Application of Kalman filter methods to event filtering and reconstruction for Neutrino Telescopy ⋆

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Abstract

Event reconstruction in underwater neutrino telescopes suffers from a high background noise due to the ^{40}K decays. Adaptive algorithms are able to suppress automatically such a noise and therefore are considered as good candidates for track fitting at the KM3NeT environment. In this note we describe an iterative event filtering and track reconstruction technique, employing Kalman Filter methods and we present results from a detailed simulation study concerning the KM3NeT detector. We evaluate the accuracy of this technique and we compare its efficiency with other standard track reconstruction methods.

Key words: Neutrino Telescope, KM3NeT, Kalman Filter, Track Reconstruction
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1. Introduction

The KM3NeT consortium is currently working on a conceptual design for a future Mediterranean neutrino telescope, which will have an instrumented volume of a scale of one km^3 [1,2]. The main background counting rate in the optical modules of an undersea neutrino detector originates from the decay of radioactive elements in the water. Sea water contains small amounts of the naturally occurring radioactive potassium isotope, ^{40}K. This isotope decays mostly through $\beta - decay$ releasing electrons that produce Cerenkov light and produce a steady, isotropic background of photons with rates of the order of 100 Hz per square centimeter of photocathode area. Although the induced number of photoelectrons per photomultiplier during the time it takes a muon to pass the detector (a few microseconds) is moderate there is still a chance that these hits may mimic the signature of a muon or shower or, more importantly, contaminate the hit pattern of a neutrino induced event. This random background can be reduced by coincidence methods to an acceptable level. But even after the application of such methods the level of the contamination is of the order of the signal itself. Adaptive algorithms are able to suppress automatically such a noise and therefore are considered as good candidates for track fitting at the KM3NeT environment. Adaptive algorithms, based on Kalman Filter methods, are extensively used in accelerator particle physics experiments, for event filtering, track reconstruction and vertex definition [3].

2. KM3NeT Detector Simulation

2.1. *Detector Description*

In this study, the neutrino telescope was assumed to consist of 80 strings, 125m apart, in hexagonal geometry as in the IceCube detector [4]. Each string

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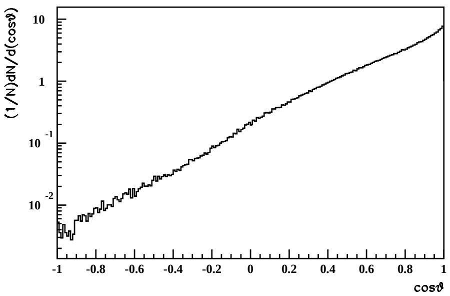

Fig. 1. The PDF of the space angle between the active PMT's direction of the MultiPMT OM and the direction of the incident wavefront.

was carrying 60 storeys, with a vertical distance of 17m between them. Each storey of the detector consisted of two Optical Modules (OMs) one looking up and the other looking down, while the OM consisted of 20 cylindrical PMTs 3" in diameter inside a 17" benthos sphere, covering 2π in solid angle [5,6]. Detailed Monte Carlo description [1] of the MultiPMT OM has shown a directional sensitivity with a median of about 20 degrees. Figure 1 presents the probability distribution function of the space angle between the active PMT's direction of the OM and the wavefront direction of incidence. The information of the directionality of the hits can be used to form a direction likelihood for each reconstructed event. This likelihood can be used as a candidate track quality criterion.

2.2. *Simulation*

The track reconstruction resolution and efficiency of the techniques described in this note, were quantified by a Monte Carlo study using KM3Sim [7], a GEANT4 [8] based simulation package to describe the passage (energy losses, electromagnetic shower production, multiple scattering, Cerenkov light emission) of muons through the water, the optical background generation due to the ^{40}K radioactive decays and optical photon absorption [2]. In this study muons generated in the energy range from 500GeV to 100TeV were fully simulated. Figure 2 presents the mean number of signal hits versus the muon energy. A constant background noise of 6.4kHz on each 3" PMT was assumed, due to PMT dark current and ^{40}K decays. To reduce the number of noise hits to an acceptable level the coincidence of 2 PMTs in an OM was required in a time window of 20nsec. This results in a background noise of 310Hz per Optical Module. The maximum distance any muon can travel in the detector is 1500 meters leading to a maximum time window of 4.5μ sec. Subsequently each simulated event contained on average 13.5 noise hits.

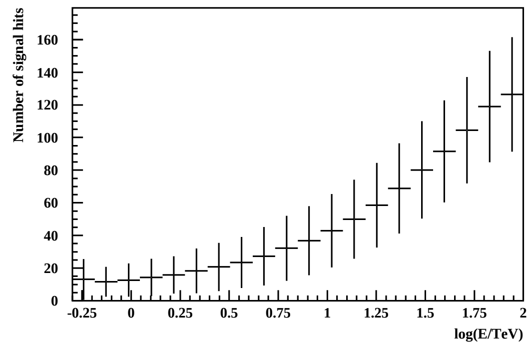

Fig. 2. The mean number of active OMs as a function of muon energy. The vertical error bars represent the RMS of the number of active OMs.

