_T(q^2, W)|^2$$

$$|G_T(q^2; M_{\Delta})|^2 = |G_M^*(q^2; W)|^2 + 3|G_E^*(q^2; W)|^2 + \frac{q^2}{2W^2} |G_C^*(q^2; W)|^2$$

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- Then

$$\Gamma_{\gamma N}(W) \equiv \Gamma_{\gamma^*N}(0; W)$$

$$\Gamma_{e^+e^-N}(W) = \frac{2\alpha}{3\pi} \int_{2m_e}^{W-M} \Gamma_{\gamma^*N}(q; W) \frac{dq}{q}$$

threshold:  $2m_e$  ( $\gamma^* \rightarrow e^+e^-$ ); upper limit  $q^2 = (W - M)^2$

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$$\Gamma_{\gamma N}(W) \equiv \Gamma_{\gamma^*N}(0; W)$$

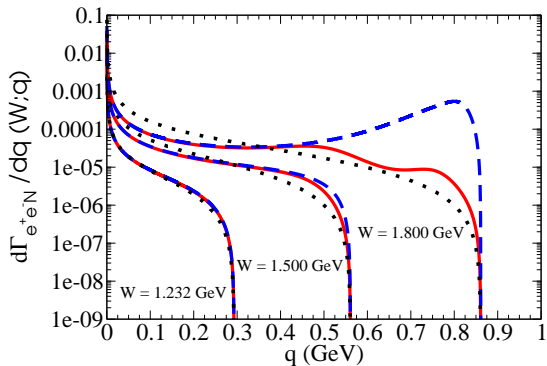
$$\Gamma_{e^+e^-N}(W) = \frac{2\alpha}{3\pi} \int_{2m_e}^{W-M} \Gamma_{\gamma^*N}(q; W) \frac{dq}{q}$$

threshold:  $2m_e$  ( $\gamma^* \rightarrow e^+e^-$ ); upper limit  $q^2 = (W - M)^2$

- $|G_M^*|^2$  model  $\Rightarrow$  model for  $\Gamma_{\gamma N}$  and  $\Gamma_{e^+e^-N}$

- Constant form factor model:  $G_M^*(q^2; W) \equiv G_M^*(0, M_\Delta)$   
M. Zetenyi and G. Wolf, PRC 67, 044002 (2003);  
F. Dohrmann et al, EPJA 45, 401 (2010)
- Two-component quark model (core/pion cloud decomposition)  
F. Iachello and Q. Wan, PRC 69, 055204 (2004);  
R. Bijker and F. Iachello, PRC 69, 068201 (2004);  
F. Dohrmann et al, EPJA 45, 401 (2010)
- Vector meson dominance  
M. I. Krivoruchenko, et al., Annals Phys. 296, 299 (2002)
- **Spectator quark model**

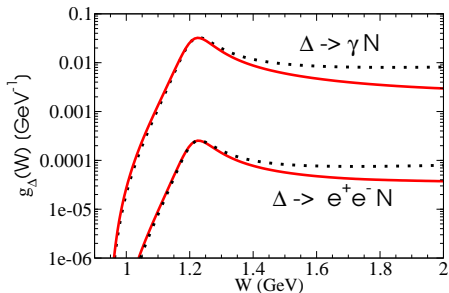
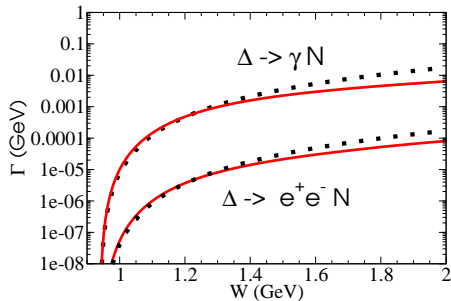
# Dalitz decay: differential width