3. Muon Track Reconstruction

3.1. *Initial prefit and filtering*

The stage before the actual track reconstruction is a prefit and filtering based on clustering of track segments. A track segment is defined as the line between the positions of two active OMs and a candidate track is defined as a track which passes between these two points in space if at least 4 other hits are consistent with this track (the corresponding residual must be between -60ns and 60ns). If no candidate track is found the event is rejected. The next step after the definition of all the candidate tracks is the clustering in direction. For each candidate track the number of other neighbouring candidate tracks with a maximum space angle of 15 degrees is found. The track with the largest number of neighbours is chosen as the best candidate solution. Hits with residuals between -40ns and 40ns are considered consistent with the best candidate solution, while the rest are rejected. Figure 3 presents the filtering efficiency of this rejection technique. The filled (open) circles represent the percentage of noise hits before (after) the application of the prefit.

[1] The description included the absorption length and refractive index of the benthos and PMT glass, as well as the optical properties of the Gell used for the optical coupling.
[2] The absorption length we have used has a maximum of 60m at 480nm. Optical photon scattering was not included in the simulation.

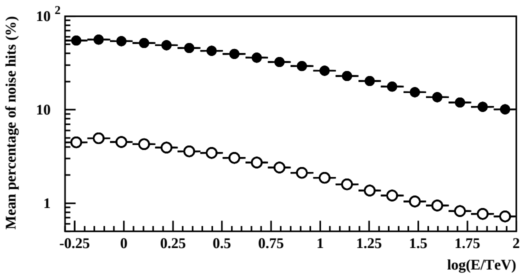

Fig. 3. The mean percentage of noise hits before (black dots) and after (open circles) the application of the prefit rejection technique.

3.2. χ^2 *fit*

The arrival time of the selected hits are used in a χ^2 minimization in order to estimate the track parameters vector, $\boldsymbol{x} = (\boldsymbol{V}, \theta, \phi)$, where the vector $\boldsymbol{V}$ is the pseudovertex, while θ and ϕ are the zenith and azimuth angles of the muon track. The χ^2 estimator is defined as:

$$\chi^2 = \sum_{i=1}^{N_{hit}} \left(\frac{t_i^{exp} - t_i}{\sigma_i}\right)^2 \tag{1}$$

where N_{hit} is the number of the hits used for the track reconstruction,
$t_i^{exp} \equiv t_i^{exp}(\boldsymbol{x})$ is the expected arrival time of the i^{th} hit, assuming that the pulse is the PMT responce to the Cerenkov light produced by a muon track with parameter vector $\boldsymbol{x}$,
t_i and σ_i is the arrival time and the error of the i^{th} hit. The t^{exp} is evaluated as:

$$t^{exp} = (L + D \cdot \tan(\theta_c))/c \tag{2}$$

$$L = \boldsymbol{d} \cdot (\boldsymbol{h} - \boldsymbol{V})$$

$$D = |\boldsymbol{h} - \boldsymbol{V} - L \cdot \boldsymbol{d}|$$

where $\boldsymbol{h}$ is the position vector of the active OM, $\boldsymbol{d}$ is the direction unit vector of the track and θ_c is the Cerenkov angle.

3.3. *Kalman Filter*

A way to perform both track fitting and background noise filtering is the Kalman Filter technique. In the most general formulation Kalman Filter can incorporate process noise (multiple scattering) and track evolution while the muon is moving through the detector. For energetic muons multiple scattering has a negligible effect and the track is considered as a straight line. In this study Kalman filter is used as a recursive track fitting method and is statistically equivalent to the Least Squares Method. With the use of Kalman Filter the muon track parameter vector $\boldsymbol{x} = (\boldsymbol{V}, \theta, \phi)$ are recursively updated taking into account one measurement (hit) after the other. At each step one can decide whether to include a hit or not. The decision is based on the value of the filtered χ^2 contribution of the hit, as explained in the following. The algorithm developed with the use of Kalman filter is the following:
a)At the starting point of the filter an initial estimation of the state, $\boldsymbol{x}_0$, and its covariance matrix, $\boldsymbol{C}_0$, are calculated using five hits on five different storeys that provide an exact solution to the equation 2. These five hits are chosen randomly with probability analog to their charge.
b)For each other hit the state vector and its covariance matrix is updated using the update equations:

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{K}_k(t_k - t_k^{exp}(\boldsymbol{x}_{k-1})) \tag{3}$$

$$\boldsymbol{C}_k = (\boldsymbol{1} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{C}_{k-1} \tag{4}$$

where $\boldsymbol{K}_k$ is the Kalman gain matrix and $\boldsymbol{H}_k$ is the derivative matrix of the t^{exp}, equation 2, with respect the state vector $\boldsymbol{x}_{k-1}$. The Kalman gain matrix is calculated as:

$$\boldsymbol{K}_k = \boldsymbol{C}_{k-1}\boldsymbol{H}_k^T(V_k + \boldsymbol{H}_k\boldsymbol{C}_{k-1}\boldsymbol{H}_k^T)^{-1} \tag{5}$$

where $V_k = \sigma_k^2$ is the measurement covariance.
c)The next step is the calculation of the filtered residual, its covariance matrix and the filtered χ^2 contribution of the hit:

$$r_k = t_k - t_k^{exp}(\boldsymbol{x}_k) \tag{6}$$

$$R_k = (1 - \boldsymbol{K}_k\boldsymbol{H}_k)V_k \tag{7}$$

$$\chi_k^2 = r_k^2/R_k \tag{8}$$

The filtered χ^2 contribution of the hit is used as a criterion to decide the quality of the hit. The hit is rejected and the state vector and covariance resumes the values of the previous step if $\chi_k^2 > 3$. The steps (b) and (c) are repeated for every available hit. The total χ^2 is the sum of the individual chi-squares of all the steps.

A large number of candidate solutions are found, each using a different set of initial state estimations. The best candidate solution is chosen using the value of the χ^2 per degrees of freedom, the direction likelihood described in Section 2.1 and the number of hits that are used in the fit. The candidate solutions that collected more hits during the fitting procedure are more likely to be closest to the real track. If there is no candidate solution with more that 7 hits the event is rejected.

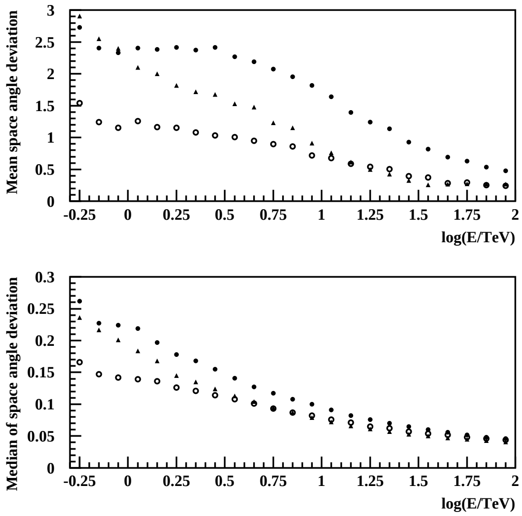

Fig. 4. The upper (lower) plot represents the mean (median) of the space angle between the reconstructed and simulated track direction as a function of muon energy. Solid dots correspond to the LSM, while open circles correspond to the Kalman Filter technique. The triangles correspond to the Kalman filter without the use of the prefit rejection algorithm.

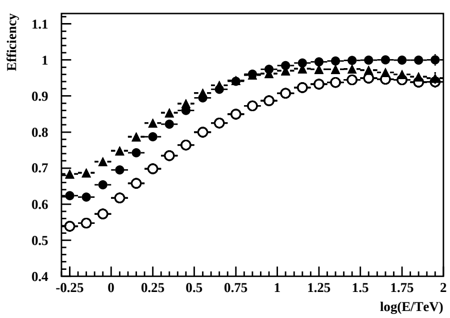

Fig. 5. The efficiency of the reconstruction techniques as a function of muon energy. The black dots correspond to the LSM, while the open circles correspond to the Kalman filter. The triangles correspond to the Kalman filter method without the use of the prefit rejection algorithm

4. Results

Figure 4 presents the median and mean of the space angle deviation between the reconstructed and simulated muon track directions as a function of the muon energy. The black dots correspond to the LSM and the open circles to the Kalman filter method. The majority of the background noise hits have been rejected by the prefilter. Likewise, Figure 5 presents the efficiency of each method as a function of the muon energy. The efficiency of the Kalman filter method in the presence of background noise, i.e. without the use of the prefit rejection algorithm is presented on the same plots with triangles.

5. Future work

A promising extension to the application of the Kalman Filter method in an underwater neutrino telescope is the simultaneous reconstruction of multiple tracks. A significant percentage of muons produced by Extensive Air Showers arrive to the underwater detector in bundles. The identification of multiple tracks can benefit atmospheric muon rejection efficiency and calibration techniques that use downcoming muons [9]. Moreover, shower reconstruction and muon energy reconstruction with the use of Kalman filter methods is studied.

6. Conclusions

The Kalman filter techniques, widely used in accelarator physics, is a promising new way of event reconstruction and filtering in an underwater neutrino telescope. Detailed Monte Carlo studies, presented in this paper, have shown that in absence of background noise the Kalman filter technique is more accurate compared to the LSM, while it maintains its resolution and efficiency in the presence of noise.

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