... const; - - - Model 1; — Model 2

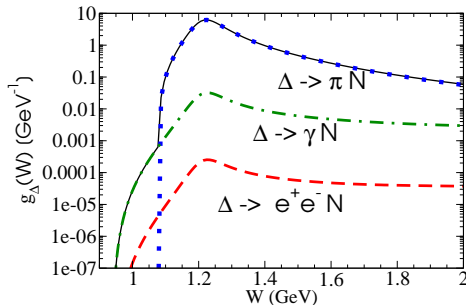
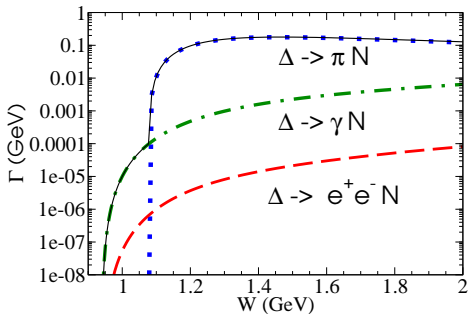


# Dalitz decay: $\Gamma(W)$ , $g_{\Delta}(W)$

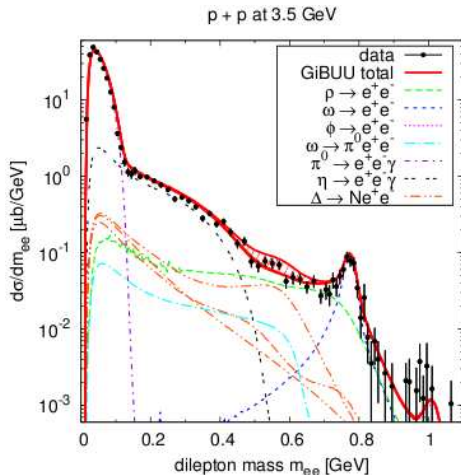


... const; — Model 2

# Dalitz decay: $\Gamma(W)$ , $g_\Delta(W)$ - Model 2 (with $\pi N$ ) <sup>†</sup>



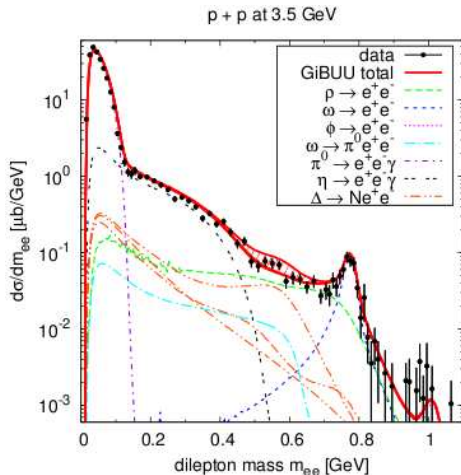
# Dalitz decay: cross-section (GiBUU model)



Courtesy of Janus Weil

Model from J. Weil, H. van Hees and U. Mosel, EPJA 48, 111 (2012)

# Dalitz decay: cross-section (GiBUU model)



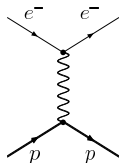
$\Delta \rightarrow \gamma^* N$  models very important for  $0.4 \text{ GeV} < q = m_{ee} < 0.7 \text{ GeV}$

Model from J. Weil, H. van Hees and U. Mosel, EPJA 48, 111 (2012)

# Nucleon form factors (1)

$$J^\mu = F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M}$$

Spacelike

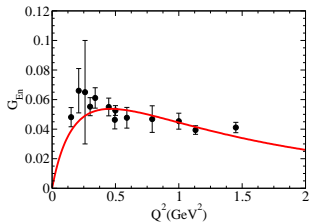
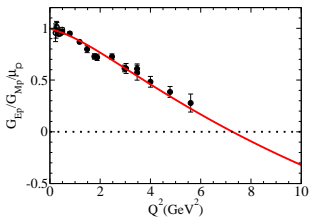
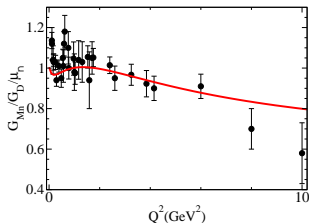
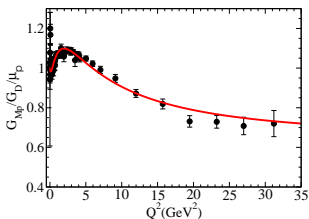


$$q^2 \leq 0$$

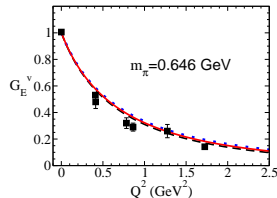
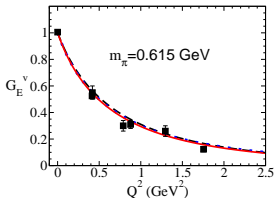
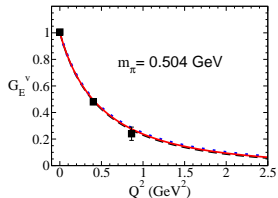
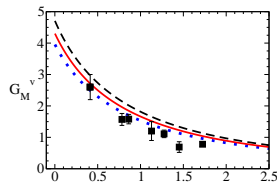
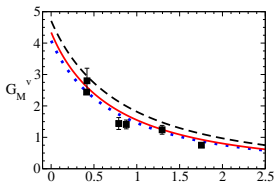
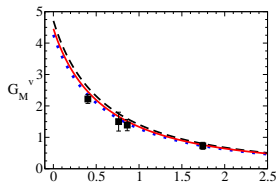
# Nucleon SL form factors

F Gross, GR and MT Peña, PRC 77, 015202 (2008) – **model II** – **No pion cloud**

Nucleon form factors:  $G_E = F_1 - \tau F_2$ ,  $G_M = F_1 + F_2$ ;  $\tau = \frac{Q^2}{4M_N^2}$



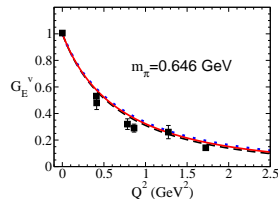
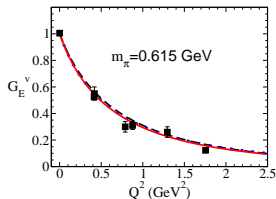
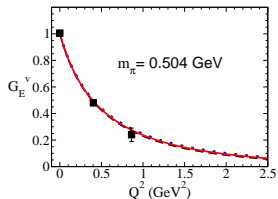
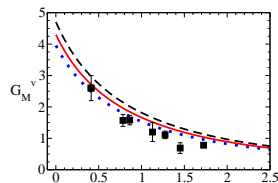
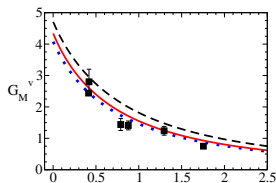
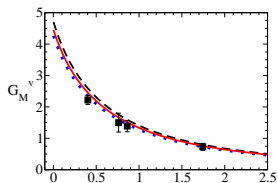
# Nucleon SL form factors in lattice $G_E^v, G_M^v$



--- GR and M. T. Peña, JPG 36, 15011 (2009)

Data: Gökeler et al, PRD 71, 034508 (2005)

# Nucleon SL form factors in lattice $G_E^v, G_M^v$



--- Description of lattice data (no refit)

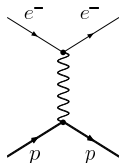
Model valid for **physical** and **lattice** regimes (no pion cloud)



# Nucleon form factors (2)

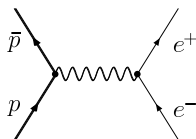
$$J^\mu = F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M}$$

Spacelike



$$q^2 \leq 0$$

Timelike

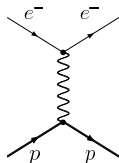


$$q^2 \geq 4M^2$$

# Nucleon form factors (2)

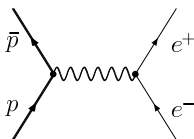
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Spacelike



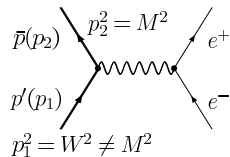
$$q^2 \leq 0$$

Timelike



$$q^2 \geq 4M^2$$

Unphysical



$$4m_e^2 \leq q^2 \leq 4M^2$$

# Nucleon form factors in timelike region [ $0 < q^2 < 4M^2$ ]

Unphysical form factors:  $4m_e^2 < q^2 < 4M^2$  can be accessed by:

- $\gamma N \rightarrow e^+e^-N$ ,  $\pi N \rightarrow e^+e^-N$   
Schäfer, Dönges and Mosel, PLB 342, 13 (1995);  
Dieperink and Nagorny, PLB 397, 20 (1997)
- $NN \rightarrow e^+e^-NN$   
Schäfer, Dönges, Engel and Mosel, NPA 575, 429 (1994)
- $\bar{N}N \rightarrow e^+e^- \pi$   
Gakh, Gustafsson, Dbeyssi and Gakh, PRC 86, 025204 (2012)

# Nucleon form factors in timelike region [ $0 < q^2 < 4M^2$ ]

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Comments:

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Comments:

- It is necessary to define **off-shell** form factors (extra parameters ...)

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- Nucleon form factors contributions are mixed with other processes (Bethe-Heitler mechanism, Virtual Compton Scattering, higher resonances, ...)

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Comments:

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- Nucleon form factors contributions are mixed with other processes (Bethe-Heitler mechanism, Virtual Compton Scattering, higher resonances, ...)
- **Two-photon exchange effects neglected in 1st approximation**

## Models:

- **Vector meson dominance:** (FF as combination of VM poles)  
Iachello, Jackson and Landé, PLB 43, 191 (1973); Gari and Krümpelmann PLB 274, 159 (1992); Lomon, PRC 64, 035204 (2001); Gustafsson, Lacroix, Duterte and Gakh EPJA 24, 419 (2005); Krivorunchenko and Martemyanov, Ann. Phys. 296, 299 (2002)  
Analytical extension for  $q^2 \geq 4M^2$ ; **Interpolation for  $0 < q^2 < 4M^2$**
- **Dispersion Analysis** (using Dispersion Relations)  
Frazer and Fulco PR 117, 1609 (1960); Mergell, Meißner and Drechsel, NPA 596 (1996);  
Hammer and Meißner, EPJA 20, 469 (2004)

$$F_i(q^2) = \frac{1}{\pi} \int_{t_0}^{\infty} dq' \frac{\text{Im}F_i(q'^2)}{q'^2 - q^2}$$

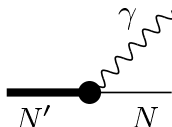
$t_0$  threshold dependent of the channel (isoscalar/isovector)

Spectral function  $\text{Im}(F_i)$  for  $q^2 > 0$  (TL)  $\Leftrightarrow$  FF for  $q^2 < 0$  (SL)



## Alternative Model:

- $N'$  as a  $qqq$  system with mass  $W$  (on-shell)



$$0 \leq q^2 \leq (W - M)^2$$

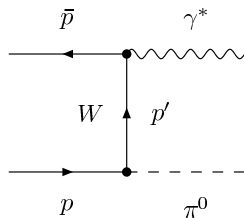
analytical continuation of **baryon wave functions** and **quark currents** (follow extension to lattice regime)

- Calculate of form factors:**

$G_E(Q^2; W)$ ,  $G_M(Q^2; W)$  for the region  $0 \leq q^2 \leq (W - M)^2$   
(analogous to the  $\gamma^* N \rightarrow \Delta$  calculations)

# Unphysical nucleon form factors [ $0 < q^2 \leq (W - M)^2$ ]

Reaction  $\bar{p}p \rightarrow \pi^0 \gamma^*$



Intermediate  $p$  with mass  $W$  (vertex  $p' \rightarrow p\gamma^*$ )

$$J^\mu = F_1^*(Q^2) \left( \gamma^\mu - \frac{\not{q}q^\mu}{q^2} \right) + F_2^*(Q^2) \frac{i\sigma^{\mu\nu}q_\nu}{M + W}$$

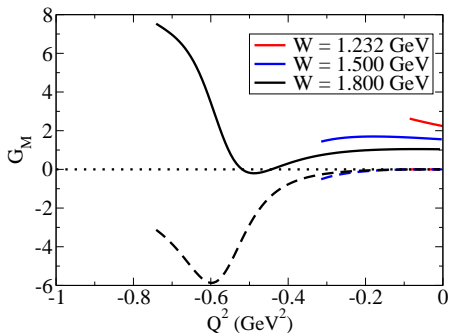
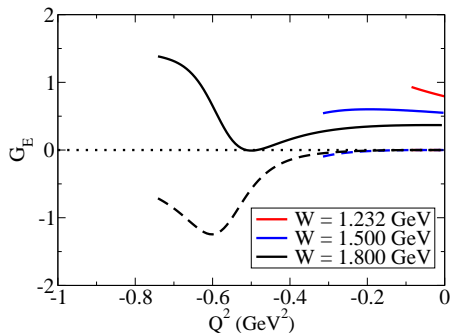
$$G_+(Q^2) \equiv \left\langle N', +\frac{1}{2} \left| \epsilon^{(+)} \cdot J \right| N, -\frac{1}{2} \right\rangle \propto \overbrace{F_1^*(Q^2) + F_2^*(Q^2)}^{G_M}$$

$$G_0(Q^2) \equiv \left\langle N', +\frac{1}{2} \left| \epsilon^{(0)} \cdot J \right| N, +\frac{1}{2} \right\rangle \propto \underbrace{F_1^*(Q^2) - \frac{Q^2}{(M + W)^2} F_2^*(Q^2)}_{G_E}$$

# Nucleon timelike form factors (preliminary)

Proton

Extension of Model II/ PRC77 015202 (2008)



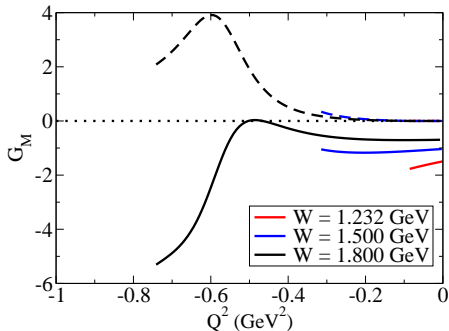
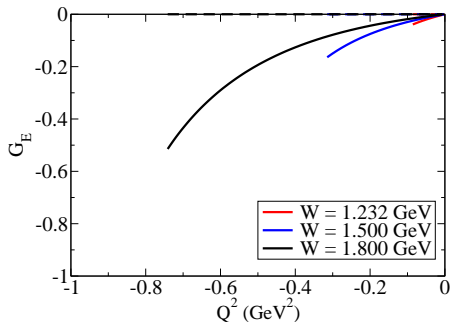
Real (solid); Imaginary (dashed)

Model with **no pion cloud**

# Nucleon timelike form factors (preliminary)

Neutron

Extension of Model II / PRC77 015202 (2008)



Real (solid); Imaginary (dashed)

Model with **no pion cloud**

- **Spectator QM:** valence quark model  $\oplus$  effective meson cloud  
Model applied to the study of  $\gamma^* N \rightarrow R$  in the timelike region  $q^2 > 0$   
Nucleon and  $R$  as  $qqq$  on-shell systems

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Model applied to the study of  $\gamma^* N \rightarrow R$  in the timelike region  $q^2 > 0$   
Nucleon and  $R$  as  $qqq$  on-shell systems
- $\gamma^* N \rightarrow \Delta$  reaction:
  - very rich structure in TL regime
  - $q^2$  dependence of the form factors is very important in the study of the  $\Gamma_{e^+e^- N}(q, W)$  and cross-section  $\frac{d\sigma_{NN}}{dq}(q)$

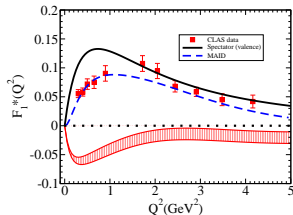
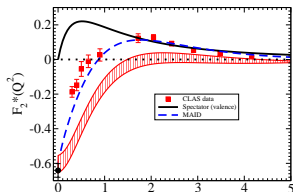
- **Spectator QM: valence quark model**  $\oplus$  **effective meson cloud**  
Model applied to the study of  $\gamma^* N \rightarrow R$  in the timelike region  $q^2 > 0$   
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- $\gamma^* N \rightarrow \Delta$  reaction:
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  - $q^2$  dependence of the form factors is very important in the study of the  $\Gamma_{e^+e^-N}(q, W)$  and cross-section  $\frac{d\sigma_{NN}}{dq}(q)$
- Possible extension to the **nucleon unphysical** form factors  
 $\gamma^* N \rightarrow N'$  (on-shell mass  $W$ )

- **Spectator QM: valence quark model**  $\oplus$  **effective meson cloud**  
Model applied to the study of  $\gamma^* N \rightarrow R$  in the timelike region  $q^2 > 0$   
Nucleon and  $R$  as  $qqq$  on-shell systems
- $\gamma^* N \rightarrow \Delta$  reaction:
  - very rich structure in TL regime
  - $q^2$  dependence of the form factors is very important in the study of the  $\Gamma_{e^+e^-N}(q, W)$  and cross-section  $\frac{d\sigma_{NN}}{dq}(q)$
- Possible extension to the **nucleon unphysical** form factors  
 $\gamma^* N \rightarrow N'$  (on-shell mass  $W$ )
- Formalism can be extended to other resonances:  
 $N^*(1440), N^*(1535), \Delta(1600)$ ,  
depending of the pion cloud component  
[ $\Delta(1232)$  system very well constrained]

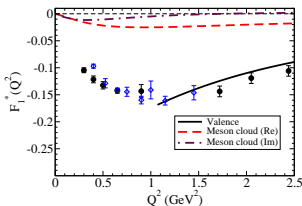
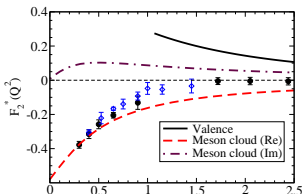


# $N^*(1440)$ , $N^*(1535)$ , $\Delta(1600)$ form factors

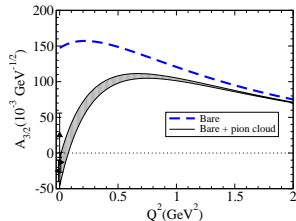
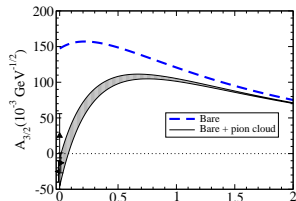
## $N^*(1440)$



## $N^*(1535)$



## $\Delta(1600)$



# Selected bibliography (part 1)

- **Timelike  $\gamma^* N \rightarrow \Delta$  form factors and Delta Dalitz decay**,  
G. Ramalho and M. T. Pena, Phys. Rev. D **85**, 113014 (2012)  
[arXiv:1205.2575 [hep-ph]].
- **A pure S-wave covariant model for the nucleon**,  
F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 015202 (2008)  
[arXiv:nucl-th/0606029].
- **A covariant formalism for the  $N^*$  electroproduction at high momentum transfer**,  
G. Ramalho, F. Gross, M. T. Peña and K. Tsushima,  
**Exclusive Reactions and High Momentum Transfer IV, 287 (2011)**  
[arXiv:1008.0371 [hep-ph]].
- **A Covariant model for the nucleon and the  $\Delta$** ,  
G. Ramalho, M. T. Peña and F. Gross, Eur. Phys. J. A **36**, 329 (2008)  
[arXiv:0803.3034 [hep-ph]].

- **D-state effects in the electromagnetic  $N\Delta$  transition,**  
G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D **78**, 114017 (2008)  
[arXiv:0810.4126 [hep-ph]].
- **Nucleon and  $\gamma N \rightarrow \Delta$  lattice form factors  
in a constituent quark model,**  
G. Ramalho and M. T. Peña, J. Phys. G **36**, 115011 (2009)  
[arXiv:0812.0187 [hep-ph]].
- **Valence quark contribution for the  $\gamma N \rightarrow \Delta$  quadrupole transition  
extracted from lattice QCD,**  
G. Ramalho and M. T. Peña, Phys. Rev. D **80**, 013008 (2009)  
[arXiv:0901.4310 [hep-ph]].
- **Electromagnetic form factors of the  $\Delta$  with D-waves,**  
G. Ramalho, M. T. Pena and F. Gross, Phys. Rev. D **81**, 113011 (2010)  
[arXiv:1002.4170 [hep-ph]].

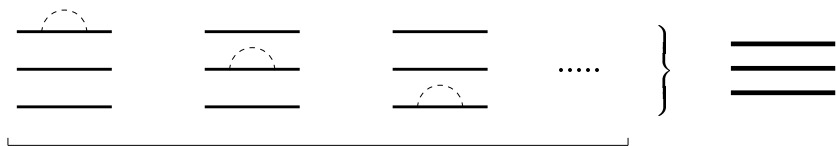
## Selected bibliography (part 3)

- **Valence quark contributions for the  $\gamma N \rightarrow P_{11}(1440)$  form factors**,  
G. Ramalho and K. Tsushima, Phys. Rev. D **81**, 074020 (2010)  
[arXiv:1002.3386 [hep-ph]].
- **A model for the  $\Delta(1600)$  resonance and  $\gamma N \rightarrow \Delta(1600)$  transition**,  
G. Ramalho and K. Tsushima, Phys. Rev. D **82**, 073007 (2010)  
[arXiv:1008.3822 [hep-ph]].
- **A covariant model for the  $\gamma N \rightarrow N(1535)$  transition at high momentum transfer**,  
G. Ramalho and M. T. Pena, Phys. Rev. D **84**, 033007 (2011)  
[arXiv:1105.2223 [hep-ph]].
- **Valence quark and meson cloud contributions for the  $\gamma^* \Lambda \rightarrow \Lambda^*$  and  $\gamma^* \Sigma^0 \rightarrow \Lambda^*$  reactions**,  
G. Ramalho, D. Jido and K. Tsushima, Phys. Rev. D **85**, 093014 (2012)  
[arXiv:1202.2299 [hep-ph]].

- **Octet baryon electromagnetic form factors in a relativistic quark model**  
G. Ramalho and K. Tsushima, Phys. Rev. D **84**, 054014 (2011) [arXiv:1107.1791 [hep-ph]].
- **Octet baryon electromagnetic form factors in nuclear medium**,  
G. Ramalho, K. Tsushima and A. W. Thomas, arXiv:1206.2207 [hep-ph].
- **Covariant nucleon wave function with S, D, and P-state components**,  
F. Gross, G. Ramalho and M. T. Pena, Phys. Rev. D **85**, 093005 (2012) [arXiv:1201.6336 [hep-ph]].

## Backup slides

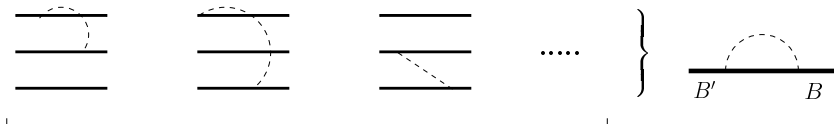
# Quark structure and electromagnetic interaction (I)



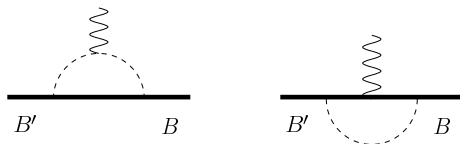
$\gamma$  coupling:



# Quark structure and electromagnetic interaction (II)

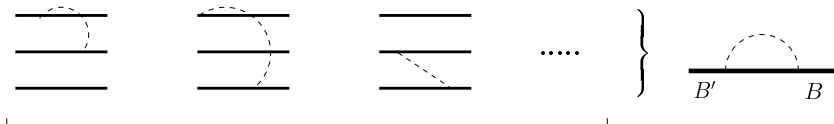


$\gamma$  coupling:

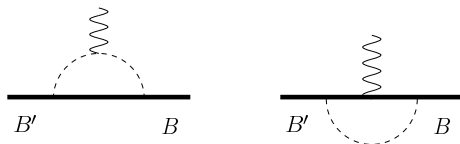




# Quark structure and electromagnetic interaction (II)

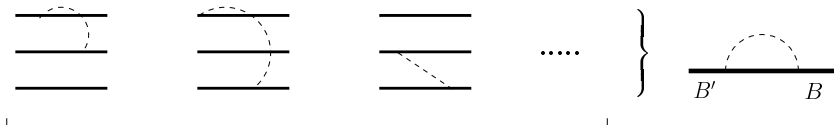


$\gamma$  coupling:

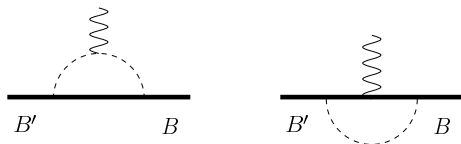


- **Not important** at high  $Q^2$  [pQCD: suppression  $1/Q^4$ ],  
**Very important** at low  $Q^2$

# Quark structure and electromagnetic interaction (II)



$\gamma$  coupling:



- **Not important** at high  $Q^2$  [pQCD: suppression  $1/Q^4$ ],  
**Very important** at low  $Q^2$
- **Assume NO interference** with quark dressing processes

$$F = F^B + F^{mc}$$

(bare  $\oplus$  meson cloud)